Endogenous Banking Networks, Systemic Risk, and Firm Investment∗

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Abstract

In this paper, we investigate the network formation in the banking system, and its connection to the real economy through loans to firms. We will approach the problem of endogenous network formation under incomplete information as a problem of mechanism design and we will focus on the possibility of mechanism design that induces banks to report their types truthfully, and the possibility of a central bank to achieve the same outcome by choosing a profile of sets of allowable ways to connect and then delegating the choice of level of loans to each bank-bank pair and bank-firm pair. We show, under relatively mild conditions on our game-theoretic model of network formation, that strategic network formation with incomplete information, implemented via a mechanism and centralized reporting, is equivalent to implementation via delegated networking with monitoring.

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1 Introduction

One of the main lessons of the financial crisis of 2008-2009 is the importance of understanding interconnectedness in financial markets and between financial markets and the real economy, and how this interconnectness enters into the determination of risk. The recent financial crisis was followed by low GDP growth. However, not all financial crises lead to severe economic downturns. For example, according to Tallman and Wicker (2010) [12], there is little evidence showing a depression after the banking panic of 1873 in the United States. With different bank sizes, banks’ lending behavior, and networks of banking system, the propagation of risks in the financial system will be different, and the mechanisms linking financial shocks and real output will be different.

There is literature studying the relationship between bank size, banks’ lending behavior and the real effect of banking crises. George (2013) [5] constructs a macroeconomic model with a heterogeneous banking sector to show that a more concentrated banking sector and more heterogeneity among big and small banks will deepen financial downturns. George does not provide an environment where network effects play a role in interbank lending. However, as indicated in Tallman and Wicker (2010), interbank lending is an important channel of transmission of systemic risk. Campello, Graham, and Harvey (2010) [2] indicates that firms are credit constraint during financial crises and cut their technology spending and capital spending. Kroszner, Laeven, and Klingebiel (2007) [6] and Dell’Ariccia, Detragiache, and Rajan (2008) [3] indicate that industries that depend more on external finance tend to be affected more during financial crises. Pagano (1993) [7] shows that financial sector not only affect real output level but growth rate of an economy.

Besides the interconnection in financial markets, even more fundamentally important is the need to understand the strategic underpinnings of financial network formation. Understanding the foundations of network formation in the strategic behavior of the players in the financial markets is absolutely essential if we are to craft policies which ensure that financial networks are well-functioning and robust against shocks. Here we will focus on the interbank loan network and the interconnections between the functioning of the banking network and real economic activity. How does a bank default affect the banking network and in turn how does a default in the banking network affect real economic activity. Conversely, how does a default in the real economy affect the banking network. Finally, are there regulatory policies that can be implemented by the central bank that will incentivize network formation in such a way that the interconnectedness which emerges from the strategic behavior of firms and banks in forming contractual financial connections exhibits a minimum acceptable level of systemic risk?

Our approach is new. In particular, we will approach the difficult problem of endogenous network formation under incomplete information as a problem of mechanism design and we will focus on the following two questions: (1) if we suppose that there is principal (in our case a central banker) who is endowed with the power to put into place player incentives for connecting (i.e., for forming a network), is it possible for the principal to design a mechanism that links the reporting of players’ private information (to the principal) and the set of connections allowed and recommended by the principal via the mechanism in such a way that players truthfully reveal their private information to the principal and follow the recommendations specified by the mechanism. (2) An even more fundamental question we address is whether or not it is possible for the principal to achieve the same outcome (as that achieved via a mechanism and centralized reporting) by instead choosing a profile of sets of allowable ways to connect (here modeled as arc types, with one set of arc types for each pair of individuals) and then delegating the choice of
an arc type to each pair of individuals. We call this approach to network formation with incomplete information delegated networking and we show, under relatively mild conditions on our game-theoretic model of network formation, that strategic network formation with incomplete information, implemented via a mechanism and centralized reporting, is equivalent to implementation via delegated networking with monitoring. Thus, we show that the delegation principle of contracting theory holds for games of network formation with incomplete information.

The fact that delegated networking works provides us with a way to analyze the problem of designing optimal bank regulations. Under delegated networking the central banker’s problem reduces to a problem of choosing a profile of arc catalogs - where each arc catalog can represent possible borrowing or lending levels or capital requirements. Thus rather than design a mechanism requiring centralized reporting, the central bank can instead design a profile of catalogs with each catalog consisting of the relevant banking decisions the central bank wishes banks to choose.

2 Financial Network and Bank-Firm Networks

I consider an economy with a set $B$ of $m$ banks and a set $F$ of $n$ firms. Banks are endowed with deposits and capital. They borrow and lend among each other, and provide loanable funds to firms. To simplify the bank-firm networks, I assume that each firm can only connect to one bank, but each bank can connect with multiple firms.

All firms produce one type of consumable goods using different intermediate goods as inputs. Each firm has a Research and Development (R&D) department that conducts multiple projects to create new versions of the intermediate goods. The new versions of intermediate goods are more productive, thus, the R&D department can sell the technology of producing the new intermediate goods to other firms to earn a profit. In order to achieve innovation, the R&D departments have to borrow money from banks to invest in research projects. Higher the investment level, more projects are likely to succeed and the firm is more likely to earn a higher profit.

2.1 Primitives

Consider a directed network. Let each bank $b$ and firm $f$ be an individual or node, associated with its own type $t$. Type of a bank includes information about its balance sheet, health condition, investment, etc. Type of a firm includes its capital, technology level, productivity of the R&D department, etc. Assume type of a bank is private information within the bank, while type of a firm is common knowledge to everyone in the economy.

To describe inter-bank lending and bank lending to firms, treat any two individuals $i, j$ as a player, and use three clubs, $c_0, c_1$, and $c_2$, to represent connection between $i$ and $j$. Club $c_0$ is the “no transaction” club, $c_1$ is the “lending” club, and $c_2$ is the “borrowing” club. If player $ij$ joins club $c_0$, there is no transaction between $i$ and $j$. If player $ij$ joins club $c_1$, individual $i$ lends money to individual $j$. If player $ij$ joins club $c_2$, $i$ borrow money from $j$. Each player will and can join one club. For bank-bank connections, if bank $i$ lends to bank $j$, then player $ij$ joins club $c_1$ and player $ji$ joins club $c_2$.

Once two individuals $i, j$ are connected, we assign an arc type $a_c$ to player $ij$ to denote how they are connected after joining a club $c$. If $ij$ joins club $c_0$, $a_c = 1$. This set-up helps us to count the number of node pairs that are not connected. If a player joins club $c_1$ or $c_2$, then the arc type $a_c$ represents the amount of money transferred.
Assumption 1. (1) \( N \) is a finite set of individuals (nodes) consisting of two subsets, \( B \) and \( F \). \( B \) has typical element \( b \) and \( F \) has typical element \( f \). \( N \) is equipped with the discrete metric, \( \eta_N \);
(2) Let \( C = \{c_0, c_1, c_2\} \) being the finite set of clubs, equipped with discrete metric, \( \eta_C \), and typical element \( c \);
(3) \( (T, B(T), \lambda) \) is the type space with typical element \( t \) where \( T \) is a complete, separable metric space with metric \( \rho_T \). \( B(T) \) is the Borel \( \sigma \)-field, and \( \lambda \) is a probability measure;
(4) \( A_c \) is the space or category of arc types, \( a_c \). For each \( c \in C \), \( A_c \) is a compact, convex subset of locally convex Hausdorff topological vector space \( E_c \), metrizable with metric \( \rho_{A_c} \) for the relative topology inherited from \( E_c \);
(5) \( A(\cdot) \) is the feasible arc correspondence, a set-value mapping from the set of all ordered pairs, \( (ij, c) \), taking values in the hyperspace of nonempty, closed subsets of \( A_c \) such that for all pairs, \( (ij, c) \),
\[ 0 \in A(ijc) \subset A_c. \]

In assumption \([A-1](5)\), zero belongs to \( A(ijc) \) means that we allow for “weak” or “inactive” connection. This set-up can be useful when one assume all banks are connected in the financial system, but there is no positive or “active” money transfer between two banks. In this case, one only needs to care about the arc type between each two banks, and use a matrix to represent inter-bank lending.

In order to define distance between two different networks, we need to find an appropriate metric for the space \( K_c := A_c \times ((N \times N) \times c) \). Consider the Cartesian product \( C \times A_c \) consisting of club-c-arc pairs, and equip \( C \times A_c \) with the sum metric \( \rho_{C \times A_c} := \eta_C + \rho_{A_c} \), where \( \rho_T \) and \( (A_c, \rho_{A_c}) \) are compact metric spaces.
Thus, \( (C \times A_c, \rho_{C \times A_c}) \) is a compact metric space. Denote by \( 2^{C \times A_c} \) the collection of all \( \rho_{C \times A_c} \)-closed subsets of \( C \times A_c \), including the empty set. For \( H \in 2^{C \times A_c} \) and \( (c, a) \in C \times A_c \), define the distance from \( (c, a) \) to \( H \) as follows:
if \( H \in 2^{C \times A_c} \) is nonempty, then
\[
\text{dist}_{\rho_{C \times A_c}}((c, a), H) = \min_{(c', a') \in H} \rho_{C \times A_c}((c, a), (c', a'));
\]
if \( H \in 2^{C \times A_c} \) is empty, then
\[
\text{dist}_{\rho_{C \times A_c}}((c, a), H) = \max_{(c', a') \in H} \rho_{C \times A_c}((c, a), (c', a')) = \text{diam}_{\rho_{C \times A_c}}(C \times A_c).
\]

The Hausdorff distance, \( h_{\rho_{C \times A_c}} \) on \( 2^{C \times A_c} \) is defined as:
if \( H^1 \) and \( H^2 \) in \( 2^{C \times A_c} \) are nonempty, then
\[
h_{\rho_{C \times A_c}}(H^1, H^2) = \max\{\max_{(c, a) \in H^1} \text{dist}_{\rho_{C \times A_c}}((c, a), H^2), \max_{(c', a') \in H^2} \text{dist}_{\rho_{C \times A_c}}((c', a'), H^1)\},
\]
if \( H \in 2^{C \times A_c} \) is empty, then
\[
h_{\rho_{C \times A_c}}(H, \emptyset) = h_{\rho_{C \times A_c}}(\emptyset, H) := \text{diam}_{\rho_{C \times A_c}}(C \times A_c);
\]
if \( H^1 \) and \( H^2 \) in \( 2^{C \times A_c} \) are empty, then
\[
h_{\rho_{C \times A_c}}(H^1, H^2) = h_{\rho_{C \times A_c}}(\emptyset, \emptyset) = 0.
\]

Note that the Hausdorff distance is a metric on the collection of \( \rho_{C \times A_c} \)-closed subsets of \( C \times A_c \). \( (2^{C \times A_c}, h_{\rho_{C \times A_c}}) \) is a compact metric space. Using similar reasoning, we can define the Hausdorff
distance on the collection of all \( \rho_{K_c} \)-closed subsets of \( K_c \), a collection denoted by \( 2^{K_c} \), and we have that \( (2^{2C \times K_c}, h_{\rho_{K_c}}) \) is a compact metric space of all \( c \)-club networks.

Take as the set of players in our game of network formation the set of all possible ordered pairs of individuals,

\[ ij := (i, j) \in B \times (B \cup F) = B \times N. \]

Thus, the set of players is given by

\[ B \times N := \{ij : i \in B, j \in N\}, \]

and the number of players is given by \( n \ast (m + n) \). A coalition of players would then be given by subset \( g \subset B \times N \). We will call such a subset, a pre-network.

In addition to the set nodes representing players, \( ij \in B \times N \), there is the set of nodes representing clubs, \( C \). Thus for our club networks we will take as our set of nodes the set

\[ (B \times N) \cup C, \]

implying that the maximum number of nodes in our network is \( n \ast (m + n) + k \).

### 2.2 \( c \)-Connections, \( c \)-Networks, and Layered Club Networks

In our bipartite club networks, each player takes a particular feasible action \( a \in A_c \) from the set of actions relevant to the club \( c \) he joins. Thus, a typical connection is given by \( (a, (ij, c)) \in A_c \times ((B \times N) \times C) \), indicating that player \( ij \) is in club \( c \) and that in this club player \( ij \) takes feasible action \( a \in A_c \). Thus, in our club networks the first set of nodes consists of the players, \( B \times N \), and the second set of nodes - the nodes to which the players send feasible arcs - is given by the set of clubs, \( C \). Thus, club networks are layered networks.

A **pre-connection** is an ordered pair of nodes, \( (ij, c) \in (B \times N) \times C \) consisting of a player node, \( ij \), and a club node, \( c \). A **ca-connection** or a **connection of type** \( a \) **in** club \( c \) **from** node \( ij \) **to** node \( c \) is an ordered pair,

\[ (a, (ij, c)) \in \cup_c A \times ((B \times N) \times C), \]

where \( a \in A_c \) is an arc type in club \( c \) and \( (ij, c) \in (B \times N) \times C \) is a pre-connection. Thus, a connection is a pre-connection to which an arc from the appropriate club has been assigned. To describe connection \( (a, (ij, c)) \) in words we would say that an arc of type \( a \) from club \( c \) runs from node \( ij \) to node \( c \).

The connection, \( (a, (ij, c)) \), is feasible provided,

\[ a \in A(ijc) \subset A_c, \]

where

\[ A(\cdot) : ((B \times N) \times C) \rightarrow \cup_c 2^{A_c}, \]

is the feasible arc correspondence defined on the set of all pre-connections, \( ((B \times N) \times C) \), and taking values in \( \cup_c 2^{A_c} \), the union of the hyperspaces, \( 2^{A_c} \), the collection of all closed subsets of \( A_c \).

Thus, a feasible connection is a pre-connection, \( (ij, c) \), to which a feasible \( c \)-arc, \( a \in A(ijc) \subset A_c \), has been assigned.

The set of all connections is given by

\[ K := \cup_c [A_c \times ((B \times N) \times C)], \]
while the set of all c-connections is given by
\[ K_c := A_c \times ((B \times N) \times \{c\}). \]

We will equip \( K_c \) with the sum metric,
\[ \rho_{K_c} := \rho_{A_c} + \eta N + \eta \eta N. \]

A c-club network \( G_c \) is a \( \rho_{K_c} \)-closed subset of c-connections, which can also be seen as a layer of the network \( G \). A club network \( G \) is the union of c-club networks. Thus, each club network is given by \( G := \cup_{c \in C} G_c \), where \( G_c \) is nonempty if and only if \( c \in D(G_{\cdot c}) := \{c \in C : |G_c| = 1\} \). Thus, if in club network \( G \) club \( c \) is in the domain of the mapping: \( c \rightarrow G_c \), then \( G_c \in P_{\rho_{K_c}}(K_c) \), where \( P_{\rho_{K_c}}(K_c) \) is the hyperspace of all nonempty, \( \rho_{K_c} \)-closed subsets of c-connections. The collection \( P_{\rho_{K_c}}(K_c) \) is an \( h_{\rho_{K_c}} \)-closed subset of \( 2^{K_c} \), and hence \( (P_{\rho_{K_c}}(K_c), h_{\rho_{K_c}}) \) is also a compact metric space of c-networks. If club \( c' \notin D(G_{\cdot c}) \), then this club has no members in network \( G \), for example, if \( c_1 \notin D(G_{\cdot c}) \), there is no inter-bank lending and no bank is lending to firms, and then we have \( G_{c'} = \emptyset \).

Each c-club network, \( G_c \), making up network \( G \) is distinguished by the arc space \( A_c \), used by club c’s members in making connections. This fact requires that in measuring the distance between networks that we do so layer by layer - with each club network, \( G_c \), being one layer in network \( G \) given by \( \cup_{c \in C} G_c \). Thus, we will define the distance between two club networks, \( G^1 := \cup_{c \in C} G^1_c \) and \( G^2 := \cup_{c \in C} G^2_c \) to be
\[ h(G^1, G^2) := \sum_{c} h_{\rho_{K_c}}(G^1_c, G^2_c). \]

Since we assume that each player \( ij \) belongs to one and only one club, the club layers partition the players into disjoint sets. For each player \( ij \), the graph of the club-arc correspondence \( c \rightarrow A(ijc) \subset A_c \) is given by
\[ GrA(ij, \cdot) := \{(c, a) \in C \times \cup A_c : a \in A(ijc)\} \subset 2^{C \times \cup A_c}. \]

Given club network \( G \), the section of \( G \) at player-club pair \((ij, c)\) is such that \( G(ijc) \subset A(ijc) \). Moreover, \( GrG(ij, \cdot) \subset GrA(ij, \cdot) \).

Now, given network \( G \in P_{\rho_{K_c}}(K) \), we can define following notations that we will use later:
\[ G(ij) := \{(c, a) \in GrA(ij, \cdot) : (a, (ij, c)) \in G\}, \]
\[ G(ijc) := \{a \in A(ijc) : (a, (ij, c)) \in G\}, \]
\[ G(c) := \{(a, ij) \in A_c \times B \times N : (a, (ij, c)) \in G\}, \]
\[ G(ca) := \{ij \in B \times N : (a, (ij, c)) \in G\}. \]

\( G(ij) \) is the section of network \( G \) at player \( ij \) and therefore, \( G(ij) \) is the set of club-arc pairs assigned to player \( ij \) in club network \( G \). If \( i, j \in B \), \( G(ij) \) describes inter-bank lending behavior. If \( i \in B, j \in F \), \( G(ij) \) describes lending from bank \( i \) to firm \( j \). For the bank-firm connections, we can define a bank-firm pre-network as a nonempty subset \( g_{BF} \) of \( B \times F \) such that its cardinality \(|g_{BF}(f)| = 1 \) where \( g_{BF}(f) := \{b \in B : (b, f) \in g_{BF}\} \). To simplify our model, we assume
that the bank-firm pre-network is exogenous. Thus, $g_{BF}$ is the set of players in the game of bank-firm network formation.

A bank-firm network, $G_{BF}$, is a nonempty subset of $A \times (g_{BF} \times \{c_0, c_1\})$ where

$$G_{BF}(bfc) := \{a \in A(bfc) : bf \in g_{BF}, (a, (bf, c)) \in G_{BF}\},$$

is nonempty for some $c = c_0$ or $c_1$. And as mentioned above, if $c = c_0$, $a = 1$ and there is no transaction between firm $f$ and bank $b$. If $c = c_1$, $a = (l, r) \in [0, l_{bf}] \times [0, r_{bf}]$ and bank $b$ provides loan $l$ to firm $f$ with an interest rate $r$ not exceeding $r_{bf}$. The upper-bound $l_{bf}$ for loans can be regulated by central bank introduced later.

$G(ca)$ is the section of network $G$ at club-arc pair $ca$, and $G(ca)$ is the set of players assigned to take action $a$ in club $c$ in network $G$.

Let $|G(ijc)| = \text{the cardinality of } G(ijc)$, and

$$|G(ca)| = \text{the cardinality of } G(ca).$$

Assume that a set has cardinality zero if and only if the set is empty. Then we can define a feasible club network as:

$$G \in P_{\rho K f}(K) \text{ is feasible if and only if } G(ijc) \subset A(ijc),$$

$$|G(ijc)| \leq 1 \text{ for all } (ij, c) \in (B \times N) \times C,$$

$$|G(ij)| = 1 \text{ for all } ij \in B \times N.$$
Equip $T_N$ with the sum metric $\rho_{T_N} := \rho_T + \eta_N + \eta_N$.

A types network, $\tau := \{\tau_i \in T \times (\{i\} \times \{i\}) : i \in N\}$, is a nonempty closed subset of $T_N$ such that for each pre-connection $(ij)$, the cross section of $\tau$ at the pre-connection $(ij)$ is such that

$$|\tau(ij)| := \begin{cases} 1 & \text{if } i = j, \\ 0 & \text{if } i \neq j. \end{cases}$$

Therefore, a types network only contains loops for each node. Note that the domain of any types network is the diagonal of the set of all pre-connections. Thus a types network $\tau$ is a set of connections of the form $\tau_i$, such that $\tau_i \in T \times (\{i\} \times \{i\})$ for all $i \in N$, and $\tau(ii) = t_i \in T$ for all $i \in N$. Let $T$ denote the collection of all type networks and let $T^{m+n} := T \times \cdots \times T$, be the space of type profiles.

Think of each pair of individuals, or each pre-connection, $(i, j) \in B \times N$, as a player $ij$, then we can think of each player $ij$ as having a two-dimensional type description, $(t_i, t_j)$, where each type $t_i$ is the arc type of the type connection $\tau_i$ corresponding to individual $i$. To simplify notations, we will write $t_{ij} := (t_i, t_j) \in T \times T$, and $t_{B \times N} := (t_{ij})_{(i, j) \in B \times N}$. For example, a bank $b$ and firm $f$ joining club $c$ is displayed in Figure 2.

![Figure 2: Bank b and Firm f as a Player bf](image)

When bank $b$ provides loans to firm $f$, player $bf$ joins the lending club $c_1$, and the firm and bank will make an agreement on interest rate and amount of loanable funds transferred, which are captured by the arc type $a_{bf}$.

### 2.4 An Example of Bank-Firm Players

To show the layered network, we provide an example of three banks and seven firms in an economy in Figure 3. It only display part of the bank-firm pairs. The complete figure should include $3 \times (3 + 7) = 30$ players. The figure shows the single club membership since each player is connected with one and only one club. The network can be sliced into three layers, each layer associated with players join one club. The first layer contains bank 2 and firm 5 joining club $c_0$, meaning there is no fund transfer. Player $b_2f_5$ is the only member in club $c_0$ in this part of figure. The second layer contains bank 2, firm 3; bank 3, firm 6; and bank 3, firm 4 joining club $c_1$, meaning these banks lend money to the corresponding firms. The third layer contains bank 1, firm 7; bank 1, firm 2; and bank 3, firm 1 joining club $c_2$, meaning these banks borrow money from corresponding firms. In a general setup, we allow all players to choose any one club. However, we can exclude the borrowing club from the choices of bank-firm pairs.

Our setup of club membership is flexible for application. We can add more clubs to represent “short-term debt” club, “long-term debt” club to our model. Then we can model banks’ flight to liquidity during financial crises.
3 Incentive Compatible Networks

3.1 Payoffs of Banks, Firms, and Players

Let \( G \in \mathbb{K} \) be a feasible club network. In a feasible club network each player \( i,j \) in club \( c \) is constrained to take only those actions that are contained in the arc set given by feasible arc correspondence for the player-club pair \((i,j,c)\), \(A(ijc) \subset A_c\). Since every players’ actions and types will affect each player \( i,j \)’s payoff, but not his best response function, we write the player \( i,j \)’s payoff as

\[
V_{ij}(t_{ij}, t_{-ij}, (c_{ij}, a_{ij}), (c_{-ij}, a_{-ij})) := \sum_{i',j' \in B \times N} u_{ij,i'j'}(t_{ij}, t_{i'j'}, (c_{ij}, a_{ij}), (c_{i'j'}, a_{i'j'})),
\]

where \( c_{ij} \) and \( a_{ij} \) represents player \( i,j \)’s action given type \( t_{ij} \), and we use \(-ij\) to represent all players except for player \( ij \).

This payoff function is based on our assumption that each player’s payoff is additively coupled in connections, which means when each player makes decision on \( c \) and \( a \) to maximize his payoff, he takes other players’ types and actions as given, without counting the externalities each player creates to the economy. This is the key assumption that distinguish each player’s payoff to a central planner’s payoff.

**Assumption 2.** Assume that all payoff functions over connections are Caratheodory, meaning that all payoff functions are measurable in types and continuous in club-arc pairs.

The measurability of payoff function with respect to types makes sure that we can take expectation on types when we calculate payoff. The continuity in club-arc pairs means that when two networks are closed to each other, the payoffs are also closed to each other. According to our definition about distance between two networks, the only situation that we have two “very closed” networks is that all the banks and firms connected in the same way in the two networks, only the arc types of some connection is slightly different. In terms of players, all players join
the three clubs in the same way in the two networks, but some are from some player to a club is slightly different in the two networks.

Given a player $i'j''$'s action, player $ij$'s payoff $u_{ij,i'j''}(t_{ij},t_{i'j''},(c_{i'j''},a_{i'j''}))$ can be calculated as a weighted sum of the payoff of individual $i$ and individual $j$. The weights on the two payoffs depends on the negotiate power of each individual. When a player $ij$ makes decision, individual $i$ and $j$ cooperate and try to achieve Pareto optimum. For an individual bank’s net payoff, it equals to the interest earned from loaning money to firms and loaning money to other banks minus interest paid to banks that the individual borrows money from.

If player $ij$ belongs to a bank-firm pre-connection, denote the player type as $t_{bf} := (t_b,t_f)$.

Firm payoff or profit depends on the type of the firm $t_f$, the level of investment, $I_f$, through the stochastic technology, $p_f(dz|t_f,I_f) \in \mathcal{P}([0, +\infty))$.

Assume that if the realized revenue $z$ is less than the interest firm $f$ should pay to bank $b$, firm $f$ will only pay $z$ and bank $b$ will suffer from some lost. Thus, if firm $f$ of type $t_f$ invests $I_{bf}$ in its R&D projects with stochastic technology $p_f(dz|t_f,I_{bf})$ borrowing the full amount of the investment $I_{bf}$ from bank $b$, for a rate of $r_{bf}$, then firm $f$’s expected profit is given by

$$\int_{r_{bf}I_{bf}}^{+\infty} (z - r_{bf}I_{bf})p_f(dz|t_f,I_{bf}),$$

and its random profit is

$$\max\{\tilde{z} - r_{bf}I_{bf}, 0\}.$$

Thus, in bank-firm network $G_{BF}$, if the section of the network at player-club pair $(bf,c_1)$ is $G_{BF}(bf,c_1)$, then

$$G_{BF}(bf,c_1) := \{(r_{bf}I_{bf}) : (r_{bf}I_{bf},(bf,c_1)) \in G_{BF}\}.$$

Given bank-firm network $G_{BF}$, firm $f$’s random return is

$$\max\{\tilde{z} - G_{BF}(bf,c_1), 0\},$$

while bank $b$’s expected profit from lending to firm $f$ is given by

$$r_{bf}I_{bf}(1 - P_f(r_{bf}I_{bf}|t_f,I_{bf})) + \int_0^{r_{bf}I_{bf}} z p_f(dz|t_f,I_{bf}).$$

Player $bf$’s expected payoff can be defined as a weighted sum of (2) and (3), where weights can be set according to the bargaining powers of the bank and the firm.

### 3.2 Mechanisms

Now we introduce a central bank (CB) to the banking system, whose job is to discover the types of the players, i.e., the types of the pre-connections, and to induce each player to establish a connection by choosing the arc recommended by the CB. We will begin by assuming that the CB seeks to accomplish their objectives of controlling systemic risk via a mechanism with central reporting. What we mean by this is that the CB presents to all players simultaneously a profile of direct mechanisms

$$(c_{ij}(\cdot), a_{ij}(\cdot))_{ij \in B \times N},$$
one mechanism \((c_{ij}(\cdot), a_{ij}(\cdot))\) for each player \(ij\), where \(a_{ij}(\cdot) : T \times T \rightarrow A(ij, c_{ij}(t_{ij}))\) for all \(t_{ij}\) is a measurable mapping from players’ type space into the space of feasible arc types for player \(ij\). If player \(ij\) reports his type to be \(t'_ij\), then the mechanism recommends to player \(ij\) that \(ij\) join club \(c_{ij}(t'_{ij})\) and take action \(a_{ij}(t'_{ij}) \in A(ij, c_{ij}(t'_{ij}))\), i.e., if a player \(ij\) lies about its type, the recommended arc type and club to join may both change.

If all players report their true types and choose the arc-club pair recommended by the mechanism, then each profile of mechanisms will give rise to a feasible club network,

\[
M(t) := M(t_{ij}, t_{-ij}),
\]
with typical connection \((a_{ij}(t_{ij}), (ij, c_{ij}(t_{ij})))\), or equivalently,

\[
M(t_{ij}, t_{-ij})(ij) := \{ac \in \text{GrA}(ij, \cdot) : (a, (ij, c)) \in M(t_{ij}, t_{-ij})\},
\]
and

\[
|M(t_{ij}, t_{-ij})(ij)| = 1, \forall ij \in B \times N.
\]
Thus, for all possible types \(t_{ij}\),

\[
M(t_{ij}, t_{-ij})(ij) = (c_{ij}(t_{ij}), a_{ij}(t_{ij})) \in \text{GrA}(ij, \cdot).
\]

### 3.3 Incentive Compatible Network Formation Mechanisms

An incentive compatible network formation mechanisms is defined as below:

**Definition 1.** \((\text{Incentive Compatible Club Connection Recommendation (IC-CCR) Mechanisms})\)

We say that a club connection recommendation mechanism, \((c_{ij}(\cdot), a_{ij}(\cdot))_{ij \in B \times N}\), for player \(ij\) is incentive compatible provided \((c_{ij}(\cdot), a_{ij}(\cdot))_{ij \in B \times N}\) satisfies the following condition: For all player \(ij\) types \(t_{ij}\) and \(t'_{ij}\) in \(T \times T\),

\[
u_{ij,ij}(t_{ij}, c_{ij}(t_{ij}), a_{ij}(t_{ij})) \geq u_{ij,ij}(t_{ij}, c_{ij}(t'_{ij}), a_{ij}(t'_{ij})).
\]

Let \(IC_{ij}\) denote the collection of all incentive compatible club connection recommendation mechanisms in \(M_{ij}(T \times T, \text{GrA}(ij, \cdot))\).

The importance of IC-CCR mechanisms is that under such a mechanism player \(ij\) has no incentive to report his type falsely - and no incentive to not follow the recommendation of the mechanism. Under assumption 2, this remains true if each player’s payoff function is additively coupled in connection. In particular, player \(ij\)’s payoff function under mechanisms is given by

\[
V_{ij}(t_{ij}, t_{-ij}, (m_{ij}(t_{ij}), m_{-ij}(t_{-ij}))) := \sum_{t'_{ij} \in B \times N} u_{ij,ij}(t_{ij}, t'_{ij}, m_{ij}(t'_{ij})), \tag{4}
\]

where

\[
m_{ij}(t'_{ij}) := (c_{ij}(t'_{ij}), a_{ij}(t'_{ij})).
\]

Note that the generalized payoff function \(V_{ij}(\cdot, \cdot)\) is also Caratheodory. Moreover, if for each player \(ij\),

\[
m_{ij}(\cdot) := (a_{ij}(\cdot), c_{ij}(\cdot)) \in IC_{ij},
\]

then for player \(ij\), we have for \((t_{ij}, t_{-ij})\) and \((t'_{ij}, t_{-ij})\)

\[
V_{ij}(t_{ij}, t_{-ij}, (m_{ij}(t_{ij}), m_{-ij}(t_{-ij})) \geq V_{ij}(t_{ij}, t_{-ij}, (m_{ij}(t'_{ij}), m_{-ij}(t_{-ij}))),
\]
i.e. player \(ij\) has no incentive to report falsely.
4 Central Bank’s Problem

Assume central bank’s payoff function is

$$U((t_{ij})_{ij \in B \times N}, (c_{ij}, a_{ij})_{ij \in B \times N})$$

over profiles of player types and action-club pairs. As mentioned in Farboodi (2014) [4], CB wants to balance the net gain from investment of banks with the expected loss of default, and without CB, banks will form a network that involves excess contagion, more than what is necessary to support the optimal level of investment. We argue that the force the drives CB’s optimal solution to be different from equilibrium is that CB takes into account the externalities or social cost created by every player and looks at the network as a whole when he makes decision. This will make the CB’s optimal strategy be different from each player’s optimal strategy. The goal of the CB is to design a network formation mechanism to induce players to reveal their types and to follow the connection recommendations of the mechanism in order to control systemic risk in the network.

Under an incentive compatible network formation mechanism,

$$t_{B \times N} \rightarrow (c_{ij}(t_{ij}), a_{ij}(t_{ij})_{ij \in B \times N}),$$

the CB’s payoff becomes

$$U(t_{B \times N}, (c_{ij}(t_{ij}), a_{ij}(t_{ij})_{ij \in B \times N}).$$

The problem faced by CB is given by

$$\max_{(m_{ij}(\cdot))_{ij \in B \times N} \in \Pi_{ij \in B \times N} M_{ij}} \int_{T^{n(\theta + m)}} U(t_{B \times N}, m_{ij}(t_{ij})_{ij \in B \times N}) d\lambda(t), \quad (5)$$

such that for all $ij$ and for all $(t_{ij}, t_{-ij})$ and $(t'_{ij}, t_{-ij}),$

$$V_{ij}(t_{ij}, t_{-ij}, (m_{ij}(t_{ij}), m_{-ij}(t_{-ij})) \geq V_{ij}(t_{ij}, t_{-ij}, (m_{ij}(t'_{ij}), m_{-ij}(t_{-ij}))), \quad (6)$$

where $M_{ij} := M_{ij}(T \times T, GrA(ij, \cdot)).$

Note that equation (5) is the unconditional expectation of CB’s payoff over types.

5 Delegated Networking Principle

Our objective now is to characterize all incentive compatible network formation mechanisms via arc-club catalogs. We call this characterization result the Delegated Networking Principle.\(^2\)

The importance of the delegated networking principal in proving existence of an optimal network formation mechanism is that it allows us to convert the principal-agent network formation game over mechanisms with incentive compatibility constraints into an equivalent unconstrained principal-agent connection game over catalogs. With this conversion, we are able to avoid the difficult problem of searching for a topology for the function space of arc-club-valued mechanisms making the subset of incentive compatible mechanisms compact, players’ payoff functions

\(^2\)The delegation principle for principal-agent games was proved in Page (1992)[9], 1997[10]). Since then it has been extended to the case of multiple principals (i.e., to common agency games with adverse selection) in Page (2000)[8] and Page and Monteiro (2003)[11].
Since each player’s payoff function upper semicontinuous. Instead, our reformulation of the network formation game as a principal-agent game over arc-club catalogs allows us to utilize the topology already present in the space of connections to establish existence.

To begin, suppose that the principal, rather than offering each player, $ij$, a mechanism, $m_{ij}(\cdot)$, and requiring a report to determine the arc-club recommendation, instead offers each player a catalog $C_{ij}$ of arc-club pairs and then observes the player’s choice from the arc-club catalog. Let $(C_{ij})_{ij\in B\times N} \in \Pi_{ij\in B\times N}P_{\rho_A}(GrA(ij, \cdot))$ be the profile of arc-club catalogs offered to the players, and consider the network formation game

$$G(t_{ij}, C_{ij})_{ij\in B\times N} := \{C_{ij}, V_{ij}(t_{ij}, t_{-ij}, (\cdot, \cdot))\}_{ij\in B\times N},$$

where each player’s payoff function is given by equation (1).

Because the catalog is compact and the player’s payoff function $V_{ij}(t_{ij}, t_{-ij}, (\cdot, \cdot), (c_{-ij}, a_{-ij}))$ is continuous for each $(t_{ij}, t_{-ij}, (c_{-ij}, a_{-ij}))$, the player’s catalog choice problem always has a solution. Let

$$V_{ij}(t_{ij}, t_{-ij}, C_{ij}, (c_{-ij}, a_{-ij})) := \max_{(c,a)\in C_{ij}} V_{ij}(t_{ij}, t_{-ij}, (c, a), (c_{-ij}, a_{-ij})),$$

and

$$\Phi_{V_{ij}}(t_{ij}, t_{-ij}, C_{ij}, (c_{-ij}, a_{-ij})) :=
\{ca \in C_{ij} : V_{ij}(t_{ij}, t_{-ij}, (c, a), (c_{-ij}, a_{-ij})) \geq V_{ij}(t_{ij}, t_{-ij}, C_{ij}, (c_{-ij}, a_{-ij}))\}.$$

Since each player’s payoff function $V_{ij}$ is additively coupled, it follows from Proposition 1.1 and Corollary 1.2 in Balder (1997) [1] that the problems in equation (1) and

$$\max_{(c,a)\in C_{ij}} u_{ij,ij}(t_{ij}, (c, a))$$

are such that if

$$u_{ij,ij}(t_{ij}, C_{ij}) := \max_{(c,a)\in C_{ij}} u_{ij,ij}(t_{ij}, (c, a)),
$$

and

$$\Phi_{u_{ij,ij}}(t_{ij}, C_{ij}) := \{ca \in C_{ij} : u_{ij,ij}(t_{ij}, (c, a)) \geq u_{ij,ij}(t_{ij}, C_{ij})\},$$

which is player $ij$’s best response given catalog $C_{ij}$.

Then for each $(t_{ij}, C_{ij})$,

$$\Phi_{u_{ij,ij}}(t_{ij}, C_{ij}) = \Phi_{V_{ij}}(t_{ij}, t_{-ij}, C_{ij}, (c_{-ij}, a_{-ij})), \forall (t_{-ij}, (c_{-ij}, a_{-ij})).$$

Follow the proofs in Page(1992, 2010), we can prove Theorem 1:

**Theorem 1.** *(Measurability and Continuity Properties of $u_{ij,ij}$ and $\Phi_{u_{ij,ij}}$)*

Suppose assumption 1 and assumption 2 hold. The following statements are true for each player $ij$.

1. For each player type $t_{ij}$, $u_{ij,ij}(t_{ij}, \cdot)$ is $h_{\mathcal{A}_c}$-continuous on $P_{\rho_{\mathcal{A}_c}}(GrA(ij, \cdot))$, and for each arc-club catalog, $C_{ij} \in P_{\rho_{\mathcal{A}_c}}(GrA(ij, \cdot))$, $u_{ij,ij}(\cdot, C_{ij})$ is $\mathcal{B}(T) \times \mathcal{B}(T)$-measurable.
2. For each player type $t_{ij}$, $\Phi_{u_{ij,ij}}(t_{ij}, \cdot)$ is $h_{\mathcal{A}_c}$-uppersemi-continuous on $P_{\rho_{\mathcal{A}_c}}(GrA(ij, \cdot))$, and $\Phi_{u_{ij,ij}}(\cdot, \cdot)$ is jointly measurable.
Theorem 1 ensures that there will be at least one optimal solution to the maximum problem stated above.

We now state and prove the Delegated Networking Principle for a principal-multi-agent games of network formation with adverse selection.\(^3\)

**Theorem 2. (The Delegated Network Principle)**

Suppose assumption 1 and assumption 2 hold. The following statements are equivalent.

1. The club connection recommendation mechanism, \((c_{ij}(\cdot), a_{ij}(\cdot))\) is incentive compatible, that is, for all \(t_{ij}\) and \(t'_{ij}\)

\[
u_{ij,i}(t_{ij}, c_{ij}(t_{ij}), a_{ij}(t_{ij})) \geq 
u_{ij,i}(t_{ij}, c_{ij}(t'_{ij}), a_{ij}(t'_{ij})).
\]

2. There exist a unique, minimal arc-club catalog \(C_{ij} \in P_{\rho_{A,c}}(GrA(ij, \cdot))\), such that for each player \(ij\), \((c_{ij}(\cdot), a_{ij}(\cdot))\) is a measurable selection of \(\Phi_{u_{ij,i,j}}(\cdot, C_{ij})\), that is there exists for player \(ij\), an arc-club catalog \(C_{ij}\) such that \((c_{ij}(\cdot), a_{ij}(\cdot)) \in M_{ij}(T \times T, GrA(ij, \cdot))\) and \((c_{ij}(t_{ij}), a_{ij}(t_{ij})) \in \Phi_{u_{ij,i,j}}(t_{ij}, C_{ij})\) for all \(t_{ij}\), and if \((c_{ij}(t_{ij}), a_{ij}(t_{ij})) \in \Phi_{u_{ij,i,j}}(t_{ij}, C'_{ij})\) for all \(t_{ij}\) for some other arc-club catalog \(C'_{ij} \in P_{\rho_{A,c}}(GrA(ij, \cdot))\), then \(C_{ij} \subseteq C'_{ij}\).

**Proof.** (1)⇒(2): Let \((c_{ij}(\cdot), a_{ij}(\cdot))\) be an incentive compatible connection mechanism and for player \(ij\) define the arc-club catalog \(C_{ij}\) as

\[
C_{ij} := \rho_{A_c} - cl\{(c_{ij}(t_{ij}), a_{ij}(t_{ij})) : t_{ij} \in T \times T\}.
\]

Note that for each \(t_{ij}, u_{ij,i}(t_{ij}, c_{ij}(t_{ij}), a_{ij}(t_{ij})) \geq u_{ij,i}(t_{ij}, (c, a))\) for all \((c, a) \in \rho_{A_c} - cl\{(c_{ij}(t_{ij}), a_{ij}(t_{ij})) : t_{ij} \in T \times T\}.

To prove this, suppose not. Then for some agent type \(t'_{ij}\) and arc-club pair

\[
(c', a') \in \rho_{A_c} - cl\{(c_{ij}(t_{ij}), a_{ij}(t_{ij})) : t_{ij} \in T \times T\},
\]

\[
u_{ij,i}(t'_{ij}, (c', a')) \geq 
u_{ij,i}(t'_{ij}, c_{ij}(t'_{ij}), a_{ij}(t'_{ij})).
\]

Because

\[
(c', a') \in \rho_{A_c} - cl\{(c_{ij}(t_{ij}), a_{ij}(t_{ij})) : t_{ij} \in T \times T\},
\]

there exists a sequence of agent types \(t^n_{ij}\) in \(T \times T\) such that \((c_{ij}(t^n_{ij}), a_{ij}(t^n_{ij})) \rightarrow (c', a')\). However, because \(u_{ij,i}(t^n_{ij}, (c', a')) \geq u_{ij,i}(t^n_{ij}, c_{ij}(t^n_{ij}), a_{ij}(t^n_{ij}))\), by the continuity of \(u_{ij,i}(t^n_{ij}, \cdot)\), the fact that \((c_{ij}(t^n_{ij}), a_{ij}(t^n_{ij})) \rightarrow (c', a')\) implies that for \(n\) large enough,

\[
u_{ij,i}(t^n_{ij}, c_{ij}(t^n_{ij}), a_{ij}(t^n_{ij})) > u_{ij,i}(t^n_{ij}, c_{ij}(t^n_{ij}), a_{ij}(t^n_{ij}))
\]

contradicting with the fact that \((c_{ij}(\cdot), a_{ij}(\cdot))\) is incentive compatible. Therefore, we conclude that \((c_{ij}(t_{ij}), a_{ij}(t_{ij})) \in \Phi_{u_{ij,i,j}}(t_{ij}, C_{ij})\) for all \(t_{ij}\).

Now suppose that

\[
(c_{ij}(t_{ij}), a_{ij}(t_{ij})) \in \Phi_{u_{ij,i,j}}(t_{ij}, C'_{ij}), \forall t_{ij} \in T \times T
\]

for some \(C'_{ij} \in P_{\rho_{A,c}}(GrA(ij, \cdot))\), but that for some \((c', a') \in C_{ij}\), \((c', a') \notin C'_{ij}\).

Again, \((c', a') \in \rho_{A_c} - cl\{(c_{ij}(t_{ij}), a_{ij}(t_{ij})) : t_{ij} \in T \times T\}\), there exists a sequence of agent types \(t^n_{ij}\) in \(T \times T\) such that \((c_{ij}(t^n_{ij}), a_{ij}(t^n_{ij})) \rightarrow (c', a')\).

\cite{Page1989,Page1992,Page2010}
\( t^n_{ij} \) in \( T \times T \) such that \((c_{ij}(t^n_{ij}), a_{ij}(t^n_{ij})) \to (c', a')\).

However, now we have
\[
(c_{ij}(t^n_{ij}), a_{ij}(t^n_{ij})) \in \Phi_{u_{ij,ij}}(t_{ij}, C'_{ij})
\]
for all \( n \) and \( \Phi_{u_{ij,ij}}(t_{ij}, C'_{ij}) \subseteq C' \).

Because \( C'_{ij} \) is closed and \((c_{ij}(t^n_{ij}), a_{ij}(t^n_{ij})) \to (c', a')\), we must conclude that \((c', a') \in C'_{ij}\), which is a contradiction.

(2)\( \Rightarrow \) (1): The proof is straightforward.

\[\square\]

6 Future Works to Do

We plan to further specify bank’s problem and CB’s problem to analyse the externalities created by each bank without regulation. We are also interested in the dynamics of debt and default when we allow maturity structure for debt. We will also find a measure of systemic risk, then we are able to conduct some policy applications using our model. Furthermore, we will do empirical research on the inter-bank lending and its effects on firm investment.
References


