Monetary Policy and Credit Constraints

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Abstract

We investigate the effects of a monetary policy expansion in an economy populated by two types of agents. The agents are heterogeneous in their preference discount factor and only the relatively more impatient agents face a collateral borrowing constraint. We find that in the most basic set up of the New Keynesian model, the inclusion of borrowing constraints can potentially lead to very different conclusions about the effects of a monetary expansion relative to the representative agent model with no financial frictions. In particular, for a 25 b.p. surprise decrease in the nominal interest rate, the standard New Keynesian model predicts a positive response in economic activity for all the subsequent periods, with output raising as much as 0.5% upon impact. On the other hand, as all of the agents in the economy become credit constrained, our model predicts that the same surprise decrease in the nominal rate leads to a negative response in economic activity for all the subsequent periods, with output responding positively only in the first period with a modest increase of about 0.01%.

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1 Introduction

The basic New Keynesian model has become the workhorse for analyzing and understanding the effects of different monetary policy interventions in the macroeconomy. In doing so, this model has been modified and often augmented along many different dimensions in an attempt to better explain the available data. Nevertheless, the transmission mechanisms operating in these more complicated models, at their core, are enhanced or diminished versions of the main channels operating in the basic model. Therefore, the presence of certain features, or lack thereof, in the simple model might have important consequences regarding the macroeconomic implications of the more complicated models.

Recognizing the importance of understanding the basic mechanisms by which monetary policy affects the aggregate economy, a vast literature has extensively studied the macroeconomic implications of standard versions of the New Keynesian model. Authors such as [12] and [32], focus on the study of simple versions of the model that have price stickiness as the only nominal friction. Some other authors, like [11] consider slightly augmented versions that incorporate additional nominal frictions. Furthermore, extensive reviews that explain in detail the theoretical foundations of the New Keynesian framework and its applications to monetary policy analysis are readily available, for instance [33], and more recently, [15].

One of the main findings of such literature relates to the way in which the nominal interest rate affects total aggregate output in the economy. In particular, under the assumption that monetary policy is conducted through a Taylor-type rule, the consensus on the literature is that surprise shocks to this rule are negatively correlated with output. That is, a positive surprise shock to the monetary policy rule would imply an increase in the nominal interest rate and a decrease in the measure of economy activity, which is usually given by some form of output. Figure 1, which is taken from [15], illustrates this key feature. An important point to note from this figure is that the standard model predicts that an increase in the nominal rate and a decrease in real output for all periods after the positive shock.

To some extent, this result is the underlying corner stone of the believe, shared by many academics, that the monetary authority can help stimulate or contract the economy through this interest rate channel. Indeed, the term “conventional monetary policy” has been coined to refer to policy interventions that try to exploit this mechanism. However, the 2008 financial crisis in the U.S. forced an entirely new perspective on this view. The so called conventional monetary policy measures taken by the Federal Reserve did not work as expected, to the extent that some “non-conventional” procedures were put in place.

Since then, many authors, like for instance [13] and [26], have tried to evaluate the effectiveness of conventional and non-conventional monetary policy interventions. The resulting literature has concluded that the role of monetary policy through both, conventional and non-conventional channels, is of central importance in determining macroeconomic activity. Similarly, this literature emphasizes the increasing importance of the financial sector in the macroeconomy, which the basic New Keynesian model completely ignores.

Motivated by this observations, the present paper revisits the standard version of the New Keynesian model and incorporates to it one of the most basic financial frictions: a collateral constraint. We argue that even in this most simple setting, there is an important channel which has been largely ignored by the literature. In the presence of the constraint, as monetary policy interventions affect the nominal interest rate and prices, it affects the market value of the good that is pledged by collateral, and hence the degree to which the constraint is binding. This effect can enhance or offset the usual effect that monetary policy has on the agents wealth through prices. Figure 10 illustrates the effect of a surprise negative shock to the monetary policy rule on total output of the economy for the standard New Keynesian model where all agents are free to borrow (the case $\Omega_b \rightarrow 0$) and the model where all agents face a collateral constraint (the case $\Omega_b \rightarrow 1$). For the case where agents face no constraint, the model reduces to the presented in [15]. As it can be seen, the conventional monetary policy which would have led to an economic stimulus, according to the intuition from the basic model, leads to an economic contraction when the agents in the economy face the constraint.

The paper is organized as follows. Section 2 specifies in detail each of the components of the model. In Section 3, a partial characterization of the solution to this model is presented. In addition, two simplified versions of the model are analyzed: the steady state case and a two-period case. Section 4 presents the model calibration and the main results of the paper, which are obtained by solving the log-linearized version of the model. Finally, some concluding remarks are presented in Section 5. In addition, all the figures are presented in Section 6.
2 Model

The basic framework consists of an infinite horizon, two-sector, New Keynesian model with price adjustments as in [5]. The economy is populated by two agents who differ only in the following dimensions: their degree of patience, their valuation of leisure time, and their ability to access the financial markets. The agent with the smallest time preference discount parameter (i.e. the impatient agent), henceforth referred to as the borrower, faces a collateral credit constraint of the type proposed by [23]. The agent with the highest time preference discount parameter, henceforth referred to as the saver, can freely access the financial markets as long as a No-Ponzi condition is satisfied. There are two goods available for consumption, durables and non-durables, each of which is produced by one of the production sectors. Agents can transfer their wealth across time to insure against aggregate uncertainty via the durable goods and a nominal risk free bond which is available in zero net supply. Finally, monetary policy is conducted through a nominal interest rate rule of the type first introduced by [30]. The only source of uncertainty is an exogenous stochastic process in the nominal interest rate rule which captures unanticipated actions by the monetary authority.

2.1 Borrower and Saver

Given the two agents in the economy, let \( a \in \{b, s\} \) denote whether the agent is a saver (\( s \)) or a borrower (\( b \)). Agent \( a \) derives utility from consumption of the non-durable good (\( c^a \)) and the durable good (\( d^a \)). In addition, the agent is endowed with \( H^a \) hours of working time and decides how many hours of work (\( n^a \)) to supply to the producers of the intermediate goods. Since agents value leisure, their supply of working hours results in disutility for them. In particular, the utility (disutility) that agent \( a \) derives from these three activities is specified as:

\[
E_0 \left\{ \sum_{t=0}^{\infty} \beta^a_t \left[ \alpha \log (c^a_t) + (1 - \alpha) \log (d^a_t) - \nu^a \left( \frac{(n^a_t)^{1+\theta}}{1+\theta} \right) \right] \right\},
\]

where \( E_t \{ \cdot \} \) denotes the conditional expectation of the agent given his information set at time \( t \), and \( \beta^a_t \) is the agent’s time preference discount factor.\(^5\)

In order to finance the consumption of the durable and non-durable goods, agents have the following sources of income: a nominal wage rate per hour of labor supplied (\( W \)), the share of profits from the ownership of the firms that produce the intermediate goods (\( \Pi^a \)), and the holdings of the undepreciated durable good. Moreover, the agents have access to a nominal risk free bond (\( B \)) which can be used to transfer resources intertemporally. Therefore, the nominal budget constraint that agent \( a \) faces is given by:

\[
P_{c,t}c^a_t + P_{d,t} (d^a_t - (1 - \delta) d^a_{t-1}) + R_{t-1} B^a_{t-1} = B^a_t + W_t n^a_t + \Pi^a_t,
\]

where \( P_{c,t} \) and \( P_{d,t} \) refer to the price index of non-durable and durable goods, respectively, and \( \delta \) is the depreciation rate of the durable good. The nominal rate on borrowing/lending bond contracts, agreed upon at time \( t - 1 \), is denoted by \( R_{t-1} \). Note that it is assumed that labor is perfectly mobile across the two productions sectors, which in turn implies that the nominal wage rate is the same regardless of the sector to which the agent supplies his labor.

In the nominal bond market, the saver’s borrowing is only limited by a No-Ponzi condition which prevents him from indefinitely rolling debt. The borrower, on the other hand, faces an endogenous borrowing limit. As in [23], this borrowing limit takes the form of a collateral constraint. Such a limit is motivated by realizing that, in principle, it is feasible for the borrower to default on the debt or to refuse to honor the terms of the contract. However, the provision

\(^2\)The model is calibrated so that in steady state, each agent chooses to work a third of their total time endowment. This requires that the parameter \( \nu \), which indexes the preference for hours worked for an agent, be distinct across the borrower and the saver. Since \( \nu \) is just a scaling parameter, it can be interpreted as a proxy for how much each agent values leisure (dislikes work).

\(^3\)Note that despite the use of the terms borrower and saver, both agents can choose to borrow or lend (save) via the nominal bond. The use of the terms is due to the fact that in the deterministic steady state with zero inflation, the patient agent will effectively lend resources to the impatient agent through the nominal bond.

\(^4\)In the current model, the time endowment of each agent is normalized to one: \( H^a = H^b = 1 \).

\(^5\)Note that we have \( \beta^b < \beta^s \).

\(^6\)The current version of the paper does not allow agents to endogenously choose the amount of firms’ shares to hold. Furthermore, for simplicity, it is assumed that the saver owns all of the intermediate firms.
of collateral provides enough incentive so that the borrower endogenously chooses to repay the debt according to the contract. The particular form of the constraint is given by:

\[ R_t B_t^b \leq (1 - \chi) (1 - \delta) E_t \{ d_{i,t} P_{d,t+1} \}. \] (3)

For a given loan amount \( B_t^b \) that the agent decides to borrow at time \( t \), the left side of the inequality is just the repayment amount that is due at period \( t + 1 \). The expression to the right of the inequality sign is the collateral requirement: a given fraction of the expected market value of the agent’s holdings of the undepreciated durable good at period \( t + 1 \). The constraint then simply requires that the repayment value of a given loan does not exceed the collateral value. The parameter \( \chi \) represents the fraction of the durable good that can not be claimed as collateral.

With this specification, given a sequence of prices \( \{ P_{c,t}, P_{d,t}, W_t, R_{t-1} \}_{t=0}^\infty \) of intermediate firm’s profits \( \{ \Pi_t \}_{t=0}^\infty \) and initial durable good and nominal bond holdings, \( d_{-1} \) and \( B_{-1} \) respectively; agent \( a \)’s problem is to choose a sequence \( \{ e_t^a, d_t^a, n_t^a, B_t^a \}_{t=0}^\infty \), with \( e_t^a, d_t^a \geq 0, n_t^a \in [0, 1] \forall t \geq 0 \), to maximize (1) subject to (2), and (3) if \( a = b \) or a No-Ponzi condition if \( a = s \).

## 2.2 Producers of Final Goods

There are two production sectors: one producing the durable good \( (Y_d) \) and the other producing the non-durable good \( (Y_n) \). Let \( j \in \{c, d\} \) denote the sector. Firms in sector \( j \) operate under a perfectly competitive environment. Each firm in sector \( j \) produces the final good using as inputs a continuum of differentiated goods which are produced by the intermediate firms. Let these intermediate goods be indexed by \( i \in [0, 1] \), so that \( y_{j,t}(i) \) is the \( i^{th} \) intermediate variety to be used in the production of good \( Y_{j,t} \) at period \( t \). In order to obtain the inputs, firms must pay a price \( P_{j,t}(i) \) to the intermediate firm producing the variety \( y_{j,t}(i) \).

All firms within sector \( j \) have access to the same technology, which is given by the CES production function

\[ Y_{j,t} = \left( \int_0^1 y_{j,t}(i)^{\epsilon_j} di \right)^{\frac{1}{\epsilon_j}}, \] (4)

where \( \epsilon_j > 1 \) is the sector specific elasticity of substitution between differentiated varieties. Firms sell the final good \( Y_{j,t} \) to the borrower and the saver at price \( P_{j,t} \).\(^7\) Therefore, for a firm producing the final good for sector \( j \), the profits at period \( t \) are given by:

\[ \Pi_{j,t} = P_{j,t} Y_{j,t} - \int_0^1 P_{j,t}(i)y_{j,t}(i) di \] (5)

Under the previous specification, given prices \( \{ P_{j,t}, \{ P_{j,t}(i) \}_{i=0}^1 \}_{t=0}^\infty \) a firm producing the final good in sector \( j \) solves the problem of choosing an allocation \( \{ Y_{j,t}, \{ y_{j,t}(i) \}_{i=0}^1 \}_{t=0}^\infty \) with \( y_{j,t}(i) \geq 0, \forall i \in [0, 1], t \geq 0, j \in \{c, d\} \) to maximize (5) subject to (4) \forall t \geq 0.

Note that the producers of the final good face a static problem (i.e., there is no intertemporal tradeoff). The solution to the previous problem yields the demand functions for the intermediate varieties

\[ y_{j,t}(i) = \left( \frac{P_{j,t}}{P_{j,t}(i)} \right)^{\epsilon_j} Y_{j,t}, \] (6)

and sector specific price index

\[ P_{j,t} = \left( \int_0^1 P_{j,t}(i)^{1-\epsilon_j} di \right)^{\frac{1}{1-\epsilon_j}}. \] (7)

(6) is important since the intermediate firm producing variety \( y_{j,t}(i) \) needs knowledge of its demand in order to solve its problem. Otherwise, the firm wouldn’t be able to internalize the effect of its choice of price \( P_{j,t}(i) \) through the demand channel.

\(^7\)Note that from the firms’ perspective, this price is exogenous. The zero-profit condition that arises due the perfectly competitive environment is what determines the price \( P_{j,t} \). For more details refer to the Appendix.
Finally, it must be noted that the scale of the final production sector is undetermined. Under the assumption that all firms in sector \( j \) are identical in every respect, the actual number of firms is irrelevant as long as the zero-profit condition is satisfied for each firm. Therefore, in the analysis that follows, it is assumed that there is one firm in each of the final production sectors.\(^8\)

### 2.3 Producers of Intermediate Goods

Still need to modify this section to account for the fact that now I am using Calvo Pricing instead of Rotemberg

For each sector, \( j \in \{c,d\} \), there is a continuum of mass one of intermediate firms, each producing a different variety of the intermediate good \( j \). Let these firms be indexed by \( i \in [0,1] \), so that \( y_{j,t}(i) \) is the notation used to represent the intermediate variety produced by firm \( i \) of good \( j \) at period \( t \). The firms in each sector operate under monopolistic competition; each firm chooses the price \( P_{j,t}(i) \) and variety \( y_{j,t}(i) \) to sell to the producers of the final good given the demand for the variety (6).

As first introduced by \([28]\), it is assumed that for firm \( i \) to be able to reset its price every period, it must invest some real resources. The motivation behind this assumption is that, by changing the price, a firm might upset the buyers of the intermediate good ultimately leading them to buy a different variety. Therefore, the potential buyers that are lost due to the price change are reflected in this cost. In particular, following \([19]\), the adjustment cost, in terms of the final good produced in the sector, takes the quadratic form specified as:

\[
Q_{j,t}(i) = \frac{\phi_j}{2} \left( \frac{P_{j,t}(i)}{P_{j,t-1}(i)} - 1 \right)^2 Y_{j,t},
\]

so that \( \phi_j \) is the parameter that controls the price resetting cost for the entire sector \( j \). Note that the price adjustment cost is proportional to the entire sector index \( Y_{j,t} \). In good times, where the entire sector’s production is higher, a potential buyer might find it easier to switch to a different variety due to a price change of firm \( i \). Hence one can expect the cost associated with price changes to be higher. The argument is analogous in the case of bad times.

For both sectors, the only input in the production of a variety is labor. Since labor is perfectly mobile across sectors and given that the labor market is perfectly competitive, the firm producing \( y_{j,t}(i) \) must pay the worker a nominal wage rate \( W_t \) for each hour worked. Note further that, from the perspective of the individual firm, this wage rate is exogenously given. All firms within the sector have access to the same production technology. Furthermore, to keep the analysis tractable, such production technology is assumed to be linear in the labor input:

\[
y_{j,t}(i) = A_j n_{j,t}(i),
\]

where \( A_j \) is a constant that measures labor productivity in each sector.\(^9\)

Finally, to simplify the problem, it is assumed that the intermediate firms are solely owned by the saver. Furthermore, shares of these firms can not be traded in the market. Given this assumption, for any \( \tau > t \), the intermediate firms discount the future nominal profit at date \( \tau \) back to date \( t \) using the saver’s nominal stochastic discount factor:

\[
\Lambda_{\tau,t} = \frac{\lambda^e_{\tau}}{\lambda^e_t} = \beta^{\tau-t} \frac{c^e_{\tau} P_{c,\tau}}{c^e_t P_{c,t}},
\]

where \( \lambda^e_{\tau} \) is the shadow price of a unit of the nominal asset for the saver at period \( \tau \).\(^{10}\)

Under this specification, the expected discounted nominal profits of the firm producing variety \( i \) in sector \( j \) are given by:

\[
\Pi_{j}(i) = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \Lambda_{t,0} \left[ P_{j,t}(i) y_{j,t}(i) - W_t n_{j,t}(i) - Q_{j,t}(i) P_{j,t} \right] \right\},
\]

\(^8\)Note that this assumption does not violate the perfect competition environment. One can think of this firm as being a representative firm for a large number of identical small firms in each sector.

\(^9\)In the current analysis, this constant is set to one in both sectors: \( A_d = A_c = 1 \).

\(^{10}\)This shadow price is obtained by taking the FOC’s of the saver’s problem. For details refer to the Appendix.
where $E_t \{ \cdot \}$ denotes expectation given the firm’s information set at time $t$. Furthermore, given that the firms are solely owned by the saver, the share of profits for the agents at period $t$ is
\[
\Pi_s^t = \sum_{j \in \{c,d\}} \left\{ \int_0^1 \left[ P_{j,t}(i) y_{j,t}(i) - W_{t} n_{j,t}(i) - Q_{j,t}(i) P_{j,t} \right] \, di \right\} \tag{12}
\]
\[
\Pi_b^t = 0 \tag{13}
\]

Finally, given sequences of nominal wage rates $\{W_t\}_{t=0}^\infty$, sector specific output and price indexes $\{\{Y_{j,t}, P_{j,t}\}_{t=0}^\infty\}_{j \in \{c,d\}}$, and initial variety price $P_{j,-1}(i)$, the intermediate firm $i$ in sector $j$ faces the problem of choosing an allocation $\{y_{j,t}(i), n_{j,t}(i)\}_{t=0}^\infty$ to maximize (11) subject to (6), (8), and (9).

## 2.4 Monetary Authority

As proposed by [30], monetary policy is conducted through the use of a nominal interest rate rule of the form
\[
\frac{R_t}{R} = \left( \frac{\pi_t}{\bar{\pi}} \right)^{\phi_{\pi}} z_t, \tag{14}
\]
where $R_t$ is the interest rate on nominal bond contracts agreed on at time $t$ and $\pi_t$ is a composite inflation index that weights the inflation in the durable ($\pi_{d,t}$) and non-durable ($\pi_{c,t}$) final sectors according to
\[
\pi_t = \pi_{c,t}^{1-\gamma} \pi_{d,t}^{\gamma},
\]
with
\[
\pi_{j,t} = \frac{P_{j,t}}{P_{j,t-1}}, \quad j \in \{c,d\}.
\]
The notation $\bar{x}$ refers to the value of the variable $x$ in the deterministic steady state. Given this rule, the monetary policy operates under inflation targeting, with the particular target being the steady state level of the inflation index.\textsuperscript{11} Furthermore, it is assumed that the interest rule follows the Taylor principle with $\phi_{\pi} > 1$. This assumption is imposed for two main reasons. First, some authors, like [31] and [12], have argued that the empirical evidence suggests that the failure of the monetary authority to follow this principle has led to episodes of greater macroeconomic instability in the US. Second, and more relevant to this paper, as explored by [32] and [33], the Taylor principle usually arises as a necessary and sufficient condition for the existence of a unique stable solution in the class of New Keynesian forward looking models.

The variable $z_t$ is an exogenous stochastic component that captures the unanticipated actions of the monetary authority.\textsuperscript{12} It is assumed that such process follows an AR(1) of the form
\[
\log(z_t) = \rho \log(z_{t-1}) + \epsilon_t, \tag{15}
\]
where $\rho \in (0,1)$ is the autocorrelation coefficient that controls the degree of persistence of the process and $\epsilon_t \sim$ WN(0, 1).

## 3 Solution

This section presents a partial characterization of the general equilibrium of the model presented in section 2. In order to get a better understanding of the model and its implications, two simplified version of the model are discussed and their analytical solution is presented in detail: the steady state case and the two-period case.

\textsuperscript{11}For the current model, this target is set to one. That is, $\bar{\pi}_d = \bar{\pi}_c = 1$.

\textsuperscript{12}For instance, one can think of this term as the monetary authority’s reaction to exogenous variables that are not currently model such as financial risk.
3.1 Equilibrium Concept

Let the initial holdings of the durable good and nominal debt for the agents, \( d_{-1}^{d}, d_{-1}^{b}, B_{-1}^{d}, \) and \( B_{-1}^{b}, \) the initial prices of the intermediate varieties for each sector \( \{ P_{j,-1}(i) \}_{j \in \{c,d\}} \), and the initial nominal interest rate \( R_{-1} \) be given. An equilibrium for this economy is defined as sequences

\[
\begin{align*}
(i) & \quad \{ (c_{t}^{a}, d_{t}^{a}, n_{t}^{a}, B_{t}^{a})_{t=0}^{\infty} \}_{a \in \{b,s\}} \\
(ii) & \quad \left\{ \left\{ (y_{j,t}(i), n_{j,t}(i), P_{j,t}(i))_{i \in [0,1]} \right\}_{t=0}^{\infty} \right\}_{j \in \{c,d\}} \\
(iii) & \quad \{ Y_{c,t}, Y_{d,t} \}_{t=0}^{\infty} \\
(iv) & \quad \{ P_{c,t}, P_{d,t}, W_{t}, R_{t} \}_{t=0}^{\infty} \\
(v) & \quad \{ \Pi_{t}^{c}, \Pi_{t}^{d} \}_{t=0}^{\infty}
\end{align*}
\]

such that the following hold:

1. Given the exogenous process (15), the sequence (iv) satisfies (14).
2. Given (ii), (iii), (iv), the sequence (v) satisfies (12) and (13).
3. Given (iv), (v), the allocation \( \{ c_{t}^{b}, d_{t}^{b}, n_{t}^{b}, B_{t}^{b} \}_{t=0}^{\infty} \) is a solution to the borrower’s problem.
4. Given (iv), (v), the allocation \( \{ c_{t}^{s}, d_{t}^{s}, n_{t}^{s}, B_{t}^{s} \}_{t=0}^{\infty} \) is a solution to the saver’s problem.
5. Given (iii), (iv), the sequence (ii) gives the solution to the intermediate firms’ problem.
6. Given \( \left\{ \left( P_{j,t}, \{ P_{j,t}(i) \}_{i \in [0,1]} \right)_{t=0}^{\infty} \right\}_{j \in \{c,d\}} \) in (iv) and (ii), the sequences (iii) and \( \left\{ \left\{ (y_{j,t}(i))_{i \in [0,1]} \right\}_{t=0}^{\infty} \right\}_{j \in \{c,d\}} \) in (ii) are a solution to the problem faced by the final good producers.
7. The final non-durable good market clears

\[
Y_{c,t} = c_{t}^{b} + c_{t}^{s} + \int_{0}^{1} Q_{c,t}(i)di, \quad \forall \ t \geq 0
\]  
(16)

8. The final durable good market clears

\[
Y_{d,t} + (1 - \delta) \left( d_{t-1}^{b} + d_{t-1}^{s} \right) = d_{t}^{b} + d_{t}^{s} + \int_{0}^{1} Q_{d,t}(i)di, \quad \forall \ t \geq 0
\]  
(17)

9. The nominal debt market clears

\[
B_{t}^{b} + B_{t}^{s} = 0, \quad \forall \ t \geq 0
\]  
(18)

10. The labor market clears

\[
n_{t}^{b} + n_{t}^{s} = \int_{0}^{1} [n_{c,t}(i) + n_{d,t}(i)]di, \quad \forall \ t \geq 0
\]  
(19)

\[\text{Note that the initial prices for the intermediate varieties implicitly define the initial price index for each sector as } P_{j,-1} = \left( \int_{0}^{1} P_{j,-1}(i)^{1-\epsilon}di \right)^{\frac{1}{1-\epsilon}}.\]

\[\text{Note that market clearing for each of the intermediate varieties is embedded in the model specification.}\]
3.2 Equilibrium Characterization

Still need to update this section to reflect the switch from Rotemberg pricing to Calvo pricing

A partial characterization of the general equilibrium is presented next. The analysis that follows focuses on the particular case of a symmetric equilibrium in which we impose that

\[ P_{j,-1}(i) = P_{j,-1}(k) = P_j, \quad \forall \ i, k \in [0, 1], \ j \in \{c, d\}. \]  

That is, all the varieties within a sector have the same predetermined price at the beginning of time.

The behavior of the firms producing the final good is completely summarized by the demand functions for intermediate varieties, as given in (6), and the sectorial price index, expression (7). Given the symmetric equilibrium assumption, the final good sectors do not display any interesting features worthwile discussing. The only implications are the now trivial relations that \( \forall \ i \in [0, 1] \):

\[ P_{j,t}(i) = P_{j,t} \]
\[ y_{j,t}(i) = Y_{j,t} \]

That is, one can refer to the varieties of a sector or to final sector output interchangeably. Furthermore, given the linear production function of the intermediate firms, this implies that for each \( j \in \{c, d\} \) and \( i \in [0, 1] \)

\[ n_{j,t}(i) = Y_{j,t} \]  

(21)

\[ Q_{j,t}(i) = \frac{\phi_j}{2} (\pi_{j,t} - 1) Y_{j,t} \]  

(22)

The optimal price setting behavior of the intermediate firms yields the \( j \) sector price dynamics equation

\[ (\pi_{j,t} - 1) \pi_{j,t} = \left( \frac{\epsilon_j}{\phi_j} \right) \left( \omega_{j,t} - \frac{\epsilon_j - 1}{\epsilon_j} \right) + \mathbb{E}_t \left\{ \Gamma_{j,t} \left( \pi_{j,t+1} - 1 \right) \pi_{j,t+1} \right\} \]

\[ = \left( \frac{\epsilon_j}{\phi_j} \right) \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \prod_{l=0}^{k} \Gamma_{j,t-l+1} \left( \omega_{j,t+k} - \frac{\epsilon_j - 1}{\epsilon_j} \right) \right\} \]

\[ + \lim_{T \to \infty} \mathbb{E}_t \left\{ \prod_{k=0}^{T} \Gamma_{j,t+k} \left( \pi_{j,t+1+T} - 1 \right) \pi_{j,t+1+T} \right\}, \]  

(23)

where \( \omega_{j,t} = \frac{W_t}{P_{j,t}} \) is the real wage rate in units of the final good of sector \( j \), and \( \Gamma_{j,t+\tau} \) is a discount factor with \( \Gamma_{j,t+T} = 1, \ j \in \{c, d\} \) if \( \tau = -1 \) and

\[ \Gamma_{j,t+\tau} = \begin{cases} \beta_s & \text{if } j = c, \\ \frac{c_{t+\tau}^d}{c_{t+\tau}^d + \eta_{j+t+\tau+1}} Y_{c,t+\tau+1}^{c_{t+\tau}^d} & \text{if } j = d, \end{cases} \]

for \( \tau \geq 0 \). The variable \( q_t = \frac{P_{d,t}}{P_{c,t}} \) denotes the relative price of a unit of durable good in terms of the non-durable.

Note that, in the deterministic steady state with zero inflation, (23) implies that the real wage rate, in units of the output of sector \( j \), is given by \( \omega_j = (\epsilon_j - 1) \epsilon_j^{-1} \). As seen on the the first term on the right hand side of (23), the driving force behind the pricing behavior is the expected discounted value of the sum of deviations of the real wage rate in sector \( j \) from its steady state value. When will inflation in the current period increase? In a neighborhood of the zero inflation deterministic steady state and as long as the limit in (23) is equal to zero, if firms expect that the overall real cost of hiring workers will increase in the future, they will choose to set a higher price in the current period. Observe also that intermediate firms choose to change the price “smoothly”. That is, rather than having a one...
time change in the current period price to offset the future expectations of the labor costs, they optimally adjust the prices of all the periods between the current period and the period in which the deviation of the labor cost is expected. In this sense, only unanticipated deviations of labor costs create sudden changes in prices. This behavior is a direct consequence of the quadratic price adjustment costs.

Now, is the second term to the right of the equal sign on (23) zero? Consider the transversality condition for the intermediate firm in sector \( j \) which is given by:

\[
\lim_{T \to \infty} \mathbb{E}_t \left\{ \phi_j P_{c,t} \beta_s^{T-t} \frac{c_t}{c_t^{T-t}} \frac{p_{j,t+T}}{p_{c,t+T}} Y_{j,t+T} (\pi_{j,t+T} - 1) \pi_{j,t+T} \right\} = 0. \tag{24}
\]

Note that as long as \( q_t, Y_{j,t}, P_{c,t} > 0 \) for \( j \in \{c, d\} \), we can divide (24) by these variables to get

\[
0 = \lim_{T \to \infty} \mathbb{E}_t \left\{ \beta_s^{T} \frac{c_t}{c_t^{T-t}} \frac{p_{j,t+T}}{p_{j,t}} \frac{P_{c,t}}{P_{c,t+T}} (\pi_{j,t+T} - 1) \pi_{j,t+T} \right\}
\]

\[
= \lim_{T \to \infty} \mathbb{E}_t \left\{ \prod_{k=0}^{T-1} \Gamma_{j,t+k} (\pi_{j,t+T} - 1) \pi_{j,t+T} \right\},
\]

which is precisely the limit term on (23).

To analyze the optimal behavior of the agents, consider first the intratemporal tradeoff that agent \( a \) faces between consumption and leisure, which is summarized in the following condition:

\[
\alpha \frac{(c_t^a)}{c_t^a} \omega_{c,t} = \nu (n_t^a)^\theta, \quad \forall t \geq 0
\]

This is a standard condition which states that the agent optimally equates the marginal benefit of labor supplied (left hand side) to its marginal cost (right hand side).

Next, since the agent can transfer wealth across time via the durable good and the nominal bond, there are two intertemporal conditions that the optimal allocation of the agents must satisfy. The condition related to the durable good is

\[
\alpha (c_t^a)^{-1} q_t = \frac{(1 - \alpha) (d_t^a)^{-1} + \beta_a (1 - \delta) \mathbb{E}_t \left\{ \alpha (c_{t+1}^a)^{-1} q_{t+1} \right\}}{1 - (1 - \chi) (1 - \delta) \zeta_t^a \mathbb{E}_t \left\{ \pi_{d,t+1} \right\}}, \tag{26}
\]

where \( \zeta_t^a = \psi_t^a (\lambda_t^a)^{-1} \), the ratio of the constraint multipliers. Again, this condition is just equating the marginal cost of the last unit of durable good to its marginal benefit. In choosing to consume an additional unit of the durable good, the agent is forgoing consumption of the non-durable good, which he values according to the left hand side of (26). What benefit does the agent get from this additional unit of durable good? Well, this marginal benefit is composed of three terms: the immediate value the agent attaches to the consumption of the good, the potential benefit of allowing the agent to borrow more through the nominal asset by increasing its holdings of collateral, \( (1 - \chi) (1 - \delta) \zeta_t^a \mathbb{E}_t \left\{ \pi_{d,t+1} \right\} \). This last term is relevant only if the credit constraint is binding (i.e. only if \( c_t^a > 0 \)).

On the other hand, the condition summarizing the intertemporal tradeoff of the nominal bond is

\[
1 = R_t \left( \zeta_t^a + \beta_a \mathbb{E}_t \left\{ \frac{c_t^a}{c_{t+1}^a} \frac{1}{\pi_{c,t+1}} \right\} \right). \tag{27}
\]

---

15 This condition can also we written in terms of the real wage expressed in units of the durable good as \( \omega_{d,t} q_t = \nu \alpha^{-1} (n_t^a)^\theta c_t^a, \quad \forall t \geq 0 \).
16 In particular, \( \psi_t^a \) is the multiplier on the collateral credit constraint (3) and \( \lambda_t^a \) is the multiplier on the budget constraint (2). Note that since the saver doesn’t face a collateral constraint, \( \psi_t^a = 0, \quad \forall t \geq 0 \).
17 The marginal unit of the durable good could allow the agent to borrow more in terms of the nominal bond. This additional borrowing would ultimately allow the agent to get \( (1 - \chi) (1 - \delta) \zeta_t^a \mathbb{E}_t \left\{ \pi_{d,t+1} \right\} \) units more of the non-durable good, which give him an additional per unit benefit of \( \alpha (c_t^a)^{-1} q_t \).
When borrowing an additional unit of the bond, the agent is able to increase his current period consumption to obtain and additional benefit of \( \alpha (P_{c,t} c_t) \). Since the agent must repay the borrowed amount in the next period, he forgos future consumption which would bring him an expected benefit of \( \alpha \beta \alpha R_{t+1} \). To the extent that the agent is credit constrained (i.e., the collateral constraint is binding and \( \zeta \)), the agent needs to give up current resources in order to increase his holdings of collateral. This additional cost is captured by the term \( R_t \psi t \). Equating these costs to the marginal benefit and rearranging one gets (27).

Equations (25)-(27) are obtained using the FOC’s for the agent’s problem. In addition, transversality conditions are needed as part of the solution characterization. The general form of the transversality condition for agent \( a \) is

\[
\lim_{t \to \infty} E_0 \left\{ \alpha \beta_a \left( q_t (1 - \delta) d_{t-1}^a - c_t^a + R_{t-1} \frac{b_{t-1}^a}{\pi_{c,t}^a} \right) \right\} = 0,
\]

(28)

where \( b_t^a = B_t^a (P_{c,t})^{-1} \) is the real debt holdings of agent \( a \) in terms of the non-durable good. Manipulating the previous condition and using each of the agents’ FOC, we can rewrite (28) for the saver as

\[
\lim_{t \to \infty} E_0 \left\{ \beta_a d_t^a + b_t^a \right\} = 0,
\]

while for the borrower it takes the form

\[
\lim_{t \to \infty} E_0 \left\{ \beta_b q_t d_t^b (1 - \gamma c_{d,t+1}) + b_t^b (1 - R_t c_t^b) \right\} = 0,
\]

where \( \gamma = (1 - \chi) (1 - \delta) \).

The characterization of the agent’s behavior is completed via the complementary slackness conditions. Given that the nominal budget constraint (2) holds with equality for both agents, the multiplier on this constraint (\( \lambda_c^a \)) will be strictly positive for all periods \( t \). Note that we can rewrite the nominal budget constraint to express it in real units of the non-durable good as

\[
e_t^a + q_t (d_{t-1}^a - (1 - \delta) d_{t-1}^a) + \frac{R_{t-1}}{\pi_{c,t}} b_{t-1}^a = b_0^a + \omega_{c,t} n_t^a + \frac{\Pi_t^a}{P_{c,t}}.
\]

(29)

The last term stands for the share of real profits agent \( a \) gets from the intermediate firms. This term is zero for the borrower and for the saver it is given by

\[
\frac{\Pi_t^a}{P_{c,t}} = Y_{c,t} \left( 1 - \omega_{c,t} - \frac{\phi_d}{2} (\pi_{c,t} - 1)^2 \right) + Y_{d,t} q_t \left( 1 - \omega_{d,t} - \frac{\phi_d}{2} (\pi_{d,t} - 1)^2 \right).
\]

Since the saver faces no credit constraint, the only relevant complementary slackness condition is the one imposed on the borrower

\[
\psi_t^b \left[ R_t B_t^b - (1 - \chi) \right] (1 - \delta) E_t \left\{ d_{t+1}^b P_{d,t+1} \right\} = 0.
\]

(30)

Before proceeding any further, some more insight can be gained from analyzing the intertemporal conditions and imposing the restrictions implied by (28). In particular, by noting that \( \lambda_t^a = \alpha (c_t P_{c,t})^{-1} \), one can rewrite (26) as

\[
\alpha (c_t^a)^{-1} q_t = \sum_{k=0}^{\infty} \left[ \beta_a (1 - \delta) \right] \frac{1}{k!} \left( 1 - \chi \right) (1 - \delta) \psi_t^a \left\{ \frac{q_t + T + 1}{c_t^a + T + 1} \right\} \left( \alpha (c_t^a)^{-1} \right)
\]

\[
+ \lim_{t \to \infty} \left[ \beta_a (1 - \delta) \right] T + 1 E_t \left\{ \frac{q_t + T + 1}{c_t^a + T + 1} \right\}.
\]

(31)
When is the the limit term in the last expression zero? In what follows, assume that \( \exists \tau \in \mathbb{N} \) such that
\[
\prod_{k=0}^{T-1} \frac{d_{t+k}^a}{d_{t+k}} \geq (1 - \delta)^T \text{ a.s., } \forall T \geq \tau. \quad (32)
\]
Intuitively, this condition imposes a lower bound on the average long run growth rate of the sample paths for the durable good. Given this condition, (28) implies that
\[
0 = \lim_{T \to \infty} \beta_a^{T+1} (1 - \delta) \mathbb{E}_t \left\{ \frac{q_{t+T+1}}{c_{t+T+1}} \right\}
\]
\[
= \lim_{T \to \infty} \beta_a^{T+1} (1 - \delta) \mathbb{E}_t \left\{ \left( \prod_{k=0}^{T-1} \frac{d_{t+k+1}^a}{d_{t+k}} \right) \frac{q_{t+T+1}}{c_{t+T+1}} \right\}
\]
\[
\geq \lim_{T \to \infty} \beta_a (1 - \delta) \mathbb{E}_t \left\{ \frac{q_{t+T+1}}{c_{t+T+1}} \right\} \geq 0,
\]
where the last line uses the fact that \( d_{t}^a, c_{t}^a, q_{t} > 0 \) for all periods \( t \). Thus the limit term in (31) is zero.

Similarly, the intertemporal condition related to the nominal bond, equation (27), can be iterated forward as long as \( \zeta_a^0 > 0 \). Therefore, for the borrower and in a neighborhood of the deterministic steady state, we have
\[
\frac{\zeta_a}{c_t^1} = \sum_{k=0}^{\infty} \mathbb{E}_t \left\{ \begin{array}{c}
\beta_{b_k}^b \left( \sum_{j=0}^{k} R_{t+j}^b \frac{c_{t+k+1}^b}{c_{t+k+1}^b} \right) \psi_{t+k}^b P_{c,t+k+1}^b \\
\lim_{T \to \infty} \alpha \beta_a^{T+1} \mathbb{E}_t \left\{ \left( \prod_{k=0}^{T-1} \frac{R_{t+k}^b}{c_{t+k+1}^b} \right) \frac{1}{c_{t+T+1}^b} \right\}
\end{array} \right\}. \quad (33)
\]
To have the limit term in this expression equal to zero, assume that \( \forall \tau' \in \mathbb{N} \) such that
\[
\prod_{k=0}^{T-1} \frac{b_{t+k+1}^b}{b_{t+k}^b} \geq \prod_{k=0}^{T-1} \frac{R_{t+k}^b}{c_{t+k+1}^b} \text{ a.s., } \forall T \geq \tau'. \quad (34)
\]
This condition is roughly imposing a lower bound on the average long run growth rate of the sample paths for the real bond holdings of the borrower. Given this condition, (28) implies that
\[
0 = \lim_{T \to \infty} \alpha \beta_a^{T+1} \mathbb{E}_t \left\{ \frac{R_{t+T}^b}{c_{t+T+1}^b} \right\}
\]
\[
= \lim_{T \to \infty} \alpha \beta_a^{T+1} \mathbb{E}_t \left\{ \frac{R_{t+T}^b}{c_{t+T+1}^b} \left( \prod_{k=0}^{T-1} \frac{b_{t+k+1}^b}{b_{t+k}^b} \frac{1}{c_{t+T+1}^b} \right) \right\}
\]
\[
\geq \lim_{T \to \infty} \alpha \beta_a^{T+1} \mathbb{E}_t \left\{ \left( \prod_{k=0}^{T} \frac{R_{t+k}^b}{c_{t+k+1}^b} \right) \frac{1}{c_{t+T+1}^b} \right\} \geq 0,
\]
where the last line uses the fact that \( R_{t}, \pi_{c,t}, c_{t}^b > 0 \) for all periods \( t \). Thus the second term on the right hand side of (33) is zero.

To finish the characterization of the solution, note that using some of the previous results in this section, the market clearing conditions (16)-(19) can be written as:
\[
Y_{c,t} = \Omega_c c_t^b + \Omega_s c_t^s \quad (35)
\]
\[
Y_{d,t} = \Omega_b (d_t^b - (1 - \delta) d_{t-1}^b) + \Omega_s \left( d_t^s - (1 - \delta) d_{t-1}^s \right) \quad (36)
\]
\[
0 = \Omega_b b_t^b + \Omega_s b_t^s \quad (37)
\]
\[
Y_{c,t} + Y_{d,t} = \Omega_b n_t^b + \Omega_s n_t^s, \quad (38)
\]
\(^{18}\)The current version of the paper does not provide conditions under which this assumption is true, but this will be explored in future versions. Also, the imposed condition is rather strong, and there might be weaker conditions that are needed for the desired result.
\(^{19}\)As with (32), this assumption is rather strong and one might be able to find weaker assumptions that ensure the desired result. Also, note that explicitly providing restrictions on the primitives that will ensure that the assumption is true is left for future versions of the paper.
which must hold \( \forall t \geq 0 \).

Therefore, the solution to the symmetric equilibrium for the model is given by sequences

1. \( \left\{ \left\{ c^a_t, d^a_t, n^a_t, b^a_t \right\}_{a \in \{b,s\}}, \{ Y^j_{j,t} \}_{j \in \{c,d\}}, \zeta^b \right\}_{t=0}^{\infty} \)

2. \( \left\{ \left\{ \pi^j_{j,t}, \omega^j_{j,t} \right\}_{j \in \{c,d\}}, R, q \right\}_{t=0}^{\infty} \)

such that given the initial conditions \( b^s_0, b^b_0, d^s_0, d^b_0, \pi^c_0, \pi^d_0, R_0, q_0 \) and the exogenous stochastic process specified by (15), satisfy (14), (23), (24), (25), (26), (27), (28), (29), (30), (35), (36), (37), and (38).

**3.3 Deterministic Steady State**

A deterministic steady state of the model is defined as \( x \in \tilde{S} \) s.t. \( x^t = x^t+1 = \tilde{x} \), where

\[
\tilde{S} = \left\{ \left\{ c^a_t, d^a_t, n^a_t, b^a_t \right\}_{a \in \{b,s\}}, \{ Y^j_{j,t} \}_{j \in \{c,d\}}, \zeta^b, \{ \pi^j_{j,t}, \omega^j_{j,t} \}_{j \in \{c,d\}}, R, q \right\}
\]

In particular, in the analysis that follows the condition \( \tilde{\pi}_d = \tilde{\pi}_c = 1 \) is imposed. That is, attention is restricted to the deterministic steady state with zero inflation.

To ensure a well defined problem, the existence and uniqueness of such steady state are important. The current paper does not provide a formal proof of these two properties; instead, it relies on the existing literature.\(^{20}\) It suffices to note that given the heterogeneous discount factor assumption \( (\beta_s > \beta_b) \), the steady state is independent of the initial wealth distribution of the agents. As for the existence of the steady state, the presence of the collateral constraint ensures that that borrower and saver can sustain a constant consumption in steady state. Under heterogenous discount rates and perfect financial markets, it is not clear whether it is the saver’s or borrower’s discount factor which pins down the market interest rate. If the saver’s discount factor determines the market interest rate, the borrower’s consumption could not remain constant and would asymptotically decrease to zero. If the borrower’s discount factor pins down the interest rate, the saver’s consumption would be asymptotically increasing over time. Therefore, these two elements (borrowing constraint and discount factor heterogeneity) ensure the existence and uniqueness of the deterministic zero inflation steady state.

Given the assumption of zero inflation, the production sector behaves as in the case of monopolistic competitive firms under perfectly flexible pricing. This implies that in each sector, firms enjoy a markup which depends on the elasticity of substitution between the sector varieties. In particular, the intermediate firms are able to markup their prices over their production cost:\(^{21}\)

\[
\tilde{\omega}_j = 1 - \frac{1}{\epsilon_j}, \quad j \in \{c,d\}.
\]

(39)

The perfect mobility of labor across sectors implies that the nominal wage rate that agents receive is the same regardless of the sector to which they supply their labor. Given (39), this determines the relative price of the durable goods

\[
\tilde{q} = \frac{\tilde{\omega}_c}{\tilde{\omega}_d} = \frac{(\epsilon_c - 1) \epsilon_d}{(\epsilon_d - 1) \epsilon_c}.
\]

(40)

The behavior of the savers and the borrowers determines the remaining steady state variables. From (27), the interest rate is given by \( \tilde{R} = (\beta_b)^{-1} \). This in turn implies that the ratio of the shadow price of the borrowing constraint to that of the budget constraint is \( \zeta^b = \beta_s - \beta_b > 0 \). That is, the borrowing constraint is binding for the borrowers in steady state. One way to interpret the value of this ratio is as follows. Suppose the collateral constraint is relaxed by

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\(^{20}\)The determinacy of the steady state in models with heterogeneous agents has been studied by authors such as [3] and [4] among others.

\(^{21}\)Recall that given the linear form of the production function, the marginal cost of each unit is just the wage rate and the marginal revenue for the firm is just the price, which is normalized to 1.
increasing the fraction of the durable good that can be pledged as collateral (i.e., a decrease in $\chi$). In particular, suppose that this is done in such a way so that the borrowers can increase their debt holdings by $\beta s$ units without changing their holdings of the durable good. This would in turn imply that the borrowers can derive an additional utility in the current period of $\beta s \lambda t$. However, next period, the value of the debt that the borrowers must repay is equal to one, and so they must forgo a unit of their income. This unit of income would have yielded them a utility of $\beta s \lambda t$. The value of relaxing the collateral constraint is given by $\tilde{\rho}_t = (\beta s - \beta b) \lambda b$, from where the desired result follows.

Therefore, the binding collateral constraint determines the debt to durables ratio that the borrowers must hold

$$\frac{\tilde{d} s}{c s} = 1 - \chi (1 - \delta) \frac{\tilde{q}}{R},$$

(41)

Given the interest rate and the relative prices, the intertemporal condition (26) determines the ratio of the durable to non-durable consumption. For the savers this ratio is

$$\frac{\tilde{d} b}{c b} = \left(1 - \alpha \right) \left(1 - \beta s (1 - \delta)\right) \frac{1}{1 - \beta b (1 - \delta)},$$

(42)

and for the borrowers

$$\frac{\tilde{d} b}{c b} = \left(1 - \alpha \right) \left(1 - \beta s (1 - \delta) + \chi (1 - \delta) (\beta s - \beta b)\right).$$

(43)

It is clear from this expressions that

$$\frac{\tilde{d} b}{c b} \leq \frac{\tilde{d} s}{c s},$$

with the inequality holding strictly for $\chi > 0$ and $\delta < 1$. For the case of full depreciation, $\delta = 1$, the durable good can’t be used to store wealth for future periods. Therefore, it is no different than the non-durable good and the ratio between the two is completely determined by their relative, static, marginal utilities. Given the agents are homogeneous in this dimension, the ratio is the same for the borrowers and the savers. To see the effect of the collateral requirement on the durables to non-durables ratio for the borrowers, note that the third term on the denominator of (43) can be interpreted as a measure of the cost of holding non-durable good units that can’t be pledged as collateral. When a fraction $\chi$ of every unit of the undepreciated durable good can’t be pledged as collateral, the borrowers are foregoing the excess return of this unit whose next best alternative would be to serve as collateral.22 Given that the borrowers face this additional cost, for every unit of non-durable, they choose to hold a smaller amount of the durable compared to the saver. For the case in which all of the durable good holdings can be pledged as collateral, $\chi = 0$, there is not foregone value of durable goods and the last term drops out. In this case, the ratio of durables to non-durables is the same for both agents.

Given the ratios (41), (43), the ratio of hours of labor supplied to units of non-durable good consumed for the borrowers can be obtained from their budget constraint. It is given by

$$\frac{\tilde{n} b}{\tilde{c} b} = \frac{1 + \tilde{q} (1 - \chi (1 - \delta) - (1 - \chi) (1 - \delta) \beta s)}{\tilde{\omega} c} \frac{\tilde{d} b}{c b}. $$

(44)

This equation can be interpreted in terms of how the borrowers are using their labor income. A fraction of the durable good is self sustained. That is, a part of the borrowers’ consumption of the durable good comes from what they had stored of that good in the previous period (the term $\chi (1 - \delta)$). Another fraction of the durable good consumption is sustained by debt holdings (the last term in the numerator). Hence, the borrowers use labor income to finance their consumption of the non-durable good and whatever fraction of the durable good they were unable to finance through debt and durable savings.

22Note that the shadow value of income is given by $\lambda b = \beta b \alpha (c b)^{-1}$. However, in steady state we have $c b = c b \forall t$; so that $\lambda b = \beta b \lambda b$.

23The market yields a return of $\beta s$ for every unit of durable good, which the borrower values at $\beta b$. So the excess return (value) of each undepreciated unit of durable good is just $(1 - \delta) (\beta s - \beta b)$. Alternatively, one can think of the additional unit of durable good as relaxing the collateral constraint, which in turn has a value of $(1 - \delta) \xi b$. 

12
The market clearing conditions can then be used to determine the consumption of the non-durable good by the borrower relative to that of the saver. This expression is given by:

\[
\frac{\tilde{c}_s}{\tilde{c}_b} = \frac{1}{\tilde{c}_b} \left[ \frac{\Omega_b \tilde{n}_b + \Omega_s \tilde{n}_s}{\Omega_s \left( 1 + \delta \frac{\tilde{d}_s}{\tilde{c}_s} \right)} \right] - \frac{\Omega_b}{\Omega_s} \left[ 1 + \delta \frac{\tilde{d}_b}{\tilde{c}_b} \right].
\]

(45)

Note that under the assumptions that \(\Omega_s + \Omega_b = 1\) and \(\tilde{n}_s = \tilde{n}_b = \tilde{n}\), the previous expression becomes

\[
\frac{\tilde{c}_s}{\tilde{c}_b} = \left[ (1 - \Omega_b) \left( 1 + \delta \frac{\tilde{d}_s}{\tilde{c}_s} \right) \right]^{-1} \left[ \frac{\tilde{n}}{\tilde{c}_b} - \Omega_b \left( 1 + \delta \frac{\tilde{d}_b}{\tilde{c}_b} \right) \right].
\]

(46)

Therefore, given the parameter values \(\alpha, \chi, \delta, \beta_s, \beta_b\), and hours of labor supplied by the agents \(\tilde{n}_b\) and \(\tilde{n}_s\); expressions (39)-(46) can be used to solve for the vector of steady state variables.

### 3.4 Two Period Model

The analytical solution of a two period version of the model is presented next. The analysis of this simple model will aid in providing intuition about the basic mechanisms at work. In the discussion to follow, the term resetting firms refers to the intermediate firms in each sector which are able to change the price and choose the optimal price that maximizes the discounted value of their profits.

#### 3.4.1 Final Period

In the last period \(t = 1\). For the resetting firms the only cost that matters when markup up their price is that of the current period production. Hence, from the optimal pricing behavior of the resetting firms, (23), it follows that

\[
\left( \hat{p}_{j,1}^* \right)^{-1} = \frac{P_{j,1}}{P_{j,1}^*} = \left( \frac{\epsilon_j}{\epsilon_j - 1} \right) w_{j,1}
\]

\[
= \left( \frac{\epsilon_j}{\epsilon_j - 1} \right) \left( \frac{1}{q_1} \right) \tilde{w}_{c,1},
\]

(47)

where, for notational convenience, \(\hat{p}_{j,1}^*\) is the defined as the inverse ratio of the price chosen by the resetting firms relative to the aggregate price level.

This price setting behavior in turn implies that inflation in each sector is a monotone increasing function of the real wage rate \(w_{c,1}\). From the definition of the aggregate price index it follows that

\[
P_{j,1}^{1-\epsilon_j} = \int_0^1 P_{j,1}(i)^{1-\epsilon_j} di
\]

\[
\implies 1 = \int_0^1 \left( \frac{P_{j,1}(i)}{P_{j,1}^*} \right)^{1-\epsilon_j} di
\]

\[
= (1 - \Phi_j) \left( \hat{p}_{j,1}^* \right)^{\epsilon_j - 1} + \int_{[0,1][Y_{j,1}]} \left( \frac{P_{j,1}(i)}{P_{j,1}^*} \right)^{1-\epsilon_j} di
\]

\[
= (1 - \Phi_j) \left( \hat{p}_{j,1}^* \right)^{\epsilon_j - 1} + \Phi_j \pi_{j,1}^{\epsilon_j - 1},
\]

(48)

---

24 This will be the case when calibrating the model.

25 In calibrating the model, rather than specifying the value of the scaling parameters \(\nu_s\) and \(\nu_b\), the scale of the economy is determined by specifying the steady state hours supplied by the agents.
where $\Upsilon_{j,1}$ is the set of resetting firms in sector $j$ and period 1. Let $\hat{p}^* (w_c)$ denote the optimal resetted inverse price ratio as a function of the real wage rate and $\pi_{j,1} (w_c)$ denote sector’s $j$ inflation as a function of the real wage rate. Consider $w^1_c < w^2_c$. Given (47) we have

$$
(1 - \pi_j) [\hat{\pi}_{j,1} (w^1_c)]^{\epsilon_j - 1} > (1 - \pi_j) [\hat{\pi}_{j,1} (w^2_c)]^{\epsilon_j - 1}
$$

which, together with (48), implies that

$$
\pi_j [\pi_{j,1} (w^2_c)]^{\epsilon_j - 1} > \pi_j [\pi_{j,1} (w^1_c)]^{\epsilon_j - 1}.
$$

Thus inflation is a monote increasing function of $w_c$. Note that the non-negativity of inflation imposes an endogenous, strictly positive, lower bound on the real wage rate. This can be seen from (48) and (47) which imply that

$$
\lim_{w_c \to 0} \pi_{c,1} = -\infty.
$$

Denote this lower bound on the real wage rate by $w^{LB}_c$. This in turn implies a lower bound on the optimal pricing ratio of the firms

$$
\lim_{\pi_{c,1} \to 0} (\hat{\pi}_{j,1})^{-1} = (1 - \pi_j)^{\frac{1}{\epsilon_j - 1}}.
$$

Second, inflation in each sector has an endogenous upper bounded. From (47), as $w_{c,1} \to \infty$ then $\hat{\pi}_{j,1} \to 0$. Thus (48) implies that

$$
\lim_{w_{c,1} \to \infty} \pi_{c,1} = \left(1 - \frac{1}{\pi_c} \right)^{\frac{1}{\epsilon_j - 1}}
$$

and

$$
\lim_{w_{c,1} \to \infty} \pi_{d,1} = \left(1 - \frac{1}{\pi_d} \right)^{\frac{1}{\epsilon_j - 1}}.
$$

The intuition for this result is as follows. The sector specific price index is computed as a weighted average of the inverse of the prices of the varieties as seen in (48). There are two factors that affect this price index: the level of price chosen by the firms that are able to reset their price, and the level of the aggregate price on the previous period (which reflects the prices of the firms that can’t reset their price). The firms that can reset their price, choose to set it at a constant mark-up over the real wage rate as in (47). In this sense, the price they choose does not have an upper bound. Given that the contribution to the aggregate price index is determined by the inverse of the individual price ratios, as the wage rate increases, the contribution of the price chosen by the resetting firms becomes smaller (indeed, becomes zero in the limit). That is, as the wage rate increases, the inverse of the current aggregate price level is the fraction $\Phi_j$ of the inverse of last period’s aggregate price level. From where it follows that current inflation (current aggregate price level relative to last period’s aggregate price level) does not depend on the wage rate for high enough wages and it is instead related to inverse of the mass of firms that don’t reset their price.

Third, inflation in each sector is smaller than unity up to the steady state wage rate $\tilde{w}_c$ and larger than unity there after.\(^{26}\) For any wage rate smaller than this value, the firms that can reset their price would then choose a smaller optimal price than the steady state price, hence pushing inflation down (i.e. smaller than 1). Simmilarly, if the wage rate is larger than its steady state value, the resetting firms would choose a larger price than the steady state price, hence pusing inflation up (i.e. larger than 1). To see this more formally, consider $w^{LB}_c \leq w_c < \tilde{w}_c$. Then from (47) we have

$$
\hat{p}_{j,1} (w_c) > 1
$$

$$
\Rightarrow (1 - \pi_j) [\hat{\pi}_{j,1} (w_c)]^{\epsilon_j - 1} > (1 - \pi_j);
$$

Note that the wage rate that yields no inflation ($\pi_d = \pi_c = 1$) is the steady state wage rate $\tilde{w}_j = \frac{1}{\epsilon_j}$, which is the same in both sectors as long as $\epsilon_c = \epsilon_d$.\(^{26}\)
and from (48) we have
\[
\Phi_j > \Phi_j \left[ \pi_{j,1} (w_c) \right]^{\epsilon_j - 1} \iff \pi_{j,1} (w_c) < 1.
\]

Consider next \( \tilde{w}_c < w_c \). Then from (47) we have
\[
1 > \tilde{p}_{j,1}^* (w_c) \implies (1 - \Phi_j) > (1 - \Phi_j) \left[ \tilde{p}_{j,1}^* (w_c) \right]^{\epsilon_j - 1}
\]
and from (48) we have
\[
\Phi_j \left[ \pi_{j,1} (w_c) \right]^{\epsilon_j - 1} > \Phi_j \iff \pi_{j,1} (w_c) > 1.
\]

Fourth, the distortion in inflation is larger in the relatively more flexible sector.\(^{27}\) What drives the optimal price is the change in the real wage rate in units of the good of the specific sector, as seen in the first line of (47). Suppose that the aggregate price level was the same in both sectors and consider a nominal wage rate smaller than in steady state. This would lead to the resetting firms, in both sectors, choosing the same optimal price level. However, in the more flexible sector, the mass of the firms that can change the price is larger. Therefore, there is a larger aggregate effect in the more flexible sector. That in turn implies that, for a given nominal wage rate, the real wage rate in sector specific units \( w_{j,1} \) is actually larger in the more flexible sector (i.e. the decrease in the nominal wage rate is somewhat offset by the decrease in the aggregate price level). Hence, although the price chosen by the resetting firms in both sectors is smaller than one, the resetted price in the more flexible sector must be larger.

Overall, there are two competing effects when \( w_{c,B} < \tilde{w}_c < w_{c,1} \). In the intensive margin, each firm in the more flexible sector chooses a price smaller than one but larger than the price of a resetting firm in the other sector. In the extensive margin, there are more firms changing the price in the more flexible sector. The net effect of both margins is that the aggregate price index is smaller in the more flexible sector. To see this, note that the combination of these two effects can’t lead to a smaller aggregate price level in the less flexible sector. If this was the case, a nominal wage rate smaller than in steady state would lead to a smaller \( w_{j,1} \) in the relatively more flexible sector. Resetting firms in this sector would choose to set an optimal price smaller than that of the firms in the less flexible sector. The end result would be that in the relatively more flexible sector, more firms reset their price and each of them chooses a smaller price than in the other sector. But this would necessarily imply a smaller aggregate price level for the more flexible sector. Thus leading to a contradiction.

The same reasoning applies to the case when \( \tilde{w}_c < w_{c,1} \). Each firm in the more flexible sector would choose a price larger than one but smaller than the price chosen by the firms in the less flexible sector. On aggregate however, there are more firms in the flexible sector that reset their price, so aggregate price level in relatively more flexible sector ends up being larger. Finally, note that for large enough wages, the resetting behavior of firms is irrelevant and inflation is just determined by the firms that can’t reset their price, as pointed out earlier.

More formally, combining equations (47), (48) and noting that
\[
q_1 = \left( \frac{\pi_{d,1}}{\pi_{c,1}} \right) q_0 \implies \frac{1}{\pi_{j,1}^{\epsilon_j - 1}} = (1 - \Phi_j) \left[ \left( \frac{\epsilon_j - 1}{\epsilon_j} \right) \left( \frac{\tilde{q}_{j,d}}{\tilde{w}_c \pi_{c,1}} \right) \right]^{\epsilon_j - 1} + \Phi_j,
\]
and using the definition of the real wage rate we have
\[
\frac{1}{\pi_{j,1}^{\epsilon_j - 1}} = (1 - \Phi_j) \left[ \left( \frac{\epsilon_j - 1}{\epsilon_j} \right) \left( \frac{P_{j,0}}{W_1} \right) \right]^{\epsilon_j - 1} + \Phi_j, \tag{49}
\]
where \( W_1 \) is the nominal wage in period 1. This expression can be interpreted as stating that there are two sources that drive the ratio of the aggregate price level between the current period and the previous period. The first component is

\(^{27}\)Under the current parametrization, the relatively more flexible sector is the durable sector. A larger distortion in inflation means that for \( w_c < \tilde{w}_c \) then \( \pi_{d,1} < \pi_{c,1} < 1 \); and for \( w_c > \tilde{w}_c \) then \( \pi_{d,1} > \pi_{c,1} > 1 \). Note also that the analysis assumes that the state variables relevant for price setting behavior \( (q_0, \Delta \rho_{c,0}, \Delta \rho_{d,0}) \) are at their steady state values.
due to the firms that can reset their price in the current period. It is just the ratio of the optimal price chosen by the resetting firms relative to the aggregate price level in the previous period, weighted by the mass of such firms. The second component is the contribution of the firms that are unable to reset their price. Given the i.i.d. assumption of the price signal across time and firms, the average price index for the subset of firms that can’t reset their price at \( t = 1 \) is the same as the aggregate price index at \( t = 0 \). So the contribution of the firms that can’t reset their price is just 1, weighted by the mass of such firms.

At this point, two assumptions are made: first \( P_{c,0} = P_{d,0} = P_0 \); secondly \( \epsilon_c = \epsilon_d = \epsilon \). Under these assumptions, the ratio of the optimal price chosen by the resetting firms to the nominal wage rate. This in turn implies the following relation

\[
(1 - \Phi_d) = (1 - \Phi_c) \left( \frac{\pi_{c,1}}{\pi_{d,1}} \right)^{\epsilon - 1} + (\Phi_c - \Phi_d) \pi_{c,1}^{\epsilon - 1}.
\]

Given that the durables sector is assumed to be more flexible (\( \Phi_d < \Phi_c \)), all the terms in the previous expression are positive. Consider the case where \( w_c^{LB} < w_{c,1} < \bar{w}_c \). As previously discussed, this implies \( \pi_{c,1}, \pi_{d,1} < 1 \). So that

\[
(\Phi_c - \Phi_d) \pi_{c,1}^{\epsilon - 1} < (\Phi_c - \Phi_d)
\]

and from (50) then

\[
(1 - \Phi_c) \left( \frac{\pi_{c,1}}{\pi_{d,1}} \right)^{\epsilon - 1} > (1 - \Phi_c) \iff \pi_{c,1} > \pi_{d,1}.
\]

For the case where \( \bar{w}_c < w_{c,1} \) inflation in both sectors is larger than unity, implying that

\[
(\Phi_c - \Phi_d) \pi_{c,1}^{\epsilon - 1} > (\Phi_c - \Phi_d)
\]

and from (50) then

\[
(1 - \Phi_c) \left( \frac{\pi_{c,1}}{\pi_{d,1}} \right)^{\epsilon - 1} < (1 - \Phi_c) \iff \pi_{c,1} < \pi_{d,1}.
\]

It is therefore clear that changes in the wage rate relative to its steady state value lead to deviations in inflation which are always larger in the relatively more flexible sector. The first panel of Figure 2 illustrates all the features just discussed about each sector’s inflation as a function of the real wage rate in units of the non-durable. One final comment. Given that the durable sector is the more flexible sector, this implies that inflation in the durables is more responsive to \( w_{c,1} \). Therefore, the relative price \( q_1 \) is an increasing function of the wage rate.

The second panel of Figure 2 illustrates the basic features of the price dispersion as a function of \( w_{c,1} \). As it can be seen from the figure, for \( w_c^{LB} < w_{c,1} < \bar{w}_c \), the price dispersion in both sectors is decreasing and it is always larger in the non-durable sector (the relatively less flexible sector). On the other hand, for \( \bar{w}_c < w_{c,1} \), both sectors exhibit an increasing price dispersion, which is now larger in the durable sector (the relatively more flexible sector).

To understand this features, it is useful to consider first the limiting cases of \( w_{c,1} \to w_c^{LB} \) and \( w_{c,1} \to \infty \). From the definition of the price dispersion we have

\[
\Delta_{P_{j,1}} = \int_0^1 \left( \frac{P_{j,1}(i)}{P_{j,1}} \right)^{-\epsilon_j} di.
\]

---

28 Given the i.i.d. assumption, \( E_i \left( P_{j,1}^{1-\epsilon_j} | S_j \right) = E_i \left( P_{j,1}^{1-\epsilon_j} \right) \), where \( S_j \sim Bernoulli \left( 1 - \Phi_j \right) \) is the random variable that governs the price change in sector \( j \). The subscript \( i \) refers to the expectation with respect to the distribution of firms.

29 The first assumption is satisfied if the economy starts at its steady state in the first period, or sufficiently close. The second assumption will be imposed in solving the log-linear model.
Recall that when \( w_{c,1} \rightarrow w_{c}^{LB} \), then \( \hat{p}_{j,1}^{*} > 1 \) while \( \pi_{j,1} \rightarrow 0 \). So that the ratio on the integral is much larger for price resetting firms than for the non resetting firms. Therefore, the contribution of the non resetting firms is negligible and can be ignored. On the other hand, for the case \( w_{c,1} \rightarrow \infty \), the optimal price setting behavior of the resetting firms yields \( \hat{p}_{j,1}^{*} \rightarrow 0 \) and \( \pi_{j,1} > 1 \). So one can ignore the contribution of the resetting firms.

Keeping this in mind, for small enough \( w_{c,1} \) the price dispersion is given by

\[
\Delta_{P_{j,1}} \approx \int_{\mathcal{Y}_{j,1}} (\hat{p}_{j,1}^{*})^{\epsilon_j} \, di
\]

\[
\approx (1 - \Phi_j) (\hat{p}_{j,1}^{*})^{\epsilon_j}
\]

\[
\approx \left( \frac{1}{1 - \Phi_j} \right)^{\epsilon_j - 1},
\]

where the last line follows by substituting the limit value of \( \pi_{j,1} \) as \( w_{c,1}^{LB} \) approaches its lower bound. Meanwhile, for large enough \( w_{c,1} \):

\[
\Delta_{P_{j,1}} \approx \int_{[0,1] \setminus \mathcal{Y}_{j,1}} \left( \frac{\hat{P}_{j,1}(i)}{\hat{P}_{j,1}} \right)^{-\epsilon_j} \, di
\]

\[
\approx \int_{[0,1] \setminus \mathcal{Y}_{j,1}} \left( \frac{\hat{P}_{j,0}(i)}{\hat{P}_{j,0}} \right)^{-\epsilon_j} \left( \frac{\hat{P}_{j,0}}{\hat{P}_{j,1}} \right)^{-\epsilon_j} \, di
\]

\[
\approx \Phi_j \Delta_{P_{j,0}} \pi_{j,1}^{\epsilon_j}
\]

\[
\approx \left( \frac{1}{\Phi_j} \right)^{\epsilon_j - 1} \Delta_{P_{j,0}},
\]

where the second to last line is a consequence of the i.i.d assumption of the price resetting signal across time and firms. The last line follows by substituting the limit value of \( \pi_{j,1} \) as \( w_{c,1} \rightarrow \infty \).

It is thus clear from the previous expressions that, for small enough \( w_{c,1} \), the price dispersion is larger for the less flexible sector (larger \( \Phi_j \)). Similarly, for large enough \( w_{c,1} \), the price dispersion is larger for the more flexible sector (smaller \( \Phi_j \)), conditional on \( \Delta_{P_{c,0}} = \Delta_{P_{c,0}} \).

Note that although the previous reasoning was applied to the extreme cases as \( w_{c} \rightarrow w_{c}^{LB} \) and \( w_{c} \rightarrow \infty \), the conclusion still holds true for the relevant range of wage rates. That is, for \( w_{c}^{LB} \leq w_{c} \leq w_{c} \), the price dispersion in the less flexible sector is larger; while for \( w_{c} < w_{c} \), the price dispersion in the more flexible sector is larger. To see this, one can think of the price dispersion as measuring the deviation of the individual firms’ prices relative to the aggregate price level. The price dispersion can be decomposed into the contribution due to the resetting firms and that of the non-resetting firms. If the optimal price chosen by the resetting firms is very large (small) relative to the aggregate price level, the price contribution of the non-resetting firms must be very small (large) relative to the aggregate price level. In other words, if the resetting firms are well above (below) the average, the non-resetting firms must be well below (above) the average. That is, if one of the two components of the price dispersion is large, the other one must be small. The question is then, which of the two components is relatively more important in determining the overall effect on the price dispersion? It turns out that whether the resetting or non-resetting firms are more important for the price dispersion depends on how the price ratio of the resetting firms relates to one.

From equation (48), it is clear that the aggregate price index is determined by these two components. The first term on the right hand side of this equation is the relative contribution to the aggregate price by the resetting firms; while the second term is the relative contribution by the non-resetting firms. On the other hand, the price dispersion can be written as

\[
\Delta_{P_{j,1}} = (1 - \Phi_j) (\hat{p}_{j,1}^{*})^{\epsilon} + \Phi_j (\pi_{j,1})^{\epsilon} = (1 - \Phi_j) (\hat{p}_{j,1}^{*})^{\epsilon - 1} \hat{p}_{j,1}^{*} + \Phi_j (\pi_{j,1})^{\epsilon - 1} \pi_{j,1}.
\]
That is, the price dispersion can be seen as a weighted average of the relative contributions of the resetting and non-resetting firms to the aggregate price index. For the resetting firms, the weight is given by \(\tilde{p}_{j,1}^*\) while for the non-resetting it is \(\pi_{j,1}\). Furthermore, since this relative weights must satisfy equation (48), it is clear that if one of them is larger than unity, the other one must be smaller than unity. Thus we must always have \(\Delta p_{j,1} > 1\).

For the case \(w_c^{LB} < w_{c,1} < \tilde{w}_c\), as previously discussed, the following relation holds
\[
\hat{p}_{c,1}^* > \hat{p}_{d,1}^* > 1 \quad \text{and} \quad 1 > \pi_{c,1} > \pi_{d,1}.
\]
Therefore, the price dispersion is mainly determined by the resetting firms. Given that is less than one, the contribution of the non-resetting firms gets assigned a relatively small weight. In addition to that, the actual value of the contribution which have a small weight and their contribution can be ignored. Thus, by noting that
\[
\left(\hat{p}_{c,1}^*\right)^\epsilon - \left(\hat{p}_{d,1}^*\right)^{\epsilon - 1} > \left(\hat{p}_{c,1}^*\right)^\epsilon - \left(\hat{p}_{d,1}^*\right)^{\epsilon - 1} > 0
\]
\(\implies\) \(\Delta p_{c,1} > \Delta p_{d,1}\).

That is, given that both, the weight given to the resetting firms and the actual value of their contribution, are larger in the less flexible sector; the price dispersion in this sector must be larger. On the other hand, for the case \(\tilde{w}_c < w_{c,1}\), then
\[
\tilde{p}_{c,1}^* < \tilde{p}_{d,1}^* < 1 \quad \text{and} \quad 1 < \pi_{c,1} < \pi_{d,1}.
\]
Therefore, the price dispersion is mainly determined by the non-resetting firms. Since \(p_{j,1}^*\) is less than one, the contribution of the resetting firms gets assigned a relatively small weight. In addition to that, the actual value of the contribution of the resetting firms is already small (i.e. \(\pi_{j,1}^{-1} < 1\)). Hence the non-resetting firms contribute small deviations which can be ignored. Thus, by noting that
\[
\left(\pi_{d,1}\right)^\epsilon - \left(\pi_{d,1}\right)^{\epsilon - 1} > \left(\pi_{c,1}\right)^\epsilon - \left(\pi_{c,1}\right)^{\epsilon - 1} > 0
\]
\(\implies\) \(\Delta p_{d,1} > \Delta p_{c,1}\).

That is, given that both, the weight given to the non-resetting firms and the actual value of their contribution, are larger in the more flexible sector; the price dispersion in this sector must be larger.

This intuition is useful in understanding why the price dispersion in each sector is decreasing for \(w_c^{LB} < w_{c,1} < \tilde{w}_c\) and increasing for \(\tilde{w}_c < w_{c,1}\). For wage rates smaller than its steady state value, the price dispersion is governed by the resetting firms’ behavior. The smaller the wage rate, the smaller the price ratio set by these firms, and hence the larger the price dispersion. For wage rates larger than the steady state wage, the price dispersion is governed by the non-resetting firms. By noting that the contribution of the non-resetting firms to the price dispersion is just given by the inflation, it is then clear that the larger the wage rate, the larger the inflation, and hence the larger the price dispersion.

Once the pricing behavior of the firms is completely characterized, one can start analyzing the role of the monetary policy in the determination of prices. In the last period, given that there is no longer an intertemporal tradeoff faced by the agents in the model, the nominal interest rate becomes irrelevant. Therefore, it is useless to think of the monetary policy in the determination of prices. In turn, the price dispersion of the resetting firms is solely determined by the wage rate \(w_{c,1}\). That is, by setting an inflation target, the monetary authority is effectively determining the level of \(w_{c,1}\). Now, the effect of different inflation targets
is illustrated in the first panel of Figure 3. Having an inflation target larger than the steady state inflation index of one, results in $\pi_{j,1} > 1$ for $j \in \{c,d\}$, but with inflation being larger in the more flexible sector. On the other hand, an inflation target smaller than one leads to $\pi_{j,1} < 1$ for $j \in \{c,d\}$, but this time inflation is larger in the less flexible sector.

Are all inflation targets feasible? Any $0 < \pi^* < 1$ is feasible. However, given that the pricing behavior of the firms endogenously delivers an upper bound for sectorial inflation, if $\pi^* > 1$ is too large, then the inflation target might not be feasible. In the deterministic case, this bound on the inflation target poses a minor problem. However, in the stochastic case where one assumes that the inflation target is subject to exogenous disturbances, one needs to be careful in choosing the support of such process to ensure that the resulting target is feasible. In this sense, the parameter $\phi_\pi$ plays an important role. A large value of this parameter implies a large upper bound for the feasible inflation target. That is, for a given stochastic process that governs the inflation target, existence of a unique solution is related to the parameter $\phi_\pi$.  

Next, given that the monetary authority is targeting a composite inflation index, it is natural to evaluate the effect of the relative weights of each sector in the index. To that end, Figures 4 and 5 illustrate how, for a given inflation target, the monetary authority is effectively determining the level of sectorial inflation by letting $\kappa \to 0$. Similarly, for $\pi^* < 1$, the target inflation index can be met with the least decrease in sectorial inflation by letting $\kappa \to 0$. The intuition of this result is pretty simple. As previously discussed, by setting the inflation target the monetary authority is effectively determining the level of $w_{c,1}$. Hence the price determination in the economy boils down to asking what is the required $w_{c,1}$ to achieve $\pi^*$. Now, as $\kappa \to 0$, the inflation index is driven mostly by the durable sector (which is the more flexible sector). Therefore, only small changes of $w_{c,1}$, relative to its steady state value, are required to get to the desired inflation target.

Note that this result offers some insights into how the monetary authority might want to choose the weights in the composite inflation index. Suppose, for instance, that the monetary authority is inflation targeting; its goal is to minimize the deviation of each sectors’ inflation relative to the steady state value. If there are exogenous stochastic disturbances that affect the level of the actual inflation target $\pi^*$, then the monetary authority would be able to better accomplish its goal by setting $\kappa$ to be small and assigning more weight to the flexible sector.

Having characterized the way in which prices are determined in the economy, the analysis of the optimal decisions for the borrowers and savers is presented next. To that end, consider first

$$A_1^b = q_1 (1 - \delta) d_0^b - \frac{R_0 b_0^b}{\pi_{c,1}}$$

$$A_1^s = q_1 (1 - \delta) d_0^s + \left( \Omega_b \Omega_s \right)^{-1} \sum_{j \in \{c,d\}} q_1^{j=d} Y_{j,1} (1 - w_{j,1} \Delta p_{j,1})$$

where $A_1^b$ and $A_1^s$ refer to the non-labor real income, in units of the non-durable, for the borrowers and savers, respectively. This income is composed of three sources: the first term is due to the market value of the wealth transferred by the agents from the previous period by means of the durable good; the second term refers to the repayment value of the debt obligations; and the last term is just the profits derived from ownership of the intermediate firms, which are zero for the borrower.

To analyze the overall effect of the wage rate in the non-labor income it is convenient to understand its effect on each of these three terms. Since $q_1$ is increasing in the wage rate, the value of the durable holdings for both agents

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30Given the endogenous bounds on each sector’s inflation, the bound for the inflation target is given by $\pi^* = \left( \phi_c^{\frac{1}{1+c}} \phi_d^{\frac{1}{1+d}} \right) \phi_\pi$.

31Note that this is related to the condition that $\phi_\pi > 1$ to guarantee the existence of a unique, bounded, rational expectation solution for the infinite horizon version of the model.

32Note that as pointed out in the previous discussion, having an inflation target $\pi^*$ that determines the prices in the economy is analogous to fixing a $w_{c,1}$. Therefore, understanding the optimal behavior of the agents amounts to characterizing the relevant variables as functions of $w_{c,1}$.

33The debt market clearing condition has been imposed in writing the non-labor income for the savers.

34This is the case since the durable sector is relatively more flexible. If instead, $\Phi_d > \Phi_c$, $q_1$ would instead be a decreasing function of the wage rate.
increases as inflation increases in the economy. Under the assumption that \( b_0^b \geq 0 \), an increase in \( w_{c,1} \) benefits the borrowers while hurting the savers. This is because the increase in the wage rate increases \( \pi_{c,1} \), hence decreasing the real value of the debt repayment obligations. The dependence of the profits on \( w_{c,1} \) is a little more involved. For \( w_{c,1}^L < w_{c,1} < w_{c,1}^H \), as previously shown, \( \Delta P_{j,1} \) decreases as the wage rate increases. So for this range of wages, as long as the increase in \( w_{c,1} \) is smaller than the decrease in \( \Delta P_{j,1} \), profits would be an increasing function of \( w_{c,1} \).

For the range of wages \( w_{c,1} < w_{c,1}^L \), the price dispersion in both sectors is increasing in \( w_{c,1} \), so profits are necessarily a decreasing function of the wage rate. Given the previous discussion, it is clear that \( A_1^s \) is always increasing in \( w_{c,1} \).

On the other hand, the effect of \( w_{c,1} \) on \( A_1^s \) depends on the relative size of each of the three components.

Figure 6 illustrates the dependence of non-labor income for the agents on the wage rate. The left panel of the figure shows that, as expected, non-labor income for the borrower is an increasing function of \( w_{c,1} \); while larger values of inherited debt holdings, \( d_0^b \), increase \( A_1^b \); while larger values of inherited debt holdings, \( R_0 b_0^b \), decrease it.

The right panel of Figure 6 shows the dependence of \( A_1^s \) on the wage rate for the savers. When the savers inherit no wealth from the previous period (the case \( R_0 b_0^b = d_0^s = 0 \)), their non-labor income is solely determined by the profits. It can be seen then how profits are increasing for small enough \( w_{c,1} \) and decreasing thereafter. Increasing the amount of inherited wealth, either in the form of durable asset holdings (\( d_0^s \)) or debt repayments (\( R_0 b_0^b \)), increases the level of non-labor income for any given wage rate. However, if the increase in inherited wealth is due to \( d_0^s \), it has the effect of enhancing the positive relation of \( A_1^s \) and \( w_{c,1} \) for the range of wages where the profit is increasing; while counterbalancing the decrease in profits as the wage rate increases. This effect can be seen for the case \( d_0^s = 1 \).

If the increase in non-labor income is due to debt holdings, this tends to make \( A_1^s \) decreasing in \( w_{c,1} \) for all feasible wages, as seen for the case \( R_0 b_0^b = 0.5 \). So, whether the savers’ non-labor income is hump-shaped or monotonically decreasing depends on the relative composition of his \( t = 0 \) portfolio. Overall then, larger inflation in both sectors implies a larger non-labor income for the borrowers and, in most cases, a smaller non-labor income for the savers.

Having characterized the non-labor income, the optimal decision rules for the agents can now be studied. In what follows, let \( a \in \{b, s\} \) denote the type of agent. Note that since the agents die at the end of this period, there is no intertemporal condition for debt holdings. It is optimal for the agents to choose to hold as much debt as possible, so that \( b^a \). The optimal choice of the remaining variables can be obtained by using the FOC for labor (25), the appropriate form of the FOC for durables (26), the budget constraints (29), and the market clearing conditions (35)-(38).

The solution is then given by

\[
c^b_1 = \frac{\alpha}{2} \left[ A_1^b + \sqrt{(A_1^b)^2 + \frac{4(w_{c,1})^2}{\nu^b}} \right]
\]

\[
c^s_1 = \frac{\alpha}{2K_1^s} \left[ A_1^s + \sqrt{(A_1^s)^2 + \frac{4(w_{c,1})^2}{\nu^s} - K_1^s} \right]
\]

\[
d^b_1 = \left( \frac{1 - \alpha}{\alpha} \right) \frac{c^a_1}{q_1}
\]

\[
n^a_1 = \frac{\alpha}{\nu^a} \frac{w_{c,1}}{c^a_1}
\]

\(56\)

\(57\)

\(58\)

\(59\)

Recall that the modeling convention \( b^a > 0 \) denotes that agent \( a \) is in debt; so this assumption just states that the borrowers are indeed the ones holding debt.

Although it is not formally shown here, there exists a value \( w_{c,1}^L < w_{c,1}^* < \hat{w}_c \) s.t. \( A_1^s \) is increasing for \( w_{c,1} < w_{c,1}^* \) and decreasing for \( w_{c,1} > w_{c,1}^* \).

It can be shown that at the steady state level of the wage rate, \( A_1^s \) is always decreasing. Hence in a neighborhood of the steady state, an increase in inflation always results in a decrease in the savers non-labor income.

To make the two period version of the model well defined, at \( t = 1 \) it is assumed that both agents face the equivalent of a No-Ponzi condition for the finite horizon case: \( R_1 b_1^a \geq 0 \).
where

\[ K_1^w = 1 - \left[ \alpha (1 - w_{c,1} \Delta p_{s,1}) + (1 - \alpha) (1 - w_{d,1} \Delta p_{s,1}) \right] \]

\[ A_1^w = q_1 (1 - \delta) \left[ \left( \frac{\Omega_0 d_0^b}{\Omega_x} + d_0^s \right) w_{d,1} \Delta p_{s,1} - \Omega_0 d_0^b \right] + R_0 b_0^d + \Omega_0 c_0^d (1 - K_1^w). \]

The variable \( K_1^w \) is just defined for notational convenience and \( A_1^w \) is the adjusted non-labor income for the saver. This adjusted non-labor income takes into account the fact that the savers’ consumption \( c_1^w \) indirectly affects \( A_1^w \). Note that when making the optimal consumption choice, the savers do not take into consideration this indirect effect of their consumption on profits. This indirect effect of \( c_1^w \) is purely a consequence of the general equilibrium.

Consider the case of the borrower first. Note that when \( A_1^b = 0 \), equations (56) and (59) imply that \( n_1^b \) is constant and independent of the wage rate. In other words, the substitution and income effects of the wage rate due to the labor income exactly offset each other. \(^{39}\) When non-labor income is non-zero, \( b_0^d > 0 \) or \( d_0^b > 0 \), the income and substitution effects of an increase in the wage rate operate in opposite directions. As \( w_{c,1} \) increases, the borrowers’ non-labor income increases and the income effect would provide an incentive for the agents to work less. On the other hand, as leisure becomes relatively more costly, the incentive for the borrowers to supply more work increases. For large enough \( w_{c,1} \), as \( \pi_{j,1} \) and \( q_1 \) approach their upper bounds, non-labor income becomes relatively constant and independent of the wage. Hence the substitution effect dominates and the labor supply is upwards sloping. \(^{40}\) For small enough \( w_{c,1} \), inflation in both sectors is very responsive to changes in the wage rate, as seen in Figure 2. In turn, this implies that small increases in \( w_{c,1} \) lead to a substantial increase in non-labor income, which enhances the income effect. If this channel is large enough, the income effect will dominate the substitution effect leading to a downwards sloping supply of labor. The left panel of figure 7 illustrates the interaction of these two effects. Interestingly, the non-labor income that contributes the most to the income effect is debt holdings, \( d_0^b \). That is, if the increase in non-labor income is due solely to an increase in durable savings \( d_0^b \), the income effect is relatively small and it is still dominated by the substitution effect. The income effect becomes relevant only for large enough values of \( b_0^d \). This is because, for \( w_{c,1}^L \leq w_{c,1} < w_{c,1}^N \), the relative price \( \pi_{c,1} \) is more responsive to the wage rate than the relative price \( q_1 \). So that changes in the wage rate get amplified enough to have a substantial income effect only through the response of non-durable inflation.

The right panel of Figure 7 shows the savers’ labor supply for different compositions of non-labor income. As previously discussed, non-labor income for the savers is decreasing in the wage rate for most of the relevant range of wages. Therefore, the indirect income effect and the substitution effect enhance each other, which leads to an upwards sloping supply of labor. \(^{41}\) For small enough values of \( w_{c,1} \), for which non-labor income is increasing, the income and substitution effects operate in opposite directions. If the income effect is large enough, the labor supply curve can be downwards sloping. Since the income effect is decreasing in the level of \( A_1^w \), as non-labor income increases the downwards sloping portion of the labor supply curve eventually becomes upwards sloping.

Equation (36) states that the borrowers devote a fraction \( \alpha \) of their effective income to the consumption of the non-durable good. It is clear from this expression that the consumption of the non-durable good is increasing in \( w_{c,1} \). As the wage rate increases, both labor and non-labor income increase and so these agents can enjoy a larger consumption. This situation is illustrated in left panel of Figure 8. Given that non-labor income is very responsive to changes in the wage for small values of \( w_{c,1} \), non-durable consumption is in turn very responsive to changes in the \( w_{c,1} \) for this range of wages. As \( w_{c,1} \to \infty \), non-labor income approaches its fixed upper bound and consumption is solely driven by labor income.

The borrowers devote the remaining \((1 - \alpha)\) fraction of their effective income to the consumption of the durable good, as seen in equation (58). Hence durable consumption has a similar profile than non-durable consumption with

\(^{39}\)Note that the wage rate affects the agents’ decision problem through its direct effect on the labor income and indirectly through its effect on the non-labor income. In this sense, the income effect associated with the wage rate has two components: the direct effect (labor income) and the indirect effect (non-labor income). The substitution effect associated with the wage rate is solely due to labor income.

\(^{40}\)This is true as long as \( \lim_{w_{c,1} \to \infty} A_1^b > 0 \). For the case where non-labor income is negative in the limit (say if the portfolio of the borrower is mainly composed of debt holdings), the income effect dominates and even for large wage rates the labor supply is downwards sloping. One can think of this as a result of the income effect being decreasing in \( A_1^b \).

\(^{41}\)The earlier remark that the income effect is decreasing in non-labor income is somewhat clearer for this case. As the level of non-labor income increases, it can be seen from the right panel of Figure 7 that the slope of the labor supply decreases for any given \( w_{c,1} \).
the caveat that as $w_{c,1}$ increases, the relative price of durables, $q_1$, increases as well. That is, the larger effective income that comes with larger wage rates is somewhat offset by this price increase. Thus, durable consumption is an increasing function of the wage rate, but its rate of increase is smaller than that of durable consumption. The left panel of Figure 9 illustrates the previous discussion.

For the savers,

3.4.2 Initial Period

Still working on this section

4 Results

This section presents the numerical solution of a log-linearized version of the model characterized in terms of impulse response functions. In particular, the system of equations (14),(15),(23)-(30), and (35)-(38) is log-linearized around the deterministic steady state with zero inflation. The solution to the log-linear system is computed following the method proposed by [29].

4.1 Calibration

Include table of calibrated parameters

4.2 Impulse Response functions

The main result is given Figure 10. The figure illustrates the response of total output, $Y^T_t$, to a 25 b.p. surprise decrease in the monetary policy rule. For this exercise, total output is defined as $Y^T_t = Y_{c,t} + q_1 Y_{d,t}$; so that it is a measure of the value of total production in the economy in units of the non-durable good.

As $\Omega_b \to 0$, we have the standard model with no borrowing constraint (baseline economy); while as $\Omega_b \to 1$, we have the limit case in which all of the agents in the economy become credit constrained (constrained economy). As it can be seen, the baseline economy replicates the result of the canonical New Keynesian model. That is, a negative monetary policy shock estimulates the economy by inducing a positive response in total output. This response is largest upon impact, but it remains positive for all subsequent periods.

On the other hand, for the constrained economy, a negative monetary policy shock has a much different effect. Although this shock induces a positive response of output upon impact, this stimulus is very modest compared to that of the baseline economy (indeed, it is an order of magnitude smaller). More strickingly, for all the subsequent periods the policy shock induces a negative response in total output. That is, contrary to the conventional intuition, the economy contracts.

For the intermediate cases in which the fraction of credit constrained agents lies between zero and one, the response of total output lies somewhere within these two extreme scenarios. As more agents in the economy face the collateral constraint, the positive stimulis upon impact decreases. Similarly, as $\Omega_b$ increases, the output response becomes negative some periods after impact, with this reversal happening faster form for larger $\Omega_b$.

In order to understand the intuition behind these results, Figures 11-13 present the impulse response functions of the model variables for the 25 b.p. negative shock to the monetary policy rule. To illustrate the mechanism at work, consider the symmetric case of $\Omega_b = 0.5$. As in the baseline economy, the effect of the shock is to increase the composite inflation index. Recall from the two period model that this implies that inflation in each sector increases, but the increase is larger in the durables sector (the relatively more flexible sector). Thus, the relative price $q_1$ increases as well. Note that larger prices imply higher production costs; which is evinced in an increase in the wage rate. Given that the nominal wage is the same in both sectors but the price level increases by more in the durables sector, the increase in the real wage rate in units of the durable good $w_d$ is smaller than its counterpart in units of the non-durable good $w_c$.

42The complete log-linearized system is presented in the Appendix.
This price adjustments lead to the response of the agents’ behavior seen in Figure 12. Given the steady state levels of debt holdings and durable consumption, both types of agents have an upwards sloping supply of labor in a neighborhood of the steady state.\footnote{As discussed for the two period model, the income effect of an increase in the wage rate is inversely related to the level of inherited non-labor income. So for values of inherited debt holdings $b^{d}_{t-1}$ and durable savings $d^{c}_{t-1}$ that imply a sufficiently large level of non-labor income, the income effect is small.} Hence the increase in wage rate leads to in increase in $n_{b}$ and $n_{a}$. Recall from the discussion of the two period model that larger inflation is beneficial for the borrowers but hurts the savers due to its effect on non-labor income. Note then that the borrowers work more, receive higher wages and have a larger non-labor income. Hence their total income is undoubtedly larger, which allows them to increase their consumption of both, durable and non-durable goods. Although $q_{1}$ increases as well, the increase in total income for this agents is large enough so that despite the larger relative price of the durable good, they can still consume more of this good. For the savers on the other hand, labor income increases but non-labor income decreases. If the overall effect is an increase in total income, savers are also able to consume more of the non-durable good. However, since their increase in income is relatively modest, the higher relative price of the durable good might end up preventing them from consuming more of the durable good.

Now we turn attention to the effects of the shock on the credit constraint. As seen in Figure 13, the value of the credit constraint multiplier $\zeta^{b}$ decreases. That is, the negative monetary policy shock relaxes the collateral constraint. This is because the market value of the durable good pledged as collateral increases even if the actual amount of durable good pledged remains fixed. Given the persistence of the shock, higher inflation today implies higher expected inflation for the next period. In addition, the relative price of the durable good increases. Therefore, the value of the durable good is now larger solely to this increase in prices. As the constraint is relaxed, the borrowers are able to access more debt. Thus, in equilibrium, the savers must be willing to lend more to the borrowers, which is then driving the increase of the nominal interest rate $R_{t}$. Note that this increase in the nominal rate partly offsets the price effect that is relaxing the constraint.

This change in debt holdings provides and additional channel that affects agents’ total income. The increased debt gives the borrowers additional resources which in turn allows them to increase their consumption of both, the durable and non-durable goods. So overall, this channel enhances the effect of their increased labor and non-labor income. On the other hand, the savers have now less resources due to the increase in lending. That is, the decrease in non-labor income for the savers is exacerbated by the outflow of resources in the form of debt. This channel then ends up offsetting the increase of labor income for the savers. With less resources at hand, the savers must now choose how much of those resources to allocate in durable and non-durable consumption. At this point, the nominal interest rate plays a crucial twofold role. First, the nominal interest rate increases by enough so that savers are willing to give up current resources in the form of loans in return for resources in the future. Secondly, the change in the interest rate provides an incentive for savers to choose lending as the primary vehicle for saving rather than holding durable assets. Therefore, as the durable good becomes less attractive, the savers shift resources into the consumption of the non-durable good. That is why for the savers Figure 12 shows an increase in $c^{a}$ and a decrease in $d^{a}$.\footnote{Note that if the non-labor income decrease and the outflow of resources due to lending are larger than the increase in labor income, total income for the savers would actually decrease. If this is the case, consumption of the durable and the non-durable good must decrease.} Now, as both, savers and borrowers, increase their consumption of the non-durable good, $Y_{c}$ increases.

Given this general idea of how the mechanism in the model operates, one can turn to the issue of analyzing the effect of varying the relative mass of borrowers in the economy. In the discussion that follows, the case $\Omega_{b} = 0.5$ will be used as the baseline case. Consider increasing the fraction of borrowers. Suppose that the response of the variables remained unchanged. Given that there are more borrowers in the economy, and noting that the increase in $c^{b}$ is larger than that of $c^{a}$, it follows that $Y_{c}$ must increase by more than in the baseline case. A larger increase in production implies a larger increase in labor hired and hence a larger increase in the wage rate. This larger increase in costs then results in a larger inflation in both sectors. This in turn enhances the channels by which borrowers’ total income increases. That is, each borrower is now able to consume more and at the same time reduce the number of hours they work. This reasoning explains why $w_{d}$, $w_{c}$, $\pi_{d}$, $\pi_{c}$, $q$, $c^{b}$, and $Y_{c}$ are increasing functions of $\Omega_{b}$ and $n^{b}$ is decreasing.

The larger increase in prices in turn implies that the borrowing constraint is relaxed by more (i.e. $\zeta^{b}$ decreases by more), allowing borrowers to increase their debt holdings. By general equilibrium effects, this would require a larger increase in the nominal interest rate than in the baseline case. This provides a stronger incentive for the savers
to use loans as the savings channel instead of using durable assets. Note that this effect extends to the borrowers. That is, the change in prices also makes the durable good less attractive for the borrowers as a savings channel. Hence the borrower shifts resources from the consumption of the durable good to that of the non-durable good. Given the increased number of borrowers, the total amount of loans required is larger and there are less savers supplying those loans. Thus each saver must lend a larger amount, and in doing so, give up a larger amount of the durable good. At the same time, total income for the savers is substantially decreased since inflation is larger and they have to extend bigger loans. Hence their consumption of the non-durable good can’t increase as much as in the baseline case. Also, to offset this decrease in income, savers have a larger increase in the number of hours they work. The previous reasoning explains why $\zeta_b$, $c^s$, $d^s$ and $d^b$ are decreasing in $\Omega_b$ while $R$ and $n^s$ are increasing.

Finally, to see why total output in the durable sector is decreasing in $\Omega_b$, one must keep in mind that the relevant variable in determining this output is not the level of durable consumption but its change relative to the previous period. In this sense, the savers contribute positively to output in the durable sector while the opposite is true for the borrowers. The reason behind the profile of $d^b$ being downward sloping has to do with the impatient of these agents and with the collateral constraint. Given that this agents are impatient, they prefer to front load consumption. In this sense, they favor consumption of the non-durable good. However, in the presence of the constraint, they are forced to hold the durable good in order to borrow and front load consumption of the non-durable. As time elapses, these agents don’t value consumption as much so that need to hold smaller amounts of the durable good to claim as collateral. Now, as $\Omega_b \to 1$, there are two effects. Firstly, as previously stated, the value of the durable good as a vehicle for saving and transferring wealth decreases. This implies that the value for the borrowers of holding this good is mainly due to its use as collateral. Secondly, the behavior of total output in the durables sector is mainly determined by the borrowers. Thus, it is the combination of this two effects that explains why $Y_d$ is decreasing in $\Omega_b$.

5 Conclusions

Still working on this section
6 Figures

![Effects of a Monetary Policy Shock (Interest Rate Rule)](image)

Figure 1: Taken from Galí (2008)
Figure 2: Inflation and price dispersion behavior in the last period
Figure 3: Price determination through monetary policy in the last period, $\kappa = 0.5$.

Figure 4: Effect of inflation index weight on price determination in the last period, $\pi^* = 1.1$. 
Figure 5: Effect of inflation index weight on price determination in the last period, $\pi^* = 0.9$

Figure 6: Non-labor income for the agents in $t = 1$ for different levels of wealth inherited from $t = 0$; for the case $\Omega_s = \Omega_b = 0.5$
Figure 7: Optimal labor supplied by the agents in $t = 1$ for different levels of wealth inhereted from $t = 0$; for the case $\Omega_s = \Omega_b = 0.5$.

Figure 8: Optimal consumption of the non-durable by the agents in $t = 1$ for different levels of wealth inhereted from $t = 0$; for the case $\Omega_s = \Omega_b = 0.5$. 
Figure 9: Optimal consumption of the durable by the agents in $t = 1$ for different levels of wealth inherited from $t = 0$; for the case $\Omega_s = \Omega_b = 0.5$.

Figure 10: IRF of total aggregate output to a surprise decrease of 25 b.p. in the MP rule.
Figure 11: IRF of model variables to a surprise decrease of 25 b.p. in the MP rule.

Figure 12: IRF of model variables to a surprise decrease of 25 b.p. in the MP rule.
Figure 13: IRF of model variables to a surprise decrease of 25 b.p. in the MP rule
References


