Optimal Employment Contracts with Hidden Search

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Optimal employment contracts backload compensation when workers can engage in non-contractable job search.

Gives rise to increasing wage profiles and decreasing separation hazards within jobs.

In addition, model provides rich wage dynamics easily fitting wage dispersion moments even in absence of worker heterogeneity.

- In contrast to wage dispersion arguments in Hornstein, Krusell, and Violante (2010).
- But, consistent with HKV, calibration implies quite moderate NPV dispersion in offer distribution.
Standard random on-the-job search model.

Hidden search intensity choice.

Employment contracts specify history conditional compensation and response to outside competition.

Renegotiation proof contract responds to outside competition similar to simple outside offer matching.

Framework very flexible and can incorporate rich productive heterogeneity on both worker and firm side.

- Lentz and Roys (2014) study firm provided training in the framework.
Backloading

- Key inefficiency: Part of incentive to search on the job is to extract rents from current match.
- Wages are backloaded to address costly rent seeking behavior within match.
Discrete time. Discount factor, $\beta$. Common to both firms and workers.

Two state model: workers can be either employed or unemployed.

In a given period, a worker meets a single vacancy with probability $\lambda$.

A vacancy is characterized by its permanent productivity $p$.

The distribution of productivity across vacancies is given by the CDF $\Phi(p)$ with support $[0, 1]$. Offer distribution.
Per period utility function, $U(w, s) = u(w) - c(\lambda)$, where $w$ is income.

$u(\cdot)$ increasing and concave.

$c(\lambda)$ increasing and convex. At least twice differentiable.

An unemployed worker receives normalized income flow $b = 0$.

No saving or borrowing.
Think of firms as a collection of independent matches.

The per period profit flow of a productivity \( p \) match is \( \pi = p - w \).

A match ends at exogenous destruction probability \( \delta \) or if the worker quits.

Vacancy creation: The analysis does not need to take a stand on this. Simply view the offer distribution as well as the cost function \( c(\cdot) \) as equilibrium outcomes.
Denote by $\sigma$ the history of the match.

A contract: $(w, \lambda)(\sigma)$.

- Denote by $V^o$ the contract’s continuation utility value conditional on revelation of outside meeting.

A worker chooses whether to reveal an outside meeting. Upon revelation:

- Firm types become common knowledge.
- $V^o$ specifies the contract’s continuation utility value.
- Outside firm can respond to $V^o$.

Special cases:
- Offer matching: $V^o$ equal to outside firm’s willingness to pay.
- No response to outside meetings: Contract not affected by outside meetings.
Limited commitment/Participation constraints:
- Worker can at any point costlessly quit to unemployment.
- Firm can costlessly lay off worker should it want to.

Incentive constraints:
- The worker’s search intensity cannot be contracted upon.
- The contract must specify a search intensity level that coincides with the worker’s optimal choice at that point.
Additional Design Constraints

- Contract must be renegotiation proof.
- No side payments.
  - Efficiency obtained if worker could “purchase” job up front.
  - Bonding or "non-compete clause" combined with firm-to-firm payments could possibly also resolve inefficiencies.

Formulate firm’s employment contract design problem with worker lifetime utility promise as state variable.
Useful object: Denote by $\tilde{V}(p)$ firm $p$’s willingness to pay for a worker.

By definition firm $p$’s match profit value at willingness to pay is zero, $\Pi(\tilde{V}(p)|p) = 0$.

Result in paper: $\tilde{V}(p)$ is monotonically increasing in $p$.
- Useful general result in paper: Wage contract is flat at $\tilde{V}(p)$. Here, worker extracts all match rents and search is jointly efficient.
- Typically, $\tilde{V}(p)$ can be calculated prior to solving for actual optimal contracts.

Define $F(\tilde{V}(p)) = \Phi(p)$. Willingness to pay offer distribution.
Renegotiation proof requirement has sharp implication for resolution of competition between firms:

- Optimal renegotiation proof contract matches outside offers similar to Postel-Vinay and Robin (2002).

- $V^o(\tilde{V}', p) = \min(\tilde{V}(p), \tilde{V}')$: continuation utility conditional on $\tilde{V}'$ meeting.

- A worker reveals any meeting where outside firm has willingness to pay greater than value of current contract.

Result: An employed worker meets an outside firm with willingness to pay greater than value of current contract:

- If current firm is more productive, worker continues with firm and contract increases continuation utility promise to the outside firm’s willingness to pay.

- If current firm is less productive, worker moves to outside firm with a utility promise equal to old firm’s willingness to pay.
The discounted lifetime utility value of a contract with a firm characterized by willingness to pay $\bar{V}$ can be written recursively by,

$$V_t = u(w_t) - c(\lambda_t) + \beta(1 - \delta) \left[ \lambda_t \int_{V_{t+1}}^{\bar{V}} V dF(V) + \lambda_t \bar{V} \hat{F}(\bar{V}) + [\lambda_t F(V_{t+1}) + (1 - \lambda_t)] V_{t+1} \right] + \beta \delta U,$$

where $U$ is the worker’s valuation of unemployment. $V_{t+1}$ is next period’s utility promise in the contract, implied by the $(w_\tau, \lambda_\tau)_{\tau=t}^{\infty}$ path. $\hat{F}(\cdot) = 1 - F(\cdot)$

Implicit assumption: Firms do not offer a worker more than that dictated by Bertrand competition.

Result: Bounded marginal utility and no lower wage bound are sufficient conditions.

Extension: Minimum wages can result in ex ante rent extraction.
By integration by parts,

\[ V_t = u(w_t) - c(\lambda_t) + \beta \left\{ \delta U + (1 - \delta) \left[ \delta_t \int_{V_{t+1}}^{\hat{V}} \hat{F}(V) dV + V_{t+1} \right] \right\}. \]

Unemployed workers have no gains to search. The value of unemployment is,

\[ U = \frac{u(0)}{1 - \beta}. \]
Incentive compatible search

- Mechanism design problem uses first order approach to the incentive compatibility problem. Given the regularity assumptions this is a valid approach.

- The first order condition for the lifetime utility maximizing search choice is,

\[
c' (\lambda_t) = \beta (1 - \delta) \int_{V_{t+1}}^{\bar{V}} \hat{F} (V) \, dV.
\]

- The key determinant of search intensity is then next period’s utility promise in the contract.
The time $t$ match value to a type $p$ firm under some contract $C$ can be written recursively by,

$$\Pi (V_t) = p - w_t + \beta (1 - d) \left\{ s_t \int_{V_{t+1}}^{V(p)} \Pi (V) dF (V) + \left[ 1 - s_t \hat{F} (V_{t+1}) \right] \Pi (V_{t+1}) \right\}$$

$$= p - w_t + \beta (1 - d) \left\{ s_t \int_{V_{t+1}}^{V(p)} \Pi' (V) \hat{F} (V) dV + \Pi (V_{t+1}) \right\},$$

where the last equality comes from integration by parts, and $\Pi (V)$ is defined as the firm match valuation subject to a utility promise of $V$. 
Firm $p$, optimal employment contract design problem:

$$\Pi (V) = \max_{(w,Y,\lambda) \in \Gamma (V)} \left[ p - w + \beta (1 - \delta) \left\{ \lambda \int_{V} \Pi' (V) \hat{F} (V) dV + \Pi (Y) \right\} \right],$$

where feasible choice set is given by,

$$\Gamma (V) = \left\{ (w,Y,\lambda) \in \mathbb{R}^2 \times [0,1] \mid u (w) - c (\lambda) + \beta \{ \delta U + (1 - \delta) \left[ \lambda \int_{Y} \hat{F} (V') dV' + Y \right] \} = V \right\}.$$

$$c' (\lambda) = \beta (1 - \delta) \int_{Y} \hat{F} (V') dV'$$

$$U \leq Y \right\}. $$
Assumption A5: \(c''(\lambda)\) is decreasing in \(\lambda\).

**Proposition 1** Given assumptions A1-A5, there exists for any productivity \(p \in [0, 1]\) a unique profit function \(\Pi(V|p)\). The profit function is differentiable, strictly decreasing and strictly concave over the support \(V \in [U, \bar{V}(p)]\), where the support bounds are given by,

\[
\bar{V}(p) = \frac{u(p) + \beta dU}{1 - \beta (1 - d)}
\]

\[
U = \frac{u(0)}{1 - \beta}.
\]

**Proposition 2** Given assumptions A1-A5, the optimal employment contract of a productivity \(p \in [0, 1]\) firm is characterized as follows: For any pair \((V_0, V_1) \in [U, \bar{V}(p)]\) such that \(V_0 < V_1\) it must be that,

\[
w(V_0) < w(V_1)
\]

\[
s(V_0) < s(V_1)
\]

\[
Y(V_0) > V_0.
\]
Existence, Uniqueness, and Characterization

- Paper provides proof of existence and uniqueness of equilibrium.
  - Key insight: Problem can be written as a contraction.

- Paper provides characterization.
  - Contract is backloaded: \( Y(V) > V, \forall V \in [U, \bar{V}) \).
  - Wages increasing in tenure, \( w(V) \) strictly increasing in \( V \).
  - Search intensity decreasing in tenure, \( \lambda(V) \) strictly decreasing in \( V \).
Proof Outline of Proposition 1

► The mapping,

\[
T (\Pi) (V) = \max_{(w,Y,\lambda) \in \Gamma(V)} \left[ p - w + \beta (1 - d) \left\{ \lambda \int_{Y} \Pi (V) \, dF (V) + [1 - \lambda \hat{F} (Y)] \Pi (Y) \right\} \right],
\]

is a contraction (by Blackwell). Therefore, existence and uniqueness.

► In addition, by a Benveniste and Scheinkman like argument, if \( T (\Pi) (V) \) is concave, then it is also differentiable.
Proof Outline of Proposition 1...

- Characterization proof:
  - Can show that mapping takes a differentiable, weakly decreasing and weakly concave function $\Pi$ and maps it into a strictly decreasing, strictly concave and therefore also differentiable function $T(\Pi) (V)$.
  - By the contraction mapping theorem this means that the fixed point must be strictly decreasing and strictly concave.
  - Differentiability is a byproduct of the concavity and is therefore maintained in the sequence.
  - Assumption A5 is a sufficient condition to prove concavity.
The optimal contract must satisfy (again suppressing dependence on $p$),

\[ \Pi'(V) = \frac{-1}{u'(w(V))} \quad (1) \]

\[ \Pi'(V) - \Pi'(Y(V)) = \Psi(Y(V)) \geq 0 \quad (2) \]

\[ \Psi(Y(V)) = \frac{\mu(Y(V)) \hat{F}(Y(V))}{1 - s(Y(V)) \hat{F}(Y(V))} \geq 0 \quad (3) \]

\[ \mu(Y(V)) = \frac{-\beta (1 - d) \int_{Y(V)}^{\bar{V}} \Pi'(V') \hat{F}(V') dV'}{\hat{c}'' (s(Y(V)))} \geq 0, \quad (4) \]

where $\mu(V)$ is the Lagrange multiplier on the incentive compatibility constraint in the problem.

With concavity of the profit function established, equation (??) gives that wages are increasing in $V$.

Assumption 5 is sufficient for $\Psi(Y)$ to be decreasing in $Y$ with $\Psi(Y) = 0$ for any $Y \leq \bar{V}$.

- With concavity of $\Pi(V)$, this delivers $Y(V) > V$ for all $V < \bar{V}$ and $Y(\bar{V}) = \bar{V}$. 

There exists for any \( p \) a unique contract for any initial utility promise.

Contracts are backloaded;
- Wages are increasing in tenure.
- Separation hazard is decreasing in tenure.

By it being a contraction a simple numerical solution algorithm is to just do value function iteration.
Continuous time version. $\lambda$ offer arrival. $\delta$ lay off rate.

Utility and search cost specifications,

$$u(w) = \frac{1 - \exp(-\alpha w)}{\alpha}$$

$$c(\lambda) = \frac{(c_0 \lambda)^{1+c_1}}{1 + c_1}.$$ 

Firm type CDF, $\Phi(\cdot)$ is a beta distribution with parameters $(\beta_0, \beta_1)$.

Minimum wage $w \geq 0$.

Parameterization,

<table>
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<tr>
<th>$\alpha$</th>
<th>$c_0$</th>
<th>$c_1$</th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
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Example with minimum wage

\[ \tilde{V}(p), \underline{V}(p) \text{ and } \tilde{V}(p) \]

\[ \lambda(\underline{V}(p), p) \text{ and } \lambda(\tilde{V}(p), p) \]
Median firm type contract

\[ \Pi(V) \]

\[ \lambda(V) \]

\[ w(V) \]

\[ \dot{V}(V) \]
Unemployed worker’s contract with median firm

Monthly separation hazard

Monthly earnings

Steady state

- Denote by $u = 1 - e$ the steady state equilibrium unemployment.
- Let $g(V, p)$ be the mass of workers employed at utility promise $V$ with a type $p$ firm, and $G(V, p)$ is the associated CDF.
- Continuous time steady state condition,

$$u \lambda_0 \Phi \left( \min \left( p, p(V) \right) \right) = e \left\{ \hat{F}(V) \int_0^p \int_U^{\min[\bar{V}_0(p'), V]} \lambda(V', p') \, dG(V', p') \ight. \\
+ \delta G(V, p) + \int_{\bar{V}_0^{-1}(V)}^p \hat{V}(V, p') \, g(V, p') \, dp' \right\}$$
Model can fit Hornstein, Krusell, and Violante (2010) preferred wage dispersion moments without including worker heterogeneity in the model.

Three core measures: Mean-min wage ratio, $b / E[w]$ and unemployed job finding rate ($EU$).

Use minimum wage $w$ in calibration to provide both incentives for unemployed workers to search as well as fitting mean-min wage ratio.

utility function CRRA, $u (w) = w^{1-\theta} / (1 - \theta)$ with $\theta = 2$.

Match productivity $f (p) = \max[b, w] + \alpha_1 p$.

Unemployed income normalized at $b = 1$.

Set search cost to be quadratic.

Firm productivity distribution maintained beta distributed with $(\beta_0, \beta_1) = (2, 3)$.

Following Shimer, job destruction generally set at $\delta = 0.24$ (monthly rate of 0.02).
### Calibration scenarios

<table>
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<th>Scenario</th>
<th>$c_0$</th>
<th>$\bar{w}$</th>
<th>$\alpha_1$</th>
<th>$\delta$</th>
<th>$E[w]/\bar{w}$</th>
<th>$b/E[w]$</th>
<th>EU rate</th>
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</tbody>
</table>

- Scenario 1 is HKV preferred moments, which model fits using the shown parameters for $(c_0, \bar{w}, \alpha_1)$.
- Scenario 2 fits to a more reasonable EU rate with average unemployment duration closer to 6 months (Fujita and Moscarini).
- Scenario 3 adjusts job destruction to the lower EU rate to maintain same steady state unemployment rate as in Scenario 1 ($u = 0.044$).
- Scenario 4 and 5 calibrate to greater $b/E[w]$ ratios.
- Scenario 6 calibrates to a much higher mean-min ratio.
- In all calibrations, the model matches the targets perfectly.
Scenario 1 Steady State

Steady state wage density

Steady state match and offer density

Wage conditional separation rate

Steady state average firm wage
Concluding Remarks

- Optimal employment contracts backload wages to reduce worker’s job search.
- Wages increasing and separation hazard decreasing in tenure.
- Introduction of search intensity into framework is helpful to produce empirically reasonable wage variation across firm types.
- Model fits HKV wage dispersion in absence of worker heterogeneity using only minimum wages for ex ante rent extraction.
- Substantial wage variation associated with little NPV dispersion in offer distribution
  - Offer distribution mean-min ratio in consumption equivalents is 1.02.