Abstract

We present novel insights on the role of international trade following unanticipated government spending and income tax changes in a flexible exchange rate environment. First, the domestic effectiveness of fiscal policy can be larger in economies more open to trade, irrespective of the trade balance dynamics. Second, when trade linkages strengthen the expansionary effects of government spending, they reduce the effects of income tax cuts, and vice versa. Third, trade openness can imply that domestic multipliers are larger with distortionary rather than non-distortionary financing. We demonstrate these results analytically in a simple two-country, two-goods model. We then combine Bayesian prior and posterior analyses on Canadian and U.S. data to obtain empirically relevant predictions in quantitative international business-cycle models, including a version that features a microfounded trade structure with endogenous tradability and firm heterogeneity. Across models, posterior estimates imply medium-run Canadian government spending multipliers are higher than in a counterfactually closed economy. Income tax cuts generate lower multipliers but are more effective in inducing positive cross-country comovement.

JEL Codes: E62, F41, C11

Keywords: Trade integration, government spending, income tax, Bayesian estimation
1 Introduction

How does trade in goods and services affect the transmission of fiscal policies domestically and abroad in a flexible exchange rate environment? According to conventional wisdom, trade linkages reduce the domestic effectiveness of fiscal policy. Real exchange rate appreciation and/or rising aggregate income increase imports and (potentially) reduce exports, mitigating the expansionary effects of the policy. Domestic losses translate to gains for trading partners, as expenditure switching boosts output abroad.¹

This paper reassesses and challenges this view, presenting fresh insights on the role of international trade following unanticipated fiscal changes under a flexible exchange rate. First, the domestic effectiveness of fiscal policy can be larger in economies more open to trade, irrespective of the trade balance dynamics. Second, when trade linkages strengthen the expansionary effects of government spending, they reduce the effects of income tax cuts, and vice versa. Third, trade linkages can reverse the conventional wisdom in the closed-economy environment: domestic multipliers can be larger with distortionary rather than non-distortionary financing. We first illustrate these arguments analytically in a two-country, one-period model. We then show that the same wisdom prevails in richer quantitative international business-cycle models using a Bayesian prior and posterior analysis on Canadian and U.S. data. Posterior estimates imply that an increase in public spending yields higher fiscal multipliers when trade linkages are stronger; income tax cuts generate lower multipliers but are more effective in inducing positive cross-country comovement. Our results have direct implications for the effectiveness of fiscal policy in the global economy, including the desirability of fiscal consolidations, balanced-budget versus debt-financed fiscal actions, and incentives for international fiscal policy coordination.

A priori, the ambiguous theoretical relationship between international trade and the effectiveness of fiscal expansions stems from international price movements in general equilibrium and changes in cross-country relative wealth. We demonstrate the core intuition in a simple one-period, open-economy variant of the model in Woodford (2011). Absent trade linkages, public expenditure and income tax changes result in familiar closed-economy messages: GDP multipliers are always less than one, and private consumption falls with increases in either government spending or income taxes.²

¹See Frenkel and Mussa (1981) and Chinn (2013) for a more formal discussion.
²See Drautzburg and Uhlig (2015), Leeper, Traum, and Walker (2017) and Ramey (2011) for detailed discussions of government spending multipliers in a closed economy. In the quantitative version of our model, we follow the literature and permit public and private consumption goods to be complements, allowing private consumption to rise
Analytical solutions demonstrate that trade in goods implies much richer dynamics. Trade linkages can either increase or decrease domestic fiscal multipliers and induce positive or negative cross-country spillovers. The extent of trade openness (i.e., the trade-to-GDP ratio) does not intrinsically determine fiscal outcomes; for the same value of the trade share, a priori the expansionary effect of the fiscal policy can be larger or smaller relative to the closed economy. Three key factors determine the domestic effectiveness and international spillovers: (i) the relative share of public and private imports (and not just the extent of home bias in government consumption), (ii) the trade elasticity, and (iii) the financing of the government budget. These factors shape the general-equilibrium response of the terms of trade (i.e., the relative price of exports to imports). In particular, when the terms of trade appreciate sufficiently, the relative increase in domestic wealth leads to higher output at the expense of trading partners. In turn, distortionary financing can impart additional stimulus, when the financing itself leads to a terms-of-trade appreciation.

The intuition is best illustrated assuming financial autarky. With balanced trade, the response of the terms of trade is a sufficient statistic for the qualitative effects on domestic and foreign variables. In contrast, whether the real exchange rate appreciates or depreciates is not central for the qualitative results. For public spending increases, a higher private-to-public import share reduces the crowding out of domestic private consumption relative to the closed economy, provided that traded goods are not too complementary (i.e., above an endogenously determined cutoff for the trade elasticity). In this case, the terms of trade appreciate in equilibrium, since domestic prices increase relative to the rest of the world to clear the excess demand for domestic goods; as the private import share increases, the positive wealth effect stemming from the favorable terms-of-trade movement strengthens, partly offsetting the decline in private domestic demand induced by higher domestic prices. When traded goods are sufficiently complementary, the opposite result holds: trade linkages result in higher domestic output when the relative share of government imports is sufficiently high.

Since income tax cuts and public spending increases have the opposite effect on domestic prices, and in turn the terms of trade, the role of trade linkages reverses with tax cuts. However, while only the trade elasticity matters with non-distortionary financing, in the more empirically relevant scenario where public spending finances the government budget (e.g. Corsetti, Meier, and Muller, 2012), the relative size of the public-private import share again is pivotal for equilibrium outcomes. International trade in assets and net exports dynamics do not overturn the key insights highlighted with an increase in government spending.
under balanced trade.

We then assess and quantify the extent to which terms-of-trade dynamics shape fiscal multipliers in a state-of-the-art international business-cycle model that includes additional, competing forces for the fiscal transmission—endogenous physical capital accumulation, pricing frictions, intertemporal trade in assets, and complementarity between private and public consumption.\(^3\) In addition, we consider a distinct version that also features endogenous goods’ tradability and firm heterogeneity, capturing key micro features of international trade, as highlighted by Melitz (2003), Ghironi and Melitz (2004), and the subsequent literature. When taking the model to the data, we focus on a benchmark small open economy, Canada. Since 80% of Canadian trade occurs with the U.S., the latter provides a realistic characterization of the rest of the world from the perspective of Canada. Accordingly, we consider a two-country model in which one country (Canada) is of measure zero relative to the rest of the world (U.S.). Consistent with recent empirical evidence (Goldberg and Tille, 2008 and Gopinath, 2015), we assume U.S. dollar exports invoicing. We employ a Bayesian prior-predictive analysis to uncover the full range of fiscal outcomes implied by the model structure before confronting the data (see Geweke, 2010). A priori both versions of the model are agnostic about the role of trade linkages for the effectiveness of fiscal actions, as well as the size and sign of the responses of the terms of trade, real exchange rate, and international macroeconomic spillovers.

To obtain empirically relevant predictions, we estimate the model with Bayesian inference. We first estimate a version of the model abstracting from endogenous tradability and firm heterogeneity. We include data on fiscal measures and bilateral trade flows. The estimates reproduce the positive cross-country correlation for several international macroaggregates, stemming from U.S. shocks accounting for a nontrivial portion of the variability of Canadian series. In addition, we find the data informative about the trade elasticity, with posterior estimates centered slightly above one.

We find medium-run government spending multipliers in Canada are higher than in a counterfactually closed economy, whereas income tax multipliers are always smaller. U.S. tax cuts entail persistent positive spillovers for Canada, while U.S. government spending increases imply short-lived positive comovement. The gains of trade linkages are relatively higher in a counterfactual scenario where the government runs a balanced-budget rule.

We then estimate the model incorporating endogenous entry of heterogeneous producers in the domestic and export markets. We augment our measurement equation with two additional annual series: the number of varieties exported from the U.S. to Canada and imported to the U.S.

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\(^3\) The literature has shown these features are key determinants of fiscal multipliers in closed economy models.
from Canada and employ mixed-frequency estimation. We find the posterior estimate of the trade elasticity remains robust to this richer trade structure and ancillary data. Endogenous entry in the export market dampens initial terms of trade movements following fiscal shocks and increases its persistence. We show this stems from endogenous fluctuations in the number of exported goods, affecting multipliers quantitatively in the medium run. Nevertheless, our key messages still hold.

**Related Literature** This paper is related to several strands of the literature. Seminal contributions focused on whether fiscal policies are beggar or prosper-thy-neighbor (e.g. Obstfeld and Rogoff, 1995, Corsetti and Pesenti, 2001, and Betts and Devereux, 2001), while more recent works address how financial arrangements and monetary/exchange rate regimes affect the fiscal transmission (e.g., Beetsma and Jensen, 2005, Gali and Monacelli, 2008, Erceg and Linde, 2012, Corsetti, Kuester, and Muller, 2013, Born, Juessen, and Muller, 2013, Muller, 2008). In addition, a few early quantitative studies examine government spending and tax changes in flexible exchange-rate models (e.g. Baxter, 1995 and Erceg, Guerrieri, and Gust, 2005). In contrast to these studies, we focus on the role of trade linkages (and its determinants) for the domestic and international transmission of fiscal policy. Farhi and Werning (2016) also compare analytically fiscal multipliers in open and closed economies but focus on a currency union in a liquidity trap.

The role of trade openness for the transmission of fiscal shocks has been explored in the context of structural panel vector autoregressions. Ilzetzki, Mendoza, and Vegh (2013) find that on average government spending multipliers are smaller in economies with trade-to-GDP ratios exceeding 60%. While Canada and the U.S. are included in their analysis, both countries fall in the “closed” classification (less than 60% trade-to-GDP ratios). Our results suggest an important avenue for future empirical analysis is to condition on the relative composition of private-public imports.

A few recent empirical studies examine cross-country spillovers from expansionary fiscal policies (Faccini, Munitaz, and Surico, 2016 and Auerbach and Gorodnichenko, 2013). Our estimates are consistent with this literature, which generally finds positive international spillovers. While a sizable empirical literature has examined the real exchange rate response to government spending shocks, the response of the terms of trade has received limited attention. The literature tends to find an increase in government spending leads to a real depreciation of the exchange rate, yet for Canada there is no conclusive evidence.\(^4\)

\(^4\)Monacelli and Perotti (2008) find the terms of trade appreciate with an increase in public spending, while Enders, Muller, and Scholl (2011) find the terms of trade depreciate.

\(^5\)Canada is the only country for which Monacelli and Perotti (2010) find no conclusive linkage, while Bouakez, Chihi, and Normandin (2014) find the real exchange rate depreciates. Kim (2015) compares the effects for 19
This paper also is related to the literature on the so-called “twin deficits,” studying the causal role of primary fiscal deficits on current account deficits. Model-based evaluations often assign a small role for fiscal policy, see Erceg, Guerrieri, and Gust (2005), Chen, Imrohoroglu, and Imrohoroglu (2009), and Ferrero (2010). Empirical evidence varies but can attribute a closer connection between the two deficits, e.g. Chinn (2017), Monacelli and Perotti (2010), Kim and Roubini (2008), and Normandin (1999). Our focus is not on providing a structural interpretation of current account dynamics. We show that fiscal induced trade deficits can coincide with higher fiscal stimulus.

Finally, this paper combines two strands of the literature that carries out likelihood-based analyses: on fiscal policy in closed economies and on international transmission of business-cycles (e.g., Drautzburg and Uhlig, 2015, Leeper, Traum, and Walker, 2017, Adolfson, Laseen, Linde, and Villani, 2005, Justiniano and Preston, 2010). To our knowledge, we are the first to estimate a model incorporating a micro-founded trade structure with endogenous entry of heterogeneous producers in domestic and export markets. Furthermore, we exploit data on the number of varieties bilaterally traded together with measures of trade flows and fiscal aggregates.

The rest of the paper is organized as follows. Section 2 presents the simple analytical model and a full characterization of the equilibrium dynamics following a public spending increase and tax cut. Section 3 describes the quantitative business-cycle model, while section 4 presents the prior-predictive analysis, estimation details, and posterior analysis. Section 5 describes the results from an extension with an endogenously determined trade structure, while section 6 concludes.

## 2 Building Intuition: A One-Period Model

To understand the core implications of trade linkages, we begin our analysis in a one-period real business cycle model that abstracts from endogenous physical capital accumulation. The closed economy is a variant of Woodford’s (2011) model, whose dynamics have been extensively analyzed in the literature. We revisit the closed-economy result and then consider a two-country version of the model. We show analytically three main results. First, in contrast to the consensus view, trade linkages can either increase or decrease domestic fiscal multipliers and induce positive or negative cross-country spillovers. Second, when trade linkages strengthen the expansionary effects of government spending, they reduce the effects of income tax cuts, and vice versa. Third, unlike in the closed-economy, domestic multipliers can be larger with distortionary rather than non-

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OECD countries. The working paper version includes individual country estimates, showing the real exchange rate appreciates significantly following an increase in Canadian government spending.
distortionary financing. All three results hold independent of the real exchange rate dynamics. The overall size of trade linkages does not intrinsically determine these outcomes. Holding the trade share constant, the spectrum of results depends on the relative import content of private and public consumption and the trade elasticity, as these features crucially define the terms-of-trade and net exports responses.

Closed Economy

A representative agent maximizes $\ln C_t - L_t^{1+\omega}/(1 + \omega)$, where $C_t$ is consumption and $L_t$ denotes hours worked. The agent produces output with linear technology $Y_t = L_t$. A government distributes a lump-sum transfer $T_t$ and imposes a tax on income $\tau$ to finance an exogenous level of public expenditures $G_t + T_t = \tau_t L_t$. The agent’s resource constraint implies that $C_t = (1 - \tau_t) L_t + T_t$.

Optimal hours supply yields $L_t^\omega / C_t = (1 - \tau_t)$, while goods market clearing implies $L_t = C_t + G_t$.

Log-linearizing the equilibrium conditions and solving the model leads to the well-known policy function

$$\hat{Y}_{t, closed} = \frac{G/Y}{1 + \omega(1 - G/Y)} \hat{G}_t - \frac{\tau(1 - G/Y)}{(1 - \tau)(1 + \omega(1 - G/Y))} \hat{\tau}_t,$$

which shows that output increases less than one-for-one with an increase in government spending. A reduction in the distortionary income tax leads to an increase in output as the household increases labor when its after-tax return is higher. The response of consumption is given by:

$$\hat{\dot{C}}_{t, closed} = \frac{\omega G/Y}{1 + \omega(1 - G/Y)} \hat{G}_t - \frac{\tau}{(1 - \tau)(1 + \omega(1 - G/Y))} \hat{\tau}_t,$$

which shows that consumption is crowded out with public spending increases, and rises following income tax reductions.

Open Economy

Consider now two symmetric countries, Home and Foreign. Each country produces a distinct, homogeneous good using labor. International trade is frictionless and trade is balanced in each period (i.e. financial autarky).

We use the subscript $D$ to denote quantities and prices of a country’s own tradable goods consumed domestically, and the subscript $X$ to denote quantities and prices of exports. Consumption $C_t$ now aggregates Home and Foreign tradable consumption sub-baskets in Armington form with
an exogenous elasticity of substitution $\phi > 0$:

$$C_t = \left[ (1 - \alpha_X)^{\frac{1}{\phi}} (C_{D,t})^{\frac{\phi - 1}{\phi}} + \alpha_X \left( (C_{X,t})^{\frac{\phi - 1}{\phi}} \right) \right]^{\frac{1}{\phi-1}}, \quad 0 \leq \alpha_X \leq 1,$$

where $1 - \alpha_X$ is the weight attached to the country’s own good. Preferences are biased in favor of domestic goods whenever $\alpha_X < 1/2$. The price index that corresponds to the basket $C_t$ is given by $P_t = \left[ (1 - \alpha_X) (P_{D,t})^{1-\phi} + \alpha_X \left( (P_{X,t})^{1-\phi} \right) \right]^{1/(1-\phi)}$. Foreign households derive utility from an analogous consumption bundle of domestic and imported consumption goods, $C_{D,t}^*$ and $C_{X,t}^*$. Home private demand for domestic and imported goods is, respectively,

$$C_{D,t} = (1 - \alpha_X) \rho_{D,t}^{-\phi} \rho_{G,t}^{\phi} G_t, \quad C_{X,t} = \alpha_X \rho_{X,t}^{-\phi} \rho_{G,t}^{\phi} G_t^*,$$

where $\rho_{G,t} \equiv P_{G,t}/P_t$ ($\rho_{G,t}^*$) denotes the Home (Foreign) price of government consumption relative to Home (Foreign) consumption. The Home government budget constraint is given by $\rho_{G,t} G_t + T_t = \tau_t \rho_{D,t} L_t$.

Balanced trade implies that the value of exports is equal to the value of imports:

$$Q_t \rho_{X,t} (C_{X,t} + G_{X,t}) = \rho_{X,t}^* (C_{X,t}^* + G_{X,t}^*).$$

The resource constraint is $L_t = C_{D,t} + C_{X,t} + G_{D,t} + G_{X,t}$. Real GDP is defined as $Y_t = C_t + \rho_{G,t} G_t$, where we deflate the nominal value of output using the CPI index $P_t$. Thus, $Y_t$ measures the consumption value of domestic output, which differs from units of production ($L_t$) due to differences
in international relative prices. Finally, we define the terms of trade as the Home price of exports relative to the price of imports (both expressed in Home currency): \( TOT_t \equiv \rho_{D,t} / \left( \rho_{P,t} Q_t \right) \).

Notice that in the symmetric steady state (where \( Q = 1 \)), the total trade to GDP ratio is given by

\[
\text{trade share} \equiv \frac{2(\rho_X C_X + \rho_X G_X)}{Y} = 2 \left[ \alpha^g_X \frac{G}{Y} + \left( 1 - \frac{G}{Y} \right) \alpha_X \right].
\]

Consequently, openness increases monotonically in either \( \alpha_X \) or \( \alpha^g_X \).

We solve for the competitive equilibrium, log-linearize the resulting system of equations, and use the method of undetermined coefficients to solve the model. Appendix A lists the complete set of nonlinear and log-linearized equilibrium conditions. In what follows, we consider solutions in the face of exogenous changes to either domestic government spending or taxes, \( \hat{G}_t \neq 0 \) or \( \hat{\tau}_t \neq 0 \), while assuming foreign government spending and taxes are always held constant at steady state, \( \hat{G}_t^* = 0 \) or \( \hat{\tau}_t^* = 0 \) for all \( t \).

**Full Home-Bias in Government Spending**

We start by studying the case of full home-bias in government spending, \( \alpha^g_X = 0 \), as it is the most common assumption in the literature.\(^6\) We first consider the case of a unitary trade elasticity \( \phi \), since this is a benchmark assumption in the literature. Under these knife-edge assumptions, the real exchange rate equals the ratio of the marginal utilities of consumption across the two countries (Cole and Obstfeld (1991)). Thus, balanced trade coincides with full international risk sharing. We analyze a general version of the model with complete international asset markets in an extension below.

**Proposition 1** Suppose that \( \phi = 1 \) and \( \alpha^g_X = 0 \). Then the solutions for GDP, consumption, labor, the terms of trade, and the real exchange rate following a 1% increase in government spending are given by

<table>
<thead>
<tr>
<th>Policy Functions for a 1% Government Spending Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Home</strong></td>
</tr>
<tr>
<td>( \hat{Y}_t = \frac{\hat{G}_t}{1 + \omega (1 - \frac{\phi}{Y})} &gt; \hat{Y}^*_t )</td>
</tr>
<tr>
<td>( \hat{C}_t = \frac{\hat{G}_t \omega (\alpha_X - 1)}{1 + \omega (1 - \frac{\phi}{Y})} &gt; \hat{C}^*_t )</td>
</tr>
<tr>
<td>( \hat{L}_t = \frac{\hat{G}_t}{1 + \omega (1 - \frac{\phi}{Y})} = \hat{L}^*_t )</td>
</tr>
<tr>
<td><strong>Foreign</strong></td>
</tr>
<tr>
<td>( \hat{Y}^*_t = -\frac{\hat{G}_t \alpha_X \omega}{1 + \omega (1 - \frac{\phi}{Y})} &lt; 0 )</td>
</tr>
<tr>
<td>( \hat{C}^*_t = -\frac{\hat{G}_t \alpha_X \omega}{1 + \omega (1 - \frac{\phi}{Y})} &lt; 0 )</td>
</tr>
<tr>
<td>( \hat{L}^*_t = 0 )</td>
</tr>
<tr>
<td><strong>Relative Prices</strong></td>
</tr>
<tr>
<td>( \hat{TOT}_t = \frac{\hat{G}_t \omega}{1 + \omega (1 - \frac{\phi}{Y})} &gt; 0 )</td>
</tr>
<tr>
<td>( \hat{Q}_t = \frac{\hat{G}_t \omega (2 \alpha_X - 1)}{1 + \omega (1 - \frac{\phi}{Y})} \leq 0 )</td>
</tr>
</tbody>
</table>

\(^6\) See for instance Corsetti and Pesenti (2001), Erceg, Guerrieri, and Gust (2005), Corsetti, Meier, and Muller (2012), Born, Juessen, and Muller (2013), Cook and Devereux (2013), and Farhi and Werning (2016).
It follows that

1. **Domestic GDP and consumption responses are increasing in openness (measured by $\alpha_X$ since $\alpha_X^g = 0$).**

2. **Foreign GDP and consumption responses are decreasing in openness (measured by $\alpha_X$ since $\alpha_X^g = 0$).**

3. **The domestic terms of trade always improve, while the real exchange rate only appreciates when $\alpha_X < 0.5$.**

Higher government expenditure raises the relative price of the Home good $\rho_{D,t}$, which, ceteris paribus, crowds out private consumption (lowering demand for domestic goods). With full home bias in government consumption, the increase in public spending creates an excess world demand for the Home good, leading to an appreciation of the terms of trade to clear the market. This favorable relative price movement induces a positive wealth effect for the Home economy as more foreign goods are traded per domestic good. In turn, this allows domestic households to consume more of their own private consumption good, reducing the crowding out of private consumption and boosting GDP in equilibrium. While $\phi = 1$ implies that the terms of trade do not depend on $\alpha_X$, both the domestic gain from this wealth effect and the foreign loss increase monotonically when trade linkages are stronger. Intuitively, as $\alpha_X$ increases, Home households are proportionally wealthier without reducing the relative world demand for the Home good in equilibrium. Indeed, government spending multipliers for Home GDP even can be larger than one.\(^7\)

While the consumption value of GDP changes because of relative price movements, the response of domestic hours is identical in open and closed economies ($\hat{L}_t = \hat{L}_t^{\text{closed}}$), and foreign hours are constant ($\hat{L}_t^* = 0$). Both results depend on the assumption of logarithmic utility coupled with full home bias in government consumption ($\alpha_X^g = 0$). Domestic hours are not affected by trade openness because logarithmic utility implies that $\hat{L}_t$ is proportional to $\check{G}_t$—income and substitution effects cancel out only when $\check{G}_t = 0$. Since $\check{G}_t = \check{G}_{D,t}$ when $\alpha_X^g = 0$, it follows that labor responds the same in open and closed economies. By the same token, foreign hours are constant as income and substitution effects in foreign hours supply exactly offset each other in the absence of foreign

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\(^7\)This is possible when government spending is a sufficiently large share of GDP. The domestic multiplier $\Delta Y_t/\Delta G_t$ can be larger than one despite an unambiguous decline in private consumption. This is possible because the increase in the relative price of government goods, $\rho_{G,t}$, dominates when $G/Y > 1 - \alpha_X$. 

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fiscal shocks. As shown below, \( \hat{L}_t \neq \hat{L}_t^{\text{closed}} \) and \( \hat{L}_t^* \neq 0 \) as long as \( \alpha_X^g > 0 \).

Notice that while the response of the terms of trade is unambiguous following an increase in public consumption, the real exchange rate can either appreciate or depreciate, depending on the degree of Home bias in private consumption. Intuitively, \( Q_t \) adjusts to maintain balanced trade. When \( \alpha_X > 0.5 \), the higher price of Home goods hurts Foreign consumers relatively more. Absent any change in the real exchange rate, this would result in a Home trade deficit. Accordingly, the real exchange rate depreciates in equilibrium to restore balanced trade. By the same logic, the real exchange rate must appreciate when \( \alpha_X < 0.5 \).

**Proposition 2** Suppose that \( \phi = 1 \) and \( \alpha_X^g = 0 \). Then the solutions for GDP, consumption, labor, the terms of trade, and the real exchange rate following a 1% tax cut are given by

<table>
<thead>
<tr>
<th>POLICY FUNCTIONS FOR A 1% TAX CUT</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Home</strong></td>
</tr>
<tr>
<td>( \hat{Y}_t = \frac{\tau(1-\frac{\alpha_X}{2} - \alpha_X)}{1+\omega(1-\frac{\alpha_X}{2})} \frac{1}{(1-\tau)} )</td>
</tr>
<tr>
<td>( \hat{C}_t = \frac{\tau(1-\alpha_X)}{1+\omega(1-\frac{\alpha_X}{2})} \frac{1}{(1-\tau)} )</td>
</tr>
<tr>
<td>( \hat{L}_t = \frac{\tau(1-\frac{\alpha_X}{2})}{1+\omega(1-\frac{\alpha_X}{2})} \frac{1}{(1-\tau)} )</td>
</tr>
</tbody>
</table>

It follows that

1. The domestic GDP and consumption responses are decreasing in openness (measured by \( \alpha_X \) since \( \alpha_X^g = 0 \)).

2. The foreign GDP and consumption responses are increasing in openness (measured by \( \alpha_X \) since \( \alpha_X^g = 0 \)).

3. The domestic terms of trade always deteriorate, while the real exchange rate only depreciates when \( \alpha_X < 0.5 \).

Following a tax cut, domestic households have more after-tax income, leading domestic consumption and GDP to rise and the domestic price to decrease. Parts 1 and 2 state that trade openness dampens the expansionary effects domestically, as the foreign economy enjoys a benefit from the deterioration in the terms of trade, leading to a positive spillover. The larger the private import share, the stronger this effect.

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8 Additionally, with alternative preferences (either non-logarithmic or non-separable), hours can increase relative to the closed economy while preserving the results of proposition 1. Results are available upon request.
The above results hold with a restriction on the elasticity of substitution: $\phi = 1$. In the following proposition, we generalize results for an arbitrary $\phi$. The intuition about the role of the trade elasticity is similar to Corsetti, Dedola, and Leduc (2008), who focus on the transmission of productivity shocks in a two-country, two-good endowment economy.

**Proposition 3** Assume full home-bias in government consumption, $\alpha^g_X = 0$. Then there exists a non-negative cut-off value $\phi^* = \left\{ \begin{array}{ll} 0 & \text{if } \alpha_X \geq \frac{1}{2} \\
\frac{1-2\alpha_X}{2(1-\alpha_X)} & \text{if } \alpha_X < \frac{1}{2} \end{array} \right.$ such that

1. When $\phi > \phi^*$, an increase in Home government spending improves the domestic terms of trade ($\hat{\text{TOT}}_t > 0$), leading to:
   - (a) $\hat{Y}_t > \hat{Y}^{\text{closed}}_t$, $\hat{C}_t > \hat{C}^{\text{closed}}_t$ and $\hat{L}_t = \hat{L}^{\text{closed}}_t$.
   - (b) $\hat{Y}_t^* < 0$, $\hat{C}_t^* < 0$ and $\hat{L}_t^* = 0$.
2. When $\phi > \phi^*$, a tax cut has the opposite effects ($\hat{\text{TOT}}_t < 0$).
3. As $\phi \to \infty$, all responses converge to their closed economy counterparts.

See appendix A for the proof. Part 1 demonstrates that for any $\phi > \phi^*$, domestic GDP and consumption responses following public spending increases are relatively higher in an open economy. With home bias ($\alpha_X < 0.5$), a terms-of-trade appreciation that increases Home wealth relative to Foreign results in relatively higher relative world demand for the domestic good. When goods are not too complementary ($\phi > \phi^*$), the ability of Home households to substitute domestic goods with cheaper imported goods softens the crowding out of private consumption for a given terms of trade. This substitution effect dominates the negative Home income effect from higher domestic prices, leading to higher consumption and GDP. However, conditional on $\phi > \phi^*$, the gains relative to the closed economy decrease monotonically in $\phi$, since a higher trade elasticity dampens the terms of trade appreciation—when $\phi \to \infty$, terms of trade are constant, and allocations in open and closed economy coincide. When $\alpha_X > 0.5$, output gains are higher in the open economy regardless of the value of $\phi$, since $\phi^* = 0$; absent Home bias in private consumption, a higher Home price hurts Foreign consumers more, leading to an unambiguous increase in the terms of trade.

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9 Conversely, higher public spending crowds out more private consumption when $\phi < \phi^*$, since households must continue to consume more costly Home goods (reducing their real income). The reduction in Home private demand is sufficiently strong to reduce the relative price of Home goods in equilibrium, leading to a deterioration in the terms of trade.
The same reasoning applies to a tax cut, although with reversed logic: a sufficiently high elasticity of substitution lowers GDP and consumption relative to a closed economy (Part 2). The role of $\phi$ is sensitive to the presence of international trade in assets, as $\phi$ shapes both the terms-of-trade and net exports responses.

**Generalizing the Composition of Government Spending**

In practice, governments import goods. Previous work has found that Home bias in government spending leads to higher multipliers (e.g. Blanchard, Erceg, and Linde (2016) and Leeper, Traum, and Walker (2011)). We now qualify and generalize this result, highlighting two aspects: 1) the overall effectiveness of public spending depends on the relative size of public-private import shares and 2) for a given public import share ($\alpha^g_X$), the expansionary effects of government spending are increasing in the private import share ($\alpha_X$). Put differently, the relative composition of public and private imports, $\nu \equiv \alpha^g_X/\alpha_X$ (and not $\alpha^g_X$ per se) is key for the effects of government spending in open economies.

We first derive analytical results under some parametric restrictions: a unitary trade elasticity, a unitary Frisch elasticity, home bias in private consumption and $\nu \leq 1$. We then show numerically the results generalize when these restrictions are relaxed.

**Proposition 4** Assume $0 < \alpha_X \leq \frac{1}{2}$, $\phi = 1$, $\omega = 1$, and $\nu \leq 1$. Then there exists a $\nu^* = \frac{(1 - \frac{G}{Y})}{(2 - \frac{G}{Y})} \in (0, \frac{1}{2})$ such that,

1. When $\nu < \nu^*$, an increase in Home government spending improves the domestic terms of trade ($\hat{TOT}_t > 0$), leading to:

   (a) $\hat{Y}_t > \hat{Y}_t^{\text{closed}}$, $\hat{C}_t > \hat{C}_t^{\text{closed}}$, and $\hat{L}_t < \hat{L}_t^{\text{closed}}$.

   (b) $\hat{C}_t^* < 0$, $\hat{Y}_t^* < 0$, and $\hat{L}_t^* > 0$.

2. Tax responses do not depend on $\nu$.

As long as the government does not consume too high of a share of total imports relative to private goods (formally $\nu < \nu^*$), the terms of trade improve, leading to higher GDP and consumption relative to the closed economy. To understand this result, notice first that when $\alpha^g_X > 0$, part of the increase in $G_t$ falls on Foreign goods, leading to an increase in Foreign output and prices, other things equal. As a result, the Home terms of trade increase by less. Moreover, the consumption
value of government goods \((\rho_{G,t})\) is lower, directly lowering Home GDP. However, proposition 4 shows that what matters is the value of \(\alpha^g_X\) relative to the private consumption import share \(\alpha_X\), i.e. for a given \(\alpha^g_X\), Home GDP responses are larger or smaller than the closed economy depending on \(\alpha_X\). As in proposition 1, since goods are are not too complementary \((\phi = 1)\), a higher private consumption import share reduces consumption crowding out.\(^{10}\) Income tax responses do not depend on \(\nu\), since government spending are assumed not respond to a tax shock; below we show how distortionary public expenditure financing alters this result.

The relative size of public-private import shares continues to determine responses once relaxing the parametric restrictions on \(\phi\), \(\alpha_X\), and \(\nu\) from proposition 4. However, outcomes now depends on a \(\phi^*\) threshold as in proposition 3, where the \(\phi^*\) boundary is now conditional on \(\nu\). When \(\phi > \phi^*\), the results and intuition of proposition 4 hold. When \(\phi < \phi^*\), the results are reversed; Home GDP responses are increasing in \(\nu\) and \(\alpha^g_X\).\(^{11}\)

Figure 1a demonstrates the robustness of proposition 4 provided that \(\phi > \phi^*\) (see appendix A for the \(\phi < \phi^*\) scenario). The figure depicts the responses of Home and Foreign GDP and the terms of trade following a 1% increase in government spending for various \(\nu\) and \(\phi\) values. As the composition of private and public imports varies (a change in \(\nu\)), the overall level of openness (i.e. the trade share) can differ. To control for this effect, the figure holds the trade share constant at 0.5; we consider a grid for \(\alpha^g_X \in [0,0.35]\) and let \(\alpha_X\) adjust to maintain a constant trade share.\(^{12}\) We consider a standard range of values for the trade elasticity: \(\phi \in [0.5, 2.5]\). The plane in each panel plots the response in the closed economy, which is independent of \(\nu\) and \(\phi\). Consistent with proposition 4, Home GDP responses decline in \(\nu\), as increased public demand for imports keeps the foreign price from falling substantially relative to the domestic price, causing private consumption to be crowded out more. As in proposition 3, home GDP and the terms-of-trade responses are decreasing in \(\phi\).

\(^{10}\) The \(\phi^*\) bound is decreasing in the steady state government spending-to-GDP ratio, \(\frac{\zeta}{\gamma}\). Intuitively, the larger the government’s share of GDP, the more domestic prices respond to an increase in public expenditure, crowding out more private consumption. Thus, the crowding out effect dominates in equilibrium sooner (i.e. for a smaller \(\nu\)).

\(^{11}\) A higher \(\alpha^g_X\) implies the initial increase in government spending falls more symmetrically on Home and Foreign goods, dampening relative price adjustments. When goods are sufficiently complementary, this softens the crowding out effect on Home private consumption, discussed in proposition 3. For a sufficiently high relative share of public to private imports (\(\nu\)), the reduction in crowding out is strong enough to raise Home GDP relative to the closed economy.

\(^{12}\) Results hold regardless of varying \(\alpha^g_X\) independently and calculating \(\alpha_X\) to maintain a constant trade share or varying \(\alpha_X\) and letting \(\alpha^g_X\) adjust. In addition, similar results hold when both \(\alpha_X\) and \(\alpha^g_X\) vary independently and the trade share endogenously changes. Results available upon request.
For simplicity, so far our analysis has assumed lump-sum transfers finance the government budget. In a closed economy, it is well known that distortionary financing can lower the domestic effectiveness of expansionary fiscal policies (e.g. Woodford (2011) and Leeper, Plante, and Traum (2010)). Here we demonstrate that trade linkages can overturn this conventional insight: an increase in government spending (an income tax cut) financed by higher income taxes (lower government spending) can be more expansionary than non-distortionary financing. Moreover, distortionary financing augments the effects of trade linkages following fiscal shocks.

Figure 1b repeats the numerical analysis of figure 1a assuming income-tax financing following a 1% government spending increase. The plane in each panel plots the response in the closed economy. Absent trade linkages, Home GDP falls when government spending increases, due to the negative effect of higher taxes on labor supply. By contrast, the response of GDP is positive for several combinations of $\phi$ and $\nu$, and is above the closed economy in the vast majority of cases. Ceteris paribus, the increase in the income tax rate raises the terms of trade, and this additional wealth effect reduces the crowding out of private consumption. As explained before, the terms-of-trade effect is stronger for lower $\nu$ and $\phi$ combinations, which can more than offset the negative response of labor supply. In addition, for sufficiently low $\nu$ and $\phi$ combinations, terms-of-trade dynamics imply that home GDP increases more with income-tax financing relative to lump-sum transfer financing (as seen by comparing the GDP responses of figures 1a and 1b).

When an income tax cut is financed by lower government expenditures, again the relative share of public and private imports, $\nu$, crucially determines the domestic effectiveness. Provided that the decrease in government spending falls sufficiently on Foreign imports, the terms of trade can improve in equilibrium, boosting Home GDP relative to the closed economy or compared to a scenario with non-distortionary financing. For the majority of parameterizations however, income tax cuts are less effective with stronger trade linkages when financed with lower public expenditures (see appendix A for more discussion).

Extensions: Non-Traded Goods and International Trade in Financial Assets

We consider two extensions to assess the robustness of our results. We briefly discuss the results here and relegate all analytical derivations and details to appendix B.

First, the introduction of non-traded goods does not alter any of the previous conclusions. In
Figure 1. Impact responses following a 1% increase in government spending. The plane in each panel denotes the response in the closed economy. In all cases, $G/Y = 0.2$, $\tau = 0.25$, $\omega = 1$, and trade share = 0.5.
particular, results are isomorphic to the benchmark model provided that the elasticity of substitution between traded and non-traded goods equals the trade elasticity. Moreover, full home bias in government consumption, as in proposition 3, yields identical allocations to full bias in non-traded government consumption. Finally, absent parameter restrictions, the relative import content of private and public consumption and the trade elasticity continue to determine the effectiveness of fiscal policy in the open economy.

Second, we relax the assumption of financial autarky and assume complete international financial markets. This adds an additional determinant to trade flows in general equilibrium. As in Obstfeld and Rogoff (2000), we continue to abstract from dynamics and consider a one-period portfolio problem. Full international risk-sharing implies that cross-country consumption levels are tied to the real exchange rate, replacing the balanced-trade condition from the benchmark model. As a result, trade balance dynamics additionally affect GDP. In addition, perfect consumption insurance optimally insulates relative wealth from any price movements.

In general, our results continue to hold in this environment; trade linkages can either increase or decrease domestic fiscal multipliers and induce positive or negative cross-country spillovers. As in the benchmark model, the outcomes depend on the relative share of public and private imports and the trade elasticity. However, three caveats emerge: 1) risk-sharing implies Home consumption is always higher (lower) than in the closed economy for a government spending increase (tax cut); 2) barring the knife-edge conditions of propositions 1 and 2, the terms of trade are no longer a sufficient statistic for the open-economy responses; and 3) the value of the trade elasticity bears different implications as it affects the extent of net-export crowding out. Terms-of-trade appreciation does not necessarily lead to higher domestic multipliers, as cross-country risk-sharing can imply an offsetting decline in net exports following a fiscal expansion. While a \( \phi \) cut-off continues to exist, following a government spending increase, Home GDP responses are higher relative to the closed economy when \( \phi < \phi^* \), with the reverse for a tax cut. To understand this result, consider the limiting case of \( \phi \to \infty \). In this case, there are no relative price movements since the goods are perfectly substitutable. Even so, a shock to Home government spending alters domestic allocations relative to the closed economy. Risk-sharing implies a decline in Home net exports (i.e. a transfer of resources from Foreign to Home), reducing GDP relative to the closed economy. As \( \phi \) decreases, the appreciation of the terms of trade reduces the crowding out of net exports, boosting home GDP.\(^{13}\)

\(^{13}\)Notice that this implies home GDP responses are now decreasing monotonically in the relative share of private-
3 A Quantitative Model

We turn to a quantitative exploration using a state-of-the-art international business-cycle model that introduces competing forces for the fiscal transmission absent from the one-period model. First, we include features proven to be important for the transmission of fiscal shocks in a closed economy—endogenous physical capital accumulation, complementarity between private and public consumption, and pricing frictions. Second, we introduce intertemporal trade in international assets, thus allowing fiscal shocks to affect the current account position of trading partners. Third, consistent with recent empirical evidence (e.g., Goldberg and Tille, 2008 and Gopinath, 2015), we assume that both exports and imports are invoiced in U.S. dollars (the dominant currency). In section 5, we introduce an endogenously determined trade structure along the lines of Melitz (2003), addressing the role of producer heterogeneity and extensive margin dynamics for the international transmission of fiscal policy.

In our quantitative exercises, we focus on the U.S. and Canada. This country pair is particularly suited for the analysis, since 80% of Canadian trade occurs with the U.S., implying that the latter provides a realistic characterization of the rest of the world from the perspective of Canada.

Since Canada is small relative to the U.S., we follow the standard approach in the literature and build a two-country model in which one country (the small open economy, also referred to as Home) is of measure zero relative to the rest of the world (Foreign henceforth). As a consequence, the policy decisions and macroeconomic dynamics of the small open economy have no impact on Foreign. We abstract from monetary frictions that would motivate a demand for cash currency and resort to a cashless model following Woodford (2003). We present in detail below the problems facing households and firms in the small open economy. Variables without a time subscript denote non-stochastic values along the balanced growth path.

Households

Each country is populated by a unit mass of atomistic households. The representative household, indexed by $j \in [0,1]$, maximizes the expected intertemporal utility function

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \tilde{\beta}_t \left[ \log \left( \tilde{C}_{jt} - h_C \tilde{C}_{t-1} \right) - \tilde{h}_t \frac{L_{jt}^{1+\omega}}{1+\omega} \right] \right\},$$

(5)

public imports $\nu$. 

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17
where $\beta \in (0, 1)$ is the discount factor, $C_{jt}$ is a consumption basket that consists of private and public consumption as described below, and $L_{jt}$ is the number of hours worked. The household values consumption relative to a habit stock defined in terms of lagged aggregate consumption $h_C\tilde{C}_{t-1}$, where $h_C \in [0, 1)$. $\tilde{\beta}_t$ denotes an exogenous shock to the discount factor, which evolves according to $\log \tilde{\beta}_t = \rho_{\tilde{\beta}} \log \tilde{\beta}_{t-1} + \varepsilon_{\tilde{\beta}t}$ with $\varepsilon_{\tilde{\beta}t} \sim N(0, \sigma_{\tilde{\beta}}^2)$; $\tilde{h}_t$ denotes an exogenous shock to the marginal disutility of hours worked, which evolves according to $\log \tilde{h}_t = \rho_{\tilde{h}} \log \tilde{h}_{t-1} + \varepsilon_{\tilde{h}t}$ with $\varepsilon_{\tilde{h}t} \sim N(0, \sigma_{\tilde{h}}^2)$. Consumption utility is logarithmic to ensure balanced growth in the presence of non-stationary technological progress.

Total consumption, $\tilde{C}_{jt}$ is the sum of private $C_{jt}$ and public $G_t$ consumption goods, $\tilde{C}_{jt} = C_{jt} + \omega_G G_t$, as in Fève, Matheron, and Sahuc (2013). Parameter $\omega_G$ governs the degree of substitutability of the consumption goods: when $\omega_G < 0$, private and public consumption are complements; when $\omega_G > 0$, the goods are substitutes. A priori, we do not restrict public and private goods to be complements or substitutes, as the literature has found evidence for both.\(^{14}\)

Market consumption $C_t$ aggregates Home and Foreign consumption sub-baskets as described by equation (3) in the previous section. We now allow for exogenous fluctuations in the elasticity of substitution across Home and Foreign baskets, $\phi_t$, capturing in reduced form fluctuations in the relative price of imported goods. We assume that $\log \phi_t = \rho_{\phi} \log \phi_{t-1} + \varepsilon_{\phi t}$, where $\varepsilon_{\phi t} \sim N(0, \sigma_{\phi}^2)$.\(^{14}\)

In addition, we now assume that the sub-baskets $C_{D,t}$ and $C_{X,t}$ aggregate differentiated varieties in Dixit-Stiglitz form. The Home tradable consumption $C_{D,t}$ aggregates domestic consumption varieties $C_{D,t}(i)$: $C_{D,t} = \left[ \int_0^1 C_{D,t}(i)(\theta_t^{-1})/\theta_t \, di \right]^{\theta_t/(\theta_t-1)}$, where $\theta_t > 1$ is the exogenous elasticity of substitution across domestic goods. We assume that $\log \theta_t = \rho_{\theta} \log \theta_{t-1} + \varepsilon_{\theta t}$, where $\varepsilon_{\theta t} \sim N(0, \sigma_{\theta}^2)$, which following the literature, we refer to as a Home price markup shock. A similar basket describes consumption of Foreign goods: $C_{X,t} = \left[ \int_0^1 C_{X,t}(i)(\tilde{\theta}_t^{-1})/\tilde{\theta}_t \, di \right]^{\tilde{\theta}_t/(\tilde{\theta}_t-1)}$. The corresponding price indexes are $P_{D,t} = \left[ \int_0^1 P_{D,t}(i) \, di \right]^{1/(1-\tilde{\theta}_t)}$ and $P_{X,t} = \left[ \int_0^1 P_{X,t} \, di \right]^{1/(1-\tilde{\theta}_t)}$, both expressed in Home currency.

International asset markets are incomplete, as non-contingent nominal bonds denominated in Foreign currency are the only internationally traded asset. The representative household also can invest in non-contingent nominal bonds denominated in Home currency, which are traded domestically. Let $A_{j_{t+1}}^j$ and $A_{k_{t+1}}^j$ denote, respectively, nominal holdings of Home and Foreign bonds

\(^{14}\)For instance, for the U.S., Aschauer (1985) and Ercolani and e Azevedo (2014) find substitutability between private and government consumption while Bouakez and Rebei (2007) and Fève, Matheron, and Sahuc (2013) find the two goods are complements.
for the representative Home household $j$. To ensure a determinate steady-state equilibrium and stationary responses to temporary shocks in the model, we follow Turnovsky (1985) and assume a quadratic cost of adjusting Foreign bond holdings $\psi \left[ A^{*j}_{t+1}/P_t \right]^2 / 2$. These costs are paid to financial intermediaries whose only function is to collect these transaction fees and rebate the revenue to households in lump-sum fashion in equilibrium.

Households also have access to one-period, riskless nominal government bonds $B_{t+1}^j$. Moreover, the household accumulates physical capital and rents it to intermediate input producers in a competitive capital market. Investment aggregates domestic and imported investment goods $I_{KD,t}$ and $I_{KX,t}$, in Armington form:

$$I_{K,t} = \left[ (1 - \alpha^j_X) \frac{1}{\phi_t} (I_{KD,t})^{\phi_{t-1}} + \left( \alpha^j_X \right) \frac{1}{\phi_t} \left( I_{KX,t} \right)^{\phi_{t-1}} \right]^{\phi_t / (\phi_t - 1)}, \quad 0 \leq \alpha^j_X \leq 1,$$

where $1 - \alpha^j_X$ is the weight attached to the country’s own investment good. The investment sub-baskets $I_{KD,t}$ and $I_{KX,t}$ have the same composition as the private consumption sub-baskets $C_{D,t}$ and $C_{X,t}$: $I_{KD,t} = \left[ \int_0^1 I_{KD,t}(i) (\hat{\theta}_t - 1) / \hat{\theta}_t \, di \right]$ and $I_{KX,t} = \left[ \int_0^1 (I_{KX,t}(i)) (\hat{\theta}_t - 1) / \hat{\theta}_t \, di \right]$. As a result, the prices of domestic and imported sub-baskets coincide with the prices of the domestic and imported consumption goods. It follows that the price index for $I_{K,t}$ is given by $P^{IK}_{t} = \left[ (1 - \alpha^j_X) (P_{D,t})^{1 - \phi_t} + \alpha^j_X (P^2_{X,t})^{1 - \phi_t} \right]^{1/(1 - \phi_t)}$.

We introduce convex adjustment costs in physical investment and variable capital utilization. The utilization rate of capital is set by the household. Thus, effective capital rented to firms, $K_{I,t}^j$, is the product of physical capital, $\tilde{K}_{I,t}^j$, and the utilization rate, $u_{K,t}^j$: $K_{I,t}^j = u_{K,t}^j \tilde{K}_{I,t}^j$. Utilization incurs a cost of $\Psi(u_{K,t}^j)$ per unit of physical capital. In steady state, $u_K = 1$ and $\Psi(1) = 0$. We define the parameter $\psi \in [0, 1)$ such that $\Psi'(1)/\Psi'(1) \equiv \psi / (1 - \psi)$. Physical capital, $\tilde{K}_{I,t}$, obeys a standard law of motion:

$$\tilde{K}_{I,t+1}^j = (1 - \delta_K) \tilde{K}_{I,t}^j + \tilde{P}_{K,t} \left[ 1 - \frac{\nu_K}{2} \left( \frac{I_{K,t}^j}{I_{K,t-1}^j} - \bar{z} \right) \right] I_{K,t}^j, \quad (6)$$

where $\nu_K > 0$ is a scale parameter, and $\tilde{P}_{K,t}$ is an investment specific shock with log $\tilde{P}_{K,t} = \rho \tilde{P}_K \log \tilde{P}_{K,t-1} + \varepsilon_{\tilde{P}_K,t}^{iid} \sim N \left( 0, \sigma^2_{\tilde{P}_K} \right)$. This shock is a source of exogenous variation in the efficiency with which the final good can be transformed into physical capital, and thus into

\[15\] Since only Foreign bonds are traded across borders, defining an adjustment cost for Foreign bond holdings is sufficient to pin down a unique steady state and ensure stationarity of the model.
tomorrow’s capital input.\footnote{Justiniano, Primiceri, and Tambalotti (2010) suggests that this variation might stem from technological factors specific to the production of investment goods, but also from disturbances to the process by which these investment goods are turned into productive capital.}

Household’s income (the sum of rental capital, labor income, and rebated profits) is taxed at the rate $\tau_I^t$. Moreover, the household pays consumption taxes $\tau_C^t$. The household’s period budget constraint is:

$$B_{j,t+1} + A_{j,t+1}^I + \epsilon_t A_{j,t+1}^r + \frac{\psi}{2} \epsilon_t P_t^r \left( \frac{A_{j+1}^r}{P_t^r} \right)^2 + P_t^C (1 + \tau_C^t) + P_t I_{j,K,t} + \Psi (u_{j,K,t}^I) \hat{K}_t = (1 + i_t) B_{j,t} + (1 + i_t^* A_{j,t}^r + (1 - \tau_I^t) \left( w_{jt} L_{jt} + P_t r_{K,t} K_{j,t}^r + P_t T_{d,t}^j \right) + P_t \left( T_{A,t}^j + T_{C,t}^j \right),$$

where $i_t$ and $i_t^*$ are, respectively, the nominal interest rates on Home and Foreign private bond holdings between $t$ and $t+1$, known with certainty as of $t-1$. $T_{A,t}^j$ denotes a sector lump-sum rebate of the cost of adjusting bond holdings, $T_{d,t}^j$ is a lump-sum rebate of profits from final producers, and $T_{C,t}^j$ is a lump-sum transfer from the government.

The household maximizes its expected intertemporal utility subject to (6) and (7). To simplify notation, we appeal to symmetry in household preferences and omit the index $j$ henceforth.

Optimal labor supply implies: $(1 - \tau_I^t) w_t = \beta_t h_t L_t^C$, where $w_t \equiv w_{jt}^r / P_t$. The Euler equation for capital accumulation requires $\zeta_{K,t} = E_t \left\{ \beta_{t,t+1} \left[ r_{K,t+1} u_{K,t+1} (1 - \tau_I^t) - \Psi (u_{j,K,t+1}) + (1 - \delta_{t+1}) \zeta_{K,t+1} \right] \right\}$, where $\zeta_{K,t}$ denotes the shadow value of capital (in units of consumption), defined by the first-order condition for investment $I_{K,t}$:

$$\zeta_{K,t}^{-1} = \left[ 1 - \frac{\nu_K}{2} \left( \frac{I_{K,t}^r}{I_{K,t-1}^r} - 1 \right)^2 - \nu_K \left( \frac{I_{K,t}^r}{I_{K,t-1}^r} - 1 \right) \left( \frac{I_{K,t}^r}{I_{K,t-1}^r} \right)^2 \right] + \nu_K \beta_{t,t+1} E_t \left[ \frac{\zeta_{K,t+1}}{\zeta_{K,t}} \left( \frac{I_{K,t+1}}{I_{K,t}} \right)^r - 1 \right] \left( \frac{I_{K,t+1}}{I_{K,t}} \right)^2 .$$

The optimal condition for capital utilization implies: $(1 - \tau_I^t) r_{K,t} = \Psi (u_{j,K,t}^I)$.

Let $a_{t+1} \equiv A_{t+1}/P_t$ denote real holdings of Home bonds (in units of Home consumption) and let $a_{t+1}^* \equiv A_{t+1}^r/P_t^r$ denote real holdings of Foreign bonds (in units of Foreign consumption). The Euler equations for bond holdings are:

$$1 + \psi a_{t+1} = (1 + i_{t+1}) E_t \left( \frac{\beta_{t,t+1}}{1 + \pi_{C,t+1}} \right),$$

where $\beta_{t,t+1}$ is the discount rate.
\[
\frac{1 + \psi a_{t+1}^*}{\Lambda_{at}} = (1 + i_{t+1}^*) E_t \left( \frac{\beta_{t,t+1} Q_{t+1}}{1 + \pi_{C,t+1} Q_t} \right),
\]
where the term $\Lambda_{at}$ captures a risk-premium shock that is effectively a shock to the uncovered interest rate parity (adjusted for the presence of bond adjustment costs). We follow Smets and Wouters (2007) and subsequent literature in specifying this shock as an exogenous term appended to the Euler equations for bonds.\(^{17}\) We assume that $\tilde{\Lambda}_{at}$ follows a zero-mean autoregressive process:
\[
\tilde{\Lambda}_{at} = \rho_{\tilde{\Lambda}a} \tilde{\Lambda}_{a, t-1} + \varepsilon_{\tilde{\Lambda}a, t}, \text{ where } \varepsilon_{\tilde{\Lambda}a, t} \sim N \left( 0, \sigma^2_{\tilde{\Lambda}a} \right).
\]

**Production**

In each country, there are two vertically integrated production stages. At the upstream level, perfectly competitive firms use capital and labor to produce a non-tradable intermediate input. At the downstream level, monopolistically competitive firms use the intermediate input to produce tradable final consumption goods.

**Homogeneous Intermediate Input Production**

There is a unit mass of perfectly competitive intermediate producers. The representative intermediate firm produces output $Y^I_t = K_t^\alpha (\bar{Z}_t L_t)^{1-\alpha}$, where $\bar{Z}_t$ is exogenous productivity and $K_t$ is physical capital. $\bar{Z}_t$ and $\bar{Z}^*_t$ are non-stationary and cointegrated stochastic processes.\(^{18}\) The growth rate of Foreign productivity $\bar{z}^*_t \equiv \bar{Z}^*_t / \bar{Z}_{t-1}$ follows
\[
\log \bar{z}^*_{t} = \rho_{\bar{z}^*} \log \bar{z}^*_{t-1} + (1 - \rho_{\bar{z}^*}) \bar{z}^* + \varepsilon^{\bar{z}^*}_t, \text{ where } \varepsilon^{\bar{z}^*}_t \sim N \left( 0, \sigma^2_{\bar{z}^*} \right).
\]

Home productivity $\bar{Z}_t$ features the same stochastic trend up to a stationary stochastic disturbance $\tilde{\zeta}_t$: $\bar{Z}_t / \bar{Z}^*_t = \tilde{\zeta}_t$, where $\log \tilde{\zeta}_t = \rho_{\tilde{\zeta}} \log \tilde{\zeta}_{t-1} + \varepsilon^{\tilde{\zeta}}_t$ and $\varepsilon^{\tilde{\zeta}}_t \sim N \left( 0, \sigma^2_{\tilde{\zeta}} \right)$.

As a result, the growth rate of Home productivity $\bar{z}_t \equiv \bar{Z}_t / \bar{Z}_{t-1}$ evolves according to
\[
\log \bar{z}_t = \log \bar{z}^*_t + \log \tilde{\zeta}_{t-1} - \log \tilde{\zeta}_t.
\]

Let $\varphi_t$ be the real price (in units of final consumption) of the intermediate input. The Home firm chooses $L_t$ and $K_t$ to maximize the value of per-period profit: $\varphi_t Y^I_t - \left( w^n_t / P_t \right) L_t - r_{K,t} K_t$, where $w^n_t$ is the nominal wage and $r_{K,t}$ is the real rental rate of capital. The first-order conditions for $L_t$ equates the value of the marginal product of labor to the real wage: $(1 - \alpha) \varphi_t Y^I_t / L_t = w^n_t / P_t$.

The first-order conditions for capital yields $\alpha \varphi_t Y^I_t / K_t = r_{K,t}$.

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\(^{17}\)Following Fisher (2015), the shock $\tilde{\Lambda}_{at}$ can also be interpreted as a structural shock to the demand for safe and liquid Foreign assets, i.e., $\tilde{\Lambda}_{at}$ captures, in reduced form, stochastic fluctuations in household’s preferences for holding one-period nominally risk-free Foreign assets.

\(^{18}\)Rabanal and Rubio-Ramirez (2015) and Rabanal, Rubio-Ramirez, and Tuesta (2011) show this specification for TFP improves a business-cycle model’s ability to match properties of real exchange rate data.
**Final Producers**

There is a continuum of symmetric firms that produce tradable consumption varieties indexed by \( i \in (0, 1) \). Domestic demand for producer \( i \) is \( Y_{D,t}(i) = (P_{D,t}(i)/P_{D,t})^{-\theta} Y_{D,t} \), while export demand is \( Y_{X,t} = (P_{X,t}/P_{X,t})^{-\theta} Y_{X,t} \), where \( P_{X,t} \) denotes the price (in Foreign currency) of the exported variety. The terms \( Y_{D,t} \) and \( Y_{X,t} \) denote, respectively, domestic and export aggregate demand for the Home basket.

We introduce price-setting frictions following Rotemberg (1982) and assume that final producers must pay a quadratic adjustment cost when changing domestic and export prices. Consistent with recent empirical evidence, we assume dollar currency pricing: both export and import prices in the small open economy are denominated in Foreign currency (U.S. dollars). We also allow for price indexation by assuming that final producers index price changes to past CPI inflation in the destination market, \( \pi_{C,t} \equiv P_t/P_{t-1} - 1 \) and \( \pi_{C,t}^* \equiv P_t^*/P_{t-1}^* - 1 \), respectively. Assuming zero trend inflation, the nominal cost of adjusting the domestic price is

\[
\frac{\nu_p}{2} \left[ \frac{P_{D,t}}{P_{D,t-1}} (1 + \pi_{C,t-1})^{-\tau_p} - 1 \right]^2 P_{D,t} Y_{D,t}^i,
\]

where \( \nu_p \geq 0 \) determines the size of the adjustment costs (domestic prices are flexible if \( \nu_p = 0 \)). The nominal cost (in units of Home currency) of adjusting the export price is

\[
\frac{\nu_T}{2} \left[ \frac{P_{X,t}}{P_{X,t-1}} (1 + \pi_{C,t-1}^*)^{-\tau_p} - 1 \right]^2 \varepsilon_t P_{X,t} Y_{X,t}^i,
\]

where \( \varepsilon_t \) is the nominal exchange rate (units of Home currency per unit of Foreign currency).

The representative producer chooses \( P_{D,t}^i \) and \( P_{X,t}^i \) in order to maximize the expected present discounted value of the stream of real profits \( E_t \sum_{s=1}^{\infty} \beta_{s,t} d_t^s \), where \( \beta_{s,t+1} \) denotes the households’ stochastic discount factor and

\[
d_t^i = \left\{ \left[ 1 - \frac{\nu_p}{2} \left( \frac{P_{D,t}}{P_{D,t-1}} (1 + \pi_{C,t-1})^{-\tau_p} - 1 \right)^2 \right] P_{D,t}^i - \varphi_t \right\} Y_{D,t}^i + \left\{ \left[ 1 - \frac{\nu_T}{2} \left( \frac{P_{X,t}}{P_{X,t-1}} (1 + \pi_{C,t-1}^*)^{-\tau_p} - 1 \right)^2 \right] \varepsilon_t P_{X,t}^i - (1 + \tau_t) \varphi_t \right\} Y_{X,t}^i,
\]

where \( \tau_t \geq 0 \) is an iceberg trade cost. In the symmetric equilibrium, the real price of Home output for domestic sales, \( \rho_{D,t} \equiv P_{D,t}/P_t \), is a time-varying markup \( \mu_{D,t} \) over the marginal cost \( \varphi_t \):
\( \rho_{D,t} = \mu_{D,t} \phi_t \). The time-varying domestic markup, \( \mu_{D,t} \), is given by:

\[
\mu_{D,t} = \frac{\bar{\theta}_t}{(\bar{\theta}_t - 1) \left(1 - \frac{\nu_T}{2} \Delta^2_{D,t}\right) + \nu_T \left\{ \frac{(1 + \pi_{D,t}) \Delta_{D,t} (1 + \pi_{C,t-1})^{-\tau_{ip}}}{\beta_{t+1} (1 + \pi_{D,t+1}) \Delta_{D,t+1} (1 + \pi_{C,t})^{-\tau_{ip}} \frac{Y_{D,t+1}}{Y_{D,t}}} \right\}}, \tag{8}
\]

where \( \pi_{D,t} = P_{D,t}/P_{D,t-1} - 1 \) and \( \Delta_{D,t} \equiv (1 + \pi_{D,t}) (1 + \pi_{C,t-1})^{-\tau_{ip}} - 1 \). The real price of Home output for export sales \( \rho_{X,t} \equiv P_{X,t}/P_{t}^* \) (in units of Home consumption) is a time-varying markup \( \mu_{X,t} \) over the marginal cost of the export bundle \( \phi_t \): \( \rho_{X,t} = \mu_{X,t} (1 + \tau_t) \phi_t/Q_t \), where \( Q_t \) denotes the real exchange rate (units of Home consumption per unit of Foreign consumption). The time-varying export markup, \( \mu_{X,t} \), is given by:

\[
\mu_{X,t} = \frac{\bar{\theta}_t}{(\bar{\theta}_t - 1) \left[1 - \frac{\nu_T}{2} \Delta^2_{X,t}\right] + \nu_T \left\{ \frac{(1 + \pi_{X,t}) \Delta_{X,t} (1 + \pi_{C,t-1})^{-\tau_{ip}}}{\beta_{t+1} (1 + \pi_{X,t+1}) \Delta_{X,t+1} (1 + \pi_{C,t})^{-\tau_{ip}} \frac{Y_{X,t+1}}{Y_{X,t}}} \right\}}, \tag{9}
\]

where \( \pi_{X,t} = P_{X,t}/P_{X,t-1} - 1 \) and \( \Delta_{X,t} \equiv (1 + \pi_{X,t}) (1 + \pi_{C,t-1})^{-\tau_{ip}} - 1 \). As expected, price stickiness introduces endogenous markup variations both in the domestic and export markets.

**Monetary and Fiscal Policy**

The monetary authority follows a Taylor-type rule, in which the nominal interest rate responds to its lagged value, deviations of CPI inflation, and GDP, from their long-run targets. We denote a variable in percentage deviations from steady state by a hat. The interest rate obeys

\[
\frac{\hat{i}_t}{1 + i} = \vartheta_i \hat{i}_{t-1}/1 + i + (1 - \vartheta_i) \left[ \vartheta_\pi \hat{\pi}_{C,t} + \vartheta_Y \hat{Y}_t \right], \tag{10}
\]

where \( 1 + i \) is the steady state of the gross nominal interest rate. GDP is defined as \( Y_t \equiv C_t + \rho_{I_K,t} I_{K,t} + \rho_{G,t} G_t + TB_t \) where \( \rho_{I_K,t} \equiv P^{I_K}_t/P_t \) and \( \rho_{G,t} \equiv P^G_t/P_t \).

The government collects tax revenues from capital, labor, profits, and consumption taxes, and sells the nominal bond portfolio, \( B_t \), to finance interest rate payments to finance its interest payments and expenditures, \( G_t \) and \( T_{G,t} \).

As in the one-period model of section 2, government consumption \( G_t \) aggregates Home and
Foreign government consumption sub-baskets. The government consumption sub-baskets $G_{D,t}$ and $G_{X,t}$ have the same composition of the private consumption sub-baskets, i.e., $G_{D,t} = \int_0^1 G_{D,t}(i)(\bar{\theta}_t - 1)/\bar{\theta}_t di$ and $G_{X,t} = \int_0^1 G_{X,t}(i)(\bar{\theta}_t - 1)/\bar{\theta}_t di$. The price index for the government consumption basket is therefore:

$$P_t^G = \left[ (1 - \alpha_X^g) (P_{D,t})^{1-\phi_t} + \alpha_X^g (P_{X,t})^{1-\phi_t} \right]^{1-\sigma_t}.$$

Fiscal choices satisfy the government’s per-period budget constraint:

$$B_{t+1} + \tau_t^l (r_{K,t} P_t K_t + w_t^n L_t + P_t T_{d,t}) + P_t \tau_t^c C_t = B_t + P_G t G_t + P_t T_{G,t}.$$

Fiscal rules dictate the evolution of policy instruments, $X = \{G, T_G, \tau^K, \tau^L\}$, and include an autoregressive term to allow for serial correlation and a response to the market value of the debt-to-GDP ratio $S_t \equiv B_t / (P_t Y_t)$—to ensure that policies stabilize debt. Specifically,

$$\dot{X}_t = \varrho_X \dot{X}_{t-1} - (1 - \varrho_{T_X}) \gamma_X \dot{X}_{t-1} + \varepsilon_{X,t},$$

where $\varepsilon_X \sim iid N(0, \sigma_X^2)$.

**Equilibrium**

Total domestic demand for the Home output basket, $Y_{D,t}$, is the sum of private-sector and government demand:

$$Y_{D,t} = \left[ 1 - \frac{\nu}{2} (\Delta_{D,t})^2 \right]^{-1} \left[ (1 - \alpha_X) \rho_{D,t}^{\gamma_\phi} C_t + \left( 1 - \alpha_X^{I_X} \right) \left( \frac{\rho_{D,t}}{\rho_{K,t}} \right)^{-\phi_t} I_{K,t} + (1 - \alpha_X^g) \left( \frac{\rho_{D,t}}{\rho_{G,t}} \right)^{-\phi_t} G_t \right],$$

Similarly, total export demand is the sum of private-sector and government Foreign demand:

$$Y_{X,t} = \left[ 1 - \frac{\nu}{2} (\Delta_{X,t})^2 \right]^{-1} \left[ \alpha_X \rho_{X,t}^{\gamma_\phi} C_t^{*} + \alpha_X^{I_X} \left( \frac{\rho_{X,t}}{\rho_{K,t}} \right)^{-\phi_t} I_{K,t}^{*} + \alpha_X^g \left( \frac{\rho_{X,t}}{\rho_{G,t}} \right)^{-\phi_t} G_t^{*} \right].$$

Goods market clearing requires $K_t^{\alpha} \left( Z_t L_t \right)^{1-\alpha} = Y_{D,t} + Y_{X,t}$. The price index of private consumption implies: $1 = (1 - \alpha_X) \rho_{D,t}^{1-\phi_t} + \alpha_X \rho_{X,t}^{1-\phi_t}$. Similarly, the price index of the investment good satisfies $\rho_{I_{K,t}}^{1-\phi_t} = (1 - \alpha_X^{I_X}) \rho_{D,t}^{1-\phi_t} + \alpha_X^{I_X} \rho_{X,t}^{1-\phi_t}$, while the price index of government consumption satisfies $\rho_{G,t}^{1-\phi_t} = (1 - \alpha_X^g) \rho_{D,t}^{1-\phi_t} + \alpha_X^g \rho_{X,t}^{1-\phi_t}$. The private-sector lump-sum transfers are $T_{A,t} = (\psi/2) Q_t (a_{s,t+1})^2$. Finally, private Home bonds are in zero net supply, which implies the
equilibrium condition \( a_{t+1} = a_t = 0 \) in all periods. Home net foreign assets are determined by:

\[
Q_t a_{*,t+1} = Q_t \frac{1 + i_t^*}{1 + \pi_{C,t}^*} a_{*,t} + T B_t,
\]

where \( T B_t \equiv Q_t \rho_{X,t} Y_{X,t} - \rho_{X,t}^* Y_{X,t}^* \) is the trade balance. The change in net foreign assets between \( t \) and \( t+1 \) is determined by the current account:

\[
Q_t (a_{*,t+1} - a_{*,t}) = C A_t \equiv Q_t r_t^* a_{*,t} + T B_t,
\]

where the foreign real interest rate \( r_t^* \) is defined by

\[
1 + r_t^* = \frac{(1 + i_t^*)}{(1 + \pi_{C,t}^*)}
\]

Foreign Economy and Model Solution

Eight Foreign variables directly affect macroeconomic dynamics in the small open economy: \( C_t^*, i_{t+1}^*, \pi_{C,t}^*, I_{K,t}^*, \rho_{X,t}^*, Y_{X,t}^* \). Foreign consumption, \( C_t^* \), government consumption \( G_t^* \), the nominal interest rate, \( i_{t+1}^* \), and inflation, \( \pi_{C,t}^* \), are determined by treating the rest of the world (Foreign) as a closed economy that features the same production structure, technology, and frictions that characterize the small open economy. To determine price and quantities related to Foreign exports and imports, we assume that Foreign producers solve a profit maximization problem that is equivalent to that faced by Home producers. See Appendix C for details.

We rewrite the model in terms of detrended variables and compute the log-linear approximation around the non-stochastic steady state. We solve the resulting linear system of rational expectation equations to obtain the model transition equation, which is linked to data with an observation equation to form the state-space model used for estimation.

4 Assessing Trade Transmissions

Model Predictions

Although the frictions and detailed structure of the quantitative model help to match properties of the data, its large size prevents analytical study. To get a sense of the range of model predictions, we employ a prior-predictive analysis in the spirit of Geweke (2010, Chapter 3). We propose independent prior density functions for structural parameters, which define the range and probability of values for each parameter. Table 1 lists our priors, which are similar to those employed for Bayesian estimation of structural models (e.g. Lubik and Schorfheide, 2005, Forni, Monteforte, and Sessa, 2009, Justiniano and Preston, 2010, Leeper, Plante, and Traum, 2010, and Drautzburg and Uhlig, 2015). We set the price stickiness parameter, \( \nu \), to a value that would replicate the frequency of
price adjustment in a Calvo-type Phillips curve in the absence of strategic price complementarities. For comparability with the literature, we directly estimate the related Calvo parameter \( \xi \).\(^{19}\) Notably, we adopt a uniform distribution for the elasticity of substitution between Home and Foreign goods (\( \phi \)) over the interval 0.05 to 3. We do not have strong prior beliefs about whether in the aggregate this elasticity of substitution is small or large. Adopting a uniform prior ensures the data inform the parameter’s estimate. Likewise, we adopt a uniform prior for the elasticity of substitution between public and private goods (\( \omega_G \)), as the literature has shown government spending responses to be sensitive to its value (e.g., Fève, Matheron, and Sahuc (2013) and Leeper, Traum, and Walker (2017)).

In addition, we impose dogmatic priors for a few parameters. We set the discount factor \( \beta \) equal to 0.99, the share of capital in the Cobb-Douglas production function of the upstream intermediate sector, \( \alpha \) equal to 0.33, and the capital depreciation rate \( \delta_K \) equal to 0.025. We set the elasticity of substitution of tradable varieties \( \theta \) equal to 6 to generate a 20 percent markup in steady state. We calibrate the size of the Home tradable sector, \( \alpha_X \), to match a steady-state trade-to-GDP ratio of 35 percent, consistent with bilateral trade data between Canada and the U.S. We set the home bias in government consumption \( \alpha^q_X \) equal to 0.1, as computed by Dion, Laurence, and Zheng (2005). Finally, we set iceberg trade costs, \( \tau \) and \( \tau^* \), equal to 0, reflecting the absence of tariff barriers between Canada and the U.S.

We take 20,000 draws from our priors and calculate model-implied impulse response functions and present-value multipliers, defined as:

\[
\text{Present Value Multiplier}(k) = \frac{E_t \sum_{j=0}^{k} \left( \prod_{i=0}^{k} (1 + r)^{-1} \right) \Delta X_{t+j}}{E_t \sum_{j=0}^{k} \left( \prod_{i=0}^{k} (1 + r)^{-1} \right) \Delta F_{t+j}}
\]

where \( r \) is the steady-state real interest rate, \( X \) stands for output, consumption, or investment and \( F \) denotes government spending or income tax revenue.

Given our priors, the prior predictive analysis produces the entire range of model outcomes possible before confronting the data. We first examine multipliers when government spending is neither a substitute nor a complement to private consumption (\( \omega_g = 0 \)), for comparability to the one-period model.\(^{20}\) Figure 2 plots 90-percent probability bands for Home government spending multipliers in this case under various assumptions about \( \alpha_X \) and \( \alpha^q_X \). In both panels, diamond and

\(^{19}\) \( \xi \) is related to \( \nu \) via the mapping \( \nu = [(\theta - 1)/\bar{\theta}] \xi/(1 - \xi)/(1 - \xi\beta) \).

\(^{20}\) As illustrated below, complementarity ensures that higher spending initially raises consumption even though long-run real interest rates also rise.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior</th>
<th>Posterior</th>
<th>90% Int</th>
<th>90% Int</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preferences</strong></td>
<td></td>
<td></td>
<td>Canada</td>
<td>U.S.</td>
</tr>
<tr>
<td>h&lt;sub&gt;c&lt;/sub&gt;, habit formation</td>
<td>B   0.5 0.1</td>
<td>0.46 [0.38, 0.54]</td>
<td>0.56 [0.47, 0.64]</td>
<td></td>
</tr>
<tr>
<td>ω, inverse Frisch</td>
<td>G   2 0.5</td>
<td>1.92 [1.43, 2.47]</td>
<td>1.64 [1.11, 2.19]</td>
<td></td>
</tr>
<tr>
<td>ω&lt;sub&gt;G&lt;/sub&gt;, substitutability of private/public cons.</td>
<td>U   0 1.36</td>
<td>-0.26 [-0.49, -0.02]</td>
<td>-0.73 [-1.10, -0.38]</td>
<td></td>
</tr>
<tr>
<td>φ, substitutability of home/foreign</td>
<td>U   1.52 0.85</td>
<td>1.12 [0.96, 1.30]</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Frictions and Production</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100 log z, growth rate</td>
<td>N   0.45 0.03</td>
<td>0.45 [0.40, 0.50]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ&lt;sub&gt;K&lt;/sub&gt;, investment adj. cost</td>
<td>N   4 1.5</td>
<td>5.56 [3.83, 7.36]</td>
<td>3.35 [2.01, 4.99]</td>
<td></td>
</tr>
<tr>
<td>ω&lt;sub&gt;G&lt;/sub&gt;, substitutability of capital utilization</td>
<td>B   0.5 0.2</td>
<td>0.40 [0.23, 0.60]</td>
<td>0.53 [0.31, 0.74]</td>
<td></td>
</tr>
<tr>
<td>ξ, Calvo price stickiness</td>
<td>B   0.5 0.2</td>
<td>0.42 [0.34, 0.56]</td>
<td>0.79 [0.74, 0.84]</td>
<td></td>
</tr>
<tr>
<td>ι&lt;sub&gt;p&lt;/sub&gt;, price partial indexation</td>
<td>B   0.5 0.16</td>
<td>0.46 [0.24, 0.69]</td>
<td>0.19 [0.08, 0.34]</td>
<td></td>
</tr>
<tr>
<td><strong>Monetary Policy</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>̺&lt;sub&gt;i&lt;/sub&gt;, resp. to lagged interest rate</td>
<td>B   0.75 0.1</td>
<td>0.71 [0.64, 0.78]</td>
<td>0.68 [0.60, 0.74]</td>
<td></td>
</tr>
<tr>
<td>̺&lt;sub&gt;i&lt;/sub&gt;, interest resp. to inflation</td>
<td>N   1.7 0.3</td>
<td>2.32 [2.00, 2.66]</td>
<td>2.03 [1.73, 2.37]</td>
<td></td>
</tr>
<tr>
<td>̺&lt;sub&gt;i&lt;/sub&gt;, interest resp. to Y</td>
<td>G   0.15 0.1</td>
<td>0.01 [0.00, 0.03]</td>
<td>0.03 [0.01, 0.05]</td>
<td></td>
</tr>
<tr>
<td><strong>Fiscal Policy</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>γ&lt;sub&gt;G&lt;/sub&gt;, debt response for G</td>
<td>N   0.3 0.1</td>
<td>0.36 [0.28, 0.45]</td>
<td>0.24 [0.12, 0.37]</td>
<td></td>
</tr>
<tr>
<td>γ&lt;sub&gt;T&lt;/sub&gt;, debt response for τ&lt;sub&gt;i&lt;/sub&gt;</td>
<td>N   0.3 0.1</td>
<td>0.29 [0.15, 0.43]</td>
<td>0.25 [0.10, 0.41]</td>
<td></td>
</tr>
<tr>
<td>γ&lt;sub&gt;TG&lt;/sub&gt;, debt response for τ&lt;sub&gt;G&lt;/sub&gt;</td>
<td>N   0.3 0.1</td>
<td>0.12 [-0.00, 0.30]</td>
<td>0.22 [0.10, 0.35]</td>
<td></td>
</tr>
<tr>
<td>̺&lt;sub&gt;G&lt;/sub&gt;, lagged response for G</td>
<td>B   0.8 0.1</td>
<td>0.88 [0.83, 0.92]</td>
<td>0.88 [0.80, 0.95]</td>
<td></td>
</tr>
<tr>
<td>̺&lt;sub&gt;G&lt;/sub&gt;, lagged response for τ&lt;sub&gt;i&lt;/sub&gt;</td>
<td>B   0.8 0.1</td>
<td>0.89 [0.78, 0.96]</td>
<td>0.88 [0.76, 0.96]</td>
<td></td>
</tr>
<tr>
<td>̺&lt;sub&gt;G&lt;/sub&gt;, lagged response for τ&lt;sub&gt;G&lt;/sub&gt;</td>
<td>B   0.5 0.2</td>
<td>0.27 [0.11, 0.43]</td>
<td>0.14 [0.04, 0.26]</td>
<td></td>
</tr>
<tr>
<td><strong>Shock Processes</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ&lt;sub&gt;Λ&lt;/sub&gt;, risk premium</td>
<td>B   0.5 0.2</td>
<td>0.86 [0.81, 0.91]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ&lt;sub&gt;θ&lt;/sub&gt;, price mark-up</td>
<td>B   0.5 0.2</td>
<td>0.92 [0.87, 0.96]</td>
<td>0.95 [0.89, 0.99]</td>
<td></td>
</tr>
<tr>
<td>ρ&lt;sub&gt;h&lt;/sub&gt;, hours supply</td>
<td>B   0.5 0.2</td>
<td>0.86 [0.78, 0.92]</td>
<td>0.84 [0.76, 0.90]</td>
<td></td>
</tr>
<tr>
<td>ρ&lt;sub&gt;φ&lt;/sub&gt;, subst home/foreign</td>
<td>B   0.5 0.2</td>
<td>0.59 [0.21, 0.97]</td>
<td>0.97 [0.92, 0.99]</td>
<td></td>
</tr>
<tr>
<td>ρ&lt;sub&gt;ζ&lt;/sub&gt;, subtotal preference</td>
<td>B   0.5 0.2</td>
<td>0.99 [0.98, 0.99]</td>
<td>0.95 [0.90, 0.99]</td>
<td></td>
</tr>
<tr>
<td>ρ&lt;sub&gt;PK&lt;/sub&gt;, investment</td>
<td>B   0.5 0.2</td>
<td>0.32 [0.12, 0.33]</td>
<td>0.96 [0.84, 0.98]</td>
<td></td>
</tr>
<tr>
<td>100σ&lt;sub&gt;Λ&lt;/sub&gt;, risk premium</td>
<td>IG  1 1</td>
<td>0.28 [0.22, 0.35]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100σ&lt;sub&gt;θ&lt;/sub&gt;, price mark-up</td>
<td>IG  1 1</td>
<td>2.08 [1.65, 2.60]</td>
<td>1.71 [1.27, 2.24]</td>
<td></td>
</tr>
<tr>
<td>100σ&lt;sub&gt;h&lt;/sub&gt;, hours supply</td>
<td>IG  1 1</td>
<td>1.42 [1.15, 1.75]</td>
<td>1.20 [0.95, 1.49]</td>
<td></td>
</tr>
<tr>
<td>100σ&lt;sub&gt;φ&lt;/sub&gt;, subtotal preference</td>
<td>IG  1 1</td>
<td>0.66 [0.34, 1.31]</td>
<td>1.44 [0.97, 1.90]</td>
<td></td>
</tr>
<tr>
<td>100σ&lt;sub&gt;ζ&lt;/sub&gt;, subtotal preference</td>
<td>IG  1 1</td>
<td>3.40 [2.89, 3.99]</td>
<td>2.74 [2.40, 3.14]</td>
<td></td>
</tr>
<tr>
<td>100σ&lt;sub&gt;PK&lt;/sub&gt;, investment</td>
<td>IG  1 1</td>
<td>0.70 [0.60, 0.81]</td>
<td>1.39 [1.21, 1.61]</td>
<td></td>
</tr>
<tr>
<td>100σ&lt;sub&gt;i&lt;/sub&gt;, monetary policy</td>
<td>IG  1 1</td>
<td>0.85 [0.68, 1.03]</td>
<td>0.60 [0.48, 0.74]</td>
<td></td>
</tr>
<tr>
<td>100σ&lt;sub&gt;G&lt;/sub&gt;, gsp spending</td>
<td>IG  1 1</td>
<td>0.35 [0.29, 0.43]</td>
<td>0.20 [0.18, 0.24]</td>
<td></td>
</tr>
<tr>
<td>100σ&lt;sub&gt;C&lt;/sub&gt;, income tax</td>
<td>IG  1 1</td>
<td>1.17 [1.01, 1.35]</td>
<td>0.75 [0.65, 0.87]</td>
<td></td>
</tr>
<tr>
<td>100σ&lt;sub&gt;C&lt;/sub&gt;, income tax</td>
<td>IG  1 1</td>
<td>1.83 [1.60, 2.08]</td>
<td>0.73 [0.66, 0.84]</td>
<td></td>
</tr>
<tr>
<td>100σ&lt;sub&gt;τG&lt;/sub&gt;, transfer</td>
<td>IG  1 1</td>
<td>1.86 [1.61, 2.12]</td>
<td>1.70 [1.49, 1.96]</td>
<td></td>
</tr>
</tbody>
</table>

*Distributions: N: Normal; G: Gamma; B: Beta; U: Uniform; IG: Inverse Gamma.
Figure 2. Present-value home (Canada) government spending multipliers at various horizons with 90-percent probability bands. In all cases, government spending is neither a substitute nor complement for private consumption.
Figure 3. Present-value home (Canada) income tax multipliers at various horizons with 90-percent probability bands.

Square lines refer to the open economy scenarios. Star lines represent results for a closed economy ($\alpha_X = \alpha^g_X = 0$). We distinguish two open economy scenarios. The top panel assumes full home bias in government consumption ($\alpha^g_X = 0$) while setting $\alpha_X$ to generate the observed trade-to-GDP ratio in Canada. The bottom panel considers the case where trade involves both private and public goods, setting $\alpha_X$ and $\alpha^g_X$ to generate the empirical targets for the trade-to-GDP ratio and the share of imported government consumption.

The intuition from the one-period model is preserved: multipliers are larger when the difference between the import content of private and public consumption increases (ultimately determined by $\alpha_X - \alpha^g_X$). For instance, when $\alpha^g_X = 0$, consumption multipliers range from $[-0.73, -0.35]$ after 20 periods in the open economy with a 35% trade share, whereas they range from $[-0.81, -0.43]$ in the closed economy. Differences for GDP are even larger. Comparing responses within a column, we see that multipliers also are decreasing in $\alpha^g_X$ conditional on a given $\alpha_X$.

Figure 3 plots 90-percent probability bands for home income tax multipliers. For brevity, we only report results for the scenario in which $\alpha_X$ and $\alpha^g_X$ are calibrated to reproduce the empirical trade targets. As in the one-period model, the absolute value of responses are higher in the closed economy. Again, the differences can be quantitatively significant; GDP multipliers range from $[-1.35, -0.18]$ after 20 periods in the open economy, whereas they range from $[-1.51, -0.26]$ in the closed economy.

Turning to our model for estimation, inclusive of government spending in the utility function ($\omega_G \neq 0$), figure 4 displays 90-percent probability bands for various domestic and international

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21Domestic multiplier range from zero to negative numbers because of the well-known result that imposing zero complementarity between public and private consumption (in the absence of financially constrained households or productive government spending) results in a large crowding out of private consumption in response to an increase in government spending.
variables following domestic and foreign government spending increases and tax decreases. The first row of the figure plots the domestic (Canadian) multipliers relative to a counterfactually closed economy, demonstrating that trade linkages can enhance or reduce multipliers a priori. The second row plots the values of the domestic multipliers, showing government spending multipliers can be positive or negative. Domestic multipliers following an income tax change are decisively negative, as tax cuts always imply increases in after-tax income. International spillovers from foreign shocks can be positive or negative for both shocks. As the figure makes clear, the model is fairly agnostic about the size and sign of key international variables following fiscal shocks. In most instances, the response of the terms of trade, real exchange rate and trade balance can be positive or negative, particularly following changes in government spending. Qualitative differences arise now due to 1) the wide range of values for the elasticity of substitution between Home and Foreign goods, as in the one-period model; 2) the degree of complementarity between public and private goods; and 3) various forms of fiscal financing, which Corsetti, Meier, and Muller (2012) notes affect real exchange rate responses. To discern which channels are most favored empirically, we turn to the estimation of the model.

**Estimation**

We estimate the model using quarterly Canadian and U.S. data over the period 1990-2007. For each country, we include a series of the log first-difference of aggregate consumption, investment, real government consumption, income tax revenue, consumption tax revenue, the real market-value of government debt, and the consumer price index, as well as log hours worked and a short-term interest rate. In addition, we include the log first-difference of the bilateral nominal exchange rate between Canada and the U.S., U.S. bilateral exports in goods to Canada, and U.S. bilateral imports in goods to Canada. Details of the data construction and linkage to observables are in Appendix D.

We use Bayesian methods, whereby the data are used to update our priors through the likelihood function, calculated using the Kalman filter. These updates give us draws from the posterior distribution. We take 1.5 million draws from the posterior distribution using the random walk

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22 Government spending multipliers have a large range of values due to the large range in the degree of complementarity/substitutability of public and private consumption. See Leeper, Traum, and Walker (2017) for a more thorough discussion of the domestic transmission in a closed economy.

23 Enders, Muller, and Scholl (2011) also show numerically this parameter can affect the sign of the real exchange rate following a government spending shock.

24 ? show deep habits also can alter the exchange rate response, while ? show a small open economy with incomplete international asset market and a debt elastic interest rate premium affects the range of exchange rate responses.
Figure 4. 90 prior percentile credible intervals for select present-value multipliers and impulse responses following various 1% fiscal shocks. X-axis measures quarters. Y-axis denotes present value multiplier values and percentage changes in the terms of trade, real exchange rate, and trade balance.
Metropolis-Hastings algorithm. For inference, we discard the first 500,000 draws and keep one out of 100 draws to obtain a sample of 10,000.\textsuperscript{25}

Table 1 displays the posterior mean and 90 percent credible sets for parameters. Posterior credible sets are tighter than the priors, and in line with estimates from the literature. Public and private goods are complements, as in Bouakez and Rebei (2007) and Fève, Matheron, and Sahuc (2013). We estimate a fairly high degree of complementarity between public and private consumption for the U.S., but the estimate is consistent with some estimates of Leeper, Traum, and Walker (2017).

Notably, the elasticity of substitution between Home and Foreign goods is tightly estimated to be slightly greater than one. Lubik and Schorfheide (2005) show estimates of this parameter are sensitive to its prior when bilateral exports and imports are not included as observables. In contrast, we find the data to be informative about this parameter once accounting for data on trade in goods.

Also of note, a nontrivial component of Canadian series’ variance shares are explained by U.S. shocks (most notably the TFP shock), which in turn helps the model account for the cross-correlation between Canadian and U.S. series. See appendix E for variance decompositions and correlograms. Justiniano and Preston (2010) stress the difficulty in reproducing these features of the data, while Adolfson, Laseen, Linde, and Villani (2005) also ascribe minimal Home variance shares to Foreign shocks. In addition, the posterior estimates imply a small role for the risk premium shock in explaining the variance of the growth rate in the nominal exchange rate.\textsuperscript{26}

**Posterior Analysis**

Figure 5 displays posterior 90-percent intervals for GDP present-value multipliers as well as impulse responses for the terms of trade, real exchange rate, and the trade balance following various fiscal shocks. Column one presents responses to a 1% increase in government spending in Canada. The increase in the demand for goods raises Canadian inflation (not pictured), leading to an increase in relative international prices and an improvement in the terms of trade. Since the Canadian monetary authority does not completely offset the increase in inflation on impact, the Canadian real

\textsuperscript{25}We set the step size to achieve an acceptance rate of 26 percent. Diagnostics to determine chain convergence include cumulative sum of the draws (CUMSUM) statistics and Geweke’s Separated Partial Means (GSPM) test.

\textsuperscript{26}Lubik and Schorfheide (2005) suggest the difficulty in accounting for exchange rate variation when abstracting from risk premium shocks.
interest rate also rises, leading to a modest appreciation in the real exchange rate.\footnote{Although the empirical literature tends to find an increase in government spending leads to a real depreciation of the exchange rate, for Canada there is no conclusive evidence. Canada is the only country for which Monacelli and Perotti (2010) find no conclusive linkage, while Bouakez, Chihi, and Normandin (2014) find the real exchange rate depreciates. Kim (2015) compares the effects for 19 OECD countries. The working paper version includes individual country estimates, showing the real exchange rate appreciates significantly following an increase in Canadian government spending.} The Canadian trade balance deteriorates significantly for one and a half years, as the government directly increases demand for imports and the improvement in the terms of trade leads to an increase in imports.

The same international transmission follows for a U.S. 1% increase in public expenditure (column 2). The Canadian terms of trade deteriorates due to the relative increase in U.S. prices and the Canadian trade balance improves (as more imports are consumed in the U.S.). In addition, the increase in U.S. demand for Canadian goods results in a positive spillover to Canadian GDP, as seen by the increase in the Canadian GDP multiplier. However, the increase in the present-value GDP multiplier quickly dissipates in Canada, as the deterioration in the Canadian terms of trade leads to a reduction in private domestic demand in Canada.

The international transmission of a 1% income tax cut differs substantially from the government...
spending increase. Column 3 plots responses following a reduction in the Canadian income tax. Lowering the income tax leads to lower equilibrium factor costs of production, leading Canadian inflation to decrease. This in turn implies lower relative prices in Canada and a reduction in Canadian real interest rates, which lead the Canadian terms of trade to deteriorate and real exchange rate to depreciate. Over time, the Canadian trade balance deteriorates as the increase in after-tax income strengthens private demand for both domestic and imported goods. The same dynamics follow from a 1% income tax cut in the U.S. (column 4). In this case, there is a persistent, positive co-movement between Canadian and U.S. GDP; negative present-value multipliers for both countries imply a tax cut generates an expansion in GDP. Since the U.S. income tax cut results in more favorable terms of trade for Canada, Canadian private domestic demand increases, boosting Canadian GDP.

Overall, our results suggest that there are larger domestic gains from expansionary government spending policies in an open economy, as they produce favorable terms of trade dynamics for the domestic economy relative to income tax cuts.\textsuperscript{28} Public spending increases imply larger positive impact spillovers than tax reductions, but they are more short-lived, as a trading partner benefits over time from more favorable terms of trade dynamics with tax cuts.

\textbf{Uncovering Key Transmissions}

To disentangle the importance of various margins for the model’s dynamics, we consider several counterfactual scenarios starting from the posterior mean estimates. Figure 6 displays Canadian GDP present-value multipliers following a 1% increase in Canadian government spending (top row) and U.S. government spending (bottom row). The first column of the first row plots Canadian GDP present-value multipliers for the open economy (solid line) and counterfactually closed economy (dashed line). Although on impact the expansionary effect is greater in the closed economy, an improvement in the terms of trade boosts domestic demand in the open economy, implying larger multipliers after a couple quarters.

Full home bias in government spending (second column, dotted line) implies larger multipliers than the closed economy counterpart (first column, dashed line) at all horizons, a result reminiscent

\textsuperscript{28} Basu and Kollmann (2013) show that increases in government investment can lead to a deterioration in the terms of trade, as its transmission is similar to an increase in productivity. Over our sample, government investment is 14% of total government purchases in Canada and 21% in the U.S.
of the one-period model. When $\alpha_G^X = 0$, there is greater impact demand for Canadian goods, strengthening the terms of trade improvement. As seen from the second row, full home bias in government goods weakens the Canadian spillover from a U.S. spending increase as well. To gauge the importance of the dynamics of the terms of trade, we consider a counterfactual where exporters receive a subsidy such that price adjustments leave the terms-of-trade unchanged (second column, starred line). Without the improvement in the terms of trade, present-value GDP multipliers are comparable to the closed-economy counterparts after 20 periods following a Canadian spending shock (top row). Without the terms of trade deterioration following a U.S. spending shock, the Canadian GDP multiplier is persistently positive (bottom row), reflecting the increase in demand for Canadian goods.

The last column of figure 6 examines how various domestic factors influence Canadian GDP multipliers. Without complementarity in public and private goods (squared line), multipliers are significantly lower, as private consumption is crowded out following an increase in public spending. With flexible prices (x-line), domestic prices increase more substantially, crowding out private demand and lowering GDP multipliers. When Ricardian equivalence holds and only lump-sum

\[29\text{We assume this subsidy is directly financed by a lump-sum tax that is non-distortionary.} \]
transfers finance debt (triangle line), multipliers are higher at longer horizons, as private demand is stimulated more without the increases in income and consumption taxes to finance debt.

Figure 7 displays Canadian GDP present-value multipliers for the same set of counterfactual scenarios following a Canadian income tax change (top row) and U.S. income tax change (bottom row). Multipliers are substantially more negative in the counterfactually closed economy at all horizons (dashed line), implying a tax cut results in a smaller expansion in the open economy (solid line), echoing the one-period model results. When the terms-of-trade are held constant (second column, starred line), the GDP multiplier is remarkably similar to the closed-economy counterpart at all horizons, showing that the international transmission of tax changes is predominately affected by the terms-of-trade movement (and its corresponding wealth effects). All other counterfactual
scenarios do not significantly alter present-value multipliers.

5 Endogenous Tradability and Firm Heterogeneity

Model Extension

Despite accounting well for the dynamics of aggregate trade flows, the model in the previous section ignores the role of firm-level dynamics. First, as extensively documented by the trade literature since Melitz (2003), the opening of trade induces within-industry reallocation of market shares from relatively less productive to relatively more productive producers. As a result, the strength of international trade affects the number of exporting plants and their average productivity. Second, for a given level of trade openness, the international propagation of fiscal shocks depends on the response of both trade margins—the number of exporters and the average quantity exported by a producer. In turn, extensive-margin dynamics affect endogenously the composition of goods that are traded, i.e., the relative size of the tradable sector.

In this section, we address these issues by incorporating a general equilibrium version of the Melitz (2003) trade model that features endogenous entry of heterogeneous producers into domestic and export markets. Our approach follows closely Ghironi and Melitz (2004) and Cacciatore and Ghironi (2014).

The only change in the model involves the production of tradable goods. We now assume there is a continuum of symmetric tradable sectors. In each sector $i$, there is a representative, monopolistically competitive firm that produces a bundle $Y^i_t$ of differentiated product varieties. Product varieties, indexed by $\omega$, are defined over a continuum $\Omega$: $Y^i_t = \int_0^\infty Y^i_t(\omega) (\theta_\omega - 1) / \theta_\omega d\omega$, where $\theta_\omega > 1$ denotes the symmetric elasticity of substitution across product varieties.\(^\text{30}\)

The number of products (or features) created and commercialized by the producer is endogenous. At each point in time, only a subset of varieties $\Omega_t \subset \Omega$ is actually available to consumers. To create a new product, the producer pays a sunk investment cost, $f_{E,t} = \bar{Z}_t f_E$, in units of intermediate input. We let the sunk entry cost drift with the level of technology to ensure balanced growth. Each product is produced by a plant, and plants produce with different technologies indexed by relative productivity $z$.\(^\text{31}\) To save notation, we identify a variety with the corresponding plant productivity $z$, omitting $\omega$. Upon product creation, the productivity level of the new plant $z$

\(^{30}\)Sectors (and sector-representative firms) are of measure zero relative to the aggregate size of the economy. $Y^i_t$ also can be interpreted as a bundle of product features that characterize the final product $i$.

\(^{31}\)Alternatively, we could decentralize product creation by assuming that monopolistically competitive firms produce product varieties (or features) that are sold to final producers, in this case interpreted as retailers. The two models are isomorphic. Details are available upon request.
is drawn from a common distribution \( G(z) \) with support on \([z_{\text{min}}, \infty)\). This relative productivity level remains fixed thereafter. Each plant uses intermediate inputs to produce its differentiated product variety, with real marginal cost \( \varphi_t(z) = \varphi_t/z \).

At time \( t \), the Home producer \( i \) commercializes \( N^i_{D,t} \) varieties and creates \( N^i_{E,t} \) new products that will be available for sale at time \( t + 1 \). New and incumbent plants can be hit by a “death” shock with probability \( \delta \in (0, 1) \) at the end of each period. The law of motion for the stock of producing plants is \( N^i_{D,t+1} = (1 - \delta) \left( N^i_{D,t} + N^i_{E,t} \right) \).

When serving the Foreign market, each producer faces per-unit iceberg trade costs, \( \tau_t > 1 \), and fixed export costs (in units of intermediate inputs) \( f_{X,t} = \bar{Z}_t \bar{f}_{X,t} \), where \( \log \bar{f}_{X,t} = \rho \bar{f}_{X,t-1} + \varepsilon_{f_{X,t}} \), where \( \varepsilon_{f_{X,t}} \sim \text{iid} N \left( 0, \sigma^2_{f_X} \right) \).\(^{32}\) Fixed export costs are paid for each exported variety. The total fixed cost is then \( N^i_{X,t} f_{X,t} \), where \( N^i_{X,t} \) denotes the number of product varieties exported to Foreign. Absent fixed export costs, each producer would find it optimal to sell all its product varieties in Home and Foreign. Fixed export costs imply that only varieties produced by plants with sufficiently high productivity (above a cutoff level \( z_{X,t} \), determined below) are exported.

Define two special “average” productivity levels (weighted by relative output shares): an average \( \bar{z}_D \) for all producing plants and an average \( \bar{z}_{X,t} \) for all plants that export:

\[
\bar{z}_D = \left[ \int_{z_{\text{min}}}^{\infty} z^{\theta_{\omega} - 1} dG(z) \right]^{\frac{1}{\theta_{\omega} - 1}}, \quad \bar{z}_{X,t}(i) = \left[ \frac{1}{1 - G\left( \bar{z}_{X,t}(i) \right)} \right] \left[ \int_{z_{X,t}(i)}^{\infty} z^{\theta_{\omega} - 1} dG(z) \right]^{\frac{1}{\theta_{\omega} - 1}}.
\]

The output bundles for domestic and export sale, and associated unit costs, are defined as follows:

\[
Y^i_{D,t} = \left[ \int_{z_{\text{min}}}^{\infty} Y^i_{D,t}(z) z^{\theta_{\omega} - 1} dG(z) \right]^{\frac{1}{\theta_{\omega} - 1}}, \quad Y^i_{X,t} = \left[ \int_{z_{X,t}}^{\infty} Y^i_{X,t}(z) z^{\theta_{\omega} - 1} dG(z) \right]^{\frac{1}{\theta_{\omega} - 1}}, \quad (12)
\]

\[
\varphi^i_{D,t} = \left[ \int_{z_{\text{min}}}^{\infty} (\varphi_t(z))^{1 - \theta_{\omega}} dG(z) \right]^{\frac{1}{1 - \theta_{\omega}}}, \quad \varphi^i_{X,t} = \left[ \int_{z_{X,t}}^{\infty} (\varphi_t(z))^{1 - \theta_{\omega}} dG(z) \right]^{\frac{1}{1 - \theta_{\omega}}} \quad (13)
\]

Assume that \( G(\cdot) \) is Pareto with shape parameter \( k_p > \theta_{\omega} - 1 \). As a result, \( \bar{z}_D = \alpha^{1/(\theta_{\omega} - 1)} z_{\text{min}} \) and \( \bar{z}_{X,t} = \alpha^{1/(\theta_{\omega} - 1)} \bar{z}_{X,t} \), where \( \alpha \equiv k_p / (k_p - \theta_{\omega} + 1) \). The share of exporting plants \( N^i_{X,t} \equiv

\(^{32}\)Empirical micro-level studies have documented the relevance of plant-level fixed export costs—see, for instance, Bernard and Jensen (2004). Although a substantial portion of fixed export costs are probably sunk upon market entry, we follow Ghironi and Melitz (2005) and do not model the sunk nature of these costs explicitly. We conjecture that introducing these costs would further enhance the persistence properties of the model. See Alessandria and Choi (2007) for a model with heterogeneous firms, sunk export costs and frictionless labor markets.
\[1 - G(z_{X,t}^i)\] \(N_{D,t}^i\) is given by:

\[N_{X,t}^i = \left(\frac{z_{\text{min}}}{z_{i,X,t}}\right)^{-k_p} \frac{k_p}{\alpha \omega - 1} N_{D,t}^i.\]  

(14)

Using equation (13), the real costs of producing the bundles \(Y_{D,t}^i\) and \(Y_{X,t}^i\) can be expressed as:

\[\varphi_{D,t}^i = N_{D,t}^i \frac{\varphi_t}{z_D}, \quad \varphi_{X,t}^i = N_{X,t}^i \frac{\varphi_t}{z_{i,X,t}}.\]  

(15)

We continue to assume that producers must pay quadratic price adjustment costs when changing domestic and export prices. The representative producer chooses \(P_{D,t}, P_{X,t}, N_{E,t}^i,\) and \(z_{X,t}^i\) in order to maximize the expected present discounted value of the stream of real profits, \(E_t \left[ \sum_{s=1}^{\infty} \beta_t \delta_{s,t} d_s \right] \), subject to the constraints (14), (15), and \(z_{i,X,t}^i = \frac{\alpha^{1/(\theta_s - 1)}}{\theta_s - 1} z_{i,X_t}^i\).\(^{33}\)

Per-period profits are now given by

\[d_t^i = \left\{ \begin{array}{ll}
1 - \frac{\nu_F}{2} \left( \frac{P_{D,t}^i}{P_{X,t}^i} \left( 1 + \pi_{C,t-1} \right)^{-\nu_C} - 1 \right)^2 & Y_{D,t}^i \\
1 - \frac{\nu_F}{2} \left( \frac{P_{X,t}^i}{P_{X,t-1}^i} \left( 1 + \pi_{C,t-1} \right)^{-\nu_C} - 1 \right)^2 & -\varphi_t \left( N_{i,E,t}^i Y_{E,t}^i + N_{i,X,t}^i Y_{X,t}^i \right)
\end{array} \right\} Y_{D,t}^i + Y_{X,t}^i \varphi_t \frac{f_{X,t}^i}{N_{X,t}^i} \tau_t = f_{X,t}^i \varphi_t.\]

The above conditions states that, at the optimum, the marginal revenue from adding a variety with productivity \(z_{X,t}^i\) to the export bundle has to be equal to the fixed cost. Thus, varieties produced by plants with productivity below \(z_{X,t}^i\) are distributed only in the domestic market. The composition of the traded bundle is endogenous and the set of exported products fluctuates over time with changes in the profitability of export.

The first-order condition with respect to \(N_{D,t+1}\) determines product creation:

\[\varphi_t f_{E,t} = E_t \left\{ (1 - \delta) \beta_{t+1} \frac{f_{X,t+1}}{N_{X,t+1}} \left( \frac{\varphi_{t+1} f_{E,t+1} - N_{X,t+1} f_{X,t+1}}{N_{X,t+1} f_{X,t+1}} \right) + \frac{1}{\omega} \left( \varphi_{D,t+1} N_{D,t+1}^i + \varphi_{X,t+1} \right) \right\}.\]  

Equation (14) implies that by choosing \(z_{X,t}^i\) the producer also determines \(N_{X,t}^i\).
In equilibrium, the cost of producing an additional variety, $\varphi_t f_{E,t}$, must be equal to its expected benefit (which includes expected savings on future sunk investment costs augmented by the marginal revenue from commercializing the variety, net of fixed export costs, if the good is exported). Finally, the (real) price of Home output for domestic sales is $\rho_{X,t} = \mu_{X,t} \varphi_{X,t}$, while the (real) price of Home output for export sales is $\rho_{D,t} = \mu_{D,t} \varphi_{D,t}$. The expressions for $\mu_{D,t}$ and $\mu_{X,t}$ are identical to equations (8) and (9) derived in the previous section.\(^{34}\)

To summarize, endogenous producer entry and firm heterogeneity introduce three additional variables ($N_{E,t}$, $N_{X,t}$, and $\bar{z}_{X,t}$). In addition, the presence of sunk and fixed export costs modifies the goods market clearing condition: $K_t^a (\bar{Z}_t L_t)^{1-\alpha} = Y_{D,t} + Y_{X,t} + N_{E,t} f_{E,t} + N_{X,t} f_{X,t}$.

**Estimation and Posterior Analysis**

We estimate the model using the same Canadian and U.S. time series as the previous section. We augment this data set with two additional measures: the number varieties exported from the U.S. to Canada and imported from Canada to the U.S. These measures are available at annual frequency from the U.S. Census, and we conduct inference with mixed frequency data, combining these annual measures with the other quarterly observables. Details of the data construction are in Appendix D.

Table 2 in Appendix F presents the posterior estimates. Several parameter estimates remain similar to the previous model that abstracted from the extensive margin of trade. Notably, the estimate of elasticity of substitution between home and foreign goods ($\phi$) is virtually unaffected. Of the structural parameters, the estimates of the inverse Frisch elasticity and capital utilization cost increase relative to the model abstracting from endogenous selection in the export market. This in turn dampens hours and effective capital dynamics relative to the baseline model.

Figure 8 presents posterior 90-percent intervals for GDP present-value multipliers as well as impulse responses for the terms of trade, real exchange rate, and the trade balance following various 1% shocks to fiscal instruments (each column denotes a different shock). The impulse responses show the main message of the previous section is preserved. Expansionary government spending shocks produce favorable terms of trade dynamics for the domestic economy while domestic income tax cuts imply deteriorations in the terms of trade. U.S. public spending increases give larger positive impact spillovers than tax reductions, but they are more short-lived, as Canada benefits

\(^{34}\)To conclude, notice that with flexible prices, the tradable sector becomes isomorphic to Ghironi and Melitz (2004). See Cacciatore and Ghironi (2014) and for details.
over time from the more favorable terms of trade dynamics with tax cuts.

From a quantitative standpoint, some important differences emerge in the posterior impulse response estimates with endogenous selection in the export market. In particular, for both fiscal shocks, terms of trade dynamics are dampened on impact. This, in turn, affects differences in open and counterfactually-closed economy multipliers. To see this, figure 9 plots the Canadian GDP present-value multiplier following a 1% increase in Canadian government spending at the posterior mean under various counterfactuals. The first column of the first row shows multipliers for the open economy (solid line) now become larger than the closed-economy counterpart (dashed line) after two years, much later than the model estimates abstracting from endogenous tradability in goods. This difference stems from the variation in the terms-of-trade dynamics. With firm heterogeneity and endogenous producer entry, terms of trade appreciate less in response to the increase in government spending. To understand this result, notice first that in the presence of endogenous tradability and
firm heterogeneity, the real exchange rate is given by

\[
Q_t = \left[ 1 - (1 - \alpha_X) N^{\frac{1-\phi}{1-\theta}} X_t \left( \mu_X, \zeta_X^t \right)^{1-\phi} \right]^{1-\theta},
\]

which shows that \( \partial Q_t / \partial N_{X,t} < 0 \) and \( \partial Q_t / \partial N^*_X,t > 0 \) i.e., the real exchange rate appreciates following an increase in the number of Home exporters (given that our estimates imply that \( \phi > 1 \)), since a higher \( N_{X,t} \) lowers the foreign price index other things equal. Similarly, \( \partial Q_t / \partial \tilde{z}_{X,t} < 0 \) and \( \partial Q_t / \partial \tilde{z}^*_X,t > 0 \), since higher Home average productivity reduces export prices, also leading to a lower Foreign price index other things equal.

These results show why endogenous fluctuations in the fraction of traded goods affect the response of the real exchange rate and thus the terms of trade \( TOT_t = \rho_{D,t} / \left( Q_t \rho^*_D,t \right) \) following fiscal shocks. Consider an increase in government spending. As in the benchmark model, higher government spending increases demand for domestic goods, resulting in a higher real marginal cost. In turn, the rise in the real marginal cost makes sunk and fixed costs more costly, reducing the number of traded goods relative to the Foreign economy. The negative response of the extensive margin dampens results in a higher \( Q_t \) (a depreciation), dampening the \( TOT_t \) improvement. For this reason, it takes longer for the government spending expansion to become more beneficial with strong trade linkages.

A similar logic is at work following a tax cut. In this case, the tax cut reduces the real marginal cost of production, stimulating producer entry in both domestic and export markets. The increase in \( N_{X,t} \) leads to an appreciation of the real exchange rate, which (partly) offsets the depreciation pressure brought about the decline in the real marginal cost.

6 Conclusion

This paper assesses the role of international trade following unanticipated fiscal expansions under a flexible exchange rate. We combine insights from a simple one-period analytical model with Bayesian prior and posterior analyses on quantitative international business-cycle models, including one that includes key micro-level features of international trade.

We show that more openness to trade can increase fiscal multipliers, contrary to conventional wisdom. In addition, when trade linkages strengthen the expansionary effects of government spending, they reduce the effects of income tax cuts, and vice versa. Finally, trade openness can imply
Figure 9. Home GDP PV Multiplier response following an increase in home government spending. Benchmark response at posterior mean and counterfactuals vary parameter/model structure listed.
that domestic multipliers are larger with distortionary rather than non-distortionary financing. The extent of trade linkages does not intrinsically determine fiscal outcomes; for the same value of the trade share, the effects of fiscal policy can be larger or smaller relative to the closed economy. The relative share of public and private imports, the trade elasticity, and the financing of the government budget shape the general-equilibrium effects. Posterior estimates imply that an increase in public spending yields higher fiscal multipliers when trade linkages are stronger; yet, tax cuts are more effective in inducing positive cross-country comovement.

Our results have direct implications for the effectiveness of fiscal policy in the global economy, including the desirability of fiscal consolidations, balanced-budget versus debt-financed fiscal actions, and incentives for international fiscal policy coordination. In addition, our results suggest an important avenue for future empirical analysis is to condition on the relative composition of private-public imports.
References


A One-Period Model Details

Equilibrium Non-Linear System of Equations:

\[ AL_t = \rho_{D,t}^{-\phi} \left[ (1 - \alpha_X) C_t + (1 - \alpha_X^g) \rho_{G,t}^\phi \tilde{G}_t + \alpha_X C_t^* \rho_{G,t}^\phi \tilde{G}_t \right] \]  

\[ A^* L_t^* Q_t^\phi = \rho_{D,t}^{-\phi} \left[ (1 - \alpha_X) C_t^* Q_t^\phi + (1 - \alpha_X^g) \rho_{G,t}^\phi \tilde{G}_t^* + \alpha_X C_t + \alpha_X^g \rho_{G,t}^\phi \tilde{G}_t \right] \]  

\[ 1 = (1 - \alpha_X) \rho_{D,t}^{1-\phi} + \alpha_X \left( \frac{\rho_{D,t}}{Q_t} \right)^{1-\phi} \]  

\[ 1 = (1 - \alpha_X) \rho_{D,t}^{1-\phi} + \alpha_X \left( \frac{\rho_{D,t}}{Q_t} \right)^{1-\phi} \]  

\[ \rho_{G,t}^{(1-\phi)} = (1 - \alpha_X^g) \rho_{D,t}^{1-\phi} + \alpha_X^g \left( \frac{\rho_{D,t}}{Q_t} \right)^{1-\phi} \]  

\[ \rho_{D,t}^{1-\phi} Q_t^\phi \left[ \alpha_X C_t^* + \alpha_X^g \rho_{G,t}^\phi \tilde{G}_t^* \right] = (\rho_{D,t}^* Q_t)^{1-\phi} \left[ \alpha_X C_t + \alpha_X^g \rho_{G,t}^\phi \tilde{G}_t \right] \]  

\[ Y_t = C_t + \rho_{G,t} \tilde{G}_t \]  

\[ Y_t^* = C_t^* + \rho_{G,t}^* \tilde{G}_t^* \]  

\[ L_t^\omega = C_t^{-\sigma}(1 - \tau) \rho_{D,t} A \]  

\[ L_t^{\omega*} = C_t^{*-\sigma}(1 - \tau^*) \rho_{D,t}^* A \]  

For the case in which income taxes and/or government spending balance the government budget we must include the government budget constraint:

\[ \rho_{G,t} G_t + T_t = \tau_t \rho_{D,t} A \]  

\[ \rho_{G,t}^* G_t^* + T_t^* = \tau_t^* \rho_{D,t}^* A \]  

Equilibrium log-linearized system of equations:

\[ \dot{L}_t + \phi \dot{\rho}_{D,t} = (1 - \alpha_X) (1 - \frac{G}{Y}) \dot{C}_t + (1 - \alpha_X^g) \frac{G}{Y} (\phi \dot{G}_t + \dot{\tilde{G}}_t) + \alpha_X (1 - \frac{G}{Y}) (\phi \dot{Q}_t + \dot{\tilde{G}}_t^*) + \alpha_X^g \frac{G}{Y} (\phi \dot{Q}_t + \dot{\tilde{G}}_t^* + \dot{\tilde{G}}_t) \]
\[ L_t^* + \phi \hat{Q}_t + \phi \hat{\rho}_{D,t} = (1 - \alpha_X)(1 - \frac{G}{Y}) (\phi \hat{Q}_t + \hat{C}_t^*) + (1 - \alpha_X^g) \frac{G}{Y} (\phi \hat{\rho}_{G,t} + \phi \hat{Q}_t + \hat{G}_t^*) + \alpha_X (1 - \frac{G}{Y}) \hat{C}_t + \alpha_X^g \frac{G}{Y} (\phi \hat{\rho}_{G,t} + \hat{G}_t) \]  

(29)

\[ 0 = (1 - \alpha_X) \hat{\rho}_{D,t} + \alpha_X (\hat{Q}_t + \hat{\rho}_{D,t}) \]  

(30)

\[ 0 = (1 - \alpha_X) \hat{\rho}_{D,t}^* + \alpha_X (\hat{\rho}_{D,t} - \hat{Q}_t) \]  

(31)

\[ \hat{\rho}_{G,t} = (1 - \alpha_X^g) \hat{\rho}_{D,t} + \alpha_X^g (\hat{Q}_t + \hat{\rho}_{D,t}) \]  

(32)

\[ \hat{\rho}_{G,t}^* = (1 - \alpha_X^g) \hat{\rho}_{D,t}^* + \alpha_X^g (\hat{\rho}_{D,t}^* - \hat{Q}_t) \]  

(33)

\[ 0 = \left[ \alpha_X \left( 1 - \frac{G}{Y} \right) + \alpha_X^g \frac{G}{Y} \right] \left[ (1 - \phi) \hat{\rho}_{D,t} + \phi \hat{Q}_t - (1 - \phi) \left( \hat{\rho}_{D,t}^* + \hat{Q}_t \right) \right] \]  

+ \alpha_X \left( 1 - \frac{G}{Y} \right) (\hat{C}_t^* - \hat{C}_t) + \alpha_X^g \frac{G}{Y} \left( \phi \hat{\rho}_{G,t}^* - \phi \hat{\rho}_{G,t} - \hat{G}_t + \hat{G}_t^* \right) \]  

(34)

\[ \hat{Y}_t = (1 - \frac{G}{Y}) \hat{C}_t + \frac{G}{Y} (\hat{\rho}_{G,t} + \hat{G}_t) \]  

(35)

\[ \hat{Y}_t^* = (1 - \frac{G}{Y}) \hat{C}_t^* + \frac{G}{Y} (\hat{\rho}_{G,t}^* + \hat{G}_t^*) \]  

(36)

\[ \omega \hat{L}_t = -\sigma \hat{C}_t - \frac{\tau}{1 - \tau} \hat{\tau}_t + \hat{\rho}_{D,t} \]  

(37)

\[ \omega \hat{L}_t^* = -\sigma \hat{C}_t^* - \frac{\tau}{1 - \tau} \hat{\tau}_t^* + \hat{\rho}_{D,t}^* \]  

For the case in which income taxes and/or government spending balance the government budget we include the condition:

\[ \sum_{\tau} \frac{G}{\tau} \left( \hat{\rho}_{G,t} + \hat{G}_t \right) = \hat{\tau}_t + \hat{\rho}_{D,t} + \hat{L}_t \]  

(38)

\[ \sum_{\tau} \frac{G}{\tau} \left( \hat{\rho}_{G,t}^* + \hat{G}_t^* \right) = \hat{\tau}_t^* + \hat{\rho}_{D,t}^* + \hat{L}_t^* \]
Proof of 3

Suppose that there is full home-bias in government consumption, \( \alpha_X^g = 0 \). Then the solutions for output, consumption, labor, the terms of trade, and the real exchange rate are given by

\[
\begin{align*}
\dot{Y}_t &= \frac{\phi(2\phi - 1 + \alpha_X(2L - 2\phi + \omega))}{(1 + \omega(1 - \frac{\phi}{\phi^*}))\Omega_1} \dot{C}_t + \frac{-\tau(2\phi - 1 + \alpha_X - 2\phi \alpha_X + 2\phi(\phi - 1)(\alpha_X - 1)}{(1 + \omega(1 - \frac{\phi}{\phi^*}))\Omega_1} \dot{\tau}_t, \\
\dot{C}_t &= \frac{-\phi(\omega(1 - \alpha_X)(2\phi - 1)}{(1 + \omega(1 - \frac{\phi}{\phi^*}))\Omega_1} \dot{C}_t + \frac{-\tau(1 - \alpha_X)(2\phi - 1)}{(1 + \omega(1 - \frac{\phi}{\phi^*}))\Omega_1} \dot{\tau}_t \\
\dot{L}_t &= \frac{\omega}{1 + \omega(1 - \frac{\phi}{\phi^*})} \dot{C}_t + \frac{-\tau(1 - \frac{\phi}{\phi^*})}{(1 + \omega(1 - \frac{\phi}{\phi^*}))\Omega_1} \dot{\tau}_t, \\
\dot{Y}_t^* &= \frac{-\phi(\alpha_X \omega)}{(1 + \omega(1 - \frac{\phi}{\phi^*}))\Omega_1} \dot{C}_t + \frac{-\tau(\alpha_X \omega)}{(1 + \omega(1 - \frac{\phi}{\phi^*}))\Omega_1} \dot{\tau}_t \\
\dot{C}_t^* &= \frac{-\phi(2\alpha_X - 1) \omega}{(1 + \omega(1 - \frac{\phi}{\phi^*}))\Omega_1} \dot{C}_t + \frac{-\tau(2\alpha_X - 1)}{(1 + \omega(1 - \frac{\phi}{\phi^*}))\Omega_1} \dot{\tau}_t \\
\dot{\tau}_t &= \frac{-\phi \alpha_X \omega}{(1 + \omega(1 - \frac{\phi}{\phi^*}))\Omega_1} \dot{C}_t + \frac{-\alpha_X \tau}{(1 + \omega(1 - \frac{\phi}{\phi^*}))\Omega_1} \dot{\tau}_t
\end{align*}
\]

where \( \Omega_1 \equiv 2\phi - 1 - 2\alpha_X(\phi - 1) \). When \( \phi > \phi^* \), \( \Omega_1 > 0 \) and when \( \phi < \phi^* \), \( \Omega_1 < 0 \). The signs of the terms of trade and foreign variables immediately follow based on the sign of \( \Omega_1 \).

The domestic consumption policy function can be written as:

\[
\dot{C}_t = \Delta^{g, closed}_{\Omega_1} \frac{(1 - \alpha_X)(2\phi - 1)}{\Omega_1} \dot{C}_t + \Delta^{\tau, closed}_{\Omega_1} \frac{(1 - \alpha_X)(2\phi - 1)}{\Omega_1} \dot{\tau}_t
\]

where \( \Delta^{g, closed}, \Delta^{\tau, closed} < 0 \) are the responses in the closed economy. Notice that \( \Omega_1 < 0 \) only for values of \( \phi < \frac{1}{2} \), implying \( (1 - \alpha_X)(2\phi - 1) < 0 \). When \( \Omega_1 > 0 \), \( (1 - \alpha_X)(2\phi - 1) > 0 \). For the open economy responses to be greater than the closed economy, we need the response to be less negative. When \( \Omega_1 > 0 \ (\phi > \phi^* \) ), we need

\[
(1 - \alpha_X)(2\phi - 1) < 2\phi - 1 - 2\alpha_X(\phi - 1)
\]

\[
\rightarrow 2\phi - 2\alpha_X \phi - 1 + \alpha_X < 2\phi - 1 - 2\alpha_X \phi + 2\alpha_X
\]

\[
\rightarrow \alpha_X < 2\alpha_X
\]

which is always satisfied. When \( \Omega_1 < 0 \), for the open economy response to be less negative than
the closed economy, we need \((1 - \alpha_X)(2\phi - 1) > 2\phi - 1 - 2\alpha_X(\phi - 1)\), which is never possible. It follows that the open economy response is less negative than the closed economy only when \(\phi > \phi^*\).

The domestic output policy function can be written as:

\[
\dot{Y}_t = \Delta_{g,\text{closed}} \frac{2\phi - 1 + \alpha_X(2 - 2\phi + \omega)}{\Omega_1} \dot{G}_t + \frac{-\tau(2\phi - 1)(1 - \alpha_X) - \tau^G(1 - 2\alpha_X + 2\phi(\alpha_X - 1))}{(2 - 2\phi)(1 - \tau)\Omega_1} \dot{\tau}_t
\]

where \(\Delta_{g,\text{closed}} > 0\) is the response in the closed economy.

Consider the response to a government spending shock. \(2\phi - 1 + \alpha_X(2 - 2\phi + \omega) > 0\) when \(\phi > (1 - \alpha_X(2 + \omega))/(2(1 - \alpha_X))\). Since \(1 - \alpha_X(2 + \omega) < 1 - 2\alpha_X\), it follows that any \(\phi > \phi^*\) necessarily satisfies this bounds. Thus, when \(\phi > \phi^*\), both numerator and denominator are positive.

It follows that the open economy response is larger than the closed economy response when

\[
2\phi - 1 + \alpha_X(2 - 2\phi + \omega) > 2\phi - 1 - 2\alpha_X(\phi - 1)
\]

\[
\rightarrow 2\phi - 1 + \alpha_X(2 + \omega) - 2\phi\alpha_X > 2\phi - 1 - 2\alpha_X\phi + 2\alpha_X
\]

\[
\rightarrow \alpha_X(2 + \omega) > 2\alpha_X,
\]

which is always satisfied. If \(\phi < \phi^*\), there is a range of \(\phi\) that implies the numerator is positive while \(\Omega_1\) and the denominator are negative. Thus, for these \(\phi\), the open economy multiplier is necessarily smaller than the closed. There is a range of \(\phi\) for which the numerator is also negative. In this case, the open economy multiplier is smaller than the closed if

\[
2\phi - 1 + \alpha_X(2 - 2\phi + \omega) > 2\phi - 1 - 2\alpha_X(\phi - 1)
\]

\[
\rightarrow \alpha_X(2 + \omega) > 2\alpha_X,
\]

which is always satisfied. It follows that the open economy response is larger than the closed economy only when \(\phi > \phi^*\).

Consider the response to a tax shock. When \(\dot{G} = 0\), the response is the same as domestic consumption and the same results for consumption carry through.

**Proof of Proposition 4.** Suppose that there is home bias in goods, \(\phi = 1, \omega = 1,\) and \(\alpha^G_X = \nu\alpha_X\) for \(\nu \in [0,1)\). Then the solutions for output, consumption, labor, the terms of trade,
and the real exchange rate are given by
\[
\hat{Y}_t = \frac{(2 - \hat{\nu})\hat{\nu} \Omega_4}{(2 - \hat{\nu})\Omega_2} \hat{G}_t - \frac{\tau(1 - \hat{\nu})(2(1 - \alpha_X) + \hat{\nu}(1 - \nu) + 2\nu - 3 + \alpha_X(1 - \nu)(1 + 2\nu(2 - \hat{\nu}))}{(1 - \tau)(2 - \hat{\nu})\Omega_2} \hat{\tau}_t,
\]
\[
\hat{C}_t = \frac{\hat{\nu}(2 - \hat{\nu})\Omega_4 + \hat{\nu}\alpha_X \Omega_3(2 - \hat{\nu}(1 + \nu))}{(2 - \hat{\nu})\Omega_2} \hat{G}_t - \frac{\tau(1 - \hat{\nu})[2 + \hat{\nu}(2 + \alpha_X(3 - 4\nu) - 3 + \hat{\nu}(1 - \nu)(1 - 2\nu\alpha_X))]}{(1 - \tau)(2 - \hat{\nu})\Omega_2} \hat{\tau}_t,
\]
\[
\hat{L}_t = \frac{\hat{\nu}(2 - \hat{\nu})\Omega_4 + \hat{\nu}\alpha_X \Omega_3}{(2 - \hat{\nu})\Omega_2} \hat{G}_t - \frac{\tau(1 - \hat{\nu})[2 + \hat{\nu}(2 + \alpha_X(3 - 4\nu) - 3 + \hat{\nu}(1 - \nu)(1 - 2\nu\alpha_X))]}{(1 - \tau)(2 - \hat{\nu})\Omega_2} \hat{\tau}_t,
\]
\[
\hat{Y}_t^* = \frac{-\hat{\nu}\alpha_X (2 - \hat{\nu}(1 + \nu)) \Omega_3}{(2 - \hat{\nu})\Omega_2} \hat{G}_t - \frac{\tau(1 - \hat{\nu})\hat{\nu}\alpha_X (2 - \hat{\nu}(1 + \nu))}{(1 - \tau)(2 - \hat{\nu})\Omega_2} \hat{\tau}_t,
\]
\[
\hat{C}_t^* = \frac{-\hat{\nu}\alpha_X (2 - \hat{\nu}(1 - \nu)) \Omega_3}{(2 - \hat{\nu})\Omega_2} \hat{G}_t - \frac{\tau(1 - \hat{\nu})\hat{\nu}\alpha_X (2 - \hat{\nu}(1 - \nu))}{(1 - \tau)(2 - \hat{\nu})\Omega_2} \hat{\tau}_t,
\]
\[
\hat{L}_t^* = \frac{-\hat{\nu}^2 \alpha_X \nu \Omega_3}{(2 - \hat{\nu})\Omega_2} \hat{G}_t + \frac{\tau(1 - \hat{\nu})\hat{\nu}\alpha_X \nu}{(1 - \tau)(2 - \hat{\nu})\Omega_2} \hat{\tau}_t,
\]
\[
\hat{TOT}_t = \frac{\hat{\nu} \Omega_3}{\Omega_2} \hat{G}_t + \frac{\tau(1 - \hat{\nu})}{(1 - \tau)\Omega_2} \hat{\tau}_t,
\]
\[
\hat{Q}_t = \frac{\hat{\nu} \Omega_3(2\alpha_X - 1)}{\Omega_2} \hat{G}_t + \frac{\tau(1 - \hat{\nu})(2\alpha_X - 1)}{(1 - \tau)\Omega_2} \hat{\tau}_t
\]
where \(\Omega_2 = (2 - \frac{\hat{\nu}}{\bar{\nu}})(1 - \frac{\hat{\nu}}{\bar{\nu}}) + \nu \frac{\hat{\nu}}{\nu} [2 - \frac{\hat{\nu}}{\nu} + 2\alpha_X (1 - \frac{\hat{\nu}}{\nu} - \nu(2 - \frac{\hat{\nu}}{\nu}))\) and \(\Omega_3 = (1 - \frac{\hat{\nu}}{\nu} - \nu(2 - \frac{\hat{\nu}}{\nu}))\) and \(\Omega_4 = 1 - \frac{\hat{\nu}}{\nu}(1 - \nu) > 0\). Notice that we can rewrite \(\Omega_2\) as:
\[
\Omega_2 = \left(2 - \frac{\hat{\nu}}{\nu}\right)(1 - \frac{\hat{\nu}}{\nu}) + 2\alpha_X \nu \frac{\hat{\nu}}{\nu} \left(1 - \frac{\hat{\nu}}{\nu}\right) + \nu \frac{\hat{\nu}}{\nu} \left(2 - \frac{\hat{\nu}}{\nu}\right) (1 - 2\alpha_X \nu)
\]

A sufficient condition for this to be positive is that \(\nu \leq 1\) and \(\alpha_X \leq \frac{1}{2}\), which is assumed by the proposition. Thus \(\Omega_2 > 0\).

Consider a positive shock to government spending. \(\Omega_2 > 0\) (and thus the numerator of \(\hat{TOT}_t\)) if \(\nu < \nu^*\) and less than zero if \(\nu > \nu^*\). The signs of \(\hat{TOT}_t\), \(\hat{L}_t^*, \hat{C}_t^*, \) and \(\hat{L}_t\) immediately follow from the relative size of \(\nu\) and \(\nu^*\). \(\hat{Y}_t^* < 0\) and \(\hat{C}_t < 0\) as well in this case if \(2 > \frac{\hat{\nu}}{\nu}(1 + \nu)\) or \(\nu < (2 - \frac{\hat{\nu}}{\nu})/(\frac{\hat{\nu}}{\nu})\) which is always satisfied for \(\nu < 1\) (assumed in the proposition).

Consider a positive shock to taxes. Given the proposition’s assumptions on \(\nu\) and \(\alpha_X\), \(\hat{TOT}_t > 0\), \(\hat{C}_t^* < 0\), \(\hat{Y}_t^* < 0\), \(\hat{L}_t^* < 0\) always.

Given the complexity of the domestic responses, we resort to a numerical proof. We take five million draws from \(\alpha_X \sim U(0, 0.5), \nu \sim U(0, 1), \tau \sim U(0, 1),\) and \(\frac{\hat{\nu}}{\nu} \sim U(0, 1)\). For all draws satisfying \(\nu < \nu^*, \hat{Y}_t > \hat{Y}_t^{\text{closed}}, \hat{C}_t > \hat{C}_t^{\text{closed}},\) and \(\hat{L}_t < \hat{L}_t^{\text{closed}}\) following a positive government spending shock, and vice versa for all draws satisfying \(\nu > \nu^*\). For all draws, \(\hat{Y}_t > \hat{Y}_t^{\text{closed}}, \hat{C}_t > \hat{C}_t^{\text{closed}},\) and \(\hat{L}_t < \hat{L}_t^{\text{closed}}\) following a positive tax shock.

53
Additional Numerical Analysis

Figure 11 demonstrates the robustness of proposition 4 when we relax the parametric restriction on $\omega$ from section 2 of the text. As $\omega$ increases, Home labor becomes less responsive to a change in the domestic price level, inducing a larger response in the price and the terms of trade. In turn, the larger response of the terms of trade leads to a larger wealth effect and increase in Home GDP relative to the closed economy.\footnote{In absolute value, the response of Home GDP decreases in $\omega$ as labor and production are less responsive.}

Figure 12 repeats the numerical analysis of section 2 of the text for a 1\% cut in the income tax and shows that income tax cuts are less effective with stronger trade linkages. When a tax cut is financed by lump-sum transfers, as in figure 12a, the terms of trade unambiguously deteriorate and Home GDP is lower relative to the closed economy for all parameterizations depicted, since there is no $\nu$ cut-off in this case and the $\phi$ boundary lies below the values depicted. When a tax cut is financed by lower government expenditures, as in figure 12b, the terms of trade are likely to substantially deteriorate, as both fiscal actions lead to a decline in the terms of trade. However, as shown in the main text, a boundary for $\nu$ exists for government spending, and once past the $\nu$ boundary, a decrease in government spending leads to an increase in the terms of trade, ceteris paribus. Thus, when an income tax cut is accompanied by a reduction in public expenditures, it is possible for the terms of trade to increase when $\nu$ is large; in turn, Home GDP can be higher than in the closed economy. Furthermore, in these cases, home GDP increases more with distortionary financing relative to lump-sum transfer financing (as seen by comparing the home GDP responses in figures 12a and 12b). These results mirror those following a government spending increase considered in the main text. Thus, when open economies benefit from pursuing expansionary government spending, tax cuts are less effective, and vice versa.

B Alternative Model Specifications for the One-Period Model

Adding Non-Tradables

In this case, there are the following new variables: $\rho_{T,t}$, $\rho_{T,t}^\phi$, $\rho_{NT,t}$, $\rho_{NT,t}^\phi$, $\rho_{Gt,t}$, $\rho_{Gt,t}^\phi$. New parameters: $\phi_N$, $\alpha_N$ and $\alpha_N^\phi$. The endogenous variables are determined by the following system of
Figure 11. Impact responses following a 1% increase in government spending. In all cases, $G/Y = 0.2$, $\tau = 0.25$, trade share = 0.5, and $\phi = 1$. 

(a) Lump-Sum Transfers Financing

(b) Income-Tax Financing
Figure 11. Impact responses following a 1% increase in government spending. In all cases, $G/Y = 0.2$, $\tau = 0.25$, trade share = 0.5, and $\phi = 1$. 

(a) Lump-Sum Transfers Financing

(b) Income-Tax Financing
Figure 12. Impact responses following a 1% cut in the income tax. The plane in each panel denotes the response in the closed economy. In all cases, $G/Y = 0.2$, $\tau = 0.25$, $\omega = 1$, and the trade share = 0.5.
equations:

\[ C_{N,t} = \alpha_N \rho_{N,t}^{-\phi N} C_t \quad (39) \]

\[ C^*_{N,t} = \alpha_N (\rho^*_N)^{-\phi N} C^*_t \quad (40) \]

\[ G_{N,t} = \alpha_N^g \left( \frac{\rho_{N,t}}{\rho_{G,t}} \right)^{-\phi N} G_t \quad (41) \]

\[ G^*_{N,t} = \alpha_N^g \left( \frac{\rho^*_N}{\rho^*_G} \right)^{-\phi N} G^*_t \quad (42) \]

\[ C_{T,t} = (1 - \alpha_N) \rho_{T,t}^{-\phi N} C_t \quad (43) \]

\[ C^*_{T,t} = (1 - \alpha_N) (\rho^*_T)^{-\phi N} C^*_t \quad (44) \]

\[ G_{T,t} = (1 - \alpha_N^g) \left( \frac{\rho_{G,T,t}}{\rho_{T,t}} \right)^{-\phi N} G_t \quad (45) \]

\[ G^*_{T,t} = (1 - \alpha_N^g) \left( \frac{\rho^*_G}{\rho^*_T} \right)^{-\phi N} G^*_t \quad (46) \]

\[ C_{D,t} = (1 - \alpha_X) \left( \frac{\rho_{D,t}}{\rho_{T,t}} \right)^{-\phi} C_{T,t} \quad (47) \]

\[ C^*_{D,t} = (1 - \alpha_X) \left( \frac{\rho^*_D}{\rho^*_T} \right)^{-\phi} C^*_{T,t} \quad (48) \]

\[ G_{D,t} = (1 - \alpha_X^g) \left( \frac{\rho_{G,T,t}}{\rho_{D,t}} \right)^{-\phi} G_{T,t} \quad (49) \]

\[ G^*_{D,t} = (1 - \alpha_X^g) \left( \frac{\rho^*_G}{\rho^*_D} \right)^{-\phi} G^*_{T,t} \quad (50) \]

\[ C_{X,t} = \alpha_X \left( \frac{\rho_{X,t}}{\rho^*_T} \right)^{-\phi} C^*_{T,t} \quad (51) \]

\[ C^*_{X,t} = \alpha_X \left( \frac{\rho^*_X}{\rho_{T,t}} \right)^{-\phi} C_{T,t} \quad (52) \]

\[ G_{X,t} = \alpha_X^g \left( \frac{\rho_{X,t}}{\rho^*_G} \right)^{-\phi} G^*_{T,t} \quad (53) \]

\[ G^*_{X,t} = \alpha_X^g \left( \frac{\rho^*_X}{\rho^*_G} \right)^{-\phi} G_{T,t} \quad (54) \]
\[ L_t = C_{D,t} + C_{X,t} + C_{N,t} + G_{D,t} + G_{X,t} + G_{N,t} \]  
(56)

\[ L_t^* = C_{D,t}^* + C_{X,t}^* + C_{N,t}^* + G_{D,t}^* + G_{X,t}^* + G_{N,t}^* \]  
(57)

\[ 1 = (1 - \alpha_N) \rho_{T,t}^{1-\phi_N} + \alpha_N \rho_{N,t}^{1-\phi_N} \]  
(58)

\[ 1 = (1 - \alpha_N) (\rho_{T,t}^*)^{1-\phi_N} + \alpha_N (\rho_{N,t}^*)^{1-\phi_N} \]  
(59)

\[ \rho_{G,t}^{1-\phi_N} = (1 - \alpha_N) \rho_{G,t}^{1-\phi_N} + \alpha_N \rho_{N,t}^{1-\phi_N} \]  
(60)

\[ (\rho_{G,t}^*)^{1-\phi_N} = (1 - \alpha_N) (\rho_{G,t}^*)^{1-\phi_N} + \alpha_N (\rho_{N,t}^*)^{1-\phi_N} \]  
(61)

\[ \rho_{T,t}^{1-\phi} = (1 - X) \rho_{D,t}^{1-\phi} + X (\rho_{X,t}^*)^{1-\phi} \]  
(62)

\[ (\rho_{T,t}^*)^{1-\phi} = (1 - X) (\rho_{D,t}^*)^{1-\phi} + X (\rho_{X,t}^*)^{1-\phi} \]  
(63)

\[ \rho_{G,t}^{1-\phi} = (1 - \alpha_N) \rho_{D,t}^{1-\phi} + \alpha_N (\rho_{X,t}^*)^{1-\phi} \]  
(64)

\[ (\rho_{G,t}^*)^{1-\phi} = (1 - \alpha_N) (\rho_{D,t}^*)^{1-\phi} + \alpha_N (\rho_{X,t}^*)^{1-\phi} \]  
(65)

\[ \rho_{N,t} = \rho_{D,t} \]  
(66)

\[ \rho_{N,t}^* = \rho_{D,t}^* \]  
(67)

\[ \rho_{X,t} = \frac{\rho_{D,t}}{\phi_t} \]  
(68)

\[ \rho_{X,t}^* = \rho_{D,t}^* \phi_t \]  
(69)

\[ \tilde{L}_t^\omega = C_t^{-\sigma} (1 - \tau_t) \rho_{D,t} \]  
(70)

\[ (\tilde{L}_t^\omega)^\omega = (C_t^*)^{-\sigma} (1 - \tau_t) \rho_{D,t}^* \]  
(71)

\[ Y_t = C_t + \rho_{G,t} G_t \]  
(72)

\[ Y_t^* = C_t^* + \rho_{G,t}^* G_t^* \]  
(73)

\[ Q_t \rho_{X,t} (C_{X,t} + G_{X,t}) = \rho_{X,t}^* (C_{X,t}^* + G_{X,t}^*) \]  
(74)

We assume that the countries are completely symmetric, so that in steady state the values across the countries are the same, i.e. \( C = C^* \). Note that in steady state all relative prices are 1.

Then: \( C_N = \alpha_N C, \quad C_T = (1 - \alpha_N) C, \quad C_D = (1 - \alpha_X) C_T = (1 - \alpha_N)(1 - \alpha_N) C, \quad C_X = \alpha_X C_T = \alpha_X (1 - \alpha_N) C \). \( G_N = \alpha_N G, \quad G_T = (1 - \alpha_N) G, \quad G_D = (1 - \alpha_X) G_T = (1 - \alpha_N)(1 - \alpha_N) G, \quad G_X = \alpha_X G_T = \alpha_X (1 - \alpha_N) G \). Log-linearizing the model around the steady state results in the

59
following system:

\[
\begin{align*}
\dot{C}_{N,t} &= -\phi_N\dot{\rho}_{N,t} + \dot{C}_t \\
\dot{C}_{N,t}^* &= -\phi_N\dot{\rho}_{N,t}^* + \dot{C}_t^* \\
\dot{G}_{N,t} &= -\phi_N\dot{\rho}_{N,t} + \phi_N\dot{\rho}_{G,t} + \dot{G}_t \\
\dot{G}_{N,t}^* &= -\phi_N\dot{\rho}_{N,t} + \phi_N\dot{\rho}_{G,t}^* + \dot{G}_t^* \\
\dot{C}_{T,t} &= -\phi_N\dot{\rho}_{T,t} + \dot{C}_t \\
\dot{C}_{T,t}^* &= -\phi_N\dot{\rho}_{T,t} + \dot{C}_t^* \\
\dot{G}_{T,t} &= -\phi_N\dot{\rho}_{G,t} + \phi_N\dot{\rho}_{G,t} + \dot{G}_t \\
\dot{G}_{T,t}^* &= -\phi_N\dot{\rho}_{G,t} + \phi_N\dot{\rho}_{G,t}^* + \dot{G}_t^* \\
\dot{G}_{D,t} &= -\phi_N\dot{\rho}_{D,t} + \dot{\rho}_{T,t} + \dot{C}_t \\
\dot{G}_{D,t}^* &= -\phi_N\dot{\rho}_{D,t} + \dot{\rho}_{T,t} + \dot{C}_t^* \\
\dot{G}_{D,t} &= -\phi_N\dot{\rho}_{D,t} + \phi_N\dot{\rho}_{G,t} + \dot{G}_t \\
\dot{G}_{D,t}^* &= -\phi_N\dot{\rho}_{D,t} + \phi_N\dot{\rho}_{G,t}^* + \dot{G}_t^* \\
\dot{G}_{X,t} &= -\phi_N\dot{\rho}_{X,t} + \dot{\rho}_{T,t} + \dot{C}_t \\
\dot{G}_{X,t}^* &= -\phi_N\dot{\rho}_{X,t} + \dot{\rho}_{T,t} + \dot{C}_t^* \\
\dot{G}_{X,t} &= -\phi_N\dot{\rho}_{X,t} + \phi_N\dot{\rho}_{G,t} + \dot{G}_t \\
\dot{G}_{X,t}^* &= -\phi_N\dot{\rho}_{X,t} + \phi_N\dot{\rho}_{G,t}^* + \dot{G}_t^* \\
\dot{G}_{N,t} &= \left(1 - \alpha_N\right)\left(1 - \alpha_N\right)\frac{C}{Y} \dot{C}_{D,t} + \alpha_N \left(1 - \alpha_N\right) \frac{C}{Y} \dot{C}_{X,t} + \alpha_N \frac{C}{Y} \dot{C}_{N,t} \\
&\quad + \left(1 - \alpha_N^2\right) \left(1 - \alpha_N^2\right) \frac{G}{Y} \dot{G}_{D,t} + \alpha_N \left(1 - \alpha_N^2\right) \frac{G}{Y} \dot{G}_{X,t} + \alpha_N \frac{G}{Y} \dot{G}_{N,t} \\
\dot{L}_t &= \left(1 - \alpha_N\right) \left(1 - \alpha_N\right) \frac{C}{Y} \dot{C}_{D,t}^* + \alpha_N \left(1 - \alpha_N\right) \frac{C}{Y} \dot{C}_{X,t}^* + \alpha_N \frac{C}{Y} \dot{C}_{N,t}^* \\
&\quad + \left(1 - \alpha_N^2\right) \left(1 - \alpha_N^2\right) \frac{G}{Y} \dot{G}_{D,t}^* + \alpha_N \left(1 - \alpha_N^2\right) \frac{G}{Y} \dot{G}_{X,t}^* + \alpha_N \frac{G}{Y} \dot{G}_{N,t}^* \\
\dot{L}_t^* &= \left(1 - \alpha_N\right) \left(1 - \alpha_N\right) \frac{C}{Y} \dot{C}_{D,t}^* + \alpha_N \left(1 - \alpha_N\right) \frac{C}{Y} \dot{C}_{X,t}^* + \alpha_N \frac{C}{Y} \dot{C}_{N,t}^* \\
&\quad + \left(1 - \alpha_N^2\right) \left(1 - \alpha_N^2\right) \frac{G}{Y} \dot{G}_{D,t}^* + \alpha_N \left(1 - \alpha_N^2\right) \frac{G}{Y} \dot{G}_{X,t}^* + \alpha_N \frac{G}{Y} \dot{G}_{N,t}^* \\
0 &= \left(1 - \alpha_N\right) \dot{\rho}_{T,t} + \alpha_N \dot{\rho}_{N,t} \\
0 &= \left(1 - \alpha_N\right) \dot{\rho}_{T,t}^* + \alpha_N \dot{\rho}_{N,t}^* \\
\dot{\rho}_{G,t} &= \left(1 - \alpha_N^2\right) \dot{\rho}_{G,t} + \alpha_N \dot{\rho}_{N,t} \\
\dot{\rho}_{G,t}^* &= \left(1 - \alpha_N^2\right) \dot{\rho}_{G,t} + \alpha_N \dot{\rho}_{N,t}^*
\end{align*}
\]
\[\hat{p}_{T,t} = (1 - \alpha_X)\hat{p}_{D,t} + \alpha_X\hat{p}_{X,t}\]  
(97)

\[\hat{p}_{T,t}^* = (1 - \alpha_X)\hat{p}_{D,t}^* + \alpha_X\hat{p}_{X,t}\]  
(98)

\[\hat{p}_{G,t} = (1 - \alpha_X^g)\hat{p}_{D,t} + \alpha_X^g\hat{p}_{X,t}\]  
(99)

\[\hat{p}_{G,t}^* = (1 - \alpha_X^g)\hat{p}_{D,t}^* + \alpha_X^g\hat{p}_{X,t}\]  
(100)

\[\hat{p}_{N,t} = \hat{p}_{D,t}\]  
(101)

\[\hat{p}_{N,t}^* = \hat{p}_{D,t}^*\]  
(102)

\[\hat{p}_{X,t} = \hat{p}_{D,t} - \hat{Q}_t\]  
(103)

\[\hat{p}_{X,t}^* = \hat{p}_{D,t}^* + \hat{Q}_t\]  
(104)

\[\omega\hat{L}_t = -\sigma\hat{C}_t + \hat{p}_{D,t} - \frac{\tau}{1 - \tau}\hat{C}_t\]  
(105)

\[\omega\hat{L}_t^* = -\sigma\hat{C}_t^* + \hat{p}_{D,t}^* - \frac{\tau}{1 - \tau}\hat{C}_t^*\]  
(106)

\[\dot{Y}_t = \frac{C}{Y}\hat{C}_t + \frac{G}{Y}(\hat{p}_{G,t} + \hat{G}_t)\]  
(107)

\[\dot{Y}_t^* = \frac{C}{Y}\hat{C}_t^* + \frac{G}{Y}(\hat{p}_{G,t}^* + \hat{G}_t^*)\]  
(108)

\[\dot{Q}_t + \hat{p}_{X,t} + \frac{\alpha_X(1 - \alpha_N)\hat{C}_t}{\alpha_X(1 - \alpha_N) + \alpha_X^g(1 - \alpha_N)} + \frac{\alpha_X^g(1 - \alpha_N)\hat{G}_t}{\alpha_X(1 - \alpha_N) + \alpha_X^g(1 - \alpha_N)} = \hat{p}_{X,t} + \frac{\alpha_X(1 - \alpha_N)\hat{C}_t^*}{\alpha_X(1 - \alpha_N) + \alpha_X^g(1 - \alpha_N)} + \frac{\alpha_X^g(1 - \alpha_N)\hat{G}_t^*}{\alpha_X(1 - \alpha_N) + \alpha_X^g(1 - \alpha_N)}\]  
(109)

**Case 1:** \(\phi = \phi_N\)

Define \(\tilde{\alpha}_X = \alpha_X(1 - \alpha_N), (1 - \tilde{\alpha}_X) \equiv (1 - \alpha_X)(1 - \alpha_N) + \alpha_N, \tilde{\alpha}_X^g \equiv \alpha_X^g(1 - \alpha_N),\) and \((1 - \tilde{\alpha}_X^g) \equiv (1 - \alpha_X^g)(1 - \alpha_N) + \alpha_N^g.\) Then when \(\phi = \phi_N,\) we can rewrite the above system of equations as

\[\hat{L}_t + \phi\hat{p}_{D,t} = (1 - \tilde{\alpha}_X)(1 - \frac{G}{Y})\hat{C}_t + (1 - \tilde{\alpha}_X^g)\frac{G}{Y}(\phi\hat{p}_{G,t} + \hat{G}_t) + \alpha_X(1 - \frac{G}{Y})(\phi\hat{Q}_t + \hat{C}_t^*) + \tilde{\alpha}_X^g\frac{G}{Y}(\phi\hat{Q}_t + \phi\hat{p}_{G,t} + \hat{G}_t)\]  

\[\hat{L}_t^* + \phi\hat{Q}_t + \phi\hat{p}_{D,t}^* = (1 - \tilde{\alpha}_X)(1 - \frac{G}{Y})(\phi\hat{Q}_t + \hat{C}_t^*) + (1 - \tilde{\alpha}_X^g)\frac{G}{Y}(\phi\hat{p}_{G,t} + \phi\hat{Q}_t + \hat{G}_t) + \alpha_X(1 - \frac{G}{Y})\hat{C}_t + \tilde{\alpha}_X^g\frac{G}{Y}(\phi\hat{p}_{G,t} + \hat{G}_t)\]  

\[0 = (1 - \tilde{\alpha}_X)\hat{p}_{D,t} + \alpha_X(\hat{Q}_t + \hat{p}_{D,t})\]  

\[0 = (1 - \tilde{\alpha}_X)\hat{p}_{D,t}^* + \alpha_X(\hat{Q}_t + \hat{p}_{D,t})\]  

\[\hat{p}_{G,t} = (1 - \tilde{\alpha}_X^g)\hat{p}_{D,t} + \alpha_X^g(\hat{Q}_t + \hat{p}_{D,t})\]
\[ \hat{\rho}_{G,h} = (1 - \hat{\alpha}_X^g) \hat{\rho}_{D,h} + \alpha_X^g (\hat{\rho}_{D,h} - \hat{Q}_t) \]

\[ (1 - \phi) \hat{\rho}_{D,h} + \phi \hat{Q}_t = \frac{\hat{\alpha}_X (1 - \frac{\hat{G}}{\hat{G}_t^*})}{\hat{\alpha}_X (1 - \frac{\hat{G}}{\hat{G}_t^*}) + \hat{\alpha}_X^g \hat{G}^*_t} \hat{C}_t^* + \frac{\hat{\alpha}_X^g \hat{G}}{\hat{\alpha}_X (1 - \frac{\hat{G}}{\hat{G}_t^*}) + \hat{\alpha}_X^g \hat{G}^*_t} \left( \frac{\hat{G}_t^*}{1 - \tau} + \hat{\rho}_{G,h} \right) \]

\[ = (1 - \phi) \hat{\rho}_{D,h} + (1 - \phi) \hat{Q}_t + \frac{\hat{\alpha}_X (1 - \frac{\hat{G}}{\hat{G}_t^*})}{\hat{\alpha}_X (1 - \frac{\hat{G}}{\hat{G}_t^*}) + \hat{\alpha}_X^g \hat{G}^*_t} \hat{C}_t^* + \frac{\hat{\alpha}_X^g \hat{G}}{\hat{\alpha}_X (1 - \frac{\hat{G}}{\hat{G}_t^*}) + \hat{\alpha}_X^g \hat{G}^*_t} \left( \frac{\hat{G}_t^*}{1 - \tau} + \hat{\rho}_{G,h} \right) \]

\[ \omega \hat{L}_t = -\hat{C}_t - \frac{\tau}{1 - \tau} \hat{\tilde{t}}_t + \hat{\rho}_{D,t} \]

\[ \omega \hat{L}_t^* = -\hat{C}_t^* - \frac{\tau}{1 - \tau} \hat{t}_t^* + \hat{\rho}_{D,t}^* \]

This system is isomorphic to the benchmark model. Thus, Home GDP responses are increasing in either the share on non-tradable government spending \((\alpha^g_X)\) or domestic tradables \((1 - \alpha^g_X)\), as both increase \(1 - \alpha^g_X\).

**Case 2:** \(\alpha^g_X = 0, \alpha^g_N = 0\) or \(\alpha^g_X = 0, \alpha^g_N = 1\)

Full home bias in tradable government goods \((\alpha^g_X = 0, \alpha^g_N = 0)\) and full bias in non-traded government goods \((\alpha^g_X = 0, \alpha^g_N = 1)\) yield identical allocations. Thus, as long as the government consumes one good, it does not matter if the good is domestic tradables or non-tradables.

To see this, notice that in either case \(\hat{\rho}_{G,t} = \hat{\rho}_{G,t} = \hat{\rho}_{D,t}\). Substituting into equations (77) and (81) and using the fact that \(\hat{\rho}_{N,t} = \hat{\rho}_{D,t}\) implies \(\hat{G}_{N,t} = \hat{G}_t\) and \(\hat{G}_{D,t} = \hat{G}_t\). Since both enter equation (91) the same way, it follows that either results in the same equilibrium allocation.

**Case 3: Generalization**

Figure 13 displays Home and Foreign GDP responses, as well as the terms of trade, following a 1% increase in government spending when we relax the parametric restrictions on \(\phi_N, \alpha^g_N,\) and \(\alpha^g_X\). Figure 13a shows Home GDP and the terms-of-trade responses are increasing in the share of public non-tradables and decreasing in the share of public imported tradables. A threshold exists such that Home GDP is greater than the closed economy provided the share of public imported tradables is not too high relative to public non-tradables. Figure 13b shows Home GDP and the terms-of-trade responses are increasing in the elasticity of substitution between tradables and non-tradables \((\phi_N)\). As \(\phi_N\) increases, private consumption goods are closer substitutes, softening the crowding out of private consumption and boosting GDP.
Figure 13. Impact responses following a 1% increase in government spending. In all cases, $G/Y = 0.2$, $\tau = 0.25$, $\omega = 1$, $\alpha_X = 0.3$, $\alpha_N = 0.6$, and $\phi = 1$. 

(a) $\phi_N = 0.75$

(b) $\alpha_X^q = 0.1$
Complete International Asset Markets

C Foreign Imports and Exports

D Data Description

The following data series mainly are taken from the Federal Reserve Economic Data (FRED), U.S. Bureau of Economic Analysis (BEA), and Statistics Canada CANSIM database. Exact sources are listed below.

1. **CPI**
   - **Canada:** Core CPI (CANSIM Table 176-0003), log transformed and first differenced to get domestic consumer inflation.
   - **U.S.:** Core CPI (FRED Series CPILFESL), log transformed and first differenced to get domestic consumer inflation.

2. **Population**
   - **Canada:** total population (CANSIM Table 051-0005).
   - **U.S.:** total civilian noninstitutional population (series CNP16OV from FRED database).

3. **GDP**
   - **Canada:** Gross domestic product at market prices (CANSIM Table 380-0064), deflated with CPI, divided by population, and log transformed.
   - **U.S.:** Gross domestic product (BEA Table 1.1.5), deflated with CPI, divided by population, and log transformed.

4. **Consumption**
   - **Canada:** Household final consumption expenditure at market prices on nondurables and services (CANSIM Table 380-0064), deflated with CPI, divided by population, and log transformed.
   - **U.S.:** Personal consumption expenditures on nondurables and services (BEA Table 1.1.5, lines 5 and 6), deflated with CPI, divided by population, and log transformed.

5. **Investment**
   - **Canada:** The sum of business gross fixed capital formation, investment in inventories, and

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36 A quarterly data series for working age population (15-64) is available from the OECD only starting in 1994. These two series correlation is above 0.99. Population by age groups is available at annual frequency in CANSIM.
household final consumption expenditure on durables and semidurables (CANSIM Table 380-0064), deflated with CPI, divided by population, and log transformed.

U.S.: Gross private domestic investment and personal consumption expenditures on durables (BEA Table 1.1.5, lines 4 and 7), deflated with CPI, divided by population, and log transformed.

6. **Government Spending**
   Canada: General governments final consumption expenditure plus gross fixed capital formation (CANSIM Table 380-0064), deflated with CPI, divided by population, and log transformed.
   U.S.: Government consumption expenditures and gross investment (BEA Table 1.1.5, line 22), deflated with CPI, divided by population, and log transformed.

7. **Income Tax Revenue**
   Canada: The sum of general government revenue on taxes on income, contributions to social insurance plans, and the sum of other current transfers (CANSIM Table 380-0080), deflated with CPI, divided by population, and log transformed.
   U.S.: Taxes on production and imports (BEA Table 3.1), deflated with CPI, divided by population, and log transformed.

8. **Consumption Tax Revenue**
   Canada: General government tax revenue on production and imports (CANSIM Table 380-0080), deflated with CPI, divided by population, and log transformed.
   U.S.: The sum of personal current taxes, taxes on corporate income, contributions for government social insurance, and taxes from the rest of the world (BEA Table 3.1), deflated with CPI, divided by population, and log transformed.

9. **Government Debt**
   Canada: General government market value of net financial assets (CANSIM Table 378-0121), multiplied by a minus sign, deflated with CPI, divided by population, and log transformed.
   U.S.: The end of period value of market debt (FRED series MVGFD027MNFRBDAL), deflated with CPI, divided by population, and log transformed.

10. **Hours Worked**
    Canada: Hours worked for total economy (CANSIM Table 383-0012), divided by population
and log transformed.

U.S.: Economy-wide total hours (BLS, \url{www.bls.gov/lpc/special_requests/us_total_hrs_emp.xlsx}),
divided by population and log transformed.

11. **Interest Rate**

   Canada: The quarterly average of interest rates on Treasury Bills for Canada divided by 4
   (FRED series INTGSTCAM193N).
   
   U.S.: The quarterly average of daily figures of the Federal Funds Rate (from the Board of
   Governors of the Federal Reserve System) divided by 4.

12. **Nominal Exchange Rate (Can/US$)**

   Nominal bilateral exchange rate (series DEXCAUS from FRED database).

13. **US Exports to Canada**

   We seasonally adjust the monthly figures of U.S. trade in goods with Canada reported by the
   Census Bureau and take quarterly averages. The series is then deflated with CPI, divided by
   population, and log transformed.

14. **US Imports from Canada**

   We seasonally adjust the monthly figures of U.S. trade in goods with Canada reported by the
   Census Bureau and take quarterly averages. The series is then deflated with CPI, divided by
   population, and log transformed.

15. **Number of US Exported Varieties to Canada**

   The number of distinct exported HS-level codes reported by the United States. Annual data
   from Schott (2008).

16. **Number of US Imported Varieties from Canada**

   The number of distinct imported HS-level codes reported by the United States. Annual data
   from Schott (2008).

E  **Quantitative Model Estimation Fit**

F  **Model with Endogenous Tradability and Firm Heterogeneity Details**

Table 2 lists the posterior estimates.
Table 2: Prior and Posterior Distributions for Estimated Parameters in the Model with Endogenous Tradability and Firm Heterogeneity.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior</th>
<th>Posterior</th>
<th>Prior</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Canada</td>
<td>U.S.</td>
</tr>
<tr>
<td></td>
<td>Dist *</td>
<td>Mean</td>
<td>Std.</td>
<td>Mean</td>
</tr>
<tr>
<td>Preferences</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>h, habit formation</td>
<td>B</td>
<td>0.5</td>
<td>0.1</td>
<td>0.47</td>
</tr>
<tr>
<td>ω, inverse Frisch</td>
<td>G</td>
<td>2</td>
<td>0.5</td>
<td>3.96</td>
</tr>
<tr>
<td>ωG, substitutability of private/public cons.</td>
<td>U</td>
<td>0</td>
<td>1.16</td>
<td>-0.17</td>
</tr>
<tr>
<td>φ, substitutability of home/foreign</td>
<td>U</td>
<td>1.52</td>
<td>0.85</td>
<td>1.18</td>
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<tr>
<td>Frictions and Production</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100 log z, growth rate</td>
<td>N</td>
<td>0.45</td>
<td>0.03</td>
<td>0.45</td>
</tr>
<tr>
<td>νK, investment adj. cost</td>
<td>N</td>
<td>4</td>
<td>1.5</td>
<td>4.77</td>
</tr>
<tr>
<td>ϕ, capital utilisation</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.86</td>
</tr>
<tr>
<td>ω, price stickiness</td>
<td>G</td>
<td>50</td>
<td>7.5</td>
<td>[38.63, 63.64]</td>
</tr>
<tr>
<td>ι, price partial indexation</td>
<td>B</td>
<td>0.5</td>
<td>0.15</td>
<td>0.59</td>
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<tr>
<td>Monetary Policy</td>
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<td></td>
</tr>
<tr>
<td>φρ, resp. to lagged interest rate</td>
<td>B</td>
<td>0.75</td>
<td>0.1</td>
<td>0.74</td>
</tr>
<tr>
<td>φκ, interest resp. to inflation</td>
<td>N</td>
<td>1.7</td>
<td>0.3</td>
<td>1.87</td>
</tr>
<tr>
<td>φγ</td>
<td>G</td>
<td>0.15</td>
<td>0.1</td>
<td>0.02</td>
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<tr>
<td>Fiscal Policy</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>γG, debt response for G</td>
<td>N</td>
<td>0.3</td>
<td>0.1</td>
<td>0.31</td>
</tr>
<tr>
<td>γf, debt response for f</td>
<td>N</td>
<td>0.3</td>
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<td>0.26</td>
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<tr>
<td>γC, debt response for C</td>
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<td>0.11</td>
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<tr>
<td>γTf, debt response for Tf</td>
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<td>0.3</td>
<td>0.1</td>
<td>0.31</td>
</tr>
<tr>
<td>eG, lagged response for G</td>
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<td>0.1</td>
<td>0.89</td>
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<td>0.1</td>
<td>0.80</td>
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<td>0.1</td>
<td>0.83</td>
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<td>0.2</td>
<td>0.25</td>
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<tr>
<td>Shock Processes</td>
<td></td>
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<td></td>
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<tr>
<td>ρA, risk premium</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.87</td>
</tr>
<tr>
<td>ρp, price mark-up</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.47</td>
</tr>
<tr>
<td>ρb, preference</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.72</td>
</tr>
<tr>
<td>ρB, hours supply</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.99</td>
</tr>
<tr>
<td>ρQ, subst home/foreign</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.98</td>
</tr>
<tr>
<td>ρZ, tpf growth</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.42</td>
</tr>
<tr>
<td>ρPK, investment</td>
<td>IG</td>
<td>1</td>
<td>1</td>
<td>0.32</td>
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<tr>
<td>100ρA, risk premium</td>
<td>IG</td>
<td>1</td>
<td>1</td>
<td>0.38</td>
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<tr>
<td>100ρp, price mark-up</td>
<td>IG</td>
<td>1</td>
<td>1</td>
<td>7.40</td>
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<td>100ρB, preference</td>
<td>IG</td>
<td>1</td>
<td>1</td>
<td>1.00</td>
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<tr>
<td>100ρB, hours supply</td>
<td>IG</td>
<td>1</td>
<td>1</td>
<td>3.48</td>
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<td>100ρQ, subst home/foreign</td>
<td>IG</td>
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<td>1</td>
<td>3.66</td>
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<td>100ρZ, tpf growth</td>
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<td>1</td>
<td>1</td>
<td>0.37</td>
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<td>100ρPK, investment</td>
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<td>1</td>
<td>0.94</td>
</tr>
<tr>
<td>100ρf, monetary policy</td>
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<tr>
<td>100ρG, gov spending</td>
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<tr>
<td>100ρf, income tax</td>
<td>IG</td>
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<tr>
<td>100ρG, transfer</td>
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<td>100ρX, fixed export cost</td>
<td>IG</td>
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</table>

*Dist: N: Normal; G: Gamma; B: Beta; U: Uniform; IG: Inverse Gamma.*