Uncertainty, Attention Allocation and Monetary Policy

Asymmetry*

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Abstract

We provide a theoretical framework and empirical evidence that the monetary policy effect is stronger when the economy faces heightened uncertainty about its productivity. Specifically, when the standard deviations of aggregate and idiosyncratic productivity increase after uncertainty shocks arrive, firms, which are constrained by the information capacity, allocate more attention to productivities and less to the monetary policy shock. In this case, firms under-react to the monetary policy shock, and this results in an increase in the real effect of the monetary policy shock. A threshold vector autoregression (TVAR), which incorporates instrumental variables to identify the monetary policy shock is used to provide empirical evidence on this reaction. The result shows that the monetary policy shock becomes more effective when uncertainty, measured by VIX, increases.

Key words: Monetary policy asymmetry, Uncertainty shock, Rational inattention

JEL classification: E31, E52, D8

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1 Introduction

There has been much empirical research on the existence of the asymmetry of monetary policy effects along the business cycles. In this literature, there have been mixed results regarding whether the monetary policy is more (or less) effective during recessions; it appears that there is no consensus yet on this question. These mixed results suggest that there may be a determinant, other than the business cycle itself, that can lead to a rise in the monetary policy asymmetry that is closely related to business cycles. In this paper, we demonstrate that uncertainty shocks, which are correlated with business cycles, can result in the monetary policy asymmetry. Then, we justify the findings of the theoretical model with a structural econometric analysis.

In this research, we investigate a novel channel that can explain the monetary policy asymmetry and show empirical evidence that provides a basis for the theoretical channel proposed in this paper. Although various properties or environments have been suggested as a mechanism that is capable of generating the monetary policy asymmetry, they can be summarized as the environments that can generate non-linear or state-dependent Phillips curves. In contrast to the previous research that has focused on the state-dependent behaviors and resulting transmitting mechanisms, we document that imperfect information and changes in the magnitudes of perceived fundamental shocks can be a source of the monetary policy asymmetry based on the rational inattention theory.

Imperfect information results in different consequences than the perfect information case. There is a large body of literature that documents different responses of macro variables to exogenous shocks when economic agents face imperfect information (Lucas, 1972; Sims, 2003; Woodford, 2003; Mankiw and Reis, 2002; Maćkowiak and Wiederholt, 2009). Therefore, agents may react to shocks differently as the structure of information changes; this would result in different propagations to shocks.

Thus, we study the macroeconomic consequences of changing the informational characteristics of exogenous shocks. Specifically, this paper documents that changes in uncertainty about exogenous shocks affect the propagation of these shocks. This environment is not an
exotic; a growing body of literature on the uncertainty shock can be one example of this environment. In this literature, the uncertainty shock is usually defined as the second moment shock to aggregate or idiosyncratic productivities in accordance with Bloom (2009). Facing these shocks, agents become less certain about their forecasts; this disturbs their actions under a perfect information environment through the wait-and-see channel, for instance. Under an imperfect information environment, this second moment shock can exert an additional propagation channel. Agents acquire and process information to determine the economic conditions via a certain type of learning mechanism facing imperfect information. When uncertainty shocks hit the economy, the effectiveness of the learning mechanism can be affected. For instance, agents use a Kalman filter to obtain the true state in a noisy information environment, and the Kalman gain, or equivalently the informational rigidity, can be changed when the variances of exogenous shocks and their noises shift; this can alter the propagations of exogenous shocks.

Imperfect information models can be divided into sticky information models and noisy information models as described in Coibion and Gorodnichenko (2012). In sticky information models, agents update their information sets infrequently. Conversely, agents revise their information sets continuously but receive only noisy signals about the true state in noisy information models. In this paper, we build a model based on noisy information assumption in two reasons. First, Coibion and Gorodnichenko (2012) show that the basic noisy information model à la Sims (2003) can explain the empirical regularities of survey expectations best. They derive conflicting predictions from various models of information rigidities and show that sticky information models cannot match some empirical regularities found in data. Second, it is straightforward to implement changes in uncertainty under a noisy information assumption. In sticky information models, fixed proportion of agents

\[1\] Berger et al. (2017) distinguish the uncertainty shocks and the realized volatility and find that uncertainty shocks which are defined as second moment news shocks do not affect the macroeconomic outcomes. This distinction can be applied in this research by manipulating the stochastic process of the second moment but this is not tried here as this will not change the main results while complicating the model and the solution method.

\[2\] Specifically, disagreement among agents should increase when shocks arrive in sticky information model whereas the degree of disagreement is independent of shocks in noisy information model. Coibion and Gorodnichenko (2012) show that data is consistent with predictions under noisy information model.
obtain perfect information about fundamentals each period. Therefore, the effect of changes in the second moment of shocks is limited. However, variations in the second moment can endogenously affect the rate with which agents process the noisy signals about fundamentals in noisy information models.

As the second moments of fundamentals evolve following the uncertainty shock, those of noises should also be changed. It is important to model how the second moments of noises evolve because they affect information flow. Thus, we introduce a rational inattention framework to discipline endogenous changes in information flow. The role of information flow on the monetary policy effect has been widely explored after a seminal paper of Lucas (1972) and the rational inattention framework can be considered as a micro foundation that gives a rise in endogenous and persistent imperfect information structure. The information flow constraint forces economic agents to allocate their attention. There has been empirical evidence that economic agents are actually inattentive to publicly available information and show incentive-based behavioral patterns predicted by rational inattention theory. (Kumar et al., 2015; Andrade and Le Bihan, 2013)

This paper shows analytically and numerically that monetary policy becomes more effective when uncertainty about productivity is relatively higher than that about monetary policy. Because this condition tends to hold during recessions, this result suggests that the monetary policy may be more effective during recession periods. However, this result also suggests one possible reason why empirical studies on this topic have provided mixed results. Because every recession is different from the others in that they have different volatilities of underlying shocks, attention allocations to shocks can vary. This variance can result in different dynamics of endogenous variables to the same shock because the composition of volatilities of underlying exogenous disturbances affects the attention allocation of agents.

In this paper, this is explained by firms’ optimal allocations of attentions. Firms take their attention away from the monetary policy and focus on acquiring more accurate information on productivities when they face an increase in uncertainty about their productivities. As firms are less attentive to monetary policy actions, they become less responsive to changes
in monetary policy. This can be considered as the mechanism that endogenously creating Lucas island for each firm by increasing the degree of information asymmetry between firms and the central bank. As a result, monetary policy effects on the output get stronger when the economy is hit by uncertainty shocks.

Then, we provide empirical evidence on the main result. A threshold vector autoregression model (TVAR) that switches between two regimes depending on the uncertainty measure is implemented. To identify the monetary policy shock, external instrumental variables are used following Gertler and Karadi (2015). The result confirms the theoretical prediction, and the monetary policy shocks are more effective when uncertainty rises. This result is robust regardless of the choices of the policy indicator variable, the instrumental variable and the uncertainty measure.

Literature

This paper is related to the three strands of literature. Primarily, this research is closely linked to the literature that documents the asymmetry of the monetary policy effects. The empirical research in this literature has provided somewhat mixed results on the existence and nature of the asymmetry. Specifically, the empirical works has focused on three types of asymmetry: (1) asymmetry related to the direction of the monetary policy, (2) asymmetry related to the phase of the business cycle, and (3) asymmetry related to the magnitude of the monetary policy shock. Most do not consider these types simultaneously and provide mixed results, in turn. Lo and Piger (2005) and Weise (1999) consider these three types in a unified framework and conclude that the second type of asymmetry is the most significant and policy actions taken during the recessions have larger effects. The previous research that focuses on the asymmetry related to the business cycle phase, such as Garcia and Schaller (2002), Kaufmann (2002), Höppner et al. (2008) and Peersman and Smets (2002), also find a similar result. Conversely, there have also been research that concludes the opposite. For example, Tenreyro and Thwaites (2016) and Thoma (1994) argue that the effectiveness of monetary policy on the output diminishes during recessions. In addition, while these research
focused on the asymmetry related to the business cycle phase, Aastveit et al. (2017) examine
the asymmetry that is closer in spirit to ours. They show that US monetary policy shocks
affect economic activity less when uncertainty is high. Our empirical methodology differs
from theirs as they identify the monetary policy shock by imposing an ordinary recursive
restriction or a sign restriction whereas we use external instruments.

There have been theoretical works that attempt to provide a rationale for monetary pol-
icy effect asymmetry. Ball and Mankiw (1994) provides an explanation for the asymmetry
related to the direction of monetary policy actions and shows that shocks that increase firms’
desired prices generate larger price responses than shocks that decrease desired prices in a
menu-cost model with a positive trend inflation. Bernanke and Gertler (1995) show that
monetary policy has stronger effects when the balance sheet channel augments and acceler-
ates the interest rate channel to the greatest extent as changes in short-term interest rate
affect the external finance premium. This effect is likely to be during recessions; therefore,
it predicts that monetary policy effect is asymmetric over the business cycle. Santoro et al.
(2014) provide the model that can explain the state-dependent Phillips curve and the asym-
metric response of output to monetary policy and investigate the empirical evidence that
the output gap shows more pronounced responses during economic contractions.

On the other hand, Vavra (2014) predicts the opposite; the monetary policy is less ef-
effective during recessions. This research finds that the dispersion of price changes is counter-
cyclical and the frequency of price adjustment comoves with the dispersion of price changes.
Combining these finding, the study concludes that the monetary policy becomes less effec-
tive during recessions as the nominal demand shock is absorbed by changes in price level.³
Bachmann et al. (2013) also find that firms change their prices more frequently when firm-
level volatility increases based on German micro data. The researchers also document that a
standard New Keynesian model predicts that monetary policy is less effective as uncertainty
increases. However, the authors also show that this change is quantitatively small.

³Klepacz (2016) also analyzes the price setting behavior using a Ss framework but finds that an increase in
the aggregate uncertainty decreases the effectiveness of the monetary policy by small amount; the response
of the output to the nominal shock decreases less than 1 percent. The result lends support for that of
Bachmann et al. (2013).
This research contributes to the rational inattention literature by extending the previous framework to incorporate the changes in volatilities. To the best of our knowledge, this paper is the first research that incorporates time-varying volatility in a general equilibrium macro model under rational inattention. There is minimal research that documents the relation between the attention allocation and time-varying volatility. In finance, Peng et al. (2007) empirically shows that the comovement of asset prices increases with the arrival of market wide macroeconomic shocks as investors are more attentive to the aggregate conditions. Zhang (2017) is closer to our research. By implementing a Markov switching factor augmented vector autoregression model (MS-FAVAR), it shows that a significant positive correlation between volatility and firms’ responsiveness to shocks exist.

Lastly, this research is also related to the literature on the propagation of uncertainty shocks. The role of the uncertainty shock as a potential driver of business cycles has been widely documented after the seminal work by Bloom (2009). This paper, however, does not focus on this role and lacks mechanisms that generates business cycles, such as a non-convex adjustment cost combined with irreversibility (Bloom et al., 2012) and financial friction with a costly state verification constraint (Christiano et al., 2014). Instead, we provide a new role of the uncertainty shock; it can result in an asymmetry of macro responsiveness to shocks when economic agents are rationally inattentive to shocks.

This paper is organized as follows. The information flow constraint and optimality of the signal are explained in Section 2. The micro-foundations of the model are introduced in Section 3. Section 4 contains analytical results based on the white noise case. Sections 5 describes the full model with persistent shocks and provides quantitative results. Section 6 connects the paper to other related papers. Specifically, we find that the model is capable of matching characteristics of individual price changes documented in Vavra (2014). It is also shown that the result remains robust if price rigidity is introduced in the model, although it attenuates the monetary policy effects as explained in the previous research. Empirical evidence from TVAR is provided in Section 7. Finally, Section 8 concludes.
2 Uncertainty shock and Information flow

In this paper, we quantify information as reduction in uncertainty which is measured by entropy following previous research on the rational inattention\footnote{This setup has been widely used in Economics after \cite{Sims2003}. See \cite{Cover2012} for details on Information theory.} The entropy of a discrete random variable $X$ can be expressed by

$$H(X) = -\sum_i p_i \log_2 p_i \tag{1}$$

where $p_i$ is a probability of a realization $i$. For instance, let $X$ be the outcome of tossing a fair coin. Then, the entropy of $X$ is simply $-\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2} = 1$. The entropy of a random variable $X$ which follows a normal distribution with standard deviation $\sigma$ is

$$H(X) = \frac{1}{2} \log_2 2\pi e \sigma^2 \tag{2}$$

The entropy of a random vector $X = (X_1, \ldots, X_T)$ that follows a multivariate normal distribution with covariance matrix $\Omega$ is given by

$$H(X) = \frac{1}{2} \log_2 (2\pi e)^T \det \Omega \tag{3}$$

The amount of information is quantified in ‘bit’ unit. One bit contains a one-digit number that can take the value of 0 or 1. Therefore, an information channel with one bit of channel capacity can transmit 0 or 1 in a given time interval. In the previous fair coin experiment, an information channel with one-bit capacity can totally resolve the uncertainty arises in the experiment. By transmitting 0 and 1 for head and tail, one can send information about the realized outcome and resolve uncertainty. In sum, uncertainty with entropy one can be resolved with one bit of information.

Similarly, conditional uncertainty is measured by conditional entropy. For example, the
conditional entropy of $X$ given $Y$ can be expressed as

$$H(X|Y) = \frac{1}{2} \log_2(2\pi e)^T \det \Omega_{X|Y}$$

(4)

where $\Omega_{X|Y}$ is the conditional covariance of $X$ given $Y$ and $X = (X_1, \ldots, X_T)$ and $Y = (Y_1, \ldots, Y_T)$ follow a multivariate normal distribution. Under aforementioned setup, the amount of information that signal $Y = (Y_1, \ldots, Y_T)$ contains about $X = (X_1, \ldots, X_T)$ can be defined as

$$I(X; Y) = H(X) - H(X|Y)$$

(5)

Based on the information theory outlined above, we define the information flow constraint as below.

**Definition** (Information flow). Let $X_t$ be the vector tracked by agents and $S_t$ be the signal vector tracking $X_t$. $S^t$ is defined as $S^t = (S_t, S_{t-1}, \ldots, S_0)$. Then, the information flow constraint is

$$H(X_t|S^{t-1}) - H(X_t|S^t) \leq \kappa$$

(6)

where $\kappa$ is the information capacity.

In words, the information flow constraint restricts the amount of information that agents can obtain from the additional signal at any given time. In some of previous research on the rational inattention, information flow between two stochastic processes is quantified as the average per-period amount of information that one process contains about another in asymptotic sense as following:

$$\lim_{T \to \infty} \frac{1}{T} I(X_1, \ldots, X_T; S_1, \ldots, S_T) \leq \kappa$$

(7)

Since these research assumed stationary processes, this limit is well-defined and asymptotic conditional variance is a constant. Therefore, above two characterizations are identical in stationary environments. \cite{Mackowiak2016} However, we will introduce uncertainty

\footnote{See \cite{Sims2003} and \cite{Mackowiak2009}, for instance.}
shocks and it results in a time-varying, history-dependent conditional variance for the tracked variables so we cannot apply the latter information flow constraint.

In this research, we will assume that the fundamental shocks follow AR(1) processes with normally distributed disturbances which are subject to the second moment shocks. Therefore, we will restrict our exposition on AR(1) case only. Since rational agents choose the forms of signals that they will receive in each period optimally, it will be valuable to derive the optimal signal that is relevant for our case. The proposition below provides that the optimal signal for an AR(1) process with time-varying volatility has the ‘target plus time-varying volatility noise’ form.

**Proposition 1** (Optimal signal form). For an AR(1) process, any optimal signal process can be expressed as \( S_t = X_t + \psi_t \) where \( \psi_t \sim N(0, \sigma_{\psi t}) \) and \( \sigma_{\psi t} = f(\sigma_t^X, \kappa) \).

**Proof.** See Appendix A.1.

### 3 Model

In this section, we implement the results discussed in the previous section into a version of New Keynesian model. The model period is a quarter. This model consists of three types of agents: representative household, monopolistically competitive firms, and government. In this research, we assume that only firms face the information flow constraint in two reasons. First, price setting behavior is a crucial determinant of the real effect of the monetary policy. As this paper examines the effect of the monetary policy confronting uncertainty shocks, we focus on the changes in firms’ price setting behavior under the information flow constraint. Second, this assumption can reduce complexity of the model, while the qualitative results remain largely unchanged. Specifically, Maćkowiak and Wiederholt (2015) showed that the qualitative results are similar under inattentive firm case and both inattentive household and firm case. This strategy was also pursued by Paciello (2012).^6

^6For the inattentive households case, the discrepancy between information sets held by households and firms continues to exist because they have different incentives to choose their information sets. Therefore, the assumption of complete information households can be considered a case in which this discrepancy is large.
3.1 Representative household

The representative household derives utility from the consumption and supplies differentiated labor variety. The representative household maximizes her lifetime utility.

$$E_t \sum_{s=0}^{\infty} \beta^s \left( \frac{C_{t+s}^{1-\gamma} - 1}{1 - \gamma} - \phi L_{t+s} \right)$$  \hspace{1cm} (8)

where $\beta$, $\gamma$, $\phi$, $C_t$ and $L_t$ denote the discount factor, inverse of inter-temporal elasticity of substitution, disutility of labor, aggregate consumption and aggregate labor supply, respectively. The representative household buys differentiated goods $C_{it}$ from the firms $i \in [0, 1]$ and consumes them by forming the aggregate good from the following technology:

$$C_t = \left( \int_0^1 C_{it}^{-1} \, di \right)^{\frac{1}{\theta}}$$  \hspace{1cm} (9)

The representative household supplies differentiated labor indexed by $j \in [0, 1]$:

$$L_t = \left( \int_0^1 L_{jt}^{-1} \, dj \right)^{\frac{1}{\eta}}$$  \hspace{1cm} (10)

$\theta$ and $\eta$ control the elasticity of substitution of differentiated consumption and labor. Since the representative household has market power in the labor market, she can set the wages for each variety of differentiated labor and supply any quantity of differentiated labor at the wage set by her. The representative household faces the following budget constraint and cash-in-advance constraint.

$$M_t + B_t = R_{t-1}B_{t-1} + P_t W_t L_t + D_t + (M_{t-1} - P_{t-1} C_{t-1})$$

$$P_t C_t \leq M_t$$  \hspace{1cm} (11)

We assume that the representative household supplies differentiated labor to grant market power in the labor market. This assumption simplifies the analysis as firms take wages as given.

This linear disutility of labor implies infinite Frisch elasticity. This is assumed for simplicity as this parameter plays no rule in the qualitative result of the paper.
$M_t$, $B_t$, $R_t$, $P_t$, $W_t$ and $D_t$ denote money holdings, nominal bond holdings, interest rate, aggregate price level, aggregate real wage, and profits transferred from monopolistically competitive firms, respectively. We assume that $R_t > 1$ for all $t$ such that the cash-in-advance constraint is always binding. We introduce the cash-in-advance constraint because the explicit mapping from the money supply to the nominal interest rate can be obtained from this constraint. Without this constraint, infinitely many equilibria exist in the model.\footnote{Firms choose the price level but this is indeterminate in the cashless economy. In order to determine the equilibrium, we need to introduce the quantity of money explicitly. This strategy is also followed by Paciello (2012).}

The optimal demand of good $i$ is given by

$$C_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\theta} C_t$$

where $P_{it}$ is the price of good $i$.

By log-linearizing the first order conditions and the cash-in-advance constraint and imposing the aggregate resource constraint $C_t = Y_t$, we can obtain following equilibrium relations:

$$y_t = E_t y_{t+1} - \frac{1}{\gamma} (r_t - E_t p_{t+1} + p_t)$$
$$w_t = \gamma y_t$$
$$p_t + y_t = m_t$$

(13)

### 3.2 Firms

Firms $i \in [0, 1]$ are subject to the following technology:

$$Y_{it} = e^{a_t} e^{z_{it}} L_{it}^\alpha$$

$Y_{it}$, $L_{it}$, $a_t$ and $z_{it}$ denote the output of firm $i$, labor demanded by firm $i$, aggregate productivity and idiosyncratic productivity of firm $i$. $\alpha$ is the labor share. We assume that idiosyncratic productivity is independent across firms. Aggregate and idiosyncratic productivities are subject to the uncertainty shock as in Bloom (2009) and Bloom et al. (2012).
In these previous research, the role of the second moment shock as a business cycle driver has been emphasized. However, this paper differs from those as we focus on changes in informativeness of signals and their implications on the role of the second moment shock as a determinant of asymmetric monetary policy effect along the business cycle. Therefore, we abstract from those mechanisms that generate business cycle fluctuations as it is not necessary to consider them to study the current topic.

Firms are subject to imperfect information. That is, they cannot directly observe aggregate and idiosyncratic productivities, monetary policy shock and aggregate variables. Firms’ information set consists of the history of current and past signals about aggregate and idiosyncratic productivity shock, monetary policy shock and the history of variances of these shocks.

\[ I_{it} = \{ S_{i}^{a}, S_{zi}^{a}, S_{iq}^{a}, S_{i-1}^{a}, S_{i-1}^{zi}, S_{i-1}^{iq}, \ldots, \sigma_{\varepsilon it}, \sigma_{\varepsilon zit}, \sigma_{\varepsilon qt}, \sigma_{\varepsilon at}, \sigma_{\varepsilon zit-1}, \sigma_{\varepsilon qt-1}, \ldots \} \]  

(15)

There is no physical rigidity when firms set prices of their own variety of goods. However, prices set by firms will show sluggish adjustments due to imperfect information, as will be shown later.

The real profit of the firm \( i \) is given by

\[ Y_{it} - \int_{0}^{1} W_{jt} L_{ijt} dj \]  

(16)

where \( W_{jt} \) and \( L_{ijt} \) are the real wage of the labor type \( j \) set by the household and labor type \( j \) demanded by the firm \( i \). The labor demand schedule can be derived from the firm’s optimality condition.

\[ L_{ijt} = \left( \frac{W_{jt}}{W_{t}} \right)^{-\eta} L_{it} \]  

(17)

Assuming \( L_{it} = \left( \int_{0}^{1} L_{ijt}^{-\eta} dj \right)^{-\frac{1}{\eta-1}} \), the above relation also implies that

\[ L_{jt} = \left( \frac{W_{jt}}{W_{t}} \right)^{-\eta} L_{t} \]  

(18)
by summing over $i$.

Since there are no physical rigidity or capital accumulation, firms’ profit maximization is a static problem.\footnote{As we are focusing on the asymmetric monetary policy effect which has a short horizon, we abstract from non-flexible inputs such as physical capital.} We use a second order Taylor approximation of the real profit function as below:}\footnote{The derivation of the real profit function can be found in Appendix B.}

\[
\text{Profit}_{it} \approx Y - WY^{\frac{1}{\alpha}} + Y(1 - \theta)p_t - WY^{\frac{1}{\alpha}} \left( -\frac{\theta}{\alpha} \right) p_t + \frac{1}{2} Y (1 - \theta)^2 p_t^2 \\
- \frac{1}{2} WY^{\frac{1}{\alpha}} \left( \frac{\theta}{\alpha} \right)^2 p_t^2 + Y(1 - \theta)(\theta - 1)p_t p_t - WY^{\frac{1}{\alpha}} \left( -\frac{\theta^2}{\alpha^2} \right) p_t^3 + Y (1 - \theta)p_t y_t \\
- WY^{\frac{1}{\alpha}} \left( -\frac{\theta}{\alpha^2} \right) p_t y_t - WY^{\frac{1}{\alpha}} \left( -\frac{\theta}{\alpha} \right) (1 - \eta)p_t w_{jt} - WY^{\frac{1}{\alpha}} \left( -\frac{\theta}{\alpha} \right) \eta p_t w_t \\
- WY^{\frac{1}{\alpha}} \left( \frac{\theta}{\alpha^2} \right) p_t (a_t + z_{it}) + \text{terms independent of } p_t
\]

(19)

Here, we use the facts that $P_i = P$ and $W_j = W$ for all $i$ and $j$ at the steady state.

We can obtain the profit maximizing price of firm $i$, $p^*_i$, by taking derivative of Equation 19 with respect to $p_t$ as below:

\[
\xi_1 p^*_i = \xi_1 p_t - \xi_2 y_t - \xi_3 w_t + \xi_4 (a_t + z_{it})
\]

(20)

where $\xi_1 = Y (1 - \theta)^2 - WY^{\frac{1}{\alpha}} \left( \frac{\theta}{\alpha^2} \right)^2$, $\xi_2 = Y (1 - \theta) - WY^{\frac{1}{\alpha}} \left( \frac{\theta}{\alpha} \right)$, $\xi_3 = WY^{\frac{1}{\alpha}} \left( \frac{\theta}{\alpha} \right)$ and $\xi_4 = WY^{\frac{1}{\alpha}} \left( \frac{\theta}{\alpha^2} \right)$. We use the fact that the household sets the same wage for all variety of labor on the equilibrium so that $w_{jt} = w_t$ holds. Rearranging and putting the intra-temporal Euler equation into Equation 20, the profit maximizing price can be written as

\[
p^*_i = p_t - \left( \frac{\xi_2 + \xi_3 \gamma}{\xi_1} \right) y_t + \left( \frac{\xi_4}{\xi_1} \right) (a_t + z_{it})
\]

(21)
By integrating Equation 21 over $i$, we can determine the aggregate price level as

$$p_i^\diamond = p_t - \left(\frac{\xi_2 + \xi_3\gamma}{\xi_1}\right)y_t + \frac{\xi_4}{\xi_1}a_t$$

(22)

Since firms face the information flow constraint, they do not know the exact value of the profit maximizing price. Instead, firms set their prices which are equal to the expectation of the profit maximizing price conditional on their information set $I_{it}$. That is, the price set by firm $i$ at time $t$, $p_{it}^*$, is given by

$$p_{it}^* = E[p_i^\diamond|I_{it}]$$

(23)

The loss in profit due to a suboptimal price can be approximated from the second order approximation to the profit function, Equation 19, as follows:

$$\text{Profit}_{it}(p_{it}^\diamond) - \text{Profit}_{it}(p_{it}^*) = -\frac{\xi_1}{2}(p_{it}^\diamond - p_{it}^*)^2$$

(24)

Firms choose their current and future stream of attention allocations to minimize the expected sum of discounted loss due to suboptimal price. Specifically, firms solve the following problem\footnote{In contrast to the price setting problem, the attention allocation problem is a dynamic problem because the choice of attention allocation at current period affects the attention allocations in future periods.}

$$\min_{I_{it}, I_{i_{t+1}}, \ldots} \sum_{j=t}^\infty \beta^{j-t}E\left[-\frac{\xi_1}{2}(p_{ij}^\diamond - p_{ij}^*)^2|I_{it}\right]$$

subject to $I_{it} \in \Omega_t$ for all $t$

where $\Omega_t$ is the set of all information sets satisfy the information flow constraint.

### 3.3 Monetary policy

The government sets the money supply according to the following rule:

$$\ln M_t = \phi_p \ln P_t + \phi_x \ln Y_t + \ln Q_t$$

(26)
\( Q_t \) represents the monetary policy shock. We implement the money supply rule for certain reasons. First, the implementation allows for a clear analytic solution of the model for the white noise case by combining with the cash-in-advance constraint. Second, a version of nominal interest rate rule can be approximated by the money supply rule. Specifically, the nominal interest rate can be obtained by combining the inter-temporal Euler equation with the money supply rule:

\[
    r_t = (\gamma \phi_p + (1 - \gamma)) E_t \pi_{t+1} + \gamma \phi_x E_t (y_{t+1} - y_t) + \gamma E_t (q_{t+1} - q_t) \tag{27}
\]

Lastly, this monetary policy rule is comparable to the rules considered in Mackowiak and Wiederholt (2009) and Woodford (2003) when we impose \( \phi_p = \phi_x = 0 \).

### 3.4 Information structure

Firms have imperfect information about the realizations of the exogenous processes \( \{a_t\} \), \( \{z_{it}\} \) and \( \{q_t\} \) since they face the information flow constraint. We assume that these exogenous processes are independent to each other. Then, signals about each process are also independent as noises are independent to each other. Based on this, the signal can be partitioned into three subvectors which contain only information about aggregate productivity, monetary policy shock and idiosyncratic productivity. Therefore, we can write the information flow constraint as

\[
    H(a_t|S_{a}^{t-1}) - H(a_t|S_{a}^{t}) + H(z_{it}|S_{zi}^{t-1}) - H(z_{it}|S_{zi}^{t}) + H(q_t|S_{q}^{t-1}) - H(q_t|S_{q}^{t}) \leq \kappa \tag{28}
\]

where \( S_{kt} \) is the signal about the process \( k \in \{a, z_i, q\} \) at time \( t \). In addition, we assume that the exogenous processes are normally distributed. In this paper, we only consider AR(1) or white noise exogenous processes with time-varying volatility. In these cases, signals of ‘true target plus noise’ form are optimal as shown in Proposition 1. Therefore, \( \{a_t, S_{at}\}, \{z_{it}, S_{zit}\} \) and \( \{q_t, S_{qt}\} \) are jointly normally distributed. Under this condition, we can rewrite the
information flow constraint as

\[ \frac{1}{2} \log_2 \left( \frac{\sigma_{at|t-1}^2}{\sigma_{at|t}^2} \right) + \frac{1}{2} \log_2 \left( \frac{\sigma_{qt|t-1}^2}{\sigma_{qt|t}^2} \right) + \frac{1}{2} \log_2 \left( \frac{\sigma_{zt|t-1}^2}{\sigma_{zt|t}^2} \right) \leq \kappa \]  

(29)

where \( \sigma_{kt|t}^2 \) and \( \sigma_{kt|t-1}^2 \) are the conditional variance of \( k_t \) given information up to time \( t \) and \( t - 1 \) for a process \( k \in \{a, z, q\} \). Since \( \{a_t, S_{at}\}, \{z_{it}, S_{zt}\} \) and \( \{q_t, S_{qt}\} \) are independent, we can treat paying attention to aggregate productivity, monetary policy shock and idiosyncratic productivity are separate activities and consider each activity separately. Let \( \kappa_1 \), \( \kappa_2 \) and \( \kappa_3 \) denote the information capacities devoted to aggregate productivity, monetary policy shock and idiosyncratic productivity. Then, the information flow constraint can be also expressed as

\[ \frac{1}{2} \log_2 \left( \frac{\sigma_{at|t-1}^2}{\sigma_{at|t}^2} \right) = \kappa_1, \quad \frac{1}{2} \log_2 \left( \frac{\sigma_{qt|t-1}^2}{\sigma_{qt|t}^2} \right) = \kappa_2, \quad \frac{1}{2} \log_2 \left( \frac{\sigma_{zt|t-1}^2}{\sigma_{zt|t}^2} \right) = \kappa_3 \]  

(30)

\[ \kappa_1 + \kappa_2 + \kappa_3 \leq \kappa \]

Firms need to choose the information set with which the loss due to suboptimal price is minimized. As the form of the optimal signal is known from Proposition 1, the problem of choosing the information set in Equation 25 boils down to choosing the variances of noises, or equivalently the attention allocated to each exogenous processes. As the information flow constraint gets looser and the information capacity increases, firms can obtain more accurate signals about these processes.

### 3.5 Equilibrium

The model is solved through a log-linearization of the first order conditions around the non-stochastic steady state. Solving this model requires solving for a fixed point as the attention allocation problem depends on the model dynamics and vice versa. We consider two cases in subsequent sections. First, we restrict our attention to white noise shock case in Section 4. In this case, the solution does not depend on the history of volatilities. Moreover, it is possible
to obtain analytic results. In Section 5, we allow persistent exogenous shocks. To obtain the numerical solution of the model, we truncate the lag length of the exogenous shocks that appear on the solution following Maćkowiak and Wiederholt (2009) and other research in this literature. We denote $\kappa_t^{t+i} = \{\kappa_{it}, \kappa_{it+1}, \kappa_{it+2}, \ldots\}$ as the collection of the current and future attention allocated for the activity $i$. Now, we define a competitive equilibrium as follows.

**Definition (Competitive equilibrium).** A competitive equilibrium consists of policy functions $c_t, l_t, p_t, m_t, b_t, r_t$ which depend on the aggregate states $a_t, q_t$ and history of variances and $y_{it}, l_{it}, p^\diamond_{it}, p^*_{it}$ which depend on the aggregate states, $z_{it}$ and the realizations of idiosyncratic signals and $\kappa_t^{t+1}, \kappa_t^{t+2}, \kappa_t^{t+3}$ which depend on the history of variances such that:

1. $\{c_t, l_t, w_t, m_t, b_t\}$ solves the household maximization problem
2. $\{\kappa_t^{t+1}, \kappa_t^{t+2}, \kappa_t^{t+3}\}$ solves firms’ optimal signal problem
3. $p^\diamond_{it}$ maximizes a firm’s profit and $p^*_{it}$ is the conditional expectation of this price
4. $p_t = \int_0^1 p^*_{it} di$, $c_t = \int_0^1 y_{it} di$ and $l_t = \int_0^1 l_{it} di$ hold

### 4 Analytic result: White noise case

In subsequent sections, we assess the impact of the uncertainty shock that increases the variances of productivities. We begin with the case that exogenous processes are white noise since it allows analytic results of the model and so helps reveal the mechanism underneath the results. Exogenous processes are defined as

$$a_t \sim N(0, \sigma_{at}^2), \quad q_t \sim N(0, \sigma_{qt}^2), \quad z_{it} \sim N(0, \sigma_{zt}^2)$$

(31)

where $\sigma_{at}^2, \sigma_{qt}^2$ and $\sigma_{zt}^2$ are time-varying variances. Then, signals about each exogenous process can be expressed as

$$S_{ait} = a_t + \psi_{ait}, \quad S_{qit} = q_t + \psi_{qit}, \quad S_{zit} = z_{it} + \psi_{zit}$$

(32)
where $\psi_{kit}$s are noises included in the signals and they are independent across firms. Noises are distributed as

$$\psi_{ait} \sim N(0, \sigma_{\psi_{ait}}^2), \quad \psi_{qit} \sim N(0, \sigma_{\psi_{qit}}^2), \quad \psi_{3it} \sim N(0, \sigma_{\psi_{3it}}^2)$$ (33)

The information flow constraint can be rewritten as

$$\frac{1}{2} \log_2 \left( \frac{\sigma_{ait}^2}{\sigma_{\psi_{ait}}^2} + 1 \right) + \frac{1}{2} \log_2 \left( \frac{\sigma_{qit}^2}{\sigma_{\psi_{qit}}^2} + 1 \right) + \frac{1}{2} \log_2 \left( \frac{\sigma_{3it}^2}{\sigma_{\psi_{3it}}^2} + 1 \right) \leq \kappa$$ (34)

We solve the model by guess and verify. We guess the solution of price and output as

$$p_t = \Gamma_1 a_t + \Gamma_2 q_t, \quad y_t = \Gamma_3 a_t + \Gamma_4 q_t$$ (35)

where $\Gamma_k$s are coefficients to be determined. Specifically, $\Gamma_k$s are time-varying coefficients because the responses of price and output depends on the attention allocated to each process as shown below. However, the entire problem is static and does not depend on the history; therefore, we omit the time subscript $t$ and treat the variances of exogenous processes as parameters. Putting the guesses into the profit maximizing price given in Equation 21, we can express the optimal price as the function of the exogenous processes as

$$p^*_it = \Delta_1 a_t + \Delta_2 q_t + \Delta_3 z_{it}$$ (36)

where $\Delta_1 = \Gamma_1 - \xi_2 + \xi_3 \gamma, \Delta_2 = \Gamma_2 - \xi_2 + \xi_3 \gamma, \Delta_3 = \xi_4 \xi_1$.

4.1 Perfect information case

For comparison, we solve the model under the perfect information case. In this case, firms are not constrained by attention allocation such that they can set the profit maximizing price. The result is summarized in the following proposition. The derivation of the solution

13They do not depend on idiosyncratic productivity as it is integrated out in the aggregate level.
Proposition 2 (Neutrality of money). Under the perfect information assumption, $\Gamma_4 = 0$ holds. Thus, money is neutral and does not have real effect.

Proof. We can obtain the solution by determining coefficients $\Gamma_1$, $\Gamma_2$, $\Gamma_3$ and $\Gamma_4$.

\[
\begin{align*}
\Gamma_1 &= \Gamma_1 - \frac{\xi_2 + \xi_3\gamma}{\xi_1} \Gamma_3 + \frac{\xi_4}{\xi_1}, \quad \Gamma_2 = \Gamma_2 - \frac{\xi_2 + \xi_3\gamma}{\xi_1} \Gamma_4 \\
\Gamma_3 &= \frac{\phi_p - 1}{1 - \phi_x} \left( \Gamma_1 - \frac{\xi_2 + \xi_3\gamma}{\xi_1} \Gamma_3 + \frac{\xi_4}{\xi_1} \right), \quad \Gamma_4 = \frac{\phi_p - 1}{1 - \phi_x} \left( \Gamma_2 - \frac{\xi_2 + \xi_3\gamma}{\xi_1} \Gamma_4 \right) + \frac{1}{1 - \phi_x}
\end{align*}
\]

Solving the system of equations, we can derive the solution as

\[
\begin{align*}
\Gamma_1 &= \frac{1 - \phi_x}{(1 - \phi_p)(\alpha - 1 - \gamma\alpha)}, \quad \Gamma_2 = \frac{1}{1 - \phi_p}, \quad \Gamma_3 = \frac{1}{1 - \alpha + \gamma\alpha}, \quad \Gamma_4 = 0
\end{align*}
\]

Therefore, changes in the money supply shock do not affect the aggregate level of output and money neutrality holds.

If we calibrate $\phi_x < 1$ and $\phi_p < 1$, the price level moves in the same direction with changes in the money supply shock, as expected by empirical evidence. Moreover, $\Gamma_1$ is less than zero and an increase in the aggregate productivity reduces the price level. Lastly, $\Gamma_3$ is greater than zero; therefore, the level of output rises as the aggregate productivity hikes.

4.2 Rational inattention case

Now we proceed to the rational inattention case in which firms face the information flow constraint. Firm $i$ sets its price as the conditional expectation of the profit maximizing price given above:

\[
p_{it}^* = E[p_{it}^* | I_{it}]
\]

\[
= \Delta_1 \frac{\sigma_{at}^2}{\sigma_{at}^2 + \sigma_{\psi at}^2} S_{ait} + \Delta_2 \frac{\sigma_{qt}^2}{\sigma_{qt}^2 + \sigma_{\psi qt}^2} S_{qit} + \Delta_3 \frac{\sigma_{zt}^2}{\sigma_{zt}^2 + \sigma_{\psi zt}^2} S_{zit}
\]

\[
= \Delta_1 \left(1 - 2^{-2\kappa_{it}} \right) S_{ait} + \Delta_2 \left(1 - 2^{-2\kappa_{zt}} \right) S_{qit} + \Delta_3 \left(1 - 2^{-2\kappa_{zt}} \right) S_{zit}
\]

\[
= \Delta_1 \left(1 - 2^{-2\kappa_{it}} \right) S_{ait} + \Delta_2 \left(1 - 2^{-2\kappa_{zt}} \right) S_{qit} + \Delta_3 \left(1 - 2^{-2\kappa_{zt}} \right) S_{zit}
\]
Combining Equation 24, 36 and 39 we can express the profit loss due to the suboptimal price setting in terms of coefficients, variances and attention allocated:

\[- \xi_1 \frac{1}{2} E \left[ (p^*_it - p^it)^2 \right] = - \xi_1 \frac{1}{2} \left[ \Delta_1^2 \sigma_{at}^2 2^{2-2\kappa_{1t}} + \Delta_2^2 \sigma_{qt}^2 2^{2-2\kappa_{2t}} + \Delta_3^2 \sigma_{zt}^2 2^{2-2\kappa_{3t}} \right] \quad (40)\]

At the beginning of each period \( t \), firms choose the optimal signals. As is proven in Proposition \[1\] the form of the optimal signal is known. Therefore, choosing the optimal signal reduces to choosing the variances of noises related to exogenous shocks. In this paper, we will focus on the interior solution when we solve the optimal attention allocation since corner solutions are unlikely to exist in reality as they imply that economic agents are not responsive to at least one of the exogenous shocks. Firms know that the form of the optimal signal is ‘true target plus noise’ such that firms only need to select the precisions of noises.

**Lemma (Optimal attention allocation).** Firms’ attention allocation that minimizes the profit loss due to the suboptimal price is given as

\[
\kappa^*_{1t} = \kappa + \frac{1}{3} \log_2 \Delta_1^2 \sigma_{at}^2 - \frac{1}{6} \log_2 \Delta_2^2 \sigma_{qt}^2 - \frac{1}{6} \log_2 \Delta_3^2 \sigma_{zt}^2 \\
\kappa^*_{2t} = \kappa - \frac{1}{6} \log_2 \Delta_1^2 \sigma_{at}^2 + \frac{1}{3} \log_2 \Delta_2^2 \sigma_{qt}^2 - \frac{1}{6} \log_2 \Delta_3^2 \sigma_{zt}^2 \\
\kappa^*_{3t} = \kappa - \frac{1}{6} \log_2 \Delta_1^2 \sigma_{at}^2 - \frac{1}{6} \log_2 \Delta_2^2 \sigma_{qt}^2 + \frac{1}{3} \log_2 \Delta_3^2 \sigma_{zt}^2 \quad (41)\]

**Proof.** See Appendix \[A.2\]

Note that \( \Delta_k \)s are also time-varying as they are determined by the allocation of attention at each period. The derivation and solution of the model are described in Appendix \[B\].

We begin the analyses by considering extreme values for attention allocations to obtain clear intuition. If we increase the information capacity to infinity, the solution should be the same as with the perfect information case. In this case, money neutrality should be restored.

---

\[1^{14}\] See Appendix \[A.2\] for its derivation.

\[15\] Under white noise assumption, attention choices at the current period do not affect the future information structure and firms’ behavior. Hence, it is sufficient to consider the loss due to suboptimal price at the current period.
Proposition 3. *As the information capacity approaches infinity, the solution converges to that of the perfect information counterpart, and money is neutral.*

*Proof.* Suppose $\kappa_{1t}^*$ and $\kappa_{2t}^*$ approach to infinity. Then,

\[
\lim_{\kappa_{1t}^* \to \infty} \Gamma_1 = \frac{1 - \phi_x}{(1 - \phi_p)(\alpha - 1 - \gamma \alpha)}, \quad \lim_{\kappa_{2t}^* \to \infty} \Gamma_2 = \frac{1}{1 - \phi_p} \tag{42}
\]

\[
\lim_{\kappa_{1t}^* \to \infty} \Gamma_3 = \frac{1}{1 - \alpha + \gamma \alpha}, \quad \lim_{\kappa_{2t}^* \to \infty} \Gamma_4 = 0
\]

holds. Since $\Gamma_4$ approaches to zero, money supply shock does not affect the real activity so that money is neutral.

It is also interesting to consider the environment that either $\kappa_{1t}^*$ or $\kappa_{2t}^*$ converges to zero. That is, firms do not allocate their attention to aggregate productivity or money supply shock at all. This behavior may occur when, for example, the volatility of idiosyncratic productivity increases tremendously compared to that of aggregate productivity or money supply shock.

Proposition 4. *As $\kappa_{1t}^*$ converges to zero, the price level does not respond to the aggregate productivity shock and the output level also does not respond to the aggregate productivity.*

*Proof.* Suppose $\kappa_{1t}^*$ approaches to zero. Then,

\[
\lim_{\kappa_{1t}^* \to 0} \Gamma_1 \to 0, \quad \lim_{\kappa_{1t}^* \to 0} \Gamma_3 \to 0 \tag{43}
\]

holds.

Because this shock originates from the supply side and firms do not utilize changes in the aggregate productivity at all, the aggregate productivity does not affect the economy.

Proposition 5. *As $\kappa_{2t}^*$ converges to zero, the price level does not respond to the monetary policy shock. Thus, the output level fully absorbs the money supply shock.*
Proof. Suppose $\kappa^*_2t$ approaches to zero. Then,

$$\lim_{\kappa^*_2t \to 0} \Gamma_2 \to 0, \quad \lim_{\kappa^*_2t \to 0} \Gamma_4 \to \frac{1}{1 - \phi_x}$$

holds. Therefore, money supply shock fully passes through the output level. \hfill \Box

To express the solution in exogenous variables only, we need to solve the fixed-point problem between Equation 41 and 77. The changes in the responsiveness to monetary policy shock to changes in uncertainty or volatility can be analyzed by taking derivatives of $\Gamma_4$ with respect to $\kappa_k$s. The price responsiveness to each shock, $\Delta_k$, varies as volatilities change but we treat them as fixed in the following comparative statics analysis. This treatment ignores the indirect effect of volatility change but qualitative results are not affected by this.

**Proposition 6.** As uncertainty, measured by variances of productivities $\sigma_{at}$, $\sigma_{zt}$, or both, increases, $\Gamma_4$ increases. That is, the responsiveness of the output to money supply shock grows stronger, and monetary policy becomes more effective.

Proof.

$$\frac{\partial \Gamma_4}{\partial \sigma_{at}^2} = \frac{\partial \Gamma_4}{\partial \kappa_2} \frac{\partial \kappa_2}{\partial \sigma_{at}^2} = \frac{2^{1-2\kappa_2} - \phi_x}{1 - \phi_x} \frac{1-\alpha+\alpha\gamma}{\alpha - \alpha\theta + \theta} \frac{1}{6\sigma_{at}^2} \left(2 - 2\kappa_2 + (1 - 2 - 2\kappa_2) \frac{1 - \phi_x}{1 - \phi_x} \frac{1-\alpha+\alpha\gamma}{\alpha - \alpha\theta + \theta}\right)^2 > 0$$

$$\frac{\partial \Gamma_4}{\partial \sigma_{zt}^2} = \frac{\partial \Gamma_4}{\partial \kappa_2} \frac{\partial \kappa_2}{\partial \sigma_{zt}^2} = \frac{2^{1-2\kappa_2} - \phi_x}{1 - \phi_x} \frac{1-\alpha+\alpha\gamma}{\alpha - \alpha\theta + \theta} \frac{1}{6\sigma_{zt}^2} \left(2 - 2\kappa_2 + (1 - 2 - 2\kappa_2) \frac{1 - \phi_x}{1 - \phi_x} \frac{1-\alpha+\alpha\gamma}{\alpha - \alpha\theta + \theta}\right)^2 > 0$$

Lastly, we provide a numerical result based on the analytical model derived above. The model parameters are given in Section 5. We solve the model numerically and show the relationship between the standard deviation of aggregate productivity and the attention allocated to the monetary policy shock. Then, the mapping between the standard deviation of aggregate productivity and the response coefficient of the output gap to the monetary policy shock $\Gamma_4$ is analyzed.
Figure 1: Attention allocations and the output gap response to the monetary policy shock over the range of standard deviation of the aggregate productivity

As the standard deviation of aggregate productivity increases, attention allocated to aggregate productivity increases while that to monetary policy shock and idiosyncratic shock decrease. Facing more volatile aggregate productivity, firms need to allocate more attention to aggregate productivity to process information about it. Otherwise, firms will suffer higher profit loss due to larger noise about aggregate productivity. At the same time, firms should reduce attention allocated to the monetary policy shock due to the information capacity constraint. Because firms cannot notice the exogenous change in the money supply in timely manner due to a decrease in attention allocated to the monetary policy, the real effect of the monetary policy shock increases as the standard
deviations of aggregate productivity shock increases.

The same argument holds for an increase in the standard deviation of idiosyncratic productivity as is shown in Figure 2. As the standard deviation of idiosyncratic shock increases, the attention allocated to the monetary policy shock decreases, and the responsiveness of the output gap to the monetary policy shock increases.

![Figure 2: Attention allocations and the output gap response to the monetary policy shock over the range of standard deviation of the idiosyncratic productivity](image)

5 Persistent Model

In this section, we extend the model to consider persistent exogenous shocks. First, the model is solved while assuming that there has been no uncertainty shock in the recent periods.
Then, we inject an uncertainty shock to this model and compare the model dynamics with and without uncertainty shock.

In this section, the exogenous processes are assumed to follow AR(1) processes:

\[ a_t = \rho_a a_{t-1} + \varepsilon_{at}, \quad q_t = \rho_q q_{t-1} + \varepsilon_{qt}, \quad z_{it} = \rho_z z_{it-1} + \varepsilon_{zit} \quad (47) \]

where disturbances are normally distributed as below:

\[ \varepsilon_{at} \sim N(0, \sigma_{at}), \quad \varepsilon_{qt} \sim N(0, \sigma_{qt}), \quad \varepsilon_{zit} \sim N(0, \sigma_{zit}) \quad (48) \]

The disturbance on the idiosyncratic productivity is independent across firms; however, the dispersion of the idiosyncratic productivity faced by each firm are synchronized and thus, identical.

Next, we describe the nature of the uncertainty shock. Specifically, we assume that the standard deviations of aggregate and idiosyncratic productivity follow AR(1) processes as below:

\[ \sigma_{at+1}^2 = (1 - \rho_a) \bar{\sigma}_{\varepsilon_a}^2 + \rho_a \sigma_{at}^2 + \varepsilon_{\sigma at+1}, \quad \sigma_{zt+1}^2 = (1 - \rho_z) \bar{\sigma}_{\varepsilon z}^2 + \rho_z \sigma_{zt}^2 + \varepsilon_{\sigma zt+1} \quad (49) \]

where the disturbances are distributed as

\[ \varepsilon_{\sigma at} \sim N \left(0, \frac{\sigma_{\varepsilon at}^2}{2}\right), \quad \varepsilon_{\sigma zt} \sim N \left(0, \frac{\sigma_{\varepsilon z}^2}{2}\right) \quad (50) \]

and \( \bar{\sigma}_{\varepsilon a}^2 \) and \( \bar{\sigma}_{\varepsilon z}^2 \) denote the steady state variances of the aggregate and idiosyncratic productivities.

The parameters are calibrated as in Table 1. The labor share \( \alpha \) and elasticity of substitution between differentiated goods \( \theta \) are borrowed from Paciello (2012), which builds the similar underlying model. There values are also in comfortable ranges in Macroeconomic literature. The value of \( \beta \) is also common in the literature. The inverse of intertemporal

\[ \text{We assume that two processes have the same autoregressive coefficient.} \]
elasticity is assumed to be 3. The value varies from 1 to 5 in the literature. In this paper, we choose 3 to guarantee that the infinite sums of higher-order expectations are well-defined.

The money supply rule that endogenously reacts to the output, and the price is not common in the literature. In this reason, we estimated the money supply rule using M1, the output gap and the de-trended GDP deflator index from 1982Q3 to 2007Q4. Due to the endogeneity problem, we implemented 2SLS estimation. The result is given below.

$$\hat{m}_t = -0.25y_t - 4.87p_t$$

\[ t\text{-values are given in parentheses. This rule shows that M1 decreases as the price level increases above the trend and the output gap increases. We import the estimated coefficients into the model.} \]

The information capacity \( \kappa \) is chosen based on two criteria. First, a calibrated \( \kappa \) should result in the impulse responses that are in accordance with the empirical analyses. Specifically, \( \kappa \) should be sufficiently large such that the attention allocation to each shock should be strictly greater than zero. Hence, \( \kappa \) should be greater than 5, approximately. Second, this parameter controls the degree of information rigidity. Hence, we can discipline this from data that hints to the degree of information rigidity. \cite{Coibion_and_Gorodnichenko_2012} report that it takes approximately three quarters to reduce the forecast error by a half. Figure 5 shows that the distance between prices under full information and \( \kappa = 5 \) case decreases by half in three quarters after the monetary policy shock hits the economy. Based on these observations, we choose \( \kappa = 5 \) as our baseline calibration.\footnote{We check the robustness of the result by choosing different values of \( \kappa \) later.}

The parameters that govern the evolutions of exogenous processes are largely obtained from \cite{Bloom_et_al_2012}, except those of the monetary policy shock. \( \rho_q \) and \( \sigma_q \) are estimated from the residuals obtained from the above regression. \cite{Bloom_et_al_2012} calibrated their model as the duration of the high uncertainty regime becomes 12.5 quarters, on average. The calibrated value of \( \rho_\sigma \) implies that the uncertainty shock remains less than 1 percent after 12.5 quarters.
Preference and technology parameters

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Exogenous processes parameters

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Table 1: Parameter calibration

5.1 Solution method

Under an imperfect information environment, optimal individual choices are determined by expected aggregate variables such as aggregate price level and/or output level based on that individual’s information set. Because aggregate dynamics are determined by the other agents, who use the different information set to behave optimally, it is necessary to address higher-order expectations to solve the model. We apply the solution method provided by Woodford (2003).

First, we solve the model under a ‘stationary’ case in that there has been no uncertainty shock in the recent periods; therefore, the variances remain fixed at the steady state level. Then, we inject an uncertainty shock to this model under a stationary history and analyze the transition of endogenous variables. We compute the solution when an uncertainty shock hits the economy at time \(t\) assuming that the economy has a stationary history until time \(t - 1\). We compute the impulse response functions to exogenous fundamental shocks in addition to an arrival of the uncertainty shock. In this section, we assume that the standard

\[^{18}\text{This method is also used by Paciello (2012).}\]

\[^{19}\text{In this paper, we use terms ‘stationary’ and ‘low uncertainty’ interchangeably. Similarly, ‘uncertain’ and ‘high uncertainty’ are interchangeable.}\]

\[^{20}\text{Numerically, it is sufficient to assume that the uncertainty shock has not been there for more than, approximately, 20 quarters under the baseline calibration.}\]
deviations of aggregate and idiosyncratic productivity become 1.91 and 3.33 times larger, respectively, than the stationary periods when an uncertainty shock hits the economy. These values are borrowed from Bloom et al. (2012). See Appendix C for details about solving the model with persistent shocks.

5.2 Results

First, we discuss how the variations in information capacity change the dynamics of endogenous variables such as output gap, price level and nominal interest rate. In this experiment, we simulate the model under the stationary environment.

![Figure 3: Attention allocation](image)

Figure 3 depicts how attention allocations change as the information capacity varies. We find observations as follows.

**Observation 1.** *Firms allocate their attention to the idiosyncratic productivity the most regardless of the total capacity.*

That is, the profit loss due to inaccurate information about the idiosyncratic productivity
is relatively larger than the aggregate productivity and monetary policy. We can consider two reasons for this finding. First, price elasticities to the aggregate and idiosyncratic shock are greater than that to the monetary policy shock. Therefore, the same amount of noises about productivities affect the price level more than that of monetary policy shock. Second, the unconditional variance of the idiosyncratic shock is much larger than that of the aggregate productivity and monetary policy.

Observation 2. Firms allocate their attention to the monetary policy shock the least regardless of the total capacity. However, the proportion of attention allocated to the monetary policy increases as the total capacity increases.

The unconditional variance of the monetary policy shock is greater than that of the aggregate productivity; however, it continues to obtain the least amount of attention. That is because the price elasticity to the monetary policy shock is much smaller than those of the other shocks. However, the proportion of attention allocated to the monetary policy shock grows as the total information capacity increases. As the attention allocated to one activity increases, the marginal gain from additional information decreases. That is, a portion of additional information has different gains conditional on the current information amount. Increasing information is more valuable when the current information is smaller. In this regard, firms allocate more attention to productivities when the total capacity is small. However, as the total capacity increases, allocating more attention to productivities becomes less beneficial and firms begin to allocate more attention to the monetary policy shock.

Observation 3. Firms under-react to the aggregate productivity shock when the information capacity constraint binds.

Figure 4 presents how impulse responses to the one standard deviation negative aggregate shock change over different amounts of total information capacity. In general, the results obtained from the white noise model continues to hold. When the total capacity is zero, all endogenous variables do not respond to an increase in the aggregate productivity. Because firms are not aware of an increase in the aggregate productivity at all, they do not change
Figure 4: Impulse responses to the one standard deviation negative aggregate productivity shock

the price, and households remain at the current consumption level as nothing has been changed in their constraints. Conversely, firms adjust their price immediately when they have perfect information.\textsuperscript{21} Therefore, households can consume more, and consumption hikes. As we decrease the total capacity, firms’ initial reactions decrease, and it takes longer to dissipate information. Firms under-react to current shocks because the current signals are contaminated by noises, and firms need more information to discriminate the shocks from noises. As time passes, firms receive more signals and information about the past shocks gets

\textsuperscript{21}We treat $\kappa = 100$ case as perfect information benchmark as the magnitudes of noises are extremely small.
more accurate. In this regard, the reactions become closer to those in the perfect information case.

Impulse responses show that firms under-react to the current aggregate productivity shock when the information capacity constraint is binding. Firms delay their reactions due to imperfect information until information is processed. Therefore, the price and the output gap show hump-shaped behavior and this becomes clearer as the total capacity \( \kappa \) decreases. The one standard deviation negative aggregate productivity shock decreases the output gap nearly one percent, whereas this increases the price level by approximately 0.2 percent under perfect information. Note that these endogenous variables achieve their full effects immediately. In contrast, they achieve their full effects in the latter periods when the information capacity binds. Specifically, when \( \kappa = 5 \), the maximum effect of the output gap arrives in three quarters. At this period, the output gap decreases approximately 0.75 percent.

**Observation 4.** The information capacity constraint can produce the real effect of the monetary policy shock. Specifically, the real effect strengthens as the total information capacity decreases.

Figure 5 shows impulse responses to the one standard deviation positive monetary policy shock for a different amount of total information capacity. As is predicted by the white noise model, money is neutral when firms have sufficiently large information capacity. Firms are fully aware of the monetary policy shock and adjust their price by one-to-one immediately under perfect information. Therefore, an increase in the quantity of money is absorbed by an increase in price, and the consumption level is not affected in the \( \kappa = 100 \) case. On the other hand, the monetary policy shock affects the output the most when the firms’ information capacity is zero. Firms do not react to an increase in the quantity of money and maintain the price level. Therefore, households can purchase more by the amount equivalent to an increase in the quantity of money. Firms adjust their prices slower as the total information capacity decreases. This finding implies that the real effect of monetary policy shock becomes greater as the information capacity decreases as predicted in the white noise model. It is noteworthy
Figure 5: Impulse responses to the one standard deviation positive monetary policy shock that the real effect of the monetary policy shock is generated by imperfect information. This result is reminiscent of Lucas (1972) and Mankiw and Reis (2002, 2007) as money non-neutrality is derived by stickiness in information. The rational inattention theory can provide a theoretical basis for the existence of endogenously determined information stickiness.

The monetary policy shock generates hump-shaped reactions to the price level under imperfect information. Impulse responses clearly show that firms under-react to the monetary policy shock when they set their prices. Under perfect information, the output gap does not respond to the monetary policy shock whereas the output gap increases approximately 0.6 percent under the baseline calibration ($\kappa = 5$) when one standard deviation positive money supply shock arrives.

Lastly, we present how the uncertainty shock changes the dynamics of endogenous vari-
In this section, we will consider the baseline model assuming $\kappa = 5$ but main results will continue to hold with different values of $\kappa$.\footnote{See Appendix D.1 for detailed discussion.}

<table>
<thead>
<tr>
<th></th>
<th>$\kappa_{1t}$</th>
<th>$\kappa_{2t}$</th>
<th>$\kappa_{1t+1}$</th>
<th>$\kappa_{2t+1}$</th>
<th>$\rho_{\kappa_1}$</th>
<th>$\rho_{\kappa_3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stationary</td>
<td>0.6996</td>
<td>0.4511</td>
<td>0.6996</td>
<td>0.4511</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uncertainty shock</td>
<td>1.6448</td>
<td>0.0082</td>
<td>1.6641</td>
<td>0.0151</td>
<td>0.5156</td>
<td>0.1811</td>
</tr>
</tbody>
</table>

Table 2: Change in attention allocation

Figure 6: Change in attention allocation

**Observation 5.** *When the uncertainty shock arrives, firms are less attentive to the monetary policy shock.*

We proceed by presenting the changes in attention allocation. As explained in Appendix C, instead of solving for the entire future sequences of the attention allocations, we assume that the attention allocations return to their stationary levels in constant rates after first two quarters. Table 2 shows how attention allocation is changed after an uncertainty shock hits the economy at time $t$. When uncertainty shocks hit the economy, the standard deviations

\footnote{See Appendix D.1 for detailed discussion.}
of aggregate and idiosyncratic productivity increased considerably. Due to the uncertainty shock, firms allocate significantly more attention to the aggregate productivity whereas they are less attentive to the monetary policy and the idiosyncratic productivity shock. Specifically, Figure 6 presents the future expected sequences of $\kappa_{it}$. Attention to the aggregate productivity increases in the first two periods then declines to the stationary level. Attention to the monetary policy shock decreases significantly in the first period then returns to the stationary level slowly. Lastly, attention to the idiosyncratic shock decreases slightly at the first two periods then returns to the stationary level in a few periods.

Figure 7: Impulse responses when the uncertainty shock arrives. The first row contains impulse responses to one standard deviation negative aggregate productivity shock, and the second row contains impulse responses to one standard deviation positive monetary policy shock.

**Observation 6.** The real effect of the monetary policy shock becomes larger when an uncertainty shock arrives.
Figure 7 contains impulse responses to the one (steady state level) standard deviation negative aggregate productivity shock and positive monetary policy shock.23 Since attention allocated to the aggregate productivity increases, both the price and the output gap are more responsive to the aggregate productivity shock. This result suggests that rational inattention may be a possible contribution to well-known business cycle asymmetry regarding steepness and deepness. Sichel (1993) defines ‘steepness’ as referring contractions are steeper than expansions. Similarly, ‘deepness’ refers an asymmetry that troughs are deeper than peaks are tall. When the economy is hit by an uncertainty shock, firms respond more strongly to the aggregate shock. Hence, the output gap drops more in faster speed after hit by the same magnitude of shock as is evident in Figure 7. Because uncertainty tend to increase during recessions, this mechanism is capable of explaining business cycle asymmetry regarding steepness and deepness.

Impulse responses to the monetary policy shock also differ considerably when uncertainty about productivities changes, because attention allocated to the monetary policy shock shows sizeable drop. When the uncertainty shock arrives, firms take their attention away from the monetary policy shock. Hence, the price response becomes muted. In this regard, households can increase their consumption more compared to the stationary environment as an increase in their money holdings is less neutralized by a smaller price change. Specifically, the output gap increases 0.2 percent point more when the economy is hit by the uncertainty shock compared to the stationary environment. Moreover, the monetary policy shock has a prolonged effect when the productivities are more uncertain. In sum, the real effect of the monetary policy shock in addition to the uncertainty shock is greater and persistent than that under the stationary environment.

As it is clear from Appendix D.1 these results are robust with different parameterization of $\kappa$. The monetary policy shock exerts a stronger and more persistent effect on the output gap when the uncertainty shock hits the economy regardless of the information capacity. However, the discrepancy of impulse responses between the stationary and the uncertain environment becomes smaller as the information capacity increases.

23We feed the same magnitude of shock to both stationary and uncertainty shock cases.
6 Discussion

In this section, we compare the result with that of Vavra (2014) and Bachmann et al. (2013) which document apparently contradictory results to this paper. In this section, we document that this paper actually complements these research; they focus on the cyclical property of the price change frequency but the benchmark model in this paper assumes flexible price. However, this model can continue to match the other dimensions of individual price micro data. It is also shown that introducing procyclical price stickiness generates the opposite effect and the monetary policy becomes less effective, as previous research assert. However, attention reallocation channel is quantitatively more important and dominates the price stickiness channel.

We begin by showing that the statistics of individual price changes generated from our model are in line with empirical findings in Vavra (2014). Next, the result in Vavra (2014) that the monetary policy is less effective on stimulating the output originates from more frequent price adjustments during high uncertainty periods. Hence, we introduce an ad hoc price stickiness to match the changes in price adjustment frequency observed in data and show that only focusing on the price adjustment frequency may be misleading.

First, we simulate the model to obtain the set of individual price changes. As a benchmark, we simulate the persistent model discussed in Section 5. The simulated result from the white noise model is also contained in Appendix E. The qualitative results are consistent with the benchmark simulation. The benchmark model is simulated for 2000 firms and 100 periods. The parameters are calibrated as in Table 1 except for the correlations among the second moment shocks and aggregate productivity shock. Previously, we assume independence among all shocks but need correlations to match the data. For brevity, we assume that the second moment shocks to the aggregate and idiosyncratic productivities are perfectly and positively correlated. Following Bloom et al. (2012), the aggregate productivity shock is generated to match the target correlation coefficient of -0.458 between the uncertainty shock series and the output series.

An example of simulated aggregate and idiosyncratic price (Firm 1) level is given in
As Maćkowiak and Wiederholt (2009) documented, the individual price changes are more volatile than the aggregate price changes. Moreover, the actual aggregate price level is more sluggish than the aggregate price under perfect information. Specifically, the standard deviation of the aggregate price level under perfect information is approximately 10 percent greater than that of the actual aggregate price level. However, the difference between two series are small. That is, the economy-wide profit loss due to the information flow constraint is not substantial. This result is also in line with Maćkowiak and Wiederholt (2009).

Figure 9 depicts the distributions of the price changes. As Vavra (2014) documented, the mode of the distribution moves to the left when recessions arrive. In addition, the standard deviation of price changes in recessions is 8.4 percent greater than that in expansions. Despite the simple construction of the model, the simulated price changes can match data well in this dimension. Vavra (2014) reports that the correlation coefficient between the cross sectional...
standard deviation of the standardized price changes and the industrial production is -0.33, while this model provides the correlation of -0.21 which is close to data. Hence, we can conclude that this model can explain the changes in price change dispersion observed in the data.

Next, we introduce an ad hoc price stickiness in our model to show how the changes in price adjustment frequency affect the real effect of the monetary policy shock. Specifically, we explicitly impose following price change rule:

\[ p_t = \eta p_{t-1} + (1 - \eta) p^*_t \]  

(52)

where \( \eta \) controls the price stickiness. In the baseline model, there is no price stickiness (\( \eta = 0 \)) and the price is always equal to the conditional expectation of profit maximizing price (\( p_t = p^*_t \)). Although we do not introduce a micro foundation for this price stickiness,
we impose this restriction to gauge the effect of changes in the price adjustment frequency.

Figure 10: Changes in the price adjustment frequency: The top and bottom panels are impulse responses to the one standard deviation negative aggregate productivity and positive monetary policy shock. The panels in the first and second column contrast the impulse responses from the baseline stationary environment (Flexible) with those under sticky price adjustment assumption (Sticky price). In these impulse responses, $\eta = 0.67$ is assumed. The third column presents the baseline impulse responses and the impulse responses obtained by imposing price adjustment frequency changes. $\eta$ is assumed to be 0.684 and 0.677 when the economy is in its stationary and uncertain periods, respectively.

Figure 10 depicts how introducing price stickiness changes the model dynamics. The bottom panels show the impulse responses to the monetary policy shock. Overall, introducing price stickiness delays the price changes and dampens the magnitude of the price changes. As predicted, price stickiness amplifies the real effect of the monetary policy. Therefore, this result confirms that an increase in price adjustment frequency can mitigate the real effect of the monetary policy.

Next, we impose the realistic frequency parameter $\eta$ across different levels of uncertainty
and analyze the quantitative significance of the frequency changes. Specifically, we assume that $\eta$ equals 0.684 and 0.677 during low and high uncertainty periods, respectively. These values originate from a German micro data documented in [Bachmann et al. (2013)]. The last column of Figure 10 contains the result. In both cases, introducing price stickiness amplifies the real effect of the monetary policy. The price adjustment frequency increases during high uncertainty periods, but the monetary policy remains more effective during these periods. That is, this result shows that changes in attention allocation are quantitatively more important than the changes in price adjustment frequency. This result is robust under a different parameterization. Even if we assume an extremely high $\eta$ in the stationary environment, say 0.9, the result does not change, and the monetary policy is more effective to boost the output during high uncertainty periods. This result continues to hold even the information capacity is very close to the perfect information case.\footnote{See Appendix D.3 for details}

This finding reconciles the result from this paper and those from [Vavra (2014)] and [Bachmann et al. (2013)]. First, we find that more frequent price changes can mitigate the real effect of the monetary policy as argued in [Vavra (2014)]. However, the focus of our exposition is orthogonal to the mechanism explained in that paper. In this paper, we introduce a different channel that connects changes in volatilities and the real effect of the monetary policy. Second, it is shown that changes in price adjustment frequency affect the effectiveness of the monetary policy but the quantitative significance of that channel is questionable. This finding is also in accordance with the result provided in [Bachmann et al. (2013)].

7 Empirical evidence

In this section, we provide empirical evidence that the monetary policy effect is asymmetric along the business cycle due to the information flow constraint as discussed in previous sections. Thus, the impulse responses of macro variables to monetary policy shocks are computed. There has been empirical evidence that monetary policy shocks exert different impacts on macroeconomic variables over the different phases of the business cycles. We add
to the existing literature by investigating the influence of uncertainty to cyclical asymmetries in monetary policy effects.

Our empirical methodology is a monthly threshold vector autoregression (TVAR, hereafter) with threshold variables regarding economic uncertainty. In addition, we use external instruments to identify monetary policy shock following Gertler and Karadi (2015).

7.1 Econometric model

The structural form of the TVAR that we are considering is given as

\[ A^1 I(w_{t-d} > w_{\text{thres}})Y_t + A^2 I(w_{t-d} < w_{\text{thres}})Y_t = \left( \sum_{j=1}^{p} C^1_j Y_{t-j} \right) I(w_{t-d} > w_{\text{thres}}) + \left( \sum_{j=1}^{p} C^2_j Y_{t-j} \right) I(w_{t-d} < w_{\text{thres}}) + \varepsilon_t \] (53)

where \( Y_t, w_t \) and \( \varepsilon_t \) are macro variables used in VAR, the threshold variable and structural shocks. We also allow possible delay, captured by parameter \( d \), in the threshold variable affecting the timing of switching. Following Gertler and Karadi (2015), \( p = 12 \) is chosen. The reduced form can be obtained by pre-multiplying \( A^{-1} \) matrix in each case.

\[ Y_t = \sum_{j=1}^{p} B^1_j Y_{t-j} + u^1_t, \text{ if } w_{t-d} > w_{\text{thres}} \]  
\[ Y_t = \sum_{j=1}^{p} B^2_j Y_{t-j} + u^2_t, \text{ if } w_{t-d} < w_{\text{thres}} \] (54)

where \( B^i_j = (A^i)^{-1} C^i_j \) and \( u^i_t = (A^i)^{-1} \varepsilon_t = S^i \varepsilon_t \). Note that TVAR is piecewise linear within each regime. The variance-covariance matrix of the reduced form shocks is given as

\[ E[u^i_t u^i_t'] = E[S^i S^i'] = \Sigma^i \] (55)

The usual triangular assumption is not plausible in this model because this model contains a financial variable and it is difficult to impose a contemporaneous restriction. As the theoretical model involves changes in volatilities of exogenous variables, one may question whether the identification strategy from Brunnermeier et al. (2016) is available. That model is impossible to implement, however, in this paper because their strategy utilizes heteroskedasticity to identify the homogeneous contemporaneous matrix that results in the same endogenous dynamics across different volatility regimes.
Let $y^p_t \in Y_t$ be the policy indicator variable with the associated policy shock $\varepsilon^p_t$. Following Gertler and Karadi (2015), we allow a possible distinction between the policy instrument (e.g., Federal Funds rate) and the policy indicator because this can capture shocks to forward guidance in the measure of the policy innovation when a government bond rate with longer maturity is used as a policy indicator.

Let $s^i$ denote the column in matrix $S^i$ associated with impact on the reduced form residual $u^i_t$ of the policy shock $\varepsilon^p_t$. Therefore, we need to estimate the following regression for each $i$ to obtain the impulse responses to the monetary policy shock.

\[
Y_t = \sum_{j=1}^{p} B^1_j Y_{t-j} + s^1 \varepsilon^p_t, \text{ if } w_{t-d} > w_{\text{thres}}
\]

\[
Y_t = \sum_{j=1}^{p} B^2_j Y_{t-j} + s^2 \varepsilon^p_t, \text{ if } w_{t-d} < w_{\text{thres}}
\]  (56)

Because computing the impulse responses to other shocks is beyond the scope of this paper, we can avoid identifying all the coefficients of $S$.

Then, we specify how external instruments are incorporated in this empirical study. After estimating the arranged reduced form TVAR models given the threshold, we introduce external instruments for the monetary policy shock to identify $s^i$ matrices. Let $Z_t$ be instrumental variables and $\varepsilon^q_t$ be a structural shock other than the policy shock. To obtain the estimate of $s^i$, we apply the two stage least squares (2SLS) method for each arranged regression $i$. First, compute estimates of reduced form residuals $u^i_t$ by OLS regressions of each arranged reduced form VAR. Then, let $u^{pi}_t$ be the reduced form residual from the equation for the policy indicator and $u^{qi}_t$ be the reduced form residuals from the remaining equations. In addition, let $s^{qi}$ and $s^{pi}$ be the responses of $u^{qi}_t$ and $u^{pi}_t$ to a unit increase in the policy shock $\varepsilon^p_t$. Then, it is possible to obtain an estimate of the ratio $s^{qi}/s^{pi}$ from the 2SLS regressions of $u^{qi}_t$ on $u^{pi}_t$, using the instrumental variables $Z_t$.

\[
u^{qi}_t = \frac{s^{qi}}{s^{pi}} \hat{u}^{pi}_t + \xi_t
\]  (57)
where $\hat{u}_{t}^{\pi i}$ is the fitted variable from the first stage regressing $u_{t}^{\pi i}$ on the instrument $Z_{t}$. Lastly, an estimate for $s^{\pi i}$ can be derived from the estimated reduced form variance-covariance matrix using Equation 55 and 57. Then, we can automatically obtain $s^{q i}$. Given estimates of $s^{i}$ and $B_{j}^{i}$, we can derive the impulse responses to the monetary policy shock.

Following [Gertler and Karadi (2015)], the set of potential external instruments consists of surprises in Federal Funds rate futures on FOMC dates. To ensure that the surprises in futures rates reflect only news about the FOMC decision, these shocks are constructed within a thirty minute window of the FOMC announcement.

### 7.2 Data

We borrow macro and instrumental variables data from [Gertler and Karadi (2015)]. The sample period covers from 1979:07 to 2012:06. The main VAR variables include log industrial production, log CPI, excess bond premium from [Gilchrist and Zakrajšek (2012)] and the policy indicator. In this paper, we consider two potential policy indicators; the Federal Funds rate (FF, hereafter) and one-year government bond yield (GS1, hereafter). The potential instrument set consists of the surprises in the current month’s Federal Funds futures (FF1, hereafter) and the three month ahead Federal Funds futures (FF4, hereafter). Because instruments are available after 1991:01, which is shorter than the sample period for VAR variables, we use the instrument and reduced form residuals from the corresponding period to identify the vector $s^{i}$. The choice of the instrumental variable originates from [Gertler and Karadi (2015)]. Specifically, policy indicators, the Federal Funds rate and one-year government bond yield are instrumented by FF1 and FF4, respectively.

Next, the potential set of uncertainty indicators that will be used as the threshold indicator includes monthly average VIX, the growth rate of monthly average VIX and lagged VIX and VIX growth rate. The lagged variables are included to incorporate possible delayed effect ($d > 0$) in TVAR model. Following [Bloom (2009)], we combined VIX index computed by the Chicago Board Options Exchange (CBOE) and monthly average volatility of daily S&P 500 return. VIX index is available from 1990, and we take monthly average in our sample.
Before 1990, we calculated monthly return volatility as the monthly standard deviation of the daily S&P 500 normalized to the same mean and variance as the VIX index between 1990 and 2003. The correlation between two volatility measures during the overlapping period is 0.86. The daily S&P 500 return series are taken from Bloom (2009).

### 7.3 Results

The estimated results show that the log-likelihood of the TVAR model is maximized when the lagged VIX and the Federal Fund rate are used as the uncertainty and policy indicators. Therefore, we provide impulse responses of log industrial production to the monetary policy shock associated with the lagged VIX and the Federal Fund rate as the uncertainty and policy indicators as the baseline result. The lagged VIX growth rate and one-year government bond yield are also used as alternatives, and the results are robust. The impulse responses associated with the lagged VIX growth and one-year government bond yield can be found in Appendix D.2.

Figure 11 presents the impulse responses of log industrial production to the monetary policy shock with the lagged VIX uncertainty indicator, the Federal Funds rate policy indicator and the current month Federal Funds futures instrument. The top panel is the impulse response from the pooled simple VAR. This finding clearly shows that a monetary policy contraction decreases the output significantly with some delays. Approximately, the monetary policy shock has statistically significant effect after 20 months through 40 months and reaches the trough in 30 months. The middle panel shows impulse response within high uncertainty regime, which corresponds to periods with the uncertainty indicator greater than the threshold. Overall, this finding is very similar to the top panel; the monetary policy shock becomes statistically significant after 20 months through 40 months and get to the trough in 30 months. However, the magnitude is much more amplified when the uncertainty indicator is higher. The trough is approximately 3.9 times deeper than the top panel. The bottom panel depicts impulse response within low uncertainty regime. Overall, this finding is not statistically significant and the estimated response is close to zero. Hence, this suggests
Figure 11: Impulse responses of log IP to the monetary policy shock with the uncertainty indicator VIX(-1), the policy indicator FF and the instrument FF1; the top panel depicts the impulse responses from the simple VAR without regime shifting. The middle and bottom panels represent the impulse responses within high and low uncertainty regime. The dashed lines show bootstrap 90 percent confidence intervals.

that the monetary policy is more effective during high uncertainty periods and supports the main result in this paper.

It is still questionable whether the threshold variable that proxies economy-wide uncertainty captures changes in uncertainty well. For instance, an increase in VIX index may be contaminated by recessionary effect as VIX and the business cycle comove. In this case, responses during high uncertainty periods may partially capture recessionary effects. To check the robustness, we isolated changes in uncertainty by projecting VIX index onto the
industrial production gap and use the orthogonal part as the threshold variable in the baseline specification. Figure 18 in Appendix D.2 contains the result of this regression. This supports the previous result and the difference in responses across uncertainty regimes is originated from changes in uncertainty, not from changes in business cycle phases.

8 Conclusion

In this paper, we provide a theoretical framework and empirical evidence that the monetary policy is more effective during high uncertainty periods. When firms are constrained by the information flow capacity, they cannot observe the exogenous disturbances with perfect accuracy and need to allocate their information capacity to different kinds of disturbances. In this circumstance, firms reallocate their attention more to productivities when the uncertainty shock hits the economy. Hence, it becomes more difficult to disseminate the monetary policy shock, and the monetary policy can affect the real activity stronger. Intuitively, this mechanism can be interpreted as endogenously creating the Lucas (1972) island and changing the speed of resolving the information friction.

Based on the theoretical prediction, we build a TVAR that incorporates external instrumental variables to identify the monetary policy shock. The monetary policy has stronger and more prolonged effect when the uncertainty measure, lagged VIX or VIX growth, is greater than the threshold.

In the future research, some extensions may be considered. Aligning the information set of households to that of firms is the first possibility. Having different information sets is prevalent in the literature and the assumption in this paper is an extreme example of this kinds. Imposing a more reasonable information set will be able to provide more reliable quantitative results. Second, incorporating the central bank signal channel will be an important topic. The signaling effect of the monetary policy gets more attention in the literature recently. If the central bank’s signals can affect uncertainty related to the course of monetary policy, this may also affect the attention allocation of private agents and change the effectiveness of the monetary policy.
References


A Proofs

A.1 Proof of Proposition 1

The proof is due to Maćkowiak et al. (2016). Let $\xi_t$ be the vector that current and sufficiently large number of past $X_t$ are stacked. That is, $\xi_t = [X_t, X_{t-1}, X_{t-2}, \ldots, X_{t-m}]'$. $\xi_t$ and the signal $S_t$ can be expressed as following state-space form:

$$\begin{align*}
\xi_{t+1} &= F\xi_t + v_{t+1} \\
S_t &= G'\xi_t + \psi_t
\end{align*}$$

The first row of $F$ describes the law of motion of $X_t$ and the rest of rows depict identity relations $X_{t-1} = X_{t-1}$, $X_{t-2} = X_{t-2}$, and so on. The vector $v_{t+1}$ only contains one element, the independent disturbance with time-varying variance to $X_t$, at the first row. The matrix $G'$ controls the weights on the current and past $X_t$s on the signal. $\psi_t$ is the noise term. Let $\Sigma_{t|t-1}$ and $\Sigma_{t|t}$ denote the conditional covariance matrix of $\xi_t$ given information set $I$ up to $t-1$ and $t$ respectively. The information set is defined as $I_t = \{S_t, \sigma_{xt+1}\} \cup I_{t-1}$. That is, the information set contains the history of variances of underlying shocks and signals received.

By applying Kalman filter, we can obtain followings:

$$\begin{align*}
E(\xi_t | I_{t-1}) &= FE(\xi_{t-1} | I_{t-1}) \\
y_t &= S_t - G'E(\xi_t | I_{t-1}) \\
Var(\xi_t | I_{t-1}) &= FVar(\xi_{t-1} | I_{t-1})F' + Var(v_t) = \Sigma_{t|t-1} \\
Var(S_t | I_{t-1}) &= G'Var(\xi_t | I_{t-1})G + Var(\psi_t) \\
K_t &= Var(\xi_t | I_{t-1})G(Var(S_t | I_{t-1}))^{-1} \\
E(\xi_t | I_t) &= E(\xi_t | I_{t-1}) + K_t y_t \\
Var(\xi_t | I_t) &= (I - K_t G') Var(\xi_t | I_{t-1}) = \Sigma_{t|t}
\end{align*}$$

(59)
Conditional normality implies that

$$ H(\xi_t|I_{t-1}) - H(\xi_t|I_t) = \frac{1}{2} \log_2 \left( \frac{\det \Sigma_{t|t-1}}{\det \Sigma_{t|t}} \right) $$

(60)

Split \( \xi_t \) into two sub-vectors \( \xi_{t}^{up} \) and \( \xi_{t}^{down} \). Let \( \xi_{t}^{up} = X_t \) and \( \xi_{t}^{down} \) consists of the remaining. The variance of \( \xi_{t}^{up} \) given \( I_t \) is the upper-left (1 \times 1) sub-matrix of \( \Sigma_{t|t} \). The objective of signal choice problem is solving the following:

$$ \min E \left[ (X_t - E(X_t|I_t))^2 | I_t \right] $$

(61)

Note that the objective above is exactly the same with the upper-left (1 \times 1) sub-matrix of \( \Sigma_{t|t} \). Due to the entropy chain rule, the information flow constraint can be written as

$$ H(\xi_{t}^{up}|I_{t-1}) - H(\xi_{t}^{up}|I_t) + H(\xi_{t}^{down}|I_{t-1}, \xi_{t}^{up}) - H(\xi_{t}^{down}|I_t, \xi_{t}^{up}) \leq \kappa $$

(62)

Suppose we have two signals that yield the same upper-left (1 \times 1) sub-matrix of \( \Sigma_{t|t} \). One signal is given as \( S_t = X_t + \psi_t \) and the other signal is a linear combination of the current and past \( X_t \) plus noise \( \psi_t \). Note that both signals imply the same value of the objective because the objective is the upper-left (1 \times 1) sub-matrix of \( \Sigma_{t|t} \). The difference \( H(\xi_{t}^{down}|I_{t-1}, \xi_{t}^{up}) - H(\xi_{t}^{down}|I_t, \xi_{t}^{up}) \) is non-negative, since acquiring additional information weakly reduces entropy and it is equal to zero if and only if \( S_t = X_t + \psi_t \) holds. Therefore, the second signal is associated with strictly more information flow while both signals result in the same value of the objective.

Next, we show that for any signal that is not \( X_t + \psi_t \), there exists a signal which is \( X_t + \psi_t \) that yields the same upper-left (1 \times 1) sub-matrix of \( \Sigma_{t|t} \). For any covariance matrices \( \tilde{\Sigma}_{t|t-1} \) and \( \tilde{\Sigma}_{t|t} \), there exists a signal which generates the posterior covariance matrix \( \tilde{\Sigma}_{t|t} \) from the prior covariance matrix \( \tilde{\Sigma}_{t|t-1} \) if and only if \( \tilde{\Sigma}_{t|t-1} - \tilde{\Sigma}_{t|t} \) is positive semi-definite. Consider a signal that is not \( X_t + \psi_t \) and yields the covariance matrix \( \tilde{\Sigma}_{t|t-1} \) and \( \tilde{\Sigma}_{t|t} \). Then, \( \tilde{\Sigma}_{t|t-1} - \tilde{\Sigma}_{t|t} \) must be positive semi-definite as \( \Sigma_{t|t} \) is generated from \( \Sigma_{t|t-1} \) by the signal. The upper-left
(1 × 1) sub-matrix of $\Sigma_{t|t-1} - \Sigma_{t|t}$ is also positive semi-definite and it implies that there exists a signal $X_t + \psi_t$ that generates the upper-left (1 × 1) sub-matrix of $\Sigma_{t|t}$ from the upper-left (1 × 1) sub-matrix of $\Sigma_{t|t-1}$.

Lastly, we show that the variance of the noise term $\sigma^2_{\psi_t}$ is the function of the information capacity $\kappa$ and the history of current and past variance of $X_t$. It can be shown from the information flow constraint.

\[
\kappa = H(X_t|I_{t-1}) - H(X_t|I_{t}) = \frac{1}{2} \log_2 \frac{\det \Sigma_{t|t-1}}{\det \Sigma_{t|t}}
\]

\[
2\kappa = \log_2 \det \Sigma_{t|t-1} - \log_2 \det \Sigma_{t|t}
= \log_2 \det \Sigma_{t|t-1} - \log_2 \det(I - K_tG') \Sigma_{t|t-1}
= \log_2 \det \Sigma_{t|t-1} - \log_2 \det(I - K_tG') \det \Sigma_{t|t-1}
= - \log_2 \det(I - K_tG')
\]

\[
2^{2\kappa} = [\det(I - K_tG')]^{-1}
= [\det(I) \det(1 - G'K_t)]^{-1}
= [1 - G'V\text{ar}(\xi_t|I_{t-1})GV\text{ar}(S_t|I_{t-1})^{-1}]^{-1}
\]

\[
2^{-2\kappa} = 1 - \text{V\text{ar}}(X_t|I_{t-1}) \text{V\text{ar}}(X_t + \psi_t|I_{t-1})^{-1}
\]

\[
1 - 2^{-2\kappa} = \frac{\text{V\text{ar}}(X_t|I_{t-1})}{\text{V\text{ar}}(X_t|I_{t-1}) + \text{V\text{ar}}(\psi_t)}
\]

\[
\text{V\text{ar}}(\psi_t) = (1 - 2^{-2\kappa})^{-1} \text{V\text{ar}}(X_t|I_{t-1}) - \text{V\text{ar}}(X_t|I_{t-1})
= \frac{2^{-2\kappa}}{1 - 2^{-2\kappa}} \text{V\text{ar}}(X_t|I_{t-1})
\]

Therefore, the variance of the noise can be obtained from $\kappa$ and the history of current and past variance of $X_t$ recursively.
A.2 Proof of Lemma 4.2

$$-\frac{\xi_1}{2} E \left[ (p_{it} - p_{it}^*)^2 | I_t \right] = -\frac{\xi_1}{2} E \left[ (\Delta_1 (2^{-2\kappa_1 t} a_t - (1 - 2^{2\kappa_1 t}) \psi_{1it}) + \Delta_2 (2^{-2\kappa_2 t} q_t - (1 - 2^{2\kappa_2 t}) \psi_{2it}) \right.
\left. + \Delta_3 (2^{-2\kappa_3 t} z_{it} - (1 - 2^{2\kappa_3 t}) \psi_{3it}) \right]^2 | I_t$$

\[= -\frac{\xi_1}{2} \left[ \Delta_1^2 2^{4\kappa_1 t} \sigma_{a_t}^2 + \Delta_1^2 (1 - 2^{-2\kappa_1 t})^2 \sigma_{\psi_{1it}}^2 + \Delta_2^2 2^{4\kappa_2 t} \sigma_{q_t}^2 \right.
\left. + \Delta_2^2 (1 - 2^{-2\kappa_2 t})^2 \sigma_{\psi_{2it}}^2 + \Delta_3^2 2^{4\kappa_3 t} \sigma_{z_{it}}^2 + \Delta_3^2 (1 - 2^{-2\kappa_3 t})^2 \sigma_{\psi_{3it}}^2 \right]
\]

\[= -\frac{\xi_1}{2} \left[ \Delta_1^2 2^{4\kappa_1 t} \sigma_{a_t}^2 + \Delta_1^2 (1 - 2^{-2\kappa_1 t})^2 \frac{\sigma_{a_t}^2}{2^{2\kappa_1 t} - 1} + \Delta_2^2 2^{4\kappa_2 t} \sigma_{q_t}^2 \right.
\left. + \Delta_2^2 (1 - 2^{-2\kappa_2 t})^2 \frac{\sigma_{q_t}^2}{2^{2\kappa_2 t} - 1} + \Delta_3^2 2^{4\kappa_3 t} \sigma_{z_{it}}^2 + \Delta_3^2 (1 - 2^{-2\kappa_3 t})^2 \frac{\sigma_{z_{it}}^2}{2^{2\kappa_3 t} - 1} \right]
\]

\[= -\frac{\xi_1}{2} \left[ \Delta_1^2 \sigma_{a_t}^2 2^{-2\kappa_1 t} + \Delta_2^2 \sigma_{q_t}^2 2^{-2\kappa_2 t} + \Delta_3^2 \sigma_{z_{it}}^2 2^{-2\kappa_3 t} \right] \tag{64} \]

Firms minimize the profit loss given the information flow constraint

$$\kappa_{1t} + \kappa_{2t} + \kappa_{3t} \leq \kappa \tag{65}$$

Since the information flow constraint should be binding at the equilibrium, we can replace \(\kappa_{3t}\) in Equation \(64\) with \(\kappa - \kappa_{1t} - \kappa_{2t}\). Taking the derivatives of Equation \(64\) with respect to \(\kappa_{1t}\) and \(\kappa_{2t}\), we can derive the first order conditions:

$$\Delta_1^2 \sigma_{a_t}^2 2^{-2\kappa_1 t} = \Delta_2^2 \sigma_{q_t}^2 2^{-2(\kappa - \kappa_{1t} - \kappa_{2t})}, \quad \Delta_2^2 \sigma_{q_t}^2 2^{-2\kappa_2 t} = \Delta_3^2 \sigma_{z_{it}}^2 2^{-2(\kappa - \kappa_{1t} - \kappa_{2t})} \tag{66}$$

\[55\]
B Detailed derivations

B.1 Firm’s profit function

Firm $i$’s real profit function is given by

$$
Profit_{it} = \frac{P_{it}}{P_t} Y_{it} - \int_0^1 W_{jt} L_{ijt} dj
$$

$$
= \frac{P_{it}}{P_t} \left( \frac{P_{it}}{P_t} \right)^{-\theta} Y_t - \int_0^1 W_{jt} L_{ijt} dj
$$

$$
= P_{it}^{1-\theta} P_t^{\theta-1} Y_t - \int_0^1 W_{jt}^{1-\eta} W_t^{\eta} L_{ijt} dj
$$

$$
= P_{it}^{1-\theta} P_t^{\theta-1} Y_t - \int_0^1 W_{jt}^{1-\eta} dj W_t^{\eta} \left( \frac{Y_{it}}{e^{at} e^{x_{it}}} \right)^{\frac{1}{\alpha}}
$$

$$
= P_{it}^{1-\theta} P_t^{\theta-1} Y_t - \int_0^1 W_{jt}^{1-\eta} dj W_t^{\eta} P_{it}^{\frac{-\theta}{\alpha}} P_t^{\frac{\theta}{\alpha}} Y_t^{\frac{1}{\alpha}} e^{-\frac{\alpha + 2 \alpha t}{2}}
$$

$$
= P_{it}^{1-\theta} e^{(1-\theta) \rho_{it}} P_t^{\theta-1} e^{(\theta-1) \rho_t} Y e^{y_t}
$$

$$
- \int_0^1 W_{jt}^{1-\eta} e^{(1-\eta) w_{jt}} dj W_t^{\eta} w_t P_{it}^{\frac{-\theta}{\alpha}} e^{-\frac{\theta}{\alpha} \rho_{it}} P_t^{\frac{\theta}{\alpha}} e^{\frac{\theta}{\alpha} \rho_t} Y_t^{\frac{1}{\alpha}} e^{\frac{1}{\alpha} y_t} e^{-\frac{\alpha t + x_{it}}{\alpha}}
$$

The second, third and fifth line follow from the demand schedule of differentiated good and labor and the fourth line originates from the production function. Lastly, the sixth line is expressed in terms of log-deviations from the steady state where letters without subscript $t$ denote the steady state values, and lower case letters represent log-deviations from the steady state.
B.2 Profit loss due to a suboptimal price

\[
\text{Profit}_{it}(p^*_it) - \text{Profit}_{it}(p^it) = \left\{ \frac{1}{2} Y(1 - \theta)^2 - \frac{1}{2} W Y^\frac{1}{\alpha} \left( \frac{\theta}{\alpha} \right)^2 \right\} (p^it - p^2it) \\
+ \left\{ Y(1 - \theta)(\theta - 1) - W Y^\frac{1}{\alpha} \left( -\frac{\theta^2}{\alpha^2} \right) \right\} p_t(p^it - p^*_it) \\
+ \left\{ Y(1 - \theta) - W Y^\frac{1}{\alpha} \left( -\frac{\theta}{\alpha} \right) \right\} y_t(p^it - p^*_it) \\
- W Y^\frac{1}{\alpha} \left( -\frac{\theta}{\alpha} \right) w_t(p^it - p^*_it) \\
- W Y^\frac{1}{\alpha} \left( \frac{\theta}{\alpha^2} \right) (a_t + z_t)(p^it - p^*_it) \\
= \frac{\xi_1}{2}(p^o^2it - p^{*2}it) - \xi_1 p^o^it(p^o^it - p^*_it) = -\frac{\xi_1}{2}(p^o^it - p^*_it)^2
\]

(68)

B.3 Solving the perfect info white noise model

Integrating Equation 36 over \(i\), we can obtain the aggregate price level as

\[
p_t = p^\circ_t = \left( \Gamma_1 - \frac{\xi_2 + \xi_3 \gamma}{\xi_1} \Gamma_3 + \frac{\xi_4}{\xi_1} \right) a_t + \left( \Gamma_2 - \frac{\xi_2 + \xi_3 \gamma}{\xi_1} \Gamma_4 \right) q_t
\]

(69)

Combining the cash-in-advance constraint and the money supply rule, the aggregate output level can be expressed as

\[
y_t = y^\circ_t = m_t - p^\circ_t = \phi_p p^\circ_t + \phi_x y^\circ + q_t - p^\circ_t
\]

(70)

Solving for \(y_t\), we can express the aggregate output as the function of exogenous processes as

\[
y_t = \frac{\phi_p - 1}{1 - \phi_x} p^\circ_t + \frac{1}{1 - \phi_x} q_t \\
= \frac{\phi_p - 1}{1 - \phi_x} \left( \Gamma_1 - \frac{\xi_2 + \xi_3 \gamma}{\xi_1} \Gamma_3 + \frac{\xi_4}{\xi_1} \right) a_t + \left\{ \frac{\phi_p - 1}{1 - \phi_x} \left( \Gamma_2 - \frac{\xi_2 + \xi_3 \gamma}{\xi_1} \Gamma_4 \right) + \frac{1}{1 - \phi_x} \right\} q_t
\]

(71)
B.4 Solving the rational inattention white noise model

Integrating equation 39 over $i$, the aggregate price level can be derived as

$$p_t^* = \Delta_1 \left(1 - 2^{-\kappa_1^*}t\right) a_t + \Delta_2 \left(1 - 2^{-2\kappa_2^*}t\right) q_t$$  \hspace{1cm} (72)

due to the independent idiosyncratic productivity and signal assumptions. Therefore, we can impose the following on the coefficients that need to be determined.

$$\Gamma_1 = \left(\Gamma_1 - \frac{\xi_2 + \xi_3 \gamma}{\xi_1} \Gamma_3 + \frac{\xi_4}{\xi_1}\right) \left(1 - 2^{-2\kappa_2^*}t\right)$$  \hspace{1cm} (73)

Combining the cash-in-advance constraint and the money supply rule, the aggregate output level is given by

$$y_t = y_t^* = m_t - p_t^* = \phi_p p_t^* + \phi_x y_t^* + q_t - p_t^*$$  \hspace{1cm} (74)

Solving for $y_t^*$, we can derive the aggregate output as the function of exogenous processes and attention allocation as

$$y_t^* = \frac{\phi_p - 1}{1 - \phi_x} p_t^* + \frac{1}{1 - \phi_x} q_t = \frac{\phi_p - 1}{1 - \phi_x} (\Gamma_1 a_t + \Gamma_2 q_t) + \frac{1}{1 - \phi_x} q_t$$  \hspace{1cm} (75)

Therefore, we can obtain the following relations to determine the coefficients.

$$\Gamma_3 = \frac{\phi_p - 1}{1 - \phi_x} \Gamma_1, \quad \Gamma_4 = \frac{\phi_p - 1}{1 - \phi_x} \Gamma_2 + \frac{1}{1 - \phi_x}$$  \hspace{1cm} (76)
Combining Equation 73 and 76, we can fully determine the coefficients.

\[
\begin{align*}
\Gamma_1 &= \frac{1 - 2^{-2\kappa_{1t}^*}}{(\alpha\theta - \theta - \alpha)2^{-2\kappa_{1t}^*} + (1 - 2^{-2\kappa_{1t}^*})(\alpha - 1 - \alpha\gamma)\frac{1-\phi_p}{1-\phi_x}} \\
\Gamma_2 &= \frac{1}{1-\phi_x} + \frac{2^{-2\kappa_{1t}^*}}{1-\phi_x} + \frac{1-\phi_p}{1-\phi_x} \\
\Gamma_3 &= \frac{1}{\phi_{p}^{-1}}\frac{1-\phi_p}{1-\phi_x} (\alpha\theta - \theta - \alpha)2^{-2\kappa_{1t}^*} - (1 - 2^{-2\kappa_{1t}^*})(\alpha - 1 - \alpha\gamma) \\
\Gamma_4 &= \frac{1}{2^{-2\kappa_{1t}^*} + (1 - 2^{-2\kappa_{1t}^*})\frac{1-\phi_p}{1-\phi_x} \frac{1-\alpha + \alpha\gamma}{1-\alpha + \theta + \theta}}
\end{align*}
\] (77)

When we impose \(\phi_x < 1\) and \(\phi_p < 1\), as in the previous subsection, all coefficients but \(\Gamma_1\), which is negative, are positive.
C Numerical method

In this paper, we follow Woodford (2003); we represent the model as a function of exogenous variables and their iterated average expectations.

Firms’ optimal price is determined by

\[ p^*_{it} = E_{it}p - \bar{\xi}E_{it}y_t + \frac{\xi_4}{\xi_1}E_{it}(a_t + z_{it}) \] (78)

where \( \bar{\xi} = \frac{\xi_2 + \xi_3 \gamma}{\xi_1} \) and \( E_{it} \) is mathematical expectation of firm \( i \) based on her information set at \( t \). Integrating the individual prices over \( i \) results in following:

\[ p_t = p_{t|t} - \bar{\xi}y_{t|t} + \frac{\xi_4}{\xi_1}a_{t|t} \] (79)

where \( x_{t|t} = \int E_{it}x_{i|t}di \) and it represents the average expectation on the variable \( x \) over all \( i \). By combining the money supply rule and cash-in-advance constraint and rearranging, the output gap can be expressed as

\[ y_t = \phi_p - \frac{1}{1 - \phi_x}p_t + \frac{1}{1 - \phi_x}q_t \] (80)

Similarly, the output gap can be expressed as a function of the average expectations:

\[ y_{t|t} = \phi_p - \frac{1}{1 - \phi_x}p_{t|t} + \frac{1}{1 - \phi_x}q_{t|t} \] (81)

Substituting Equation 81 into Equation 79, we can obtain the following:

\[ p_t = p_{t|t} - \bar{\xi}\phi_p - \frac{1}{1 - \phi_x}p_{t|t} + \frac{1}{1 - \phi_x}q_{t|t} + \frac{\xi_4}{\xi_1}a_{t|t} \] (82)

Introducing the notation

\[ x_{t}^{(k)} \equiv x_{t|t}^{(k-1)} \quad \text{for each} \quad k \geq 1 \]

\[ x_{t}^{(0)} \equiv x_t \] (83)
for higher-order average expectations and iterating Equation (82), we can obtain:

\[ p_t = -\bar{\xi} + \sum_{k=1}^{\infty} \left(1 + \frac{1 - \phi}{1 - \phi x}\right)^{k-1} q_t^{(k)} + \frac{\xi}{\xi_1} \sum_{k=1}^{\infty} \left(1 + \frac{1 - \phi}{1 - \phi x}\right)^{k-1} a_t^{(k)} \]  (84)

Next, we cast the model into a Kalman filter. Let \( X_t \) and \( S_{it} \) be the state vector and observable vector as below:

\[
X_t = \begin{bmatrix} a_t \\ q_t \end{bmatrix} = \begin{bmatrix} \rho_a & 0 \\ 0 & \rho_q \end{bmatrix} \begin{bmatrix} a_{t-1} \\ q_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{at} \\ \epsilon_{qt} \end{bmatrix} \]

(85)

\[
S_{it} = \begin{bmatrix} S_{ait} \\ S_{qit} \end{bmatrix} = \begin{bmatrix} a_t + \psi_{ait} \\ q_t + \psi_{qit} \end{bmatrix} \]

(86)

We introduce the extended state vector \( \bar{X}_t \) as below:

\[
\bar{X}_t = \begin{bmatrix} a_t \\ q_t \\ \sum_{k=1}^{\infty} \left(1 + \frac{1 - \phi}{1 - \phi x}\right)^{k-1} a_t^{(k)} \\ \sum_{k=1}^{\infty} \left(1 + \frac{1 - \phi}{1 - \phi x}\right)^{k-1} q_t^{(k)} \end{bmatrix} \]

(87)

Then, the transition and measurement equation can be expressed as

\[
\bar{X}_t = M \bar{X}_{t-1} + N \bar{\varepsilon}_t
\]

(88)

\[
S_{it} = O \bar{X}_t + \psi_{it}
\]

(89)

where \( \bar{\varepsilon}_t = [\epsilon_{at} \epsilon_{qt}]' \), \( \psi_{it} = [\psi_{ait} \psi_{qit}]' \) and \( O = [I_{2x2} \ 0_{2x2}] \). The price level \( p_t \) can be determined by \( \begin{bmatrix} 0 & 0 & \xi_4 & -\bar{\xi} \end{bmatrix} \bar{X}_t \) and the output gap can be obtained by Equation (80) when the price level is provided. Therefore, the model dynamics can be solved when matrices \( M \) and \( N \) are

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26 The absolute value of \( 1 + \frac{1 - \phi}{1 - \phi x} \) should be less than one.

27 At this stage, we don’t need to consider the signals about idiosyncratic productivity, because it does not affect the aggregate dynamics.

61
determined. $\bar{X}_t$, $M$ and $N$ can be partitioned as follow:

\[
\bar{X}_t = \begin{bmatrix} X_t \\ F_t \end{bmatrix}, \quad M = \begin{bmatrix} M_{12 \times 2} & 0_{2 \times 2} \\ G_{2 \times 2} & H_{2 \times 2} \end{bmatrix}, \quad N = \begin{bmatrix} I_{2 \times 2} \\ h_{2 \times 2} \end{bmatrix}
\] (90)

where

\[
M_1 = \begin{bmatrix} \rho_a & 0 \\ 0 & \rho_q \end{bmatrix}, \quad F_t = \begin{bmatrix} \sum_{k=1}^{\infty} \left(1 + \bar{\xi} \frac{1-\phi_y}{1-\phi_x} \right)^{k-1} a_t^{(k)} \\ \sum_{k=1}^{\infty} \left(1 + \bar{\xi} \frac{1-\phi_y}{1-\phi_x} \right)^{k-1} q_t^{(k)} \end{bmatrix}
\] (91)

We need to determine matrices $G$, $H$ and $h$ to solve the model. From the above transition and measurement equation, we can formulate a Kalman filter as below:

\[
\bar{X}_{i,t|t} = \bar{X}_{i,t|t-1} + k[S_{it} - O\bar{X}_{i,t|t-1}]
\] (92)

\[
\bar{X}_{i,t|t-1} = M\bar{X}_{i,t-1|t-1}
\] (93)

where $k$ is the Kalman gain matrix. Combining Equation (90), (91), (92) and (93) and integrating over $i$, $\bar{X}_{t|t}$ can be expressed as

\[
\bar{X}_{t|t} = \bar{X}_{t|t-1} + k[O\bar{X}_t - OM\bar{X}_{t-1|t-1}]
\]

\[
= M\bar{X}_{t-1|t-1} + k[OM\bar{X}_{t-1} + ON\varepsilon_t - OM\bar{X}_{t-1|t-1}]
\] (94)

\[
= kOM\bar{X}_{t-1} + (I - kO)M\bar{X}_{t-1|t-1} + kON\varepsilon_t
\]

$\bar{X}_{t|t}$ can be turned into $F_t$ by the relationship below:

\[
F_t = \eta \bar{X}_{t|t} = \begin{bmatrix} 1 & 0 & 1 + \bar{\xi} \frac{1-\phi_y}{1-\phi_x} & 0 \\ 0 & 1 & 0 & 1 + \bar{\xi} \frac{1-\phi_y}{1-\phi_x} \end{bmatrix} \bar{X}_{t|t}
\] (95)
Combining Equation (94) and (95), we can obtain the transition equation of $F_t$.

\[ F_t = \eta kO M \bar{X}_{t-1} + \eta (I - kO) M \bar{X}_{t-1|t-1} + \eta k O N \varepsilon_t \]

\[ = \eta k M 1 X_{t-1} - \eta k M 1 X_{t-1|t-1} + \eta M \bar{X}_{t-1|t-1} + \eta k \varepsilon_t \]

\[ = \eta k M 1 X_{t-1} + \left( M 1 + \left( 1 + \xi \frac{1 - \phi_p}{1 - \phi_x} \right) G X_{t-1|t-1} \right) X_{t-1|t-1} + \left( 1 + \xi \frac{1 - \phi_p}{1 - \phi_x} \right) H F_{t-1|t-1} \]

\[ + \left( 1 + \xi \frac{1 - \phi_p}{1 - \phi_x} \right) H F_{t-1|t-1} + \eta k \varepsilon_t \]

\[ = \eta k M 1 X_{t-1} + \left( M 1 + \left( 1 + \xi \frac{1 - \phi_p}{1 - \phi_x} \right) G - \eta k M 1 \right) X_{t-1|t-1} \]

\[ + \left( 1 + \xi \frac{1 - \phi_p}{1 - \phi_x} \right) H F_{t-1|t-1} + \eta k \varepsilon_t \]

\[ = \eta k M 1 X_{t-1} + \left( M 1 + \left( 1 + \xi \frac{1 - \phi_p}{1 - \phi_x} \right) G - \eta k M 1 - H \right) X_{t-1|t-1} \]

\[ + HF_{t-1} + \eta k \varepsilon_t \]

The last equality of Equation (96) comes from lagging and rearranging Equation (95). Matching Equation (88) and (96), we can determine the matrices $G$, $H$ and $h$ as below.

\[ G = \eta k M 1, \quad h = \eta k, \quad H = M 1 + \left( 1 + \xi \frac{1 - \phi_p}{1 - \phi_x} \right) G - \eta k M 1 \]

(97)

Therefore, we can fully solve the model when the Kalman gain matrix $k$ is determined.

The Kalman gain matrix can be obtained as below

\[ k_t = Var_{t-1}(\bar{X}_t)O'(Var_{t-1}(S_t))^{-1} \]

(98)

where $Var_t(X)$ is a conditional variance of $X$ based on information up to $t$. Conditional

\[ ^{28}\text{So far, we have suppressed time subscript in } k. \text{ However, it is time-varying as we allow changes in dispersions of exogenous shocks. It implies that matrices which determine the model dynamics, } M \text{ and } N, \text{ are also time-varying, as is obvious from Equation (97). Thus, we will be explicit about time subscripts afterwards.} \]
variances are given by

\[ \text{Var}_{t-1}(\bar{X}_t) = M_{t-1}\text{Var}_{t-1}M'_t + \text{Var}_{t-1}(N_{t-1}\epsilon_t) \]
\[ \text{Var}_{t-1}(S_t) = O\text{Var}_{t-1}(\bar{X}_t)O' + \text{Var}_{t-1}(\psi_t) \]
\[ \text{Var}_t(\bar{X}_t) = (I - k_tO)\text{Var}_{t-1}(\bar{X}_t) \]

(99)

First, we solve the model under a ‘stationary’ case in a sense that there has been no uncertainty shock in the recent periods so the variances are remain fixed, and are matrices \( M \) and \( N \). Then, we inject a uncertainty shock to this model under a stationary history and analyze the transition of endogenous variables.

Under a stationary case, attention allocations, conditional variances, and matrices \( M \) and \( N \) are not time-varying. Therefore, we solve the model under a stationary environment as follow:

1. Given \( \kappa_1, \kappa_2 \) and \( \kappa_3 \), determine \( \text{Var}_{t-1}(\psi_t) \).

From Equation (30), we can show that

\[ \kappa_1 = \frac{1}{2} \log_2 \frac{\text{Var}_{t-1}(a_t)}{\text{Var}_t(a_t)} = \frac{1}{2} \log_2 \frac{\text{Var}_{t-1}(a_t)}{\text{Var}_{t-1}(a_t) - \text{Var}_{t-1}(a_t)\text{Var}_{t-1}(S_{at})^{-1}\text{Var}_{t-1}(a_t)} \]
\[ = \frac{1}{2} \log_2 \frac{1}{1 - \text{Var}_{t-1}(S_{at})^{-1}\text{Var}_{t-1}(a_t)} = \frac{1}{2} \log_2 \frac{1}{1 - [\text{Var}_{t-1}(a_t) + \text{Var}_{t-1}(\psi_{at})]^{-1}\text{Var}_{t-1}(a_t)} \]

(100)

Therefore, we can obtain the following from Equation (100).

\[ 2^{2\kappa_1}\frac{\text{Var}_{t-1}(a_t) + \text{Var}_{t-1}(\psi_{at})}{\text{Var}_{t-1}(\psi_{at})} \]

(101)

Next, conditional variances of exogenous processes can be computed as below

\[ \text{Var}_{t-1}(a_t) = \rho_a \text{Var}_{t-1}(a_{t-1}) + \text{Var}_{t-1}(\varepsilon_{at}) \]
\[ = \rho_a \text{Var}_{t-1}(a_{t-1}) + \sigma^2_{\varepsilon_{at}} \]

(102)

\[ \text{We only describe deriving } \text{Var}_{t-1}(\psi_{at}). \text{ The variances of noises about } q_t \text{ and } z_{it} \text{ can be computed in the way.} \]
\[
\begin{align*}
\text{Var}_t(a_t) &= \text{Var}_{t-1}(a_t)2^{-2\kappa_1 t} \\
&= [\rho_a^2 \text{Var}_{t-1}(a_t) + \sigma_{\epsilon a t}^2]2^{-2\kappa_1 t} \\
&= \sigma_{\epsilon a t}^2 2^{-2\kappa_1 t} + \rho_a^2 \sigma_{\epsilon a t-1}^2 2^{-2\kappa_1 t-1} - 2\kappa_1 - 1 + \rho_a^2 \sigma_{\epsilon a t-2}^2 2^{-2\kappa_1 t-2} - 2\kappa_1 t - 1 + \ldots \tag{103}
\end{align*}
\]

Combining above equations, we can obtain

\[
\text{Var}_{t-1}(a_t) = \sigma_{\epsilon a t}^2 + \rho_a^2 \sigma_{\epsilon a t-1}^2 2^{-2\kappa_1 t-1} + \rho_a^4 \sigma_{\epsilon a t-2}^2 2^{-2\kappa_1 t-2} - 2\kappa_1 t - 1 \ldots \tag{104}
\]

Under a stationary history, Equation (104) can be simplified as

\[
\text{Var}_{t-1}(a_t) = \sigma_{\epsilon a t}^2 (1 + \rho_a^2 2^{-2\kappa_1} + \rho_a^4 2^{-4\kappa_1} + \ldots) \quad \tag{105}
\]

Combining Equation (102) and (105), we can derive the conditional variance under a stationary environment:

\[
\text{Var}_{t-1}(\psi_{at}) = \frac{\sigma_{\epsilon a t}^2}{(2\kappa_1 - 1)(1 - \rho_a^2 2^{-2\kappa_1})} \quad \tag{106}
\]

2. Given \(M\) and \(N\), calculate \(\text{Var}_{t-1}(\bar{X}_t)\).

\(\text{Var}_{t-1}(\bar{X}_t)\) can be obtained by solving the following Riccati equation by iteration:

\[
V = M[(I - VO'(VO' + \text{Var}_{t-1}(\psi_t))^{-1}O)V]M' + NV\text{Var}_{t-1}(\varepsilon_t)N' \quad \tag{107}
\]

where \(V\) is \(\text{Var}_{t-1}(\bar{X}_t)\) that solves Riccati equation. From \(\text{Var}_{t-1}(\bar{X}_t)\), we can obtain \(\text{Var}_{t-1}(S_t)\), \(\text{Var}_{t}(\bar{X}_t)\) and the Kalman gain matrix \(k\).

3. Given conditional variances and the Kalman gain \(k\), update \(M\) and \(N\) by Equation (97).

4. Repeat step 2 through 3 until \(M\) and \(N\) converge.

5. Compute impulse response functions of \(p^*_it\) and \(p^it\) to one standard deviation shock to
fundamental processes and their noises.

6. Choose $\kappa_1$, $\kappa_2$, and $\kappa_3$ that minimize the sum of all squared distance between impulse responses of $p_{it}^\gamma$ and $p_{it}^\ast$.

7. Return to step 1 and iterate until the procedure converges.

Next, we compute the solution when a uncertainty shock hits the economy at time $t$ assuming that the economy has a stationary history until time $t-1$. We compute the impulse response functions to exogenous fundamental shocks on top of an arrival of the uncertainty shock as follow. In this paper, we assume that the standard deviations of aggregate and idiosyncratic productivity become 1.91 and 3.33 times bigger compared to the stationary periods when a uncertainty shock hits the economy.

1. Compute the streams of unconditional variances of exogenous shocks, from $t$ to $t+2T$, by feeding $\varepsilon_{\sigma at}$ and $\varepsilon_{\sigma zt}$ to Equation (49) for $T$ large enough. These variances converge to their unconditional means $\bar{\sigma}_a^2$ and $\bar{\sigma}_z^2$ eventually.

2. We need to guess the sequences of $\kappa_1$ and $\kappa_2$ from $t$ to $t+2T$ based on information up to time $t$, because future allocations determine $M$ and $N$ in the future periods. Instead of guessing entire sequences, we assume that the attention allocations return to their stationary levels in constant rates after two quarters and guess the allocation at $t$ and $t+1$ and the rate of change as shown below.

\[
\text{guess } \kappa_{1t}, \kappa_{2t}, \kappa_{1t+1}, \kappa_{2t+1} \text{ then, } \kappa_{3t}, \kappa_{3t+1} \text{ determined}
\]
\[
\text{compute } \kappa_{it+j} = \rho_{\kappa_1}(\kappa_{it+j-1} - \kappa_{i,\text{stationary}}) + \kappa_{i,\text{stationary}} \text{ for } i \in \{1, 3\} \text{ and } j \geq 2
\]
\[
\kappa_{2t+j}, j \geq 1 \text{ determined from } \kappa_{1t+j}, \kappa_{3t+j} j \geq 2
\]

Under this simplifying assumption, we only need to find optimizers for $\kappa_{1t}$, $\kappa_{2t}$, $\kappa_{1t+1}$, $\kappa_{2t+1}$, $\rho_{\kappa_1}$ and $\rho_{\kappa_3}$.

\[
\text{30In this sense, the solution obtained in this problem is an approximate solution. However, we solved for } \kappa_{it+j} \text{ for } 0 \leq j \leq 9 \text{ and found that this gives very close approximation.}
\]

66
3. Compute the sequences of conditional variances of exogenous processes from \( t \) to \( t+2T \).

From Equation (104), we can compute the sequences of conditional variances based on the sequences of unconditional variances of exogenous processes and attention allocations as below.

\[
\begin{align*}
\text{Var}_{t+j}(a_{t+1+j}) &= \sigma^2_{at+1+j} + \rho_a^2(2^{-2\kappa_1 t+j})\text{Var}_{t+j-1}(a_{t+j}) \\
\text{Var}_{t+j}(q_{t+1+j}) &= \sigma^2_{qt+1+j} + \rho_q^2(2^{-2\kappa_2 t+j})\text{Var}_{t+j-1}(q_{t+j}) \\
\text{Var}_{t+j}(z_{it+1+j}) &= \sigma^2_{zt+1+j} + \rho_z^2(2^{-2\kappa_3 t+j})\text{Var}_{t+j-1}(z_{it+j})
\end{align*}
\]

for \( j \geq -1 \) and \( \kappa_{1t-1}, \kappa_{2t-1}, \kappa_{3t-1}, \text{Var}_{t-2}(a_{t-1}), \text{Var}_{t-2}(q_{t-1}), \) and \( \text{Var}_{t-2}(z_{it-1}) \) are their stationary values.

4. Compute the sequences of variances of noises by applying Equation (105) and (106).

5. Repeat step 2 through 4 from the stationary case above period by period, from \( t \) to \( t+T \) and obtain the sequences of \( M_t \) and \( N_t \). Then compute impulse responses of the extended state variable \( \bar{X} \) to aggregate productivity and monetary policy shock originated at \( t \).

\[
\bar{X}_t = N_t \epsilon_t \\
\bar{X}_{t+j} = M_{t+j} \bar{X}_{t+j-1}, \quad j \geq 1
\]

6. Compute impulse response functions of \( p^*_it \) and \( p^*_it \) to one standard deviation shock to fundamental processes and their noises based on step 5.

7. Repeat step 5 through 6 starting from \( t + j \) for \( j \in \{1, 2, 3, \cdots , T\} \).

8. Choose \( \kappa_1, \kappa_2, \) and \( \kappa_3 \) that minimize the discounted sum of all squared distance between impulse responses of \( p^*_it \) and \( p^*_it \).

9. Return to step 1 and iterate until the procedure converges.
D Robustness checks

D.1 Different information capacity

In this section, we present additional results by assuming the different information capacities. Specifically, we consider $\kappa \in \{6, 8, 10\}$.

<table>
<thead>
<tr>
<th>$\kappa = 6$</th>
<th>$\kappa_{1t}$</th>
<th>$\kappa_{2t}$</th>
<th>$\kappa_{3t}$</th>
<th>$\rho_\kappa$</th>
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</thead>
<tbody>
<tr>
<td>Stationary</td>
<td>0.9800</td>
<td>0.7414</td>
<td>4.2786</td>
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<tr>
<td>Uncertainty shock</td>
<td>0.9251</td>
<td>0.1104</td>
<td>4.9645</td>
<td>0.8073</td>
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<table>
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<th>$\kappa = 8$</th>
<th>$\kappa_{1t}$</th>
<th>$\kappa_{2t}$</th>
<th>$\kappa_{3t}$</th>
<th>$\rho_\kappa$</th>
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</thead>
<tbody>
<tr>
<td>Stationary</td>
<td>1.6610</td>
<td>1.4161</td>
<td>4.9229</td>
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<tr>
<td>Uncertainty shock</td>
<td>1.7554</td>
<td>0.6503</td>
<td>5.5943</td>
<td>0.7541</td>
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</table>

<table>
<thead>
<tr>
<th>$\kappa = 10$</th>
<th>$\kappa_{1t}$</th>
<th>$\kappa_{2t}$</th>
<th>$\kappa_{3t}$</th>
<th>$\rho_\kappa$</th>
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<tbody>
<tr>
<td>Stationary</td>
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<td>2.1682</td>
<td>5.4251</td>
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<tr>
<td>Uncertainty shock</td>
<td>2.5552</td>
<td>1.3861</td>
<td>6.0587</td>
<td>0.7475</td>
</tr>
</tbody>
</table>

Table 3: Change in attention allocation

In general, attention allocated to the idiosyncratic productivity increases and that to the monetary policy shock decreases when the uncertainty shock hits the economy. The changes in the aggregate productivity are equivocal; when the total capacity is lower, say less than 8, attention allocated to the aggregate shock decreases whereas it increases when the total capacity is large enough, greater than or equal to 8 here.

As explained in the main text, $\rho_\kappa$ gets smaller as the total capacity increases. If the total capacity becomes greater, firms can process more information and resolve uncertainty more promptly. Hence, attention allocation comes back to its stationary allocation faster.
Figure 12: Impulse responses when the uncertainty shock arrives ($\kappa = 6$): The first row contains impulse responses to one standard deviation negative aggregate productivity shock and the second row contains impulse responses to one standard deviation positive monetary policy shock.
Figure 13: Impulse responses when the uncertainty shock arrives ($\kappa = 8$)
Figure 14: Impulse responses when the uncertainty shock arrives ($\kappa = 10$)
D.2 Different uncertainty and policy indicators

Figure 15: Impulse responses of log IP to the monetary policy shock with the uncertainty indicator VIX(-1), the policy indicator GS1 and the instrument FF4; the top panel depicts the impulse responses from the simple VAR without regime shifting. The middle and bottom panels represent the impulse responses within high and low uncertainty regime. The dashed lines show bootstrap 90 percent confidence intervals.

Figure 15 presents the impulse responses of log industrial production to the monetary policy shock with the lagged VIX uncertainty indicator, the one-year government bond policy indicator and the three month ahead Federal Funds futures instrument. With longer term policy indicator and instrument, the effect of monetary policy shock becomes statistically more significant and the magnitude is also amplified as shown in the top panel. Gertler and Karadi (2015) explained that this is because the longer-term indicator and instrument contain
additional information about the future course of the policy, such as forward guidance. The middle and bottom panels present somewhat mixed result. In this case, the effect of the monetary policy is not significant in 95 percent confidence level regardless of the uncertainty level. On the other hand, the magnitude of the trough remains approximately 2 times larger when uncertainty is higher compared to low uncertainty regime.
Figures below contain impulse responses with the lagged VIX growth rate as the uncertainty indicator. In these figures, the monetary policy shock is more effective in terms of the magnitude of the response and statistically more significant when uncertainty is higher. This provides robustness of the result.

![Impulse responses of log IP to the monetary policy shock with the uncertainty indicator VIX growth(-1), the policy indicator FF and the instrument FF1; the top panel depicts the impulse responses from the simple VAR without regime shifting. The middle and bottom panels represent the impulse responses within high and low uncertainty regime. The dashed lines show bootstrap 90 percent confidence intervals.](image)

Figure 16: Impulse responses of log IP to the monetary policy shock with the uncertainty indicator VIX growth(-1), the policy indicator FF and the instrument FF1; the top panel depicts the impulse responses from the simple VAR without regime shifting. The middle and bottom panels represent the impulse responses within high and low uncertainty regime. The dashed lines show bootstrap 90 percent confidence intervals.
Figure 17: Impulse responses of log IP to the monetary policy shock with the uncertainty indicator VIX growth(-1), the policy indicator GS1 and the instrument FF4; the top panel depicts the impulse responses from the simple VAR without regime shifting. The middle and bottom panels represent the impulse responses within high and low uncertainty regime. The dashed lines show bootstrap 90 percent confidence intervals.
Figure 18: Impulse responses of log IP to the monetary policy shock with the uncertainty indicator $P(\text{VIX}(-1)|\text{IP GAP}(-1))$, the policy indicator FF and the instrument FF1; the top panel depicts the impulse responses from the simple VAR without regime shifting. The middle and bottom panels represent the impulse responses within high and low uncertainty regime. The dashed lines show bootstrap 90 percent confidence intervals.
D.3 Sticky price vs attention: looser information capacity

![Graphs showing changes in price adjustment frequency](image)

Figure 19: Changes in the price adjustment frequency: $\kappa = 10$ alternative

Although impulse responses of the output gap are very similar, the magnitude remains larger when the uncertainty shock hits the economy. In other words, the amplification from changes in price rigidity does not exceed the amplification from attention reallocation.
E  Price simulation: White noise model

Additionally, we simulate the model to obtain the set of individual price changes with the white noise model as this is easier to deal with and allows non-approximate exact solution of the model. Moreover, we can reduce the computation time by substantial amount so that we can simulate longer periods and obtain the larger number of sample. The white noise model is simulated for 2000 firms and 500 periods. The parameters are calibrated as in Table 1 except for the correlations among the second moment shocks and aggregate productivity shock. We use the same series of shocks as in Section 6.

Figure 20: Simulated aggregate and individual prices

An example of simulated aggregate and idiosyncratic price (Firm 1) level is given in Figure 20.\textsuperscript{31}

Figure 21 depicts the distributions of the price changes. During expansions, the median

\textsuperscript{31}Because the model is under white noise environment, the price level in each period can be regarded as the price change.
Table 4: Comparing statistics from the white noise and persistent models and the data is positive and the dispersion is smaller than that from recessions. On the other hand, the median is negative and the dispersion is larger during recessions as described in Vavra (2014).

Table 4 contains key statistics of price changes. As Vavra (2014) documents, the dispersion of the price changes increases during recessions. Specifically, the standard deviation of price changes in recessions is 26 percent greater than that in expansions in the white noise model. In addition, the correlation coefficient between the standard deviation of the price changes and the output is -0.36 which is very close to -0.33 observed in the data. Despite the simple construction of the model, the simulated price changes can match data very well in this dimension.