Lending in Last Resort to Governments

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Fiscal fundamentals in 2009: an international comparison

Figure: Ratio of fiscal balance and net government debt to GDP in selected advanced economies (2009). Source: WEO.
Introduction

Fiscal fundamentals in 2009: an international comparison

Figure: Ratio of fiscal balance and net government debt to GDP in selected advanced economies (2009). Source: WEO.
Why has there been a government debt crisis in Europe rather than elsewhere?

One view: the monetary authorities provide less support to government debt in the euro area than elsewhere

- The central bank should act as a “lender of last resort” to sovereigns (De Grauwe, 2011)

The purpose of this paper is to provide a theoretical and quantitative perspective on this view.
Introduction

Model

- Model of closed-economy with government and households

- Government finances utility-yielding expenditures with fiscal receipts and debt
  - debt helps to smooth impact of shocks to fiscal receipts

- Endogenous default: the government can default opportunistically

- There can be self-fulfilling increases in the interest rate that the government has to pay on its debt (Calvo, 1988)
  - loop: higher interest rate → higher default probability → higher interest rate
  - equilibrium determined by sunspot variable
Introduction

- A large lender (lender in last resort) can reduce the region of multiplicity by lending to the government at the low interest rate.

- The model without sunspot is calibrated by reference to the low-frequency dynamics of government debt in advanced economies (1950-2011).

- Sunspots and a LOLR are then introduced: impact?

- Work in progress
Introduction

Literature


Introduction

Structure

1) Model
2) Calibration
3) Impact of sunspots
4) Impact of LOLR
Model: main features

- Discrete infinite time

- Government and representative infinitely-lived household, both optimizing intertemporally

- Concave utility for government expenditure, linear utility for private consumption

- Government can default: output cost

- Self-fulfilling increase in interest rate on government debt triggered by sunspot
Model

- Infinitely-lived representative consumer with utility

\[ U_t = E_t \left\{ \sum_{j \geq t} \beta^{j-t} [c_j + u(g_j)] \right\}, \]

where \( u(g) \) is the utility from public expenditures

- Riskless real interest factor is equal to \( 1/\beta \)

- Consumer’s budget constraint

\[ c_t + b_t = y_t - \delta_t \Delta y - \tau_t + (1 - \delta_t h) R_{t-1} b_{t-1}, \]

where \( b_t \): one-period government bonds; \( y_t \): exogenous income; \( \delta_t \): dummy variable for government default; \( \tau_t \): taxes paid to the government; \( R_{t-1} \): interest factor on government debt; \( h \): haircut
The government maximizes,

\[ U_{gt} = E_t \left\{ \sum_{j \geq t} (\beta_g)^{j-t} [c_j + u(g_j)] \right\} \]

with \( \beta_g \leq \beta \)

Government budget constraint,

\[ g_t + (1 - \delta_t h)R_{t-1}b_{t-1} = b_t + \tau_t \]

Stochastic constraint on tax revenues,

\[ \tau_t \leq x_t \]

where \( x_t \) follows Markov process with transition probabilities \( \mu(x'|x) \)
Model

- Consolidated budget constraint,
  
  \[ c_t + g_t = y_t - \delta_t \Delta y \]

- Objective function of government can be decomposed in two terms,
  
  \[ U_{gt} = E_t \left\{ \sum_{j \geq t} \beta_g^{j-t} y_j \right\} + E_t \left\{ \sum_{j \geq t} \beta_g^{j-t} [u(g_j) - g_j - \delta_j \Delta y] \right\} \]

  \[ V_{gt} \]

  - The first term is exogenous
  
  - The second term is maximized under the budget constraint,
    
    \[ g_t + (1 - \delta_t h) R_{t-1} b_{t-1} \leq b_t + x_t \]

    Depends on \( x_t \) but not on \( y_t \) so that \( y_t \) is irrelevant for the government maximization problem
In the recursive formulation of equilibrium $x_t$ will be part of the state.

Another part of the state is the debt due at the beginning of period $t$,

$$d_t = R_{t-1}b_{t-1}$$

In equilibrium, the government defaults iff debt repayment is larger than a state-contingent threshold $\bar{d}$

Third state variable: sunspot
In period $t$ the government announces how much debt it wishes to issue, $b_t$

Households announce their supply schedules: how much they are ready to buy in function of $R_t$

Given the linear preferences, the households supply indeterminate quantity of funds if

$$R_t = \frac{1}{\beta [1 - h Pr (\delta_{t+1} = 1)]}$$

Hence interest factor must satisfy fixed-point equation

$$R_t = \frac{1}{\beta [1 - h Pr_t (R_t b_t > \bar{d}_{t+1})]}$$

Both sides increase with $R_t \rightarrow$ multiple solutions
The deterministic case

\[ R = \frac{1}{\beta \left(1 - h \cdot 1_{bR > \bar{d}}\right)} \]

**Figure**: Interest rate multiplicity, deterministic case
The costs of sunspots

- If sunspot triggers increase in interest rate, one option for the government is to default and exit region of multiplicity
  - the government is better off defaulting today rather than borrowing at a high interest rate and defaulting tomorrow

- Difference with multiplicity a la Cole-Kehoe: the multiplicity depends on the level of debt

- Another option for the government is to contract spending to exit the multiplicity region
Model

- Limit the attention to the two extreme solutions,

\[
R_{0t} = \min \left\{ R, R = \frac{1}{\beta \left[ 1 - h \Pr_t (R_b > d_{t+1}) \right]} \right\}
\]

\[
R_{1t} = \max \left\{ R, R = \frac{1}{\beta \left[ 1 - h \Pr_t (R_b > d_{t+1}) \right]} \right\}
\]

- Households coordinate or one or the other solution by looking at a sunspot variable \( \sigma_t \) that can take values 0 or 1

- The sunspot variable is i.i.d.: denote by \( \eta \) the probability that \( \sigma_t = 1 \)
Equilibrium interest factor can be written

\[ R_0(x, b) = \min \{ R, \beta R \left[ 1 - h \Pr (Rb > \bar{d}_{\sigma'}(x')) \right] \} = 1 \]
\[ R_1(x, b) = \max \{ R, \beta R \left[ 1 - h \Pr (Rb > \bar{d}_{\sigma'}(x')) \right] \} = 1 \]

where \( \bar{d}_{\sigma'}(x') \) is the threshold for default in the next period.

Multiple equilibria if \( R_1(x, b) > R_0(x, b) \)
Lender of last resort (LOLR)

- LOLR: Large lender set up by the households which operates under the following rules

- The LOLR can lend up to $\ell$ to the government

- The LOLR is treated in the same way as households in a default → same default threshold as before for total debt repayment
The LOLR lends at the low interest rate if $\sigma = 1$ provided that its lending removes the bad equilibrium, i.e.,

$$\max \{ R, \, \beta R \left[ 1 - h \Pr \left( R(b - \ell) + R_0(x, b)\ell > \overline{d}_{\sigma'}(x') \right) \right] = 1 \} = R_0(x, b)$$

If this condition is satisfied the LOLR does not need to lend in equilibrium and $R_1(x, b) = R_0(x, b)$

If $\ell$ is very large there is no multiplicity
Model

Equilibrium

- State at time $t$ is summarized by $(\sigma_t, x_t, d_t)$
- First, nature chooses the sunspot variable $\sigma_t$ and the level of fiscal receipts $x_t$
- Then the government decides whether to default or not, $\delta_t$, the level of public expenditures $g_t$, and the level of new borrowing $b_t$ taking $R_{\sigma_t}(x_t, b_t)$ as given
If the government decides to repay its objective function satisfies the Bellman equation,

\[
V^R_{\sigma}(x, d) = \max_{g, b} \{u(g) - g + \beta g EV_{\sigma'}(x', d')\}
\]

\[
g + d \leq x + b,
\]

\[
d' = R_{\sigma}(x, b) b
\]

with \( V_{\sigma}(x, d) = \max [V^R_{\sigma}(x, d), V^D_{\sigma}(x, d)] \)

If the government defaults

\[
V^D_{\sigma}(x, d) = V^R_{\sigma}(x, (1 - h)d) - \Delta y
\]
Default threshold satisfies

\[
V^D_{\sigma}(x, \overline{d}_\sigma(x)) = V^R_{\sigma}(x, (1 - h)\overline{d}_\sigma(x)) - \Delta y
\]

Government defaults if and only if debt repayment is larger than this threshold,

\[
[\delta_{\sigma}(x, d) = 1] \iff [d > \overline{d}_\sigma(x)]
\]
Model

- Interest factor if no sunspot

\[ R_0(x, b) = \min \{ R, \beta R \left[ 1 - h \Pr (Rb > d_{\sigma'}(x')) \right] = 1 \} \]

- Interest factor if sunspot

\[ R_1(x, b) = R_0(x, b) \]

if

\[ \max \{ R, \beta R \left[ 1 - h \Pr (R(b - \ell) + R_0(x, b)\ell > d_{\sigma'}(x')) \right] = 1 \} = R_0(x, b) \]

otherwise

\[ R_1(x, b) = \max \{ R, \beta R \left[ 1 - h \Pr (Rb > d_{\sigma'}(x')) \right] = 1 \} \]

- Equilibrium: a collection of functions \( V_{\sigma}(\cdot, \cdot), V_{\sigma}^R(\cdot, \cdot), V_{\sigma}^D(\cdot, \cdot), g_{\sigma}(\cdot, \cdot), b_{\sigma}(\cdot, \cdot), R_{\sigma}(\cdot, \cdot), \delta_{\sigma}(\cdot, \cdot) \) and \( d_{\sigma}(\cdot) \) that satisfy conditions above
Utility for government expenditures,

\[ u(g) = g + \alpha \min(g, g^*) \]

where \( \alpha \) is the surplus per unit of government spending and \( g^* \) is the first-best level of government expenditures

**Two fiscal states:** \( x_H > x_L \) with constant probability \( \mu \) of switching from one state to the other
$E(y) = 1$ so all variables should be interpreted as shares of (average) GDP

Table 1. Benchmark calibration

<table>
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<th>$\beta$</th>
<th>$\beta_g$</th>
<th>$\eta$</th>
<th>$g^*$</th>
<th>$\alpha$</th>
<th>$\Delta y$</th>
<th>$h$</th>
<th>$x_H$</th>
<th>$x_L$</th>
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<td>0.97</td>
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<td>0.1</td>
<td>0.37</td>
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This calibration allows the model **without sunspot** to match the low-frequency fiscal dynamics in G7 countries (1950-2011)
Calibration

Identifying debt increase/decrease spells in the data

Figure: US General Government Debt/GDP (% 1950-2012). Source: IMF.
Calibration

- $\beta = \beta_g = 0.97$: average real interest rated 3%

- $x_H = 0.37$ and $x_L = 0.31$: matches average level and standard deviation of government receipts/GDP

- $\mu = 0.04$: matches average length of debt increase/decrease spells (26.5 years in the data, accounting for incomplete spells with max. likelihood estimation)

- $\alpha = 0.3$ and $g^* = 0.32$: matches growth rate of debt/GDP in debt increase/decrease spells and average level of debt (to be fine-tuned)

- Solve by discrete-state-space value function iteration
Calibration

- Two hundred years of simulated debt dynamics

Figure: Illustrative simulated path for debt-to-GDP ratio
Interest rate schedule, high fiscal state

Figure: Variations of $R$ with $b$, $x = x_H$
Calibration

- Interest rate schedule, low fiscal state

**Figure**: Variations of $R$ with $b$, $x = x_L$
What comes next

- Introduce sunspots and look at implications for debt dynamics and welfare
- Conditional on sunspots, introduce LOLR, varying its size
Sunspots: quantitative implications

Impact of sunspot on debt thresholds

Figure: Variations of $\overline{d}_\sigma(x)$ with $\eta$
Sunspots: quantitative implications

Impact of sunspot on default frequency

Figure: Variations of frequency of defaults with \( \eta \)
Sunspots: quantitative implications

Impact of sunspot on average debt level

Figure: Variations of average debt level with \textit{eta}
Sunspots: quantitative implications

Impact of sunspot on welfare

Figure: Variations of welfare loss from sunspots with \( \eta \)
How should we think about the size of the LOLR?

Central bank?

Banking system (financial repression)?
Figure: Coverage of government debt by M0, M2. Selected economies, 2015.
Figure: ESCB assets and government debt of selected euro area countries (2010, bn euros). Source: ECB and eurostats.
Conditional on sunspot ($\eta = 0.05$), impact of LOLR on debt thresholds

**Figure**: Variations of debt default thresholds with LOLR size
Conditional on sunspot ($\eta = 0.05$), impact of LOLR on default frequency

![Graph: Variations of default frequency with LOLR size](image-url)
Conditional on sunspot ($\eta = 0.05$), impact of LOLR on average debt

Figure: Variations of average debt with LOLR size
Sunspots: quantitative implications

Conditional on sunspot ($\eta = 0.05$), impact of LOLR on welfare

Figure: Variations of welfare with LOLR size
Conclusions

- I look at a model in which a LOLR for governments works by construction
  - many problems are abstracted from: observability of sunspot, etc.

- The LOLR works, but it needs to be very large

Work in progress; extensions include

- Debt maturity

- Cost of large-scale LOLR (financial repression)

- Seniority of LOLR