Internal Drain and the Role of International Reserves

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Abstract
To examine the effect of international reserve accumulation policy on portfolio choice and inequality of consumers under internal drain, we construct the model by combining heterogeneous agent model and fiscal theory of price level and nominal exchange rate. By employing this model, we can conclude that, when the central bank responds to the household’s foreign asset holdings more strictly, severe portfolio rebalancing is mitigated and the inequality of households, measured by the variance of household’s consumption, decreases. However, the mean of consumption also decreases as the central bank responds strictly. Therefore, policy makers should be careful when they accumulate the international reserves.

JEL Classification: E5, E3, E6, F4

Keywords: Internal Drain, International Reserves Accumulation, International Reserve Accumulation Policy, Heterogeneous Agents

1 Introduction

The present research question emerged from the question: "Should monetary authority hold foreign international reserves?" Whether or not we can answer to this question, international reserve has been one of the major account of central bank’s balance sheet. In recent years, international reserve accumulation has accelerated rapidly, reaching more than 10 percent of global GDP. According to IMF’s report, for most emerging countries, reserve coverage has risen to high levels relative to traditional norms, reaching 10 months of imports and 475 percent of short-term external debt in 2008. Late 2000s, since the global recession triggered by the financial crisis, originating from U.S. Mortgage crisis, there is possibility that the pace of reserve accumulation will revert to the original rate as the world economy is recovering. However, considering an increase in trade volume and the openness of international financial market, the pace might accelerate. Also, some countries suffered from the crisis might want to accumulate more according to the lessen that they have received during the financial crisis.

While experiencing a financial crisis, (relatively more contagious than before,) especially emerging economies realized that higher reserves helped reduce the impact of the crisis on their markets. So, the accumulation of international reserves carries benefits in terms of financial independence and international status. Therefore, whether or not they consent the level of international reserves, all agree that international reserves can play a role as an insurance. However, the voice of criticism on this does not decline because sustained reserve accumulation is an inefficient form of insurance. Recent studies warn that monetary authorities with over-accumulated international reserves may face substantial challenge. They pointed out that, since most international reserves concentrated in U.S. dollars, excessive reliance on self-insurance might increase the risks contrary to their expectations.

Therefore, we have to find the link between international reserves and exchange rate to answer the main question that we have introduced above. Many models include the mechanisms which might cause the rapid change in exchange

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rate and, under this environment, they derived the role of international reserves as an exchange rate stabilizer or calculate the optimal level of international reserves. According to Obstfeld et al. (2010), these models can be classified into two groups, depending on the main cause of the exchange rate surge; one is internal drain and the other is external drain. Internal drain stems from domestic credit crunch and external drain is usually caused by deteriorated foreign borrowing conditions. Although the clear linkage between these two drains are not investigated yet, the occurrence of both events at the same time has been observed in many emerging countries, frequently. So, double drain (or twin crisis) concept, consisting of internal and external drain, is suggested and making a model with double drain is the desirable scenario. However, most of previous studies examined external drain and the internal drain has received relatively little attention. Therefore, inventing the concrete model which can examine the internal drain related issues may have a contribution in double drain literature.

The main cause of internal drain is in the uncharted terrain. At the fundamental level, however, there is a similar consensus on the reason, consumer’s fear. When consumers are captivated by fear, they will go to the bank to withdraw their money, bank run,(runs from bank deposits to currency) and rebalance their portfolio by purchasing more foreign assets and selling domestic assets, flight-to-quality. This domestic capital flight is financed through withdrawals of domestic bank deposits and it may be larger than sudden stop, typical example of external drain, by trade or debt-financing channels. This internal drain is not a hypothetical channel in the new era of financial globalization and to reflect this phenomena, we will use heterogeneous agents with different endowment and asset holdings. By introducing heterogeneity, we may examine portfolio rebalancing issue with sufficient accuracy.

The contents of following sections are as follows. In section 2, we will construct the model and check the existence and uniqueness of the equilibrium. In section 3, we will explain about model parameters and, in section 4, we will provide the computation procedure. In section 5, the result will be provided and we will conclude at the last section.

2 Benchmark Model

We are going to construct the simple model which can examine the role of international reserves as an exchange rate stabilizer under internal drain situation. As we have pointed out above, double drain consists of two things, internal and external drain. Although these two drains tend to happen at the same time, we will narrow down our interest on internal drain. When domestic financial institutions cannot maintain financial stability, consumers are usually captivated by fear and flight-to-quality, portfolio rebalancing, and bank run are observed. As a result of these behaviors, the portion of foreign assets in portfolio increases and that of domestic assets decreases.

To replicate this phenomena, we will start with a heterogeneous agent model with different asset holdings and individual endowment. There are two assets in the model: home government bond and foreign asset. Home government bond, issued by fiscal authority, is a risky asset because fiscal authority may default on a fraction of δt of its total

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1Obstfeld et al. (2010) introduced many cases in their paper including Argentina, Singapore, Malaysia, and Republic of Korea.
2Although the linkage between internal and external drains are not clearly investigated yet, the occurrence of both events at the same time has been observed in many emerging countries frequently. So, one important feature is to model internal and external drain, respectively. Therefore, although we narrow down our interest on internal drain, we will examine external drain shortly. External drain is usually caused by deteriorated foreign borrowings and typical examples are sudden stop or increase in premium. So, we may think in a couple of ways to include external drain in the model. One feature is related with volume of foreign currency such as sudden stop of foreign direct investment or retrieving of investment and repayment pressure due to the denial of roll-over. The other way is an increase in loan premium rate. Many candidates exist to proxy this premium and one of them is CDS (Credit Default Swap) rate.
3The main source of causing inefficiency in the economy is the market failure such as monopolistic competition or incomplete market. Therefore, the popular research topic is to introduce the policy which can remedy this market failure so that we can achieve the efficient equilibrium where several welfare theorems hold. One of the main interest of our model is portfolio rebalancing and the number of assets we can include in the model is limited. So, the number of assets is less than the number of shocks, which implies that our model bears incomplete capital market. If the aggregate shock is the only source of uncertainty, then employing the representative agent model is not a bad choice. However, in our model, home government bond is a risky asset and household has a state-dependent utility. In this situation, heterogeneous agent model is the proper model although it looks like a rich model. Also, by introducing heterogeneity, we may introduce one more dimension which examine the cause of equilibrium inefficiency. That is the aggregation failure of individual household’s decision. From the studies of An et al. (2009), we may see detailed explanation on the difference between representative and heterogeneous agent model in the view of aggregation failure and market failure. We will address these issues more closely in the following sections.
liabilities\footnote{By introducing default probability, we may reflect the consumers captivated by fear. Technically, this default rate is the only aggregate shock in our model so it plays an important role in determining the consumer’s portfolio.}. The other asset is a foreign asset with fixed interest rate. Although the interest rate is fixed, this asset is also a risky asset because, unlike other heterogeneous agent models, we deal with nominal assets and, therefore, the rate of return on this asset depends on nominal exchange rate. Each individual household is allowed to access the both home and foreign asset market without transaction cost. With these ingredients, we will derive the equilibrium of this model.

2.1 Household

Individual household makes its own portfolio to smooth consumption by accessing domestic and foreign financial markets. Both assets are one-year-maturity bonds and there is no transaction costs on accessing both financial markets\footnote{In real world, bid-ask spread exists. Therefore, in microfinance literature, there are so many studies which figure out the factors determining the spreads. Moreover, in high frequency trading literature, slippage is the main research topic. The slippage is the difference between estimated transaction costs and the amount actually paid. This is closely related with several finance factors such as liquidity risk, brokerage fees.}. However, both assets are risky and the reason is as follows. The main interest of this paper is the role of international reserves under the internal drain. As we have pointed out above, the main cause of internal drain is the consumers’ fear. So, the model should include the feature which may make consumers captivated by fear and we mitigated this requirements by introducing the uncertainty in the rate of return on home government bond. Home government may decide to default $\delta_t$ fraction of its own government liabilities and, therefore, household may lose its home government bond holdings next period with probability $\delta_t$. Also, the foreign asset is a risky asset although it bears fixed interest rate. That is because the foreign asset is a nominal asset and the rate of return is affected by nominal exchange rate. So, as long as the central bank of home country is not in the pegged exchange rate system, fixed exchange-rate system, holding foreign asset takes market risk. Therefore, households should take the risk when they purchase assets.

We have continuum of household $j, j \in [0, 1]$, who maximizes its own utility function subject to its own budget constraint.

$$
\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i u(c_{t+i}^j) 
$$

subject to

$$
P_t c_t^j + q_t B_{H,t}^j + e_t q_t^* B_{F,t}^j + P_t e_t^j \leq P_t y_t^j + B_{H,t-1}^j + e_t B_{F,t-1}^j
$$

$$
B_{H,t-1}^j = 0 \quad \text{with probability } \delta_t
$$

where $P_t$ is price level, $e_t$ is nominal exchange rate, $q_t$ is the price of home government bond and $q_t^*$ is the price of foreign asset, $c_t^j$ is individual consumption, $y_t^j$ is individual endowment, $B_{H,t}^j$ are home government bond holdings and $B_{F,t}^j$ are foreign asset holdings. $y_t^j$ follows AR(1) process. Also, as we have pointed out above, home government decides to make a default the $\delta_t$ fraction of its own liabilities so that the $\delta_t$ fraction of households receive no money from home government bond in this period. In other words, $B_{H,t-1}^j$ is zero with probability $\delta_t$. We will assume that all the markets are competitive. So, there is no market structure related inefficiency such as markup and household regard prices as given when it solves its own problem. We may rewrite the problem in a recursive way.

$$
V^j(B_{H,t}^j, B_{F,t}^j, y_t^j, T^j; P, q, q^*, e, \delta_s) = \max_{c_t^j, B_{H,t-1}^j, B_{F,t-1}^j} u(c_t^j) + \beta \mathbb{E}_t \left( V^j(B_{H,t}^j, B_{F,t}^j, y_t^j, T^j; P, q, q^*, e^t, \delta_s) \right)
$$

$$
+ \beta \mathbb{E}_t \delta^j \left( \delta_t \right) V^j(0, B_{H,t}^j, B_{F,t}^j, y_t^j, T^j; P, q, q^*, e^t, \delta_s)
$$

\footnote{Introducing default of home government bond is the simple way to introduce uncertainty. However, it does not match with our data because the Republic of Korea never default its liabilities. Therefore, I construct the model in a different way by introducing time-varying interest rate instead of introducing default rate. It does not change the main result and, therefore, we may stick with the original setup, time-varying default rate. Also, introducing default rate is more compatible with interest rate rule. So, for the future study, I have decided to introduce default rate instead of time-varying interest rate.}

\footnote{Uribe (2006) assumed that government bonds are risky assets, which implies that, at each period, the fiscal authority may default on a fraction of its total liabilities. His model is a representative agent model and it cannot examine the influence of default to household’s financial decision. That is because, when we have state-dependent utility, we have to solve the problem separately depending on state and then get the expected total utility by using the state probability as weight (Mas-colell et al. (1995)). So, introducing heterogeneity is more accurate way to examine the model which includes the defaultable bond.}
subject to \( (2) \) where superscript \( ' \) represents next period’s variables. Now, we can see the advantage of heterogeneous model more clearly. As we have pointed out above, the individual household’s problem is state dependent and, with heterogeneous model, we can solve it.

To solve the problem, household has to know the price level and nominal exchange rates and other aggregate variables. However, the price level and other variables we assumed as given are the function of aggregate variables and these aggregate variables are obtained after gathering individual household’s decision variables. The typical approach in heterogeneous agent model literature is introducing the law of motion for aggregate variables and price equations. With the law of motion of aggregate variables, household may anticipate aggregate variables and, then, household may calculate the price level and nominal exchange rate that we have assumed as given. These equations are model specific so that we will examine these equations after introducing central bank and government sector\(^8\).

### 2.2 Aggregate Discount Factor

Household can access two asset market so that discount factor should be arbitrage-free. The Euler equations that we may derive from household’s optimization problem are

\[
q_t = \beta \mathbb{E}_t (1 - \delta_{t+1}) \frac{u'\left(c_{t+1}^j\right)}{u'\left(c_t^j\right)} \frac{P_t}{P_{t+1}} \\
q_t = \beta \mathbb{E}_t \frac{c_{t+1}^j}{c_t} \frac{u'\left(c_{t+1}^j\right)}{u'\left(c_t^j\right)} \frac{P_t}{P_{t+1}}
\]

So, except individual consumption variable, all other variables are aggregate variables. This implies that each household should have same ratio between current and future marginal utilities. Gornemann et al. (2012) suggest the way to derive the aggregate discount factor from this individual discount factor. They assume that, in pricing claims, the home government bond decisions are based on the average of the individual household’s preferences, where the average is taken weighting their marginal utilities of consumption by their end-of-period home government holdings. More precisely, the stochastic aggregate discount factor is given as follows.

\[
Q_{t,t+1} = \beta \mathbb{E}_t (1 - \delta_{t+1}) \frac{b_t}{b_{t+1}} \frac{P_t}{P_{t+1}}
\]

where \( b_t \) is the individual household’s home government bond holdings and \( b_{t+1} \) is the aggregate individual household’s home government bond holdings. In our model, the ratio between current and future marginal utilities is constant across individuals and we may derive the following relationship.

\[
Q_{t,t+1} = \beta \mathbb{E}_t (1 - \delta_{t+1}) \frac{P_t}{P_{t+1}}
\]

The reason why we have this simple form is that the price of home government bond is fixed. We will use this aggregate discount factor to derive the equilibrium conditions in the following sections. The appendix provides the detailed derivation on how to connect this aggregate discount factor with equilibrium condition\(^9\).

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\(^8\)This typical approach is used with discrete time model. In continuous time model, we have different approach. That approach is based on mean field game theory and it uses two equations : Hamilton-Jacobi-Bellman equation and Fokker-Planck equation. The key feature of this approach is constructing information structure and, by doing in this way, HJB represents the behavior of agents and Fokker-Planck equation represents the distribution of households. Nuno (2013) provides the detailed explanation on simple model in continuous time.

\(^9\)Gornemann et al. (2012) includes only one asset, mutual fund, which has all the shares in that economy as its underlying assets. So, stochastic discount factor varies according to household’s financial decision. In numerical step, it is usually difficult to calculate the aggregate discount factor. So, they used the following equation to derive the stochastic discount factor.

\[
Q_{t,t+1} = \beta u' \left( \int c' d\mu' \right) / u' \left( \int c d\mu \right)
\]

which relies on the representative-agent discount factor.
2.3 Central Bank and Government

Conventional way to examine the effect of fiscal and monetary policy was consolidating the government’s flow budget constraint. The consolidated government’s flow budget constraint in conventional models is as follows.

\[
q_t B_{H,t}^G - e_t q_t^H B_{F,t}^G + s_t = \frac{(1 - \delta_t) B_{H,t-1}^G - e_t B_{F,t-1}^H}{P_t}
\]

where the fiscal surplus \( s_t \) is the lump-sum tax \( t_t \) less the fiscal authority expenditure in units of real goods. However, we assumed that government spending is zero and this assumption may not change the result. Following the balance sheet view, we may change the above budget constraint as follows.

\[
\frac{e_t \tilde{B}_{F,t}^G - e_t \tilde{B}_{F,t-1}^G}{\Delta \text{asset}} + \frac{\tilde{B}_{H,t}^G - \tilde{B}_{H,t-1}^G}{\Delta \text{stabilities}} + \frac{s_t - i_{H,t-1} \tilde{B}_{H,t-1}^G + i_{F,t-1} \tilde{B}_{F,t-1}^G}{\Delta \text{capital}}
\]

where \( i_H \) is the net nominal interest rate on home government bond, which is \( R_H = 1 + i_H \) and \( i_F \) is the risk-free net nominal interest rate on foreign bond, which is \( R_F = 1 + i_F \). Moreover, to show this view more clearly, we have changed the variables by using \( \sim \). Namely, we set \( \tilde{B}_t = q_t B_t \) and \( \tilde{B}_{t-1} = (1 - \delta_t) B_{t-1} + \frac{i_{F,t-1} B_{F,t-1}}{1 + i_{F,t-1}} \).

Given this balance sheet, government may set government surpluses for current and future periods and central bank can set the amount of international reserves. In this model, we assume that the interest rate on government bond is constant, which implies that central bank does not implement interest rate rule.\(^{10}\) As we have seen above, the main reason of holding international reserves is to prevent the exchange rate surge. In our model, the change in individual household’s financial decision, the main cause of portfolio rebalancing, is the only source of the fluctuation in exchange rate. Therefore, to stabilize the exchange rate, government should monitor the amount of foreign assets held by households and we set the following simple policy rule.

\[
B_{F,t}^G = \rho_{F,0} + \rho_{F,1} B_{F,t}^H
\]

where \( B_{F,t}^G \) is the amount of international reserves, \( B_{F,t}^H \) is the aggregate foreign assets held by households and \( \rho_{F,0} \) and \( \rho_{F,1} \) are policy parameters. Both policy parameters have their own interpretation: \( \rho_{F,0} \) is the minimum level of international reserves and \( \rho_{F,1} \) is the strictness of government policy.\(^{11}\) Another policy is government surplus policy.

\[
(s_t - s) = \rho_S (s_{t-1} - s)
\]

where \( \rho_S \) is policy parameter.\(^{12}\) So, current government surplus depends on its previous series and surplus shock, which implies that government surplus is independent of price level and exchange rate.\(^{13}\) If we assume that government spending is zero, government surplus is identical to tax. In this context, when tax is negative, we may interpret it as subsidy.

\(^{10}\) Actually, I construct the model with monetary policy, which implies that my model may have time-varying interest rate with monetary policy shock. With this extended model, however, I cannot get the stable result. In near future, I will employ interest rate rule and derive the non-linear type impulse response function of the monetary policy shock.

\(^{11}\) Research on international reserve accumulation policy is still in its early stage, as the brevity of the bibliography attests. Therefore, unfortunately, the form of policy used in this paper stands on the poor base. Therefore, it is believed that further experimentation with the methods outlined here is worthwhile. We may choose another type of international reserve accumulation policy as follows.

\[
B_{F,t}^G = \rho_{F,0} + \rho_{F,1} \frac{B_{F,t}^H}{B_{F}}
\]

The interpretation of the policy parameters is almost identical to the above policy form. However, the influence on exchange rate should be different because the amount of international reserves held by government is different. Also, we may have more sophisticated policy form with state dependent policy parameters. I will leave these policy forms for the future research.

\(^{12}\) We followed Uribe (2006)’s approach. In his paper, he assumed an exogenous, stationary, stochastic tax process and the law of motion of lump-sum taxes follows an AR(1) process of the above form.

\(^{13}\) We may construct the government surplus rule in a various different ways. If we construct the government surplus rule which are independent of price level and exchange rate, we may proceed the following sections.
2.4 Price level and Exchange Rate Determination

Most of other heterogeneous agent models are real models so that there is no price level and only the real assets are traded. However, our model is a nominal model and we cannot be free from the burden that we should derive the equilibrium price level and nominal exchange rate. Because of price level indeterminancy, however, we may not find relevant clues from conventional macroeconomic models. However, Leeper (2013) suggested fiscal theory of price level, which can determine the equilibrium price level and nominal exchange rate simultaneously by combining two intertemporal equilibrium conditions. In this section, we will draw on the work of Leeper (2013) in order to derive the equilibrium price level and nominal exchange rate.

Two intertemporal equilibrium conditions can be derived from the aggregated household’s budget constraint and government budget constraint. Let’s examine the aggregated household’s budget constraint. By aggregating individual household’s budget constraint and imposing transversality condition, we may get the following equilibrium condition.

\[
\frac{(1 - \delta_t)B_{H,t-1}^H + \epsilon_tB_{F,t-1}^H}{P_t} = \sum_{k=0}^{\infty} \beta^k E_t S_{t+k} - \sum_{k=0}^{\infty} \beta^k E_t NX_{t+k}
\]  

Therefore, given \(B_{H,t-1}^H\) and \(B_{F,t-1}^H\), price level and nominal exchange rate are determined by the discounted present value of government surpluses and net exports. In fact, to derive this interpretation, we need the following strong assumption - the government surplus and net export processes are independent of both price level and nominal exchange rate. Most of the structural form of trade models assume that net export is the function of home and foreign countries’ consumption – the government surplus and net export processes are independent of both price level and nominal exchange rate. In fact, to derive this interpretation, we need the following strong assumption.

To have positive price level and exchange rate, there are two major exchange rate determination theories, one is purchasing price parity (hereafter, PPP) and the other is interest rate parity (hereafter, IRP). The concept of PPP determines the exchange rate between two currencies in order to have same purchasing power and it is related with trade. The concept of IRP determines the exchange rate in order to have same rate or return on assets in both countries and it is related with financial markets. Therefore, PPP is relevant for the long-run level of exchange rate whereas IRP is for that of short-run. However, the Leeper (2013)’s approach can cover short- and long-run of exchange rate by considering current and expected future variables. Therefore, we will employ his approach.

The amount of outstanding bond is the liabilities of government and the international reserve can be an asset of government. Therefore, in government balance sheet view, the amount of liabilities to its debtors, such as households and financial institution, can be supported by both the sum of expected present value of current and future capital and the current market value of international reserves.

Using these two equilibrium conditions, \(5\) and \(6\), we may derive the price level and nominal exchange rate as follows.

\[
P_t = (1 - \delta_t) [1 - \gamma_t - 1 + \gamma_{t-1} \alpha_{t-1}] \frac{B_{G,H,t-1}^G}{E_t PV(S) - (1 - \alpha_{t-1}) E_t PV(NX)}
\]

\[
\epsilon_t = (1 - \delta_t) (1 - \alpha_{t-1}) \frac{B_{G,H,t-1}^G}{E_t PV(S) - (1 - \alpha_{t-1}) E_t PV(NX)}
\]

where \(\gamma\) is the fraction of home government bond held by financial institution and \(\alpha = \frac{B_{F}^P}{B_{F}^F + B_{F}^P}\). Therefore, price level and exchange rate can be calculated given predetermined bond holdings, current default rate, and future surpluses and net exports. By using \(4\), we may calculate the price level and exchange rate numerically. To have positive price level and exchange rate, there are two major exchange rate determination theories, one is purchasing price parity (hereafter, PPP) and the other is interest rate parity (hereafter, IRP). The concept of PPP determines the exchange rate between two currencies in order to have same purchasing power and it is related with trade. The concept of IRP determines the exchange rate in order to have same rate or return on assets in both countries and it is related with financial markets. Therefore, PPP is relevant for the long-run level of exchange rate whereas IRP is for that of short-run. However, the Leeper (2013)’s approach can cover short- and long-run of exchange rate by considering current and expected future variables. Therefore, we will employ his approach.

14Regarding nominal exchange rate, there are two major exchange rate determination theories, one is purchasing price parity (hereafter, PPP) and the other is interest rate parity (hereafter, IRP). The concept of PPP determines the exchange rate between two currencies in order to have same purchasing power and it is related with trade. The concept of IRP determines the exchange rate in order to have same rate or return on assets in both countries and it is related with financial markets. Therefore, PPP is relevant for the long-run level of exchange rate whereas IRP is for that of short-run. However, the Leeper (2013)’s approach can cover short- and long-run of exchange rate by considering current and expected future variables. Therefore, we will employ his approach.

15The specific assumptions we employed and derivation are provided at appendix.

16As we have pointed out, we need this assumption that \(NX_t\) and \(\epsilon_t\) processes are independent of price level and nominal exchange rate. Although these assumptions are strong, we employed these assumptions because our main interest is household’s financial decision under internal drain situation. Also, depending on government surplus policy, price level and exchange rate may vary although other predetermined variables are same.
level and exchange rate, both the following conditions should hold.

\[ \mathbb{E}_t PV(S) - (1 - \alpha_{t-1})\mathbb{E}_t PV(NX) > 0, \quad \gamma_{t-1}\mathbb{E}_t PV(S) - \mathbb{E}_t PV(NX) > 0 \]

Both \( \gamma \) and \( \alpha \) should be between 0 and 1. Therefore, when the second condition are satisfied, both price level and nominal exchange rate should always be positive.

### 2.5 Equilibrium

Equilibrium consists of a set of value function \( \{V(S)\} \), a set of decision rules for consumption, home government bond holdings and foreign asset holdings \( \{c(S), B_{Ht}(S), B_{Ft}(S)\} \), aggregate inputs \( \{B_{Ht}^H(\mu), B_{Ht}^F(\mu)\} \), factor prices \( \{P(S, \mu), e(S, \mu)\} \), and a law of motion for the distribution \( \mu^* = T(\mu) \), where \( S \) is a set of state variables, \( (B_{Ht}^H, B_{Ft}^F, y^t, T, \bar{P}, \bar{q}, \bar{q}^*, e, \delta, \bar{s}) \) and \( \mu \) is the distribution, such that

1. **Individual Optimization**:
   Given \( P(S, \mu) \) and \( e(S, \mu) \), the individual household decision rules, \( c(S), B_{Ht}^H(S) \), and \( B_{Ft}^F(S) \), solve Bellman equation.

2. **The goods market clears**:
   \[ \int_0^1 c_t(S) d\mu + g_t + NX_t = \int_0^1 y_t(S) d\mu \]  
   where \( g_t \) is government spending and \( NX_t \) is net export for all \( \mu \).

3. **The home government market clears**:
   \[ \int_0^1 B_{Ht}(S) d\mu + B_{Ht}^F = B_{Ht}^G \]  
   for all \( \mu \).

4. **The home government balance sheet**
   \[ \frac{q_t B_{Ht}^G - e_t q_t^* B_{Ft}^G}{P_t} + s_t = \frac{(1 - \delta_t) B_{Ht}^G_{t-1} - e_t B_{Ft}^G_{t-1}}{P_t} \]  
   \[ (s_t - \bar{s}) = \rho_S(s_{t-1} - \bar{s}) \]  
   \[ B_{Ft}^G = \rho_{F,0} + \rho_{F,1} B_{Ft}^F \]  

5. **Price level and nominal exchange rate equations**
   \[ P_t = (1 - \delta_t) [1 - \gamma_{t-1} + \gamma_{t-1} \alpha_{t-1}] \mathbb{E}_t PV(S) - (1 - \alpha_{t-1}) \mathbb{E}_t PV(NX) \]  
   \[ e_t = (1 - \delta_t) (1 - \alpha_{t-1}) \mathbb{E}_t PV(S) - (1 - \alpha_{t-1}) \mathbb{E}_t PV(NX) \]  

6. **Policy functions**

7. **Individual and aggregate behaviors are consistent**
   \[ \mu'(B_{Ht}^0, B_{Ft}^0, \bar{Y}^0) = \int_{B_{Ht}^0, B_{Ft}^0} \left\{ \int_{B_{Ht}^0, B_{Ft}^0} 1_{B_{Ht}^0 = B_{Ht}^F(s), B_{Ft}^0 = B_{Ft}^F(s)} d\pi_y(y | y) d\mu \right\} dB_{Ht}^0 dB_{Ft}^0 dy \]  
   for all \( B_{Ht}^0 \subset B_{Ht}, B_{Ft}^0 \subset B_{Ft} \) and \( y^0 \subset y \).
2.6 Existence and Uniqueness of Equilibrium

Conventional heterogeneous agent models include goods market and one asset market (Aiyagari (1994); Krusell and Anthony A. Smith (1998); Huggett (1993)) By using Walras’ law, all we have to check is whether one of either markets clears or not. Therefore, they derive the demand and supply function of asset market and try to prove the existence and the uniqueness of equilibrium by using graphical approach. The reason why they choose the asset market to check the existence is that their models are real models, which implies that the price of goods are normalized as 1, namely fixed, and they try to find the equilibrium by changing interest rate, which is closely related with the price of asset market. Therefore, it is highly probable to choose the asset market to check the uniqueness of equilibrium.

However, we cannot employ this approach to check the existence of our model’s equilibrium. In our model, we have one more asset market so that we have to check two market clearing conditions among three. Also, because these three markets are linked, so does the demand and supply function of each market. So, we should be careful to choose which market we will focus. The prices of home government bond and foreign asset are fixed in this model so the starting point should be goods market. Regarding supply side, our economy is an endowment economy so that individual household tries to sell its endowment to goods market. So, as long as the price level is positive, the total amount of goods market supply is the total amount of endowment. However, to conclude the shape of demand function, we need to spend more time.

The conventional demand function is the monotone decreasing function in goods price. However, in our model, the price level is the function of predetermined, current and future aggregate variables. To be specific, the price level has positive relationship with outstanding home government bond and the demand function has positive relationship with asset holdings. So, the demand function may not be a monotonically decreasing function in goods price. Therefore, we cannot guarantee the uniqueness of equilibrium. However, we can easily check the amount of demand at both extreme ends. When price level goes to zero, the demand increase to positive infinity and when price level goes to positive infinity, the demand will decrease to zero. Therefore, the demand function should cross the supply function line at least once. Therefore, we can guarantee the existence of equilibrium but cannot guarantee the uniqueness.

3 Model Parameters for Benchmark Setup

The purpose of the paper is to ascertain to what extent international reserve accumulation policy in the republic of Korea may have distributional consequences through the channels that we model. Toward that end, we first seek to closely calibrate the model to the Korean Economy. Also, we used the data of United States and European Union to set the interest rate of foreign asset. The calibration sample ranges from 2000M1 to 2013M12. One period in the model is an annual. Table on next page summarize the calibrated parameters, the choice of which we will explain.

3.1 Household

We start by calibrating the household sector. We choose the period utility function to be of the log form, \( u(c) = \ln(c) \). The time-discount factor, \( \beta \), is set as 0.98 to obtain an annualized nominal return on saving of 2 to 3%.\(^{19}\) Default rate, which we denote \( \delta_t \), has a stochastic shock. There are two possible aggregate states: either the state is normal, and \( \delta = \delta_n \), or it is crisis, and \( \delta = \delta_c \). The aggregate shock follows a first-order Markov structure given by the transition

\(^{17}\)To be specific, there are three equilibrium conditions, two market clearing conditions and one aggregate resource constraint. The aggregate resource constraint includes both goods market and bond market clearing conditions because endowment can be consumed or invested without limitation. Therefore, when one market clears, the other market clears automatically.

\(^{18}\)Our model is a small open economy model and household and government can purchase foreign assets as much bonds as they want. Therefore, the equilibrium amount of foreign asset is determined by demand side. The demand function of home government bond is the decreasing function in its price as like other demand functions. And we may assume that home government will issue as long as household can support government’s decision because tax is free variable which makes the government budget constraint clear.

\(^{19}\)The interest rate of emerging economies is higher than that of advanced countries, which is the interest rate of international reserves. Therefore, the \( \beta \) should be lower than this, if our model is a closed economy. However, our economy is an open economy and household can access to foreign asset market, too. Therefore, when we set the discount factor, we have to think about the interest rate of portfolio, which consists of home and foreign assets.
an AR(1) process in logs 

With this transition matrix, the expected durations of good and bad times are

given that it is the default this period. The process for is chosen so that the average duration of both good and bad times match with data. So, the transition matrix for default probability is

\[ \Pi = \begin{bmatrix} p_{1,1} & p_{2,1} \\ p_{1,2} & p_{2,2} \end{bmatrix} = \begin{bmatrix} 0.95 & 0.05 \\ 0.2 & 0.8 \end{bmatrix} \]

With this transition matrix, the expected durations of good and bad times are 0.8 and 0.2, respectively. To set default rate, we have to think about interest rate of home government bond and foreign asset. The default rate at normal and crisis state is 1% and around 5%, respectively. Also, I set the interest rate on home government bond as 5% and that of foreign asset as 1%.²⁰

Next important thing is the household’s individual endowment. The individual endowment \( y \) is assumed to follow an AR(1) process in logs

\[ \ln y' = \rho y \ln y + \epsilon_y, \quad \epsilon_y \sim N(0, \sigma_y^2) \quad (17) \]

In this paper, we just set \( \rho_y \) and \( \sigma_y^2 \) as 0.9 and 0.01. Using these parameters, we may employ Tauchen method suggested by Tauchen (1986). We have used 3\( \sigma_y \) to cover the above stochastic process.²²

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Households</td>
<td></td>
</tr>
<tr>
<td>( \beta = 0.98 )</td>
<td>Discount Factor</td>
</tr>
<tr>
<td>( r_h = 0.05 )</td>
<td>Interest Rate on Home Government Bond</td>
</tr>
<tr>
<td>( r_f = 0.01 )</td>
<td>Interest Rate on Foreign Asset</td>
</tr>
<tr>
<td>( \rho_y = 0.9 )</td>
<td>Persistence Parameter for the log of individual endowment.</td>
</tr>
<tr>
<td>( \sigma_y = 0.01 )</td>
<td>a variance of the innovation of the log of individual endowment.</td>
</tr>
<tr>
<td>Government</td>
<td></td>
</tr>
<tr>
<td>( \tau = 0.1 )</td>
<td>Steady State Level of Tax or Subsidy</td>
</tr>
<tr>
<td>( \delta = 0.01(0.05) )</td>
<td>Default Rate for Normal(Crisis) State</td>
</tr>
<tr>
<td>( p_{s,s'} )</td>
<td>Default Shock Transition Probabilities</td>
</tr>
<tr>
<td>( \rho_S = 0.9 )</td>
<td>Persistence Parameter for Government Surplus Policy.</td>
</tr>
<tr>
<td>( \rho_{R,0} = 0.1 )</td>
<td>Level Parameter for International Reserve Policy.</td>
</tr>
<tr>
<td>( \rho_{R,1} = 0.2 \sim 0.5 )</td>
<td>Slope Parameter for International Reserve Policy.</td>
</tr>
</tbody>
</table>

²⁰I estimated the transition matrix with detrended output by using MLE. To be specific, I have used the following simple model.

\[ \Delta y_{s,t} = \nu_s + \epsilon_{s,t}, \quad \epsilon_{s,t} \sim N(0, \sigma_s) \]

where \( \Delta y \) is detrended output and \( \nu \) and are state dependent mean and error, respectively. I referred Lecture note of macroeconomics written by Park and Chang (2012). By using the estimated transition probability matrix, we may calculate the expected duration by using the following equation.

\[ \begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix} = \begin{pmatrix} p_{1,1} & p_{2,1} \\ p_{1,2} & p_{2,2} \end{pmatrix} \begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix} \]

With this constraint \( \pi_1 + \pi_2 = 1 \), we may calculate the \( \pi_1 \) and \( \pi_2 \). By inverting these two probabilities, we may calculate the average duration.

²²Although we just set the parameters for individual endowment process, we may view \( y \) to reflect a broad measure of earning ability in the market. Chang and Kim (2006) followed this view and estimated the stochastic process of \( y \) by using the individual wages. Their model setup is slightly different from our model. They introduced individual productivity instead of individual endowment and suggested the way to estimate the productivity process by using wage process. They assumed that individual productivity process are as follows.

\[ \ln x' = \rho_x \ln x + \epsilon_x, \quad \epsilon_x \sim N(0, \sigma_x^2) \]
The last thing we have to consider is the sum of present value of current and future net exports. As we can see to have positive price level and nominal exchange rate, we have some limits on this value. So, we assume that the value of this term is \( \frac{0.01}{1-\beta} \) at normal state and \( \frac{0.001}{1-\beta} \) at crisis state.

### 3.2 Central Bank and Government

Central bank and government share budget constraint, which implies consolidated budget constraint model. As our model does not include interest rate rule, the central bank implements only the international reserve accumulation policy. We assume the simple rule with two policy parameters: one parameter sets the minimum level of international reserves and the other parameter sets the responsiveness to the household’s foreign asset holdings. We set the minimum level parameter as 0.1 and change the slope parameter from 0.2 to 0.5 with increment 0.1. As we have pointed out above, results of this study leave more to be investigated and answered, but they do throw some doubt on claim of this specific policy form.

By the way, government set the amount of government surpluses. According to ’2014 Index of Economic Freedom’, the tax to GDP of Republic of Korea is 25.9% and government spending to tax is 30.2%, which implies government deficit. So, it is hard to tell the steady state level of government surplus is positive. However, to get the solution of this model with positive price level and nominal exchange rate, we set steady state level of government surplus as 0.1.

### 4 Computation

To get the equilibrium of the heterogeneous agent model, we have to consider two major factors: one is the policy functions and the other is the distributions of heterogeneous households. We will examine why these two things are important in solving heterogeneous agent models by comparing with representative agent models. Under the representative model, the aggregate variable is the household’s individual decision variables or state variables. Therefore, we do not have to care about how to get the aggregate variables to check whether equilibrium conditions holds. However, under the heterogeneous agent model, each individual household’s decision is different and we need the distribution of households to get the aggregate variables. The difficult problem is that distribution can be obtained after aggregating policy functions and, to get the policy function, we need the price level and nominal exchange rate, which are the functions of aggregate variables, namely the functions of distribution. Therefore, there are several approaches to overcome this problem and this is the key difference between representative and heterogeneous agent model.

#### 4.1 Policy Functions

Although the computation power has been increase rapidly, the method to increase the rate of convergence is still major research topics. We may classify these studies into two: one is modifying the problem to reduce the number of state and decision variables and the other is employing advanced techniques using the characteristics of objective function such as monotonicity or concavity. Because our model has many state variables, spending time on these methods is worthwhile to increase the efficiency of the problem.

Let’s examine how to reduce the number of variables, first. There are two sorts of variable, decision and state variable. Our main interest is portfolio rebalancing and, therefore, it is hard to diminish the number of decision variables. By their model, the individual log wage can be written as \( \ln w_i^t = \ln w_t + \ln x_i^t \). By quasi-differencing, individual wage involves as follows.

\[
\ln w_i^t = \rho_x \ln w_i^{t-1} + (\ln w_t - \rho_x \ln w_{t-1}) + \epsilon_{x,t}
\]

We may estimate with time-dummy variables replacing the aggregate price term, \( \ln w_t - \rho_x \ln w_{t-1} \). To avoid bias from self-selection, they get estimates \( \rho_x \) and \( \sigma_x \) by using the maximum likelihood(ML) estimate of Heckman (1979).

\(^{23}\)A representative agent model is a heterogeneous agent model which uses Dirac delta function as its distribution of households. According to Kreyszig (1999), Dirac delta function, or \( \delta \) function, is a generalized function or distribution on the real number line that is zero everywhere except at one real number, with an integral of one over the entire real line.
variable. So, we will focus on reducing the number of state variables. To solve the value function, price level, nominal exchange rate, the interest of home government bond, the interest rate of foreign asset and default rate should be given. Using (7) and (8), equilibrium price level and nominal exchange rate can be calculated. Also, other variables are obtained by the parameter that we have calibrated above. So, we can solve this problem numerically and we will construct more simple model by using cash-at-hand method.

\[ V^j(x^j, y^j; B^H_{F,t}, B^F_{F,t}, \delta_s) = \max_{B^H_{F,t}, B^F_{F,t}} \{ u(c^j) + \beta E_t V^j(x^j, y^j; B^H_{F,t}, B^F_{F,t}, \delta_s) \} \]
\[ x^j = P y^j + B^H_{F,t} + e B^F_{F,t} \]
\[ x^j' = P' y^j + B^H_{F,t}' + e' B^F_{F,t}' \]
\[ B^H_{F,t}' = 0 \text{ with probability } \delta'(\delta_s) \]

By constructing the model in this way, we may reduce the number of state variables from 9 to 6. It gives a lot of numerical benefit and usually increase the accuracy. However, to use this method, we need assumption that household does not prefer the return on home government asset to that of foreign assets and vice versa. In other words, the source of cash at hands does not matter to households when they make a portfolio decision and a consumption decision.

Gordon (2013) provides many useful method suitable with cash-at-hand method. Given the home government holdings, we may get the optimal holdings of foreign assets. After then, we may determine the amount of home government and foreign asset holdings by comparing all these asset portfolio combinations. When we calculate the holdings, we may get the optimal holdings of foreign assets. After then, we may determine the amount of home government surplus, \( G \), to get these variables in advance. Regarding next period’s variables, we may deal with the law of motion for aggregate variables. However, it is still difficult to get price level and nominal exchange rate because it includes current period’s government surplus, \( S \). One way to deal with this problem is using price level and nominal exchange rate equations. By constructing the model in this way, we may reduce the number of state variables from 9 to 6. It gives a lot of numerical benefit and usually increase the accuracy. However, to use this method, we need assumption that household does not prefer the return on home government asset to that of foreign assets and vice versa. In other words, the source of cash at hands does not matter to households when they make a portfolio decision and a consumption decision.

4.2 Modified Individual Household’s Budget Constraint

Although we have examined the existence of equilibrium, it is difficult to solve the problem with tax, which is free variable. So, we will assume that tax is lump-sum. Based on this assumption, we will modify the individual household’s budget constraint by substituting (4).

\[ P_t c^j_t + q_t B^H_{H,t} + e_t q_t' B^H_{F,t} \leq P_t y^j_t + B^H_{F,t-1} + e_t B^j_{F,t-1} - (1 - \delta_t) B^G_{H,t-1} + e_t B^G_{F,t-1} + q_t B^G_{H,t} - e_t q_t' B^G_{F,t} \]

To get this budget constraint, we have to know several things in advance such as next period’s aggregate variables, \( B^G_{H,t}, B^G_{F,t} \), and the price level and nominal exchange rate, \( P_t \) and \( e_t \). As we have pointed out above, it is impossible to get these variables in advance. Regarding next period’s variables, we may deal with the law of motion for aggregate variables. However, it is still difficult to get price level and exchange rate because it includes current period’s government surplus, \( S \). One way to deal with this problem is using price level and nominal exchange rate equations. By substituting equation (7) and (8) into equation (3), we may derive the following equations.

\[ P_t = (1 - \delta_t) [ 1 - \gamma_{t-1} + \gamma_{t-1} \alpha_{t-1} ] \frac{B^G_{H,t-1}}{1 - \gamma_{t-1} \alpha_{t-1}} + \beta E_t PV(NX) \]
\[ e_t = (1 - \delta_t) [ 1 - \alpha_{t-1} ] B^G_{F,t-1} \frac{\gamma_{t-1} s_t + \gamma_{t-1} \beta(1-\rho_S) s_t}{1 - \beta_{t-1} \gamma_{t-1} \alpha_{t-1}} - (1 - \rho_S) E_t PV(NX) \]

If we construct model according to asset pricing literature, then we can reduce the number of decision variables by changing the problem with household who determines the total amount of portfolio. That is because the ratio is determined by the expected return and volatility of assets and the amount of each asset can be calculated by multiplying the ratio with the total amount of portfolio. However, getting the variance of the rate of return of foreign asset is too difficult to calculate because of next period’s exchange rate. Therefore, we may not employ this approach.

Judd (1998) provides many advanced techniques suitable for multivariate problems. Although we cannot guarantee that these method outperform forms that we have used here because our policy function is not sufficiently smooth, we may try these techniques in near future to convince this intuition.
where

\[ s_t = \frac{(1 - \beta S_t)(1 - \alpha_{t-1}) \Phi_2 \Phi_1}{(1 - \delta t)(1 - \beta S_t)(1 - \gamma_{t-1} + \gamma_{t-1} \alpha_{t-1})} \]

\[ \Phi_1 = q_t B_{H,t}^G - (1 - \delta t) B_{H,t-1}^G + (1 - \delta t)(1 - \alpha_{t-1}) B_{H,t-1}^G \gamma_{t-1} (B_{F,t-1}^G q_t - q_t B_{F,t}^G) \]

\[ \Phi_2 = q_t B_{H,t}^G - (1 - \delta t) B_{H,t-1}^G + (1 - \delta t)(1 - \alpha_{t-1}) B_{F,t-1}^G \gamma_{t-1} (B_{F,t-1}^G q_t - q_t B_{F,t}^G) \]

As we can see above, given the sum of present expected value of net exports and other parameters, we may derive all the equations. By using these equations, we may start to solve the value function to get the policy functions. Derivation details are provided at appendix.

### 4.3 Stable Distribution

After getting policy function, we have to calculate the stable distribution of household. If the model does not include aggregate uncertainty, we may assume that we have invariant distribution. However, our model has aggregate uncertainty due to time varying default rate, we cannot assume that distribution is invariant but we can assume that distribution is stable, instead. Approximating the equilibrium in the presence of aggregate fluctuations requires us (i) to include the measure of workers and the default shock in the list of state variables and (ii) to keep track of the evolution of the measure \( \mu \) over time. Since \( \mu \) is an infinite dimensional object, it is almost impossible to implement these tasks as they are. As we have pointed out above, it is well known that numerical solution of dynamic programming problems becomes increasingly difficult as the size of the state space increases. Krusell and Anthony A. Smith (1998) suggested the way to reduce the state space by using the moments. This method based on the assumption that distribution can be sufficiently forecasted by moments and households are boundedly rational in their perceptions of how \( \mu \) evolves over time. Also, they suggest that we have to increase the sophistication of these perceptions until the errors that households make become negligible. So, we will follow the procedure suggested by Krusell and Anthony A. Smith (1998): agents are assumed to make use of its first moment only in predicting the law of motion for \( \mu \). Therefore, computing the equilibrium with aggregate fluctuations amounts to finding the value functions, decision rules, and law of motion for the aggregate assets in \( B_{H}^G \) and \( B_{F}^G \). Although we provides the detailed procedure in the appendix, we will examine the law of motion for the aggregate asset holdings here.

\[ B_{H,t}^G = C_1 + D_{1,1} B_{H,t-1}^G + D_{1,2} B_{F,t-1}^G \]

\[ B_{F,t}^G = C_2 + D_{2,1} B_{H,t-1}^G + D_{2,2} B_{F,t-1}^G \]

Following Krusell and Anthony A. Smith (1998), the aggregate law of motions for both assets are state dependent, \( \delta_0 \) and \( \delta_0 \). Therefore, we have total four aggregate law of motions. At initial stage, we will set the coefficients with plausible criterion. Every iteration, we will get simulated paths based on policy functions and update the coefficients until it converges enough.

### 4.4 Set Grid and Steady State Value

The grids that we have used to solve are supported at the above table. We set more grids for home government bond than that of foreign asset because the range of home government bond holdings is wider than that of foreign asset.

\(^{26}\) After getting policy function, we have to calculate the stable distribution of household. If the model does not include aggregate uncertainty, we may assume that we have invariant distribution. Under this assumption, we can solve with following steps.

1) Set the grid for state and decision variables.
2) Get the policy functions by iterating value function.
3) Set initial distribution and update with the policy functions that we have obtained above. If it converges, go to next step. Otherwise update again.
4) Calculate the aggregate variables using policy function and invariant distribution. Check whether the market equilibrium conditions hold. If it holds, we reach the equilibrium and stop. Otherwise go to the first step with different market prices such as interest rate, wage rate.

\(^{27}\) This is main difference between invariant distribution assumption and stable distribution assumption. As we have seen above, if we can assume invariant distribution, we do not have to simulate the paths whereas simulation is essential step to get the equilibrium under stable distribution assumption.
Table 2: Grid for Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Range</th>
<th>Number of Grids</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_j^H$</td>
<td>Ind. Households’ Home Gov. Holdings</td>
<td>[0, 0.50]</td>
<td>60</td>
</tr>
<tr>
<td>$B_j^F$</td>
<td>Ind. Households’ Foreign Asset Holdings</td>
<td>[0, 0.12]</td>
<td>20</td>
</tr>
<tr>
<td>$y_i^j$</td>
<td>Ind. Households’ Endowment</td>
<td>$3\sigma_y$</td>
<td>7</td>
</tr>
<tr>
<td>$B_F^H$</td>
<td>Agg. Household’s Foreign Asset</td>
<td>[0.97, 1.03]</td>
<td>3</td>
</tr>
<tr>
<td>$B_H^H$</td>
<td>Agg. Household’s Home Gov. Bond</td>
<td>[0.97, 1.03]</td>
<td>3</td>
</tr>
</tbody>
</table>

The reason why we set in this way is that stable nominal exchange rate is over 10. It implies that the grid for foreign asset covers the wider range than that of home government bond covers when we multiply stable exchange rate. Under internal drain, household tends to buy more foreign asset and our model which covers different range of assets may be justified. As we have noticed above, to get the grid for individual households endowment, we used Tauchen method with $3\sigma_y$.

The other aggregate variables should be carefully determined because it is closely related with price level, nominal exchange rate and current period’s government surplus, which affects policy functions. Also, getting the theoretical value of aggregate variables at steady state is quite difficult. So, based on steady state conditions, we derived the steady state values numerically. The details are provided at appendix.

4.5 Simulation

With obtained policy function, we may simulate the paths with many agents with different asset holdings and endowments. After simulating the agents, we may update the aggregate variables by summing all the individual household’s decision variables with proper weight. Given that aggregate variables, we may simulate the paths again and, in the end, we can get multiple times series, which consists of aggregate variables. Using these multiple time series, we may update the coefficients for the law of motion. We will iterates these steps until the coefficients converges. While updating the coefficients, we should be careful to avoid the following problems.

The first problem is that simulated multiple time series are affected by initial guess on distribution. If we overlook this issue, then we may get biased results so that we usually simulate the sufficient length of periods and then cut off the first initial periods. To our knowledge, there is no certain rule for this but we set the length of simulation as 5500 periods and then cut off the first 500 periods.

The second problem is that individual shock may play a role as aggregate shock. In representative agent model, all the shocks are aggregate shock and there is no individual shock. So, they cannot say about aggregation failure. However, by introducing heterogeneous agent model, we may distinguish the aggregation failure with market failure. To utilize this advantage of heterogeneous model, we have to simulate with many individual agents to avoid this issue and we simulated with around 30000 agents.

The third problem might happen when we compare the policies. One of our aim is to examine the influence of central bank’s international reserve accumulation policy to household’s financial decision. To do this, we control everything except the policy parameters of central bank’s policy. So, we fixed seed number as 123456789, which are used to generate random number. By doing in this way, we may eliminate the risk which comes from the difference in randomness of simulated paths when we compare the policies.\(^{28}\)

\(^{28}\)When we change the grid for these aggregate variables, we used the certain criterion, which is that both price level and nominal exchange rate should be positive under parameters and specific functions that we set. We may find other sophisticated criterions to refine our initial guesses. However, we shall not dwell here on those sophisticated methods.

\(^{29}\)To check robustness, I used different seed number, namely the computer time when program starts. But, this does not change the main results that we have obtained here.
5 Numerical Results

The common consensus among studies on international reserves is that the higher reserves helped reduce the impact of the crisis by supporting financial independent status. This statement has been supported by many theoretical models which focus on external drain issues. Therefore, we will examine whether our model, which includes the portfolio rebalancing issue, can support or undermine this statement.

<table>
<thead>
<tr>
<th>Table 3: Portfolio Ratio ((\frac{\text{Home Gov. Bond}}{\text{Home Gov. Bond + Foreign Asset}}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope 0.2</td>
</tr>
<tr>
<td>Normal</td>
</tr>
<tr>
<td>High</td>
</tr>
<tr>
<td>Medium</td>
</tr>
<tr>
<td>Low</td>
</tr>
</tbody>
</table>

From this table, we may say that households change their portfolio depending on default shock. When home government bond becomes more risky, they will increase the portion of foreign asset and decrease the portion of home government bond. This is similar to the internal drain situation and we may say that our model replicate the internal drain situation quite well.

5.1 The Effect of International Reserve Accumulation Policy on Portfolio Rebalancing

Slope parameter implies how much the central bank responds to the household’s aggregate foreign asset holdings. The interesting point is that the large amount of international reserves with higher parameters cause a little change in portfolio holdings at crisis although the crucial reason for holding reserves is to prevent the surge in exchange rate at the crisis. However, with higher parameters, household purchases more home government bond at normal state. From the asset pricing literature, we may find the clue for explaining this symptom. The main objective of household is to maximize the utility by smoothing consumption. Therefore, households prefer the asset which is helpful for them to smooth consumption. For example, let’s assume that there are two risky assets, which bear state-dependent returns.

\[
\begin{align*}
\text{Rate of Return on Asset 1:} & \begin{cases} 
\text{Normal:} & 5\\ 
\text{Crisis:} & 1
\end{cases} \\
\text{Rate of Return on Asset 2:} & \begin{cases} 
\text{Normal:} & 1\\ 
\text{Crisis:} & 5
\end{cases}
\end{align*}
\]

If household is allowed to access both asset markets, then household will choose asset 2. At good state, household may receive higher income and, with high probability, household can consume smoothly without any help from the return on asset. However, at bad state, household may receive lower income and, with high probability, household cannot consume smoothly without the return on asset. This theory can be employed to explain our model. Exchange rate tends to increase at the crisis state. Then, the expected rate of return on foreign asset increases at crisis and decreases at normal even though the interest rate on foreign asset is fixed. However, the rate of return on home government bond is totally opposite. At good state, the default rate is low so that the rate of return on home government bond is high whereas at bad state, the rate of return on that is low. Therefore, foreign asset is more helpful asset to smooth consumption so that household has an incentive to hold foreign asset although the expected rate of return on foreign asset is lower than that of home government bond.

Also, this explanation can be supported by the upward trend with an increase in policy parameters. With higher reserves, the difference in exchange rate between normal and crisis is getting smaller because higher reserve can make more downward pressure on exchange rate at crisis. It implies that the benefit of holding foreign asset decreases when the central bank responds more strictly to household’s foreign asset holdings.
5.2 The Effect of International Reserve Accumulation Policy on Inequality

The heterogeneous agent model has received relatively little attention so that the effect of policy on inequality has never been a favorite of the economists, even among central bankers. However, all the policy can have the effect on inequality. For example, when interest rate increases, debtors have to repay more and creditors can get some benefit. So, wealth transfers from the debtors to the creditors. If interest rate decrease, exactly opposite situation happens. In this circumstances, where households with different wealth make their own decision in a different way, which causes the change in wealth distribution, examining the effect of policy shock to aggregate variables with representative agent model may cause substantial inaccuracy because, in this situation, there is high probability of getting aggregation failure. Therefore, in this section, we shall see the effect of policy on inequality.

At the outset, it is imperative to clarify that our model does not include policy shock so, it is impossible to draw the impulse response functions to policy shocks. However, with different policy parameter, we may get the difference in individual household’s behavior and, based on this, we will try to examine the effect of policy on inequality.

To check the inequality issue, we will calculate the variance of consumption. The summary is provided at the table 4. As we can see, the variance of consumption at both normal and crisis state decreases as the central bank responds strictly to the household’s aggregate foreign asset holdings. From figure 2, we may check this graphically. According to figure 2, the gap between the consumption of the rich and the poor decreases. That is because individual household with a little asset holdings consumes more whereas individual household with a large amount of asset holdings consumes less as the central bank responds more strictly to household’s foreign asset holdings.

Table 4: Standard Variation of Consumption with Different $\rho_s$.

<table>
<thead>
<tr>
<th>State</th>
<th>$\rho_s = 0.2$</th>
<th>$\rho_s = 0.3$</th>
<th>$\rho_s = 0.4$</th>
<th>$\rho_s = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>0.8708</td>
<td>0.7496</td>
<td>0.6104</td>
<td>0.4611</td>
</tr>
<tr>
<td>Crisis</td>
<td>0.1173</td>
<td>0.0868</td>
<td>0.0625</td>
<td>0.0444</td>
</tr>
</tbody>
</table>

We may explain this symptom with two reasons. As the international reserve increases, not only nominal exchange rate but also price level decreases. With a decrease in price level, household may enjoy more amount of real consumption. However, with decreased nominal exchange rate, the coverage for foreign asset holdings decreases and, therefore, individual household’s total nominal asset holdings decreases. So, with a decrease in nominal exchange rate, household may enjoy less amount of real consumption. So, we may conclude that, for the poor, price level effect is higher than that of nominal exchange rate effect whereas, for the rich, nominal exchange rate effect is higher than that of price level effect.

Table 5: Mean of Consumption with Different $\rho_s$.

<table>
<thead>
<tr>
<th>State</th>
<th>$\rho_s = 0.2$</th>
<th>$\rho_s = 0.3$</th>
<th>$\rho_s = 0.4$</th>
<th>$\rho_s = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>0.8499</td>
<td>0.7655</td>
<td>0.7040</td>
<td>0.6556</td>
</tr>
<tr>
<td>Crisis</td>
<td>0.3073</td>
<td>0.2505</td>
<td>0.2071</td>
<td>0.1729</td>
</tr>
</tbody>
</table>

If we employ asset pricing view, we may have another story for this situation. When the central bank accumulates more international reserves, the exchange rate fluctuation decreases, which implies that foreign assets becomes a less attractive asset for household. Therefore, household may want to purchase home government bond instead of foreign

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$^{30}$I tried to introduce monetary policy with monetary policy shock in this model. However, it was too complex for me to solve in a short time. I will extend this paper someday by introducing monetary policy with policy shock. Gornemann et al. (2012) suggested way to combine heterogeneous agent model with the New Keynesian model. Also, they introduced monetary authority which sets the policy interest rate. Using this model, they derived the effect of technology shock to aggregate variables and to inequality. Although it is very decent approach, they did not include the price level in the household’s optimization problem.
asset and it cause change in government budget constraint. In this view, government can issue more bonds and may decrease the total amount of tax. However, since this scenario happens when central bank accumulate more international reserves, central bank may want to increase tax. Therefore, the effect of international reserve accumulation policy on total amount of tax depends on parameters. According to our model, total utility from consumption decreases as the central bank increase the policy parameters.

6 Conclusion

This paper starts with the following question: "Should monetary authority hold foreign international reserves?" To answer this question, we construct the model which can examine the effect of international reserves under internal drain. Based on our model, we can give a positive answer to this question at least if the central bank tries to hedge the internal drain. Internal drain happens when the consumer is captivated by fear and, therefore, financial institution loses its efficiency.

To reflect this feature, we build a model with heterogeneous agent model and international reserve bearing central bank. Moreover, we develop international reserve accumulation policy and examined the effect of this policy to aggregate variables and inequality. From this model, we may say that higher international reserves can help to prevent the severe portfolio rebalancing under internal drain situation. That is because a foreign asset becomes less attractive as the nominal exchange rate move smoothly.

In addition, we may find another channel that international reserve can affect the household’s inequality. Higher reserves can decrease the price level and nominal exchange rate. These two changes affects the household’s consumption in opposite direction. When price level decreases, household may enjoy more real consumption. However, when nominal exchange rate decreases, the nominal value of foreign asset holdings decreases and, therefore, household have to lessen their consumption. In sum, for the poor, the effect of price change is higher than that of nominal exchange rate change whereas, for the rich, the opposite is true. Therefore, higher reserves is helpful to mitigate the consumption gap.

However, it would be serious oversight to conclude that accumulating international reserve is the panacea for financial independence and consumption inequality. That is because the sum of individual utilities decreases as the central bank responds more strictly, which implies that total consumption decreases. Therefore, we have to be careful when we set the policy parameters. Therefore, results of this study leave more to be investigated and answered.
References


## A Data

We visited the web pages of three central banks, Federal Reserve Bank at St. Louis, European Central Bank and the Bank of Korea. The web site address are as follows.

<table>
<thead>
<tr>
<th>Central Bank</th>
<th>Address</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRB</td>
<td><a href="http://research.stlouisfed.org/fred2/">http://research.stlouisfed.org/fred2/</a></td>
</tr>
<tr>
<td>BOK</td>
<td><a href="http://ecos.bok.or.kr/">http://ecos.bok.or.kr/</a></td>
</tr>
<tr>
<td>IMF</td>
<td><a href="http://www.imf.org/">http://www.imf.org/</a></td>
</tr>
<tr>
<td>WB</td>
<td><a href="http://data.worldbank.org/">http://data.worldbank.org/</a></td>
</tr>
<tr>
<td>KRX</td>
<td><a href="http://www.krx.co.kr/">http://www.krx.co.kr/</a></td>
</tr>
</tbody>
</table>

At ECOS site, we downloaded the interest rate of one-year-maturity government bond and calculated the mean of the interest rate of home government bond. Also, we visited KRX site to examine the portion of foreign investors. The time span is from February 2000 to December 2013. At FRB site, we downloaded the interest rate of one-year maturity bond to obtain the interest rate of foreign asset. Also, we checked European data at ECB site to calculate the interest rate. That is because as we can see from the below graph, around 30% of international currency is Euro. From IMF, we downloaded the countries’ international reserves holdings and, from The World Bank, we downloaded the countries’ GDP.

![Reserve Currency Graph](image)

Figure 1: Reserve Currency

The original data source of this graph is IMF and I downloaded this graph from Wikepedia.

## B Derivation of Several Equations in our model

In this section, we will provide several derivation details that we have used in the paper. The first thing is the modified household’s budget constraint. By combining the household’s budget constraint with government budget constraint,
we may get the modified household’s budget constraint. The second thing is the derivation of the intertemporal equilibrium conditions. While deriving the intertemporal equilibrium conditions, we will see how we can connect the aggregate discount factor with these two intertemporal equilibrium conditions. The last thing is the derivation of current period’s government surplus. We will substitute price equation and nominal exchange rate equation into government budget constraint. After then, we will get the simplified equation.

### B.1 Deriving Modified Household’s Budget Constraint under Equilibrium

The budget constraint of Household with lump-sum tax is

\[ P_t c_t^i + q_t B_{H,t}^j + e_t q_t^* B_{F,t}^j + P_t t_t \leq P_t y_t^i + B_{H,t-1}^j + e_t B_{F,t-1}^j \]

With zero government consumption, we may have the following government budget constraint.

\[ P_t t_t = (1 - \delta_t) B_{H,t-1}^G - e_t B_{F,t-1}^G - q_t q_t^* B_{F,t}^G \]

With these two assumptions, we may get the following individual household’s budget constraint under equilibrium.

\[ P_t c_t^i + q_t B_{H,t}^j + e_t q_t^* B_{F,t}^j \leq P_t y_t^i + B_{H,t-1}^j + e_t B_{F,t-1}^j - (1 - \delta_t) B_{H,t-1}^G + e_t B_{F,t-1}^G + q_t B_{H,t}^G - e_t q_t^* B_{F,t}^G \]

### B.2 Deriving Intertemporal Equilibrium Conditions

The GBC is

\[ \frac{(1 - \delta_t) B_{H,t-1}^G - e_t B_{F,t-1}^G}{P_t} = \frac{q_t B_{H,t}^G - e_t q_t^* B_{F,t}^G}{P_t} + s_t \]

By iterating one period forward,

\[ \frac{(1 - \delta_t) B_{H,t-1}^G - e_t B_{F,t-1}^G}{P_t} = \mathbb{E}_t q_t \frac{P_{t+1}}{P_t} \frac{1}{1 - \delta_{t+1}} \frac{(1 - \delta_{t+1}) B_{H,t}^G - (1 - \delta_{t+1}) q_{t+1} q_t^* B_{F,t}^G}{P_{t+1}} + s_t \]

Let’s assume that the following equation holds, which is UIRP.

\[ \frac{1}{q_t} \mathbb{E}_t (1 - \delta_{t+1}) = \frac{1}{q_t} \mathbb{E}_t \frac{e_{t+1}}{e_t} \]

and set

\[ q_{t,t+1} = \mathbb{E}_t \frac{P_{t+1}}{P_t} \frac{q_t}{1 - \delta_{t+1}} \]

\[ q_{t,t+k} = \mathbb{E}_t \frac{P_{t+k}}{P_t} \prod_{i=0}^{k-1} \frac{q_{t+i}}{1 - \delta_{t+i+1}} \]

By iterating forward and using the definitions above, we may get the following equation.

\[ \frac{(1 - \delta_t) B_{H,t-1}^G - e_t B_{F,t-1}^G}{P_t} = \mathbb{E}_t q_{t,t+1} \frac{(1 - \delta_{t+1}) B_{H,t}^G - e_{t+1} B_{F,t}^G}{P_{t+1}} + s_t \]

\[ = \mathbb{E}_t q_{t,t+1} \left[ \mathbb{E}_{t+1,q_{t+1,t+2}} \frac{(1 - \delta_{t+2}) B_{H,t+1}^G - e_{t+2} B_{F,t+1}^G}{P_{t+2}} + s_{t+1} \right] + s_t \]

\[ = \sum_{i=0}^{\infty} \mathbb{E}_t q_{t,i+is_{t+i}} + \lim_{T \to \infty} \mathbb{E}_t q_{t,T} \frac{(1 - \delta_{T+T}) B_{H,T+T-1}^G - e_{T+T} B_{F,T+T-1}^G}{P_{T+T}} \]
With the following transversality condition,

$$\lim_{T \to \infty} \mathbb{E}_t q_{t,t+T} \frac{(1 - \delta_{t+T})B^G_{H,t+T-1} - \epsilon_{t+T}B^G_{F,t+T-1}}{P_{t+T}} = 0$$

we can get the following intertemporal equilibrium condition that we have used in the paper.

$$\frac{(1 - \delta_t)B^G_{H,t-1} - \epsilon_tB^G_{F,t-1}}{P_t} = \sum_{i=0}^{\infty} \mathbb{E}_t q_{t,t+i} s_{t+i}$$

Next thing is the intertemporal equilibrium condition derived from household’s budget constraint. The first step is aggregating the individual household’s budget constraint. The integrated household’s budget constraint is as follows.

$$P_t \int c_i^t \, d\mu + q_t \int B^H_{i,t} \, d\mu + \epsilon_t \int q_i^t \, d\mu = P_t \int y_i^t \, d\mu + \int B^H_{i,t-1} \, d\mu + \epsilon_t \int B^H_{i,t-1} \, d\mu$$

Then, we may have the following expression.

$$P_tC_t + q_tB^H_{H,t} + \epsilon_t q_t^tB^H_{F,t} + P_tT_t = P_tY_t + (1 - \delta_t)B^H_{H,t-1} + \epsilon_tB^H_{F,t-1}$$

To proceed, we need the following assumption.

$$Y_t = C_t + G_t + NX_t$$

where $G_t$ is government spending and $NX_t$ is net exports. With this assumption, we may have the following expression.

$$\frac{(1 - \delta_t)B^H_{H,t-1} + \epsilon_tB^H_{F,t-1}}{P_t} = s_t + NX_t + \frac{q_tB^H_{H,t} + \epsilon_t q_t^tB^H_{F,t}}{P_t}$$

By iterating forward, we may have the following expression.

$$\frac{(1 - \delta_t)B^H_{H,t-1} + \epsilon_tB^H_{F,t-1}}{P_t} = \sum_{i=0}^{\infty} \mathbb{E}_t q_{t,t+i} (s_{t+i} + NX_{t+i}) + \lim_{T \to \infty} \mathbb{E}_t q_{t,t+T} \frac{(1 - \delta_{t+T})B^H_{H,t+T-1} + \epsilon_{t+T}B^H_{F,t+T-1}}{P_{t+T}}$$

With TVC, we may have the following condition.

$$\frac{(1 - \delta_t)B^H_{H,t-1} + \epsilon_tB^H_{F,t-1}}{P_t} = \sum_{i=0}^{\infty} \mathbb{E}_t q_{t,t+i} (s_{t+i} + NX_{t+i})$$

Let’s link these intertemporal equilibria conditions with aggregate discount factor. As we have seen in the paper, the aggregate discount factor under equilibrium is

$$Q_{t,t+1} = \beta \mathbb{E}_t (1 + \delta_{t+1}) \frac{P_t}{P_{t+1}}$$

and this value should be equal to $q_t$, which is the price of home government bond. The reason why we have fixed aggregate discount factor is that the interest rate of home government bond is constant. So, by substituting this equation into the definition, $q_{t,t+1}$, we can get the following equation.

$$q_{t,t+1} = \mathbb{E}_t \frac{P_{t+1}}{P_t} \frac{1}{1 - \delta_{t+1}} \beta (1 + \delta_{t+1}) \frac{P_t}{P_{t+1}} = \beta$$

Therefore, in sum, we have the following intertemporal equilibrium conditions.

$$\frac{(1 - \delta_t)B^H_{H,t-1} + \epsilon_tB^H_{F,t-1}}{P_t} = \sum_{i=0}^{\infty} \mathbb{E}_t \beta^i (s_{t+i} + NX_{t+i})$$

$$\frac{(1 - \delta_t)B^G_{H,t-1} - \epsilon_tB^G_{F,t-1}}{P_t} = \sum_{i=0}^{\infty} \mathbb{E}_t \beta^i s_{t+i}$$

20
B.3 Deriving the current government surplus

To get current period’s government surplus, we have to use three equations, government budget constraint, price level and exchange rate equations. To be specific, we will use the following equations.

\[ q_t B_{H,t}^G - \epsilon_t q_t^* B_{F,t}^G + P_t s_t = (1 - \delta_t) B_{H,t-1}^G - \epsilon_t B_{F,t-1}^G \]

\[ P_t = (1 - \delta_t) \left[ 1 - \gamma_{t-1} + \gamma_{t-1} \alpha_{t-1} \right] \frac{B_{H,t-1}^G}{E_t PV(S) - (1 - \alpha_{t-1}) E_t PV(NX)} \]

\[ \epsilon_t = (1 - \delta_t) \left[ 1 - \alpha_{t-1} \right] \frac{B_{H,t-1}^G}{B_{F,t-1}^G} \gamma_{t-1} E_t PV(S) - E_t PV(NX) \]

After substituting these two price related equations into the above equation, we may get the following equation.

\[ E_t PV(s) \left[ q_t B_{H,t}^G - (1 - \delta_t) B_{H,t-1}^G + (1 - \delta_t) (1 - \alpha_{t-1}) \right] \frac{B_{H,t-1}^G}{B_{F,t-1}^G} \gamma_{t-1} \left( B_{F,t-1}^G - q_t^* B_{F,t}^G \right) \]

\[ + (1 - \delta_t) \left[ 1 - \gamma_{t-1} + \gamma_{t-1} \alpha_{t-1} \right] B_{H,t-1}^G s_t \]

\[ = E_t PV(NX) \left[ (1 - \alpha_{t-1}) \left( q_t B_{H,t}^G - (1 - \delta_t) B_{H,t-1}^G \right) + (1 - \delta_t) (1 - \alpha_{t-1}) B_{H,t-1}^G (B_{F,t-1}^G - q_t^* B_{F,t}^G) \right] \]

We have to deal with many current and future variables which are not pre-determined variables. Let’s think about the present value of net export related terms. We assumed that present value of net exports may have two values, $\frac{0.001}{1-\beta}$ at normal and $\frac{0.001}{1-\beta}$ at crisis to reflect the trade deficit at crisis. Next thing is the sum of present value of government surpluses.

\[ E_t PV(S) = \sum_{k=0}^{\infty} \frac{\beta^k (s_{t+k} - \bar{s})}{1 - \beta \rho_S} + \sum_{k=0}^{\infty} \frac{\beta^k \bar{s}}{1 - \beta \rho_S} \]

By substituting this equation into the above equation, after some algebra, we may get the following equation.

\[ \frac{s_t}{1 - \beta \rho_S} \left[ (1 - \delta_t) (1 - \beta \rho_S) (1 - \gamma_{t-1} + \gamma_{t-1} \alpha_{t-1}) \right] B_{H,t-1}^G \Phi_1 \]

\[ = (1 - \alpha_{t-1}) \Phi_2 E_t PV(NX) - \frac{\beta (1 - \rho_S) \bar{s}}{1 - \beta \rho_S} \Phi_1 \]

where

\[ \Phi_1 = q_t B_{H,t}^G - (1 - \delta_t) B_{H,t-1}^G + (1 - \delta_t) (1 - \alpha_{t-1}) \frac{B_{H,t-1}^G}{B_{F,t-1}^G} \gamma_{t-1} \left( B_{F,t-1}^G - q_t^* B_{F,t}^G \right) \]

\[ \Phi_2 = q_t B_{H,t}^G - (1 - \delta_t) B_{H,t-1}^G + (1 - \delta_t) \frac{B_{H,t-1}^G}{B_{F,t-1}^G} \left( B_{F,t-1}^G - q_t^* B_{F,t}^G \right) \]

Therefore, we may get the current period government surplus which satisfies the government budget constraint and other price related equations.

\[ s_t = \frac{(1 - \beta \rho_S) (1 - \alpha_{t-1}) \Phi_2 E_t PV(NX) - \frac{\beta (1 - \rho_S)}{1 - \beta \rho_S} \Phi_1}{(1 - \delta_t) (1 - \beta \rho_S) (1 - \gamma_{t-1} + \gamma_{t-1} \alpha_{t-1}) B_{H,t-1}^G + \Phi_1} \]

Therefore, with this current government surplus, we may recover price level and exchange rate. The price level is

\[ P_t = (1 - \delta_t) \left[ 1 - \gamma_{t-1} + \gamma_{t-1} \alpha_{t-1} \right] \frac{B_{H,t-1}^G}{E_t PV(S) - (1 - \alpha_{t-1}) E_t PV(NX)} \]

\[ = (1 - \delta_t) \left[ 1 - \gamma_{t-1} + \gamma_{t-1} \alpha_{t-1} \right] \frac{s_t}{1 - \beta \rho_S} + \frac{\beta (1 - \rho_S) \bar{s}}{(1 - \beta \rho_S)(1 - \beta)} - (1 - \alpha_{t-1}) E_t PV(NX) \]
and, in a similar fashion, the exchange rate is

\[ e_t = (1 - \delta_t) (1 - \alpha_{t-1}) B_{H,t-1}^G \frac{\gamma_{t-1} E_t PV(S) - E_t PV(NX)}{B_{F,t-1}^G} \]

\[ = (1 - \delta_t) (1 - \alpha_{t-1}) B_{H,t-1}^G \frac{\gamma_{t-1} s_t + \frac{\beta(1-\rho_S)^S}{1-\beta}}{B_{F,t-1}^G s_t + \frac{\beta(1-\rho_S)^S}{1-\beta} - (1 - \beta \rho_S)(1 - \alpha_{t-1}) E_t PV(NX)} \]

### B.4 Deriving Steady State Values

The two aggregate resource constraint at equilibrium are as follows.

\[ P_t C_t + q_t B_{H,t}^H + e_t q_t^* B_{H,t}^H + P_t t_t = P_t Y_t + (1 - \delta_t) B_{H,t-1}^H + e_t B_{F,t-1}^H \]

\[ q_t B_{H,t}^G - e_t q_t^* B_{F,t}^G + P_t s_t = (1 - \delta_t) B_{H,t-1}^G - e_t B_{F,t-1}^G \]

Considering past period’s state, we may have the following four states, \((N, N), (N, C), (C, N)\) and \((C, C)\). By using this equation \(B_{H}^G(S) = (1 - \gamma) B_{H}^G(S)\) and international reserves accumulation rule \(B_{F}^G(S) = \rho_0 + \rho_1 B_{F}^H(S)\), where \(S = N, C\), we may reduce the unknown variables. Let’s check the equations we have to get the steady state values depending on states.

\[ q(1 - \gamma) B_{H}^G(S') + e(S', S) q^* B_{F}^H(S') + P(S', S) s(S', S) \]

\[ = P(S', S) N X(S') + (1 - \delta(S'))(1 - \gamma) B_{H}^G(S) + e(S', S) B_{F}^H(S) \]

and

\[ q B_{H}^G(S') - e(S', S) q^* (\rho_0 + \rho_1 B_{F}^H(S')) + P(S', S) s(S', S) = (1 - \delta(S')) B_{H}^G(S) - e(S', S) (\rho_0 + \rho_1 B_{F}^H(S)) \]

where \(S'\) and \(S\) are the current status and past period’s status, which can be \(N\) or \(C\). Therefore, we have eight equations. The steady state value of price level and nominal exchange rate can be derived as follows.

\[ P(S', S) = (1 - \delta(S')) \left\{ 1 - \gamma \frac{\rho_0 + \rho_1 B_{F}^H(S)}{\rho_0 + (1 + \rho_1) B_{F}^H(S)} \right\} \frac{s(S', S)}{1 - \beta \rho_S} + \frac{B_{H}^G(S)}{B_{F}^G(S)} \frac{\frac{\beta(1 - \rho_S)^S}{1 - \beta}}{(1 - \beta \rho_S)(1 - \beta)} \frac{E_t PV(NX)(S)}{\rho_0 + (1 + \rho_1) B_{F}^H(S)} \]

\[ e(S', S) = (1 - \delta(S')) \left\{ \frac{\rho_0 + \rho_1 B_{F}^H(S)}{\rho_0 + (1 + \rho_1) B_{F}^H(S)} \right\} \frac{B_{H}^G(S)}{B_{F}^G(S)} \frac{\gamma s(S', S) + \frac{\beta(1 - \rho_S)^S}{1 - \beta}}{s(S', S) + \frac{\beta(1 - \rho_S)^S}{1 - \beta} - (1 - \beta \rho_S) E_t PV(NX)(S)} \]

Therefore, since price level and nominal exchange rate are the functions of \(B_{H}^G, B_{H}^F\) and \(s(S', S)\), the number of unknown variables is eight, which are

\(B_{H}^G(N), B_{H}^G(C), B_{H}^H(N), B_{H}^H(C), s(N, N), s(N, C), s(C, N), s(C, C)\)

Since the number of variables are same as the number of equations, we can get these variables. However, the equations we have are non-linear equations so that we have to solve it numerically. I used MATLAB with fmincon function. I used non-negative constraint for bond holdings and some inequality constraints on government surpluses. Among 12 inequality constraints, only one constraint is binding.

### C Isolated Government Balance Sheet

This section draws on the work of Park (2012), which derives the quasi-fiscal policy by isolating government balance sheet. Conventional way to examine the effect of fiscal and monetary policy was consolidating the government’s flow
budget constraint. The consolidated government’s flow budget constraint in conventional models is as follows:

\[
\frac{B_{H,t}^G - e_t B_{F,t}^G}{P_t} + s_t = \frac{R_{iH,t-1} B_{H,t-1}^G - e_t R_{iF,t-1} B_{F,t-1}^G}{P_t}
\]

where the fiscal surplus \( s_t \) is the lump-sum tax \( \tau_t \) less the fiscal authority expenditure in units of real goods. However, I assumed that government spending is zero and this assumption may not change the result. Following the balance sheet view, we may change the above budget constraint as follows.

\[
\frac{e_t B_{F,t}^G - e_t B_{F,t-1}^G}{P_t} = \frac{B_{H,t}^G - B_{H,t-1}^G}{P_t} + \frac{i_{H,t-1} B_{H,t-1}^G - e_t i_{F,t-1} B_{F,t-1}^G}{P_t}
\]

where \( i_{H,t} \) is the risk-free net nominal interest rate on domestic government bond, which is \( R_{iH,t} = 1 + i_{H,t} \) and \( i_{F,t} \) is the risk-free net nominal interest rate on foreign bond, which is \( R_{iF,t} = 1 + i_{F,t} \). If we focus on the fact that central bank’s balance sheet is isolated from the balance sheet of fiscal authority, then this constraint can be divided into two as follows.

\[
\frac{B_{H,t}^G - B_{H,t-1}^G}{P_t} + \frac{e_t B_{F,t}^G - e_t B_{F,t-1}^G}{P_t} = \frac{0}{\Delta \text{assets}} + \frac{i_{H,t-1} B_{H,t-1}^G - i_{F,t-1} B_{F,t-1}^G}{P_t} + \frac{\text{cap}_t}{\Delta \text{capital}}
\]

\[
\frac{0}{\Delta \text{assets}} = \frac{B_{H,t}^G - B_{H,t-1}^G}{P_t} + \frac{s_t - \text{cap}_t}{\Delta \text{liabilities}} - \frac{i_{H,t-1} B_{H,t-1}^G}{P_t}
\]

where \( B_{H,t}^G \) refers to the total debt, \( B_{F,t}^G \) is the bonds held by central bank and \( \text{cap}_t \) refers to the transfer from the fiscal authority to the central bank in real goods unit. The key difference with conventional model is showing the capital process transfers from government to central bank explicitly. So, we may need one more policy behavior related with this capital transfer. These two authorities may have several policies as follows.

\[
\hat{R}_{H,t} = \alpha \hat{s}_t + \xi_t^R, \quad \xi_t^R = \rho \xi_{t-1}^R + \epsilon_t^R
\]

\[
\hat{s}_t = \gamma \hat{b}_{H,t-1}^G + \xi_t^s, \quad \xi_t^s = \rho \xi_{t-1}^s + \epsilon_t^s
\]

\[
\text{cap}_t = \kappa (\text{cap}_t - \hat{b}_{H,t-1}^G) + \text{cap}_t^\text{cap}, \quad \xi_t^\text{cap} = \rho \text{cap}_{t-1}^\text{cap} + \epsilon_t^\text{cap}
\]

where \( \alpha, \gamma \) and \( \kappa \) are policy parameters and \( \xi_t^R \) are shocks. The last equation shows the capital transfer from government to central bank and \( \hat{b}_{H,t-1}^G \) is the real capital of the central bank and \( \text{cap}_t \) is the central bank’s real capital target. If \( \kappa \) is positive, the real capital of central bank is less than target level, government will increase the capital transfer and vice versa. By iterating three intertemporal equilibrium conditions, household budget constraint, central bank and government budget constraint, we may derive the following equilibrium conditions for home country.

\[
\frac{R_{H,t-1} B_{H,t-1}^H + e_t R_{iF,t-1} B_{F,t-1}^H}{P_t} = \mathbb{E}_t PV(s) - \mathbb{E}_t PV(NX)
\]

\[
\frac{R_{H,t-1} B_{H,t-1}^G + e_t R_{iF,t-1} B_{F,t-1}^G}{P_t} = -\mathbb{E}_t PV(\text{cap})
\]

\[
\frac{R_{H,t-1} B_{H,t-1}^G}{P_t} = \mathbb{E}_t PV(s) - \mathbb{E}_t PV(\text{cap})
\]

As we have examine in the main paper, we may derive the price level and nominal exchange rate from these conditions. The price level can be easily derived by the last equation, GBC.

\[
P_t = \frac{R_{H,t-1} B_{H,t-1}^G}{\mathbb{E}_t PV(s) - \mathbb{E}_t PV(\text{cap})}
\]

31 Unlike the model we have examined, this model includes two risk-free assets to simplify the model.
and nominal exchange rate is derived by using the first equations and equilibrium price level. Let’s assume that \( \gamma \) fraction of home government bond is held by financial institution. Then, we may have the following expression.

\[
e_t = \frac{R_{H,t-1}}{R_{F,t-1}} \frac{B_{H,t-1}^G}{B_{F,t-1}^H + B_{F,t-1}^G} \gamma_{t-1} E_t PV(s) - \gamma_{t-1} E_t PV(cap)
\]

So, the equilibrium price level and exchange rate from isolated balance sheet model (hereafter IBS model) is slightly different from that of consolidated balance sheet model (hereafter CBS model). The main difference is that with IBS model, equilibrium price level is wholly determined by the outstanding home government bonds whereas with CBS model, equilibrium price level is affected not only by outstanding home government bond, but also by the amount of foreign asset holdings. Therefore, the determination of the model depends on the researcher’s objective. If the country is close to the big closed economy, then CBS model may be the proper choice. But, it is so general statement that it does not hold for any specific model. For example, if foreign investor are allowed to access the home government bond market and are able to purchase it, then price level also can be affected by the foreign investor.

D Computation Procedure

I referred [Chang and Kim (2006), Krusell and Anthony A. Smith (1998), and Gornemann et al. (2012)]. The computation details are as follows:

1. First, we choose the grid for home government bond holdings \( (B_{H,t}^H) \) and foreign asset holdings \( (B_{F,t}^H) \) and idiosyncratic endowment \( (y_t) \). The number of grids are denoted by \( N_{B,H}, N_{B,F} \) and \( N_y \). We use \( N_{B,H} = 60, N_{B,F} = 20, \) and \( N_y = 7 \). The home government bond holding \( B_{H,t}^H \) is in the range of \([0, 60]\), where the average holding is around 6.2. The foreign asset holding \( B_{F,t}^H \) is in the range of \([0, 12]\), where the average holdings is around 1.3. The grid points of assets are not equally spaced. We assign more points on the lower asset range to better approximate savings decisions of workers with lower assets. For the endowment, \( y_t \), we construct grid vectors of \( N_y \) equally spaced points in which \( \ln y_t \) lies on the range of \( \pm 3\sigma y \sqrt{1 - \rho^2} \).

2. Second, we will set the grids for the aggregate variables, \( B_{H,t}^H, B_{F,t}^H \) and \( B_{F,t}^G \), in range of \([0.97 X^*, 1.03 X^*]\) where \( X^* \) denotes the steady-state aggregate variables. In our numerous simulations, the aggregate variables have reached the upper or lower bound. So, to get the accurate solution, we have to change this range wider. However, if we widen the range, we frequently face the negative price level and nominal exchange rate. Therefore, we have to monitor the simulation process carefully. Also, we set the grid for default with two state, 0.01 for normal and 0.05 for crisis, with transition probability matrix.

3. Let the parametric law of motion for the aggregate assets holdings as follows.

\[
B_{H,t}^H = C_1 + D_{1,1} B_{H,t-1}^H + D_{1,2} B_{F,t-1}^H \\
B_{F,t}^H = C_2 + D_{2,1} B_{H,t-1}^H + D_{2,2} B_{F,t-1}^H
\]

We set two equations for each state, normal and crisis, and, therefore, we have four equations. In order for individuals to make their decisions on savings and on consumption, they have to know (or predict) the price level and nominal exchange rate. Since these two variables depend on previous and current aggregate variables, we may calculate the price level and nominal exchange rate from this law of motion for the aggregate assets holdings.

4. We choose the initial values for the coefficients \( C \) and \( D \)'s. God initial values may come from a representative agent model. Several authors used the steady state values that they get from representative agent models and, based on this, they set the initial guesses.

5. Given the law of motions for aggregate assets and price level and nominal exchange rate, we solve the individual optimization problem. Because we did not set any limit on the law of motion for the aggregate assets holdings, the
predicted aggregate values may be not on the grid. If the variable is not on the grid, we may interpolate the value functions when necessary.

6. Using policy functions, transition probability matrix, and the assumed law of motion for the aggregate asset holdings, we generate a set of artificial time-series data \( \{ B^H_H, B^F_H \} \) of the length of 5500 periods. Each period \( \{ B^H_H, B^F_H \} \) is calculated by aggregating the asset holdings of around 30000 individuals.

7. We obtain new values for coefficients, by the OLS from the simulated data. If updated coefficients are close enough to previous ones, we find the law of motion. Otherwise, we update the coefficients and go to step 5.

The estimated law of motion for asset holdings with different policy parameters are provided at table 6 and 7. (The number in parenthesis is the \( t \)-value.)

**D.1 Cointegrating Regression**

Although I employ the exactly same method that other studies suggest, I am not fully convinced about this result. The studies suggest us to check the \( R^2 \) and the variance of error terms. So, I check them and I got \( R^2 \) near 1 and very small standard variance. So, according to suggested method, our estimates are reliable. However, when I calculate the \( t \)-statistics, it is so high, even weird looking. So, I employ Augmented Dickey-Fuller Test to check the unit root process with the unit-root null. Sometimes we cannot reject the null hypothesis for both time series, which implies that both of processes are \( I(1) \) processes. However, sometimes we can reject the null hypothesis for one time series, which implies that one process is \( I(1) \) and the other is \( I(0) \) process. Fortunately, I can reject the null hypotheses for all the error of time series. Therefore, I am free from spurious regression. But, I have to check whether our estimates are consistent. If both variables are \( I(1) \), then estimates are consistent. If one of processes are \( I(1) \), then the estimate of \( I(1) \) is consistent. To have consistent estimator, we have to employ other methods such as CCR (Miller (2010)).

**E Consumption Results with Different Asset Holdings**

We draw the consumption graph depending on asset holdings at figure 2. Before using cash-at-hands method, we have six state variables, \( (B^F_H, B^F_F, y; B^H_H, B^H_F, \delta) \). The first step is distinguishing consumption decision into two groups depending on state, normal and crisis. The second step is making a 9 sections by dividing the each asset holdings into three groups, high, middle, and low. The third step is calculating the mean of each section across other grids such as \( y, B^H_H \) and \( B^H_F \). In sum, we derive the mean of consumption across aggregate variables and individual endowment. Regarding the figure 2, X axe is home government bond holdings and Y axe is foreign asset holdings, both of them are separated into three and we labeled with low, medium, high. Z axe is the level of consumption.
Table 6: The law of motion of aggregate home government bond holdings

<table>
<thead>
<tr>
<th>$\rho_S$</th>
<th>State</th>
<th>$C_1$</th>
<th>$D_1$</th>
<th>$D_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>Good</td>
<td>-0.6921</td>
<td>0.9193</td>
<td>0.0286</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-28.8274)</td>
<td>(38.2936)</td>
<td>(1.1903)</td>
</tr>
<tr>
<td>0.2</td>
<td>Bad</td>
<td>-0.6090</td>
<td>0.5543</td>
<td>0.0920</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-6132.2716)</td>
<td>(5587.1052)</td>
<td>(927.7248)</td>
</tr>
<tr>
<td>0.3</td>
<td>Good</td>
<td>-0.9156</td>
<td>0.9577</td>
<td>0.0303</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-53.6650)</td>
<td>(56.1343)</td>
<td>(1.7769)</td>
</tr>
<tr>
<td>0.3</td>
<td>Bad</td>
<td>-0.4393</td>
<td>0.6380</td>
<td>0.0791</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-1802.9692)</td>
<td>(2618.6143)</td>
<td>(624.6414)</td>
</tr>
<tr>
<td>0.4</td>
<td>Good</td>
<td>-0.9156</td>
<td>0.9577</td>
<td>0.0303</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-53.6650)</td>
<td>(56.1343)</td>
<td>(1.7769)</td>
</tr>
<tr>
<td>0.4</td>
<td>Bad</td>
<td>-0.4393</td>
<td>0.6380</td>
<td>0.0791</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-1802.9692)</td>
<td>(2618.6143)</td>
<td>(624.6414)</td>
</tr>
<tr>
<td>0.5</td>
<td>Good</td>
<td>-1.1338</td>
<td>0.8986</td>
<td>0.0499</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-36.8070)</td>
<td>(29.1709)</td>
<td>(1.6191)</td>
</tr>
<tr>
<td>0.5</td>
<td>Bad</td>
<td>-1.4427</td>
<td>1.0784</td>
<td>-0.0020</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-137037.2211)</td>
<td>(102431.9051)</td>
<td>(-186.0632)</td>
</tr>
</tbody>
</table>

Table 7: The law of motion of aggregate foreign asset holdings

<table>
<thead>
<tr>
<th>$\rho_S$</th>
<th>State</th>
<th>$C_1$</th>
<th>$D_1$</th>
<th>$D_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>Good</td>
<td>11.0711</td>
<td>1.2378</td>
<td>0.5515</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-0.1362)</td>
<td>(0.1809)</td>
<td>(0.0056)</td>
</tr>
<tr>
<td>0.2</td>
<td>Bad</td>
<td>13.3788</td>
<td>6.6816</td>
<td>-0.5160</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.5883)</td>
<td>(1.7921)</td>
<td>(-0.1384)</td>
</tr>
<tr>
<td>0.3</td>
<td>Good</td>
<td>11.5969</td>
<td>0.4650</td>
<td>0.6275</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-0.3427)</td>
<td>(0.3585)</td>
<td>(0.0113)</td>
</tr>
<tr>
<td>0.3</td>
<td>Bad</td>
<td>-8.6028</td>
<td>5.2775</td>
<td>-0.2391</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(143.2015)</td>
<td>(87.8485)</td>
<td>(-3.9807)</td>
</tr>
<tr>
<td>0.4</td>
<td>Good</td>
<td>-0.9156</td>
<td>0.9577</td>
<td>0.0303</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-53.6650)</td>
<td>(56.1343)</td>
<td>(1.7769)</td>
</tr>
<tr>
<td>0.4</td>
<td>Bad</td>
<td>-0.4393</td>
<td>0.6380</td>
<td>0.0791</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-1802.9692)</td>
<td>(2618.6143)</td>
<td>(624.6414)</td>
</tr>
<tr>
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<td>Good</td>
<td>13.4328</td>
<td>1.2077</td>
<td>0.4078</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.1182)</td>
<td>(0.2804)</td>
<td>(0.0947)</td>
</tr>
<tr>
<td>0.5</td>
<td>Bad</td>
<td>26.6648</td>
<td>-0.6177</td>
<td>0.6171</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(34208.6031)</td>
<td>(-792.4708)</td>
<td>(791.6310)</td>
</tr>
</tbody>
</table>
(a) Normal State, $\rho_s = 0.2$

(b) Crisis State, $\rho_s = 0.2$

(c) Normal State, $\rho_s = 0.3$

(d) Crisis State, $\rho_s = 0.3$

(e) Normal State, $\rho_s = 0.4$

(f) Crisis State, $\rho_s = 0.4$

(g) Normal State, $\rho_s = 0.5$

(h) Crisis State, $\rho_s = 0.5$

Figure 2: Plot of Consumption with Different Asset Holdings and $\rho_s$