On the Formation of Automobile Dealer Trade Networks

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Abstract

This paper analyzes the network structure of automobile inter dealer trade markets. Inter dealer trade data from Adamson Motors, a medium sized dealer in Rochester Minnesota, shows that the network of dealer relationships is not complete. Further, large trade volumes are generated by relatively few trade partners. A homogenous linking network of retail automobile sales is developed. A link represents a relationship, which lubricates trade between dealers by allowing them to trade whenever one dealer is in need. The optimal network structure is shown to depend crucially on inventory complementarity, distance, an appropriate access to global inventory and buyer substitution. Asymmetric information about buyer search/inconvenience costs allows the dealer to increase profits by limiting the number of trade partners. This reduction in the number of links raises the local buyers cost of obtaining models not currently in stock and allows the dealer to sell models with higher profit margins. These are the models in stock, that don’t incur transportation costs with sale. This effect diminishes as the number of trading relationships declines. After a certain threshold, raising the cost of obtaining distant models results in an overwhelming level of lost sales to distant dealers of the same make and local dealers of alternative makes. Because of network incompleteness, trades are routinely not completed. These buyers must travel which decreases consumer surplus due to increased search/inconvenience costs. Finally a computational model is proposed to analyze the likelihood of increasing an individual dealers profitability by simultaneously decreasing inventory size and beginning to trade with transfers.
1 Introduction

The purpose of this research is to analyze the efficiency and formation of automobile inter dealer trade networks. I propose using network theory coupled with discrete choice models of demand to analyze the network structure of the market, its efficiency properties and several policies designed to potentially improve efficiency and profitability. A simple example is given to show the conditions under which dealer trades are optimal. Then the network structure of trading relationships is shown to depend crucially on inventory complementarity, consumer substitution, an appropriate degree of access to global inventory and distance. In particular, under certain conditions symmetric equilibria exist that are characterized by intermediate access to global inventory. In other words, dealers optimally choose to trade with only a few partners. This decreases consumer surplus as buyers must travel and negotiate with more distant dealers. The paper proposes a model that could be used to analyze several policies designed to potentially improve efficiency and profitability. These are reducing the costs of completing more complicated trades and modifying the terms of trade currently in use. Specifically, almost all trades are bilateral because of costs. Dealers contact each other over the telephone and it can be difficult to negotiate even bilateral trades. If a mechanism could be developed to reduce these costs, more complicated trades could be completed to potentially improve efficiency and profitability. As for the second policy, most trades are completed at invoice or invoice minus holdback (to be explained below). Very rarely do dealers trade with transfers. Because of a large number of options offered on any given model for some makes, dealer trade business can be a very significant portion of business. At Adamson Motors, a medium sized dealership in Rochester Minnesota, the fraction of Chrysler sales generated by dealer trades is between 50 – 60 percent. A dealership like Adamson’s faces the following tradeoff. Larger inventories are beneficial not only because greater selection implies higher domestic sales, but also since the cars can be traded for vehicles at other dealerships. However, large inventories are costly since new cars are typically financed by the dealership until sold. Smaller inventories are less costly but make it more difficult to trade at the current norm of invoice or invoice minus holdback. The question then is this. Could a dealership like Adamson’s improve profitability by simultaneously reducing inventory and beginning to trade with transfers? This paper proceeds as follows. First Adamson Motors Chrysler inter dealer trade data for the past two years is analyzed. Then a model is developed to understand the structure and efficiency of the inter dealer trade market. Finally, a model is proposed to analyze the stated policies.
2 Qualitative and Empirical Insights into the Market

New cars are manufactured and sold to dealers, who then sell them to consumers. Automobile manufacturers have vast dealer networks in the United States. See Figure 1 for a snapshot of the northeastern US Chrysler dealer network. Because it takes several weeks to manufacture a new car to a particular consumer's specifications, a dealer that doesn't have the desired car in stock can look to other dealers in an attempt to get the car. If the dealer finds the car at another dealership, he can either try to buy the vehicle or attempt a dealer trade; that is, he tries to trade a vehicle on his lot for it. Dealer buys are relatively rare in normal economic times because the dealer holding the desirable car typically would rather hold it to sell himself. One period of exception was the recent US great recession. During this period dealer inventories were flush from a combination of low demand and manufacturer pressure to increase floor plan volume. In this period most dealers preferred to complete dealer buys in an attempt to reduce floor plan financing costs.

Returning to trades, if the dealer has a desirable car that the other dealer wants, the transaction is relatively easy since both parties immediately benefit. However, trades are routinely completed where only one side immediately benefits. For example, two dealers with a strong relationship will routinely make trades that are more beneficial to one party because eventually the other dealer will be repaid by performing a similar trade. They call this the two week rule, trading with a dealer today may be beneficial because on average a dealer will be repaid within two weeks. The repetition caused by these types of trades reduces the costs of completing each trade; these dealers form relationships which lubricate the negotiation process between them.

One interesting thing about the actual trade is that it never contains transfers. The two cars are literally traded for an established price (called the invoice or invoice minus holdback). Invoice is what it costs to purchase the car from the manufacturer and holdback is a fraction of the price the manufacturer pays to the dealer for selling the vehicle. For Chrysler, holdback is 3 percent. To explain this in more detail, consider the following example. Suppose dealers A and B are trading two vehicles. Dealer A’s car is worth 20,000 and Dealer B’s car is worth 10,000. Holdback is 3 percent of the invoice and is paid to the dealer from the manufacturer when the car is removed from inventory. So if the two dealers trade at invoice, then dealer A pays dealer B 10,000, receives 3 percent of 20,000 from the manufacturer and dealer B receives 3 percent of 10,000 and pays A 20,000. In this case, the dealers get paid from the manufacturer for the car they ordered. If they trade at invoice minus holdback, then dealer A pays dealer B 10,000 less holdback and Dealer B pays dealer A 20,000 less holdback. Here the dealers get paid by the manufacturer for the cars they sell. Currently in the midwestern United States, almost all trades are completed at invoice minus holdback. It changed a few years ago from invoice to invoice minus holdback when dealers were doing a lot of buys. Inventories were flush because of the recession so no one wanted a car back. In order to reduce inventories, dealers were willing to sell cars to other dealers at invoice minus holdback. This norm stuck after the inventory problem ceased.

Often times a dealer is not able to complete a dealer trade. In this case, the dealer attempts to sell the buyer a similar vehicle on the lot. Salesman are trained to do this and buyer search/inconvenience costs can be high; they might have to travel to a different city and negotiate with another dealer for the purchase of their first choice. Further, a dealer may voluntarily not disclose a potential trade to the customer in order to sell him a more profitable model on the lot. A buyer may not be willing to wait 6 weeks for the vehicle to be manufactured to their desired specification. Because of these frictions, the system as is may not be efficient. Buyers may not be matched efficiently with the global inventory of vehicles. A key result of this paper is that optimal dealer behavior reduces consumer surplus through increased buyer search/inconvenience costs.

The interesting thing about the network of dealer relationships is that it is not complete. After analyzing a data set from a particular dealer in Rochester Minnesota, I found that Adamson Motors had around 18 solid Chrysler business relationships with other dealers. These were dealers where Adamsons could locate any car on their lot and know with almost complete certainty that they could get that car. These dealers were not homogenous in their characteristics. Some were far and some were somewhat close. Some were of the same size and some were larger. Some had different product mixes in the sense that they had a different assortment of brands on the lot. What caused these relationships to form? At the outset I had several preliminary guesses as to why. They were the following.

1. Dealers very close to one another may tend not to trade, and thus wouldn’t establish relationships. Because of close proximity, each dealer would have access to the same buyers who could easily shop
around. Why share profits with close dealer by performing a dealer trade when you could advertise and compete for the buyer yourself?

2. Dealers that are far away might not trade with one another. Here I thought the answer was simple, trading with distant dealers involves high wage and fuel costs which brings down the profit margins of sale.

3. Are small dealers isolated? This could be because it would be very rare for a larger dealer to need something on a small dealer’s lot. It would be beneficial of course for the small dealer but not for any dealer of larger size.

4. Perhaps there is simply heterogeneity in dealers attitudes towards trade? If a vehicle is traded away then the dealer knows the other has it sold. But this buyer could have simply traveled and purchased the vehicle from him. Maybe some dealers simply prefer to compete directly rather than trade.

5. Business relationships could develop because of manufacturer incentives. Different regions have different incentives which transmit into different prices in different areas. Therefore, one vehicle that is desirable in one location might not be desirable in another location. The two dealers might then find it beneficial to trade these vehicles. Once the relationship is formed, the two might continue to trade in ways where individual trades might be more beneficial to one dealer over the other.

6. Medium size dealers could tend to form relationships with very large dealers. Medium size dealers know that a large dealer is more likely to have the vehicle they need, therefore they might be willing to do anything possible to establish a relationship with the large dealer.

7. Maybe dealers form these relationships for some collusionary purpose?

Although these all seem plausible, the theoretical model developed in this paper provides evidence for the development of an intermediate level of relationships dependent on inventory complementarity, distance and buyer substitution. Further, the data provided in the next section paints a slightly more complicated situation than one might expect.

2.1 Empirical Snapshot of a Node: Adamson Motors of Rochester Minnesota

The data used to motivate this paper was collected from Adamson Motors of Rochester Minnesota. The dealer trade data spans the year 2012 and includes all Chrysler trades, buys, sells and favor with Adamsons as one of the counterparties. The favor of the trade goes to the initiating side. A sample entry is Tanner Motors, of Brainerd Wisconsin, 6 trades, 2 in Tanners favor, 0 buys, 0 sells, 213 miles from Adamson Motors with Chrysler inventory as of 1/3/13 of 141. The data set contains 192 trade partners with 697 total trades. The average number of trades across partners is 3.65 with 46 percent in Adamsons favor. The average inventory size across all partners is 183 compared to Adamsons 214.

Figure 2 gives the distribution of trades. Just over 50 percent of dealers have fewer than 2 trades over the period. Approximately 20 percent of dealers fall within 2 – 4 total trades. These trades are typically completed only if both sides immediately benefit. The dealer in need must trade something immediately desirable to the other dealer. This contrasts to the trades completed at dealers with approximately 10 or more total trades. These trades are routinely completed where only one side immediately benefits. This could happen if the dealer initiating the trade doesn’t immediately have something desirable the other dealer wants. The dealer on the losing side of the transaction is regardless willing to make the trade knowing that sometime in the near future he may need to initiate the same type of trade in his favor. From Figure 2, one can see that these types of relationships are relatively rare.

Figure 3 plots percentage of dealers versus the percentage of trades. 20 percent of the total number of trades across partners account for just over 50 percent of the trades and 40 percent account for approximately 75 percent. The 18 dealers with 10 or more trades account for 31.25 percent of the total number. These numbers demonstrate the relative importance of just a few trade partners, those where the dealers complete trades where one side may only immediately benefit.

Figure 4 displays trade volumes between Adamsons and dealerships located in the Minneapolis/St. Paul area. These dealers are on average 80-90 miles from Adamsons. This area is clustered with Chrysler
dealerships and 6 of the 18 dealers with 10 or more trades are located in this area. As you can see from the figure, there is significant heterogeneity in trade volumes between Adamson’s and dealers in this area. What is the cause of this heterogeneity? This refutes the idea that distance is the only important factor in the development of trade relationships.

Figure 5 gives a snapshot of Adamsons and its trade partners with at least 7 trades. As you can see there are many nodes relatively close by with only a few trades. Some of these dealerships have very large inventories; for example, Eau Claire, La Crosse, Madison, Waterloo and Milwaukee. What is the source of this incompleteness? Further, there are close nodes with large trade volumes. Some of these dealers are large and some are small. Therefore, the formation of trade relationships is more complicated than simply distance and inventory size considerations. From the data, Adamsons trades with several small dealers. This refutes claim 3 from above. However, if the dealer is smaller then more than 50 percent of the trades are in Adamsons favor. Therefore, it appears as though the small dealer is doing everything possible to please Adamsons, the larger dealer. Claim 6 seems to be verified by the data. Adamsons trades with several large dealers, and the overall favor is in the direction of the larger dealer.

2.2 Chrysler Model Heterogeneity: Inventory Complementarity

In this section I will run you through the array of options available on a 2014 Dodge Ram pickup truck. The purpose is to display the large number of available options and resulting vehicle specifications. With such a large number of available options, estimating demand becomes very difficult. For example, suppose a dealership sold 10 Dodge Ram 1500 pickups in May of last year and there isn’t any reason to think that demand for May of this year will be significantly different. Its simply not possible to know the precise interior/exterior color combination, box size, transmission and engine type of each of the 10 models he plans to sell. Because of the small number of sales compared to model complexity, even a seasoned sales manager cannot precisely estimate demand. Therefore, trading relationships develop through trade repetition based upon inventory complimentarily. Two dealers with relatively seasoned sales managers find it difficult to precisely estimate demand. If they order similarly but in complementary ways (which arise due to randomness) then its possible that both could benefit from trade.

So lets go ahead and try to build a 2014 Dodge Ram pickup truck. I’ll follow the algorithm given to customers visiting ramtrucks.com at http://www.ramtrucks.com/hostc/bmo/CUT201413/models.do?. First, I’m asked to select a model. There are nine total; Tradesman, Express, HFE, SLT,Lone Star, Big Horn, Outdoorsman, Sport, Laramie, Laramie Longhorn and Laramie Limited. Supposed that I’m interested in the base package, the Tradesman. Next I’m asked to select 4x2 or 4x4 and box size. Since I’m interested in Minnesota sales I’ll look at the 4x4 because two wheel drive vehicles are horrible in winter conditions. Given the 4x4 selection, I have a choice of 5 box/cab sizes; Regular 6’4”, Regular 8’, Quad 6’4”, Crew 5’7”, Crew
6'4". Regular indicates two doors while quad and crew indicate four doors. The difference between the crew and quad is that the crew has more seating space and smaller bed. Given the choice of box/bed size, its time to choose interior/exterior color. There are 12 exterior colors available and only 1 interior color. Other models have more than one interior, for example the SLT has two. So far I’ve selected a Dodge Ram 1500 pickup, Tradesman, Quad 6'4" with black clear coat exterior paint and black/diesel gray interior. Up to this point the number of combinations on just the Tradesman 4x4 are 60; simply from the box/cab size and exterior color choice.

In the next set of choices there are four categories; Interior, Exterior, Power Train and Packages. Start with the interior. Several of these options can be added after the vehicle is on the lot so I need to be a little careful. I’m given the option of media screen size; either 3 or 5 inch. Its too costly to rip out the media system and replace it with a different screen so this adds to model complexity. Similarly, I’m given the option of whether or not to have a rear back up camera. Next move to the power train. I’m given 3 different engine and 3 transmission types; 3.6-Liter V6 24-Valve WT Engine, 5.7-Liter V* HEMI MDS VVT Engine, 3.0-Liter V6 Turbo Diesel, 8-Speed TorqueFlite Auto Trans 845RE, 6-Speed Automatic Trans and the 8-Speed TorqueFlite Auto Trans 8HP70. Next I can choose the rear/axle ratio; 3.21, 3.92 and 3.55. A lower rear/axel ratio implies lower RPM and greater fuel economy while a higher ratio improves towing, carrying loads and towing. However, not all engine/transmission and rear/axel ratio combinations are possible. Even if I only allow for 3 combinations here (one engine with one transmission with one rear/axel ratio), this increases the total model complexity to 180 (without the screen and camera). With the screen and camera combinations I arrive at 720. Recall that this calculation is just for the one model, the Tradesman. If instead I were interested in the Laramie Longhorn, perhaps for the Premium Bifunctional Halogen Projector Headlamps, Premium LED Taillamps, Laramie Longhorn Unique Premium 7-Inch Instrument Cluster, Real Wood Accent Interior, Uconnect 8.4AN (RA4) System with 8.4-Inch Touchscreen or any other of its Tradesman different features that 720 number would increase dramatically.

By this point I think its clear. Even a seasoned hiring manager has significant difficulty precisely matching inventory to demand. Even if he knows the Tradesman is popular in his area, if he only plans on selling 10 how does he choose which of the 720 (low estimate) to stock? This seems to be the main reason for the development of relationships between specific dealers. Inventory complementarity between dealerships is crucially important. I suspect that this occurs naturally because of model complexity. If two dealers know that the Tradesman is popular in their area, they order accordingly but are going to be off due to the sheer complexity of the vehicle. Because of this randomness and low sales relative to model complexity, dealers might benefit through natural inventory complementarity. For example, from the data set Adamsons has
large trade volumes with a distant dealer in eastern South Dakota. These trades are almost always truck for truck. Both dealers sell a lot of trucks so benefit from trade. This explains one reason why specific relationships form, but what about how many to form? To anticipate the results to come, the ability of the dealer to shift a buyer off to a similar vehicle is of significant importance. Because each of the 720 only varies in one option, they are not that different. Therefore, not only does the ranking preference of a buyer matter but also his tendency to be thrown off a particular type due to the increased cost of obtaining his first choice; say if the local dealer doesn’t have that precise model or does not/ can not trade for it. Depending on this substitution then, a dealer might be able to increase profit by voluntarily not completing a trade in order to sell the buyer off on his more profitable model in stock.

3 Model with Homogeneous Parameters and Inventory of Size One

This model will be used to analyze the optimal inter dealer trade network structure. A simple example with two dealers and two models is examined to explain the potential benefit of dealer trades. Then the \( m \) dealer case is analyzed. Of critical importance is buyer substitution, distance and a preference toward an intermediate level of links. The latter explains the incompleteness seen in Adamsons interdealer trade data. Further, this incompleteness increases dealer profits but decreases consumer surplus through search/inconvenience costs.

3.1 Model Definitions and Parameters

Let \( S_i = (s_{i1}, ..., s_{im}) \) be the inventory of Chrysler dealer \( i \) where \( m \) is the number of models and \( s_{ij} \) is the number of model \( j \) that dealer \( i \) has in stock. Assume that no two dealers have the same model in stock and global inventory of each model is nonzero. In particular, let \( s_{ii} = 1 \) and \( s_{ij} = 0 \) for all \( j \neq i \). Each dealer has only one unit of one model in stock. Each period one consumer randomly shows up at either the local Chrysler or alternative make dealership, for example Ford, at location \( i \). Let the probability the consumer shows up at location \( i \) be \( \frac{1}{m} \). Once at \( i \), the buyer randomly determines to head to either the Chrysler or alternative make dealership. Let \( \sigma_{IC} \) be the conditional probability that the buyer shows up at the Chrysler dealership at \( i \) and \( \sigma_{IAM} \) for the alternative make. If the buyer shows up at the Chrysler dealership, he randomly demands some model \( j \). Let \( \pi_{ij} \) be the probabilistic demand for model \( j \) at Chrysler dealership \( i \).
Figure 5: Adamson Motors: Chrysler Trade Network - Blue: 7-9 Trades - Black 10+ Trades

These probabilities will be formalized in a later section. The following notation will be used

\[ p = \text{price of each model} \]

\[ I = \text{invoice of model } i \]

\[ c_{ik} = \text{transport cost between locations } i \text{ and } k \]

\[ \beta = \text{dealer discount factor} \]

\[ \gamma = \text{fraction of transport cost paid by the buyer} \]

\[ \mu_1 = \text{rate at which the buyer at } i \text{ substitutes between the Chrysler and alternative make dealerships} \]

\[ \mu_2 = \text{degree of substitution between Chrysler models at dealership } i \]

\[ \delta = \text{consumer search/inconvenience cost of negotiating with a distant dealer} \]

\[ \lambda = \text{fraction of search/inconvenience cost the local dealer can recoup when trading for a model not in stock} \]

The model will proceed in three periods. In the first, dealerships will choose their trade partners. These relationships will be represented by a matrix \( g = (g_{ij}) \). The rules of network formation will be Jackson-Wolinsky. That is, both dealerships need to agree to form a link and only one is needed to destroy a link. The second and third periods are a model of auto sales. In the second period, the consumer is shown a menu of costs associated with obtaining each model. This menu depends upon the network of relationships...
developed in the first period, as explained below. The consumer chooses one model according to the discrete choice model of demand. In the third period dealers sell in autarky, again explained below.  

3.2 Discrete Choice Model of Demand

The conditional nested logit model of demand will be used to determine the probability of a consumer showing up at dealership \( i \) demanding model \( j \). This needs to be random because of model complexity; a dealer doesn’t know with complete certainty the precise model a consumer desires. The nested model is used because the substitution from the Chrysler dealership to a nearby alternative make competitor is critically important. One nest will be the available inventory at the Chrysler dealership while the other nest will signify one vehicle at its immediate alternative make neighbor. This two nest specification allows for flexibility in analyzing the substitution effect in network formation.

A consumer showing up at dealership \( i \) can travel to purchase any available model. In doing so, they pay the fraction \( \gamma \) of the transportation cost \( c_{ik} \) when purchasing a vehicle from dealership \( k \). When the consumer searches out a vehicle elsewhere, they incur the search/inconvenience cost \( \delta \) with \( \delta > 0 \). The intuition for this additional cost is that the consumer has to search out and negotiate with another dealer which can be costly.

The consumer at dealership \( i \) knows the full inventory of its local dealer and the costs associated with trading for a vehicle not currently in stock. Let \( U_{ij} \) be buyer \( i \)'s utility of consuming model \( j \) and \( v_i \)'s value of the \( j^{th} \) model. If the local dealership has model \( j \) in stock, then the consumer's utility is

\[
U_{ij} = v - p + \mu_2 \epsilon_{ij}
\]

The parameter \( \mu_2 \) controls buyer substitution across models at the Chrysler dealership \( i \). As \( \mu_2 \) goes to zero, the products become perfect substitutes and the buyer purchases the vehicle with the highest deterministic utility. The higher \( \mu_2 \), the less substitutable are the models at Chrysler dealer \( i \).

If the local dealer does not have the model in stock and is unable to trade for it, the buyer has the option to seek out the model elsewhere. This utility is

\[
U_{ij} = v - p - \gamma c_{ik} - \delta + \mu_2 \epsilon_{ij}
\]

Finally, if dealer \( i \) can trade for the vehicle then the consumer's utility is

\[
U_{ij} = v - p - \gamma c_{ik} - \lambda \delta + \mu_2 \epsilon_{ij}
\]

If dealer \( i \) doesn’t have complete information about the level of \( \delta \), he can extract at most \( \delta \) and perhaps through negotiation can only charge \( \lambda \delta \) over the distant dealer with \( 0 \leq \lambda \leq 1 \). If the local dealer tries to charge more than \( \delta \) he won’t make the sale. If \( \lambda < 1 \) then the buyer receives additional value if dealer \( i \) trades for model \( j \) over traveling to purchase it. Therefore, if dealer \( i \) decides not to trade, the effective cost of obtaining model \( j \) increases and the buyer substitutes toward the other models available. The motivation for \( \lambda < 1 \) is simple, the buyer knows \( \delta \) and the seller does not. This gives the buyer negotiating power making it difficult for the seller to extract the entire rent.

Let \( V_{ij} \) be the non random part of \( U_{ij} \). If \( (\epsilon_{ij})_{j} \) are i.i.d type double exponential across \( j \), then the probability that consumer \( i \) shows up at the Chrysler dealership demanding model \( j \) is

\[
\pi_{ij} = \exp\left\{ \frac{V_{ij}}{\mu_2} \right\} \sum_j \exp\left\{ \frac{V_{ij}}{\mu_2} \right\}
\]

Given the two nests at each location, the attractiveness of the Chrysler nest is given by the expected value

\[
ECV_i = E(\max_j \{U_{ij}\})
\]

\footnote{The assumption about the structure of sales in period three is made because the trade action in period two is enough to capture the incentives for link formation. Sales in period three is important, but autarkic payoffs are enough to capture this relevance.}
Given the assumption on errors, $ECV_i$ has closed form

$$ECV_i = \mu_2 \ln \left( \sum_{j=1}^{m} \exp \left( \frac{V_{ij}}{\mu_2} \right) \right)$$

Let $AM_i$ be the alternative make value at $i$. Then the probability that a buyer at $i$ chooses the Chrysler nest is given by

$$\sigma_{iC} = \frac{\exp \left( \frac{ECV_i}{\mu_1} \right)}{\exp \left( \frac{ECV_i}{\mu_1} \right) + \exp \left( \frac{AM_i}{\mu_1} \right)}$$

where $\mu_1$ is the substitutability parameter between nests at location $i$. It has the same interpretation as $\mu_2$ given above. Finally, the consumer surplus at location $i$ is given by

$$CS_i = \mu_1 \ln \left[ \exp \left( \frac{ECV_i}{\mu_1} \right) + \exp \left( \frac{AM_i}{\mu_1} \right) \right]$$

### 3.3 Period Three Profits: Autarky

In this period, dealers sell in autarky; that is, the buyer doesn’t travel to purchase any model and no trading takes place. Given the model set up, either the dealer has inventory of size one or zero in period three. If the dealer makes a sale in the second period, then his profit in period three is zero. If the dealer $i$ doesn’t make a sale in the second period then his profit is

$$p - (1 - h)I$$

### 3.4 Complete Period Two and Period Three Profits

In period two dealers can use their trade partners to trade for models not in stock. Let $CP^i_{kj}$ be the continuation profit of dealer $i$ if the buyer shows up at location $k$ looking for model $j$. First suppose that $k \neq i$. If $g_{ik} = 1$ then

$$CP^i_{ki} = \beta (p - (1 - h)I)$$

since dealer $k$ is able to trade dealer $i$ for model $i$. If $g_{ik} = 0$, then

$$CP^i_{ki} = p - (1 - h)I - (1 - \gamma)c_{ik}$$

since the buyer at $k$ travels to dealer $i$ to purchase the vehicle. If $k \neq i$ and $j \neq i$ then

$$CP^i_{kj} = \beta (p - (1 - h)I)$$

If the buyer shows up at location $i$, then

$$CP^i_{ii} = p - (1 - h)I$$

$$CP^i_{ij} = p - I - (1 - \gamma)c_{ij} + \lambda \delta$$

if $g_{ij} = 1$ since dealer $i$ can trade $j$ for model $j$ and

$$CP^i_{ij} = \beta (p - (1 - h)I)$$

if $g_{ij} = 0$ since dealer $i$ cannot trade dealer $j$ for model $j$. With this notation, the expected profit of dealer $i$ given network $g$ is

$$\text{Profit}_i(g) = \frac{1}{m} \left( \sum_{k} \sigma_{Ck} \pi_{kj} CP^i_{kj} + \beta \sum_{k} (1 - \sigma_{Ck})(p - (1 - h)I) \right)$$
3.5 Pairwise Stable Equilibrium

Definition Network $g = \{g_{ij}\}$ for all $i, j = 1, \ldots, m$ is pairwise stable if

- there does not exist $i$ such that $g_{ik} = 1$ and $\text{Profit}_i(g) < \text{Profit}_i(g')$ where $g'$ is equal to $g$ except for $g'_{ik} = 0$.
- there does not exist $i$ and $k$ with $g_{ik} = 0$ such that $\text{Profit}_i(g) \leq \text{Profit}_i(g')$ and $\text{Profit}_k(g) \leq \text{Profit}_k(g')$ with strict inequality for at least one where $g'$ is equal to $g$ except that $g'_{ik} = 1$.

3.6 Example: Two Dealerships and Two Models

At this stage it will be helpful to analyze a simplified case of the model to clarify how it works and gain some intuition about why relationships form. Suppose that there are two dealerships and two models. Let $S_1 = (1, 0)$ and $S_2 = (0, 1)$. That is, dealer one has model one in stock and dealer two model two. Recall that a relationship (or link) between two dealers is an obligation to trade. For example, suppose that the consumer shows up at dealer one demanding Chrysler model two. Dealer one only holds model one and thus doesn’t have model two in stock but can trade model one for it from dealer two. Since the link exists, dealer one has model one in stock so sells it to the consumer and reduces his inventory to 0 units. In

\[
\text{Profit}_1 = v - p + \mu_2 \epsilon_11
\]

\[
U_{11} = v - p + \mu_2 \epsilon_11
\]
\[
U_{12} = v - p - \gamma c_{12} - \lambda \delta + \mu_2 \epsilon_{12}
\]
\[
U_{21} = v - p - \gamma c_{12} - \lambda \delta + \mu_2 \epsilon_{21}
\]
\[
U_{22} = v - p + \mu_2 \epsilon_{22}
\]

With the link, continuation payoffs of dealer one are

\[
CP^1_{11} = p - (1 - h)I
\]
\[
CP^1_{12} = p - (1 - h)I - (1 - \gamma)c_{12} + \lambda \delta
\]
\[
CP^2_{21} = \beta(p - (1 - h)I)
\]
\[
CP^2_{22} = \beta(p - (1 - h)I)
\]

so that the expected profit of dealer one is

\[
\text{Profit}_1 = \frac{1}{2} \left[ \sigma c_1 (\pi_{11} CP^1_{11} + \pi_{12} CP^1_{12}) + \sigma c_2 (\pi_{21} CP^1_{21} + \pi_{22} CP^1_{22}) + \beta (2 - \sigma c_1 - \sigma c_2)(p - (1 - h)I) \right]
\]

$CP^1_{11}$ is the continuation payoff when the consumer shows up at dealer one demanding model 1. Dealer one has model 1 in stock so sells it to the consumer and reduces his inventory to 0 units. In $CP^1_{12}$, dealer one doesn’t have model 2 in stock but can trade model 1 for it from dealer 2. Since the link exists, dealer one conducts this trade and sells model 2 to the buyer thus reducing his inventory to 0. $CP^2_{21}$ is the continuation payoff when the consumer shows up at dealer 2 demanding model 1. Dealer 2 trades dealer 1 for model 1. Dealer one then sells model 2 in autarky in the third period. Finally, $CP^2_{22}$ is the continuation payoff when the consumer shows up at dealer 2 demanding model 2. Dealer 2 has this model in stock so no trade is made. Dealer one then sells models 1 in period three. Profits for dealer two with the link are computed in an analogous way so that

\[
\text{Profit}_1 = \frac{1}{2} \left[ \sigma c_1 (\pi_{11} CP^2_{11} + \pi_{12} CP^2_{12}) + \sigma c_2 (\pi_{21} CP^2_{21} + \pi_{22} CP^2_{22}) + \beta (2 - \sigma c_1 - \sigma c_2)(p - (1 - h)I) \right]
\]
In the absence of a link the buyer must travel to purchase the vehicle not in local stock from another dealer. Let the bold notation indicate values without the link. With this, consumer utilities are

\[
U_{11} = v - p + \mu_2 \epsilon_{11}
\]
\[
U_{12} = v - p - \gamma c_{12} - \delta + \mu_2 \epsilon_{12}
\]
\[
U_{21} = v - p - \gamma c_{12} - \delta + \mu_2 \epsilon_{21}
\]
\[
U_{22} = v - p + \mu_2 \epsilon_{22}
\]

where \(\delta\) has been added to represent the additional cost of searching and negotiating for the vehicle elsewhere.

The continuation payoffs of dealer one in absence of the link are

\[
CP_{11} = p - (1 - h)I
\]
\[
CP_{12} = \beta(p - (1 - h)I)
\]
\[
CP_{21} = p - (1 - h)I - (1 - \gamma)c_{12}
\]
\[
CP_{22} = \beta(p - (1 - h)I)
\]

\(CP_{11}\) is the continuation payoff when the consumer shows up at dealer one. \(CP_{12} = CP_{22}\) since only dealer 2 makes a sale when the buyer demands model 2 and thus dealer one is left with his model. In \(CP_{21}\), the consumer shows up at location 2 demanding model one. Dealer 2 doesn’t have this model in stock so the buyer travels to dealer 1 to purchase it. Dealer 1 then reduces his inventory to zero. With this the expected profit of dealer \(i\) in the absence of the link is

\[
Profit_i = \frac{1}{2} \left[ \sigma_{C1}(\pi_{11} CP_{11}^i + \pi_{12} CP_{12}^i) + \sigma_{C2}(\pi_{21} CP_{21}^i + \pi_{22} CP_{22}^i) + (2 - \sigma_{C1} - \sigma_{C2})(p - (1 - h)I) \right]
\]

For the network with the link to be pairwise stable, it must be true that both

\[
Profit_1 \geq Profit_1
\]
\[
Profit_2 \geq Profit_2
\]

Before leaving this section, let's try to get a feel for why the link might be pairwise stable; that is, why both dealers might benefit from trade. What makes this difficult is that we not only need to analyze how each continuation payoff changes, but also how the different utilities transfer into different probabilities of sale.

Suppose dealer one is contemplating severing the link. First consider the effects when the buyer shows up at location two. If dealer one severs the link he gets to sell model one to the buyer that shows up at dealer 2 demanding model one. This generates an immediate profit of

\[
\sigma_{C2}\pi_{21}(p - (1 - h)I - (1 - \gamma)c_{12})
\]

However, in the same state by severing the link he loses the benefit of holding model one in period 2. That is, he loses

\[
\sigma_{C2}\pi_{21}\beta(p - (1 - h)I)
\]

We need to be careful when comparing this tradeoff directly though because the probabilities that the buyer showing up at dealer two demanding model one are different in both cases. If \(\lambda < 1\), then \(\pi_{21} < \pi_{21}\). If \(\lambda = 1\) then there is no substitution at all. Smaller values of delta or larger values of lambda push these two probabilities closer together. However, the change in probability is further complicated since the increase in cost of purchasing model 1 at location 2 affects the nest choice. In other words, the buyer is more likely to purchase the car in the alternative make nest if dealer one severs the link. How about the effects in state 22, when the buyer shows up at dealer two demanding model two? Dealer one doesn’t make a sale in either scenario so we only need to compare the change in probability. Because \(\pi_{21}\) drops when dealer one severs the link, \(\pi_{22} > \pi_{22}\). However, \(\sigma_{C2} < \sigma_{C2}\) so its unclear whether the dealer is better or worse off in this state. If substitution to the alternate make nest is high, relative to substitution to the other Chrysler model then the
effect of severing the link at 22 will be negative. Finally, how about in the alternative make state in location 2? Since without the link its more likely the buyer will purchase the alternative make, $1 - \sigma C_2 < 1 - \sigma C_1$. This implies that the benefit of period two sales in autarky at this state increases without the link.

Next consider the effects if the buyer shows up at location one. In state 12, without the link dealer one no longer gets to make the sale. He simply carries model one into period two. This raises the costs of obtaining model two for buyer one so that $\pi_{12} < \pi_{11}$. Further, $\sigma C_1 < \sigma C_2$ because the alternative make nest is more attractive without the link. Therefore, he loses a sale in 12 but earns profit by holding model 1 until period 2. How about in state 11? The payout is the same in both cases but the probabilities of sale are different. Because the cost of obtaining model two increases without the link, $\pi_{11} > \pi_{12}$. However, $\sigma C_1 < \sigma C_2$ so I can’t say unambiguously that expected profit at 11 increases. The intuition is that because the cost of obtaining model two increased, the dealer can switch the buyer more easily off to the model he has in stock. However, the buyer also has an increased interest in the alternative make model so that the overall effect at the local Chrysler dealership cannot be signed. Finally, how about in the alternate make state at location one? Expected profit in this state increases without the link since the probability that the buyer purchases the alternate make increases.

As you can see, weighing the benefits and costs at each state of deleting the link is a complicated matter. It depends crucially on the tendency to substitute from one Chrysler model to the other and also to substitute away to the alternative make. However, each dealer is unambiguously better off in period 2 without the link.

To see this, the expected profit of period two sales with the link is

$$[\sigma C_2(\pi_{21} + \pi_{22}) + (1 - \sigma C_2) + (1 - \sigma C_1)] \beta(p - (1 - h)I)$$

since profit in autarky is the same regardless of the model in stock. Without the link, the expected profit of period two sales is

$$[\sigma C_1\pi_{12} + \sigma C_2\pi_{22} + (1 - \sigma C_2) + (1 - \sigma C_1)] \beta(p - (1 - h)I)$$

Since $\pi_{j1} + \pi_{j2} = 1$, $\pi_{j1} + \pi_{j2} = 1$, $\sigma C_1 = \sigma C_2$ and $\sigma C_1 = \sigma C_2$, the above two expressions simplify to

$$(2 - \sigma C_2)\beta(p - (1 - h)I)$$

From this, the difference is $(\sigma C - \sigma C)\beta(p - (1 - h)I)$ where its clear that period 2 sales are higher without the link.

Therefore, dealer one is better off severing the link if the benefit of selling a vehicle in 21, possible increased benefit at 11, and increase in profit from period 3 outweigh the cost of not selling a vehicle at 12. The benefit of selling a vehicle at 21 is less than that of selling at 12 because of asymmetric information about search/inconvenience costs (different probabilities of sale); however, because of the increased buyer cost at 12 dealer one is more able to switch this buyer to model one. This could increase profit at 11 depending on the tendency to switch to the alternative make option. This leads to the following theorem.

**Theorem 3.1.** The decision to sever the link when $m = 2$ depends on the sign of

$$(\sigma C - \sigma C)\beta(p - (1 - h)I) + (\sigma C \pi_{11} - \sigma C \pi_{11})(p - (1 - h)I) - \gamma C_{11}(p - (1 - h)I - \gamma C_{12}) - \sigma C \pi_{12}(p - (1 - h)I - \gamma C_{12} + \lambda \delta)$$

That is, the optimality of dealer trades in the two dealer case depends on the sign of the above expression. A negative sign implies that dealer trades are optimal, while a positive sign implies that the empty network is pairwise stable.

The logic for the optimality of dealer trades in this simple example is straightforward. If the dealer raises the cost of obtaining the distant model by not trading and the buyer simply heads down the street to purchase the Ford then its not optimal to sever the link. If instead, by raising the cost the dealer is able to sell the buyer on his more profitable model in stock, then its optimal to sever the link. To gain some intuition for the comparative statics of the above expression, consider the following simulation. Let $v = 250$, $p = 150$, $I = 10$, $\lambda = 0$, $\gamma = .5$, $c = 100$, $\beta = .5$ and $h = 0$. The following will analyze the role of the substitution parameters; $\mu_1$, $\mu_2$ and $\delta$ in the decision to form the link. First, let $\mu_1 = 50$, $\mu_2 = 20$ and $\delta = 10$. At this parametrization, $\mu_1$ is high enough, relative to $\mu_2$ so that severing the link increases profit.
The substitution effect toward the more profitable Chrysler model dominates the substitution effect toward the Ford. As seen in figure 6, increasing δ here increases profit because of this domination.

Next, let δ = 10 and increase μ₂ from 0 to 100. As seen in 7, increasing μ₂ initially raises the change in profit but eventually decreases it. For μ₂ small, the Chrysler models are so substitutable that there is no positive substitution effect of severing the link. Therefore, increasing μ₂ eventually makes the Chrysler models heterogeneous enough so that severing the link increases profit. Eventually, the products become so heterogeneous that severing the link doesn’t change the probabilities of Chrysler sale at all. This of course eliminates the positive Chrysler substitution effect of severing the link. Therefore, at some point increasing μ₂ further makes it optimal to trade since the positive Chrysler substitution effect gets dominated by the substitution effect towards the Ford.

Finally, set μ = 50 and μ₂ = 80 and vary δ. As seen in figure 8, because μ₂ is high enough relative to μ₁, the substitution to the outside option dominates the substitution across the Chrysler models. Therefore, increasing δ here exaggerates this tendency and decreases the change in profit.

3.7 Optimal Access to Global Inventory

This section explains the incompleteness of the inter dealer trade network as being a property of each dealer’s preferred access to global inventory. Too many trade partners implies too many trades; a dealer can raise profits by severing a link, effectively raising the buyers cost of obtaining that model so he can more likely be sold on other models in stock. Models on the lot typically generate higher profit margins since they don’t incur transportation costs. Too few trade partners result in an overwhelming amount of lost sales by triggering the same increase in buyer costs. Rather than increasing profit, raising costs again by restricting trade results in too many lost sales to other Chrysler dealerships and to the alternative make. This preference toward an intermediate level of trade partners decreases consumer surplus from that obtained with the complete network; buyer costs to obtain some models rises through search/inconvenience costs. The incompleteness of the inter dealer trade network is seen explicitly in Figure 5 and was revealed through discussions with sales managers in charge of dealer trades.

To reveal the intuition for this result in the clearest possible way, consider symmetric equilibria and assume that both β = 0 and λ = 0. In this environment, the probabilities of Chrysler sale σᵢ and probabilities of Chrysler model sale given Chrysler sales πᵢⱼ are completely determined by the number of trade partners. Each will be given an argument (·) that indicates the number of trade partners. Let Mˣ be the current candidate for the symmetric equilibrium number of trade partners. Suppose that the buyer shows up at
location \(i\) demanding a Chrysler. Dealer \(i\) has model \(i\) in stock so that

\[
\pi_i(M^x) = \frac{\exp\left\{\frac{v - p}{\mu_2}\right\}}{\exp\left\{\frac{v - p}{\mu_2}\right\} + M^x\exp\left\{\frac{v - p - \gamma c}{\mu_2}\right\} + (M - M^x - 1)\exp\left\{\frac{v - p - \gamma c - \delta}{\mu_2}\right\}}
\]

If dealer \(i\) doesn’t have model \(j\) in stock, but can trade for it then

\[
\pi_{ij}(M^x) = \frac{\exp\left\{\frac{v - p - \gamma c}{\mu_2}\right\}}{\exp\left\{\frac{v - p}{\mu_2}\right\} + M^x\exp\left\{\frac{v - p - \gamma c}{\mu_2}\right\} + (M - M^x - 1)\exp\left\{\frac{v - p - \gamma c - \delta}{\mu_2}\right\}}
\]

Let \(\pi_h\) indicate this probability. If dealer \(i\) doesn’t have model \(j\) in stock and can’t trade for it then

\[
\pi_j(M^x) = \frac{\exp\left\{\frac{v - p - \gamma c - \delta}{\mu_2}\right\}}{\exp\left\{\frac{v - p}{\mu_2}\right\} + M^x\exp\left\{\frac{v - p - \gamma c}{\mu_2}\right\} + (M - M^x - 1)\exp\left\{\frac{v - p - \gamma c - \delta}{\mu_2}\right\}}
\]

Let \(\pi_a\) indicate this probability. The expected value of the chrysler nest is given by

\[
ECV_i(M^x) = \mu_2 \ln \left( \exp\left\{\frac{v - p}{\mu_2}\right\} + M^x\exp\left\{\frac{v - p - \gamma c}{\mu_2}\right\} + (M - M^x - 1)\exp\left\{\frac{v - p - \gamma c - \delta}{\mu_2}\right\} \right)
\]

Finally, given the expected value of the Chrysler next at location \(i\), the probability that the buyer chooses a Chrysler at \(i\) is given by

\[
\sigma_C(M^x) = \frac{\exp\{ECV_i(M^x)\}}{\exp\{ECV_i(M^x)\} + \exp\{AM_i\}}
\]

Let \(AM_i = \sum_{j=0}^{M-1} ECV_i(j)\). Since ECV is sensitive to the value of \(\mu_2\), this simply anchors the alternative make value.

What follows analyzes the optimal number of trade partners in the symmetric equilibrium; that is, where all dealers have the same number of trade partners. By assumption, \(i\) doesn’t care with whom he trades, only the total number of options available through trade. The focus here will be the intuition for the optimality of intermediate access to global inventory. Dealer \(i\)’s expected profit with access to \(M^x\) trade partners is

\[
\sigma_C(M^x) \left[ \pi_i(M^x)(p - I) + M^x\pi_h(M^x)(p - I - \gamma c) \right] + (M - M^x - 1)\sigma_C(M^x)\pi_a(M^x)(p - I - \gamma c)
\]
Dealer 1 is able to sell model $i$ from his lot, and trade for $M^x$ models from his partners. $M - M^x - 1$ buyers travel to his location to purchase model $i$. Suppose that $M^x = M - 1$ and that dealer $i$ is contemplating severing a link. If he does, he can no longer sell one model to his domestic buyer but he might be able to sell him the model he has on the lot, at a higher profit margin. Since he has access to many models, if he can’t sell him on the model he has in stock he still might be able to sell him another model he must trade for. Worst case scenario the buyer travels or buys the alternative make but this is unlikely given the dealers wide access to global Chrysler inventory. Further, dealer 1 can now sell model 1 to the buyer that travels from his old trade partner. If dealer $i$ severs the link with dealer $j$, then his expected profit is

$$
s_C(M - 2)\left[\pi_i(M - 2)(p - I) + (M - 2)\pi_h(M - 2)(p - I - \gamma c)\right] + \sigma_C(M - 2)\pi_a(M - 2)(p - I - \gamma c)
$$

If ECV doesn’t change much, then $\sigma_C$ doesn’t change much. In words, the buyer still likes the Chrysler nest just as much as before. This is likely since the buyer at $i$ still has wide access to global Chrysler inventory. Since dealer $i$ can no longer trade for model $j$, the probability of his buyer choosing this model reduces to $\pi_a(M - 2)$. This reduction in probability is transferred over to the higher margin model and remaining $M - 2$ models the dealer can trade for. In words, the dealer can more likely sell the buyer off on his higher margin model. Given his wide access to global Chrysler inventory, even if he can’t sell him on the model he owns, its still likely he can trade for the model the buyer wants. Further, since the dealer no longer trades with one dealership, he is able to sell model $i$ to the buyer traveling from location $j$. The positive substitution effect from deleting links decreases after some threshold since its more likely the buyer will buy the alternative make or travel to purchase a Chrysler from another dealership. Deleting a link with $M^x < M - 1$ is a slightly more complicated expression given by

$$
\sigma_C(M^x - 1)\left[\pi_i(M^x - 1)(p - I) + (M^x - 1)\pi_h(M^x - 1)(p - I - \gamma c)\right] + (M - M^x - 1)\sigma_C(M^x)\pi_a(M^x)(p - I - \gamma c)
+ \sigma_C(M^x - 1)\pi_a(M^x - 1)(p - I - \gamma c)
$$

Next, consider a deviation by adding a link. Start with the empty network, so that $M^x = 0$. The expected profit of dealer $i$ is

$$
\sigma_C(0)\pi_i(0)(p - I) + (M - 1)\sigma_C(0)\pi_a(0)(p - I - \gamma c)
$$

If dealer $i$ adds a link with dealer $j$, then his expected profit is

$$
\sigma_C(1)(\pi_i(1)(p - I) + \pi_h(1)(p - I - \gamma c)) + (M - 2)\sigma_C(0)\pi_a(1)(p - I - \gamma c)
$$
That is, he is able to trade for one model for his domestic buyer but loses one sale from the buyer at \( j \). The intuition for why this should be beneficial is that lowering the cost of the buyer obtaining one of the models keeps that buyer from substituting away to the alternative make and other Chrysler dealerships, all of which the dealer gets no profit. The dealer loses some profit from the substitution away from the higher margin model, but this isn’t large since the probability is switched from all the other models he can’t obtain as well.

To gain further intuition for this result, consider the parametrization with \( p = 50, v = 100, I = 10, M = 10, \gamma = .5, c = 20, \mu_1 = 1, \mu_2 = 100 \) and \( \delta = 100 \). Price, valuation, invoice, gamma and transportation cost are held cost to understand the effect of \( \mu_1 \) and \( \mu_2 \) on the equilibrium symmetric network. Recall that higher values of \( \mu_2 \) indicate lower levels of substitution but also increase the expected Chrysler value.

To anchor the value of \( AM \) then, its set to be the average of \( ECV \) over \( M^x \). Without doing this it would be difficult to differentiate the direct effect of changing the substitutability across Chrysler models from the effect it also has on the relationship between \( ECV \) and \( AM \).

### Table 1: \( ECV \) across \( M^x \) with \( \mu_2 = 100 \)

<table>
<thead>
<tr>
<th>( M^x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ECV )</td>
<td>228.16</td>
<td>233.98</td>
<td>239.48</td>
<td>244.7</td>
<td>249.65</td>
<td>254.37</td>
<td>258.88</td>
<td>263.2</td>
<td>267.33</td>
<td>271.31</td>
</tr>
</tbody>
</table>

### Table 2: \( \sigma_C \) across \( M^x \) with \( \mu_2 = 100 \)

<table>
<thead>
<tr>
<th>( M^x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_C )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>.0016</td>
<td>.1894</td>
<td>.9633</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

### Table 3: \( ECV \) across \( M^x \) with \( \mu_2 = 1 \)

<table>
<thead>
<tr>
<th>( M^x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ECV )</td>
<td>50</td>
<td>50</td>
<td>50.0001</td>
<td>50.0001</td>
<td>50.0002</td>
<td>50.0002</td>
<td>50.0003</td>
<td>50.0003</td>
<td>50.0003</td>
<td>50.0004</td>
</tr>
</tbody>
</table>

Figures 1 and 3 show the effect of \( \mu_2 \) on \( ECV \). Higher values of \( \mu_2 \) indicate lower levels of substitutability and increase the spread in \( ECV \). Since \( AM \) is anchored at the midpoint of \( ECV \), higher values of \( \mu_2 \) create a large jump in \( \sigma_C \). For example, from Table 2 \( \sigma_C \) jumps down from 1 to .1894 from the 6th to 5th link. The probability that the buyer wants a Chrysler below the 5th link is zero. This represents the idea that after some threshold the buyer becomes so dissatisfied with the Chrysler dealers access to global inventory that he doesn’t even consider a purchase. Playing around with the substitutability across Chrysler models changes this threshold point of dissatisfaction. As \( \mu_2 \) decreases this threshold property evaporates and the buyer always wants to purchase a Chrysler with 50 percent probability. This is shown in Table 4. Intuitively, as \( \mu_2 \) decreases, the Chrysler models become perfect substitutes so that raising the cost of obtaining one of the models doesn’t change the overall Chrysler dealership value since the buyer simply substitutes to another model. Higher levels of \( \mu_2 \) indicate lower levels of substitutability so that raising the costs of obtaining one model decreases the Chrysler dealership value substantially. Therefore, one way to produce an equilibrium with intermediate access is to increase \( \mu_2 \) so that at some point deleting a link decreases profits substantially because of buyer dissatisfaction.

Figure 9 displays the symmetric equilibrium number of links for \( \mu_2 = 100 \) for \( \delta \) between 0 and 500. The tendency is clear, increasing delta decreases the optimal number of links up until the point where the buyer becomes so dissatisfied that he buyers the alternative make with probability one. The intuition for this result is clear. Higher values of delta indicate greater levels of substitution to the other Chrysler models. This can be seen in Figure 10 for the Chrysler model in stock. The \( x \) axis is the change between \( x \) and \( x - 1 \) links. Because the model in stock has a higher expected value than any of the other models that must be traded or traveled for, substitution toward the more profitable model is relatively larger than substitution towards the other models. Figure 11 shows this property. The chart measures the difference in substitution toward the in stock model to the difference in substitution towards models the buyer must travel for. That is,

\[
[\pi_{ii}(M^x - 1) - \pi_{ii}(M^x)] - [\pi_{a}(M^x - 1) - \pi_{a}(M^x)]
\]
Table 4: $\sigma_C$ across $M^z$ with $\mu_2 = 1$

<table>
<thead>
<tr>
<th>$M^z$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_C$</td>
<td>.5</td>
<td>.5</td>
<td>.5</td>
<td>.5</td>
<td>.5</td>
<td>.5</td>
<td>.5</td>
<td>.5</td>
<td>.5</td>
<td>.5</td>
</tr>
</tbody>
</table>

This difference increases with higher search/inconvenience costs. Therefore, the arbitrage gain from deleting links lasts longer with higher levels of delta. This produces symmetric equilibria with lower levels of global inventory access.

### 3.8 Preference to Trade: Near or Far?

This section analyzes the effect of distance on trade holding inventory complementarity constant. Suppose there are three dealers indexed $A, B$ and $C$ with inventories $S_A = (1,0)$ and $S_B = S_C = (0,1)$. Further, assume that $c_{AB} < c_{AC}$ and $\lambda = 0$. There is perfect inventory complementarity between dealers $A$ and $B$ and $A$ and $C$, the only question is how distance affects $A$’s decision on with whom to trade. There is no reason for $B$ to trade with $C$ because they have the same inventory. In what follows, bold indicates the probabilities of sale when $A$ trades with $B$ and no bold when $A$ trades with $C$. $\pi = \frac{1}{2}$ will not be included in the expected profit given below because it has no effect on the results. First, suppose that dealer $A$ trades with dealer $B$. Expected profit at dealership $A$ when the buyer shows up in location $A$ is

$$\sigma_{CA} [\pi_{A1}(p - I) + \pi_{A2}(p - I - \gamma_{CAB})] + (1 - \sigma_{CA}) \beta \text{Profit}^A$$

Dealer $A$ is able to sell to both his consumers by trading with dealer $B$. If the buyer shows up at dealership $B$ then $A$’s expected profit is

$$\beta(p - (1-h)I)$$

since $B$ is able to meet the demands of his buyer. Finally, if the buyer shows up at location $C$ dealer $A$’s expected profit is

$$\sigma_{CC} [\pi_{C1}(p - I - \gamma_{AC}) + \pi_{C2}\beta(p - (1-h)I)] + (1 - \sigma_{CC}) \beta(p - (1-h)I)$$

since now the buyer showing up at location $C$ travels to dealer $A$ to purchase model one. Note that this probability contains the additional search/inconvenience cost. The expected profit for $A$ if he trades with $B$ is very similar. $A$ prefers to trade with $B$ over $C$ if

$$\text{Profit}_A - \text{Profit}_A > 0$$
This leads to the following result,

**Theorem 3.2.** Dealer $A$ prefers to trade with dealer $B$, the closer dealer if

\[
\left[ \sigma_{CA} \pi_{A1} - \sigma_{CA} \pi_{A2} \right](p - I - \gamma c_{AB}) > 0
\]

4 Model with Heterogeneous Parameters and Two Types of Trades

This section proposes a model that could be used to analyze the likelihood of increasing and individual dealers profits by simultaneously beginning to trade with transfers while decreasing inventory size. Larger inventories are beneficial not only because greater selection implies higher domestic sales, but also since the cars can be traded for vehicles at other dealerships. However, large inventories are costly since new cars are typically financed by the dealership until sold. Smaller inventories are less costly but make it more difficult to trade at the current norm of invoice or invoice minus holdback. This question is quantitative, so a more realistic model of automobile sales and inter dealer trading is needed. In particular, the following model adds heterogeneity in pricing, inventory costs and two different types of trades. Heterogeneous pricing is needed because dealers can and do compete on price. Further, manufacturer incentives differ based on location. Inventory costs are included because the focus is on the tradeoff between these costs and transfer payments. Finally, the empirical snapshot given above provides evidence that two different types of trade relationships exist, each of which are characterized by large trade volumes. Reducing inventory size could alter the composition of trade partners between these two classes and thus affect the tradeoff under study. The two types of trade relationships are given in the following section.

4.1 Types of Trades

There are two types of trades in the model.

1. Mutually Beneficial: In this market organization, dealers only trade if both parties benefit.

2. One Sided Immediate Benefit: In this market organization, dealers are willing to trade two vehicles even if only one side benefits immediately. It is partially competitive in the sense that the dealer not needing to trade today attempts to trade for a desirable car from the other dealer.
In reality, dealers are always willing to make trades that are mutually beneficial and sometimes willing to make trades when only one side immediately benefits. The latter trade is beneficial if the dealer hopes to be paid back in the future. In the model, the opportunity to make trades of this type will be represented by a link between the two dealers. If a link exists, it will be said that a relationship exists between the two. A dealer can attempt to get a car using either type of trade. Next, the model is specified. It is similar to the homogenous case but different in several ways. There will be some overlap in the presentation only to avoid confusion.

4.2 Model Definitions and Parameters

Let $S_i = (s_{i1}, ..., s_{in})$ be the inventory of Chrysler dealer $i$. Each dealer has either one unit in stock or none at all. Each period one consumer randomly shows up at either the local Chrysler or alternative make dealership at location $i$. Let the probability the consumer shows up at location $i$ be $\pi_i$. Once at $i$, the buyer randomly determines to head to either the Chrysler or alternative make dealership. Let $\sigma_{Ci}$ be the conditional probability that the buyer shows up at the Chrysler dealership at $i$ and $\sigma_{iAM}$ for the alternative make. If the buyer shows up at the Chrysler dealership, he randomly demands some model $j$. Let $\pi_{ij}$ be the probabilistic demand for model $j$ at Chrysler dealership $i$. These probabilities will be formalized in a later section. The following notation will be used

- $p_{ij} = \text{price of model } j \text{ at dealership } i$
- $I_i = \text{invoice of model } i$
- $c_{ik} = \text{one way transport cost between location } i \text{ and } k$
- $\beta = \text{dealer discount factor}$
- $\gamma_H = \text{fraction of transport cost paid by the buyer if they locate the vehicle}$
- $\gamma_A = \text{fraction of transport cost paid by the buyer if the dealer locates the vehicle}$
- $N = \text{number of dealerships}$
- $M = \text{number of models}$
- $M_i = \text{cardinality of dealer } i's \text{ inventory}$
\[ h = \text{holdback} \]
\[ r_i = \text{rate at which dealer } i \text{ finances its inventory} \]
\[ \mu^i_1 = \text{rate at which the buyer at } i \text{ substitutes between the Chrysler and alternative make dealerships} \]
\[ \mu^i_2 = \text{degree of substitution between Chrysler models at dealership } i \]

**Assumption:** The inventory of dealership \( k \) is fixed at \( M_k \).

**Assumption:** The transportation cost between dealers \( i \) and \( k \) is constant across models.

The model will proceed in three periods. In the first, dealerships will choose their trade partners. These relationships will be represented by a matrix \( g = (g_{ij}) \). The existence of a link allows for trades where potentially only one side immediately benefits. The rules of network formation are Jackson-Wolinsky. The second and third period will be a model of auto sales. In the second period, one consumer will randomly show up at some dealership \( k \) looking for some Chrysler model \( m \) or the alternative make. If the dealership has the model in stock, they will immediately make the sale. If not, then they can look to their trade partners and remaining dealerships for the desired model. After all of the sales and trading takes place in period one the model proceeds to period three where dealers sell in autarky.

### 4.3 Period 3: Autarky

Each dealership is isolated. If it has the desired vehicle in stock, it sells the vehicle and if not then no sale is made. The expected profit of dealer \( i \) is

\[
\text{Profit}^A_i = \sum_j s_{ij} \pi_{ij}(p_{ij} - (1 - h)i_j) - \sum_j s_{ij}(1 - \pi_{ij})r_ir_j
\]

Given that inventories and prices are exogenous, a dealer in autarky doesn’t take any endogenous action. The assumption of autarky in period three greatly simplifies payoffs because updating probabilities in period two without it would require the dealer to know something about global inventories after trading and selling in the first period. This of course is not unreasonable, but would greatly complicate the basic structure of the model.

### 5 Complete Period 2 and 3 Profits

Let \( g = (g_{ij}) \) be the network. \( g_{ij} = 1 \) if a link exists between dealer \( i \) and dealer \( j \) with the property that \( g_{ij} = 1 \) if and only if \( g_{ji} = 1 \). If a link does not exist between dealers \( i \) and \( j \), then \( g_{ij} = 0 \). Suppose that dealer \( i \) is requesting model \( j \) from dealer \( k \); that is the consumer is demanding model \( j \) and \( s_{ij} = 0 \). If \( g_{ik} = 1 \) and \( s_{kj} = 1 \), then dealer \( i \) can demand a trade from \( k \) even if \( k \) doesn’t immediately benefit. If \( g_{ik} = 0 \) and \( s_{kj} = 1 \), then dealer \( i \) can only request a trade where \( k \) immediately benefits. In this case, what does dealer \( k \) want in return? Dealer \( k \) wants a model currently not in his inventory that increases his expected discounted profit tomorrow. Here I’ve made the following subtle assumption. Further, suppose that only the Chrysler nest exists in period 2.

**Assumption:** Dealer \( i \) and \( k \) make their trade decision holding \( i \)’s demand constant.

For any contemplated trade \( I \) need to be careful because probabilities of sale at dealership \( i \) in the first period depend upon the costs associated with purchasing each vehicle. In particular, a dealer contemplating trading dealer \( i \) model \( j \) should take into account the buyers behavior if he doesn’t trade. However, if he does, then every other trade decision must be accounted for to determine the change in probability. The same holds for \( i \) when determining with whom and what to trade. This greatly complicates the structure of the model though so I assume it away for now. The results that follow don’t depend on the assumption, as will be carefully explained later.

First consider trades of immediate benefit. Suppose that model \( j \) is such that there does not exist a \( k \) such that \( g_{ik} = 1 \) and \( s_{kj} = 1 \). That is, none of \( i \)'s regular trade partners have the desired vehicle in stock. Let \( D \) be the set of all dealers with \( k \) in stock. Rank the elements in \( D = \{d_1, d_2, \ldots, d_m\} \) by distance, closest
to furthest from $i$. For each $d_k \in D$ with $k > 1$, let $D_{d_k} = \{t_1, \ldots, t_m\}$ be the set of all $t$ such that $s_{d_k,t} = 0$ and

$$r_{d_k}I_j + \beta_{d_k}\text{Profit}^A_{\text{No Trade}} < r_{d_k}I_t + \beta_{d_k}\text{Profit}^A_{ij}$$

Here the profits in period 2 are computed in autarky, depending on whether or not a trade takes place. In particular, $\text{Profit}^A_{\text{No Trade}}$ is profit in autarky if no trade is made and $\text{Profit}^A_{ij}$ is profit in autarky if model $j$ is traded away for $t$. Dealer $d_k$ with $k > 1$ is willing to trade $j$ for any model in $D_{d_k}$. Why $k > 1$? Because even if these dealers decide not to trade, the buyer will not purchase $j$ from them. If dealer $i$ cannot get model $j$ through trade, then the buyer will travel to $d_1$ to purchase it. For dealer $d_1$, let $D_{d_1}$ be the set of all $t$ such that $s_{d_1,t} = 0$ and

$$p_{ij} - (1-h)I_j - \gamma_{Acid} + \beta_{d_k}\text{Profit}^A_{\text{add j}} < r_{d_k}I_t + \beta_{d_k}\text{Profit}^A_{ij}$$

Where $\text{Profit}^A_{\text{add j}}$ is profit in autarky provided that model $j$ is sold out of $d_k$'s inventory. The difference here is that if $d_1$ decides not to trade then the buyer interested in model $j$ will travel to his dealership to purchase it. Note that dealer $d_1$ receives the price from location $i$. This reflects the fact that manufacturer incentives are based on the customers place of address. If $i$ can get model $j$ from one of his regular trade partners then all trades of immediate benefit are determined using the first inequality.

Dealer $d_k$ is willing to trade dealer $i$ model $j$ for any model $t \in D_k$. Which model will dealer $i$ chose to trade to $k$ for model $j$?

**Assumption**: Dealer $i$ has negotiating power to choose any $t \in D_k$ when trading with dealer $k$ where both sides immediately benefit.

Dealer $i$ chooses model $t_{ijk}$ to trade to $d_k$ such that

$$t_{ijk} = \text{argmax}_{t \in D_k} \{-r_{d_k}I_{tk} + \beta_{d_k}\text{Profit}_{\text{lose }t_k}^A\}$$

That is, dealer $i$ trades the model that leaves him the highest expected profit tomorrow. $t_{ijk}$ indicates the model $i$ trades to $k$ when requesting model $j$. 

What if $g_{ik} = 1$ and $s_{kj} = 1$? Recall that in this case dealer $i$ can demand a trade in which only $i$ immediately benefits. If there is a trade that benefits both, assume it is executed as if $g_{ik} = 0$. If not, I make the following assumption.

**Assumption**: Dealer $i$ offers to trade the car in his inventory that gives $k$ the highest expected profit tomorrow. This assumption seems reasonable since dealer $k$ is already taking a hit by trading model $j$. Let $t_{ijk}$ be the index of the model that dealer $i$ trades to dealer $k$ when requesting model $j$. If a trade does not exist between the two, let $t_{ijk} = 0$.

**Assumption**: The dealer $i$ initiating the trade with $k$ is able to pass on $\gamma_{Hcik}$ of the transportation cost to the buyer.

Now dealer $i$ trying to get model $j$ knows what each dealer wants in return. In deciding with whom to trade, $i$ must consider the transportation cost and required trade of dealing with each dealer $k$. That is, he must consider the entire continuation payoff when choosing with whom to complete the trade. Suppose that dealer $i$ trades model $t_{ijk}$ to $k$. Let $S_{ik}^{jk}$ be his inventory after the trade. That is,

$$S_{ik}^{jk} = S_i - e_{t_{ijk}}$$

where $e_{t_{ijk}}$ is the $M_i$ dimensional vector with all zeros except for a one in entry $t_{ijk}$.

Dealer $i$ chooses to trade with dealer $k_{ij}$ that solves

$$k_{ij} = \text{argmax}_{k \in \mathbb{N}} \{p_{ij} - (1-h)I_j - \gamma_{Hcik} - \sum_t s_{ikt}r_{it}I_t + \beta_i\text{Profit}_{\text{lose }t_{ijk}}^A\}$$

Provided that this continuation payoff is larger than $i$ would get by not trading at all. In other words, the maximum continuation payoff is larger than

$$\beta_i \left(\sum_t \bar{\pi}_{it}s_{it}(p_{it} - (1-h)I_{it}) - \sum_t s_{it}(1-\bar{\pi}_{it})r_{it}I_t\right)$$
Where the bar notation indicates probabilities in autarky. If dealer \( i \) cannot get model \( j \) let \( k_j^i = 0 \). Let the maximum continuation payoff of dealer \( i \) if consumer \( j \) shows up at his dealership \( i \) be denoted \( CP_{ij}^i \). If \( s_{ij} = 0 \), then \( CP_{ij}^i \) is the value just configured. If \( s_{ij} = 1 \) then

\[
CP_{ij}^i = (p_{ij} - (1 - h)I_j) - \sum_{t \neq j} s_{it}r_tI_t + \beta_i \text{Profit}_{\text{loc}_j}^i
\]

With this the expected profit of dealer \( i \) in the event that a consumer shows up at his location is

\[
\sum_j \pi_{ij}CP_{ij}^i
\]

All that’s left is to configure expected profit if the consumer shows up elsewhere. This is simply an accounting of the changes and resulting costs associated with \( i \)’s inventory after trade takes place. Suppose that some consumer \( q \) shows up demanding model \( j \) with \( s_{qj} = 0 \). If \( k_j^q \neq i \) or 0 then dealer \( i \)’s continuation profit is

\[
CP_{qj}^i = -\sum_{t \neq j} s_{it}r_tI_t + \beta_i \text{Profit}_{\text{no trade}}^A
\]

Alternatively, if \( k_j^q = i \) then dealer \( k \) wants to trade model \( t_{qji} \) for model \( j \) from dealer \( i \) so that the continuation payoff is

\[
CP_{qj}^i = -\sum_{t \neq j} s_{it}r_tI_t - r_tI_{qji} + \beta_i \text{Profit}_{\text{trade}}^A
\]

Finally, if \( k_j^q = 0 \) and \( i \) is the closest dealership to \( q \) then the continuation payoff is

\[
CP_{qj}^i = p_{ij} - (1 - h)I_j - \gamma_A c_{iq} - \sum_{t \neq j} s_{it}r_tI_t + \beta_d \text{Profit}_{\text{sold}}^A
\]

With all of this, the expected profit of dealer \( i \) is

\[
\text{Profit}_i = \sum_k \pi_k \sigma_{CK} \sum_j \pi_{kj}CP_{kj}^i + \sum_k (1 - \sigma_{CK})CP_{AM}^i
\]

where \( CP_{AM}^i \) indicates profit in autarky when the dealer carries his original inventory over to period three.

5.1 Discrete Choice Model of Demand

The conditional nested logit model of demand will be used to determine the probability of a consumer showing up at dealership \( i \) demanding model \( j \). This needs to be random because of model complexity: a dealer doesn’t know with complete certainty the precise model a consumer desires. The nested model is used because the substitution from the dealership of interest to a nearby alternative make competitor is critically important. One nest will be the available inventory at the dealership in the network while the other nest will signify one vehicle at its immediate alternative make neighbor. The intuition for this set up is the following. A consumer showing up at the Chrysler dealer attempts to locate his most preferred model. However, because dealerships of competing makes tend to locate very close to one another, there could be an alternative model of another make relatively close to the Chrysler dealership. Because the substitution from Chrysler to this alternative make is critically important in the analysis that follows, the alternate car is given its own nest. This two nest specification allows for flexibility in analysing the substitution effect in network formation.

The following two assumptions specify consumer knowledge about the system and buyer search/hassle costs of seeking out a model elsewhere.

**Assumption**: A consumer showing up at dealership \( i \) can travel to purchase any available model. In doing so, they pay the fraction \( \gamma_A \) of the transportation cost \( c_{ik} \) when purchasing a vehicle from dealership \( k \). When the consumer searches out a vehicle elsewhere, they incur the search/hassle cost \( \delta \) with \( \delta > 0 \). The intuition for this additional cost is that the consumer has to search out and negotiate with another dealer which can
be costly. Since a consumer receives the same manufacturer incentives no matter where he purchases the vehicle, it will be assumed that the search/hassle costs outweigh any potential benefit from negotiating with another dealer. Therefore, a consumer will always purchase a vehicle from the home dealership provided they have the vehicle in stock or are able to trade for it.

**Assumption**: The consumer at dealership \( i \) knows the full inventory of its local dealer and the costs associated with trading for a vehicle not currently in stock.

With these assumptions we can determine consumer utility of buying each model at the Chrysler dealership. Let \( U_{ij} \) be buyer \( i \)'s utility of consuming model \( j \) and \( v_{ij} \) \( i \)'s value of the \( j \)'th model. If the local dealership has model \( j \) in stock, then the consumers utility is

\[
U_{ij} = v_{ij} - p_{ij} + \mu_i^2 \epsilon_{ij}
\]

If the local dealer does not have the model in stock and is unable to trade for it, the buyer has the option to seek out the model elsewhere. Since it is assumed that the buyer pays the domestic price no matter where he purchases from, he buys the vehicle from the closest dealership. Let this dealership be \( k \). This utility is

\[
U_{ij} = v_{ij} - p_{ij} - \gamma A c_{ik} + \mu_i^2 \epsilon_{ij}
\]

Finally, if dealer \( i \) can trade for the vehicle then the consumers utility is

\[
U_{ij} = v_{ij} - p_{ij} - \gamma H c_{ikj} + \mu_i^2 \epsilon_{ij}
\]

where \( k^j_i \) is the optimal dealer with whom \( i \) can trade for model \( j \).

**Assumption**: Global inventory of each model \( j \) is non-zero.

If \( \epsilon_{ij} \) are i.i.d type double exponential across, then the probability that consumer \( i \) shows up at the Chrysler dealership demanding model \( j \) is

\[
\pi_{ij} = \frac{\exp \{ U_{ij} / \mu_i^2 \}}{\sum_j \exp \{ U_{ij} / \mu_j^2 \}}
\]

The parameter \( \mu_i^2 \) controls buyer substitution across models at the Chrysler dealership \( i \). As \( \mu_2 \) goes to zero, the products become perfect substitutes and the buyer purchases the vehicle with the highest deterministic utility. The higher \( \mu_2 \), the less substitutable are the models at Chrysler dealer \( i \).

Given the two nests at each location, the attractiveness of the Chrysler nest is given by the expected value

\[
ECV_i = E(\max_j \{ U_{ij} \})
\]

where \( ECV_i \) indicates the expected Chrysler value at dealer \( i \). Given the assumption on errors, \( ECV_i \) has closed form (up to an additive constant?)

\[
ECV_i = \mu_i^2 \ln \left( \sum_{j=1}^{M_i} \exp \left( \frac{U_{ij}}{\mu_i^2} \right) \right)
\]

Let \( AM_i \) be the alternative make value at \( i \). Then the probability that a buyer at \( i \) chooses the Chrysler nest is given by

\[
\sigma_{Ci} = \frac{\exp \left( \frac{ECV_i}{\mu_i^1} \right)}{\exp \left( \frac{ECV_i}{\mu_i^1} \right) + \exp \left( \frac{AM_i}{\mu_i^1} \right)}
\]

where \( \mu_i^1 \) is the substitutability parameter between nests at location \( i \). It has the same interpretation as \( \mu_i^2 \) given above. Finally, the consumer surplus at location \( i \) is given by

\[
CS_i = \mu_i^1 \ln \left[ \exp \left( \frac{ECV_i}{\mu_i^1} \right) + \exp \left( \frac{AM_i}{\mu_i^1} \right) \right]
\]

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5.2 Trading with Transfers

In this section I evaluate the likelihood of increasing an individuals dealers profitability by trading with transfers while simultaneously reducing inventory size. Recall that inventories are costly, typically financed with interest only loans, and a large percentage of sales are generated by dealer trades. Therefore, it seems possible that inventories could be reduced while lubricating the system of trades through transfers to increase profitability. Smaller inventories reduce costs but also make it less likely that the dealer will have the customers most desired vehicle. Further, smaller inventories make it less likely that another dealer negotiating a trade will want something from the requesting dealers floor plan. To lubricate trade then, the dealer could share profits per trade with the dealer in which he is requesting the trade. This is much more complicated than any of the results previously derived and requires a slightly more complex model. In particular, because inventory costs are of primary focus, the duration of a period must be fully understood. The cost of holding inventory for one hour is very different than the cost for holding that same inventory for a month. Suppose that I want the model to last for one week. Then, I need to pick some number of periods $T$ to run the model and $\pi_i$ so that the expected number of sales matches some level of expected weekly sales. There is one major complication however. The probabilities of sale depend on global inventories and the trade agreements negotiated between dealers. Therefore, every network structure has a different level of expected sales for any $T$ and $(\pi_i)_i$. To benchmark the model, $T$ and $(\pi_i)_i$ can be selected according to some fixed probability of sale when the buyer shows up at dealer $i$. There are many ways to choose the fixed probability. For example, it can be chosen by considering sales in autarky over one period. Alternatively, the probabilities can be obtained by solving the two period model given above. The model will be tested using these different methods.

Let $W_i$ be the weekly sales of dealer $i$ and $q_i$ the probability of sale given the buyer shows up at $i$. The solution $(T, \pi)$ solves the following equation for every $i$

$$\pi_i q_i T = W_i$$

which simply states the expected number of sales equals the desired number for each dealer $i$. The solution to this set of equations is

$$T = \sum_i \frac{W_i}{q_i}$$

$$\pi_i = \frac{W_i}{q_i \sum_i W_i}$$

For example, if $i = 1, 2$, $q_i = 1$ and $W_i = 30$ for each $i$, then the number of periods is 60 and the probability the buyer shows up at dealer $i$ is $\frac{1}{2}$. If $W_i$ is sales per month then each period represents half a day and the probability that the buyer shows up at either dealer is $\frac{1}{2}$ each half day.

By figuring out the length of each day, the inventory cost can be better specified in the algorithm specified above. For example, if the length of the model is one month and each period is half a day, the prime rate can be used to specify the precise inventory cost per period. That is, for model $i$ the inventory cost in period $t$ is

$$r \cdot \frac{5}{365} I_i$$

where $r$ is the annual prime rate.

Let $S^t$ be the $MxN$ matrix of dealer inventories in period $t$. Alternatively, $S^t = (s^t_1, ..., s^t_N)$. The state space in every period $t$ is $S$, the set of all $MxN$ dimensional matrices with $S_{ij} \in \{0, 1\}$ for all $i, j$ and $S \in S$. Let $V_i^t(S)$ be the expected discounted profit of dealership $i$ at period $t$ when global inventories are $S \in S$. In the final period, as before each dealership sells in autarky. This specifies $V_i^t$ for all $i$. $V_i^{t-1}$ is computed in exactly the same way as the two period model, where the profit in autarky tomorrow is simply the the value function at $T$. Similarly, the value function at $t$ is computed in the same was as the two period model except that $V_i^{t+1}$ is used in place of the profit in autarky in every possible buyer/model state.

The extended $T$ period model proceeds in exactly the same way as the two period model. In period 0, the dealers form links and then use them to trade and sell in the next $T - 1$ periods. In the final period the dealers trade in autarky.

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6 Conclusion

This paper analyzed the structure and efficiency of automobile trade networks. Network structure is dependent on inventory complementarity, distance, buyer substitution and a preference toward an intermediate access to global inventory. Because of a large number of options for any given Chrysler model, even seasoned sales managers have a difficult time ordering inventory to completely match buyer demand. Trade relationships develop because of this natural randomness in demand. Dealers have preferences however on access to global inventory, preferring an intermediate access. Network incompleteness is optimal due to asymmetric information about buyer search/inconvenience costs. Intermediate access increases dealer profits but reduces consumer surplus through the introduction of search/inconvenience costs. Further research includes using the model outlined in the paper to analyze the likelihood of increasing an individual dealers profits by simultaneously reducing inventory size and beginning to trade with transfers.

7 References