Liquidity in Asset Markets with Search Frictions

Ricardo Lagos
New York University

Guillaume Rocheteau
University of California–Irvine
Motivation

- Trade in asset markets is often distinctively non-Walrasian

- Over-the-counter markets
  - completely decentralized, no formal organization
  - trade is bilateral, prices and quantities negotiated

- Many assets are traded in over-the-counter markets
  - Real assets
  - Financial assets
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Trade in asset markets is often distinctively *non-Walrasian*. Over-the-counter markets are:

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Many assets are traded in over-the-counter markets, including:

- Real assets
  - Cars
  - Real estate
- Financial assets
  - Some stocks
  - Currencies
  - Derivatives
  - Corporate bonds
  - Government bonds
  - Federal funds
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The broad question

How do over-the-counter frictions in the trading process affect the performance of asset markets?
Why do we care?

1. Volume traded in over-the-counter markets is large

2. Some key markets have over-the-counter structure, e.g., federal funds market (Ashcraft and Duffie, 2007)
Why do we care?

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Related work, and what we do

- Duffie, Gârleanu and Pedersen (2005)
  
  Asset holdings in \(\{0, 1\}\) and limited heterogeneity

- We introduce:
  - Unrestricted choices of asset holdings
  - Free-entry of dealers
  - More general forms of heterogeneity and preferences

- The model remains tractable...
  - Full characterization of dynamic and steady-state equilibria
  - Analytical distribution of asset holdings / trade sizes / prices

- The class of models is now ready for quantitative analysis
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What we find

- \( \{0, 1\} \) restrictions \( \Rightarrow \) existing theories neglect a critical feature of illiquid markets:
  - Agents can mitigate trading frictions by choosing asset holdings in order to reduce their trading needs.

This mechanism:

- Shapes the distribution of asset holdings.
- Is a key determinant of the measures “market liquidity”: 
  - Trade volume
  - Transaction costs (bid-ask spreads)
  - Execution delays
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Population and Technology

- Continuous time, infinite horizon
- Two types of infinitely-lived agents
  - unit measure of dealers
  - unit measure of investors
- An asset (e.g., a tree)
  - perfectly divisible, in fixed supply $A \in \mathbb{R}_+$
  - yields a *nontradable* dividend flow to its owner (e.g., fruit)
- A *numéraire* good
  - consumed and produced by all agents (linear disutility)
Preferences

- Investors’ instantaneous utility: $u_i(a) + c$
  - $a \in \mathbb{R}_+$ is the dividend flow generated by $a$ units of asset
  - $c \in \mathbb{R}$ is the net consumption of numeraire good
  - $i \in \mathcal{X} = \{1, \ldots, I\}$ indexes an idiosyncratic preference shock

- Idiosyncratic preference shocks at Poisson rate $\delta$

- Probability of preference type $i$ is $\pi_i$

- Dealers’ instantaneous utility function: $c$

- All agents discount at rate $r$
Trading arrangement

- **Dealers**
  - have continuous access to a competitive interdealer market
  - do not hold asset positions

- **Investors**
  - contact dealers at random with Poisson rate $\alpha$
  - may hold any nonnegative asset position

- When a dealer and an investor make contact, they trade
  - bilaterally
  - with terms of trade determined by Nash bargaining
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Example: market for corporate bonds
($3 trillion in 1998)

- No formal organization, completely decentralized
- Dealers and investors trade bilaterally
- To trade, a counterparty contacted over the telephone
- Contacted dealer quotes a price
- Initial quote is indicative: negotiations ensue regarding the price and quantity to be traded
- Dealers typically have access to brokered networks where they manage their positions
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- Competitive interdealer market
Investors

\[ V_i(a, t) = \mathbb{E}_i \left[ \int_t^T e^{-r(s-t)} u_k(s)(a) ds \right. \\
\left. + e^{-r(T-t)} \left\{ V_k(T)[a_k(T)(T), T] \right. \\
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- \( p(t) \): competitive price of the asset at time \( t \)
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Dealers

\[ W(t) = \mathbb{E} \left\{ e^{-r(T-t)} \left[ \int \phi_i(a, T) dH_T + W(T) \right] \right\} \]

- \( H_t(A, I) \) is the time-\( t \) measure of investors with asset holding \( a \) in the set \( A \subseteq \mathbb{R}_+ \) and preference type \( i \) in the set \( I \subseteq \mathcal{X} \)
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Nash bargaining

Dealer’s bargaining power: $\eta \in [0, 1]$

\[
[a_i(t), \phi_i(a, t)] = \arg \max_{(a', \phi)} [V_i(a', t) - p(t)(a' - a) - \phi - V_i(a, t)]^{1-\eta} \phi^\eta
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**Lemma**

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a_i(t) = \arg \max_{a' \geq 0} [V_i(a', t) - p(t)a']
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\phi_i(a, t) = \eta \{V_i[a_i(t), t] - V_i(a, t) - p(t) [a_i(t) - a]\}
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- \( a_i(t) \) maximizes the total gains from trade
- \( \phi_i(a, t) \) splits the gains from trade according to \( \eta \)
Nash bargaining

Dealer’s bargaining power: $\eta \in [0, 1]$

$$[a_i(t), \phi_i(a, t)] = \arg \max_{a', \phi} [V_i(a', t) - p(t) (a' - a) - \phi - V_i(a, t)]^{1-\eta} \phi^\eta$$

**Lemma**

$$a_i(t) = \arg \max_{a' \geq 0} [V_i(a', t) - p(t)a']$$

$$\phi_i(a, t) = \eta \{ V_i[a_i(t), t] - V_i(a, t) - p(t) [a_i(t) - a] \}$$

- $a_i(t)$ maximizes the total gains from trade
- $\phi_i(a, t)$ splits the gains from trade according to $\eta$
Lemma

An investor with preference type $i$ and asset holdings $a$ who readjusts his asset position at time $t$ solves

$$\max_{a' \geq 0} \left[ \bar{u}_i(a') - q(t)a' \right]$$

where

$$\bar{u}_i(a) = \frac{(r + \kappa) u_i(a) + \delta \sum_j \pi_j u_j(a)}{r + \kappa + \delta}$$

$$q(t) = (r + \kappa) \left[ p(t) - \kappa \int_0^\infty e^{-(r+\kappa)s} p(t + s) ds \right]$$

$$\kappa \equiv \alpha (1 - \eta)$$
Lemma

\[ rp(t) - \dot{p}(t) = q(t) - \frac{\dot{q}(t)}{r + \kappa} \]

If \( \lim_{t \to \infty} e^{-rt} p(t) = 0 \), then the price of the asset is

\[ p(t) = \int_t^\infty e^{-r(s-t)} \left[ q(s) - \frac{\dot{q}(s)}{r + \kappa} \right] ds \]
Intermediation fees

\[ \phi_i(a, t) = \frac{\eta \left\{ \bar{\mu}_i [a_i(t)] - \bar{\mu}_i (a) - q(t) [a_i(t) - a] \right\}}{r + \kappa} \]
Distribution of investors over individual states

- $n_i(t)$: time-$t$ density of investors with preference type $i$

\[
n_i(t) = e^{-\delta t} n_i(0) + (1 - e^{-\delta t}) \pi_i
\]

- State of an investor: $(a, i) \in \mathbb{R}_+ \times \mathcal{X}$

- $n_{ji}(\tau, t)$: time-$t$ density of investors of type $i$ who are holding asset position $a_j(t - \tau)$ (i.e., those investors whose last trade was at time $t - \tau$ when their preference type was $j$, and who have preference type $i$ at time $t$)

\[
n_{ji}(\tau, t) = e^{-\alpha \tau} \left[ (1 - e^{-\delta \tau}) \pi_i + e^{-\delta \tau} \mathbb{1}_{\{i=j\}} \right] n_j(t - \tau)
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Distribution of investors

Lemma

The measure of investors across individual states at time $t$ is

$$H_t(A, I) = \sum_{i \in I} \sum_{j=1}^{l} \left[ n_{ji}^0(A, t) + \int_0^t \mathbb{I}_{\{a_j(t-\tau) \in A\}} n_{ji}(\tau, t) d\tau \right]$$

for all $(A, I) \subseteq \Sigma$, where

$$n_{ji}^0(A, t) = e^{-\alpha t} \left[ (1 - e^{-\delta t}) \pi_i + e^{-\delta t} \mathbb{I}_{\{i=j\}} \right] H_0(A, \{j\})$$
**Definition**

Given an initial condition $H_0$, an equilibrium is a time-path $\langle \{a_i(t)\}, q(t), p(t), \{\phi_i(a, t)\}, H_t \rangle$ that satisfies:

$$
\bar{u}_i'[a_i(t)] \leq q(t), \quad "=" \quad \text{if } a_i(t) > 0
$$

$$
\sum_{i=1}^{l} n_i(t)a_i(t) = A
$$

$$
p(t) = \int_{t}^{\infty} e^{-r(s-t)} \left[ q(s) - \frac{\dot{q}(s)}{r + \kappa} \right] ds
$$

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\phi_i(a, t) = \frac{\eta \left\{ \bar{u}_i[a_i(t)] - \bar{u}_i(a) - q(t) [a_i(t) - a] \right\}}{r + \kappa}
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$$
Existence and the frictionless limit

Proposition

There exists a unique equilibrium
Existence and the frictionless limit

**Proposition**

There exists a unique equilibrium

**Proposition**

As $\alpha \to \infty$, $\langle \{a_i(t)\}, q(t), p(t), \{\phi_i(a, t)\}, H_t \rangle$ converges to the Walrasian allocation, $\langle \{a^*_i(t)\}, q^*(t), p^*(t), \{\phi^*_i(a, t)\}, H^*_t \rangle$

\[ u'_i [a^*_i(t)] = q^*(t) \]  \[\sum_{i=1}^{l} n_i(t) u'_i^{-1} [q^*(t)] = A \]

$\phi^*_i(a, t) = 0$ for all $a$, $i$ and $t$

$H^*_t (A, I) = \sum_{i \in I} \mathbb{I} \{a^*_i(t) \in A\} n_i(t)$
Efficiency

Social planner:

- maximizes the sum of agents’ utilities subject to trading frictions
- can allocate $\alpha A$ assets among $\alpha$ investors distributed according to $f_i(t)$

Proposition

The equilibrium is efficient if and only if $\eta = 0$

- A bargaining inefficiency:
  Investors anticipate that they will have to pay fees for rebalancing their portfolios in the future
  $\Rightarrow$ They choose asset positions that are too compressed
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- A bargaining inefficiency:
  Investors anticipate that they will have to pay fees for rebalancing their portfolios in the future
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Proposition

For any $H_0$, the equilibrium allocations and prices, $\langle \{a_i(t)\}, q(t), p(t), \{\phi_i(a, t)\}, H_t \rangle$, converge to the unique steady-state allocations and prices $\langle \{a_i\}, q, p, \{\phi_i(a)\}, n_{ij} \rangle$

\[
\bar{u}_i'(a_i) \leq q \quad "=" \quad \text{if } a_i > 0
\]

\[
\sum_{i=1}^{l} \pi_ia_i = A
\]

\[
p = \frac{q}{r}
\]

\[
\phi_i(a) = \frac{\eta [\bar{u}_i(a) - \bar{u}_i(a) - q(a_i - a)]}{r + \kappa}
\]

\[
n_{ij} = \frac{\delta \pi_i \pi_j + \mathbb{1}_{\{i=j\}} \alpha \pi_i}{\alpha + \delta}
\]
Numerical example

**Distribution of preference shocks**

**Optimal Portfolio**

**Stationary distribution**

**Intermediation fees**
Dimensions of liquidity

1. Trade volume

\[ V = \frac{\alpha}{2} \sum_{i,j=1}^{I} n_{ij} |a_j - a_i| \]

2. Transaction costs

\[ \phi_i(a) = \eta \left[ \bar{u}_i(a_i) - \bar{u}_i(a) - q(a_i - a) \right] \frac{1}{r + \kappa} \]

3. Dealer revenue

\[ \Phi = \sum_{i,j=1}^{I} n_{ji} \phi_{ji} \]

where \( \phi_{ji} \equiv \phi_i(a_j) \)
Dimensions of liquidity

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   \mathcal{V} = \frac{\alpha}{2} \sum_{i,j=1}^{l} n_{ij} |a_j - a_i|
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Dimensions of liquidity: with and without restrictions on asset holdings

DGP

\[ V = \alpha \delta \min\{A(1-\pi),(1-A)\pi\} \]

This paper
Dimensions of liquidity: with and without restrictions on asset holdings

\[ V = \alpha \frac{\delta \min\{A(1-\pi),(1-A)\pi\}}{\alpha+\delta} \]

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for \( a_j > a_i \),

\[ p_{ij} = p + \frac{\phi_{ij}}{a_j - a_i} > p, \]

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\sum_{i,j=1}^l n_{ji} \frac{\eta[\bar{u}_i(a_i) - \bar{u}_j(a_j)]}{r+\kappa}
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Proposition

Let \( u_i(a) = \epsilon_i a^{1-\sigma} / (1 - \sigma) \) with \( \sigma > 0 \). An increase in \( \kappa \) causes the equilibrium distribution of asset holdings to become riskier, in the second-order stochastic sense.

Corollary

Let \( u_i(a) = \epsilon_i a^{1-\sigma} / (1 - \sigma) \) with \( \sigma > 0 \), and \( I = 2 \). Trade volume, \( V \), increases with \( \kappa \).

Proposition

Let \( u_i(a) = \epsilon_i a^{1-\sigma} / (1 - \sigma) \) with \( \sigma \geq 1 \). For any pair \( (\kappa, \kappa') \) such that \( \kappa' > \kappa \), the distribution of trade sizes associated with \( \kappa' \) dominates the one associated with \( \kappa \) in the first-order stochastic sense.
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Transaction costs

- Transaction costs increase with the size of the trade:

**Lemma**

Consider an investor who holds asset position \( a \geq 0 \) and wishes to trade \( a_i - a > 0 \). (i) \( \partial \phi_i(a) / \partial a \) has the same sign as \( a - a_i \) and (ii) \( \frac{\partial}{\partial a} \left[ \frac{\phi_i(a)}{a_i - a} \right] < 0 \)

- In equilibrium, transaction costs are non-monotonic in \( \kappa \):

**Proposition**

For each \( (i, j) \in \mathbb{X}^2 \), there exists \( \bar{r} > 0 \), such that for all \( r < \bar{r} \) and \( \eta \in (0, 1) \), \( \phi_{ji} \) is non-monotonic in \( \kappa \)

- Intuition:
  - Competition effect
  - Reallocation effect
Transaction costs

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Consider an investor who holds asset position \( a \geq 0 \) and wishes to trade \( a_i - a > 0 \). (i) \( \partial \phi_i(a) / \partial a \) has the same sign as \( a - a_i \) and (ii) \( \frac{\partial}{\partial a} \left[ \frac{\phi_i(a)}{a_i - a} \right] < 0 \)

- In equilibrium, transaction costs are non-monotonic in \( \kappa \):

**Proposition**

For each \( (i, j) \in \mathcal{X}^2 \), there exists \( \bar{r} > 0 \), such that for all \( r < \bar{r} \) and \( \eta \in (0, 1) \), \( \phi_{ji} \) is non-monotonic in \( \kappa \)

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  - Competition effect
  - Reallocation effect
Transaction costs

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There exists $\bar{r} > 0$, such that for all $r < \bar{r}$ and $\eta \in (0, 1)$, the dealers’ expected revenue, $\Phi$, is non-monotonic in $\kappa$.

Intuition: Dealers’ revenue is maximum in markets that are neither too liquid nor too illiquid.
**Corollary**

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- **Intuition:**
  - In very “liquid” markets (\( \alpha \to \infty \)) : investors have good outside options to immediate trade
  - In very “illiquid” markets (\( \alpha \to 0 \)) : \( a_i \to A \) for all \( i \)
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Dealer revenue

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Investors with a higher contact rate $\alpha$ tend to trade larger quantities and at a lower cost per unit.
Endogenous trading delays

1. Free-entry of dealers

2. Equilibrium
   1. Existence
   2. Multiplicity / uniqueness

3. Allocative efficiency
Why free entry?

“In competitive dealer markets, dealer spreads ultimately depend on the costs that dealers incur in running their business. The free entry and exit of dealers ensures that spreads will adjust so that dealers just earn normal profits. When spreads are too high, their competition for order flow will cause spreads to fall, and as spreads fall, so do expected profits.”

Harris (Trading and Exchanges, 2003, p. 298)
Model with free entry

- A large measure of dealers choose whether to participate (Utility of not participating is normalized to 0)
- Flow cost of participating: $\gamma > 0$
- Investors contact dealers with Poisson rate $\alpha(v)$
  - $\alpha(0) = 0$, $\alpha'(v) > 0$, $\alpha''(v) < 0$
- Steady-state value of a dealer:
  \[
  rW = -\gamma + \frac{\alpha(v)}{v} \sum_{i,j} n_{ji} \phi_{ji}
  \]
- Free-entry implies
  \[
  \frac{\alpha(v)}{v} \Phi = \gamma
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**Proposition**

Assume $\eta > 0$. There exists a steady-state equilibrium with free entry of dealers, and it has $\nu > 0$. 
Existence

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Unique equilibrium

- Linear preferences, $u_i(a) = \varepsilon_i a$

- “Monopolist” dealers, $\eta = 1$

- Intuition:
  - no asset reallocation effects $\Rightarrow$ unique equilibrium
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Multiple equilibria

Proposition

Assume $\eta \in (0, 1)$ and $\alpha(\nu) = \nu^\theta$, with $\theta \in (0, 1]$.

There is $\bar{r} > 0$, $\bar{\theta} \in (0, 1)$, $\bar{\gamma} > 0$ and $\underline{\gamma} \in (0, \bar{\gamma})$ such that

for all $(r, \theta) \in [0, \bar{r}) \times (\bar{\theta}, 1]$:

- there is no active steady-state equilibrium if $\gamma > \bar{\gamma}$
- there are multiple active steady-state equilibria if $\gamma \in (\underline{\gamma}, \bar{\gamma})$

Intuition:

- if $\alpha(\nu)/\nu$ is not too elastic, the liquidity externality that operates through the reallocation effect leads to a positive feedback to dealers' profit.
Multiple equilibria

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Multiple equilibria

\[ \frac{\alpha(\nu)}{\nu} \sum n_{ji} \phi_{ji} - \gamma \]
Self-fulfilling liquidity

1. If investors believe that the market is liquid (i.e., short trading delays, low transaction costs...)

2. Investors with high (low) marginal utility demand large (small) quantities of the asset

3. Transaction sizes are large on average

4. A large measure of dealers find it profitable to enter

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Regulatory changes
Liquidity and dealers’ market power
Technological innovations in trading
Investors gain access to ECNs
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Investors gain access to ECNs

- Expected profit (for $\beta=0$)
- Equilibrium measure of dealers
- Average execution delay
- Optimal portfolios
Efficiency

- Social welfare at the steady state \( r \approx 0 \)

**Proposition**

*Equilibrium with free-entry is inefficient*

- Asset allocation is efficient iff \( \eta = 0 \)
- Search externalities are internalized iff \( \frac{\nu \alpha'(\nu)}{\alpha(\nu)} = \eta \)

\( \Rightarrow \)

- Impossible to simultaneously
  - eliminate the "hold-up problem"
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Summary of results

\{0, 1\} restrictions \Rightarrow \text{existing theories neglect a critical feature of illiquid markets}

1. Reductions in trading frictions increase dispersion of asset holdings

2. \Rightarrow \text{larger transaction sizes} \Rightarrow \text{larger trade volume}

3. \Rightarrow \text{dealers’ incentives to make markets are non-monotonic (because spreads increase with trade size)}

4. \Rightarrow \text{Multiple equilibria:}
   
   \text{scarce liquidity arises naturally as a self-fulfilling phenomenon}
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4. ⇒ Multiple equilibria: scarce liquidity arises naturally as a self-fulfilling phenomenon
Weston (2000): regulatory reforms in NASDAQ have reduced bid-ask spreads but increased trading volume and prompted a net entry of dealers

- Consistent with a decrease in $\eta$

Stoll (2006, p. 163-162): electronic trading in stock markets has led to tighter bid-ask spreads, higher asset turnover, larger volume and larger aggregate revenues of securities firms

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(reduction in the market power of dealers)

- May 24, 1994: Charges of collusion (ignoring odd eights) leveled by Christie and Schultz in a Vanderbilt University press release
- May 24, 1994: NASD officials and members meet and spreads begin to fall dramatically.
- NASD adopts the “Manning Rules”: A dealer cannot trade ahead of his customers’ limit orders
  - June 24, 1994: Manning I (for orders dealer gets directly)
  - May 22, 1995: Manning II (for orders dealer gets through other dealers or brokers)

Example: dealer quotes 100 bid, 102 ask. If a customer places a limit order to buy at 101, the dealer could ignore it, and even continue to buy for his own account at 100. The customer was only entitled to an execution when the dealer’s ask price dropped to 101.
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Regulatory changes in NASDAQ 1996–2001
(public access to interdealer markets through ECNs)

- 1996–1997: SEC’s rule on Order Execution Obligations
  - Display Rule: Requires dealer to display customer limit orders priced better than his own quote
  - Quote Rule: Requires dealer to make publicly available any superior prices he privately quotes through ECNs that where previously used exclusively by marketmakers and large institutions

- 2001: SEC’s rule on Disclosure of Order Execution and Routing Practices

These best execution practices have effectively allowed investors to trade with each other directly through the ECNs
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  - Display Rule: Requires dealer to display customer limit orders priced better than his own quote
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These best execution practices have effectively allowed investors to trade with each other directly through the ECNs.
Regulatory changes in NASDAQ 1996–2001
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What does our utility function stand for?

1. **Literal interpretation:** the utility of the services provided by a real asset (car, house, land)

2. **Alternative interpretation:** $a$ is a capital and $u_i(a)$ the output (in terms of numeraire good) produced using this capital stock (e.g., Cavalcanti, 2004, ET)
   - $i$ represents an idiosyncratic productivity shock

3. **Reduced form for the various services provided by a financial asset (DGP):**
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Numerical example

\[ u_i(a) = \varepsilon_i a^{\frac{1-\sigma}{1-\sigma}}, \quad \varepsilon_i = \frac{i-1}{i-1} \text{ for } i = 1, \ldots, I \]

\[ \pi_i = \frac{\lambda^{i-1} / (i - 1)!}{\sum_{j=1}^{I} \lambda^{j-1} / (j - 1)!} \text{ for } i = 1, \ldots, I \]

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- Implied turnover rate \( \approx 8 \) (yearly)
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- Average effective volume-weighted spread \(\approx 0.2\) basis points of the asset price