Abstract

This paper develops an empirical approach to optimal income taxation design within an equilibrium collective marriage market model. Taxes distort labour supply and time allocation decisions, as well as marriage market outcomes, and the within household decision process. Using data from the American Community Survey and American Time Use Survey we structurally estimate our model and explore empirical design problems. We consider the optimal design problem when the planner is able to condition taxes on marital status, as in the U.S. tax code, but for married couples we allow for an arbitrary form of tax jointness.

1 Introduction

Tax and transfer policies often depend on family structure, with the tax treatment of married and single individuals varying significantly both across countries and over time. In the United States there is a system of joint taxation where the household is taxed based on total family income. Given the progressivity of the tax system, it is not neutral with respect to marriage and both large marriage penalties and marriage bonuses coexist. In

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contrast, the majority of OECD countries have individual income taxation where each individual is taxed separately based on his/her income. In such a system married couples are treated as two separate individuals and hence there is no subsidy or tax on marriage. But what is the appropriate choice of tax unit and how should individuals and couples be taxed? A large and active literature concerns the optimal design of tax and transfer policies. In an environment where taxes affect the economic benefits from marriage, such a design problem has to balance redistributive objectives with efficiency considerations, whilst recognizing that the structure of taxes may affect who gets married, and to whom they get married, as well as the intra-household allocation of resources.

Following the seminal contribution of [34], a large theoretical literature has emerged that studies the optimal design of tax schedules for single individuals. This literature casts the problem as a one-dimensional screening problem, recognizing the asymmetry of information that exists between agents and the tax authorities.\(^1\) The analysis of the optimal taxation of couples has largely been conducted in environments where the form of the tax schedule is restricted to be linearly separable, but with potentially distinct tax rates on spouses (see Boskin and Sheshinski [9], Apps and Rees [4, 5, 6], and Alesina, Ichino and Karabarbounis [3] for papers in this tradition). A much smaller literature has extended the Mirrleesian approach to study the optimal taxation of couples as a two-dimensional screening problem. Most prominently, Kleven, Kreiner and Saez [30, 31] consider a unitary model of the household, in which the primary earner makes a continuous labour supply decision (intensive only margin) while the secondary worker makes a participation decision (extensive only margin), and characterize the optimal form of tax jointness.\(^2\) By taking the married unit as given the optimal nonlinear tax system analyses in Kleven, Kreiner and Saez [30, 31] ignores the distortionary effect of couple’s taxation on who gets married and to whom they get married. It is also ill equipped to analyze the distortionary effect of taxation on the intra-household allocation of resources. Moreover, the primary/secondary earner asymmetry ignores the potential role of the tax system in inducing specialization in couples.\(^3\)

The theoretical optimal income taxation literature provides many important insights that are relevant when considering the design of a tax system. However, the quantitative

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\(^{1}\)See Brewer, Saez and Shephard [11] and Piketty and Saez [35] for recent surveys.

\(^{2}\)See also Brett [10], Cremer, Lozachmeur and Pestieau [21], Frankel [23], and Immervoll et al. [29].

\(^{3}\)The large growth in female labour force participation has made the traditional distinction between primary and secondary earners much less clear. Women now make up around half of the U.S. workforce, with an increasing fraction of households in which the female is the primary earner. See, e.g. Blau and Kahn [7] and Gayle and Golan [26].
empirical applicability of optimal tax theory is dependent upon a precise measurement of the key behavioral margins: How do taxes affect market work, the amount of time devoted to home production, and the patterns of specialization within the household? How do taxes influence the within household allocation of resources? What is the effect of taxes on the decision to marry and to whom? In order to examine both the optimal degree of progressivity and jointness of the tax schedule, and to empirically quantify the importance of the marriage market in shaping these, we follow Blundell and Shephard [8] by developing an empirical structural approach to non-linear income taxation design that centres the entire analysis around a rich micro-econometric model.

Our model integrates the collective model of Chiappori [13, 14] with the empirical marriage-matching model developed in [18]. Individuals make marital decisions that comprise extensive (to marry or not) and intensive (i.e. sorting) margins based on utilities that comprise both an economic benefit and a idiosyncratic non-economic benefit. The economic utilities are micro-founded and are derived from the household decision problem. We consider an environment that allows for very general non-linear income taxes, and which features for both intensive and extensive labour supply margins, home production time, and both public and private good consumption. As in Galichon, Kominers and Weber [25] we allow for utilities to be imperfectly transferable across spouses.

Using data from the American Community Survey and the American Time Use Survey we structurally estimate our equilibrium model, exploiting variation across markets in terms of both tax and transfer policies, and population vectors. We then use our estimated model directly to examine problems related to the optimal design of the tax system, while acknowledging that taxes may distort labour supply and time allocation decisions, as well as marriage market outcomes, and the within household decision process. Our taxation design problem is based on an individualistic social welfare function, with inequality both within and across households adversely affecting social welfare. We allow for a very general specification of the tax schedule for both singles and married couples, that nests both individual and fully joint taxation, but also allows for very general forms of tax jointness.

The remainder of the paper proceeds as follows. In Section 2 we describe our equilibrium model of marriage, consumption, and time allocation. Section 3 introduces the

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4Other papers that integrate a collective time allocation model within an empirical marriage-matching model include Chiappori, Costa Dias and Meghir [16] who consider an equilibrium model of education and marriage with labour supply and consumption in a transferable utility model, and Choo and Seitz [19].
analytical framework that we use to study taxation design within our equilibrium collective model. Section 4.3 describes our microeconometric specification, while Section 4 discusses the data and estimation procedure, as well as detailing our main estimation results. In Section 5 we present our main optimal taxation design results, both allowing for very general forms for the tax schedule, as well as forms which restrict the form of jointness. Finally, Section 6 concludes.

2 A model of marriage and time allocation

We present an empirical model of marriage-matching and intrahousehold allocations by consider a static equilibrium model of marriage with imperfectly transferable utility, labour supply, home production, and potentially joint and non-linear taxation. While there are two previous papers in the literature that analyses collective decisions model with imperfectly transferable utility (see 19, and 25) those papers do not deal explicitly with joint taxation and public good produced with time inputs. Both of these additions leads to important differences in the formulation and outcomes of the model. In the presence of joint taxation – even without public goods – the nice clear separation of the income sharing problem and the maximization of leisure is lost. Intuitively joint taxation by itself introduces a element of “public goods” into the problem.

2.1 Environment

We consider an economy comprising of $K$ separate markets. Given that there are no interactions across markets we suppress explicit conditioning on market unless such a distinction is important and proceed to describe the problem for a given market. In such a market there are $I$ types of men and $J$ types of women. The population vector of men is given by $\mathcal{M}$, whose element $m_i$ denotes the measure of type $i$ males. Similarly, the population vector of women is given by $\mathcal{F}$, whose element $f_j$ denotes the measure of type $j$ females. Associated with each male and female type is a utility function, a distribution of wage offers, a productivity of home time, a distribution of preference shocks, a value of non-labour income, and a demographic transition function (which is defined for all possible spousal types). While we are more restrictive in our empirical application, in principle all these objects may vary across markets. Moreover, these markets may differ in their tax system $T$ and the economic/policy environment more generally.
We make the timing assumption that the realizations of wage offers, preference shocks, and demographic transitions only occurs following the clearing of the marriage market. There are therefore two (interconnected) stages to our analysis. First, there is the characterization of a marriage matching function, which is an $I \times J$ matrix $\mu(T)$ whose $\langle i, j \rangle$ element $\mu_{ij}(T)$ describes the measure of type $i$ males married to type $j$ females, and which we write as a function of the tax system $T$. The second stage of our analysis which follows marriage market decisions is then concerned with the joint time allocation and resource sharing problem for households. These two stages are linked through the decision weight in the household problem: these affect the second stage problem and so the expected value of an individual from any given marriage market position. These household decision (or Pareto) weights will adjust to clear the marriage market, such that there is neither excess demand nor supply of any given type.

### 2.2 Time allocation problem

We first describe the decision of single individuals and married couples once the marriage market has cleared. At this stage, all uncertainty (wage offers, preference shocks, and demographic transitions are realized) and time allocation decisions are made. Individuals have preferences defined over leisure, consumption of a market private good (whose price we normalize to one), and a non-marketable public good produced with home time.

#### 2.2.1 Time allocation problem: single individuals

Consider a single male of type $i$. His total time endowment is $L_0$ and he chooses the time allocation vector $a^i = (\ell^i, h^i_w, h^i_Q)$ comprising hours of leisure $\ell^i$, market work time $h^i_w$, and home production time $h^i_Q$, to maximize his utility. Time allocation decisions are discrete, with all feasible time allocation vectors described by the set $A^i$. All allocations that belong to this set necessarily satisfy the time constraint $L_0 = \ell^i + h^i_w + h^i_Q$. Associated with each possible discrete allocation is the additive state specific error $\epsilon_{a^i}$. Excluding

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5 Individuals may also choose to remain unmarried and we use $\mu_{i0}(T)$ and $\mu_{0j}(T)$ to denote the respective measures of single males and females. The marriage matching function must satisfy the usual feasibility constraints. Suppressing the dependence on $T$ we require that: $\mu_{i0} + \sum_j \mu_{ij} = m_i$ for all $i$, $\mu_{0j} + \sum_i \mu_{ij} = f_j$ for all $j$, and $\mu_{i0}, \mu_{0j}, \mu_{ij} \geq 0$ for all $i$ and $j$.

6 A description of the choice set used in our empirical implementation, together with the parameterisation of the utility function and the complete stochastic structure, are provided in Section 4.3.
any additive idiosyncratic payoff from remaining single, the individual decision problem
may formally be described by the following utility maximization problem:

$$\max_{a^i \in A^i} u^i(\ell^i, q^i, Q^i, X^i) + \epsilon_{a^i},$$

subject to,

$$q^i = y^i + w^ih^i_w - T(w^ih^i_w, y^i; X^i) - FC(h^i_w; X^i),$$

$$Q^i = h^i_Q. \quad (2b)$$

Equation 2a states that consumption of the private good is simply equal to net family
income (the sum of earnings and non-labour income, minus net taxes) and less any
possible fixed work of market work, $FC(h^i_w; X^i) \geq 0$. These fixed costs (as in 20)
are non-negative for positive values of working time, and zero otherwise. Equation 2b says
that total production/consumption of the home good is equal to home time.

The solution to this constrained utility maximization problem is described by the
incentive compatible time allocation vector $a^i_{0^i}(w^i, y^i, X^i, \epsilon^i; T)$, which upon substitution into equation 1 (and including the state specific preference term associated with this allocation) yields the indirect utility function for type $i$ males that we denote as $v^i_{0^i}(w^i, y^i, X^i, \epsilon^i; T)$. The decision problem for single women of type $j$ is described similarly and yields the indirect utility function $v^j_{0^j}(w^j, y^j, X^j, \epsilon^j; T)$.

2.2.2 Time allocation problem: married individuals

Married individuals are egoistic and we consider a collective model that assumes an
efficient allocation of intra-household resources (13, 14). An important economic benefit
of marriage is given by the publicness of some consumption. We assume that the home
produced good (that is produced by combining male and female home time) is public
within the household, which both members may consume equally. Consider an $\langle i, j \rangle$
couple and let $\lambda_{ij}$ denote the Pareto weight on female utility in such a union.$^7$ The household chooses a time allocation vector for each adult, as well as determining how
total private consumption is divided between the spouses. Note that the state specific

$^7$That the Pareto weights only depend on the types $\langle i, j \rangle$ is a consequence of our timing assumptions and efficient risk sharing within the household. See Section ?? for a discussion. The parameterization of the utility function in our empirical implementation will imply a very close connection between the Pareto weight and the endogenous consumption share (see Section 4.3).
errors $\epsilon_{a_i}$ and $\epsilon_{a_j}$ for any individual depend only on their own time allocation, and not on the time allocation of their spouse. Moreover, the distributions of these preference terms, as well as the form of the utility function, do not change with marriage. We formally describe the household problem as:

$$
\max_{a_i \in A_i, a_j \in A_j, s_{ij} \in [0,1]} (1 - \lambda_{ij}) \times \left[ u^i(\ell^i, q^i, Q; X^i) + \epsilon_{a_i} \right] + \lambda_{ij} \times \left[ u^j(\ell^j, q^j, Q; X^j) + \epsilon_{a_j} \right],
$$

subject to:

$$
q = q^i + q^j = y^i + y^j + w^i h^i_w + w^j h^j_w - T(w^i h^i_w, w^j h^j_w, y^i, y^j; X) - FC(h^i_w, h^j_w; X),
$$

$$
q^j = s_{ij} \cdot q,
$$

$$
Q = \bar{Q}_{ij}(h^i_Q, h^j_Q; X).
$$

In turn, this set of equality constraints describe i) that total family consumption of the private good equals family net income with the tax schedule here allowed to depend very generally on the labour market earnings of both spouses, less any fixed work-related costs; ii) the wife receives the endogenous consumption share $0 \leq s_{ij} \leq 1$ of the private good; iii) the public good is produced using home production time with the production function $\bar{Q}_{ij}(\cdot, \cdot; \cdot)$, which may also depend upon family demographics.

Letting $w = [w^i, w^j]$, $y = [y^i, y^j]$, $X = [X^i, X^j]$, and $\epsilon = [\epsilon^i, \epsilon^j]$, the solution to the household problem is the incentive compatible time allocation vectors $a^*_{ij}(w, y, X, \epsilon; T, \lambda_{ij})$ and $a^*_{ij}(w, y, X, \epsilon; T, \lambda_{ij})$, together with the private consumption share $s^*_{ij}(w, y, X, \epsilon; T, \lambda_{ij})$. Upon substitution into the individual utility functions (and including the state specific error that is associated with the individual’s own time allocation decision) we obtain the respective male and female indirect utility functions $v^i_{ij}(w, y, X, \epsilon; T, \lambda_{ij})$ and $v^j_{ij}(w, y, X, \epsilon; T, \lambda_{ij})$.

### 2.3 Marriage market

We embed our time allocation model in a frictionless empirical marriage market model. As noted above, an important timing assumption is that marriage market decisions are made prior to the realization of wage offers, preference shocks, and demographic transitions. Thus, decisions are made based upon the expected value of being in a given marital position, together with an idiosyncratic component that we describe below.
2.3.1 Expected values

Anticipating our later application, we write the expected values from remaining single for a type $i$ single male and type $j$ single female (excluding any additive idiosyncratic payoff that we describe below) as explicit functions of the tax system $T$. These respective expected values are:

\[
U_{i0}^i(T) = \mathbb{E}[v_{i0}^i(w^i, y^i, X^i, \epsilon^i; T)],
\]

\[
U_{0j}^j(T) = \mathbb{E}[v_{0j}^j(w^j, y^j, X^j, \epsilon^j; T)],
\]

where the expectation is taken over wage offers, demographics, and the preference shocks. For married individuals, their expected values (again excluding any additive idiosyncratic utility payoffs) may similarly be written as a function of the both the tax system $T$ and a candidate Pareto weight $\lambda_{ij}$ associated with a type $\langle i, j \rangle$ match:

\[
U_{ij}^i(T, \lambda_{ij}) = \mathbb{E}[v_{ij}^i(w, y, X, \epsilon; T, \lambda_{ij})],
\]

\[
U_{ij}^j(T, \lambda_{ij}) = \mathbb{E}[v_{ij}^j(w, y, X, \epsilon; T, \lambda_{ij})].
\]

Note that the Pareto weight within a match does not depend upon the realization of uncertainty. This implies full commitment and efficient risk sharing within the household. It is straightforward to show that the expected value of a type $i$ man when married to a type $j$ woman is strictly decreasing in the wife’s Pareto weight $\lambda_{ij}$, while the expected value of his wife is strictly increasing in $\lambda_{ij}$. We have a even stronger results in our framework, that is

\[
\frac{\partial U_{ij}^i(\lambda_{ij})}{\partial \lambda} = -\frac{\lambda_{ij}}{1 - \lambda_{ij}} \frac{\partial U_{ij}^j(\lambda_{ij})}{\partial \lambda} < 0.
\]

This relationship between the effect of the wife’s Pareto on the husband’s and wife’s expected value will be useful is proving identification of the wife’s Pareto weight.

2.3.2 Marriage decision

As in Choo and Siow [18] we assume that in addition to the systematic component of utility (as given by the expected values above) a given male $g$ of type $i$ receives an idiosyncratic payoff that is specific to him, and the type of spouse $j$ that he marries but
not her specific identity. These idiosyncratic payoffs are denoted $\theta^i_{ij}$ and are observed prior to the marriage decision. Additionally, each male also receives an idiosyncratic payoff from remaining unmarried which depends on his specific identity and is similarly denoted as $\theta^i_{i0}$. The initial marriage decision problem of a given male $g$ is therefore to choose to marry one of the $J$ possible types of spouses, or to remain single. His decision problem is therefore:

$$\max_j \{ U^i_{i0}(T) + \theta^i_{i0}, U^i_{i1}(T, \lambda_{i1}) + \theta^i_{i1}, \ldots, U^i_{ij}(T, \lambda_{ij}) + \theta^i_{ij} \},$$

(5)

where the choice $j = 0$ corresponds to the single state.

We assume that the idiosyncratic payoffs follow the Type-I extreme value distribution with a zero location parameter and the scale parameter $\sigma$. This assumption implies that the proportion of type $i$ males who would like to marry a type $j$ female (or remain unmarried) are given by the conditional choice probabilities:

$$p^i_{ij}(T, \lambda^i) = \Pr[U^i_{ij}(T, \lambda_{ij}) + \theta^i_{ij} > \max_j \{ U^i_{i0}(T), U^i_{i1}(T, \lambda_{i1}) + \theta^i_{i1}, \ldots, U^i_{ij}(T, \lambda_{ij}) + \theta^i_{ij} \} \quad \forall h \neq j] = \frac{\mu^d_{ij}(T, \lambda^i)}{m_i} = \frac{\exp[U^i_{ij}(T, \lambda_{ij})/\sigma]}{\exp[U^i_{i0}(T)/\sigma] + \sum_{h=1}^J \exp[U^i_{ih}(T, \lambda_{ih})/\sigma]},$$

(6)

where $\lambda^i = [\lambda_{i1}, \ldots, \lambda_{ij}]^\top$ is the $J \times 1$ vector of Pareto weights associated with different spousal options for a type $i$ male, and $\mu^d_{ij}(T, \lambda^i)$ is the measure of type $i$ males who “demand” type $j$ females (the conditional choice probabilities $p^i_{ij}(T, \lambda^i)$ multiplied by the measure of men of type $i$). Women also receive idiosyncratic payoffs associated with the different marital states and their marriage decision problem is symmetrically defined. With identical distributional assumptions, the proportion of type $j$ females who would like to marry a type $i$ male is given by:

$$p^j_{ij}(T, \lambda^j) = \frac{\mu^s_{ij}(T, \lambda^j)}{f_j} = \frac{\exp[U^j_{ij}(T, \lambda_{ij})/\sigma]}{\exp[U^j_{0j}(T)/\sigma] + \sum_{g=1}^I \exp[U^j_{gj}(T, \lambda_{gj})/\sigma]},$$

(7)

where $\lambda^j = [\lambda_{ij}, \ldots, \lambda_{ij}]^\top$ is the $I \times 1$ vector of Pareto weights for a type $j$ female, and $\mu^s_{ij}(T, \lambda^j)$ is the measure of type $j$ females who would choose type $i$ males. We also refer to this measure as the “supply” of type $j$ females to the $\langle i, j \rangle$ sub-marriage market.
2.3.3 Marriage market equilibrium

An equilibrium of the marriage market is characterized by $I \times J$ matrix of Pareto weights $\lambda = [\lambda^1, \lambda^2, \ldots, \lambda^J]$ such that for all $(i, j)$ the measure of type $j$ females demanded by type $i$ men is equal to the measure of type $j$ females supplied to type $i$ males. That is,

$$
\mu_{ij}(T, \lambda) = \mu_{ij}^d(T, \lambda^i) = \mu_{ij}^s(T, \lambda^j) \quad \forall i = 1, \ldots, I, j = 1, \ldots, J.
$$

Along with the usual regularity conditions, which are formally stated in Appendix A, for existence and uniqueness of equilibrium we need that as consumption of the private good approaches zero individual utility approaches negative infinity. This condition is standard in literature on collective household models and will be enforced in the numerical algorithm we use for solving for equilibrium.

**Proposition 1.** If the idiosyncratic payoffs follow the Type-I extreme value distribution,

$$
\lim_{q^i \to 0} u^i(\ell^i, q^i, Q; X^i) = \lim_{q^j \to 0} u^j(\ell^j, q^j, Q; X^j) = -\infty,
$$

and the regularity conditions in Appendix A hold. Then there exist a unique marriage market equilibrium.

In Appendix C we describe the numerical algorithm that we apply when solving for an equilibrium given $T$, and note important properties regarding how the algorithm scales as the number of markets is increased.

3 Optimal taxation framework

In this section we present the analytical framework that we use to study tax reforms that are optimal under a social welfare function. The social planners problem is to choose a tax system $T$ to maximize a social welfare function subject to a revenue requirement, the individual/household incentive compatibility constraints, and the marriage market equilibrium conditions. The welfare function is taken to be *individualistic*, and is based on individual maximized (incentive compatible) utilities following both the clearing of the marriage market, and the realizations of wage offers, state specific preferences, and demographic transitions. Note that inequality both within and across households will adversely affect social welfare.
In what follows, we use \( G^i_{i0}(w^i, X^i, \epsilon^i) \) and \( G^j_{0j}(w^j, X^j, \epsilon^j) \) to respectively denote the single type \( i \) male and single type \( j \) female joint cumulative distribution functions for wage offers, state specific errors, and demographic transitions. The joint cumulative distribution function within an \( \langle i, j \rangle \) match is similarly denoted \( G_{ij}(w, X, \epsilon) \). It is also necessary to describe the endogenous distribution of idiosyncratic payoffs for individuals within a given marital position. These differ from the unconditional \( \text{EV}(0, \sigma_\theta) \) distribution for the population as a whole, because individuals non-randomly select into different marital positions on the basis of these. They are also a function of tax policy. We let \( H^i_{i0}(\theta^i; T) \) denote the cumulative distribution function of these payoffs amongst single type \( i \) males and similarly define \( H^j_{0j}(\theta^j; T) \) for single type \( j \) females. Amongst married men and women in an \( \langle i, j \rangle \) match these are given by \( H^i_{ij}(\theta^i; T) \) and \( H^j_{ij}(\theta^j; T) \) respectively. We provide a characterization of these distributions in Appendix B.

Our simulations will consider the implications of alternative redistributive preferences for the planner, which we will capture through the utility transformation function \( Y(\cdot) \). The social welfare function is defined as the sum of these transformed utilities:

\[
W(T) = \sum_i \mu_{i0}(T) \int Y \left[ v_{i0}^i(w^i, y^i, X^i, \epsilon^i; T) + \theta^i \right] dG^i_{i0}(w^i, X^i, \epsilon^i) dH^i_{i0}(\theta^i; T)
\]

\[
+ \sum_j \mu_{0j}(T) \int Y \left[ v^j_{0j}(w^j, y^j, X^j, \epsilon^j; T) + \theta^j \right] dG^j_{0j}(w^j, X^j, \epsilon^j) dH^j_{0j}(\theta^j; T)
\]

\[
+ \sum_{i,j} \mu_{ij}(T) \int Y \left[ v^i_{ij}(w, y, X, \epsilon; T, \lambda_{ij}(T)) + \theta^i \right] dG_{ij}(w, X, \epsilon) dH^i_{ij}(\theta^i; T)
\]

\[
+ \sum_{i,j} \mu_{ij}(T) \int Y \left[ v^j_{ij}(w, y, X, \epsilon; T, \lambda_{ij}(T)) + \theta^j \right] dG_{ij}(w, X, \epsilon) dH^j_{ij}(\theta^j; T).
\]

The maximization of \( W(T) \) is subject to a number of constraints. Firstly there are the usual incentive compatibility constraints that require that time allocation and consumption decisions for individuals and households are optimal given \( T \). We embed this in our formulation of the problem through the inclusion of the indirect utility functions. Second, individual’s optimally select into different marital positions based upon expected values and their realized idiosyncratic payoffs (equation 5). Third, we obtain a marriage
market equilibrium so that given $T$ there is neither excess demand or excess supply of spouses in each sub-marriage market (equation 8). In Proposition 1 we provide sufficient conditions to ensure that a unique equilibrium exists given $T$. Fourth, there is the requirement that an exogenously determined revenue amount $\bar{T}$ is raised, as given by the revenue constraint:

$$\mathcal{R}(T) = \sum_i \mu_{0i}(T) \int R_{i0}^i(w^i, y^i, \epsilon^i; T) \, dG_{i0}^i(w^i, X^i, \epsilon^i) \tag{11}$$

$$+ \sum_j \mu_{0j}(T) \int R_{0j}^j(w^j, y^j, \epsilon^j; T) \, dG_{0j}^j(w^j, X^j, \epsilon^j)$$

$$+ \sum_{ij} \mu_{ij}(T) \int R_{ij}(w, y, X, \epsilon; T, \lambda_{ij}(T)) \, dG_{ij}(w, X, \epsilon) \geq \bar{T},$$

where $R_{i0}^i(w^i, y^i, X^i, \epsilon^i)$ describes the amount of revenue raised from a single type $i$ male given $w^i, y^i, X^i$, and $\epsilon^i$, and that his time allocation decision is optimal given $T$. We similarly define $R_{0j}^j(w^j, y^j, \epsilon^j; T)$ for single type $j$ women, and $R_{ij}(w, y, X, \epsilon; T, \lambda_{ij}(T))$ for married $\langle i, j \rangle$ couples.

Note that taxes affect the problem in the following way. First, they have a direct effect on welfare and revenue holding behavior and the marriage market fixed. Second, there is a behavioral effect such that time allocations within any given match change affecting both welfare and revenue. Third, there is a marriage market effect that changes who marries with whom, the allocation of resources within the household (through adjustments in the Pareto weights), and the distribution of the idiosyncratic payoffs within any given match.

4 Data, identification and estimation

4.1 Data

We use two data sources for our estimation. Firstly, we use data from the 2006 American Community Survey (ACS). This provides us with information on education, marital patterns, demographics, incomes, and labour supply. We supplement this with pooled
American Time Use Survey (ATUS) data, which we use to construct a broad measure of home time. Following Aguiar and Hurst [1] and [2], we segment the total endowment of time into three broad mutually exclusive time use categories: work activities, home production activities, and leisure activities. Home production hours contain core home production, activities related to home ownership, obtaining goods and services, and care of other adults. It also contains childcare hours that measure all time spent by the individual caring for, educating, or playing with their children.

For both men and women we define three broad education groups for our analysis: less-than high school, high school graduate, and college and above. These constitute the types for the purposes of marriage market matching. Our sample is restricted to single individuals who are aged 25–35 (inclusive). For married couples, we include all individuals where the reference person householder, as defined by the Census Bureau, belongs to this same age band.

Our estimation allows for market variation in the population vectors and the economic environment (taxes and transfers). We define a market at the level of the Census Bureau-designated division, with each division comprising a small number of states. Within these markets we calculate accurate tax schedules (defined as piecewise linear functions of family earnings) prior to estimation using the National Bureau of Economic Research TAXSIM calculator (see 22), including both federal and state tax rates (including the Earned Income Tax Credit), and supplemented with detailed program rules for major welfare programs. The inclusion of welfare benefits is important in order to reflect the financial incentives for lower-income households. We describe our calculation of the

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8The American Time Use Survey (ATUS) is a nationally representative cross-sectional time-use survey launched in 2003 by the U.S. Bureau of Labor Statistics (BLS). The ATUS interviews a randomly selected individual age 15 and older from a subset of the households that have completed their eighth and final interview for the Current Population Survey (CPS), the U.S. monthly labor force survey.

9See Aguiar, Hurst and Karabarbounis [2] for a full list of the time use categories contained in the ATUS data and a description of how there are categorized.

10This type of educational categorisation is standard in the marriage market literature. Papers that have used similar categories are [18], [19], [28], [17], [15], among others.

11Similar age selections are common in the literature. See [17], Chiappori, Salanié and Weiss [15], Galichon and Salanié [24] for examples.

12There are nine Census Bureau divisions: New England (Connecticut, Maine, Massachusetts, New Hampshire, Rhode Island, and Vermont); Mid-Atlantic (New Jersey, New York, and Pennsylvania); East North Central (Illinois, Indiana, Michigan, Ohio, and Wisconsin); West North Central (Iowa, Kansas, Minnesota, Missouri, Nebraska, North Dakota, and South Dakota); South Atlantic (Delaware, Florida, Georgia, Maryland, North Carolina, South Carolina, Virginia, Washington D.C., and West Virginia); East South Central (Alabama, Kentucky, Mississippi, and Tennessee); West South Central (Arkansas, Louisiana, Oklahoma, and Texas); West Mountain (Arizona, Colorado, Idaho, Montana, Nevada, New Mexico, Utah, and Wyoming); Pacific (Alaska, California, Hawaii, Oregon, and Washington).
empirical tax and transfer schedules in Appendix D.

4.2 Identification

The estimation will be of a fully specified parametric model. It is still important to explore non-/semi-parametric identification of the model because it indicates what is the source of variation in the data that is filtered through the economic model that gives rise to the parameter estimates versus which parameter estimates arise from the functional form imposed in estimation. Here we explore semi-parametric identification. That is, as is standard in the identification literature of discrete choice and equilibrium marriage market models, we assume that the distribution of the state specific errors and the marital shocks are known. The identification result does not rely on the actual distribution of the state specific errors, however, the Type I extreme value assumption is essential to the identification result in this paper and the frictionless marriage market literature in general.\footnote{See Choo and Siow (2006), Choo and Seitz (2013), Chiappori, Dias, and Meghir (2015) and Galichon and Salanie (2015) for examples.}

The intuition behind our identification result is straightforward. Using the marriage market equilibrium and variation in the distribution of female and male types across market we are able to identify the wife’s Pareto weight $\lambda_{ij}$. Then given the wife’s Pareto weight $\lambda_{ij}$ and observation of time allocation the identification of the utility and the home production functions follow from known results in the static discrete choice models. Our identification result is a formalization of the assumption of the existence of distributional factor, defined as any factor that influences the decision process but does not affect preferences or budget sets, used to establish identification in empirical application of collective models of household behavior. A difference in sex ratio is a popular distributional factor used in empirical application. Our identification result is derived from the equilibrium conditions of a fully specified equilibrium marriage market. In Appendix E we demonstrate the identification of the wife’s Pareto weight $\lambda_{ij}$ from variation across markets and the Type 1 extreme value assumption on the marital shocks.

4.3 Empirical specification

In Section 4.5 we will see that there are important differences in labour supply and the time spent on home production activities for men and women. Moreover, there are large
differences between those who are single and those who are married (and with whom). Our aim is to construct a credible and parsimonious model of time allocation decisions that adequately allows for individual heterogeneity in preferences and can well describe these facts. Moreover, we want to be able to do this whilst maintaining that individual preferences are unchanged by marriage.

We define the time allocation sets $A^i$ and $A^j$ symmetrically for all individuals. The total time endowment $L_0$ is set equal to 112 hours per week. To construct these sets, we assume that there are a non-discretionary four hours spent on home production and define a discrete grid comprising 10 equi-spaced values. A unit of time is therefore given by $(112 - 4)/(10 - 1) = 12$ hours. Given time may be devoted to three alternatives (with the sum of leisure, market work and home work equal to $L_0$) there are a total of 55 possible time allocation choices for an individual. For couples there are $55^2 = 3025$ discrete alternatives. To reduce the dimension of the choice slightly set we assume that there is a minimal amount of leisure consumed (the first strictly positive amount on our grid), and that maximum work and (non-discretionary) home production time is 60 hours per week. This then implies 33 alternatives for single individuals, and so $33^2 = 1089$ alternatives for couples. This dimension reduction is of little consequence as we only remove alternatives that are never practically chosen.

All the estimation and simulation results presented here assume individual preferences that are separable in the private consumption good, but non-separable in leisure and household public good consumption. Moreover, these preferences are unchanged by the state of marriage. We allow preferences to depend upon individual type, as well as upon demographics. In our empirical application the demographic realizations correspond to the presence of children, which will depend on both own type and (for married individuals) on spousal type.\textsuperscript{14} For type $i$ males we assume preferences of the form:

$$u^i(\ell^i, q^i, Q; X^i) = \frac{q^i[1-\sigma_q]}{1-\sigma_q} + \beta^i(X^i)\left[\frac{[\ell^i Q^i]^{1-\sigma_q}(X^i)}{1-\sigma_q} - 1\right],$$

and with preferences defined symmetrically for women. This preference specification allows us to derive an analytical expression for the private good consumption share $s_{ij}$ for any joint time allocation in the household (i.e. the solution to equation 3). Given our parameterization, the share is independent of $q$ and is tightly connected to the household

\textsuperscript{14}This implies that the marriage market stage of our problem can be thought of as individuals making joint marriage and (uncertain) fertility decisions.
Pareto weight. It is straightforward to show that the female share of private consumption is given by:

\[ s_{ij}(\lambda_{ij}) = \left[ 1 + \left( \frac{\lambda_{ij}}{1 - \lambda_{ij}} \right)^{-1/\sigma_q} \right]^{-1}, \]

which is clearly increasing in the female weight. In the case that \( \sigma_q = 1 \) this reduces to \( s_{ij}(\lambda_{ij}) = \lambda_{ij} \). To ensure that the sufficient conditions required for the existence and uniqueness of a marriage market equilibrium are satisfied (as described in Proposition 1), we require that \( \sigma_q \geq 1 \).

A potential economic benefit from marriage is in terms of home production. We assume a constant elasticity of substitution (CES) production technology that depends on the time inputs of both spouses, \( h^i_Q \) and \( h^j_Q \). The relative importance of male and female time may depend upon demographics, as given by the share parameter \( \alpha(X) \). Additionally, we allow for a match specific term \( A_{ij} \) that determines the overall efficiency of production within an \( \langle i, j \rangle \) match. We therefore have:

\[ \tilde{Q}_{ij}(h^i_Q, h^j_Q; X) = A_{ij} \times \left[ \alpha(X) \cdot (h^i_Q)^\nu + (1 - \alpha(X)) \cdot (h^j_Q)^\nu \right]^{\frac{1}{\nu}}, \]

where the elasticity of substitution is given by \( 1/(1 - \nu) \).

The state specific errors associated with the discrete individual time allocation decisions \( \epsilon_{a_i} \) and \( \epsilon_{a_j} \) are assumed Type-I extreme value, with the scale parameter \( \sigma_{\epsilon} \). The marriage decision depends upon the expected value of a match. For couples, the maximization problem of the household is not the same as the utility maximization problem of an individual. As a result, the well-known convenient results for expected utility and conditional choice probabilities in the presence of extreme value errors (see, e.g. 32) do not apply for married individuals. We therefore evaluate these objects numerically. Log-wage offers are normally distributed, with the parameters of the distribution a function of both gender and education type. Finally, for singles, the demographic transitions

\(^{15}\text{In the case where the private good curvature parameter } \sigma_q \text{ varies across spouses, the endogenous consumption share } s_{ij} \text{ will also be a function of the household private good consumption.}\)

\(^{16}\text{We approximate the integral over these preference shocks through simulation. To preserve smoothness of our distance metric (in estimation), as well as the welfare and revenue functions (in out simulations) we employ a Logistic smoothing kernel. Conditional on } (w, y, X, \epsilon) \text{ and the match } \langle i, j \rangle \text{ this assigns a } \text{probability of any given joint allocation being chosen by the household. We implement this by adding an extreme value error with scale parameter } \tau_{\epsilon} > 0 \text{ which varies with all possible joint discrete time alternatives. The probability of a given joint time allocation conditional on is given by the usual conditional Logit form. As the smoothing parameter } \tau_{\epsilon} \to 0 \text{ we get the unsmoothed simulated frequency.}\)
are a function of gender and type. For couples, they depend on both own and spouse type.

4.4 Estimation

We estimate our model with a moment based estimation procedure. We employ an equilibrium constraints (or MPEC) approach to our estimation (36). This requires that we augment the estimation parameter vector to include the complete vector of Pareto weights for each market. Estimation is then performed with $I \times J \times K$ equality constraints that require that there is neither excess demand nor supply for individuals in any marriage market position and in each market (that is, equation 8 holds). In practice, this equilibrium constraints procedure is much quicker than a nested fixed point approach (which would require that we solve the equilibrium for every candidate model parameter vector in each market) and is also more accurate as it does not involve the solution approximation step that we describe in Appendix C.

A rich set of moments are targeted in our estimation. These include moments that describe marital sorting patterns by market, measures of labour supply and home time for men and women (all by the presence of children and by own and spouse type) and accepted wages. A complete description of all the moments used are provided in Appendix F.

4.5 Estimation results

We now provide a brief overview of the results of our main estimation exercise. A more complete characterization of estimates and the within sample fit is provided in Appendix G. In Table 2 we show the fit to marital sorting patterns across all markets (see the Appendix G for these at the market level) and can see that the model is capable of well replicating empirical marital sorting patterns. Recall that we do not have any parameter at the match level than can be varied to fit marital patterns independently of the time allocation behavior. In Figure 1 we present the distribution of work time for both single and married women, with and without children (here aggregated over own and spousal types, and markets). The model is able to generate the most salient features of the data: relative to single women, married women work less and have higher home time, with the

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17Given our definition of a market, and the number of male/female types, this involves $3 \times 3 \times 9 = 81$ additional parameters and non-linear equality constraints.
differences most pronounced for women with children. The corresponding relationships for men are shown in Figure 2. There are much smaller differences in both labour supply and home time between single and married men. Men with children have higher home time, although the difference is much smaller than observed in the case of women.

An important object of interest is the Pareto weight, and how this varies at the level of the match and across markets. The Pareto weights implied by our model estimates are presented in Table 1. Given our parameter estimates, these are also synonymous with the female share of total household private consumption. There are some important features from the table. Firstly, while there are exceptions it is generally the case that the female weight is increasing when she is more educated relative to her husband. For example, a college educated woman receives (on average) a share of 0.46 if she is married a man who has the same level of education. For a woman of the same education type to be willing to marry a man with less than high school education, her share must be increased to 0.75. Second, there is a dispersion in these weights across markets, which here reflects the joint impact of variation in taxes and the population vectors.

While the following optimal design exercise directly uses the behavioral model developed in Section 2, to help understand the implications of our parameter estimates for time allocation decisions, we simulate elasticities under the actual 2006 tax systems for different family types. All elasticities are calculated by increasing the net wage rate, hold the marriage market fixed, and correspond to uncompensated changes. Increasing net wages of a given adult in households that comprise multiple members means that
Figure 1: Female labour supply and home time. Figure shows empirical and predicted frequencies of female work hours and home time, aggregated over all female and spousal types and conditional on marital status and children. UN corresponds to non-employment (0 hours); PT corresponds to part-time (12, 24 hours); FT corresponds to full-time (36, 48, 60 hours). L corresponds to low home-time (4, 16 hours); ML corresponds to medium-low home-time (28 hours); MH corresponds to medium-high home-time (40 hours); H corresponds to high home-time (52, 64 hours).
Figure 2: Male labour supply and home time. Figure shows empirical and predicted frequencies of male work hours and home time, aggregated over all male and spousal types and conditional on marital status and children. UN corresponds to non-employment (0 hours); PT corresponds to part-time (12, 24 hours); FT corresponds to full-time (36, 48, 60 hours). L corresponds to low home-time (4, 16 hours); ML corresponds to medium-low home-time (28 hours); MH corresponds to medium-high home-time (40 hours); H corresponds to high home-time (52, 64 hours).
Table 2: Empirical and predicted marital sorting patterns

<table>
<thead>
<tr>
<th></th>
<th>Less than high school</th>
<th>High school</th>
<th>College and above</th>
</tr>
</thead>
<tbody>
<tr>
<td>Women</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less than high school</td>
<td>-</td>
<td>0.032</td>
<td>0.162</td>
</tr>
<tr>
<td>Men</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less than high school</td>
<td>0.033</td>
<td>0.025</td>
<td>0.023</td>
</tr>
<tr>
<td>High school</td>
<td>0.163</td>
<td>0.015</td>
<td>0.173</td>
</tr>
<tr>
<td>College and above</td>
<td>0.085</td>
<td>0.001</td>
<td>0.035</td>
</tr>
</tbody>
</table>

Notes: Table shows empirical and simulated distribution of marriage sorting patterns by education group, aggregated over all marriage markets. The statistic in brackets corresponds to the simulated value. Each cell corresponds to the fraction of households in that position. Empirical frequencies are calculated with 2006 American Community Survey using sample selection as detailed in Section 4.1. Simulated frequencies are calculated using the model estimates as presented in Appendix G.

we are perturbing the tax schedule as we move in a single dimension. Starting from a fully joint system (as is true in our estimation exercise) this is equivalent to first taxing the spouse whose net wage is not varied on the original joint tax schedule, and then reducing marginal tax rates for subsequent earnings (as then applied to the earnings of their spouse, whose net wage we are varying). The results of this exercise are shown in Table 3. For single individuals we report employment, conditional work hours, and home time elasticities in response to changes in their own wage. For married individuals we additionally report cross-wage elasticities that describe how employment, work hours, and home time respond as the wage of their spouse is varied.\(^{18}\)

Our labour supply elasticities suggest that women are more responsive to changes in their own wage (both on the intensive and extensive margin) than are men. The same pattern is true with respect to changes in the new wage of their partner. However, own-wage elasticities are always at least as large (in absolute terms) as are cross-wage elasticities. The own-wage hours and participation elasticities that we find are very much consistent with range of estimates in the labour supply literature (see, for example, 33). The evidence of cross-wage labour supply effects is more limited, although the findings

---

\(^{18}\)Own wage conditional work hours elasticities condition on being employed in the base system. As we increase an individual’s net-wage (holding that of any spouse fixed) their employment is necessarily weakly increasing. For cross wage conditional work hours elasticities, we condition on being employed both before and after the net-wage increase.
Table 3: Simulated elasticities

<table>
<thead>
<tr>
<th></th>
<th>Married</th>
<th></th>
<th>Single</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Men</td>
<td>Women</td>
<td>Men</td>
</tr>
<tr>
<td><strong>Work hours</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Own-wage elasticity</td>
<td>0.11</td>
<td>0.16</td>
<td>0.04</td>
</tr>
<tr>
<td>Cross-wage elasticity</td>
<td>-0.07</td>
<td>-0.12</td>
<td>-</td>
</tr>
<tr>
<td><strong>Participation</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Own-wage elasticity</td>
<td>0.04</td>
<td>0.17</td>
<td>0.01</td>
</tr>
<tr>
<td>Cross-wage elasticity</td>
<td>-0.04</td>
<td>-0.08</td>
<td>-</td>
</tr>
<tr>
<td><strong>Home hours</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Own-wage elasticity</td>
<td>-0.06</td>
<td>-0.09</td>
<td>-0.02</td>
</tr>
<tr>
<td>Cross-wage elasticity</td>
<td>0.04</td>
<td>0.05</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes: All elasticities simulated under 2006 federal and state tax and transfer systems, aggregated over markets, and holding the marriage market fixed. Elasticities are calculated by increasing the individual’s net wage rate by 1% (own-wage elasticity) or the wage of their spouse by 1% (cross-wage elasticity) as described in the main text. Participation elasticities measure the percentage point increase in the employment rate; work hours elasticities measure the percentage increase in hours of work amongst workers in the base system; home hours elasticities measure the percentage increase in total home time hours.

Here are consistent with [7]. In the table we also report home hours elasticities which suggest that individuals substitute away from home time for a given uncompensated change in their wage, and substitute towards home time when their spouses wage is increased. The same home-time pattern reduction (for single men and women) was reported in Gelber and Mitchell [27].

We also simulate elasticities related to the marriage market. We consider a perturbation whereby we increase the tax liability on all marriage couples. That is, we (uniformly) reduce the marriage bonus/increase the marriage penalty. Our simulations imply that a $1,000 increase in the marriage penalty would increase the fraction of single households by 1.1 percentage points.

5 Optimal taxation of the family

In this section we consider the normative implications when we adopt a social welfare function with a set of subjective social welfare weights. There are two main stages to our analysis. Firstly, we consider the case where we do not restrict the form of jointness permitted in our choice of tax schedule for married couples. Under alternative assumptions on the degree of inequality aversion, we empirically characterize the form of the
optimal tax system and show the importance of the marriage market in determining this. Second, we consider the choice of tax schedules when it is restricted to be either fully joint for married couples or completely independent. In both these cases we quantify the welfare loss relative to our more general benchmark specification.

The results presented in this section assume a single marriage market, with the population vectors for men and women defined as those corresponding to the aggregate. We consider the following form for the utility transformation function in our social welfare function:

\[ Y(v; \theta) = e^{\delta v} - \frac{1}{\delta}, \]

which is the same form as considered in Blundell and Shephard [8]. Under this specification we have that \( \delta = 0 \) corresponds to the linear case (by L'Hôpital’s rule), and \( -\delta = -Y''(v; \theta)/Y'(v; \theta) \) corresponding to the (constant) coefficient of absolute inequality aversion.

This form of utility transformation function has useful properties, and in conjunction with the additively of the idiosyncratic marital payoffs permits us to obtain the following useful result:

**Proposition 2.** Consider a married type \( i \) male in an \( (i, j) \) marriage. The contribution of such individuals to \( W(T) \) in equation 10 for \( \delta < 0 \) is given by:

\[
W_{ij}^i(T) = \int_{\theta^i} \int_{w, X, \epsilon} \left[ v_{ij}^i(w, y, X, \epsilon; T) + \theta^i \right] dG_{ij}(w, y, X) dH_{ij}^i(\theta^i)
\]

\[
\quad = p_{ij}^i(T)^{-\delta \sigma_\theta} \Gamma(1 - \delta \sigma_\theta) \int_{w, X, \epsilon} \frac{\exp[\delta v_{ij}^i(w, y, X, \epsilon; T)]}{\delta} dG_{ij}(w, X, \epsilon) - \frac{1}{\delta},
\]

where \( \Gamma(\cdot) \) is the gamma function and \( p_{ij}^i(T) \) is the conditional choice probability (equation 6) for type \( i \) males. For \( \delta = 0 \) this integral evaluates to:

\[
W_{ij}^i(T) = \gamma - \sigma_\theta \log p_{ij}^i(T) + \int_{w, X, \epsilon} v_{ij}^i(w, y, X, \epsilon; T) dG_{ij}(w, X, \epsilon),
\]

where \( \gamma = -\Gamma'(1) \approx 0.5772 \) is the Euler-Mascheroni constant. The form of the welfare function contribution is symmetrically defined in alternative positions, and for married women, single men and single women.

A proof of this proposition is provided in Appendix B. As part of that proof, we characterize the distribution of the idiosyncratic payoffs for individuals who select into
a given marital position.\textsuperscript{19} This result allows us to decompose the welfare function contributions in to parts that reflect the distribution of idiosyncratic utility payoffs from marriage and singlehood, and that which reflects the welfare from individual consumption and time allocation decisions. It is also obviously very convenient from a computational perspective as the integral over the state specific terms does not require simulating.

5.1 Specification of the tax schedule

Before presenting the results from our design simulations, we first describe the parametric specification of the tax system that we use in our illustrations. Consider the most general case. The tax system comprises a schedule for singles (varying with earnings) and a schedule for married couples (varying with the earnings of both spouses). We exogenously define a set of $N$ ordered tax brackets $0 = n_1 < n_2 < \ldots < n_N$ that apply to the earnings of a given individual. We assume, but do not require, that these brackets are the same for both members in a married couple, and also for singles. Associated with each bracket point for singles is the tax level parameter vector $t_{N \times 1}$. For married couples we have the tax level parameter matrix $T_{N \times N}$. Consistent with real-world tax systems, we do not consider gender-specific taxation and therefore impose symmetry of the tax matrix in all our simulations. Together, our tax system is therefore characterized by $N + N \times (N + 1)/2$ tax parameters defined by the vector $\beta_T = [t_{N \times N}^*, \text{vec}(T_{N \times N}^*)]$.

The parameter vector $t_{N \times 1}$ and matrix $T_{N \times N}$ define tax amounts at levels of earnings that coincide with the exogenously chosen tax brackets (or nodes). The tax liability for other earnings levels is obtained by fitting an interpolating function. For singles this is achieved through familiar linear interpolation, so that the tax schedule is of a piecewise linear form. We extend this for married couples by a procedure of polygon triangulation. This divides the surface into a non-overlapping set of triangles. Within each of these triangles, marginal tax rates for both spouses, while potentially different, are by construction constant.\textsuperscript{20}

In our application we set $N = 9$ with the earnings brackets (expressed in dollars per week in 2006 prices) as $\{0, 200, 400, 600, 800, 1200, 1600, 2000, 8000\}$. Thus, we have

\textsuperscript{19}This is a related, but distinct result compared to Proposition 1 in Blundell and Shephard [8]. That proposition does not apply to the welfare contribution conditional on a given marital state as (for individuals in couples) the maximisation problem of the household is not synonymous with the maximisation of the individual utility function.

\textsuperscript{20}The requirement that marginal tax rates can not exceed 100\% (as earnings in any feasible dimension is varied) may be incorporated by imposing $(N - 1) + N \times (N - 1)$ linear restrictions on the parameters.
a tax system that is characterized by 54 parameters. Using our estimated model, the exogenous revenue requirement $T$ is set equal to the expected state and federal income tax revenue (including EITC payments) and net of welfare transfers.21

5.2 Implications for design

We now describe our main results. In Figure 3 we present the joint (net-income) budget constraint for both singles and married couples, calculated under the parameterization $\delta = 0$. For clarity of presentation, the figure has been truncated at individual earnings greater than $2,000 a week ($104,000 a year). The implied schedule for singles is shown by the blue line. The general flattening of this line as earnings increase indicates a broadly progressive structure. In the same figure, the optimal schedule for married couples is shown by the three dimensional surface, which is symmetric by construction (i.e. gender neutrality). Within each of the shaded triangles, the marginal tax rates of both spouses are constant. As the earnings of either spouse change in any direction such that we enter a new triangle, marginal tax rates will potentially change. Holding constant the earnings of a given spouse, we can clearly see a progressive structure. Comparing these implied schedules at different levels of earnings is then informative about the degree of tax jointness. To better illustrate the implied degree of tax jointness, in Figure 4 we show the associated marginal tax rate of a given individual, as the earnings of their spouse is fixed at different levels. One of the most prominent features of this graph is that marginal tax rates tend to be lower the higher is the earnings of ones spouse. This is the negative jointness result described in Kleven, Kreiner and Saez [31].

[To be completed]

5.3 The importance of the marriage market

In the simulations presented in Section 5.2 we saw that under the range of government preference parameters considered, that the implied marital sorting pattern was relatively

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21This is calculated to be $TBC. We solve the optimal design problem numerically. Given our parameterization of the tax schedule, we solve for the optimal parameter vectors $\beta_T$ using an equilibrium constraints approach that is similar to that described by 36, in the context of estimation problems. This involves augmenting the parameter vector to include the $I \times J$ vector of Pareto weights as additional parameters, and imposing the $I \times J$ equilibrium constraints $\mu^T_{ij} (T, \lambda^T) = \mu^T_{ij} (T, \lambda^J)$ in addition to the usual incentive compatibility and revenue constraints. This approach only involves calculating the equilibrium associated with the optimal parameter vector $\beta^*_T$ (rather than any candidate $\beta_T$ as would be true in a nested fixed point procedure).
Figure 3: Optimal tax schedule with $\delta = 0$. Figure shows net-income as a function of labour earnings for both single individuals (blue line) and couples (three dimensional surface). Marginal tax rates for both spouses are constant within each of the shaded triangles.

Figure 4: Marginal tax rates with $\delta = 0$. Figure shows marginal tax rate as a function of individual labour earnings for singles and married couples with fixed spousal earnings.
close to that from our estimated model. To better understand the importance of the marriage market in determining the optimal structure of taxes and transfers we perform the following exercise. We resolve for the optimal structure holding the entire vector of Pareto weights, marriage market positions, and distributions of idiosyncratic payoffs fixed at their values from the corresponding optimum from the previous section. The extent to which the optimal schedules differ (together with any imbalance in spousal type supply/demand) once the marriage market is held fixed is directly informative about the importance of the marriage market.

5.4 Restrictions on the form of tax schedule jointness

Our previous analysis allowed for a very general form for the tax schedule. We now repeat our analysis, but with the permissible form of jointness much more restricted.

5.4.1 Individual taxation

Individual taxation: \( y_1 + y_2 - T(y_1) - T(y_2) \)

5.4.2 Joint taxation with income splitting

Income splitting: \( y_1 + y_2 - 2T\left(\frac{y_1 + y_2}{2}\right) \)

5.4.3 Joint taxation with income aggregation

Income aggregation: \( y_1 + y_2 - T(y_1 + y_2) \)

[To be completed]

6 Summary and conclusion

[To be completed]
Appendices

A Proof of proposition 1

We assume that the distribution $G_{ij}(w, y, X, \epsilon)$ is absolutely continuous and twice continuously differentiable. The utility functions $u^i(\ell^i, q^i, Q; X^i)$ and $u^j(\ell^j, q^j, Q; X^j)$ are assumed concave increasing in $\ell, q,$ and $Q,$ and with $\lim_{q^i \to 0} u^i(\ell^i, q^i, Q; X^i) = \lim_{q^j \to 0} u^j(\ell^j, q^j, Q; X^j) = -\infty.$ To proceed we define the excess demand function as:

$$ED_{ij}(\lambda) = \mu^d_{ij}(\lambda^i) - \mu^s_{ij}(\lambda^j), \quad \forall i = 1, \ldots, I, j = 1, \ldots, J.$$ (14)

Equilibrium existence is synonymous with the excess demand for all types being equal to zero at some vector $\lambda^* \in [0, 1]^I \times J,$ i.e. $ED_{ij}(\lambda^*) = 0, \forall i = 1, \ldots, I, j = 1, \ldots, J.$ Equilibrium uniqueness implies that there is a single vector that achieves this. Under our regularity conditions we have that: (i) $U_{ij}^i(\lambda_{ij})$ and $U_{ij}^j(\lambda_{ij})$ are continuously differentiable in $\lambda_{ij};$ (ii) $\partial U_{ij}^i(\lambda_{ij})/\partial \lambda = -\frac{\lambda_{ij}}{1-\lambda_{ij}} \partial U_{ij}^j(\lambda_{ij})/\partial \lambda < 0; (iii)$ $\lim_{\lambda_{ij} \to 0} ED_{ij}(\lambda_{ij}, \lambda_{-ij}) > 0,$ and; (iv) $\lim_{\lambda_{ij} \to 1} ED_{ij}(\lambda_{ij}, \lambda_{-ij}) < 0.$

A.1 Properties of the excess demand functions

We now state further properties of the excess demand functions, which we apply when we provide our proof of existence and uniqueness. Note that as we vary $\lambda_{ij}$ our excess
demand functions must satisfy:

\[
\frac{\partial ED_{ij}(\lambda)}{\partial \lambda_{ij}} = \frac{\partial \mu^d_{ij}(\lambda)}{\partial \lambda_{ij}} - \frac{\partial \mu^s_{ij}(\lambda)}{\partial \lambda_{ij}} < 0, \tag{15a}
\]

\[
\frac{\partial ED_{ik}(\lambda)}{\partial \lambda_{ij}} = \frac{\partial \mu^d_{ik}(\lambda)}{\partial \lambda_{ij}} - \frac{\partial \mu^s_{ik}(\lambda)}{\partial \lambda_{ij}} > 0; k \neq j, \tag{15b}
\]

\[
\frac{\partial ED_{kj}(\lambda)}{\partial \lambda_{ij}} = \frac{\partial \mu^d_{kj}(\lambda)}{\partial \lambda_{ij}} - \frac{\partial \mu^s_{kj}(\lambda)}{\partial \lambda_{ij}} > 0; k \neq i, \tag{15c}
\]

\[
\frac{\partial ED_{kl}(\lambda)}{\partial \lambda_{ij}} = \frac{\partial \mu^d_{kl}(\lambda)}{\partial \lambda_{ij}} - \frac{\partial \mu^s_{kl}(\lambda)}{\partial \lambda_{ij}} = 0; k \neq i, l \neq j. \tag{15d}
\]

Note that equation (15d) is a consequence of the IIA property of the Type-I extreme value distribution and is therefore critical for our proof of the uniqueness of equilibrium.

### A.2 Existence

To prove existence we construct a function, \( \Gamma(\lambda) \), as:

\[
\Gamma(\lambda) = \psi \cdot ED(\lambda) + \lambda, \tag{16}
\]

for \( \psi > 0 \) which maps \([0,1]^{I \times J}\) onto \([0,1]^{I \times J}\). Then by Tarski’s theorem, if \( \Gamma(\lambda) \) is non-decreasing in \( \lambda \) there exists a \( \lambda^* \in [0,1]^{I \times J} \) such that \( \lambda^* = \Gamma(\lambda^*) \). However, \( \lambda^* = \psi \cdot ED(\lambda^*) + \lambda^* \) iff \( ED(\lambda^*) = 0 \). Assuming that \( U_{ij}(\lambda_{ij}) \) for \( k = i, j \) is derived from the time allocation problem described in the main text then one has proven the existence of equilibrium. It is therefore sufficient to show that one can construction a \( \Gamma(\lambda) = \psi \cdot ED(\lambda) + \lambda \) such that:

1. \( \psi \cdot ED(\lambda) + \lambda \in [0,1]^{I \times J} \)

2. \( \Gamma(\lambda) \) is non-decreasing in \( \lambda \).

**Lemma 1.** The excess demand functions are continuously differentiable with \( ED(0_{I \times J}) \geq 0 \) and \( ED(1_{I \times J}) \leq 0 \).
Proof of Lemma 1. The continuously differentiability follows directly from the regularity conditions described above. \( ED(0_{I \times J}) \geq 0 \) and \( ED(1_{I \times J}) \leq 0 \) follow from our regularity conditions along with equations (15a) to (15d).  

Lemma 2. For all \( \langle i, j \rangle \) there exist a \( \psi_{ij} > 0 \) such that \( 0 \leq \Gamma_{ij}(\lambda) \leq 1 \).

Proof of Lemma 2. For each \( \langle i, j \rangle \) define the sets \( BC_{ij}^+ = \{ \lambda \in [0,1]^{I \times J} : ED_{ij}(\lambda) > 0 \} \) and \( BC_{ij}^- = \{ \lambda \in [0,1]^{I \times J} : ED_{ij}(\lambda) < 0 \} \). Then define \( \psi_{ij}^+ = \min \{ (1 - \lambda_{ij}) / ED_{ij}(\lambda) : \lambda \in BC_{ij}^+ \} \) and \( \psi_{ij}^- = \min \{ -\lambda_{ij} / ED_{ij}(\lambda) : \lambda \in BC_{ij}^- \} \). Continuity of \( ED_{ij}(\lambda) \) implies that both \( \psi_{ij}^+ \) and \( \psi_{ij}^- \) exist and are strictly positive. Then for all \( \psi_{ij} \in (0, \min \{ \psi_{ij}^+, \psi_{ij}^- \}) \) we have that

\[
0 \leq \psi_{ij} ED_{ij}(\lambda) + \lambda_{ij} \leq 1
\]  

Lemma 3. There exist a \( \psi > 0 \) such that \( 0_{I \times J} \leq \Gamma(\lambda) \leq 1_{I \times J} \) and \( \partial \Gamma(\lambda) / \partial \lambda_{ij} \geq 0_{I \times J} \).

Proof of Lemma 3. Let \( D_{ij} = \max_{k,l} \max_{\lambda} \left\{ |\partial ED_{ij}(\lambda) / \partial \lambda_{kl}| : \lambda \in [0,1]^{I \times J} \right\} \) then for each \( \langle i, j \rangle \) and for all \( \psi_{ij} \in (0, 1/D_{ij}) \) one has the following:

\[
\frac{\partial [\psi_{ij} ED_{ij}(\lambda) + \lambda_{ij}]}{\partial \lambda_{ij}} = \psi_{ij} \frac{\partial ED_{ij}(\lambda)}{\partial \lambda_{ij}} + 1 \geq -\psi_{ij} D_{ij} + 1 > 0,
\]

\[
\frac{\partial [\psi_{ij} ED_{ij}(\lambda) + \lambda_{ij}]}{\partial \lambda_{kj}} = \psi_{ij} \frac{\partial ED_{ij}(\lambda)}{\partial \lambda_{kj}} \geq 0 \text{ for } k \neq i,
\]

\[
\frac{\partial [\psi_{ij} ED_{ij}(\lambda) + \lambda_{ij}]}{\partial \lambda_{il}} = \psi_{ij} \frac{\partial ED_{ij}(\lambda)}{\partial \lambda_{il}} \geq 0 \text{ for } l \neq j,
\]

which follows from equations (15a) to (15d). Let:

\[
\overline{\psi} = \min \{ \min \{ \psi_{11}^+, \psi_{11}^- \}, \ldots, \min \{ \psi_{ij}^+, \psi_{ij}^- \}, 1/2D_{11}, \ldots, 1/2D_{IJ} \}. \]

Now choose any \( \psi \in (0, \overline{\psi}) \) and define \( \Gamma(\lambda) = \psi \cdot ED(\lambda) + \lambda \). We now have \( \Gamma : [0,1]^{I \times J} \to [0,1]^{I \times J} \) with \( \partial \Gamma(\lambda) / \partial \lambda_{ij} \geq 0_{I \times J} \) for all pairs \( \langle i, j \rangle \).

Therefore from Lemma 3 Tarski’s conditions are satisfied and an equilibrium exists.

\[\text{[22]}\text{Although } BC_{ij}^+ \text{ and } BC_{ij}^- \text{ are not compact the minimum still exist over these sets because as we approach the “open part” of the set, the objective goes to } \infty.\]
A.3 Uniqueness of Equilibrium

Uniqueness follows from the differentiability of $\Gamma(\lambda)$. That is given that $[0,1]^{I \times J}$ is closed and connected if $\Gamma(\lambda)$ is differentiable almost everywhere. Then if for some $\beta \in (0,1)$ we have that $\|\Gamma(\lambda) - \Gamma(\lambda')\| \leq \beta \|\lambda - \lambda'\|$ for all $\lambda$ and $\lambda'$ and some norm $\|\cdot\|$, then by the contraction mapping theorem there exists a unique fixed $\lambda^* \in [0,1]^{I \times J}$ such that $\Gamma(\lambda^*) = \lambda^*$. However, $\lambda^* = \psi \cdot \text{ED}(\lambda^*) + \lambda^*$ iff $\text{ED}(\lambda^*) = 0$ therefore $\lambda^*$ is also the unique equilibrium to our model.

**Lemma 4.** Under the regularity conditions there is a unique equilibrium.

**Proof of Lemma 4.** For notational ease let $\Gamma_{ij}(\lambda)$ be defined as

$$\Gamma_{ij}(\lambda) = \psi \cdot \text{ED}_{ij}(\lambda) + \lambda_{ij}.$$ 

From the proof of Lemma 3 we know that:

$$0 \geq \frac{\partial \Gamma_{ij}(\lambda)}{\partial \lambda_{ij}} = \psi \cdot \frac{\partial \text{ED}_{ij}(\lambda)}{\partial \lambda_{ij}} + 1 < 1, \quad (20)$$

since $\psi > 0$ and from equation (15a) we have that $\partial \text{ED}_{ij}(\lambda)/\partial \lambda_{ij} < 0$. Moreover, by construction:

$$\frac{1}{2} \geq \frac{\partial \Gamma_{ij}(\lambda)}{\partial \lambda_{ik}} \geq 0. \quad (21)$$

And the IIA property in (15d) implies that

$$\frac{\partial \Gamma_{ij}(\lambda)}{\partial \lambda_{kl}} = 0. \quad (22)$$

Therefore, $\Gamma$ is a contraction, since, by the mean value theorem,

$$|\Gamma_{ij}(\lambda) - \Gamma_{ij}(\lambda')| \leq \|\nabla \Gamma_{ij}(\tilde{\lambda})\| \|\lambda - \lambda'\| < \beta \|\lambda - \lambda'\| .$$

Where $\|\cdot\|$ is the sup norm, $\tilde{\lambda}$ is a point on the line between $\lambda$ and $\lambda'$, and $\beta$ is therefore a number less than 1 such that the absolute values of the derivatives of $\Gamma$ are less than $\beta$. This implies that:

$$\|\Gamma(\lambda) - \Gamma(\lambda')\| \leq \beta \|\lambda - \lambda'\|. \quad \blacksquare$$
B Proof of proposition 2

In this Appendix we derive the contribution of the marital shocks within each match to the social welfare function. We proceed in two steps. First, we characterize the distribution of marital preference shocks within a particular match, recognizing the non-random selection into a given position. Second, given this distribution we obtain the adjustment term using our specification of the utility transformation function.

Consider the first step. For brevity of notation, here we let $U_j$ denote the expected utility of a given individual from choice/spousal type $j$. Associated with each alternative $j$ is an extreme value error $\theta_j$ that has scale parameter $\sigma_\theta$. We now characterize the distribution of $\theta_j$ conditional on $j$ being chosen. Letting $p_j = (\sum_k \exp[(U_k - U_j)/\sigma_\theta])^{-1}$ denote the associated conditional choice probability it follows that:

$$
\Pr[\theta_j < x|j = \arg \max_k U_k + \theta_k] = \frac{1}{\sigma_\theta p_j} \int_{-\infty}^{x} \prod_{k \neq j} \exp \left( -e^{-\frac{\theta_j + U_j - U_k}{\sigma_\theta}} \right) \exp \left( -e^{-\frac{\theta_j}{\sigma_\theta}} \right) e^{-\frac{\theta_j}{\sigma_\theta}} d\theta_j
$$

$$
= \frac{1}{\sigma_\theta p_j} \int_{-\infty}^{x} \exp \left( -e^{-\frac{\theta_j}{\sigma_\theta}} \sum_k e^{-\frac{U_j - U_k}{\sigma_\theta}} \right) e^{-\frac{\theta_j}{\sigma_\theta}} d\theta_j
$$

$$
= \frac{1}{\sigma_\theta p_j} \int_{-\infty}^{x} \exp \left( -e^{-\frac{\theta_j}{\sigma_\theta} p_j^{-1}} \right) e^{-\frac{\theta_j}{\sigma_\theta}} d\theta_j
$$

$$
= \exp \left( -e^{-\frac{\theta_j + \sigma_\theta \log p_j}{\sigma_\theta}} \right)
$$

Hence, the distribution of the idiosyncratic payoff conditional on option $j$ being optimal, is also extreme value with the common scale parameter $\sigma_\theta$ and the shifted location parameter $-\sigma_\theta \log p_j$.

Marital payoff adjustment term: $\delta < 0$

Now consider the second step when $\delta < 0$. Using the form of the utility transformation function (equation ??), and letting $Z_j$ denote the entire vector of post-marriage realizations in choice $j$ (wages, preference shocks, demographics), it follows that the contribution to social welfare of an individual in this marital position may be written in the...
form:

\[
\int_{\theta_j} \int_{Z_j} Y[v_j(Z_j) + \theta_j] \, dG_j(Z_j) \, dH_j(\theta_j) = \int_{\theta_j} \exp(\delta \theta_j) \, dH_j(\theta_j) \int_{Z_j} \frac{\exp[\delta v(Z_j)]}{\delta} \, dG_j(Z_j) - \frac{1}{\delta},
\]

where we have suppressed the dependence on the tax system \( T \).

We now complete our proof in the \( \delta < 0 \) case by providing an analytical characterisation of the integral term over the idiosyncratic marital payoff. Using the result that \( \theta_j | j = \text{arg max}_k u_k + \theta_k \sim EV(-\sigma_0 \log p_j, \sigma_0) \) from above, we have:

\[
\int_{\theta_j} \exp(\delta \theta_j) \, dH_j(\theta_j) = \frac{1}{\sigma_0} \int_{\theta_j} \exp(\delta \theta_j) \exp(-[\theta_j + \sigma_0 \log p_j] / \sigma_0) e^{-\exp(-[\theta_j + \sigma_0 \log p_j] / \sigma_0)} \, d\theta_j
\]

\[
= \exp(-\delta \sigma_0 \log p_j) \int_{0}^{\infty} t^{-\delta \sigma_0} \, e^{-t} \, dt
\]

\[
= p_j^{-\delta \sigma_0} \Gamma(1 - \delta \sigma_0).
\]

The second equality performs the change of variable \( t = \exp(-[\theta_j + \sigma_0 \log p_j] / \sigma_0) \), and the third equality uses the definition of the Gamma function. Since we are considering cases where \( \delta < 0 \), this integral will converge.

**Marital payoff adjustment term: \( \delta = 0 \)**

The proof when \( \delta = 0 \) follows similarly. Here the contribution to social welfare of a given individual in a given marital position is simply given by:

\[
\int_{\theta_j} \int_{Z_j} Y[v_j(Z_j) + \theta_j] \, dG_j(Z_j) \, dH_j(\theta_j) = \int_{\theta_j} \theta_j \, dH_j(\theta_j) + \int_{Z_j} v(Z_j) \, dG_j(Z_j)
\]

\[
= \gamma - \sigma_0 \log p_j + \int_{Z_j} v(Z_j) \, dG_j(Z_j),
\]

with the second equality using the above result for the distribution of marital shocks within a match and then just applying the well-known result for the expected value of the extreme value distribution with a non-zero location parameter.

### C Marriage market numerical algorithm

In this Appendix we describe the iterative algorithm that we use to calculate the market clearing vector of Pareto weights. We first note that using the conditional choice prob-
abilities from equation 6 we are able to write the quasi-demand equation of type $i$ men for type $j$ spouses as:

$$
\sigma \theta \times \left[ \ln \mu^d_{ij}(T, \lambda^i) - \ln \mu^d_{i0}(T) \right] = U^i_{ij}(T, \lambda_{ij}) - U^i_{i0}(T).
$$

Similarly, the conditional choice probabilities for females from equation 7 allows us to express the quasi-supply equation of type $j$ women to the $\langle i, j \rangle$ submarket as:

$$
\sigma \theta \times \left[ \ln \mu^s_{ij}(T, \lambda^j) - \ln \mu^s_{0j}(T) \right] = U^j_{ij}(T, \lambda_{ij}) - U^j_{0j}(T).
$$

The algorithm proceeds as follows:

1. Provide an initial guess of the measure of both single males $0 < \mu^d_{i0} < m_i$ for $i = 1, \ldots, I$, and single females $0 < \mu^s_{0j} < f_j$ for $j = 1, \ldots, J$.

2. Taking the difference of the quasi-demand (equation 23) and the quasi-supply (equation 24) functions for each $\langle i, j \rangle$ submarriage market, and imposing the market clearing condition $\mu^d_{ij}(T, \lambda^i) = \mu^s_{ij}(T, \lambda^j)$ we obtain:

$$
\sigma \theta \times \left[ \ln \mu^s_{0j}(T) - \ln \mu^d_{i0}(T) \right] = U^j_{ij}(T, \lambda_{ij}) - U^j_{0j}(T) - \left[ U^i_{ij}(T, \lambda_{ij}) - U^i_{i0}(T) \right],
$$

which given the single measures $\mu^d_{i0}$ and $\mu^s_{0j}$ (and the tax schedule $T$) is only a function of the Pareto weight for that submarriage-market $\lambda_{ij}$. Given our assumptions on the utility functions there exists a unique solution to equation 25. This step therefore requires solving for the root of $I \times J$ univariate equations.

3. From Step 2, we have a matrix of Pareto weights $\lambda$ given the single measures $\mu^d_{i0}(T)$ and $\mu^s_{0j}(T)$ from Step 1. These can be updated by calculating the conditional choice probabilities (equation 6 and equation 7). The algorithm returns to Step 2 and repeats until the vector of single measures for both males and females has converged.

In practice we are able to implement this algorithm by first evaluating the expected utilities $U^i_{ij}(T, \lambda)$ and $U^j_{ij}(T, \lambda)$ for each marital match combination $\langle i, j \rangle$ on a fixed grid of Pareto weights $\lambda \in \lambda^{grid}$ with $\inf[\lambda^{grid}] \geq 0$ and $\sup[\lambda^{grid}] \leq 1$. We may then replace $U^i_{ij}(T, \lambda)$ and $U^j_{ij}(T, \lambda)$ with an approximating parametric function, so that no expected

---

23This is similar to the method described in independent work by [25].
values are actually evaluated within the iterative algorithm.\textsuperscript{24}

\section*{D \hspace{1em} Empirical tax and transfer schedule implementation}

In this appendix we describe our implementation of the empirical tax and transfer schedules for our estimation exercise. Since some program rules will vary by U.S. state, here we are explicit in indexing the respective parameters by market.\textsuperscript{25} Our measure of taxes includes both state and federal Earned Income Tax Credit (EITC) programmes, and we also account for Food Stamps and the Temporary Assistance for Needy Families (TANF) program. It does not include other transfers and non-income taxes such as sales and excises taxes. In addition to market, the tax schedules that we calculate also vary with marital status and with children. We assume joint filing status for married couples. For singles with children we assume head of household filing status.

Consider (a married or single) household $i$ in market $k$, with household earnings $E_{ik} = h_{iw}^k \cdot w_{ik}$ and demographic characteristics $X_{ik}$. As before, the demographic conditioning vector comprises marital status and children. The total net tax liability for such a household is given by $T_{ik} = \tilde{T}_{ik} - Y_{TANF}^{ik} - Y_{FSP}^{ik}$, where $\tilde{T}_{ik}$ is the (potentially negative) tax liability from income taxes and the Earned Income Tax Credit (EITC), $Y_{TANF}^{ik}$ and $Y_{FSP}^{ik}$ are the respective (non-negative) amounts of TANF and Food Stamps.

\subsection*{Income taxes and EITC}

Our measure of income taxes $\tilde{T}_{ik}$ includes both federal and state income taxes, as well as federal and state EITC. These are calculated with the NBER TAXSIM calculator, as described in Feenberg and Coutts [22]. Prior to estimation we calculate schedules for all markets and for all family types. We assume joint filing status for married couples. In practice, only around 2\% of married couples choose to file separate tax returns. For singles with children we assume head of household filing status. Note that certain state

\textsuperscript{24}Calculating the expected values within a match are (by many orders of magnitude) the most computationally expensive part of our algorithm. An implication of this is that if there are multiple markets $K$, and each market $k \leq K$ only differs by the population vectors $\mathcal{M}_k$ and $\mathcal{F}_k$ and/or the demographic transition function, then the computational cost in obtaining the equilibrium for all $K$ markets is \textit{approximately independent} of the number of markets $K$ considered. In our application we also have market variation in taxes and transfers so we are not able to exploit this property.

\textsuperscript{25}Since our definition of a market is at a slightly more aggregated level than the state level, we apply the state tax rules that correspond to the most populous state within a defined market (Census Bureau-designated division).
rules may imply discontinuous changes in tax liabilities following a marginal change in earnings. To avoid the technical and computational issues that are associated with this we (locally) modify the tax schedule in these events.\textsuperscript{26}

**Food Stamp Program**

Food Stamps are available to low income households both with and without children. For the purposes of determining the entitlement amount, net household earnings are defined as:

\[
N_{\text{FSP}}^{ik} = \max\{0, E_{ik} + Y_{\text{TANF}}^{ik} - D_{\text{FSP}}[X_{ik}]\},
\]

where \(Y_{\text{TANF}}^{ik}\) is the dollar amount of TANF benefit received by this household (see below), and \(D_{\text{FSP}}[X_{ik}]\) is the standard deduction, which may vary with household type. The dollar amount of Food Stamp entitlement is then given by:

\[
Y_{\text{FSP}}^{ik} = \max\{0, Y_{\text{max}}^{\text{FSP}}[X_{ik}] - \tau_{\text{FSP}} \times N_{\text{FSP}}^{ik}\},
\]

where \(Y_{\text{max}}^{\text{FSP}}[X_{ik}]\) is the maximum food stamp benefit amount for a household of a given size, and \(\tau_{\text{FSP}} = 0.3\) is the phase-out rate.\textsuperscript{27}

**TANF**

TANF provides financial support to families with children. Given the static framework we are considering we are not able to incorporate certain features of the TANF program, notably the time limits in benefit eligibility (see 12). For the purposes of entitlement calculation, we define net-household earnings as:

\[
N_{\text{TANF}}^{ik} = \max\{0, (1 - R_{\text{TANF}}^{k}) \times (E_{ik} - D_{\text{TANF}}^{k}[X_{ik}])\},
\]

\textsuperscript{26}These discontinuities are typically small. Our modification procedure involves increasing/decreasing marginal rates in earnings tax brackets just below the discontinuity.

\textsuperscript{27}In practice the Food Stamp Program also has a gross-earnings and net-earnings income test. These require that earnings are below some threshold that is related to the Federal Poverty Line for eligibility (see, e.g. 12). For some families, these eligibility rules would mean that there may be a discontinuous fall in entitlement (to zero) as earnings increase. While these rules are straightforward to model, we do not incorporate them for the same reason we do not allow discontinuities in the combined income taxes/EITC schedule. We also assume a zero excess shelter deduction in our calculations, and do not consider asset tests. Incorporating asset tests (even in a dynamic model) is very challenging as there exist very specific definitions of countable assets that do not correspond to the usual assets measure in life-cycle models.
where the dollar earnings disregard $D^k_{TANF}[X_{ik}]$ varies by market and household characteristics. The market-level percent disregard is given by $R^k_{TANF}$. The dollar amount of TANF entitlement is then given by:

$$Y^k_{TANF} = \min\{Y^\max_{TANF}[X_{ik}], \max\{0, r^k_{TANF} \times (Y^\max_{TANF}[X_{ik}] - N^k_{TANF})\}\}.$$  

Here $Y^\max_{TANF}[X_{ik}]$ defines the maximum possible TANF receipt in market $k$ for a household with characteristics $X_{ik}$, while $Y^\max_{TANF}[X_{ik}]$ defines what is typically referred to as the payment standard. The ratio $r^k_{TANF}$ is used in some markets to adjust the total TANF amount.\(^{28}\)

### E Identification

In equilibrium we have:

$$\ln \mu_{ij}(T, \lambda') - \ln \mu_{ij0}(T, \lambda') = \frac{U^i_{ij}(T, \lambda_{ij}) - U^i_{ij0}(T)}{\sigma_{\theta}}$$  \hspace{2cm} (26)$$

$$\ln \mu_{ij}(T, \lambda') - \ln \mu_{ij0}(T, \lambda') = \frac{U^i_{ij}(T, \lambda_{ij}) - U^i_{ij0}(T)}{\sigma_{\theta}}.$$  \hspace{2cm} (27)

The LHS of both is data. Suppose we hold $T$ fixed, and vary $m_i$ (conceptually we are looking across different markets that only differ in population vectors). Then,

$$\frac{\partial}{\partial m_i} \left[ \ln \mu_{ij}(T, \lambda') - \ln \mu_{ij0}(T, \lambda') \right] = 1 \frac{\partial U^i_{ij}(T, \lambda_{ij})}{\partial \lambda_{ij}} \frac{\partial \lambda_{ij}}{\partial m_i}.$$  \hspace{2cm} (28)$$

$$\frac{\partial}{\partial m_i} \left[ \ln \mu_{ij}(T, \lambda') - \ln \mu_{ij0}(T, \lambda') \right] = 1 \frac{\partial U^i_{ij}(T, \lambda_{ij})}{\partial \lambda_{ij}} \frac{\partial \lambda_{ij}}{\partial m_i}.$$  \hspace{2cm} (29)

Taking the ratio of the derivatives (and defining the LHS as $k_{ij}$):

$$k_{ij} = \frac{\partial U^i_{ij}(T, \lambda_{ij})/\partial \lambda_{ij}}{\partial U^i_{ij}(T, \lambda_{ij})/\partial \lambda_{ij}}$$

\(^{28}\)For reasons identical to those discussed in the case of Food Stamps, we do not consider the similar gross and net-income eligibility rules that exist for TANF, as well as the corresponding asset tests. See Footnote 27. We also do not consider the time limits in eligibility.
We then apply the envelope result:

$$(1 - \lambda_{ij}) \cdot \frac{\partial U_i^j(T, \lambda_{ij})}{\partial \lambda_{ij}} + \lambda_{ij} \cdot \frac{\partial U_i^j(T, \lambda_{ij})}{\partial \lambda_{ij}} = 0$$

So that:

$$k_{ij} = \frac{\lambda_{ij}}{\lambda_{ij} - 1} \Rightarrow \lambda_{ij} = \frac{k_{ij}}{k_{ij} - 1}.$$ 

Since $k_{ij}$ is identified so is $\lambda_{ij}$.

**F Moment list**

In this appendix we list the complete set of estimation moments (total of X moments). The fit of the model is described in Section 4.5 from the main text. Recall that there are nine markets and three education groups (types) for both men and women.

**Marriage market**

Single men households, by own education and market $[3 \times 9]$
Single women households, by own education and market $[3 \times 9]$
Married households by joint education and market $[3 \times 3 \times 9]$

**Labour supply**

Single men mean conditional hours by own education and market $[3 \times 9]$
Single women mean conditional hours by own education and market $[3 \times 9]$
Married men conditional hours by own education and market $[3 \times 9]$
Married women conditional hours by own education and market $[3 \times 9]$
Single men mean employment by own education and market $[3 \times 9]$
Single women mean employment by own education and market $[3 \times 9]$
Married men employment by own education and market $[3 \times 9]$
Married women employment by own education and market $[3 \times 9]$
Single men (UN/PT/FT) by own education and children $[3 \times 3 \times 2]$
Single women (UN/PT/FT) by own education and children $[3 \times 3 \times 2]$
Married men (UN/PT/FT) by joint education and children $[3 \times 3 \times 3 \times 2]$
Married women (UN/PT/FT) by joint education and children $[3 \times 3 \times 3 \times 2]$
Single men conditional hours (mean) by own education and children [3 × 2]
Single men conditional hours (s.d.) by own education and children [3 × 2]
Single women conditional hours (mean) by own education and children [3 × 2]
Single women conditional hours (s.d.) by own education and children [3 × 2]
Married men conditional hours (mean) by own education and children [3 × 2]
Married men conditional hours (s.d.) by own education and children [3 × 2]
Married women conditional hours (mean) by own education and children [3 × 2]
Married women conditional hours (s.d.) by own education and children [3 × 2]

Accepted wages

Single men accepted log-wages (mean) by own education [3]
Single men accepted log-wages (s.d.) by own education [3]
Single women accepted log-wages (mean) by own education [3]
Single women accepted log-wages (s.d.) by own education [3]
Married men accepted log-wages (mean) by own education [3]
Married men accepted log-wages (s.d.) by own education [3]
Married women accepted log-wages (mean) by own education [3]
Married women accepted log-wages (s.d.) by own education [3]
Married couples accepted log-wages (product) by joint education [3 × 3]

Home time

Single men home time (L/ML/MH/H) by own education and children [4 × 3 × 2]
Single women home time (L/ML/MH/H) by own education and children [4 × 3 × 2]
Married men home time (L/ML/MH/H) by own education and children [4 × 3 × 2]
Married women home time (L/ML/MH/H) by own education and children [4 × 3 × 2]
Single men home hours (mean) by own education and children [3 × 2]
Single men home hours (s.d.) by own education and children [3 × 2]
Single women home hours (mean) by own education and children [3 × 2]
Single women home hours (s.d.) by own education and children [3 × 2]
Married men home hours (mean) by own education and children [3 × 2]
Married men home hours (s.d.) by own education and children [3 × 2]
Married women home hours (mean) by own education and children [3 × 2]
Married women home hours (s.d.) by own education and children [3 × 2]
Married men home hours (mean) by joint education $[3 \times 3]$
Married women home hours (mean) by joint education $[3 \times 3]$
Single men home hours (mean) by children $[2]$
Single men home hours (s.d.) by children $[2]$
Single women home hours (mean) by children $[2]$
Single women home hours (s.d.) by children $[2]$
Married men home hours (mean) by children $[2]$
Married men home hours (s.d.) by children $[2]$
Married women home hours (mean) by children $[2]$
Married women home hours (s.d.) by children $[2]$

G Additional parameter and results tables

Let $\beta$ denote the $N \times 1$ vector of model parameters (whose true value is $\beta^*$) and let $m(\beta)$ describe the $M \times 1$ vector of moments/auxiliary parameters. Our estimation minimizes the distance between $m(\beta)$ and $m(\beta^*)$ using the $M \times M$ positive definite weighting matrix $W$. Our estimator solves:

$$
\hat{\beta} = \arg\min_{\beta} [m(\beta) - m(\beta^*)]^{\top} W [m(\beta) - m(\beta^*)],
$$

which we implement as:

$$
[\hat{\beta}, \lambda(\hat{\beta})] = \arg\min_{\beta, \lambda} [m(\beta, \lambda) - m(\beta^*, \lambda(\beta^*))]^{\top} W [m(\beta, \lambda) - m(\beta^*, \lambda(\beta^*))] \\
\text{s.t. } \mu_{ijk}(\beta, \lambda_k) = \mu_{ijk}^s(\beta, \lambda_k) \quad \forall i = 1, \ldots, I, j = 1, \ldots, J, k = 1, \ldots, K.
$$

Recall that $k$ indexes market, with $\lambda$ here defining the stacked $(I \times J \times K)$ vector of Pareto weights in all markets. The variance matrix of our estimator is given by:

$$
[D_m(\beta^*)^{\top} W D_m(\beta^*)]^{-1} D_m(\beta^*)^{\top} W \Sigma(\beta^*) W^{\top} D_m(\beta^*) [D_m(\beta^*)^{\top} W D_m(\beta^*)]^{-1},
$$

where $D_m(\beta^*) = \partial m(\beta^*)/\partial \beta$ is the $M \times N$ derivative matrix of the moment conditions with respect to the model parameter vector, and $\Sigma(\beta^*)$ is the $M \times M$ covariance matrix of $\beta^*$.

\[29\]When we include $\lambda$ as an explicit argument, this means we are evaluating the object for an arbitrary vector of Pareto weights, rather than the equilibrium weights. By definition: $m(\beta, \lambda(\beta)) = m(\beta)$. 

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the moments/auxiliary parameters. We estimate $D_m(\beta^*)$ by numerical differentiation of $m(\beta)$ evaluated at $\beta = \hat{\beta}$. We estimate the covariance matrix of our moments/auxiliary parameters using a bootstrap procedure.\textsuperscript{30} We choose $W$ to be a diagonal matrix, whose element is proportional to the inverse of the variance. Table 4 presents the estimates from our model, together with the accompanying standard errors.

\textsuperscript{30}Since we are using two independent data sources, we set the covariance of any moments calculated using alternative sources to be zero so that the matrix $\Sigma(\beta)$ may be written as a block-diagonal matrix.
Table 4: Parameter estimates

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Log-wage offers:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male, below high school: mean</td>
<td>2.528</td>
<td>0.007</td>
</tr>
<tr>
<td>Male, below high school: s.d.</td>
<td>0.485</td>
<td>0.004</td>
</tr>
<tr>
<td>Male, high school: mean</td>
<td>2.771</td>
<td>0.002</td>
</tr>
<tr>
<td>Male, high school: s.d.</td>
<td>0.506</td>
<td>0.002</td>
</tr>
<tr>
<td>Male, college: mean</td>
<td>3.198</td>
<td>0.004</td>
</tr>
<tr>
<td>Male, college: s.d.</td>
<td>0.562</td>
<td>0.002</td>
</tr>
<tr>
<td>Female, below high school: mean</td>
<td>2.080</td>
<td>0.008</td>
</tr>
<tr>
<td>Female, below high school: s.d.</td>
<td>0.650</td>
<td>0.007</td>
</tr>
<tr>
<td>Female, high school: mean</td>
<td>2.425</td>
<td>0.002</td>
</tr>
<tr>
<td>Female, high school: s.d.</td>
<td>0.574</td>
<td>0.002</td>
</tr>
<tr>
<td>Female, college: mean</td>
<td>2.957</td>
<td>0.003</td>
</tr>
<tr>
<td>Female, college: s.d.</td>
<td>0.545</td>
<td>0.002</td>
</tr>
<tr>
<td><strong>Preference parameters:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male leisure scale index, intercept</td>
<td>0.015</td>
<td>0.012</td>
</tr>
<tr>
<td>Male leisure scale index, high school</td>
<td>0.864</td>
<td>0.019</td>
</tr>
<tr>
<td>Male leisure scale index, college</td>
<td>0.778</td>
<td>0.026</td>
</tr>
<tr>
<td>Male leisure scale index, children</td>
<td>0.113</td>
<td>0.008</td>
</tr>
<tr>
<td>Male leisure curvature, no children</td>
<td>0.969</td>
<td>0.000</td>
</tr>
<tr>
<td>Male leisure curvature, children</td>
<td>0.971</td>
<td>0.000</td>
</tr>
<tr>
<td>Female leisure scale index, intercept</td>
<td>0.209</td>
<td>0.013</td>
</tr>
<tr>
<td>Female leisure scale index, high school</td>
<td>0.628</td>
<td>0.015</td>
</tr>
<tr>
<td>Female leisure scale index, college</td>
<td>0.701</td>
<td>0.019</td>
</tr>
<tr>
<td>Female leisure scale index, children</td>
<td>0.638</td>
<td>0.017</td>
</tr>
<tr>
<td>Female leisure curvature, no children</td>
<td>0.910</td>
<td>0.008</td>
</tr>
<tr>
<td>Female leisure curvature, children</td>
<td>0.946</td>
<td>0.004</td>
</tr>
<tr>
<td>Home good curvature</td>
<td>1.117</td>
<td>0.032</td>
</tr>
<tr>
<td>Male fixed costs</td>
<td>1.167</td>
<td>3.018</td>
</tr>
<tr>
<td>Female fixed costs</td>
<td>28.300</td>
<td>2.496</td>
</tr>
<tr>
<td>Children fixed costs</td>
<td>42.205</td>
<td>1.588</td>
</tr>
<tr>
<td>State specific error, s.d.</td>
<td>0.384</td>
<td>0.005</td>
</tr>
<tr>
<td>Marital shock, s.d.</td>
<td>0.223</td>
<td>0.010</td>
</tr>
<tr>
<td><strong>Home production technology:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male weight, children</td>
<td>0.047</td>
<td>0.034</td>
</tr>
<tr>
<td>Male weight, no children</td>
<td>0.097</td>
<td>0.084</td>
</tr>
<tr>
<td>Substitution elasticity parameter</td>
<td>-0.918</td>
<td>0.550</td>
</tr>
<tr>
<td>Match [below high school, below high school]</td>
<td>0.675</td>
<td>0.024</td>
</tr>
<tr>
<td>Match [below high school, high school]</td>
<td>0.325</td>
<td>0.015</td>
</tr>
<tr>
<td>Match [below high school, college]</td>
<td>0.135</td>
<td>0.013</td>
</tr>
<tr>
<td>Match [high school, below high school]</td>
<td>0.473</td>
<td>0.038</td>
</tr>
<tr>
<td>Match [high school, high school]</td>
<td>1.583</td>
<td>0.129</td>
</tr>
<tr>
<td>Match [high school, college]</td>
<td>0.209</td>
<td>0.021</td>
</tr>
<tr>
<td>Match [college, below high school]</td>
<td>0.185</td>
<td>0.045</td>
</tr>
<tr>
<td>Match [college, high school]</td>
<td>1.984</td>
<td>0.343</td>
</tr>
<tr>
<td>Match [college, college]</td>
<td>3.913</td>
<td>0.670</td>
</tr>
</tbody>
</table>

Notes: All parameters estimated simultaneously using a moment based estimation procedure as detailed in Section 4 from the main text. All incomes are expressed in dollars per-week in average 2006 prices.
Figure 5: Marginal tax rate schedules by State

Notes: Figure shows the effective combined Federal and State marginal tax rate, including income tax, payroll tax, and Earned Income Tax Credit. Figure calculated using TAXSIM under 2006 tax systems, assumes that there are no other sources of income, and (when present) there are two children. For couples we assume married filing jointly status; for singles with children we assume head-of-household status. Presented schedules correspond to most populous States in respective Census Bureau-designated division; Texas, Tennessee, and Florida do not have any State Income Tax or State EITC and so are combined.
References


