How does international trade affect inflation-unemployment dynamics in an open economy: evidence from the U.S regional Phillips Curve and E.U's cross-country Phillips curve

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"It is just not credible that the U.S can remain an oasis of prosperity unaffected by a world that has experienced greatly increased stress" (Greenspan, 1998)

1 Abstract

How has international trade affected the dynamics between inflation and unemployment? I study this dynamic by examining the slope of the Phillips Curve equation by exploiting the rich variation of the regional unemployment and inflation index. To do so, I recognize that the closed economy model doesn't support the notion of the shifting Phillips curve coefficient. I reformulate the classic New Keynesian open-economy model to multiple open-economy regions in the U.S. that trade with the rest of the world. This formulation augments the canonical Phillips curve with the terms of trade interacted with the import exposure variable, which is crucial in explaining the puzzling dynamics of inflation and unemployment in different regions of the U.S.

2 Introduction

The Phillips Curve is an economic theory that inflation and unemployment have a stable and inverse relationship. From a theoretical perspective, the New Keynesian Phillips Curve(NKPC) is one of the two main optimality conditions that arise from the New Keynesian model, the NKPC coming from the combination of the firm's price resetting and profit maximization problem. Additionally, the Phillips curve is the main constraint in solving the monetary policy's objective of minimizing the deviation from the optimal inflation and deviation from the optimal unemployment.

Theoretically, the relationship has a textbook forward-looking formulation of the inflation π_t that depends on three main factors such as expected inflation, output gap measured between unemployment and natural rate of unemployment(NAIRU) measured by CBP, and cost-push shock or supply shock:

$$\pi_t = \beta E_t \pi_{t+1} - \kappa(U_t - U_t^n) + u_t \tag{1}$$

where $\kappa = \frac{(1-\theta)(1-\theta\beta)}{\theta}$ which are constant parameters due to Calvo staggered price resetting. However such formulation is problematic since the empirical findings of the slope do not correspond to the theoretical model's constant and stable slope.

In the United States, the Federal Reserve's legal policy mandate focuses on three domestic variables: full employment, price stability, and long-term interest rates. However, the latter doesn't

(1960-83 (blue dots), 1984-99 (orange diamonds), 2000-2018q1 (green triangles))

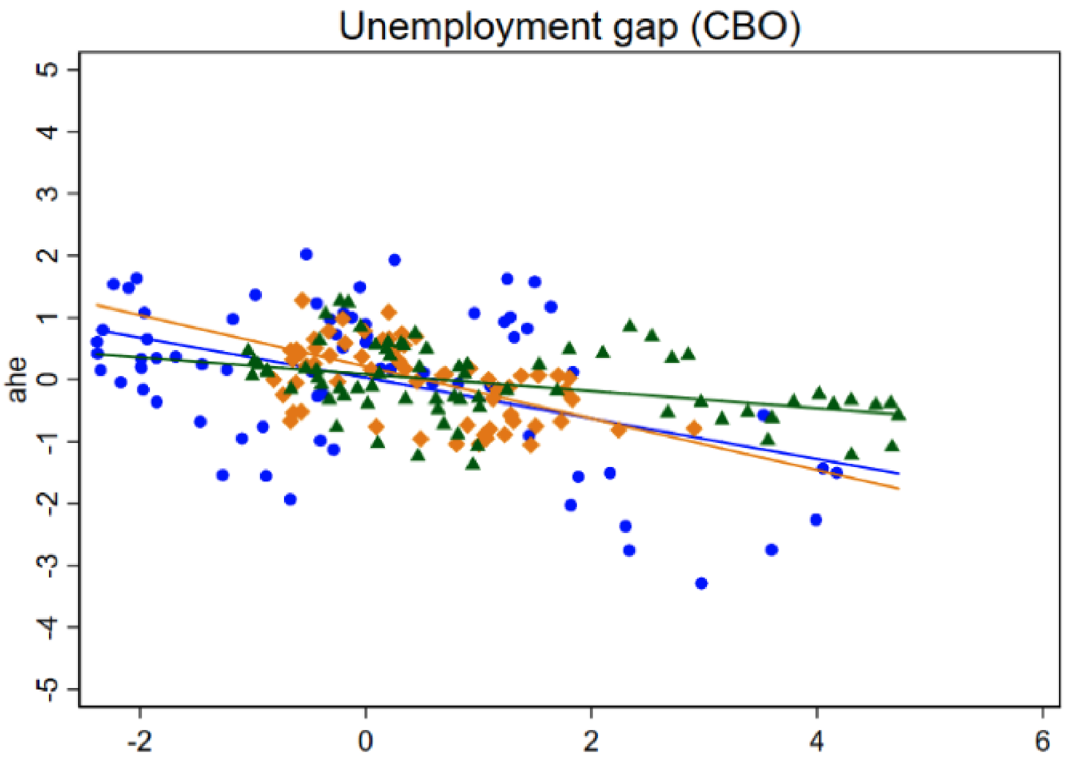
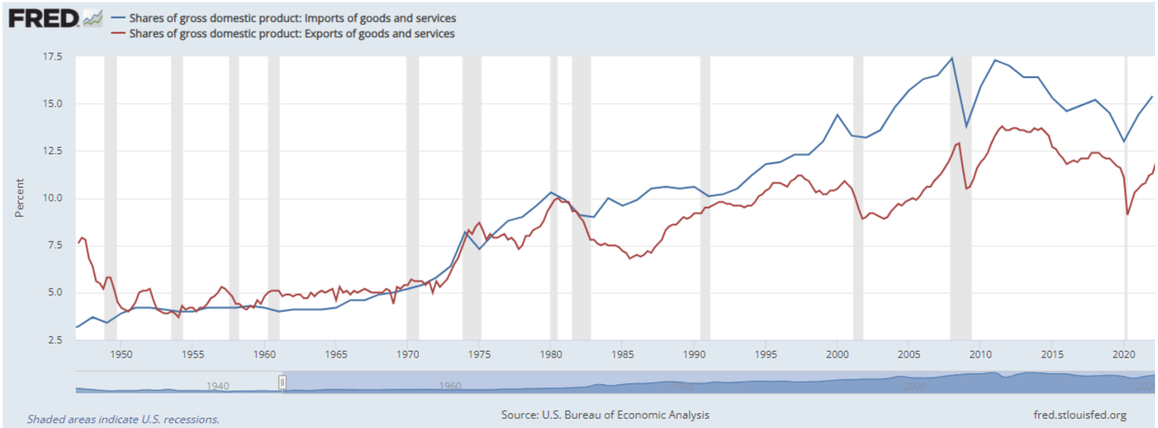


Figure 1: The aggregate accelerated Phillips Curve data from Stock and Watson calculations

exist without price stability, so the U.S. is known to adopt a dual mandate policy officiating this policy through the Humphrey-Hawkins Act of 1978. That focus, however, does not imply that foreign events are not significant drivers of Fed actions.

Empirically, many economists have noted that the effect of unemployment on price inflation has diminished in recent years, even questioning if the relationship is "dead"(Coibion, Gorodnichenko, 2015) or "hibernating"(Hooper, Mishkin, Sufi, 2019). The Phillips curve relationship has flattened so much that large changes in unemployment appear to have little effect on inflation, but there are periods where it seems like the Phillips Curve is back. This puzzling dynamics has been a topic of research for decades. Before the pandemic, the accelerationist Phillips Curve (where $\pi_{t+1} = \pi_{t-1}$) has been empirically modeled by econometricians, and the conclusion is that the flattening of the curve is due to the stable tradeable goods (Stock and Watson, 2019).

At the same time, the U.S. trade in goods and services has grown significantly in the last 50 years. Additionally the terms of trade of the U.S have deteriorated significantly with the U.S The nominal share of imported goods in the U.S. GDP has tripled since 1970 and accelerated since the 1980s. However, whether globalization has altered the inflation's sensitivity to unemployment measured by the slope of the Phillips Curve is the question that has plagued many economists.



For instance, some economists have suggested that globalization could be partially responsible for the recent challenge faced by researchers trying to explain inflation dynamics—an issue tightly linked to the growing disconnect between unemployment and inflation—known by some as the "Missing Disinflation Puzzle" (Forbes, 2019; Heise et al., 2020).

In terms of policy implications, Chairwoman Yellen emphasized the importance of the requirement for the monetary policy to be re-calibrated to account for the changes brought by globalization since the effects of trade will have an indirect effect on the ability of the country like the U.S to achieve its inflation target (Yellen, 2006). Chairman Bernanke and Chairwoman Yellen have both emphasized the dependence on factor markets from economic conditions abroad might have reduced the market power of domestic producers, how their power had declined, and how lower import prices both of final and intermediate goods might have contributed to maintaining overall low inflation.

Accordingly, the U.S. inflation has surged with annual CPI reaching 8.6 in June of 2022, the highest reading since the early 80s. Chairman Powell attributed this surge in inflation to several global unprecedented developments such as the COVID-19 pandemic, the war in Ukraine, and factory shutdowns in China -all of them are events that contribute to the global slack which are measured through the unemployment gap and the output gap.

I develop a model of an open economy with multiple small open economy regions in the U.S. and sticky prices to derive the interaction between trade openness and terms of trade and relate to inflation and unemployment dynamics. I define trade openness as the import exposure as widely known in the trade literature. My model considers multiple regions with differing degrees of import exposure towards the rest of the world. My model delivers a structural equation that shows that less trade-exposed regions would have a steeper Phillips curve. Intuitively, the pass-through of regional unemployment to regional inflation should depend on the exposure of the particular region to trade because the supply shock should depend on the global factors.

3 Literature review

Numerous studies have attempted to explain the flattening of the Phillips curve, proposing explanations such as the large component of inflation indices that is not cyclically sensitive (Stock and Watson 2018), stabilizing role of inflation expectations (Hazell, Hereno, Nakumara and Steinson, 2019)and central bank credibility (Coibion and Gorodnichenko 2015)

The role of global factors gained attention as policymakers discussed how increased imports from low-wage economies appeared to be moderating inflation (Yellen 2006). The focus on global factors gained even more prominence when Borio and Filardo (2007) showed that global slack was becoming more important than domestic slack in inflation models. This prompted a heated debate on the role of global factors—with some papers finding less support for the effect of globalization (i.e., Ball 2006 and Ihrig et al. 2010).

However, international trade research has highlighted the consequences of trade on a regional level recently. For instance, imports cause higher unemployment in import-competing manufacturing industries, which became more apparent when China was admitted to WTO(Autor, Dorn, Hansen, 2014). More recently, the empirical work supports the notion on trade liberalization's effects on Brazilian labor markets and prolonged decline in employment and earnings(Dix-Carneiro and Kovak). A growing body of research suggests the regional effects of trade liberalization, most notably using the changes across industries (Kovak, Brian, 2013)

The former head of IMF Kenneth Rogoff presented the first systematic analysis of global inflation by describing a "near-universal fall in inflation" due to international trade (Rogoff 2003) in addition to a widespread shift towards inflation targeting and more credible central banks. The main reasons he laid out for the global disinflation of the 1990s and early 2000s remain as relevant today as they were then. The debate on the role of global factors has recently reemerged, however, as a possible explanation for why inflation remained muted in the 2010s Forbes(2020) and Heise(2019).

4 Explanations to the identification of the Phillips Curve

4.1 Explanation 1: Too little variation in the aggregate data

For instance, let's look at the issues related to the Phillips Curve identification. The first reason outlined by Hooper, Sufi, Mishkin(2019) being not be enough variation in the aggregate data, according to whom the state and MSA evidence suggests that the post-1988 Phillips curve may have so few observations of very tight labor markets since 1988 and possibly because of the endogoneity of monetary policy. One possible explanation of why they don't find a steep Phillips curve with nonlinearities in the more recent data is that economic up-cycles have become more muted over time, so there has been too little variability in the data to pick up a more normal Phillips curve in the national data. As they observed in below figure, substantial labor market tightening was much more prevalent in earlier decades than it has been in more recent decades. Now let's compare this to the variation obtained from the regional unemployment of the metro area data points. We can clearly see the rich variation in metro areas to pick up a steeper Phillips curve.

4.2 Explanation 2: Endogenous Monetary policy tampers with the true Phillips Curve slope and induces a positive correlation

Endogenous monetary policy induces a positive correlation between inflation and the unemployment gap that biases the slope coefficient of the Phillips curve toward zero. Indeed, according to in the Fitzgerald and Nicolini (2014) and McLeay and Tenroyo(2018) models, optimal monetary policy completely eliminates any correlation between inflation and the output gap (or equivalently the unemployment gap). The models by Fitzgerald and Nicolini (2014) and McLeay and Tenroyo (2018) imply that empirical estimates of a flattening Phillips reflect a monetary policy that is more responsive to deviations in the unemployment gap.

The reason for instability of the Phillips curve could be attributed to even the individual chairs according to McLeaya and Tenreyro(2019).For instance, from the period of 1957 quarter to 1971, the Philips Curve was viewed as an exploitable long-run trade-off. Hence, overly accommodate fiscal and monetary policy led to unemployment falling below the natural rate of unemployment. The period of 1971 quarter 3 to 1980 quarter 4 large cost-push shocks brought by oil supply disruption and its

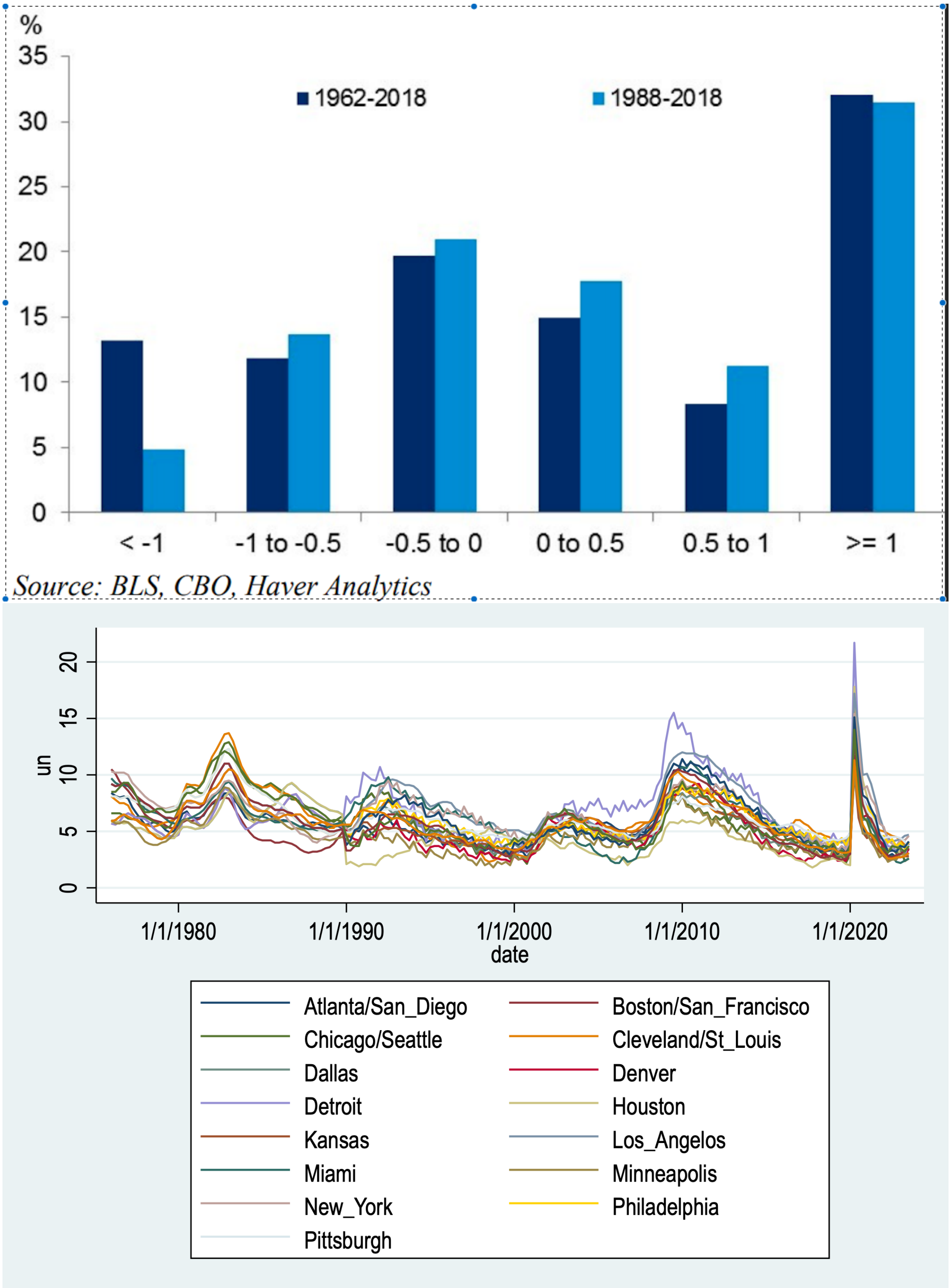


Figure 2: Variation of the aggregate the disaggregated data.

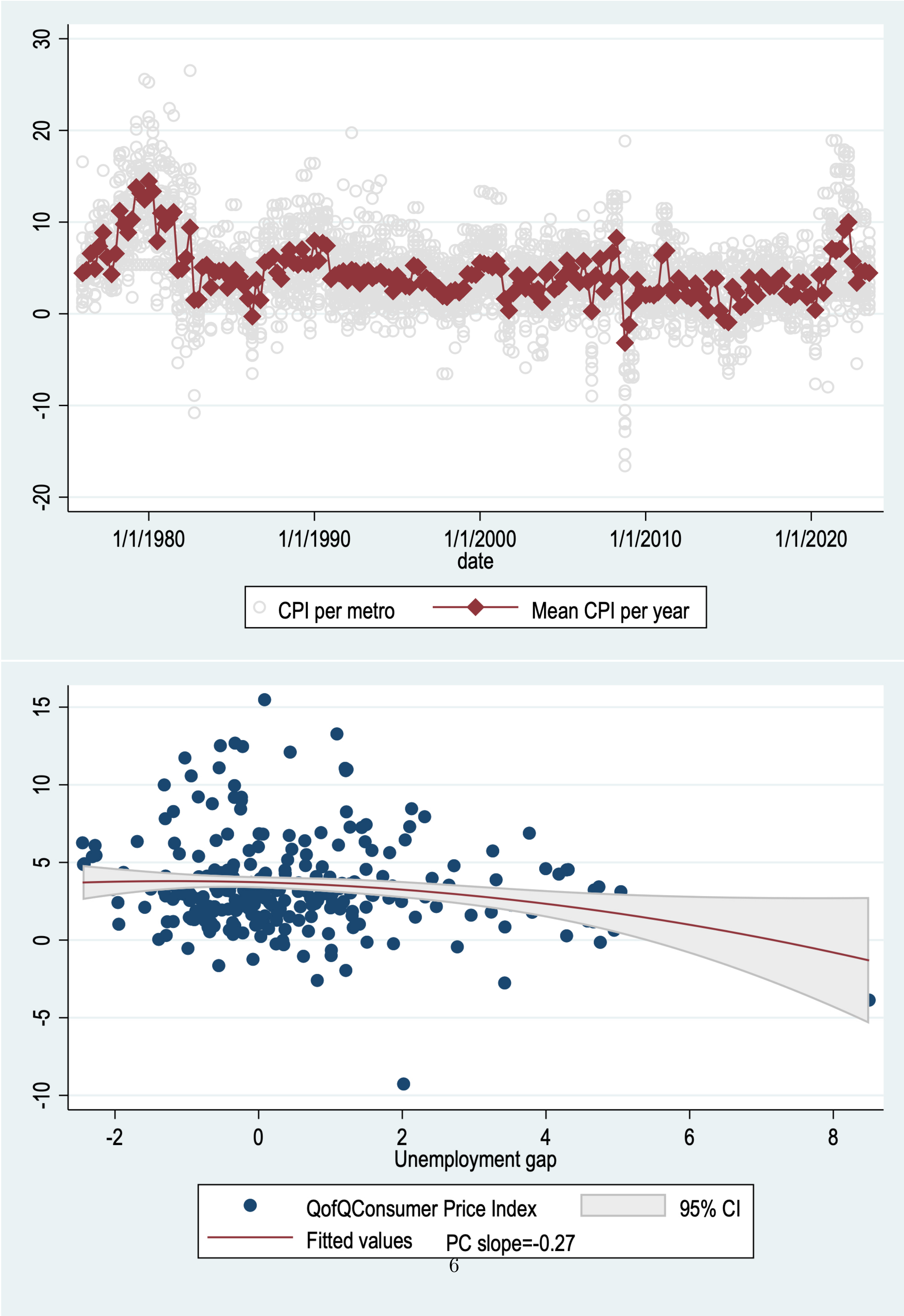
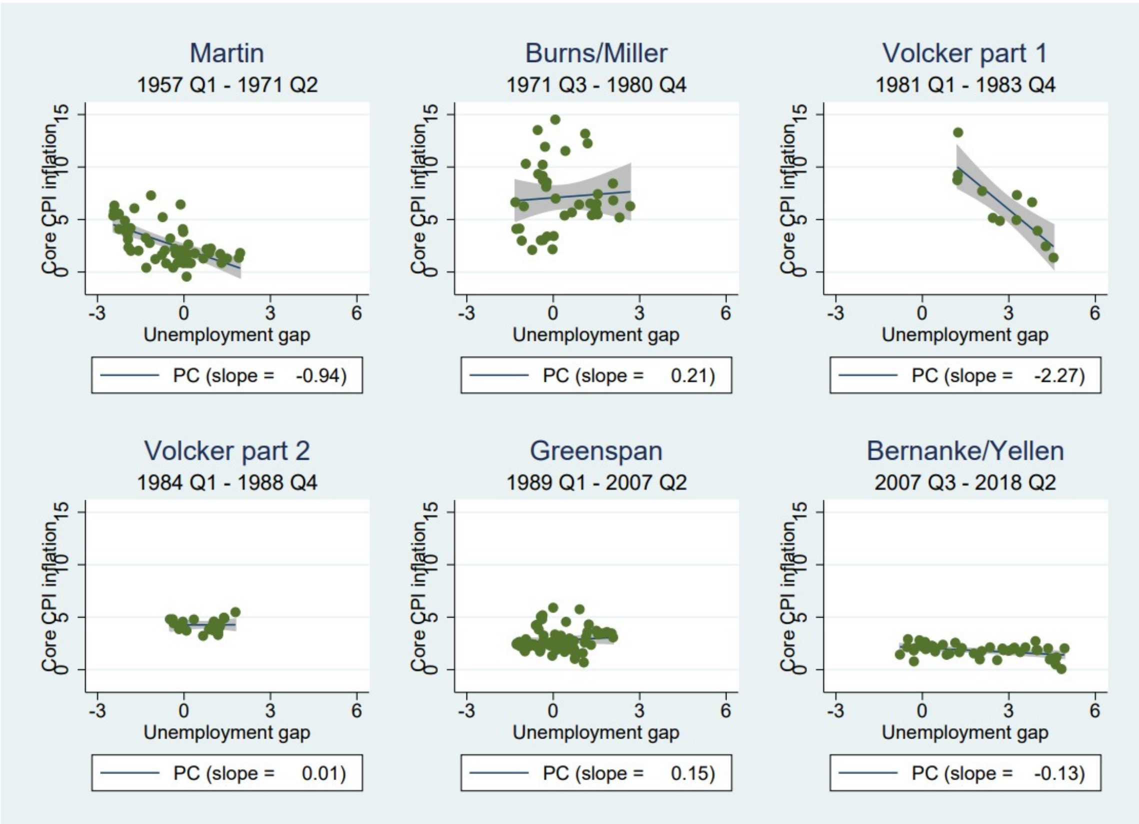


Figure 3: Variation of the aggregate the disaggregated data.

Figure 9: *Phillips correlation by Fed Chair*



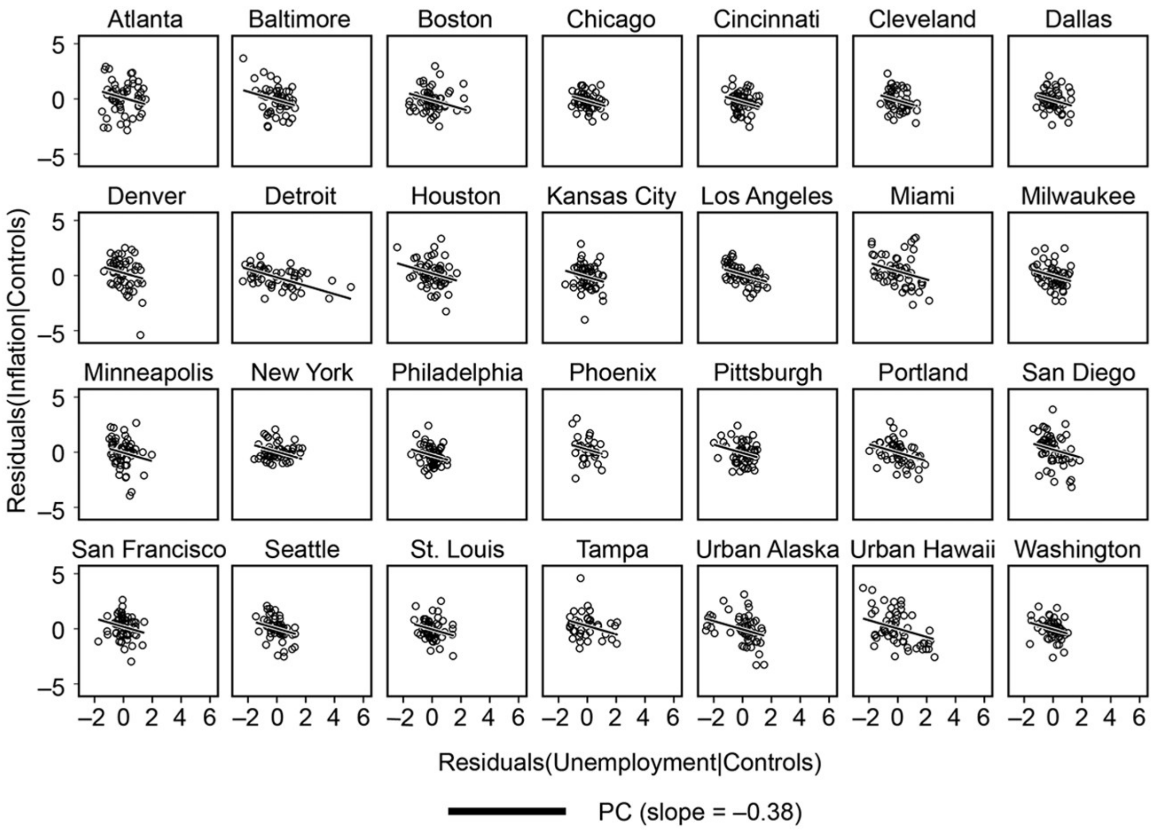


Figure 4: Phillips Curve slope across time and space

impact on inflation expectations led to both inflation and unemployment increase-disappearance of the phillips curve. From 1981 quarter 1 to 1983 quarter 4, the first half of Paul Volcker's tenure saw a re-emergence of a steep negative PC slope. However, in the second half of Paul Volcker's tenure the correlation has disappeared and the Great Moderation era starting from 1984 has started and was characterized by good policy and good luck, good luck meaning low cost push shock. Lastly, the 2008 financial crisis has started and the era of missing disinflation, which has been the subject of the concern for the Phillips Curve researchers.

Hence, the two papers both suggest a solution to this problem that is the use of the pooling of the dis-aggregated data coming from the state and metro area. Both paper adopt a notion of using an instrumental variable of unemployment lagged on the unemployment to account for the endogeneity and advocate for the time fixed effects to account for the common variables such as the monetary stance of the interest rate. Furthermore, monetary policy can be treated as exogenous in state and MSA data because monetary policy is national and so is the same for all states and MSAs.

4.3 Explanation 3: Anchoring of the long-run inflation expectations

This anchoring may have been bolstered by the Fed's adoption of a specific 2 percent inflation objective in 2012, which was preceded by a decade during which that objective was widely seen as

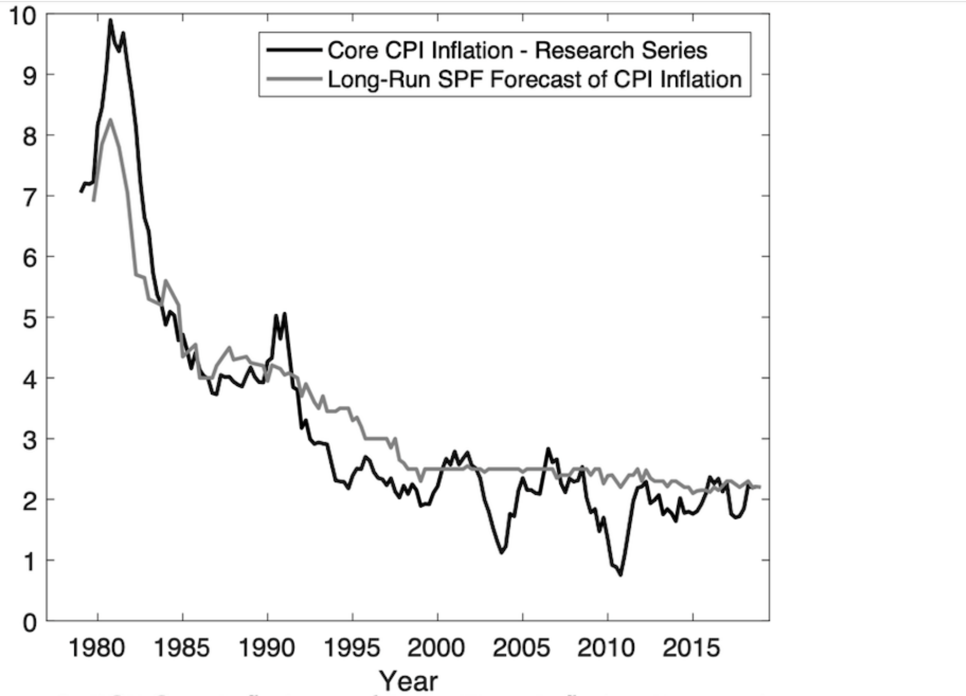


Figure 5: PCE core inflation and long-term inflation Expectations

implicit. Then the Phillips Curve formulation with the long run inflation will be:

$$\pi_t = \beta E_t \pi_{t+\infty} - \kappa(U_t - U_t^n) + u_t \tag{2}$$

Before 1980s, long-term inflation expectations fluctuated a great deal. Inflation fell rapidly over Volcker's disinflation. In contrast, since the 1990s, the long-term inflation expectations have been extremely stable. This shift in belief about the long-run inflation expectations observed can be seen in the Survey of Professional Forecasters(Hazell, Herreno, Nakamura, Steinsson, 2022). What this formulation of the Phillips curve makes clear is that changes in beliefs about the long-run monetary regime feed strongly into current inflation. Furthermore, in the presence of substantial variation in $E_t\pi_{t+\infty}$, the relationship between π_t and u_t may be essentially uninformative about the slope of the Phillips curve.

4.4 My Explanation: The shifting Phillips Curve due to the heterogenous import exposure of the regions interacted with terms of trade

It's imperative to recognize the instability of the Phillips Curve coefficient across time and space. I recognize that the closed economy model doesn't support the notion of the shifting Phillips curve coefficient and that the monetary union of the regions eliminates the need for including confounding variables such as expected inflation or monetary policy intervention, so I reformulate the textbook classic New Keynesian open-economy model to multiple open-economy regions in the U.S.that trade with the rest of the world. Additionally, I use this model as a framework to model the European Union as a multiple-country open economy. This allows me to derive the structural equation for the

Phillips Curve augmented with terms of trade interacting with the trade openness, and define the trade openness as import share relative to GDP.

4.5 My contribution and other competing literature

This paper contributes to two strands of literature: macro literature that focuses on regional inflation-unemployment dynamics mentioned above and trade literature that emphasizes the local labor influenced by trade (Autor, Dorn, Hanson, (AER 2014), Dix-Carneiro and Kovak(AER, 2017)). My model is distinct in a way of being a true open economy model that allows for the change in the slope of the Phillips Curve due to the assumptions of the differing trade activities across time and space. Second, I derive the regional Phillips curve in a single monetary union, which resolves an important limitation of the canonical models that may be confounded by monetary policy and expected inflation dynamics and is absorbed in the panel setting's time-fixed effects specification.

Unlike the previous literature, my model attempts to explain the instability of the price Phillips curve. My paper is similar to Hazell Hereno Nakumara Steinson (2020) and Hottman, Reyes-Herold, 2023) in sense of using an open economy model, but doesn't only look at the non-tradeble good inflation and unemployment dynamics, but rather focuses on the traditional price inflation and unemployment dynamics. The model employed in the analysis is similar to the Hottman, Reyes-Herold as it also transforms the classic Gali and Monacelli's open economy model to suit the regional dynamics, but doesn't remove the influence of the terms of trade variable to be replaced by the expenditure share variable as it is in Hottman and Herold, 2023.

It's clear from theese studies that the coefficient instability of the Phillips Curve slope is related to different time periods or different regional exposures to the import and terms of trade, whether it is before and after 1984 of the Great Moderation era or regional variation in import exposure. McLean and Tenreyro suggest adjusting for this with US metro area data, in which case they find evidence of a steeper and more robust Phillips curve, which is reported at -0.37. On the other hand, Nicolini and Fetzgerald find steeper and constant Phillips curve slope before and after 1984 with a Phillips curve slope of -0.3. Hazell Hereno Nakumara Steinson (2020) use open economy model and still find some flattening of the Phillips curve since the 1990s due to better inflation achnhoring with a Phillips curve slope of -0.125. Recently, Hottman and Herold(2023) find a flatter slope of the non-tradeable good inflation due to the import share heterogeneity of the U.S states with varying slopes depending on the regional heterogeneity of higher and lower import exposure(Hottman, Reyes-Herold, 2023).

5 A multi-region open New Keynesian model

The U.S is assumed to be a home country with multiple open metro areas indexed by r $\in$ [1,...I] each of which is a small open economy represented by a unit interval relative to the rest of the world. The framework extends the benchmark open economy with nominal rigidities a la Calvo (Gali and Monacelli, 2005) to multiple open economies framework. It is assumed that the regions share identical preferences, technology, and market. There is no movement between regions, but since the metro area could encompass multiple states, there is a movement between the neighboring states if the area is considered one metro area region.

5.1 Household's consumption maximization problem

The domestic economy is populated by infinitely-lived households, consuming Dixit-Stiglitz aggregates of domestic ($C_{H,rt}$) and imported ($C_{F,rt}$) goods, by domestic firms producing a differentiated good, and by a continuum of importing firms that operate as price setters in the local market. All

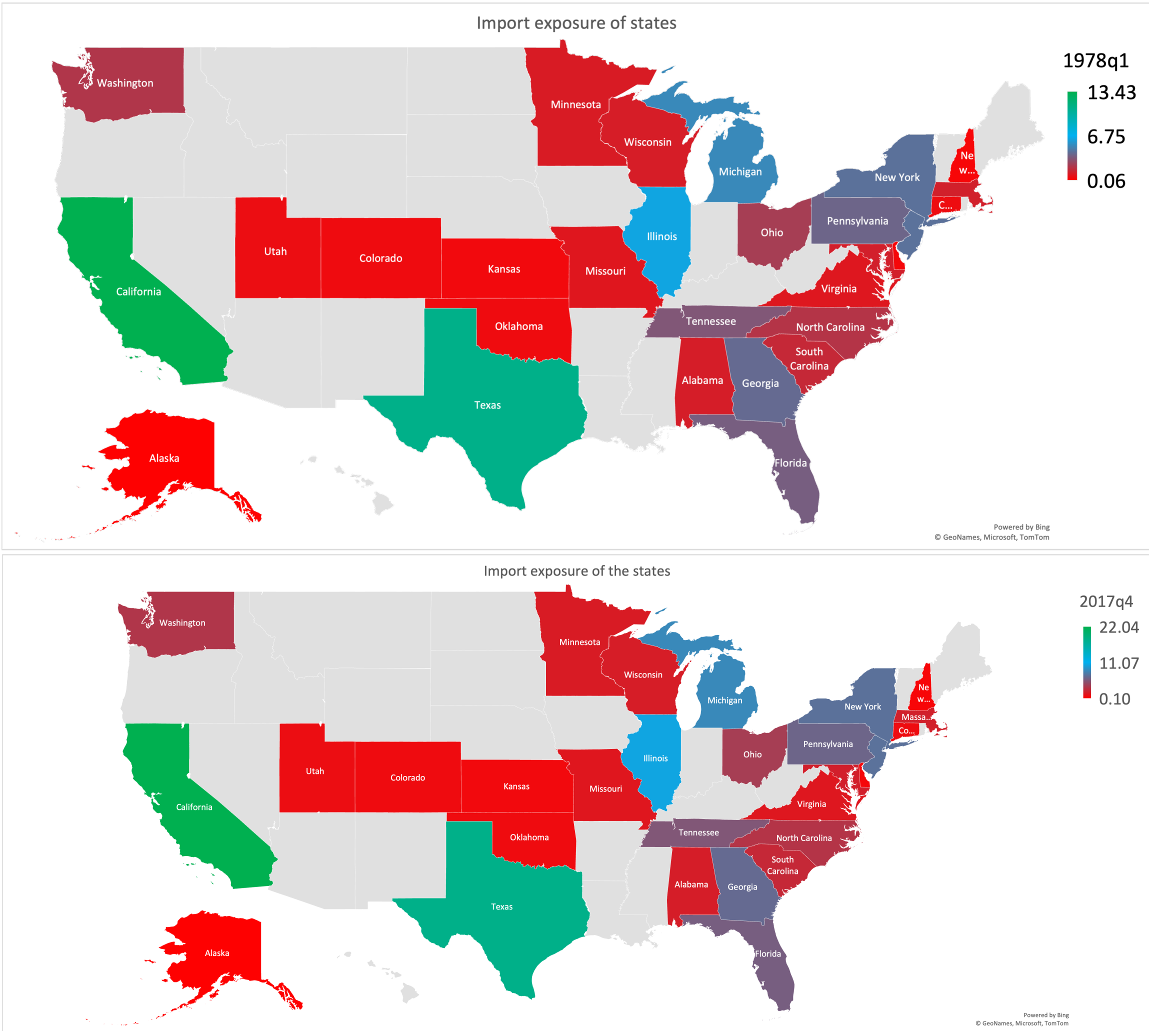


Figure 6: Caption

goods are tradeable. i ∈ [0, 1] indexes good varieties since each region produces a continuum of differentiated goods, represented by the unit interval and α_{rt} is a measure of openness and we assume it can differ across regions. More open regions will have higher values of α_{rt}, and therefore lower home bias. Region r is inhabited by a representative household that seeks to maximize:

$$E_0 \sum_{t=0}^{\infty} \underbrace{U(C_{rt}, N_{rt})}_{U(C_{rt},N_{rt})=\frac{C_{rt}^{1-\sigma}}{1-\sigma}-\frac{N_{rt}^{1+\varphi}}{1+\varphi}} \tag{3}$$

$$\int_0^1 \underbrace{P_{H,rt}(i)}C_{H,rt}(i)di + \int_0^1\int_0^1 P_{f,rt}(i)C_{f,rt}(i)didf + E_t[Q_{t,t+1}D_{r,t+1}] \leq D_{rt} + W_{rt}N_{rt} \tag{4}$$

Here N_{rt} is employment in region r, C_{rt} is consumption of a composite consumption index in region r. $P_{H,rt}(i)$ is the price index of the domestic variety i $\in [0,1]$ and $P_{f,rt}(i)$ is the price index of goods imported, D_{t+1} is the nominal payoff in t+1 of the portfolio held a the end of t. The variables are all expressed in units of domestic currency. $Q_{t,t+1}$ stochastic discount factor for one-period-ahead nominal payoffs.There are complete international financial markets. Each household has access to complete set of contingent claims traded internationally.

The composite consumption index C_{rt} is defined as a combination of the $C_{H,rt}$ and $C_{F,rt}$ which are indexes of consumption of domestic and foreign goods with $\eta > 0, \eta$ being the elasticity of substitution between domestic and foreign goods:

$$C_{rt} = [(1-\alpha_{rt})^{\frac{1}{\eta}} \underbrace{C_{Hrt}^{\frac{\eta-1}{\eta}}}_{[\int_0^1 C_{H,rt}(i)^{\frac{\epsilon-1}{\epsilon}}di]^{\frac{\epsilon}{\epsilon-1}}} + \alpha_{rt}^{\frac{1}{\eta}} \underbrace{C_{Frt}^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}}}_{C_{F,rt}=[\int_0^1 \underbrace{C_{f,rt}^{\frac{\gamma-1}{\gamma}}]^{\frac{\gamma}{\gamma-1}}}_{C_{f,rt}=[\int_0^1 C_{f,rt}(i)^{\frac{\epsilon-1}{\epsilon}}di]^{\frac{\epsilon}{\epsilon-1}}}} \tag{5}$$

where $C_{H,rt}$ is an index of consumption of goods produced in region r given by the constant elasticity of substitution (CES) function with $\epsilon \geq 1$ denoting the elasticity of substitution between varieties produced within the United States. On the other hand, $C_{F,rt}$ is the index of consumption of goods imported in each region r with γ measuring the substitutability between goods produced in different foreign countries. $C_{f,rt}$ is an index of quantity of goods imported from different countries and consumed by domestic households with $\epsilon \geq 1$ denoting the elasticity of substitution between varieties produced within a foreign country:

$$C_{H,rt} = [\int_0^1 C_{Hrt}(i)^{\frac{\epsilon-1}{\epsilon}}di]^{\frac{\epsilon}{\epsilon-1}} \quad C_{F,rt} = [\int_0^1 C_{f,rt}^{\frac{\gamma-1}{\gamma}}]^{\frac{\gamma}{\gamma-1}} \quad C_{f,rt} = [\int_0^1 C_{f,rt}(i)^{\frac{\epsilon-1}{\epsilon}}di]^{\frac{\epsilon}{\epsilon-1}}$$

The solution to the household maximization problem is the optimal allocation of the expenditures across locally produced varieties yielding the demand functions for home and foreign consumptions:

$$C_{H,rt}(i) = [\frac{P_{H,rt}(i)}{P_{H,rt}}]^{-\epsilon}C_{H,rt} \quad C_{f,rt}(i) = [\frac{P_{f,rt}(i)}{P_{f,rt}}]^{-\epsilon}C_{f,rt} \tag{6}$$

for all i, f $\in [0,1]$ where $P_{H,rt}$ is the domestic price index and $P_{f,rt}$ is the price index of goods imported from country f for all f ∈(0, 1)is given by:

$$P_{H,rt} = [\int_0^1 P_{H,rt}(i)^{1-\epsilon}di]^{\frac{1}{1-\epsilon}} \quad P_{f,rt} = [\int_0^1 P_{f,rt}(i)^{1-\epsilon}di]^{\frac{1}{1-\epsilon}} \tag{7}$$

where $C_{H,rt}$ denotes the consumption index of locally produced goods where the prices and quantities are such that $\int_0^1 P_{H,rt}(i)C_{H,rt} = P_{H,rt}C_{H,rt}$ and $\int_0^1 P_{f,rt}(i)C_{f,rt} = P_{f,rt}C_{f,rt}$

The optimal allocation of expenditures on imported goods by country of origin implies:

$$C_{f,rt} = \frac{P_{f,rt}}{P_{F,rt}}^{-\gamma} C_{F,rt}$$

for all f $\in$ [0, 1] where $P_{F,rt}$ is the price index for imported goods in domestic currency:

$$P_{F,rt} = \int_0^1 [P_{f,rt}^{1-\gamma} df]^{\frac{1}{1-\gamma}}$$

Since C_{rt} is defined as a combination of the C_{Hrt} and C_{Frt} which are indexes of consumption of domestic and foreign goods:

$$C_{rt} = [(1-\alpha_{rt})^{\frac{1}{\eta}} \underbrace{C_{H,rt}^{\frac{\eta-1}{\eta}}} + \alpha_{rt}^{\frac{1}{\eta}} \underbrace{C_{F,rt}^{\frac{\eta-1}{\eta}}}]^{\frac{\eta}{\eta-1}} \tag{8}$$

Then the optimal allocation of expenditures between domestic and imported goods:

$$C_{H,rt} = (1-\alpha)\frac{P_{H,rt}}{P_{rt}}^{-\eta} C_{rt} \quad C_{F,rt} = \alpha \frac{P_{F,rt}}{P_{rt}}^{-\eta} C_{rt} \tag{9}$$

Then, finally, the CPI equation is:

$$P_{rt} = [(1-\alpha)P_{H,rt}^{1-\eta} + \alpha P_{F,rt}^{1-\eta}]^{\frac{1}{1-\eta}} \tag{10}$$

Hence, accordingly parameter α corresponds to the share of domestic consumption allocated to imported goods. It is also in this sense that α represents a natural index of openness. Accordingly, total consumption expenditures by domestic households are given by $P_{rt}C_{rt} = P_{H,rt}C_{H,rt} + P_{F,rt}C_{F,rt}$

5.2 CPI equation

Log-linearization of the CPI expression around a steady-state yields:

$$p_{rt} = (1-\alpha)p_{H,rt} + \alpha p_{F,rt} \tag{11}$$

Bilateral terms of trade between the domestic economy and country i and $S_{f,t} = \frac{P_{f,t}}{P_{H,t}}$, the price of country i's goods in terms of home goods. The effective terms of trade are thus given by: $S_{rt} = \frac{P_{F,rt}}{P_{H,rt}} = [\int_0^1 S_{f,rt} di]^{\frac{1}{1-\gamma}}$, which is an approximation of the by the log-linear expression: $s_{rt} = \int_0^1 s_{ft} di = p_{F,rt} - p_{H,rt}$. Then $p_{rt} = p_{H,rt} - \alpha p_{H,rt} + \alpha p_{F,rt} = p_{H,rt} + \alpha s_{rt}$

Then domestic producer inflation $\pi_{H,rt} = p_{H,rt} + p_{H,rt-1}$ (defined as the rate of change in the index domestic goods prices), and CPI inflation are linked:

$$\pi_{rt} = \pi_{H,rt} + \alpha[(p_{F,rt} - p_{F,rt-1}) - (p_{H,rt} - p_{H,rt-1})]$$

$$\pi_{rt} = \pi_{H,rt} + \alpha \Delta s_{rt} \tag{12}$$

The period utility function is specialized to be of the form and reformulated as:

$$U(C_{rt}, N_{rt}) = \frac{C_{rt}^{1-\sigma}}{1-\sigma} - \frac{N_{rt}^{1+\varphi}}{1+\varphi} \tag{13}$$

s.t

$$P_{rt}C_{rt} + E_tQ_{t,t+1}D_{t+1} \leq D_t + W_tN_t \tag{14}$$

Then the regional intertemporal optimality condition by the household is given as:

$$C_{rt}^{\sigma}N_{rt}^{\varphi} = \frac{W_{rt}}{P_{rt}} \tag{15}$$

Then intertemporal optimality condition is:

$$Q_{t,t+1} = \beta\frac{C_{rt+1}}{C_{rt}}^{-\sigma}\frac{P_{rt}}{P_{rt+1}} \tag{16}$$

The usual Euler equation:

$$\beta R_tE_t[\frac{C_{rt+1}}{C_rt}\frac{P_{rt}}{P_{rt+1}}] = 1 \tag{17}$$

The first-order conditions of the consumer's problem are standard and can be written in a conventional log-linearized form as:

$$w_{rt} - p_{rt} = \sigma c_{rt} + \varphi n_{rt} \tag{18}$$

$$c_t = E_tc_{t+1} - \frac{1}{\sigma}[r_t - E_t\pi_{t+1}] \tag{19}$$

Here wt is the nominal wage, nt is labor and π_t is the CPI inflation rate

6 Government policy

The monetary authority conducts a common policy for all regions r = 1, . . . , I. We assume that the policy takes the form of the following interest rate rule:

$$\hat{r_t} = \phi_\pi[\pi_t - \overline{\pi_t}] - \phi_u[\hat{u_t} - \overline{u_t}] \tag{20}$$

Aggregate inflation and unemployment are in turn defined as population-weighted averages across regions:

$$\pi_t = \sum_{r=1}^{I}\nu_{rt}\pi_{rt} \tag{21}$$

$$\hat{u_t} = \sum_{r=1}^{I}\nu_{rt}\hat{u_t} \tag{22}$$

7 Firm's price resetting and profit maximization problem

A typical firm produces variety i indexed by i ∈ (0, 1).The firm in region r has technology A_{rt}. Then $Y_{rt}(i)$ denote total output by firm i located in region r at time t.:

$$Y_{rt}(i) = A_{rt}N_{rt}(i)$$

A_{rt} is an exogenous and stochastic productivity shock affecting region r. We assume that firms set prices in a staggered fashion, as in Calvo (1983). Given that the firms face sticky prices and can only adjust their price in period t with probability $1-\theta$ if the firm adjusts prices $P_{H,rt}$.

The firm's nominal profits at time t are given by:

$$\Pi_{rt} = P_{H,rt}(i)Y_{rt}(i) - W_t N_{rt}(i)$$

Since firms face sticky prices and they can only adjust their price in period t with probability $1-\theta$ if firm i is able to update its price $P_{H,rt}(i)$ in period t, and will maximize the objective function:

$$\sum_{k=0}^{\infty} E_t[Q_{t,t+k}(P_{H,rt}(i))Y_{r,t+k}(i) - W_{rt+k}N_{rt}] \tag{23}$$

s.t

$$Y_{H,rt}(i) = C_{H,rt}(i) + X_{rt}(i) \tag{24}$$

In addition to domestic households demanding goods in region r, there is also a demand by the rest of the world for each variety i $\in [0,1]$produced in each region.We assume that the demand for exports of good i $\in [0,1]$ produced in region r is given by:

$$X_{rt}(i) = \frac{P_{H,rt}(i)}{P_{H,rt}}^{-\epsilon} X_{rt} \tag{25}$$

then

$$X_{rt} = [\int X_{rt}(i)^{\frac{\epsilon-1}{\epsilon}} di]^{\frac{\epsilon}{\epsilon-1}} \tag{26}$$

is an index across varieties, summarizing aggregate exports from region r to the rest of the world.

Then putting everything together, firm's profit maximization problem becomes:

$$\sum_{k=0}^{\infty} E_t[Q_{t,t+k} \underbrace{(P_{H,rt}(i))Y_{r,t+k}(i) - W_{rt+k}N_{rt}}_{\Pi_{rt}(i)=P_{H,rt}(i)\underbrace{Y_{rt}(i)}_{A_{rt}N_{rt}(i)} - \underbrace{W_{rt}N_{rt}}_{MC_{rt}P_{H,rt}A_{rt}N_{rt}}}] \tag{27}$$

s.t

$$Y_{H,rt}(i) = \underbrace{C_{H,rt}(i)}_{=[\frac{P_{H,rt}(i)}{P_{H,rt}}]^{-\epsilon}C_{H,rt}} + \underbrace{X_{rt}(i)}_{\frac{P_{H,rt}(i)}{P_{H,rt}}^{-\epsilon}X_{rt}}) = [\frac{P_{H,rt}(i)}{P_{H,rt}}]^{-\epsilon}(C_{H,rt} + X_{rt}) = [\frac{P_{H,rt}(i)}{P_{H,rt}}]^{-\epsilon}Y_{H,rt} \tag{28}$$

Marginal cost is then: $MC_{rt} = \frac{W_{rt}}{P_{H,rt}A_{rt}}$

We assume that firms set prices in a staggered fashion a la Calvo (1983). Given that the firms face sticky prices and can only adjust their price in period t with probability $1-\theta$ if the firm adjusts prices $P_{H,rt}$. Then the firm's price resetting problem becomes:

$$max_{P_{H,rt}(i)} \sum_{k=0}^{\infty} \theta^k E_t[Q_{t,t+k} \underbrace{\Pi_{rt+k}(i)}_{P_{H,rt}(i)Y_{rt}(i) - MC_{rt}P_{H,rt}Y_{rt}(i)}] \tag{29}$$

s. t

$$Y_{rt}(i) = [\frac{P_{H,rt}(i)}{P_{H,rt}}]^{-\epsilon}Y_{H,rt} \tag{30}$$

Then substituting the constraint into the profit equation:

$$max_{P_{H,rt}(i)} \sum_{k=0}^{\infty} \theta^k E_t[Q_{t,t+k}[\frac{P_{H,rt}(i)^{1-\epsilon}}{P_{H,rt+k}^{-\epsilon}} Y_{H,rt+k} - MC_{rt+k} P_{H,rt+k}(i)^{-\epsilon} Y_{H,rt+k}] \tag{31}$$

and then differentiation with respect to $P_{H,rt}$:

$$\frac{\partial \Pi_{rt+k}(i)}{\partial P_{H,rt}(i)} = Y_{H,rt+k} P_{H,rt}(i)^{-\epsilon-1}[(1-\epsilon)P_{H,rt+k}^{\epsilon} P_{H,rt}(i) + \epsilon MC_{rt+k} P_{H,rt+k}^{1+\epsilon}]$$

Then the optimal condition of a firm price resetting is:

$$\sum_{k=0}^{\infty} E_t[Q_{t,t+k} \frac{\partial \Pi_{rt+k}}{\partial P_{H,rt}(i)}] = 0$$

where $Q_{t,t+k} = \beta^k \frac{U'(C,rt+k)}{U'(C,rt)} \frac{P_{rt}}{P_{rt+k}}$

After solving to find the optimal price, realize $P_{H,rt}$(i)=$P_{H,rt}$

$$P_{H,rt}(i) = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{k=0}^{\infty}[\theta\beta]^k E_t[\frac{U'(C_{rt+k})}{P_{rt+k}}[MC_{rt+k}][P_{H,rt+k}^{1+\epsilon}][Y_{H,rt+k}]}{\sum_{k=0}^{\infty}[\theta\beta]^k E_t[\frac{U'(C_{rt+k})}{P_{rt+k}}[P_{H,rt+k}^{\epsilon}][Y_{H,rt+k}]} \tag{32}$$

$$p_{H,rt}^{reset} = log\frac{\epsilon}{\epsilon - 1} + (1-\beta\theta)\sum_{k=0}^{\infty}[\theta\beta]^k E_t[mc_{r,t+k} + p_{H,rt+k}] \tag{33}$$

Reset prices and domestic prices are linked due to the Calvo-assumption in the following way:

$$P_{H,rt}^{1-\epsilon} = (1-\theta)P_{H,rt}^{reset} + \theta P_{H,rt-1}^{1-\epsilon} \tag{34}$$

Since $\pi_{H,rt} = \frac{P_{H,rt}}{P_{H,rt-1}}$ and $\pi_{H,rt}^{reset} = \frac{P_{H,rt}^{reset}}{P_{H,rt-1}^{reset}} - 1$ then: Combining 31 and 33 then log-linearizing around the zero inflation

$$\pi_{H,rt} = \frac{(1-\theta)(1-\theta\beta)}{\theta} \hat{mc}_{rt} + \beta E_t[\pi_{H,t+1}] \tag{35}$$

The parameters only depend on the readjustment of the price θ. However, the marginal cost will not be only dependent on constant parameters. Recall the Marginal Cost equation from the household's optimization problem. The regional inter-temporal optimal condition of the household was previously given as: $\frac{W_{rt}}{P_{rt}} = C_{rt}^{\sigma_{r\alpha}} N_{rt}^{\varphi_r}$. since the marginal was cost was previously defined as : $MC = \frac{W_{rt}}{P_{rt}A_{rt}} = \frac{C_{rt}^{\sigma_{ra}} N_{rt}^{\varphi_r}}{A_{rt}} = Y_{rt}^{\sigma_{ra}+\varphi_r}$ using the market clearing condition of $C_{rt} = Y_{rt}$.

Then the marginal cost is :

$$\hat{mc}_{rt} = [\sigma_{r\alpha} + \varphi_r] \underbrace{\hat{y_{rt}}}_{[y_{rt} - y_{rt}^n]}$$

$$\hat{mc}_{rt} = -[\sigma_{r\alpha} + \varphi_r] \underbrace{\hat{u_{rt}}}_{[u_{rt} - u_{rt}^n]}$$

There exists a wedge between slack(unemployment gap) and Marginal cost.

Domestic price inflation Phillips Curve's slope is not constant

$$\pi_{H,rt} = -\kappa_{r\alpha}\hat{u_{rt}} + \beta E_t[\pi_{H,rt+1}] \tag{36}$$

where $\kappa_{r\alpha} = \frac{(1-\theta)(1-\theta\beta)}{\theta}[\sigma_{r\alpha} + \varphi_r]$ and $\sigma_{r\alpha}(\alpha)$

α_r or openness of the region affects the slope of the Phillips curve. Then inflation responds to variation in the unemployment gap. Under high substitutability of goods $\sigma\eta$ >1, the slope of the PC will be flatter with higher openness index α_r. Hence, more trade-open regions should produce a flatter PC slope. Let' go back to final price inflation Phillips curve and reformulate the familiar $\pi_{rt} = \pi_{H,rt} + \alpha\Delta s_{rt}$. equation to : $\pi_{rt} = \pi_{H,rt} + \alpha\Delta s_{rt} = -\kappa_{r\alpha}\hat{u_{rt}} + \beta E_t[\pi_{H,rt+1}] + \alpha\Delta s_{rt}$.

8 Open economy New Keynesian Phillips Curve estimation

The final price Philips curve equation is then the combination of home inflation $\pi_{H,rt}$and import exposure interacted with terms of trade Δs_{rt}:

$$\pi_{rt} = \pi_{H,rt} + \alpha\Delta s_{rt} = -\kappa_{r\alpha}\hat{u_{rt}} + \beta E_t[\pi_{H,rt+1}] + \alpha\Delta s_t$$

Then the equation to estimate the regional inflation for each metro area that trades with the rest of the world(ROW) is :

$$\pi_{rt} = \underbrace{\frac{(1-\theta)(1-\theta\beta)}{\theta}}_{constant}\underbrace{[\sigma(\alpha_{rt}) + \varphi(\alpha_{rt})]}_{time-varying\alpha}\underbrace{(u_{rt} - u_{rt}^{gap})}_{u_{rt}^{gap} unobserved} + \underbrace{\beta E_t[\pi_{H,rt+1}]}_{common?} + \underbrace{\alpha\Delta s_t}_{\frac{Import}{GDP}\times\Delta TOT} \tag{37}$$

- θ and β are constant parameters? from Calvo staggered firm price resetting problem
- σ_{rt} regional elasticity of intertemporal substition of the household that varies across time due to the trade exposure of α for each region
- $\alpha = \frac{Import_{rt}}{GDP_{rt}}$ regional import exposure over time for each region
- u_{rt} Unemployment rate for each region
- u_{rt}^{gap} The natural rate of unemployment for each region
- $\beta E_t[\pi_{H,rt+1}]$ Expected inflation for the next period that should be common across region within one monetary union
- $\Delta s_r = \Delta TOT$ national terms of trade index measures the change in the purchasing power of exports relative to imports for a given country.An index of 100 and more indicates favourable conditions for the United States.

9 Empirical analysis results

I specify the following equation for the structural equation to be analyzed in the panel data setting:

$$\pi_{rt} = \beta_0 + \underbrace{\gamma_r}_{regionalFE} + \underbrace{\delta_t}_{timeFE} + \beta_1 U_{rt}[IV : UN_{rt-1}] + \beta_2 ImpExp_{rt} + \beta_3 ImExp_{rt} \times \Delta TOT_t \tag{38}$$

- π_{rt} is quarter over quarter log CPI BLS 24 metro area data(1976q1-2023q3)+ semiannual data(1987q1-1997q3) for Atlanta, Cincinnati, Denver, Kansas, Minneapolis, San Diego, Seattle, Alaska, Hawaii to account for missing data from 1987q1-1997q3. Baltimore, Pheonix areas were excluded for data limitation reasons
- u_{rt} quarterly unemployment data from BLS(1990q1-2023q3)+state unemployment data for the location of the biggest city of the metro region from BLS(1976q1-2023q3)
- $ImExp_{rt} = \frac{Import_{rt}}{GDP_{rt}}$ is calculated from state import data for the biggest city in the metro region divided by the GDP of the state where the largest metro city is located (2008q1-2023q3) $+\frac{NationalImp_t/NationalGDP_t}{numberofregions}$ as a proxy for each metro area (1976q1-2007q3) from BEA.
- Δ Terms of Trade index is a national index that comes from BEA(1976q1-2023q3)
- regional fixed effects are to control unemployment gap or omitted variable bias as similarly done in (Tenreyo and McLeaya(2019))
- the time fixed effect is to absorb the expected inflation across time that is common for regions in a monetary union following Hazell, Hereno, Nakumara, Steinson(2022)
- Instrumental variable to control for regional simultaneity bias.

Dependent variable is inflation

Regressors	(1) 1974-2019	(2) 1984-2019	(3) 1990-2019	(4) 2000-2019	(5) 2008-2019
Unemployment	-0.212***	-0.267***	-0.276***	-0.305***	-0.348***
	(0.0490)	(0.0519)	(0.0641)	(0.0902)	(0.123)
Import exposure	0.216***	0.219***	0.231***	0.237***	0.225***
	(0.0397)	(0.0366)	(0.0373)	(0.0398)	(0.0477)
TOT*ImportExposure	0.175***	0.171***	0.169***	0.168***	0.168***
	(0.0344)	(0.0312)	(0.0311)	(0.0314)	(0.0324)
Constant	5.946***	6.375***	9.220***	6.550***	8.312***
	(0.734)	(0.694)	(0.648)	(0.662)	(0.832)
Observations	3,340	2,732	2,277	1,517	911
Number of region	19	19	19	19	19
Region FE	YES	YES	YES	YES	YES
Time FE	YES	YES	YES	YES	YES
IV	Unemp lagged	Unemp lagged	Unemp lagged	Unemp lagged	

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

10 Conclusion

This open economy model is crucial in understanding the inflation and unemployment dynamics explained by the shifting Phillips curve slope. The crucial component of this shift is due to the terms of trade's interaction with import exposure of different regions. This variable is crucial in understanding the imported inflation, which can come from different countries.

My model has many advantages compared to the canonical closed-economy models. First, my model is a true open economy model that allows the U.S region to trade with the rest of the world (ROW) instead of an open economy of tradeable and non-tradeable goods inside the U.S unlike Hazell, Hereno, Nakumara, Steinson (2020). Moreover, my model allows for the change in the slope of the Phillips Curve due to the assumptions of the openness of the trade. Thus this model has a clear interpretation of the slope change due to the differing trade activities of regions. Second, I derive the regional Phillips curve augmented with the terms of trade interacting with the import exposure term, which resolves an important limitation of the canonical models that assume that the Phillips curve is constant. Then the NKPC depends on the trade openness or import exposure of each region. This framework then allows me to analyze the effect of U.S. trading partners on the New Keynesian Phillips Curve.