Trade Dynamics in the Market for Federal Funds

Gara Afonso
FRB of New York

Ricardo Lagos
New York University
The market for federal funds

A market for loans of reserve balances at the Fed.
The market for federal funds

- What’s traded?
  
  Unsecured loans (mostly overnight)

- How are they traded?
  
  Over the counter

- Who trades?
  
  Commercial banks, securities dealers, agencies and branches of foreign banks in the U.S., thrift institutions, federal agencies
The market for federal funds
Why is the fed funds market interesting?

- It is an interesting example of an OTC market
  (Unusually good data is available)

- Reallocates reserves among banks
  (Banks use it to offset liquidity shocks and manage reserves)

- Determines the interest rate on the shortest maturity instrument in the term structure

- Is the “epicenter” of monetary policy implementation
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In this paper we ... 

(1) Develop a model of trade in the fed funds market that explicitly accounts for the two key OTC frictions:

- Search for counterparties
- Bilateral negotiations
In this paper we ...

(2) Use the theory to address some elementary questions:

- **Positive:**
  - What are the determinants of the fed funds rate?
  - How does the market reallocate funds?

- **Normative:**
  - Is the OTC market structure able to achieve an efficient reallocation of funds?
In this paper we ... 

(3) Calibrate the model and use it to:

- Assess the ability of the theory to account for empirical regularities of the fed funds market:
  - Intraday evolution of reserve balances
  - Dispersion in fed funds rates and loan sizes
  - Skewed distribution of number of transactions
  - Skewed distribution of proportion of intermediated funds
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  - Intraday evolution of reserve balances
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  - Skewed distribution of number of transactions
  - Skewed distribution of proportion of intermediated funds

- Conduct policy experiments:
  
  What is the effect on the fed funds rate of a 25 bps increase in the interest rate that the Fed pays on reserves?
The model

- A trading session in continuous time, \( t \in [0, T] \), \( \tau \equiv T - t \)
- Unit measure of banks hold reserve balances
  \( k(\tau) \in \mathbb{K} = \{0, 1, \ldots, K\} \)
- \( \{n_k(\tau)\}_{k \in \mathbb{K}} \) : distribution of balances at time \( T - \tau \)
- Linear payoffs from balances, discount at rate \( r \)
- Fed policy:
  - \( U_k \) : payoff from holding \( k \) balances at the end of the session
  - \( u_k \) : flow payoff from holding \( k \) balances during the session
- Trade opportunities are bilateral and random (Poisson rate \( \alpha \))
- Loan and repayment amounts determined by Nash bargaining
- Assume all loans repaid at time \( T + \Delta \), where \( \Delta \in \mathbb{R}_+ \)
Institutional features of the fed funds market

Model

- Search and bargaining

Fed funds market
Institutional features of the fed funds market

**Model**
- Search and bargaining

**Fed funds market**
- Over-the-counter market
Institutional features of the fed funds market

**Model**
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- $[0, T]$  

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- $\{n_k(T)\}_{k \in K}$

Fed funds market
- Over-the-counter market
- 4:00pm-6:30pm
Institutional features of the fed funds market

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- \([0, T]\]
- \(\{n_k(T)\}_{k \in K}\)
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**Fed funds market**
- Over-the-counter market
- 4:00pm-6:30pm
- Distribution of reserve balances at 4:00pm
- Reserve requirements, interest on reserves...
Bank with balance $k$ contacts bank with balance $k'$ at time $T - \tau$
Bank with balance $k$ contacts bank with balance $k'$ at time $T - \tau$

- The set of feasible post-trade balances is:

$$\Pi (k, k') = \{ (k + k' - y, y) \in \mathbb{K} \times \mathbb{K} : y \in \{0, 1, \ldots, k + k'\}\}$$
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  \]

- The set of feasible loan sizes is:
  \[
  \Gamma (k, k') = \{b \in \{-K, \ldots, 0, \ldots, K\} : (k - b, k' + b) \in \Pi (k, k')\}
  \]
Bank with balance $k$ contacts bank with balance $k'$ at time $T - \tau$

- The set of feasible post-trade balances is:
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  \]

- $V_k(\tau)$: value of a bank with balance $k$ at time $T - \tau$
Bank with balance $k$ contacts bank with balance $k'$ at time $T - \tau$.

The *loan size* $b$, and the *repayment* $R$ maximize:

\[
\left[ V_{k-b} (\tau) + e^{-r(\tau+\Delta)} R - V_k (\tau) \right]^\frac{1}{2} \left[ V_{k'+b} (\tau) - e^{-r(\tau+\Delta)} R - V_{k'} (\tau) \right]^\frac{1}{2}
\]

s.t. \hspace{1cm} b \in \Gamma (k, k') \hspace{1cm} R \in \mathbb{R}
Bargaining

Bank with balance $k$ contacts bank with balance $k'$ at time $T - \tau$.

The loan size $b$, and the repayment $R$ maximize:

\[
\left[ V_{k-b}(\tau) + e^{-r(\tau+\Delta)}R - V_k(\tau) \right]^{\frac{1}{2}} \left[ V_{k'+b}(\tau) - e^{-r(\tau+\Delta)}R - V_{k'}(\tau) \right]^{\frac{1}{2}}
\]

s.t. \quad b \in \Gamma(k, k'), \quad R \in \mathbb{R}

\[
b^* \in \arg \max_{b \in \Gamma(k, k')} \left[ V_{k'+b}(\tau) + V_{k-b}(\tau) - V_{k'}(\tau) - V_k(\tau) \right]
\]

\[
e^{-r(\tau+\Delta)}R^* = \frac{1}{2} \left[ V_{k'+b^*}(\tau) - V_{k'}(\tau) \right] + \frac{1}{2} \left[ V_k(\tau) - V_{k-b^*}(\tau) \right]
\]
Value function

\[
\begin{align*}
    rV_i(\tau) + \dot{V}_i(\tau) &= \\
    &= u_i + \frac{\alpha}{2} \sum_{j,k,s \in \mathbb{K}} n_j(\tau) \phi_{ij}^{ks}(\tau) [V_k(\tau) + V_s(\tau) - V_i(\tau) - V_j(\tau)]
\end{align*}
\]
Value function

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\[ = u_i + \frac{\alpha}{2} \sum_{j,k,s \in \mathbb{K}} n_j(\tau) \phi_{ij}^{ks}(\tau) [V_k(\tau) + V_s(\tau) - V_i(\tau) - V_j(\tau)] \]

with \( V_i(0) = U_i \), and

\[ \phi_{ij}^{ks}(\tau) = \begin{cases} \tilde{\phi}_{ij}^{ks}(\tau) & \text{if } (k, s) \in \Omega_{ij} \left[ V(\tau) \right] \\ 0 & \text{if } (k, s) \notin \Omega_{ij} \left[ V(\tau) \right] \end{cases} \]
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with

\[ \Omega_{ij} [V (\tau)] \equiv \arg \max_{(k', s') \in \Pi(i, j)} [V_{k'} (\tau) + V_{s'} (\tau) - V_i (\tau) - V_j (\tau)] \]

where \( \tilde{\phi}_{ij}^{ks} (\tau) \geq 0 \) and \( \sum_{k \in \mathbb{K}} \sum_{s \in \mathbb{K}} \tilde{\phi}_{ij}^{ks} (\tau) = 1 \)
Time-path for the distribution of balances

For all \( k \in \mathbb{K} \),

\[
\dot{n}_k (\tau) = \alpha n_k (\tau) \sum_{i \in \mathbb{K}} \sum_{j \in \mathbb{K}} \sum_{s \in \mathbb{K}} n_i (\tau) \phi_{ki}^{sj} (\tau) \\
- \alpha \sum_{i \in \mathbb{K}} \sum_{j \in \mathbb{K}} \sum_{s \in \mathbb{K}} n_i (\tau) n_j (\tau) \phi_{ij}^{ks} (\tau)
\]
Definition

An equilibrium is a value function, $V$, a path for the distribution of reserve balances, $n(\tau)$, and a path for the distribution of trading probabilities, $\phi(\tau)$, such that:

(a) given the value function and the distribution of trading probabilities, the distribution of balances evolves according to the law of motion; and

(b) given the path for the distribution of balances, the value function and the distribution of trading probabilities satisfy individual optimization given the bargaining protocol.
Assumption A. For any \( i, j \in \mathbb{K} \), and all \((k, s) \in \prod (i, j)\), the payoff functions satisfy:

\[
\begin{align*}
    u\left[\frac{i+j}{2}\right] + u\left[\frac{i+j}{2}\right] & \geq u_k + u_s \\
    U\left[\frac{i+j}{2}\right] + U\left[\frac{i+j}{2}\right] & \geq U_k + U_s, \quad ">" \text{ unless } k \in \left\{\left\lfloor\frac{i+j}{2}\right\rfloor, \left\lceil\frac{i+j}{2}\right\rceil\right\}
\end{align*}
\]

where for any \( x \in \mathbb{R} \),

\[
\lfloor x \rfloor \equiv \max \{k \in \mathbb{Z} : k \leq x\}
\]

\[
\lceil x \rceil \equiv \min \{k \in \mathbb{Z} : x \leq k\}
\]
Proposition

Let the payoff functions satisfy Assumption A. Then:

(i) An equilibrium exists. The paths $V(\tau)$ and $n(\tau)$ are unique.

(ii) The equilibrium path for $\phi(\tau) = \{\phi_{ij}^{ks}(\tau)\}_{i,j,k,s \in K}$ is

$$
\phi_{ij}^{ks}(\tau) = \begin{cases} 
\tilde{\phi}_{ij}^{ks}(\tau) & \text{if } (k, s) \in \Omega_{ij}^* \\
0 & \text{if } (k, s) \notin \Omega_{ij}^*
\end{cases}
$$

where $\tilde{\phi}_{ij}^{ks}(\tau) \geq 0$ and $\sum_{(k,s) \in \Omega_{ij}^*} \tilde{\phi}_{ij}^{ks}(\tau) = 1$, with

$$
\Omega_{ij}^* = \begin{cases} 
\left\{ \left\{ \left( \frac{i+j}{2}, \frac{i+j}{2} \right) \right\}, \left\{ \left[ \frac{i+j}{2} \right], \left[ \frac{i+j}{2} \right] \right\}, \left( \left[ \frac{i+j}{2} \right], \left[ \frac{i+j}{2} \right] \right) \right\} & \text{if } i + j \text{ even} \\
\left\{ \left\{ \left( \frac{i+j}{2}, \frac{i+j}{2} \right) \right\}, \left\{ \left[ \frac{i+j}{2} \right], \left[ \frac{i+j}{2} \right] \right\} \right\} & \text{if } i + j \text{ odd.}
\end{cases}
$$
Proposition

Let the payoff functions satisfy Assumption A. Then, the equilibrium supports an efficient allocation of reserve balances.
Positive implications

The theory delivers:

1. Time-varying distribution of trade sizes, trade volume
2. Time-varying distribution of fed fund rates
3. Endogenous intermediation
Trade volume

- Flow volume of trade at time \( T - \tau \):

\[
\bar{\nu}(\tau) = \sum_{i \in K} \sum_{j \in K} \sum_{k \in K} \sum_{s \in K} \nu_{ij}^{ks}(\tau)
\]

where

\[
\nu_{ij}^{ks}(\tau) \equiv \alpha n_i(\tau) n_j(\tau) \phi_{ij}^{ks}(\tau) |k - i|
\]
Trade volume

- Flow volume of trade at time $T - \tau$:

$$\bar{v}(\tau) = \sum_{i \in K} \sum_{j \in K} \sum_{k \in K} \sum_{s \in K} v_{ij}^{ks}(\tau)$$

where

$$v_{ij}^{ks}(\tau) \equiv \alpha n_i(\tau) n_j(\tau) \phi_{ij}^{ks}(\tau) |k - i|$$

- Total volume traded during the trading session:

$$\bar{v} = \int_0^T \bar{v}(\tau) d\tau$$
Fed funds rate

If a bank with \( i \) borrows \( k - i = j - s \) from bank with \( j \) at time \( T - \tau \), the interest rate on the loan is:

\[
\rho_{ij}^{ks}(\tau) = \frac{\ln \left[ \frac{R_{ij}^{ks}(\tau)}{k-i} \right]}{\tau + \Delta} = r + \frac{\ln \left[ \frac{V_j(\tau) - V_s(\tau)}{j-s} + \frac{1}{2} S_{ij}^{ks}(\tau)}{j-s} \right]}{\tau + \Delta}
\]
Fed funds rate

- If a bank with $i$ borrows $k - i = j - s$ from bank with $j$ at time $T - \tau$, the interest rate on the loan is:

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$$

- The daily average (value-weighted) fed funds rate is:

$$
\bar{\rho} = \frac{1}{T} \int_{0}^{T} \bar{\rho}(\tau) \, d\tau
$$

where

$$
\bar{\rho}(\tau) \equiv \sum_{i \in K} \sum_{j \in K} \sum_{k \in K} \sum_{s \in K} \omega_{ij}^{ks} (\tau) \rho_{ij}^{ks} (\tau)
$$

$$
\omega_{ij}^{ks} (\tau) \equiv \nu_{ij}^{ks} (\tau) / \bar{\nu}(\tau)
$$
Endogenous intermediation

- Cumulative purchases: $O^p = \sum_{n=1}^{N} \max\{k_n - k_{n-1}, 0\}$

- Cumulative sales: $O^s = -\sum_{n=1}^{N} \min\{k_n - k_{n-1}, 0\}$
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Bank-level measures of intermediation

- Excess funds reallocation:

$$X = O^p + O^s - |O^p - O^s|$$
Endogenous intermediation

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Bank-level measures of intermediation

- Excess funds reallocation:
  $$X = O^p + O^s - |O^p - O^s|$$

- Proportion of intermediated funds:
  $$\iota = \frac{X}{O^p + O^s}$$
• Analytics for special case with $\mathcal{K} = \{0, 1, 2\}$

• Intuition for efficiency result

• Frictionless limit

• Analytical figures

• Small-scale simulations: $\mathcal{K} = \{0, 1, 2\}$
Payoff functions

\[ u_k = \begin{cases} 
(k')^{1-\epsilon} i^d_+ & \text{if } 0 \leq k' \\
(k')^{1+\epsilon} i^d_- & \text{if } k' < 0 
\end{cases} \quad \text{with } \epsilon \approx 0 
\]

\[ e^{r\Delta f} U_k = \begin{cases} 
 k' + i^e_f \bar{k} + i^e_f (k' - \bar{k}) & \text{if } \bar{k} \leq k' \\
k' + i^e_f \bar{k} - i^w_f (\bar{k} - k') & \text{if } k' < \bar{k} 
\end{cases} \quad \text{where } k' \equiv k - \bar{k}_0 
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Payoff functions

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$$e^{r\Delta_f} U_k = \begin{cases} 
k' + i^r_\var{\bar k} + i^e_\var{k'} (k' - \bar k) & \text{if } \bar k \leq k' \\
k' + i^r_\var{\bar k} - i^w_\var{k'} (\bar k - k') & \text{if } k' < \bar k 
\end{cases}$$

where \( k' \equiv k - \bar k_0 \)

\( \bar k_0 \) is a “shifter”
Distribution of balances at 16:00 for “typical day” in 2007

- Sample: $N = 142$ commercial banks that traded fed funds at least once during 2007 Q2

- $\hat{k}^i$: bank $i$’s average balance at 16:00 over a given two-week maintenance period during 2007 Q2, divided by bank $i$’s daily average required operating balance over the same period
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$$\mathbb{K} = \{0, \ldots, 250\}, \quad \bar{k} = 1, \quad \mathbb{K}' \equiv \mathbb{K} - \bar{k}_0, \quad \bar{k}_0 = 50$$
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\[ \mathbb{K} = \{0, \ldots, 250\}, \quad \bar{k} = 1, \quad \mathbb{K}' \equiv \mathbb{K} - \bar{k}_0, \quad \bar{k}_0 = 50 \]

\[ n_k (T) = \frac{1}{N} \sum_{i=1}^{N} \mathbb{I}_{\{\hat{k}^i \in [k-\bar{k}_0, k-\bar{k}_0+1)\}} \]
Distribution of balances at 16:00 for “typical day” in 2007

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  \[ n_k(T) = \frac{1}{N} \sum_{i=1}^{N} \mathbb{I}\{\hat{k}^i \in [k-\bar{k}_0, k-\bar{k}_0+1]\} \]

  \[ Q = \sum_{k=0}^{250} (k - \bar{k}_0) n_k(T) \approx 1.04 \]
Policy parameters as in 2007

<table>
<thead>
<tr>
<th>$T$</th>
<th>$\Delta_f$</th>
<th>$\Delta$</th>
<th>$i^d_+$</th>
<th>$i^d_-$</th>
<th>$i^r_f$</th>
<th>$i^e_f$</th>
<th>$i^w_f$</th>
<th>$i^c_f$</th>
<th>$i^o_f$</th>
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<tr>
<td>$\frac{2.5}{24}$</td>
<td>$\frac{2.5}{24}$</td>
<td>$\frac{22}{24}$</td>
<td>$\frac{10^{-7}}{360}$</td>
<td>$\frac{.0036}{360}$</td>
<td>$0$</td>
<td>$0$</td>
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<td>$\frac{.0725}{360}$</td>
<td>$\frac{.0925}{360}$</td>
<td>$\frac{0.0001}{360}$</td>
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</tbody>
</table>
Calibrated parameters, 2007 targets

\[(\alpha, P^w) = (100, \frac{0.0525}{360})\]

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>fed funds rate</td>
<td>.0527</td>
<td>.0525*</td>
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<tr>
<td>std. dev. of balances at 6:30 pm</td>
<td>1.2</td>
<td>1.15*</td>
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<tr>
<td>median number of counterparties</td>
<td>7</td>
<td>2</td>
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<tr>
<td>mean number of counterparties</td>
<td>7</td>
<td>4.5</td>
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<tr>
<td>intermediation index</td>
<td>.65</td>
<td>.43</td>
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</table>
Reserve balances and fed funds rates (2007)
Distributions of loan sizes and trading activity (2007)
Policy parameters as in 2011

<table>
<thead>
<tr>
<th>$i_r$</th>
<th>$i_e$</th>
<th>$i_w$</th>
<th>$i_c$</th>
<th>$i_o$</th>
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<td>$\frac{.0025}{360}$</td>
<td>$\frac{.0075}{360}$</td>
<td>$\frac{.0175}{360}$</td>
<td>$\frac{.0415}{360}$</td>
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</table>
Calibrated parameters, 2011 targets

<table>
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<th>($\alpha$, $P^w$) = (1, 0)</th>
<th>Model</th>
<th>Data</th>
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<tbody>
<tr>
<td>fed funds rate</td>
<td>.0029</td>
<td>.0025*</td>
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<tr>
<td>std. dev. of balances at 6:30 pm</td>
<td>31</td>
<td>31*</td>
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<tr>
<td>median number of counterparties</td>
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<td>mean number of counterparties</td>
<td>.1</td>
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<td>intermediation index</td>
<td>.02</td>
<td>.35</td>
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Policy experiments (baseline policy as in 2011)

<table>
<thead>
<tr>
<th>$i_f$</th>
<th>$Q/\bar{k} = 0.50$</th>
<th>$Q/\bar{k} = 1.00$</th>
<th>$Q/\bar{k} = 30$</th>
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<tbody>
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<td>38</td>
<td>36</td>
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<td>64</td>
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<th>$i^w_f$</th>
<th>$Q/\bar{k} = 0.50$</th>
<th>$Q/\bar{k} = 1.00$</th>
<th>$Q/\bar{k} = 30$</th>
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<tr>
<td>50</td>
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<td>100</td>
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<td>61</td>
<td>26</td>
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Corridor system

<table>
<thead>
<tr>
<th>$(i_f, i_f^w)$</th>
<th>$Q/\bar{k} = 0.50$</th>
<th>$Q/\bar{k} = 1$</th>
<th>$Q/\bar{k} = 30$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 25</td>
<td>14</td>
<td>13</td>
<td>1</td>
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<tr>
<td>25 – 50</td>
<td>39</td>
<td>38</td>
<td>26</td>
</tr>
<tr>
<td>50 – 75</td>
<td>64</td>
<td>63</td>
<td>51</td>
</tr>
<tr>
<td>75 – 100</td>
<td>89</td>
<td>88</td>
<td>76</td>
</tr>
<tr>
<td>100 – 125</td>
<td>114</td>
<td>113</td>
<td>101</td>
</tr>
</tbody>
</table>
Corridor system
IOR Policy intuition from the analytical example

Proposition

If \( r \approx 0 \),

\[
\rho_f (\tau) \approx \beta (\tau) i_f^e + [1 - \beta (\tau)] i_f^w \quad \text{where}
\]

1. If \( n_2 (T) = n_0 (T) \), \( \beta (\tau) = \theta \)

2. If \( n_2 (T) < n_0 (T) \), \( \beta (\tau) \in [0, \theta] \), \( \beta (0) = \theta \) and \( \beta' (\tau) < 0 \)

3. If \( n_0 (T) < n_2 (T) \), \( \beta (\tau) \in [\theta, 1] \), \( \beta (0) = \theta \) and \( \beta' (\tau) > 0 \).
Ex-ante heterogeneity

We also extend the model to allow for:

1. Heterogeneity in contact rates
2. Heterogeneity in bargaining powers
3. Heterogeneity in target balances (or non-bank participants, e.g., GSEs)
More to be done...

- Fed funds brokers
- Banks’ portfolio decisions
- Random “payment shocks”
- Sequence of trading sessions
- Quantitative work with ex-ante heterogeneity
The views expressed here are not necessarily reflective of views at the Federal Reserve Bank of New York or the Federal Reserve System.
Evidence of OTC frictions in the fed funds market

- Price dispersion
- Intermediation
- Intraday evolution of the distribution of reserve balances
- There are banks that are “very long” and buy
- There are banks that are “very short” and sell
Price dispersion

Intraday Distribution of Fed Funds Spreads, 2005

- 10th/90th Percentiles
- 25th/75th Percentiles
- Median
- Mean
Intermediation: excess funds reallocation
Intermediation: proportion of intermediated funds

Proportion of Intermediated Funds

- 2005
- 2006
- 2007
- 2008
- 2009
- 2010

- 0
- 0.1
- 0.2
- 0.3
- 0.4
- 0.5
- 0.6
Intraday evolution of the distribution of reserve balances

Normalized Balances, 2007

[Diagram showing intraday evolution of reserve balances with 10th/90th, 25th/75th percentiles, and median values.]
Banks that are “long”...and buy...
Banks that are “short”...and sell...

Sales by Banks with Negative Adjusted Balances, 2007

- 10th/90th Percentiles
- 25th/75th Percentiles
- Median
- Mean
Daily volume

Daily Amount of Federal Funds Loans

Daily Number of Federal Funds Loans
Daily volume (size distribution)
Daily distribution of the number of counterparties

- Distribution of Daily Number of Counterparties
- Distribution of Daily Number of Counterparties Banks Lend to
- Distribution of Daily Number of Counterparties Banks Borrow from

Graphs showing the distribution of daily number of counterparties, with percentile and mean values.
Intraday volume (dollar amount)
Intraday volume (number of loans)
Intraday size distribution of loans
Intraday size distribution of loans

**Intraday 90th Percentile of Loan Sizes (over time)**

- **90th Percentile, 15:45-16:15**
- **90th Percentile, 17:45-18:15**

**Intraday 10th Percentile of Loan Sizes (over time)**

- **10th Percentile, 15:45-16:15**
- **10th Percentile, 17:45-18:15**
Trading activity by time-of-day
Intraday evolution of the distribution of reserve balances
Intraday evolution of the distribution of reserve balances
Daily fed funds rate vs. FOMC target

Daily Fed Funds Rate and FOMC Target

Deviations between Fed Funds Rate and FOMC Target
Daily effective fed funds rate vs. FOMC target

- Daily effective fed funds rate vs. FOMC target

- Deviations between effective fed funds rate and FOMC target
Daily fed funds rate dispersion

Daily Dispersions of Fed Funds Rate

- FOMC Target
- 90th Percentile of Fed Funds Rate
- 10th Percentile of Fed Funds Rate
Fed funds rate vs. effective fed funds rate

Daily Spread between Fed Funds and Effective Rate

Percentage:
-2 0 0.2 0.4 0.6
Years:
2005 2006 2007 2008 2009 2010
Intraday distribution of fed funds spreads
Intraday distribution of fed funds spreads (over time)
Intraday distribution of fed funds/FOMC target spreads
Daily intermediation

Excess Funds Reallocation

Proportion of Intermediated Funds

Daily Distribution of Excess Funds Reallocation

Daily Distribution of Proportion of Intermediated Funds
Banks that are “long”...and buy...
Banks that are “short”...and sell...
Daily fed funds rate vs. IOR

Daily Dispersion of Fed Funds Rate and IOR

- 10th Percentile of Fed Funds Rate
- 90th Percentile of Fed Funds Rate
- IOR
Daily FFR and daily effective FFR vs. IOR: a puzzle
Value function (derivation)

\[ J_k(x, \tau) = \mathbb{E} \left\{ \int_0^{\min(\tau, T)} e^{-r z} u_k(z) \, dz + \mathbb{I}_{\{\tau > T\}} e^{-r T} \left( U_k + e^{-r \Delta x} \right) + \mathbb{I}_{\{\tau \leq T\}} e^{-r \tau} \int J_{k-b_{ss}(\tau, \tau)} (x + R_s s(\tau, \tau), \tau, \tau) \mu(ds', \tau, \tau) \right\} \]
Value function (derivation)

\[ J_k(x, \tau) = \mathbb{E} \left\{ \int_0^{\min(\tau, \tau_{\alpha})} e^{-rz} u_k \, dz + \mathbb{I}_{\{\tau_{\alpha} > \tau\}} e^{-r\tau} \left( U_k + e^{-r\Delta x} \right) + \right. \\
\left. \mathbb{I}_{\{\tau_{\alpha} \leq \tau\}} e^{-r\tau_{\alpha}} \int J_{k-b_{ss'}(\tau-\tau_{\alpha})} (x + R_{s's}(\tau - \tau_{\alpha}), \tau - \tau_{\alpha}) \mu(ds', \tau - \tau_{\alpha}) \right\} \]

- \( \tau_{\alpha} \): time until next trading opportunity
- \( b_{ss'}(\tau) \): balance that bank \( s = (k, x) \) lends to bank \( s' = (k', x') \) at time \( T - \tau \)
- \( R_{s's}(\tau) \): repayment negotiated at time \( T - \tau \) (due at \( T + \Delta \))
- \( \mu(\cdot, \tau) \): prob. measure over individual states, \( s' = (k', x') \)
Bargaining

Bank with \( s = (k, x) \) meets bank \( s' = (k', x') \) at \( T - \tau \).

The loan size \( b \) and the repayment \( R \) maximize:

\[
\left[ J_{k-b} (x + R, \tau) - J_k (x, \tau) \right]^{\frac{1}{2}} \left[ J_{k'+b} (x' - R, \tau) - J_{k'} (x', \tau) \right]^{\frac{1}{2}}
\]

s.t. \( b \in \Gamma (k, k') \)

\( R \in \mathbb{R} \)
Value function (derivation)

\[ J_k (x, \tau) = V_k (\tau) + e^{-r(\tau+\Delta)}x \quad \text{where} \]

\[ V_k (\tau) = \mathbb{E} \left\{ \int_0^{\min(\tau_\alpha, \tau)} e^{-rz} u_k \, dz + \mathbb{I}_{\{\tau_\alpha > \tau\}} e^{-r\tau} U_k + \mathbb{I}_{\{\tau_\alpha \leq \tau\}} e^{-r\tau_\alpha} \right\} \]

\[ \sum_{k' \in K} n_{k'} (\tau - \tau_\alpha) \left[ V_{\tau - b_{kk'}} (\tau - \tau_\alpha) (\tau - \tau_\alpha) + e^{-r(\tau+\Delta-\tau_\alpha)} R_{k'k} (\tau - \tau_\alpha) \right] \]
Value function (derivation)

\[ J_k (x, \tau) = V_k (\tau) + e^{-r(\tau+\Delta)} x \]

where

\[ V_k (\tau) = \mathbb{E} \left\{ \int_0^{\min(\tau_\alpha, \tau)} e^{-rz} u_k dz + \mathbb{I}_{\{\tau_\alpha > \tau\}} e^{-r\tau} U_k + \mathbb{I}_{\{\tau_\alpha \leq \tau\}} e^{-r\tau_\alpha} \right\} \]

\[ \sum_{k' \in K} n_{k'} (\tau - \tau_\alpha) \left[ V_{k' - b_{kk'}} (\tau - \tau_\alpha) (\tau - \tau_\alpha) + e^{-r(\tau+\Delta-\tau_\alpha)} R_{k'k} (\tau - \tau_\alpha) \right] \]

\[ b_{kk'} (\tau) \in \arg \max_{b \in \Gamma(k, k')} \left[ V_{k' + b} (\tau) + V_{k - b} (\tau) - V_{k'} (\tau) - V_k (\tau) \right] \]
Value function (derivation)

\[ J_k(x, \tau) = V_k(\tau) + e^{-r(\tau+\Delta)}x \quad \text{where} \]

\[ V_k(\tau) = \mathbb{E} \left\{ \int_0^{\min(\tau_\alpha, \tau)} e^{-rz} u_k \, dz + \mathbb{I}_{\{\tau_\alpha > \tau\}} e^{-r\tau} U_k + \mathbb{I}_{\{\tau_\alpha \leq \tau\}} e^{-r\tau_\alpha} \right\} \]

\[ \sum_{k' \in \mathcal{K}} n_{k'}(\tau - \tau_\alpha) \left[ V_{k-b_{kk'}}(\tau - \tau_\alpha) (\tau - \tau_\alpha) + e^{-r(\tau+\Delta-\tau_\alpha)} R_{k'k}(\tau - \tau_\alpha) \right] \}

\[ b_{kk'}(\tau) \in \arg \max_{b \in \Gamma(k, k')} [V_{k'+b}(\tau) + V_{k-b}(\tau) - V_{k'}(\tau) - V_k(\tau)] \]

\[ e^{-r(\tau+\Delta)} R_{k'k}(\tau) = \frac{1}{2} \left[ V_{k'+b_{kk'}}(\tau)(\tau) - V_{k'}(\tau) \right] + \frac{1}{2} \left[ V_k(\tau) - V_{k-b_{kk'}}(\tau)(\tau) \right] \]
Special case with $\mathbb{K} = \{0, 1, 2\}$

- Bank with $i = 2$ is a lender, bank with $j = 0$, a borrower
- $\theta \in [0, 1]$ : bargaining power of the borrower
- Only potentially profitable trade is between $i = 0$ and $j = 2$
- $S(\tau) \equiv 2V_1(\tau) - V_2(\tau) - V_0(\tau)$
- Conjecture $S(\tau) > 0$ for all $\tau \in [0, T]$ (to be verified later)
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- Assumption: $2u_1 - u_2 - u_0 \geq 0$ and $2U_1 - U_2 - U_0 > 0$

Given $\{n_k(T)\}$, the distribution of balances follows:

$$\dot{n}_0(\tau) = \alpha n_2(\tau) n_0(\tau)$$
$$\dot{n}_2(\tau) = \alpha n_2(\tau) n_0(\tau)$$
Time-path for the distribution of balances

\[ n_2(\tau) = n_2(T) - [n_0(T) - n_0(\tau)] \]

\[ n_1(\tau) = 1 - n_0(\tau) - n_2(\tau) \]

\[ n_0(\tau) = \frac{[n_2(T) - n_0(T)] n_0(T)}{n_2(T) e^{\alpha[n_2(T) - n_0(T)](T-\tau)} - n_0(T)} \]
The repayment $R$ solves:

$$\max_R \left[ V_1(\tau) - V_0(\tau) - e^{-r(\tau+\Delta)} R \right]^{\theta} \left[ V_1(\tau) - V_2(\tau) + e^{-r(\tau+\Delta)} R \right]^{1-\theta}$$

$$\Rightarrow$$

$$e^{-r(\tau+\Delta)} R(\tau) = \theta [V_2(\tau) - V_1(\tau)] + (1 - \theta) [V_1(\tau) - V_0(\tau)]$$
Value function

\[ rV_0(\tau) + \dot{V}_0(\tau) = u_0 + \alpha n_2(\tau) \theta S(\tau) \]

\[ rV_1(\tau) + \dot{V}_1(\tau) = u_1 \]

\[ rV_2(\tau) + \dot{V}_2(\tau) = u_2 + \alpha n_0(\tau) (1 - \theta) S(\tau) \]

\[ V_i(0) = U_i \text{ for } i = 0, 1, 2 \]
Value function

\[ rV_0 (\tau) + \dot{V}_0 (\tau) = u_0 + \alpha n_2 (\tau) \theta S (\tau) \]

\[ rV_1 (\tau) + \dot{V}_1 (\tau) = u_1 \]

\[ rV_2 (\tau) + \dot{V}_2 (\tau) = u_2 + \alpha n_0 (\tau) (1 - \theta) S (\tau) \]

\[ V_i (0) = U_i \text{ for } i = 0, 1, 2 \]

\[ \Rightarrow \]

\[ \dot{S} (\tau) + \delta (\tau) S (\tau) = 2u_1 - u_2 - u_0 \]

\[ \delta (\tau) \equiv \{ r + \alpha [\theta n_2 (\tau) + (1 - \theta) n_0 (\tau)] \} \]
Surplus

\[ S(\tau) = \left( \int_{0}^{\tau} e^{-[\bar{\delta}(\tau) - \bar{\delta}(z)]} dz \right) \bar{u} + e^{-\bar{\delta}(\tau)} S(0) \]

\[ \bar{u} \equiv 2u_1 - u_2 - u_0 \]

\[ S(0) = 2U_1 - U_2 - U_0 \]

\[ \bar{\delta}(\tau) \equiv \int_{0}^{\tau} \delta(x) dx \]

\[ \delta(\tau) \equiv \{ r + \alpha [\theta n_2(\tau) + (1 - \theta) n_0(\tau)] \} \]
Fed funds rate

\[ R(\tau) = e^{\rho(\tau+\Delta)} \times 1 \]
Fed funds rate

\[ R(\tau) = e^{\rho(\tau+\Delta)} \times 1 \]

\[ \rho(\tau) = \frac{\ln R(\tau)}{\tau + \Delta} \]

\[ = r + \frac{\ln [V_2(\tau) - V_1(\tau) + (1 - \theta) S(\tau)]}{\tau + \Delta} \]
Intuition for efficiency result

\[ rV_0(\tau) + \dot{V}_0(\tau) = u_0 + \alpha n_2(\tau) \theta S(\tau) \]

\[ rV_1(\tau) + \dot{V}_1(\tau) = u_1 \]

\[ rV_2(\tau) + \dot{V}_2(\tau) = u_2 + \alpha n_0(\tau) (1 - \theta) S(\tau) \]
Intuition for efficiency result

\[ rV_0 (\tau) + \dot{V}_0 (\tau) = u_0 + \alpha n_2 (\tau) \theta S (\tau) \]
\[ r\lambda_0 (\tau) + \dot{\lambda}_0 (\tau) = u_0 + \alpha n_2 (\tau) S^* (\tau) \]
\[ rV_1 (\tau) + \dot{V}_1 (\tau) = u_1 \]
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\[ r\lambda_2(\tau) + \dot{\lambda}_2(\tau) = u_2 + \alpha n_0(\tau) S^*(\tau) \]

\[ S(\tau) = \bar{u} \int_0^\tau e^{-[\bar{\delta}(\tau) - \bar{\delta}(z)]} dz + e^{-\bar{\delta}(\tau)} S(0) \]
\[ S^*(\tau) = \bar{u} \int_0^\tau e^{-[\bar{\delta}^*(\tau) - \bar{\delta}^*(z)]} dz + e^{-\bar{\delta}^*(\tau)} S(0) \]
Intuition for efficiency result

\[
\begin{align*}
    rV_0(\tau) + \dot{V}_0(\tau) &= u_0 + \alpha n_2(\tau) \theta S(\tau) \\
    r\lambda_0(\tau) + \dot{\lambda}_0(\tau) &= u_0 + \alpha n_2(\tau) S^*(\tau) \\
    rV_1(\tau) + \dot{V}_1(\tau) &= u_1 \\
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    rV_2(\tau) + \dot{V}_2(\tau) &= u_2 + \alpha n_0(\tau) (1 - \theta) S(\tau) \\
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\end{align*}
\]

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\end{align*}
\]

\[
\bar{\delta}^*(\tau) - \bar{\delta}(\tau) = \alpha \int_0^\tau [(1 - \theta) n_2(z) + \theta n_0(z)] dz \geq 0
\]
Intuition for efficiency result

- **Equilibrium:**

  Gain from trade as perceived by borrower: \( \theta S(\tau) \)

  Gain from trade as perceived by lender: \( (1 - \theta) S(\tau) \)

- **Planner:**

  Each of their marginal contributions equals \( S^*(\tau) \)

- \( \delta^*(\tau) \geq \delta(\tau) \) for all \( \tau \in [0, T] \), with “=” only for \( \tau = 0 \)

  \( \Rightarrow \) The planner “discounts” more heavily than the equilibrium

  \( \Rightarrow S^*(\tau) < S(\tau) \) for all \( \tau \in (0, 1] \)

  \( \Rightarrow \) Social value of loan < joint private value of loan
Intuition for efficiency result

- **Equilibrium:**
  
  Gain from trade as perceived by borrower: \( \theta S(\tau) \)
  
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- $\delta^*(\tau) \geq \delta(\tau)$ for all $\tau \in [0, T]$, with “=” only for $\tau = 0$

  ⇒ The planner “discounts” more heavily than the equilibrium
  
  ⇒ $S^*(\tau) < S(\tau)$ for all $\tau \in (0, 1]$

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Intuition for efficiency result

- Planner internalizes that searching borrowers and lenders make it easier for other lenders and borrowers to find partners.

- These “liquidity provision services” to others receive no compensation in the equilibrium, so individual agents ignore them when calculating their equilibrium payoffs.

- The equilibrium payoff to lenders may be too high or too low relative to their shadow price in the planner’s problem:

  E.g., too high if \((1 - \theta) S(\tau) > S^*(\tau)\)
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Frictionless limit

Proposition

Let $Q \equiv \sum_{k=1}^{K} k n_k (T) = 1 + n_2 (T) - n_0 (T)$.

For $\tau \in [0, T]$,

$$\rho^\infty (\tau) = \begin{cases} 
  r + \frac{\ln \left(1 - e^{-r \tau} \frac{u_1 - u_0}{r} + e^{-r \tau} (U_1 - U_0)\right)}{\tau + \Delta} & \text{if } Q < 1 \\
  r + \frac{\ln \left(1 - e^{-r \tau} \frac{u_1 - u_0 - \theta \bar{u}}{r} + e^{-r \tau} (U_1 - U_0 - \theta S(0))\right)}{\tau + \Delta} & \text{if } Q = 1 \\
  r + \frac{\ln \left(1 - e^{-r \tau} \frac{u_2 - u_1}{r} + e^{-r \tau} (U_2 - U_1)\right)}{\tau + \Delta} & \text{if } 1 < Q.
\end{cases}$$
IOR Policy: intuition from the analytical example

\[ n_2(T) = n_0(T) \]

\[ \theta i^e_f + (1 - \theta) \left( i^e_f + i^c_f + P^c \right) \]

\[ \rho_f(T - t) \]

\[ \rho_f^\infty \]

\[ T \]

\[ t \]
IOR Policy: intuition from the analytical example

\[ i^r_f + i^c_f + P^c \]

\[ \theta i^r_f + (1-\theta)(i^r_f + i^c_f + P^c) \]

\[ n_2(T) < n_0(T) \]

\[ \rho_f^\infty \]

\[ \rho_f(T-t) \]

\[ T \]

\[ t \]
IOR Policy: intuition from the analytical example

\[ n_0(T) < n_2(T) \]

\[ \theta i_f^e + (1-\theta)(i_f^r + i_f^c + P^c) \]

\[ \rho_f(T-t) \]

\[ \rho_f^\infty \]
IOR Policy: intuition from the analytical example

\[ n_0(T) = n_2(T) \]

\[ n_2(T) < n_0(T) \]

\[ n_0(T) < n_2(T) \]
Small-scale simulations: $\mathbb{K} = \{0, 1, 2\}$

$k = 1$

Two scenarios

<table>
<thead>
<tr>
<th>$n^H_0(T), n^L_2(T)$</th>
<th>$n^L_0(T), n^H_2(T)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>{0.6, 0.3}</td>
<td>{0.3, 0.6}</td>
</tr>
</tbody>
</table>

Experiments

<table>
<thead>
<tr>
<th>Bargaining Power ($\theta$)</th>
<th>Discount Rate ($i^w_f$)</th>
<th>Contact Rate ($\alpha$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>$\frac{.0050}{360}$</td>
<td>25</td>
</tr>
<tr>
<td>0.5</td>
<td>$\frac{.0075}{360}$</td>
<td>50</td>
</tr>
<tr>
<td>0.9</td>
<td>$\frac{.0100}{360}$</td>
<td>100</td>
</tr>
</tbody>
</table>
Discount-Window lending rate