ABSTRACT. Should policymakers facing weak economic performance and high government debt pursue an expansionary fiscal policy? Recent empirical studies find some evidence that government spending may not be as expansionary when the debt-to-output ratio is high, but the results are inconclusive across studies. This paper explores how government debt levels can matter for spending effects via policy expectations and fiscal adjustments. It finds that the fiscal multiplier is generally smaller in a high-debt than low-debt state when general income tax rates serve as an adjustment instrument, but the difference shrinks as the wealth effect on labor becomes strong. Uncertainties about timing, the reaction magnitude to debt, and the debt target of fiscal consolidations also matter. Expecting a higher debt target is not always expansionary. When households perceive consolidations to implement via adjusting labor tax rates, expecting a higher debt target produces a positive wealth effect, which reduces current hours worked and thus offsets positive government spending effects.

Keywords: debt-dependent fiscal policy effect; fiscal multipliers; non-linear fiscal policy effects; policy uncertainty

JEL Codes: E62; H30; H60

1. Introduction

Six years after the global financial crisis, most of advanced economies have entered an era of high government debt with a lackluster economic performance. The average net debt-to-GDP share for general governments increases from 44 percent in 2007 to 70 percent in 2014.
for the advanced economies (International Monetary Fund (2015b)). As shown in Figure 1, the fiscal position for most G7 countries has deteriorated substantially. Excluding Canada and Germany, the average net debt has reached 97 percent of GDP in 2014. Despite a large amount of stimulus through quantitative easing, the output gap remains large, at $-1.9\%$ of the potential GDP for advanced economies and $-2.8\%$ for the euro area (International Monetary Fund (2015b)). As monetary policy is still constrained, the duality of weak economic performance and high government debt presents policymakers a difficult choice about whether to pursue fiscal expansions to prompt recoveries in face of high debt.\(^1\)

Can government debt accumulation interfere with the effects of fiscal expansions? Based on numerous consolidation episodes in Europe and Canada in the 1980s to early 1990s, Giavazzi and Pagano (1990) and Alesina and Perotti (1996) find that fiscal consolidations after years of debt accumulation are expansionary, contradicting the Keynesian prediction. Subsequent estimates from larger samples find some support for state-dependent government spending effects, but the results are not always clear. Perotti (1999) finds that a government expenditure increase has a positive (Keynesian) effect on consumption in good times but negative (non-Keynesian) effect in bad times among OECD countries; the bad-time result, however, is less robust when the most influential country is dropped. Using the euro data, Kirchner et al. (2010) instead find the initial debt condition does not matter for consumption.\(^2\) As for output, Kirchner et al. (2010), Ilzetzki et al. (2013), and Nickel and Tudyka (2013) all find that the output multiplier is smaller or can turn negative in the long run for highly indebted economies, but Corsetti et al. (2012) does not find significant differences

\(^1\)Following Prime Minister Abe’s announcement to postpone a scheduled sales tax increase in response to negative GDP growth, Moody downgraded Japan’s credit rating in December 2014, citing uncertainty over the achievability of fiscal deficit reduction goals and over the effectiveness of growth enhancing policy measures (Moody’s Investors Service (2014)). In light of the lower than expected growth in advanced economies and further slowdown in emerging economies in the first half of 2015, International Monetary Fund (2015a) advises that “fiscal policy should be growth friendly,” and fiscal consolidations be anchored as a medium-term plan.

\(^2\)On the other hand, Corsetti et al. (2012) find unusual result that consumption turns positive in later periods under weak public finance conditions while remains negative under the normal conditions.
in output under various public finance conditions. Lastly, Born et al. (2015) find that government spending has a negative effect on output in the short run but a positive effect at the longer horizon, although this conclusion is less robust when using an alternative way to proxy fiscal stress.

To see why such a wide variety of empirical outcomes is likely, this paper takes a theoretical approach to understanding the effects of government spending in different states of debt. One common view—the policy expectations view, as emphasized in Blanchard (1990)—resorts to the wealth effects associated with the expectations about future fiscal adjustments. In Bertola and Drazen (1993) and Sutherland (1997), individuals expect that once certain debt threshold is reached, a sudden large tax increase is more likely to occur. Applying this logic in our context, a deficit-financed government spending increase pushes government debt higher, which can increase the perceived probability of fiscal consolidations, generating a negative wealth effect to interact with the spending effects.

To model the expectations effects, we examine a linear fiscal reaction rule to debt (commonly used in the DSGE literature, e.g., Cogan et al. (2010), Leeper et al. (2010), and Uhlig (2010)), as well as non-linear rules to account for non-systematic nature of fiscal adjustments or consolidations observed in reality. Fiscal consolidations tend to lag a debt pileup but the timing at which a consolidation would trigger is unknown. Moreover, depending on an incumbent’s preference, reaction magnitudes and debt targets can vary across episodes. Based on the consolidation data compiled in Devries et al. (2011), Figure 2 plots the debt dynamics (lines, the right axis) against the size of fiscal consolidation in percent of GDP (bars, the left axis) for Canada, Italy, Japan, and U.S. from 1980 to 2009. The plots make clear that

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3In addition to the channel of wealth effects, Perotti (1999) also examines the roles of liquidity constraints and the probability an incumbent will be in power next period to study the Keynesian and non-Keynesian fiscal policy effects in times of fiscal stress.

4For example, the political deadlocks and slow recovery from the Great Recession in the U.S. have delayed fiscal consolidations since 2011 despite its elevated debt level exceeding 80 percent of GDP. Yet a series of tax increases and government spending cuts were triggered in 1990 when the debt-to-GDP ratio was below 60 percent of GDP.

5The choice of the four countries is driven by the availability of debt-to-GDP ratios in Haver Analytics.
commonly adopted linear rules—with certain timing and a constant reaction magnitude—do not quite resemble actual practice of fiscal consolidations.

We adopt an real business cycle (RBC) framework with a shortcut that government and private consumption are complements to capture the Keynesian effect of government spending as found in the VAR evidence (e.g., Blanchard and Perotti (2002), Perotti (2005), and Galí et al. (2007)). In the model, distorting income tax rates serve as instruments to maintain debt sustainability. To capture uncertainties in implementing fiscal consolidations, we introduce a distribution of debt thresholds, defined by the collection of the maximum amount of debt that existing fiscal policy can sustain without consolidation. Fiscal reactions to debt can take two values following a regime-switching process: a high value represents a government’s consolidation efforts to reduce debt, and a low value to prevent debt from getting on an explosive path. Each period an effective debt threshold is drawn from the simulated distribution. A consolidation is triggered only if current indebtedness exceeds the effective threshold. This formulation captures some randomness in conducting fiscal consolidations without resorting to modeling complicated political factors underlying consolidation decisions. The setup implies that consolidation probabilities are higher when a government is more indebted. Uncertain debt targets are also introduced by a similar manner with a regime-switching process between two target values. Our simple structure of the model allows us to obtain a fully nonlinear solution under the rational expectations to accommodate uncertain fiscal consolidations.

The analysis first simulates government spending effects under the linear fiscal rule with the general income tax rate as an adjustment instrument. We find that the economy in the high-debt state produces a smaller spending multiplier for output than that in the low-debt state. As emphasized in the policy expectations view, expecting higher future taxes generates a negative wealth effect on current consumption. Since households also derive utility from leisure, the expectations effect on output partly depends on the wealth effect on labor supply. In general, as the wealth effect on labor supply gets stronger, the output
multiplier difference between the two debt states becomes smaller in the short run, but the difference in the two states remains large at the longer horizon.

Next, we analyze the expectations effect induced by uncertainties in conducting fiscal consolidations. With uncertainty, government spending multipliers are naturally characterized by a confidence interval, reflecting a range of possible fiscal paths in the future. Depending on the type of adjustment instruments perceived by households, we find that fiscal uncertainties can be either contractionary or expansionary for government spending effects in the high debt state. When households expect consolidations to be implemented via capital tax increases, expecting a higher capital tax rate discourages current investment and offsets the positive output effect of government spending especially at the longer horizon. In contrast, expecting a higher labor income tax rate induces households to work harder, amplifying the government spending effects.

When households expect that the government may stabilize at a higher debt target than the lower, steady-state debt level, it implies that the expected future tax liabilities can be lower. This positive wealth effect, however, is not always expansionary. Applying the reasoning earlier, we find that when the expectation is associated with a potential decline in the labor tax rate, the positive wealth effect has a negative effect on current labor, which can make an expected higher debt target contractionary.

Our analysis adds to the booming literature in state-dependent fiscal policy effects. Aside from fiscal states focused here, the literature has examined the state of business cycle conditions (e.g., Corsetti et al. (2010), Auerbach and Gorodnichenko (2012), Blanchard and Leigh (2013), and Owyang et al. (2013)) and of monetary policy (e.g., Christiano et al. (2011) and Erceg and Lindé (2014)). The paper is also related to another line that studies fiscal policy effects in a non-linear framework. For example, Dotsey and Mao (1997) and Bi et al. (2013) explore whether standard models can generate expansionary fiscal consolidations when the government debt level is high. Davig (2004) models government debt by a
regime-switching process on a hidden state and finds that agents’ investment responses tax policy depend on their inference regarding a debt regime. Davig and Leeper (2011) analyze the government spending effects, incorporating uncertain active/passive monetary and fiscal policy combinations captured by a regime-switching process. We study debt-dependent government spending effects with a focus on policy expectations effects about uncertain fiscal consolidations.

2. Model Setup

The model has a simple RBC structure. Our initial investigation discovers that the wealth effect on labor supply is important in the debt-dependent spending effects. To allow for variation in this effect, we adopt a utility function similar to Monacelli and Perotti (2008) and Jaimovich and Rebelo (2009), which can accommodate both a Greenwood, Hercowitz, and Huffman’s preference (GHH, Greenwood et al. (1988)) and King, Plosser, and Rebelo’s preference (KPR, King et al. (1988)). The baseline model described in this section has a GHH utility function. This choice is motivated by Schmitt-Grohé and Uribe’s (2012) estimates, which support the GHH utility specification.\footnote{Their specification does not have government consumption in the utility function.} Sensitivity analysis in Section 5 assumes the KPR preference.

2.1. The Baseline Model. Households derive utility from effective consumption ($\tilde{c}_t$) and leisure ($1 - l_t$). Effective consumption is assumed to be a constant-elasticity-of-substitution (CES) index of private consumption ($c_t$) and government spending ($g_t$):

$$\tilde{c}_t = \left[ \omega \left( c_t \right)^{\frac{1}{\sigma}} + (1 - \omega) \left( g_t \right)^{\frac{1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}}. \quad (1)$$

A representative household chooses private consumption, labor ($l_t$), investment ($i_t$), and capital ($k_t$) to maximize utility

$$E \sum_{t=0}^{\infty} \beta^t \frac{(\tilde{c}_t - \varphi l_t X_t)^{1-\sigma}}{1 - \sigma}, \quad (2)$$
subject to the budget constraint
\[ c_t + i_t + q_t b_t = (1 - \tau_t) \left( w_t l_t + r_t^k k_{t-1} \right) + b_{t-1} + z_t, \]  
where \( b_t \) is one-period government bond that sells at a price \( q_t \) at time \( t \) and pays one unit of goods at \( t+1 \), \( \tau_t \) is the general income tax rate, \( w_t \) is the real wage rate, \( r_t^k \) is the rental rate for capital, and \( z_t \) is transfers from the government. \( X_t \) is an index variable evolving according to
\[ X_t = c_t^\psi X_{t-1}^{1-\psi}. \]  
The law of motion for capital is
\[ k_t = (1 - \delta) k_{t-1} + i_t - \frac{\kappa}{2} \left( \frac{i_t}{k_{t-1}} - \delta \right)^2 k_{t-1}, \]  
where \( \frac{\kappa}{2} \left( \frac{i_t}{k_{t-1}} - \delta \right)^2 k_{t-1} \) is the capital adjustment cost.

When \( \omega = 1 \) or the government spending does not enter the utility, \( \psi = 0 \) (\( \psi = 1 \)) yields the standard GHH (KPR) preference. Our baseline calibration has \( \omega < 1 \) and \( \psi = 0 \). Also, to generate the short-run Keynesian effect of government spending, we have government spending enter the utility function as a complement to private consumption.\(^7\) When solving the model with \( \psi = 0 \), \( X_t \) is normalized to 1, following the standard practice in the literature.

Firms are perfectly competitive, producing with Cobb-Douglas technology,
\[ y_t = a_t k_{t-1}^{\alpha} l_t^{1-\alpha}. \]  
The technology, \( a_t \), follows an AR(1) process
\[ \ln \frac{a_t}{a} = \rho_a \ln \frac{a_{t-1}}{a} + \varepsilon_t^a, \quad \varepsilon_t^a \sim N(0, \sigma_a^2). \]  
The government collects taxes and issues debt to pay for its purchases, transfers, and debt service. The government flow budget constraint is:
\[ tax_t + q_t b_t = g_t + b_{t-1} + z, \]  
\(^7\)The alternative is to build a New Keynesian model with rule-of-thumb households and sticky prices and wages as in Colciago (2011) to generate short-run co-movement in government spending and private consumption. Such a complicated setup with numerous states is difficult to solve for a fully-nonlinear, rational expectations solution.
where $tax_t = \tau_t (w_t l_t + r^K_t k_{t-1}) = \tau_t y_t$. To derive the intertemporal budget constraint, we iterate the flow budget constraint forward and impose the transversality condition for debt to obtain that

$$b_{t-1} = \sum_{i=0}^{\infty} \beta^i E_t \frac{\lambda_{t+i}}{\lambda_t} (tax_{t+i} - g_{t+i} - z_{t+i}),$$

(9)

where $b_{t-1}$ is the beginning value of government debt at time $t$. This intertemporal budget constraint serves as a base for our definition of debt thresholds, to be introduced later.

The lump-sum transfers $z_t = z \forall t$, which is calibrated to close the government budget in the steady state. Government spending follows an AR(1) process:

$$\ln \frac{g_t}{g} = \rho_g \ln \frac{g_{t-1}}{g} + \varepsilon^g_t, \quad \varepsilon^g_t \sim N(0, \sigma^2_g).$$

(10)

In the baseline specification, the general income tax rate reacts to debt by a linear rule:

$$\tau_t = \tau + \gamma (b_{t-1} - b).$$

(11)

Parameter $\gamma$ represents a low reaction magnitude to debt. When studying uncertain fiscal consolidations, we introduce non-linear rules with two reaction magnitudes: $\gamma$ and $\gamma^H$, a high reaction magnitude.

2.2. Calibration and the Solution Method. The model is calibrated at a quarterly frequency. The discount factor $\beta$ is set to 0.99, and the depreciation rate $\delta$ is 0.025. Preferences over consumption are logarithmic, so $\sigma = 1$. The capital income share $\alpha$ is 0.36. To have a Frisch labor elasticity of 0.5, we set $\theta = 3$. Following Gourio (2012), the capital adjustment parameter has $\kappa = 1.7$. To calibrate effective consumption $\tilde{c}_t$, we follow Bouakez and Rebei’s calibration (2007) to assume that the weight of private consumption is $\omega = 0.9$ for the U.S. economy. Since the elasticity of substitution between private consumption and government spending $\nu$ is not conventionally estimated, we back out $\nu = 0.4$ to have the model-implied peak fiscal multipliers in the low-debt state roughly match the cumulative multiplier for developed countries at about 0.8, as estimated by Ilzetzki et al. (2013). In the baseline calibration, $\gamma = 0.03$ implies a slow speed to retire debt.
To calibrate the exogenous processes, we rely on reduced-form estimation using U.S. quarterly data from 1970Q1 to 2015Q1 in the National Income and Product Accounts (NIPA) Tables released by the Bureau of Economic Analysis. The fiscal data are from governments of all levels. The data for technology come from Solow residuals. The steady-state government spending-to-output ratio and the general income tax rate are set to the sample averages: $\frac{g}{y} = 0.21$ and $\tau = 0.261$. For the steady-state debt-to-annual output ratio, we calibrate it to the average of net debt-to-GDP ratio from 2001 to 2008 at 0.38. Then the model implied transfers-to-output ratio is 0.036. The steady-state technology $a$ is normalized to 1. The estimation of the three AR(1) processes yields $\rho_g = 0.73$, $\rho_\tau = 0.89$, $\rho_a = 0.78$, $\sigma_g = 0.75\%$, $\sigma_\tau = 1.8\%$, and $\sigma_a = 0.6\%$.\(^8\)

To solve the model non-linearly, we use the monotone mapping method, as adopted in Coleman (1990) and others. Appendix A lists the equilibrium system of the baseline model, and Appendix B has the details for implementing the method.

3. Debt-Dependent Government Spending Effects

We simulate the responses to a government spending increase in the low- and high-debt state, using the baseline (GHH preference) model. It also quantifies the bias associated with the linear solution technique.

3.1. Simulating a High-Debt State. To generate a high-debt state consistent with equilibrium conditions, we start from a deterministic steady state at time $t = -55$. Then, the

\(^8\)The data for government spending are defined as the sum of current expenditure and gross government investment (NIPA Table 3.1, lines 20 and 39), minus net transfers and net interest payments (NIPA Table 3.1, lines 22 and 27). The general income tax rate is the ratio of current tax receipts (NIPA Table 3.1, line 2) plus contributions for government social insurance (NIPA Table 3.1, line 7) to GDP (NIPA Table 1.1.5, line 1). Solow residuals are estimated from the data of GDP, total hours worked, and capital. Total hours are the product of the average working hour index and civilian employment. Average working hours are measured by the index of average weekly hours in nonfarm business (2005=100, seasonally adjusted, the Bureau of Labor Statistics (BLS), PRS85006023). Employment is measured by the civilian employment for persons 16 years of age and older (seasonally adjusted, BLS, CE16OV). Capital is the annual stock of fixed assets and consumer durable goods (NIPA Fixed Assets Table 1.1, line 1), adjusted by the GDP deflator and then interpolated to quarterly series.
economy is subject to a sequence of negative TFP shocks from \( t = -54 \) to \( t = -6 \). During this period, government spending is maintained at the steady-state level, and tax policy follows (11) with \( \gamma = 0.03 \). As output and the tax base fall, tax revenue decreases accordingly, leading to government debt accumulation. At \( t = 0 \), the government debt-to-annual steady-state output ratio reaches 0.85, slightly higher than the net debt-to-GDP ratio of the U.S. in 2014 (around 0.8).\(^9\)

For the low-debt state, we simply assume that no shocks hit before \( t = 0 \). Thus, the debt-to-annual output ratio is the same as in the steady state (0.38) at time 0.

### 3.2. Fiscal Multipliers in Different States of Debt.

Figure 3 compares the impulse responses to a 1-percentage increase in the government spending-to-output ratio at time 0 in the low-debt state (dashed lines) to those in the high-debt state (solid lines). As the initial states are different, the units are measured by gaps between paths with and without the government spending shock, scaled by the the deterministic steady-state values. Table 1 reports the responses in cumulative government spending multipliers for output, consumption, and investment at different horizons, computed as

\[
\sum_{i=1}^{k} \frac{r_{t+i-1}^{-1} \Delta y_{t+i-1}}{\sum_{i=1}^{k} r_{t+i-1}^{-1} \Delta y_{t+i-1}}
\]

where \( \Delta y \) and \( \Delta g \) are level changes relative to a path without government spending increases. When computing consumption and investment multipliers, \( \Delta y \) is replaced by \( \Delta c \) or \( \Delta i \).

Regardless of the debt state, the model implies that an increase in government spending has an expansionary effect on output in the short run, a positive effect on consumption and labor, and a negative effect on investment. Despite the qualitative similarities between the two states, the quantitative differences are nontrivial. The impact multiplier for output in the high-debt state is only half of that in the low-debt state (0.4 versus 0.8). In addition to output, the difference for the consumption multiplier is also big. The impact consumption

\[^9\]In our thought experiment, the high-debt state results from a sequence of negative technology shocks, which yield a below steady-state capital stock and an above steady-state debt level at time 0. The recent debt accumulation among advanced economies was mainly driven by reduced revenues from the severe recession plus costs of financial rescue and fiscal stimulus programs, which yield a similar qualitative state with a high debt level and a low capital stock relative to the deterministic growth path, similar to our model economy.
multiplier drops to 0.2 in the high-debt state from 0.8 in the low-debt state. The cumulative five-year multiplier for consumption remains positive in the low-debt state but turns negative in the high-debt state.

Government debt levels can affect its spending effects in two stages: the first stage is before the realization of fiscal adjustments or consolidations, and the second stage is after the realization. Since the baseline model has only one-quarter delay between the debt increase and fiscal adjustments, the first stage with pure expectations effects is relatively short. In the second stage, both expectations and adjustment realization effects interact to influence government spending effects.

In Figure 3, the two states have the same changes (relative to the steady state) in the tax rate, capital, and government debt, yet the responses of output, consumption, investment, and hours worked diverge on impact, indicating the existence of policy expectations effects. As well known, expecting higher future taxes induces a negative wealth effect, discouraging consumption. This negative wealth effect is stronger in the high-debt state because households anticipate a bigger tax increase, which leads to a smaller consumption increase in the high-debt state. As households consume less, investment falls less in the high-debt state. Also notice that hours worked increase less on impact in the high- than low-debt state. Although the negative wealth effect from expecting higher future taxes has a positive effect on labor supply, this effect is relatively weak in the baseline model. Unlike the standard GHH preference, our specification with government spending in the utility function keeps a small wealth effect on labor supply.\textsuperscript{10} The slightly higher labor response on impact in the low-debt state is mainly driven by the need to support higher consumption increase. With more hours worked, output on impact is higher in the low-debt than the high-debt state.

\textsuperscript{10}From (A.1) to (A.4) in Appendix A, it can be seen that when $\omega \to 1$, $\tilde{c}_t \to c_t$. Then, the labor supply equilibrium condition $\varphi \theta_t^{\sigma-1} = \omega c_t \frac{1}{\theta_t} \frac{1}{\theta_t} (1 - \tau_t) w_t \to \varphi \theta_t^{\sigma-1} = (1 - \tau_t) w_t$; i.e. the wealth effect on labor supply approaches to zero.
Starting from time 1, the response of the income tax rate diverges between the two states. The effects of a government spending increase are interfered by the realization of higher income tax rates. During the second stage, the income tax rate increases by a larger magnitude in the high-debt state than in the low-debt state. Not surprisingly, a higher income tax rate directly lowers investment and hours worked. With less capital and fewer hours worked, output in the high-debt state is lower at the longer horizon than in the low-debt state. In the baseline model, this translates to a difference of 0.8 for the five-year cumulative output multiplier (−0.2 versus −1.0).

The simulation in the baseline model confirms that policy expectations play an important role in the short-run effects of government spending. The analysis does not explore in details the different expectations effects induced by different types of adjustment instruments. In Section 4, we separate capital and labor income tax rates as fiscal adjustment instruments.

3.3. Bias from a Linear Solution Method. In addition to comparing the multipliers in various debt states, Table 1 also presents the multipliers obtained from the conventional method—solving a first-order, log-linearized equilibrium system. Fiscal multipliers computed from the linear method are constant regardless of debt levels. Parker (2011) points out that one main problem in the existing literature on studying fiscal policy in recessions is the solution method for solving a linearized dynamic system. Consistent with his critique, we show that the fiscal multipliers from a linear solution can yield nontrivial biases when an economy is highly indebted.

Figure 4 plots the decision rules of investment, labor, and consumption against the state variables of government debt (top row) and capital (bottom row) under the baseline specification. When plotting the decisions rules against a particular state, other state variables are set at their steady-state values. The solid lines are derived from the non-linear solution and

\(^{11}\)The practice of changing one state variable at a time may raise some eyebrows since it ignores the endogenous relationships among states imposed by the equilibrium conditions over time. Our purpose here is only to highlight the non-linearity in decision rules. When assessing government spending effects in a
Several observations can be made from Table 1 and Figure 4. First, the biases from the linear solution become larger when the state of either debt or capital moves away from the deterministic steady state. For example, employing the linear solution method would report the one-year cumulative output multiplier is 0.7 regardless of debt levels, while in the high-debt state the actual multiplier is only 0.3. Second, the true responses of hours worked with respect to capital are non-monotonic. A decline in capital reduces output and consumption, making households work more. However, a decline in capital also reduces the marginal product of labor, which shifts labor demand to the left and thus reduces equilibrium hours worked. When capital falls not too much below the deterministic steady-state level, the former effect dominates. As capital further falls much below, the latter effect dominates, explaining the non-monotonic responses of hours worked. The linear solution misses this non-linearity. Lastly, while our main interest in this study is the state of government debt, Figure 4 makes clear that endogenous, related states (such as capital here) are important to account for the overall effects to a government spending shock in a high-debt state. We see that in the case of investment and consumption, the biases associated with capital deviation to its steady state value are much larger than those with debt deviation. Since debt does not rise own it own, the analysis for studying debt-dependent fiscal effects must account for the causes that trigger debt accumulation.

4. Uncertain Fiscal Consolidations

The analysis so far imposes a rigid fiscal rule: at each period, households know exactly when fiscal adjustments would take place and the reaction magnitude. Fiscal adjustments or consolidations in reality are difficult decisions to make; they carry substantial political

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particular debt state as in Section 3.2 and Sections 4 and 5, we follow the procedure in Section 3.1 to ensure that the relationship among state variables in the high-debt state are internally consistent.
risks and thus are subject to a large uncertainty on the legislative voting outcomes (e.g., Posner and Sommerfeld (2012)). This section investigates the expectations effects arising from uncertain timing and a potential switch to a higher debt reaction magnitude or a higher debt target.

4.1. Simulating the Distribution of Debt Threshold. Our modeling of policy uncertainties requires simulating a distribution of government debt thresholds ($B_t^*$). A debt threshold equals the discounted infinite sum of current and future primary surpluses conditional on a future path of income tax rates, government spending, and transfers (see (C.1) in Appendix C). The distribution is defined as the collection of debt thresholds that the government is able to sustain without consolidations.\footnote{The concept of debt thresholds here differs from fiscal limits in Bi (2012). Both are simulated based on an expression similar to the intertemporal government budget constraint. Fiscal limits are defined as the maximum debt level that a government is able and willing to service under the maximum tax rates an economy can impose. Instead, debt thresholds in this paper are defined as the sustainable level of debt that can be supported without fiscal consolidations.} Appendix C describes the procedures for simulating the distribution of debt thresholds.

Figure 5 plots the cumulative density functions (CDF) of the simulated distribution. The debt level in the y-axis is scaled by the deterministic steady-state output. The distribution mean implies that the average sustainable debt level is 88 percent of the steady-state output.\footnote{If scaled by current output, the mean sustainable debt-to-annual output ratio is 1.27.} The CDF implies that the probability to conduct fiscal consolidation is 0 at the deterministic steady state with a debt-to-output ratio of 0.38. This consolidation probability climbs up to 0.3 when the debt-to-steady state output ratio reaches 0.85 in the high-debt state.

4.2. Uncertain Timing and Reaction Magnitudes to Debt. To investigate how different types of perceived adjustment instruments can affect expectations effects, for the analysis in this section we distinguish between the capital income tax rate ($\tau^k_t$) and the labor income ($\tau^l_t$) to replace the general income tax rate ($\tau_t$). The household’s budget constraint (3) is
revised as \(c_t + i_t + q_t b_t = (1 - \tau_t^k) w_t l_t + (1 - \tau_t^k) \tau_t^b k_{t-1} + b_{t-1} + z_t\) and \(tax_t\) in (8) is computed as \(\tau_t^l w_t l_t + \tau_t^k k_{t-1}\). The constant debt reaction parameter \(\gamma\) in (11) is replaced with a time-varying variable: \(\gamma_t^k\) or \(\gamma_t^l\). Specifically, when households expect that the government may implement a fiscal consolidation through raising the capital tax rate,

\[
\tau_t^k = \tau^k + \gamma_t^k (b_{t-1} - b)
\] (12)

\(\gamma_t^k\) in (12) can take one of the two values:

\[
\gamma_t^k = \begin{cases} 
\gamma, & \text{if } b_{t-1} < b_t^*, \\
\gamma^H, & \text{if } b_{t-1} \geq b_t^*,
\end{cases}
\]

where \(\gamma^H > \gamma > 0\), and \(b_t^*\) is the effective debt threshold drawn from \(B_t^*\) at the beginning of time \(t\).

Likewise, when households expect that a possible consolidation is implemented via higher labor tax rate, the labor tax rate follows

\[
\tau_t^l = \tau^l + \gamma_t^l (b_{t-1} - b)
\] (13)

where

\[
\gamma_t^l = \begin{cases} 
\gamma, & \text{if } b_{t-1} < b_t^*, \\
\gamma^H, & \text{if } b_{t-1} \geq b_t^*,
\end{cases}
\]

Since the probability to trigger a fiscal consolidation is 0 in the low-debt state, the simulations of uncertain fiscal consolidation are only conducted in the high-debt state. Starting from \(t = 1\), the government determines whether to adopt \(\gamma^H\) by drawing an effective debt threshold \((b_t^*)\) from \(B_t^*\). If \(b_t^* \leq b_0\), \(\gamma_t^k\) (or \(\gamma_t^l\)) switches from 0.03 to \(\gamma^H = 0.06\), indicating the adoption of consolidation measures. In the case where a consolidation is not implemented at \(t = 1\), an effective threshold \(b_2^*\) is drawn again next period to be compared with \(b_1\). This process continues until a consolidation is adopted. Once adopted, the probability of staying at the fiscal consolidation regime is \(\text{prob}(\gamma_t^k = \gamma^H | \gamma_{t-1}^k = \gamma^H) \equiv p^H\), while the probability of switching back to the no consolidation regime is \(1 - p^H\).

In the model with policy uncertainties, we calibrate \(\gamma^H = 0.06\). Also, \(\tau^k = \tau^l = 0.261\), which is same as \(\tau\) in the baseline model. This allows us to compare government spending
effects with those from the baseline model. To calibrate $p^H$, we intend to capture “lumpy” fiscal consolidations observed in practice, which tend to last for several years (see Figure 2) once adopted. We set $p^H = 0.9375$. This persistence implies that on average a consolidation continues for about four years, roughly the average length of episodes documented in Devries et al. (2011) for 17 countries from 1978 to 2009.

Figure 6 compares the impulse responses of a government spending increase with uncertain reaction magnitudes to debt (dashed lines) to those with no uncertainty (solid lines, the baseline model). The responses without uncertainty have $\gamma^k_t = \gamma^l_t = \gamma = 0.03 \forall t$, equivalent to the baseline model. The left (right) column assumes that households expect the capital (labor) income tax rates may switch to a higher value. To isolate the pure expectations effects, the responses plotted are from a policy path that regime switching does not realize for the 20 quarters plotted.

The figure shows that the expectations effects of uncertain debt reaction are quite different for the two tax instruments. Expecting that the capital tax rate may rise is contractionary, offsetting some positive output effect of a spending increase. The dominant channel is the negative intertemporal substitution effect, which discourages current investment. Although the negative wealth effect due to potential higher taxes makes households work harder, falling investment lowers marginal product of labor and reduces labor demand. The net effect on labor, not shown in the figure, is negligible. With lower investment and thus lower capital, output increases less with uncertainty in regime-switching $\gamma^k_t$ than without uncertainty. Also, with uncertainty, the real interest rate is lower than without. As expected future consumption is lower (from possible larger debt reaction), the expected benefit of saving is higher, lowering the equilibrium interest rate. While output and tax revenues are lower with uncertainty, government debt instead falls slightly because of less interest payment.
On the other hand, if the labor tax rate instead of the capital tax rate is expected to switch, the policy expectations become expansionary as shown in the right column. Here the dominant channel is the negative wealth effect, which induces households to reduce consumption to increase saving by investment and works more hours. Although the GHH preference assumed in the baseline model only has a small positive wealth effect on labor supply, with higher labor and less negative investment responses, overall output is higher with uncertainty than without.

Figures 7 and 8 plot the cumulative multipliers in both debt states and under various scenarios of policy expectations. The solid lines represent the baseline case—without policy uncertainty. The green dashed lines represent the case with regime-switching $\gamma^k_t$ (Figure 7) or $\gamma^l_t$ (Figure 8) but the switching does not occur throughout the horizon plotted. The red dashed line is the mean of 1000 simulations based on the regime-switching rules described in Section 4.2, and the red dotted-dashed lines are its 10-90 percent confidence bands.

Consistent with the earlier analysis, pure expectations effects from expecting regime-switching $\gamma^k_t$ are contractionary relative to the scenario without policy uncertainty. That is why the green dashed line lies below the black solid line in Figure 7. Clearing up such policy uncertainty raises the output multipliers, as the red lines lie above the green dashed line. On the contrary, expectations effects from regime-switching $\gamma^l_t$ are expansionary, as shown in Figure 8. Also, both figures show that uncertainties in implementing fiscal consolidations imply fiscal multipliers especially at longer horizons should be characterized by a confidence interval, reflecting a wide range of possible fiscal paths to implement fiscal adjustments or consolidations.

4.3. Uncertain Debt Targets. In the earlier analysis, the debt target is always set at $b$ (the deterministic steady-state level) each period without uncertainty. In reality, the debt target is a vague concept. Even for countries that have official or statutory debt ceilings, they are subject to changes, as observed in the U.S. since 2011. Also in the European Union,
the Stability and Growth Pact requires its member states to aim toward a debt level not exceeding 60 percent of GDP, but the rule is implemented with flexibility.\textsuperscript{14} So far we see that the strength of expectations effects partly depend on the perceived size of a future fiscal adjustment or consolidation, which in turn is determined by the distance from the current debt level to a debt target. The analysis below explores another type of policy uncertainty associated with debt targets.

We continue to use the model that distinguishes between capital and labor income tax rates, but assume that the debt reaction magnitudes of the two tax rates to debt are constant over time, so we can focus on the effects arising from uncertain debt targets.

To model uncertain debt targets, we assume that the debt target \( b_t^T \) can take two values:

\[
b_t^T = \begin{cases} 
  b, & \text{if } b_{t-1} < b_t^*, \\
  b^H, & \text{if } b_{t-1} \geq b_t^*,
\end{cases}
\]

where \( b^H > b \), and \( b_t^* \) is the effective debt threshold drawn from \( B^* \). The tax policy rules are revised as

\[
\tau^j_t = \tau^j(b_t^T) + \gamma^j(b_t - b_t^T), \quad j \in \{k, l\}
\]

where \( \tau^j(b_t^T) \) is the steady-state tax rate that is consistent with the debt target at \( t \). In the steady state, a higher debt target is associated with a larger \( \tau^k(b_t^T) \) or \( \tau^l(b_t^T) \). As a result, the steady-state output, consumption and investment are also state-dependent.

At time 0, the economy in the high-debt state with a debt target \( b \). After a regime switch to \( b^H \), the probability of staying at the high-debt-target regime is \( \text{prob}(b_t^T = b^H | b_{t-1}^T = b^H) = p^H \), while the probability of switching back to the low-debt-target regime is \( 1 - p^H \).

Given the very different expectations effects about the two income tax rates, in this simulation we allow one of the tax rate to serve as an adjusting instrument at a time. Figure 9 compares the impulse response of a government spending increase with uncertainty in the

\textsuperscript{14}For member states that have a debt level exceeding 60 percent of GDP, it requires that the debt to diminish each year at a satisfactory pace.
debt target (dashed lines) to those without uncertainty (solid lines). The left (right) column assumes that the capital (labor) income tax rates is the fiscal adjustment instrument and thus $\gamma_k = 0.06$ and $\gamma_l = 0$ ($\gamma_k = 0$ and $\gamma_l = 0.06$). Like the earlier exercise, the responses plotted under uncertainties are from a policy path that regime switching in the debt target does not realize throughout the simulation horizon, so we can focus on the expectations effects of uncertain debt targets.

Intuitively, increasing a debt target generates a positive wealth effect, which encourages consumption and expands output. In our model, the effect of uncertain debt targets, again, depends on adjusting instruments. Expecting that the debt target may switch to a higher level is expansionary as shown in the left column, because the capital tax rate is likely to be lower relative to the case without uncertain debt targets. Expecting lower capital tax rates induces households to invest more. Thus, the investment falls less (and turn positive in later years) in response to the spending increase, compared to the case without uncertainty. If instead the adjusting instrument is the labor tax rate, the dominant effect becomes the positive wealth effect that reduces current labor supply relative to the case without uncertainty. Also, with lower labor inputs, the marginal product of labor falls, reducing investment. Since future tax liabilities can be potentially higher, the expected higher consumption, reduces the marginal benefits of future consumption, raising the equilibrium interest rate in both cases. The higher debt service payment, plus contractionary output, leads to more debt accumulation in the case of the labor tax rate adjustment, relative to the case without uncertainty (the right column) and to the case with capital tax adjustments.

Somewhat different from our conclusion, Richter and Throckmorton (2015), using a model without distinguishing the capital and income tax rate, conclude that uncertain debt targets are expansionary. In our model economy, their results are possible when households expect

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15The responses under no uncertainty has $b_t^T = b \forall t$. This setup is slightly different from the baseline model, as we only allow one tax rate to adjust at a time.
that most of adjusting is likely to fall on reducing the capital tax rate from the level without uncertainty.

5. Sensitivity Analysis

The analysis we have conducted so far finds that the wealth effect on labor supply is important for debt-dependent government spending effects. The GHH preference assumed in the baseline model weakens this effect. In sensitivity analysis, we explore another commonly adopted preference—the KPR preference \( \psi = 1 \) in (4)\) under no policy uncertainty. For a given \( \theta \), the KPR preference has much stronger wealth effect on labor supply than the GHH preference. Figure 10 plots the impulse responses to a government spending shock. The dashed (dotted-dashed) lines are the responses in the low- (high-) debt state.

The main difference between GHH and KPR is the labor response. In contrast to Figure 3, with the KPR preference the labor response on impact is now higher in the high-debt than low-debt state, as anticipating future higher taxes makes households want to work harder to smooth future income and consumption loss. With higher labor inputs, the marginal product of capital is also higher in the high-debt state. As a result, investment declines less in the high-debt state at the beginning. While the output multipliers in the short run are the same for the low- and high-debt states, it continues to be the case that government spending has more negative output effects at the longer horizon in the high-debt state than in the low-debt state, as the effects on output are still dominated by the realization of fiscal adjustments.

Our robust finding that government spending in the high-debt state has more negative output effect than in the low-debt state is consistent with the qualitative findings in Ilzetzki et al. (2013) and Nickel and Tudyka (2013). The source for the ambiguity in the short-run spending effect we identify—the wealth effect on labor supply—may help explain inconclusive results on a less positive effect of government spending across empirical studies.
6. Conclusion

We study debt-dependent effects of fiscal expansions in a simple RBC model solved non-linearly under rational expectations. The focus is on the expectations effects of fiscal adjustments or consolidations. In addition to the common linear fiscal rules in the literature, policy uncertainty in the magnitude of debt reaction and debt targets are explored. We find that government debt can matter negatively for the expansionary effects of a government spending increase. When the wealth effect on labor supply is sufficiently small, the differences in the output multipliers between the high- and low-debt states are larger in the short run. Under our baseline model with a GHH preference and a general income tax rate as an adjustment instrument, the impact output multiplier in the high-debt state is only half of the size in the low-debt state. At the longer horizon, the multiplier difference between the two states becomes larger with either a GHH or KPR preference.

With policy uncertainty, we find that whether the uncertainty arises from the reaction magnitude to debt or debt targets, the expectations effects relative to the case without uncertainty depend on the perceived fiscal adjustment instrument. When households expect that the debt reaction magnitude of the capital (labor) tax rate can be potentially bigger, policy uncertainty is contractionary (expansionary), offsetting (amplifying) the output effect of government spending. By the same logic, when the capital tax rate is the adjustment instrument, expecting that the debt target can increase—implying a potential fall in the capital tax rate—becomes expansionary; the opposite is true for the labor tax rate.

Our findings suggest that without controlling for adjustment or consolidation instruments, a wide range of empirical estimates for government spending effects in the high-debt state is more likely than in the low-debt state or during normal times. Since an expansionary fiscal policy in the high-debt state is not all contractionary in the short run, our results advise that policymakers that would like to pursue fiscal expansion when the debt level is high should anchor expectations of fiscal adjustments.
One caveat worth mentioning is that our analysis does not consider rising sovereign risk premia when a country approaches its debt limit, beyond which a sovereign default would occur. For countries with a sovereign default history, policy expectations in that situation can go beyond fiscal consolidations considered here; they may also involve expectations about a future period of severe economic disruption, including turmoils in financial systems, trade exclusions, etc. Under those circumstances, expectations effects are likely to be much more contractionary on government spending effects than what we present in this study.
DEBT-DEPENDENT EFFECTS OF FISCAL EXPANSIONS

APPENDIX A. EQUILIBRIUM CONDITIONS OF THE BASELINE MODEL

\[ \ddot{c}_t = \left[ \omega \left( c_t \right)^{\frac{1}{\nu}} + \left( 1 - \omega \right) \left( g_t \right)^{\frac{1}{\nu}} \right]^{\frac{\nu}{\nu - 1}}, \]  
(A.1)

\[ \lambda_t = \left( \ddot{c}_t - \varphi l_t^{\theta} \right)^{-\sigma} \omega c_t^{\frac{1}{\nu}} c_t^{\frac{1}{\nu - 1}} \]  
(A.2)

\[ \left( \ddot{c}_t - \varphi l_t^{\theta} \right)^{-\sigma} \varphi \theta l_t^{\theta-1} = \lambda_t \left( 1 - \tau_t \right) w_t \]  
(A.3)

The above two equations can be reduced as:

\[ \varphi \theta l_t^{\theta-1} = \omega c_t^{\frac{1}{\nu}} c_t^{\frac{1}{\nu - 1}} \left( 1 - \tau_t \right) w_t \]  
(A.4)

Let \( \lambda_t \) and \( \xi_t \) be the Lagrangian multipliers for household’s budget constraint and law of motion for capital, define Tobin’s Q as \( TQ_t = \frac{\ddot{q}_t}{\lambda_t} \)

\[ 1 = TQ_t \left( 1 - \frac{\kappa}{2} \left( \frac{i_t}{k_{t-1}} - \delta \right) \right) \]  
(A.5)

\[ TQ_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[ (1 - \tau_{t+1}) r_{t+1} + TQ_{t+1} \left( 1 - \delta \right) + \kappa \left( \frac{i_{t+1}}{k_t} - \delta \right) \frac{i_{t+1}}{k_t} - \frac{\kappa}{2} \left( \frac{i_{t+1}}{k_t} - \delta \right) \right] \]  
(A.6)

\[ k_t = (1 - \delta) k_{t-1} + i_t - \frac{\kappa}{2} \left( \frac{i_t}{k_{t-1}} - \delta \right)^2 k_{t-1} \]  
(A.7)

\[ \lambda_t q_t = \beta E_t \lambda_{t+1} \]  
(A.8)

\[ y_t = a_t k_{t-1}^{\alpha} l_t^{1-\alpha} \]  
(A.9)
\[(1 - \alpha)y_t = w_t l_t \quad \text{(A.10)}\]

\[\alpha y_t = r_t^k k_{t-1} \quad \text{(A.11)}\]

\[y_t = c_t + i_t + g_t \quad \text{(A.12)}\]

\[
\tau_t \left( w_t l_t + r_t^k k_{t-1} \right) + q_t b_t = g_t + b_{t-1} + z_t
\]

\[\tau_t = \tau + \gamma(b_{t-1} - b) \quad \text{(A.14)}\]

\[
\ln \frac{a_t}{a} = \rho_a \ln \frac{a_{t-1}}{a} + \varepsilon^a_t \quad \text{(A.15)}
\]

\[
\ln \frac{g_t}{g} = \rho_g \ln \frac{g_{t-1}}{g} + \varepsilon^g_t \quad \text{(A.16)}
\]

**APPENDIX B. SOLVING THE MODEL NONLINEARLY**

When solving the nonlinear model, the state space is \(S_t = \{b_{t-1}, a_t, g_t, k_{t-1}, rs_t\}\). The variable \(rs_t\) indicates the regime in the model with fiscal uncertainty. In the model with regime-switching debt reaction magnitudes, \(\gamma_t\) equals \(\gamma\) if \(rs_t = 1\) and \(\gamma^H\) if \(rs = 2\). In the model with regime-switching debt targets, \(b_t^T\) equals \(b\) if \(rs_t = 1\) and \(b^H\) if \(rs = 2\). In the model with a fixed regime (i.e., no policy uncertainty), \(\gamma_t\) always equals \(\gamma\) and \(b_t^T\) always equals \(b\).

Define the decision rules for the end-of-period government bond as \(b_t = f^b(S_t)\), and consumption as \(c_t = f^c(S_t)\). The decision rules are solved as follows.
(1) Define the grid points by discretizing the state space. Make initial guesses for $f^b_0$ and $f^c_0$ over the state space.

(2) At each grid point, solve the nonlinear model and obtain the updated rules $f^b_i$ and $f^c_i$ using the given rules $f^b_{i-1}$ and $f^c_{i-1}$:

(a) Given $rs_t$, derive $\tau^{k}_t$, $\tau^{l}_t$, or $\tau_t$ using the tax rule.

(b) Compute $\tilde{c}_t$ from (A.1). Using equations (A.4) and (A.10), we can solve $l_t$ by the following equation

$$l_t = \left[ \frac{\omega(\tilde{c}_t/c_t)^{1/\sigma}(1 - \tau_t)(1 - \alpha) a_t k^a_t}{\varphi^\theta} \right]^{1/(\theta + \alpha - 1)}.$$

Then, compute $\lambda_t$ from (A.2).

(c) Compute $y_t$, $w_t$, and $r^k_t$ using (A.9), (A.10), and (A.11).

(d) Given $y_t$, $c_t$, and $g_t$, we can solve $i_t$ from the aggregate resource constraint (A.12), and then obtain $TQ_t$, $k_t$, and bond’s price $q_t$ from (A.5), (A.7), and (A.13).

(e) Use linear interpolation to obtain $f^b_{i-1}(S_{t+1})$ and $f^c_{i-1}(S_{t+1})$ with $S_{t+1} = (b_t, a_{t+1}, g_{t+1}, k_t, rs_{t+1})$.

Then follow the above steps to solve $\tau^k_{t+1}$, $\tau^{l}_{t+1}$, or $\tau_{t+1}$, $r^k_{t+1}$, $\lambda_{t+1}$, $i_{t+1}$, and $TQ_{t+1}$.

(f) Update the decision rules $f^b_i$ and $f^c_i$ using (A.6) and (A.8).

(3) Check convergence of the decision rules. If $|f^b_i - f^b_{i-1}|$ or $|f^c_i - f^c_{i-1}|$ is above the desired tolerance (set to $1e-7$), go back to Step (2); otherwise, $f^b_i$ and $f^c_i$ are the decision rules.

**APPENDIX C. SIMULATING THE DISTRIBUTION OF DEBT THRESHOLDS**

This appendix describes procedures in simulating debt threshold distributions, defined as

$$B^*_t \sim \sum_{i=0}^{\infty} \beta^i \frac{\lambda^{*}_{t+i}}{\lambda_t} \left( \tau^{*}_{t+i} y_{t+i} - g^{*}_{t+i} - z \right),$$

where $\tau^{*}_{t+i}$ and $g^{*}_{t+i}$ are future tax rates and government spending associated with computing debt thresholds. Since these fiscal variables represent policy without consolidations, we
assume a simple AR(1) process for the income tax rate:

$$\ln \frac{\tau^*_t}{\tau} = \rho \ln \frac{\tau^{*}_{t-1}}{\tau} + \varepsilon^*_t.$$  

(C.2)

which implies that the series of the tax rates is mean reverting. In the simulation, we set the mean of $\tau^*_t$ to be 0.287, the highest revenue to GDP ratio for U.S. in the sample, slightly above the average revenue to GDP ratio 0.261. Technology and government spending follow (A.15) and (A.16). The AR(1) coefficients and standard deviations are obtained from the reduced-form estimates as describe in Section 2.2.

Assume the decision rule for labor is $c^*_t = m^c(S_t)$, where $S_t = \{a_t, g_t, \tau_t, k_{t-1}\}$ indicates the initial state. Let $T^*_t \equiv \tau^*_t y_t$ be the tax revenues. After obtaining the converged rules for $m^c(.)$, the rules for $T^*_t = m^T(S_t)$ can be derived, which are consistent with the optimization conditions from the household’s and the firms’ problems.

To proceed:

(1) Define the grid points by discretizing the state space. Make initial guesses for $m^c_0$ over the state space.

(2) At each grid point, solve the nonlinear model under the assumption that the tax rate follows the specified $AR(1)$ processes (C.2), using the given rules $m^c_{i-1}$, and obtain the updated rules $m^c_i$. Specifically,

(a) Compute $\tilde{c}_t$ from (A.1). Using equations (A.4) and (A.10), we can solve $l_t$ by the following equation

$$l_t = \left[ \omega \left( \frac{\tilde{c}_t}{c_t} \right)^{1/\sigma} (1 - \tau_t) (1 - \alpha) a_t k_{t-1}^{\alpha} \right]^{\frac{1}{\delta + \sigma - 1}}.$$  

Then, compute $\lambda_t$ from (A.2).

(b) Compute $y_t$, $w_t$, and $r^k_t$ using (A.9), (A.10), and (A.11).

(c) Given $y_t$, $c_t$, and $g_t$, we can solve $i_t$ from the aggregate resource constraint (A.12), then obtain $TQ_t$, and $k_t$ from (A.5) and (A.7).
(d) Use linear interpolation to obtain \( m_{i-1}^{c}(S_{t+1}) \), where \( S_{t+1} = (a_{t+1}, g_{t+1}, \tau_{t+1}, k_{t}) \).

Then follow the above steps to solve \( r_{t+1}^{k}, \lambda_{t+1}, i_{t+1}, \) and \( TQ_{t+1} \).

(e) Update the decision rules \( m_{i}^{c} \) using (A.6).

(3) Check convergence of the decision rules. If \( |m_{i}^{c} - m_{i-1}^{c}| \) is above the desired tolerance (set to \( 1e^{-7} \)), go back to step (2). Otherwise, \( m_{i}^{c} \) is the decision rule.

(4) Use the converged rules, \( m^{c} \), to compute the decision rules for \( m^{T} \).

After solving the maximum tax revenue \( m^{T}(.) \), the distribution of debt threshold is obtained using Markov Chain Monte Carlo simulations. To proceed,

(1) For each simulation \( j \), we randomly draw the exogenous shocks for TFP \( (\varepsilon_{a,j}^{a+i}) \), government spending \( (\varepsilon_{g,j}^{g+i}) \), and tax \( (\varepsilon_{t}^{T,j}) \) for 1000 periods, \( i = \{1, 2, 3, ..., 1000\} \). At each period, we obtain \( T_{t+i}^{*,j} (i = 1, ..., 1000) \) by interpolating on the decision rules \( m^{T}(.) \). Then, the debt threshold for simulation \( j \) is computed, conditional on particular sequences of shocks,

\[
B_{t}^{*,j} = \sum_{i=0}^{\infty} \beta^{i} \frac{\lambda_{i}^{*,j}}{\lambda_{t}^{*,j}} (T_{t+i}^{*,j} - g_{t+i}^{*,j} - z) \quad (C.3)
\]

(2) Repeat the simulation for 10, 000 times (\( j = \{1, ..., 10000\} \)) to have \( \{B_{t}^{*,j}\}_{j=1}^{10000} \), which form the distribution of \( B_{t}^{*} \).
### Table 1: Cumulative government spending multipliers, the baseline model (the GHH preference).

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<tr>
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### Table 2: Cumulative government spending multipliers for output, the KPR preference.

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<td>5 years</td>
<td>−0.92</td>
<td>−1.29</td>
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Figure 1: General net government debt of G7 Countries. Data are based on the World Economic Outlook Database October 2014, International Monetary Fund (2014).
Figure 2: Debt and fiscal consolidation: bars—discretionary consolidation measures in percent of GDP, based on data in Devries et al. (2011) (left axis); lines—public debt in percent of GDP (right axis).

Figure 3: Government spending effects: the baseline model. The x-axis is in quarters after the initial increase in government spending. Except for G/Y, the y-axis is the difference between a path with and without a G shock, scaled by the the deterministic steady-state values.
Figure 4: Decision rules of hours worked (labor), investment, and consumption. Both x-axis and y-axis are in percent deviations from the deterministic values. When plotting the decisions rule for one state variable (debt—the top row, or capital—the bottom row), we hold other states at their steady-state values.

Figure 5: The Distribution of Debt Thresholds. The x-axis is the ratio of debt to steady-state annual output.
Figure 6: Government spending effects: uncertainty about the reaction magnitude to debt. The x-axis is in quarters. Except for G/Y, the y-axis is the difference between a path with and without a G shock, scaled by the deterministic steady-state values.
Figure 7: Cumulative output multipliers: “high debt, no uncertainty” is the baseline model; “high debt, rs $\gamma^K_t$, expectation” has uncertainty but $\gamma^K_t$ switching does not occur; “high debt, rs $\gamma^K_{tm}$” is the $\gamma^K_t$ switching case with 1000 draws for the policy paths and the bands are 10-90 percent intervals.

Figure 8: Cumulative output multipliers: “high debt, no uncertainty” is the baseline model; “high debt, rs $\gamma^L_t$, expectation” has uncertainty but $\gamma^L_t$ switching does not occur; “high debt, rs $\gamma^L_{tm}$” is the $\gamma^L_t$ switching case with 1000 draws for the policy paths and the bands are 10-90 percent intervals.
Figure 9: Government spending effects: uncertainties about the debt targets. The x-axis is in quarters. Except for G/Y, the y-axis is the difference between a path with and without a G shock, scaled by the deterministic steady-state values.
Figure 10: KPR specification of the utility function: impulse responses to a government spending-to-output ratio increase of 1 percentage point.
DEBT-DEPENDENT EFFECTS OF FISCAL EXPANSIONS

REFERENCES


International Monetary Fund, 2014. World economic outlook database, October.


International Monetary Fund, 2015b. World economic outlook database, April. Washington, D.C.


