Asymmetric Information, Incompleteness and Inefficiency in Automobile Dealer Trade Networks

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Abstract

This paper analyzes the incompleteness in automobile dealer trade networks. Inter dealer trade data from Adamson Motors, a medium sized dealer in Rochester Minnesota, shows that the network of dealer relationships is not complete. Further, large trade volumes are generated by relatively few trade partners. A homogenous linking network of retail automobile sales is developed. A symmetric pairwise stable equilibrium is shown to always exist. A link represents a relationship, which lubricates trade between dealers by allowing them to trade whenever one dealer is in need. Asymmetric information about buyer search/inconvenience costs allows the dealer to increase profits by limiting the number of trade partners. This reduction in the number of links raises the local buyers cost of obtaining models not currently in stock and allows the dealer to sell models with higher profit margins. These are the models in stock, that don’t incur transportation costs with sale. This effect diminishes as the number of trading relationships declines. After a certain threshold, raising the cost of obtaining distant models results in an overwhelming level of lost sales to distant dealers of the same make and local dealers of alternative makes. Network incompleteness is shown to be economically inefficient. In the absence of asymmetric information about buyer search costs, the complete network is the unique pairwise stable and efficient network.
1 Introduction

The purpose of this research is to analyze the efficiency and formation of automobile inter dealer trade networks; specifically, network incompleteness driven by asymmetric information about buyer search/inconvenience costs. A dealer trade is a swap of vehicles at cost (called invoice) between two automobile dealers. Dealer trades occur because of the vast number of options available for any given make and model. This makes it very unlikely the local dealer will have the buyers most desired specification in stock, so he may attempt to trade another dealer for it. Because of a large number of options offered on any given model, dealer trades can be a very significant portion of business. At Adamson Motors, a medium sized dealership in Rochester Minnesota, the fraction of Chrysler sales generated by dealer trades is between 50 – 60 percent. Further, this data reveals network incompleteness. Adamson’s does a lot of business with relatively few trade partners and doesn’t do any trading at all with many dealers. Network incompleteness is not generated by dealer satiation with a small number of partners, since there are far too many model specifications. This point will be clarified in more detail in section 2.4.

Because of this network in which automobile dealers are embedded, questions related to retail automobile sales should be conducted with a clear understanding of network structure and formation. For example, a problem facing any dealership is rather to set prices high and stock fewer models, or set prices low and stock fewer models. However, modifying price affects the likelihood distant buyers will travel to purchase a vehicle from you. Further, this decision affects whether other dealers want to do dealer trade business with you. Similarly, higher advertising may generate more sales but also decrease dealer trade business. There are also several other questions related to the dealer trade network that can be addressed with a careful understanding of the network. For example, almost all trades are bilateral because of costs. Dealers contact each other over the telephone and it can be difficult to negotiate even bilateral trades. If a mechanism could be developed to reduce these costs, more complicated trades could be completed to potentially improve efficiency and profitability. Another example has to do with modifying dealer terms of trade. Currently, most trades are completed at invoice or invoice minus holdback (to be explained below). Very rarely do dealers trade with transfers. A dealership like Adamson’s faces the following tradeoff. Larger inventories are beneficial not only because greater selection implies higher domestic sales, but also since the cars can be traded for a vehicles at other dealerships. However, large inventories are costly since new cars are typically financed by the dealership until sold. Smaller inventories are less costly but make it more difficult to trade at the current norm of invoice or invoice minus holdback. The question then is this. Could a dealership like Adamson’s improve profitability by simultaneously reducing inventory and beginning to trade with transfers?

A homogenous linking network of retail automobile sales is developed. A symmetric pairwise stable equilibrium is shown to always exist. A link represents a relationship, which lubricates trade between dealers by allowing them to trade whenever one dealer is in need. Asymmetric information about buyer search/inconvenience costs allows the dealer to increase profits by limiting the number of trade partners. This reduction in the number of links raises the local buyers cost of obtaining models not currently in stock and allows the dealer to sell models with higher profit margins. These are the models in stock, that don’t incur transportation costs with sale. This effect diminishes as the number of trading relationships declines. After a certain threshold, raising the cost of obtaining distant models results in an overwhelming level of lost sales to distant dealers of the same make and local dealers of alternative makes. Network incompleteness is shown to be economically inefficient. In the absence of asymmetric information about buyer search costs, the complete network is the unique pairwise stable and efficient network.

This paper proceeds as follows. First, qualitative and empirical aspects of the market are described. Dealer trade data from Adamson Motors of Rochester Minnesota is summarized. Then the model is defined, and a simple case is given to for clarity. The paper then proceeds to prove the existence of pairwise stable equilibria. A condition is given for uniqueness. Next, special emphasis is given to asymmetric information to clarify this mechanism. Computational comparative results are given to explain the generation of incompleteness related to buyer search inconvenience cost, asymmetric information, transportation cost and buyer substitution. Finally, it is shown that incompleteness always generates economic inefficiency. In the absence of asymmetric information about buyer search/inconvenience costs, the complete network is the unique pairwise stable equilibrium and unique efficient network.
2 Qualitative and Empirical Insights into the Market

2.1 Nature of Dealer Trades

New cars are manufactured and sold to dealers, who then sell them to consumers. Automobile manufacturers have vast dealer networks in the United States. See Figure 1 for a snapshot of the northeastern US Chrysler dealer network. Because it takes several weeks to manufacture a new car to a particular consumer specifications, a dealer that doesn’t have the desired car in stock can look to other dealers in an attempt to get the car. If the dealer finds the car at another dealership, he can either try to buy the vehicle or attempt a dealer trade; that is he tries to trade a vehicle on his lot for it. Dealer buys are relatively rare in normal economic times because the dealer holding the desirable car typically would rather hold it to sell himself. One period of exception was the recent US great recession. During this period dealer inventories were flush from a combination of low demand and manufacturer pressure to increase floor plan volume. In this period most dealers preferred to complete dealer buys in an attempt to reduce floor plan financing costs. This fact was obtained through discussions with automobile sales managers.

Returning to trades, if the dealer has a desirable car that the other dealer wants, the transaction is relatively easy since both parties immediately benefit. However, trades are routinely completed where only one side immediately benefits. For example, two dealers with a strong relationship will routinely make trades that are more beneficial to one party because eventually the other dealer will be repaid by performing a similar trade. They call this the two week rule, trading with a dealer today may be beneficial because on average a dealer will be repaid within two weeks. The repetition caused by these types of trades reduces the costs of completing each trade; these dealers form relationships which lubricate the negotiation process between them.

One interesting thing about the actual trade is that it never contains transfers. The two cars are literally traded for an established price (called the invoice or invoice minus holdback). Invoice is what it costs to purchase the car from the manufacturer and holdback is a fraction of the price the manufacturer pays to the dealer for selling the vehicle. For Chrysler, holdback is 3 percent. To explain this in more detail, consider the following example. Suppose dealers A and B are trading two vehicles. Dealer A’s car is worth 20,000 and Dealer B’s car is worth 10,000. Holdback is 3 percent of the invoice and is paid to the dealer from the manufacturer when the car is removed from inventory. So if the two dealers trade at invoice, then dealer A pays dealer B 10,000, receives 3 percent of 20,000 from the manufacturer and dealer B receives 3 percent of 10,000 and pays A 20,000. In this case, the dealers get paid from the manufacturer for the car they ordered. If they trade at invoice minus holdback, then dealer A pays dealer B 10,000 less holdback and Dealer B pays
dealer A 20,000 less holdback. Here the dealers get paid by the manufacturer for the cars they sell. Currently in the midwestern United States, almost all trades are completed at invoice minus holdback. It changed a few years ago from invoice to invoice minus holdback when dealers were doing a lot of buys. Inventories were flush because of the recession so no one wanted a car back. In order to reduce inventories, dealers were willing to sell cars to other dealers at invoice minus holdback. This norm stuck after the inventory problem ceased.

Often times a dealer is not able to complete a dealer trade. In this case, the dealer attempts to sell the buyer a similar vehicle on the lot. Salesman are trained to do this and buyer search/inconvenience costs can be high; they might have to travel to a different city and negotiate with another dealer for the purchase of their first choice. Further, a dealer may voluntarily not disclose a potential trade to the customer in order to sell him a more profitable model on the lot. A buyer may not be willing to wait 6 weeks for the vehicle to be manufactured to their desired specification.

2.2 Empirical Snapshot of a Node: Adamson Motors of Rochester Minnesota

The data used to motivate this paper was collected from Adamson Motors of Rochester Minnesota. The dealer trade data spans the year 2012 and includes all Chrysler trades, buys, sells and favor with Adamsons as one of the counterparties. The favor of the trade goes to the initiating side. A sample entry is Tanner Motors, of Brainerd Wisconsin, 6 trades, 2 in Tanners favor, 0 buys, 0 sells, 213 miles from Adamson Motors with Chrysler inventory as of 1/3/13 of 141. The data set contains 192 trade partners with 697 total trades. The average number of trades across partners is 3.65 with 46 percent in Adamsons favor. The average inventory size across all partners is $183.75$ compared to Adamsons $214$.

Figure 2 gives the distribution of trades. Just over 50 percent of dealers have fewer than 2 trades over the period. Approximately 20 percent of dealers fall within 2 – 4 total trades. These trades are typically completed only if both sides immediately benefit. The dealer in need must trade something immediately desirable to the other dealer. This contrasts to the trades completed at dealers with approximately 10 or more total trades. These trades are routinely completed where only one side immediately benefits. This could happen if the dealer initiating the trade doesn’t immediately have something desirable the other dealer wants. The dealer on the losing side of the transaction is regardless willing to make the trade knowing that sometime in the near future he may need to initiate the same type of trade in his favor. From Figure 2, one can see that these types of relationships are relatively rare.

Figure 3 plots percentage of dealers versus the percentage of trades. 20 percent of the total number of trade partners account for just over 50 percent of the trades and 40 percent account for approximately 75 percent. The 18 dealers with 10 or more trades account for 31.25 percent of the total number. These numbers demonstrate the relative importance of just a few trade partners, those where the dealers complete
trades where one side may only immediately benefit.

Figure 4 displays trade volumes between Adamsons and dealerships located in the Minneapolis/St. Paul area. These dealers are on average 80-90 miles from Adamsons. This area is clustered with Chrysler dealerships and 6 of the 18 dealers with 10 or more trades are located in this area. As you can see from the figure, there is significant heterogeneity in trade volumes between Adamson’s and dealers in this area. What is the cause of this heterogeneity? This refutes the idea that distance is the only important factor in the development of trade relationships.

Figure 5 gives a snapshot of Adamsons and its trade partners with at least 7 trades. As you can see there are many nodes relatively close by with only a few trades. Some of these dealerships have very large inventories; for example, Eau Claire, La Crosse, Madison, Waterloo and Milwaukee. What is the source of this incompleteness? Further, there are close nodes with large trade volumes. Some of these dealers are large and some are small. Therefore, the formation of trade relationships is more complicated than simply distance and inventory size considerations. From the data, Adamsons trades with several small dealers. This refutes claim 3 from above. However, if the dealer is smaller then more than 50 percent of the trades are in Adamsons favor. Therefore, it appears as though the smaller dealer is doing everything possible to please Adamsons, the larger dealer. Claim 6 seems to be verified by the data. Adamsons trades with several large dealers, and the overall favor is in the direction of the larger dealer.

2.3 Factors Affecting Formation of Dealer Trade Network

There are many factors affecting the dealer trade network. This paper analyzes one component of network structure; that of network incompleteness. Other potential factors not analyzed in this paper include

1. Dealers very close to one another may tend not to trade, and thus wouldn’t establish relationships. Because of close proximity, each dealer would have access to the same buyers who could easily shop around. Why share profits with close dealer by performing a dealer trade when you could advertise and compete for the buyer yourself?

2. Dealers that are far away might not trade with one another. Here I thought the answer was simple, trading with distant dealers involves high wage and fuel costs which brings down the profit margins of sale.

3. Are small dealers isolated? This could be because it would be very rare for a larger dealer to need
something on a small dealers lot. It would be beneficial of course for the small dealer but not for any dealer of larger size.

4. Perhaps there is simply heterogeneity in dealers attitudes towards trade? If a vehicle is traded away then the dealer knows the other has it sold. But this buyer could have simply traveled and purchased the vehicle from him. Maybe some dealers simply prefer to compete directly rather than trade.

5. Business relationships could develop because of manufacturer incentives. Different regions have different incentives which transmit into different prices in different areas. Therefore, one vehicle that is desirable in one location might not be desirable in another location. The two dealers might then find it beneficial to trade these vehicles. Once the relationship is formed, the two might continue to trade in ways where individual trades might be more beneficial to one dealer over the other.

6. Medium size dealers could tend to form relationships with very large dealers. Medium size dealers know that a large dealer is more likely to have the vehicle they need, therefore they might be willing to do anything possible to establish a relationship with the large dealer.

7. Maybe dealers form these relationships for some collusionary purpose?

Although these all seem plausible, the theoretical model developed in this paper focuses on evidence for the development of an intermediate level of relationships. Research into these other factors could be extensions to this research.

2.4 Chrysler Model Heterogeneity

In this section I will run you through the array of options available on a 2014 Dodge Ram pickup truck. The purpose is to display the large number of available options and resulting vehicle specifications. With such a large number of available options, estimating demand becomes very difficult. Further, the number of model specifications is so vast that no dealer could satiate access to all models with a limited number of trade partners. Network incompleteness is then not driven by dealer satiation of possible inventory through a limited number of partners.

Let's go ahead then and try to build a 2014 Dodge Ram pickup truck. I'll follow the algorithm given to customers visiting ramtrucks.com at http://www.ramtrucks.com/hostc/bmo/CUT201413/models.do?. First, I'm asked to select a model. There are nine total; Tradesman, Express, HFE, SLT, Lone Star, Big Horn, Big Horn, Big Horn, Big Horn, Big Horn.
Outdoorsman, Sport, Laramie, Laramie Longhorn and Laramie Limited. Supposed that I’m interested in the base package, the Tradesman. Next I’m asked to select 4x2 or 4x4 and box size. Since I’m interested in Minnesota sales I’ll look at the 4x4 because two wheel drive vehicles are horrible in winter conditions. Given the 4x4 selection, I have a choice of 5 box/cab sizes; Regular 6’4", Regular 8’, Quad 6’4", Crew 5’7", Crew 6’4". Regular indicates two doors while quad and crew indicate four doors. The difference between the crew and quad is that the crew has more seating space and smaller bed. Given the choice of box/bed size, its time to choose interior/exterior color. There are 12 exterior colors available and only 1 interior color. Other models have more than one interior, for example the SLT has two. So far I’ve selected a Dodge Ram 1500 pickup, Tradesman, Quad 6’4” with black clear coat exterior paint and black/diesel gray interior. Up to this point the number of combinations on just the Tradesman 4x4 are 60; simply from the box/cab size and exterior color choice.

In the next set of choices there are four categories; Interior, Exterior, Power Train and Packages. Start with the interior. Several of these options can be added after the vehicle is on the lot so I need to be a little careful. I’m given the option of media screen size; either 3 or 5 inch. Its too costly to rip out the media system and replace it with a different screen so this adds to model complexity. Similarly, I’m given the option of whether or not to have a rear back up camera. Next move to the power train. I’m given 3 different engine and 3 transmission types; 3.6-Liter V6 24-Valve WT Engine, 5.7-Liter V8 HEMI MDS VVT Engine, 3.0-Liter V6 Turbo Diesel, 8-Speed TorqueFlite Auto Trans 845RE, 6-Speed Automatic Trans and the 8-Speed TorqueFlite Auto Trans 8HP70. Next I can choose the rear/axle ratio; 3.21, 3.92 and 3.55. A lower rear/axel ratio implies lower RPM and greater fuel economy while a higher ratio improves towing, carrying loads and
towing. However, not all engine/transmission and rear/axel ratio combinations are possible. Even if I only allow for 3 combinations here (one engine with one transmission with one rear/axel ratio), this increases the total model complexity to 180 (without the screen and camera). With the screen and camera combinations I arrive at 720. Recall that this calculation is just for the one model, the Tradesman. If instead I were interested in the Laramie Longhorn, perhaps for the Premium Bifunctional Halogen Projector Headlamps, Premium LED Taillamps, Laramie Longhorn Unique Premium 7-Inch Instrument Cluster, Real Wood Accent Interior, Uconnect 8.4AN (RA4) System with 8.4-Inch Touchscreen or any other of its Tradesman different features that 720 number would increase dramatically.

By this point I think its clear. A limited number of trade partners cannot deliver complete access to all possible model configurations. Therefore, network incompleteness is not driven by dealer satiation through a limited number of trade partners.

3 The Model

This model will be used to analyze the optimal inter dealer trade network structure. A simple example with two dealers and two models is examined to explain the potential benefit of dealer trades. Then the m dealer case is analyzed. Of critical importance is buyer substitution, distance and a preference toward an intermediate level of links. The latter explains the incompleteness seen in Adamsons interdealer trade data. Further, this incompleteness increases dealer profits but decreases consumer surplus through search/inconvenience costs.

3.1 Model Definitions and Parameters

Let \( S_i = (s_{i1}, ..., s_{im}) \) be the inventory of Chrysler dealer \( i \) where \( m \) is the number of models and \( s_{ij} \) is the number of model \( j \) that dealer \( i \) has in stock. Assume that no two dealers have the same model in stock and global inventory of each model is nonzero. In particular, let \( s_{ii} = 1 \) and \( s_{ij} = 0 \) for all \( j \neq i \). Each dealer has only one unit of one model in stock. Each period one consumer randomly shows up at either the local Chrysler or alternative make dealership, for example Ford, at location \( i \). Let the probability the consumer shows up at location \( i \) be \( \frac{1}{m} \). Once at \( i \), the buyer randomly determines to head to either the Chrysler or alternative make dealership. Let \( \sigma_{i\text{C}} \) be the conditional probability that the buyer shows up at the Chrysler dealership at \( i \) and \( \sigma_{i\text{AM}} \) for the alternative make. If the buyer shows up at the Chrysler dealership, he randomly demands some model \( j \). Let \( \pi_{ij} \) be the probabilistic demand for model \( j \) at Chrysler dealership \( i \). These probabilities will be formalized in a later section. The following notation will be used

\[
p = \text{price of each model}
\]

\[
I = \text{invoice of model i}
\]

\[
c = \text{transportation cost between any two locations}
\]

\[
\gamma = \text{fraction of transport cost paid by the dealer}
\]

\[
\mu_1 = \text{rate at which the buyer at } i \text{ substitutes between the Chrysler and alternative make dealerships}
\]

\[
\mu_2 = \text{degree of substitution between Chrysler models at dealership } i
\]

\[
\delta = \text{consumer search/inconvenience cost of negotiating with a distant dealer}
\]

\[
\lambda = \text{fraction of search/inconvenience cost the local dealer can recoup when trading for a model not in stock}
\]

The model will proceed in two periods. In the first, dealerships will choose their trade partners. These relationships will be represented by a matrix \( g = (g_{ij}) \). The rules of network formation will be Jackson-Wolinsky. That is, both dealerships need to agree to form a link and only one is needed to destroy a link. The second period is a model of auto sales. The consumer is shown a menu of costs associated with obtaining each model. This menu depends upon the network of relationships developed in the first period, as explained below. The consumer chooses one model according to the discrete choice model of demand.
3.2 Discrete Choice Model of Demand

The conditional nested logit model of demand will be used to determine the probability of a consumer showing up at dealership $i$ demanding model $j$. This needs to be random because of model complexity; a dealer doesn’t know with complete certainty the precise model a consumer desires. The nested model is used because the substitution from the Chrysler dealership to a nearby alternative make competitor is critically important. One nest will be the available inventory at the Chrysler dealership while the other nest will signify one vehicle at its immediate alternative make neighbor. This two nest specification allows for flexibility in analysing the substitution effect in network formation.

A consumer showing up at dealership $i$ can travel to purchase any available model. In doing so, they pay the fraction $(1 - \gamma)$ of the transportation cost $c$ when purchasing a vehicle from dealership $k$. When the consumer searches out a vehicle elsewhere, they incur the search/inconvenience cost $\delta$ with $\delta > 0$. The intuition for this additional cost is that the consumer has to search out and negotiate with another dealer which can be costly.

The consumer at dealership $i$ knows the full inventory of its local dealer and the costs associated with trading for a vehicle not currently in stock. Let $U_{ij}$ be buyer $i$’s utility of consuming model $j$ and $v_i$’s value of the $j^{th}$ model. If the local dealership has model $j$ in stock, then the consumers utility is

$$U_{ij} = v - p + \mu_2 \epsilon_{ij}$$

The parameter $\mu_2$ controls buyer substitution across models at the Chrysler dealership $i$. As $\mu_2$ goes to zero, the products become perfect substitutes and the buyer purchases the vehicle with the highest deterministic utility. The higher $\mu_2$, the less substitutable are the models at Chrysler dealer $i$.

If the local dealer does not have the model in stock and is unable to trade for it, the buyer has the option to seek out the model elsewhere. This utility is

$$U_{ij} = v - p - (1 - \gamma)c - \delta + \mu_2 \epsilon_{ij}$$

Finally, if dealer $i$ can trade for the vehicle then the consumers utility is

$$U_{ij} = v - p - (1 - \gamma)c - \lambda \delta + \mu_2 \epsilon_{ij}$$

If dealer $i$ doesn’t have complete information about the level of $\delta$, he can extract at most $\delta$ and perhaps through negotiation can only charge $\lambda \delta$ over the distant dealer with $0 \leq \lambda \leq 1$. If the local dealer tries to charge more than $\delta$ he won’t make the sale. If $\lambda < 1$ then the buyer receives additional value if dealer $i$ trades for model $j$ over traveling to purchase it. Therefore, if dealer $i$ decides not to trade, the effective cost of obtaining model $j$ increases and the buyer substitutes toward the other models available. The motivation for $\lambda < 1$ is simple, the buyer knows $\delta$ and the seller does not. This gives the buyer negotiating power making it difficult for the seller to extract the entire rent.

Let $V_{ij}$ be the non random part of $U_{ij}$. If $(\epsilon_{ij})_j$ are i.i.d type double exponential across $j$, then the probability that consumer $i$ shows up at the Chrysler dealership demanding model $j$ is

$$\pi_{ij} = \frac{\exp\left(\frac{V_{ij}}{\mu_2}\right)}{\sum_j \exp\left[\frac{V_{ij}}{\mu_2}\right]}$$

Given the two nests at each location, the attractiveness of the Chrysler nest is given by the expected value

$$ECV_i = E(\max_j \{U_{ij}\})$$

Given the assumption on errors, $ECV_i$ has closed form

$$ECV_i = \mu_2 \ln \left( \sum_{j=1}^{m} \exp \left[ \frac{V_{ij}}{\mu_2} \right] \right)$$

Let $AM_i$ be the alternative make value at $i$. Then the probability that a buyer at $i$ chooses the Chrysler nest is given by

$$\sigma_{iC} = \frac{\exp \left( \frac{ECV_i}{\mu_1} \right)}{\exp \left( \frac{ECV_i}{\mu_1} \right) + \exp \left( \frac{AM_i}{\mu_1} \right)}$$
where $\mu_1$ is the substitutability parameter between nests at location $i$. It has the same interpretation as $\mu_2$ given above. Finally, the consumer surplus at location $i$ is given by

$$CS_i = \mu_1 \ln \left( \exp \left( \frac{ECV_i}{\mu_1} \right) + \exp \left( \frac{AM_i}{\mu_1} \right) \right)$$

If the alternative make nest is excluded,

$$CS_i = ECV_i$$

### 3.3 Period Two Dealer Profits

In period two dealers can use their trade partners to trade for models not in stock. Let $CP^i_{kj}$ be the continuation profit of dealer $i$ if the buyer shows up at location $k$ looking for model $j$. First suppose that $k \neq i$. If $g_{ik} = 1$ then

$$CP^i_{ki} = 0$$

since dealer $k$ is able to trade dealer $i$ for model $i$. If $g_{ik} = 0$, then

$$CP^i_{ki} = p - I - \gamma c$$

since the buyer at $k$ travels to dealer $i$ to purchase the vehicle. If $k \neq i$ and $j \neq i$ then

$$CP^i_{kj} = 0$$

If the buyer shows up at location $i$, then

$$CP^i_{ii} = p - I$$

if $g_{ij} = 1$ since dealer $i$ can trade $j$ for model $j$ and

$$CP^i_{ij} = 0$$

if $g_{ij} = 0$ since dealer $i$ cannot trade dealer $j$ for model $j$. With this notation, the expected profit of dealer $i$ given network $g$ is

$$Profit_i(g) = \frac{1}{m} \sum_{kj} \sigma_{C_k \pi_{kj}} CP^i_{kj}$$

### 3.4 Pairwise Stable Equilibrium

**Definition** Network $g \in G$ for all $i, j = 1, ..., m$ is pairwise stable if

- there does not exist $i$ such that $g_{ik} = 1$ and $Profit_i(g) < Profit_i(g')$ where $g'$ is equal to $g$ except for $g'_{ik} = 0$.

- there does not exist $i$ and $k$ with $g_{ik} = 0$ such that $Profit_i(g) \leq Profit_i(g')$ and $Profit_k(g') \leq Profit_k(g)$ with strict inequality for at least one where $g'$ is equal to $g$ except that $g'_{ik} = 1$.

The network $g \in G$ is a symmetric pairwise stable equilibrium if each node has the same number of links.

### 3.5 A Note on Pricing

Given the specification of the model, I would like to clarify a few points on actual automobile pricing. Every vehicle comes with a manufacturer suggested price (MSRP). From this, the manufacturer offers buyers incentives that typically vary per month. An incentive is an immediate buyer rebate. From this, dealers offer dealer discounts. Dealers are completely free to determine the level of discount. At Adamson Motors, a medium sized dealer in Rochester Minnesota, they attempt to charge more for models that are traded for. This is the rational for the $\lambda \delta$ term in traded model profit. The dealer is effectively performing a service for the buyer and attempts to extract rent. The one complication is that search/inconvenience costs are asymmetric information.
4 Example: Two Dealerships and Two Models

At this stage it will be helpful to analyze a simplified case of the model to clarify how it works and gain some intuition about why relationships form. Suppose that there are two dealerships and two models. Let $S_1 = (1, 0)$ and $S_2 = (0, 1)$. That is, dealer one has model one in stock and dealer two model two. Recall that a relationship (or link) between two dealers is an obligation to trade. If a link is formed between the dealers, then consumer utilities are

\[
\begin{align*}
U_{11} &= v - p + \mu_2 \epsilon_{11} \\
U_{12} &= v - p - (1 - \gamma)c - \lambda \delta + \mu_2 \epsilon_{12} \\
U_{21} &= v - p - (1 - \gamma)c - \lambda \delta + \mu_2 \epsilon_{21} \\
U_{22} &= v - p + \mu_2 \epsilon_{22}
\end{align*}
\]

With the link, continuation payoffs of dealer one are

\[
\begin{align*}
CP_{11} &= p - I \\
CP_{12} &= p - I - (1 - \gamma)c + \lambda \delta \\
CP_{21} &= 0 \\
CP_{22} &= 0
\end{align*}
\]

so that the expected profit of dealer 1 is

\[
\text{Profit}_1 = \frac{1}{2} \sigma_{C1}(\pi_{11} CP_{11}^{1} + \pi_{12} CP_{12}^{1})
\]

$CP_{11}^{1}$ is the continuation payoff when the consumer shows up at dealer one demanding model 1. Dealer one has model 1 in stock so sells it to the consumer and reduces his inventory to 0 units. In $CP_{12}^{1}$, dealer 1 doesn’t have model 2 in stock but can trade model 1 for it from dealer 2. Since the link exists, dealer 1 conducts this trade and sells model 2 to the buyer thus reducing his inventory to 0. $CP_{21}^{1}$ is the continuation payoff when the consumer shows up at dealer 2 demanding model 1. Dealer 2 trades dealer 1 for model 1. Dealer one thus earns zero profit in this state. Finally, $CP_{22}^{1}$ is the continuation payoff when the consumer shows up at dealer 2 demanding model 2. Dealer 2 has this model in stock so no trade is made. Dealer one thus makes nothing in this state. Profits for dealer two with the link are computed in an analogous way.

In the absence of a link the buyer must travel to purchase the vehicle not in local stock from another dealer. Let the bold notation indicate values without the link. With this, consumer utilities are

\[
\begin{align*}
U_{11} &= v - p + \mu_2 \epsilon_{11} \\
U_{12} &= v - p - (1 - \gamma)c - \delta + \mu_2 \epsilon_{12} \\
U_{21} &= v - p - (1 - \gamma)c - \delta + \mu_2 \epsilon_{21} \\
U_{22} &= v - p + \mu_2 \epsilon_{22}
\end{align*}
\]

where $\delta$ has been added to represent the additional cost of searching and negotiating for the vehicle elsewhere. The continuation payoffs of dealer one in absence of the link are

\[
\begin{align*}
CP_{11} &= p - I \\
CP_{12} &= 0 \\
CP_{21} &= p - I - (1 - \gamma)c \\
CP_{22} &= 0
\end{align*}
\]

$CP_{11}^{1}$ is the continuation payoff when the consumer shows up at dealer one. $CP_{12}^{1} = CP_{22}^{1}$ since only dealer 2 makes a sale when the buyer demands model 2 and thus dealer one is left with his model. In $CP_{21}^{1}$, the consumer shows up at location 2 demanding model one. Dealer 2 doesn’t have this model in stock so the
buyer travels to dealer 1 to purchase it. Dealer 1 then reduces his inventory to zero. With this the expected profit of dealer $i$ in the absence of a link is

$$\text{Profit}_i = \frac{1}{2} \left[ \sigma_C \pi_{11} \text{CP}_1 + \sigma_C \pi_{21} \text{CP}_2 \right]$$

For the network with the link to be pairwise stable, it must be true that both

$$\text{Profit}_1 \geq \text{Profit}_i$$

$$\text{Profit}_2 \geq \text{Profit}_i$$

Before leaving this section, let’s try to get a feel for why the link might be pairwise stable; that is, why both dealers might benefit from trade. What makes this difficult is that we not only need to analyze how each continuation payoff changes, but also how the different utilities transfer into different probabilities of sale.

Suppose dealer one is contemplating severing the link. First consider the effects when the buyer shows up at location two. If dealer one severs the link he gets to sell model one to the buyer that shows up at dealer 2 demanding model one. This generates an immediate profit of

$$\sigma_C \pi_{21} (p - I)$$

Since the dealer didn’t sell anything in this state with the link, this is a direct increase in profit. How about the effects in state 22, when the buyer shows up at dealer two demanding model two? Dealer one doesn’t make a sale in either scenario so there is nothing to compare in this state.

Next consider the effects if the buyer shows up at location one. In state 12, without the link dealer one no longer gets to make the sale. This raises the costs of obtaining model two for buyer one so that $\pi_{12} \leq \pi_{11}$. Further, $\sigma_C < \sigma_C$ because the alternative make nest is more attractive without the link. How about in state 11? The payout is the same in both cases but the probabilities of sale are different. Because the cost of obtaining model two increases without the link, $\pi_{11} > \pi_{12}$. However, $\sigma_C < \sigma_C$ so it can’t be unambiguously claimed that expected profit at 11 increases. The intuition is that because the cost of obtaining model two increased, the dealer can switch the buyer more easily off to the model he has in stock. However, the buyer also has an increased interest in the alternate make model so that the overall effect at the local Chrysler dealership cannot be signed. Finally, how about in the alternate make state at location one?

As you can see, weighing the benefits and costs at each state of deleting the link is somewhat complicated. It depends crucially on the tendency to substitute from one Chrysler model to the other and also to substitute away to the alternative make. The condition for optimality of dealer trades is given in the following theorem.

**Theorem 4.1.** The decision to sever the link when $m = 2$ depends on the sign of

$$(\sigma_C \pi_{11} - \sigma_C \pi_{11})(p - I) + \sigma_C \pi_{12} (p - I - \gamma c) - \sigma_C \pi_{12} (p - I - \gamma c + \lambda \delta)$$

A negative sign implies that dealer trades are optimal, while a positive sign implies that the empty network is pairwise stable.

The logic for the optimality of dealer trades in this simple example is straightforward. If the dealer raises the cost of obtaining the distant model by not trading and the buyer simply heads down the street to purchase the Ford, then it’s not optimal to sever the link. If instead, by raising the cost the dealer is able to sell the buyer on his more profitable model in stock, then it’s optimal to sever the link.

### 5 Existence of Symmetric Pairwise Stable Equilibria

This section establishes the existence of symmetric pairwise stable equilibria. First I introduce some notation that will be useful for the remainder of the paper.
5.1 Notation for Symmetric Pairwise Stable Equilibria

This paper will focus on symmetric pairwise stable equilibria. Because this concept will be used heavily throughout the rest of the paper, the related notation will be introduced in this section. Let $M^x = \sum_k g_{ik}$. $M^x$ indicates the number of trade partners. By definition, $M^x$ will be the same for all dealers in any symmetric equilibrium. Since I will be adding and deleting links, it will be useful to attach the argument $M^x$ to both $\pi_{ik}$ and $\sigma_c$. Suppose that Dealer $i$ has model $k$ in stock so that

$$\pi_{ii}(M^x) = \frac{\exp\left(\frac{v-p}{\mu_2}\right)}{\exp\left(\frac{v-p}{\mu_2}\right) + M^x \exp\left(\frac{v-p-\gamma c}{\mu_2}\right) + (M - M^x - 1) \exp\left(\frac{v-p-\gamma c - \delta}{\mu_2}\right)}$$

If dealer $i$ doesn’t have model $j$ in stock, but can trade for it

$$\pi_{ij}(M^x) = \frac{\exp\left(\frac{v-p-\gamma c}{\mu_2}\right)}{\exp\left(\frac{v-p}{\mu_2}\right) + M^x \exp\left(\frac{v-p-\gamma c}{\mu_2}\right) + (M - M^x - 1) \exp\left(\frac{v-p-\gamma c - \delta}{\mu_2}\right)}$$

Let $\pi_h$ indicate this probability. If dealer $i$ doesn’t have model $j$ in stock and can’t trade for it

$$\pi_{ij}(M^x) = \frac{\exp\left(\frac{v-p-\gamma c - \delta}{\mu_2}\right)}{\exp\left(\frac{v-p}{\mu_2}\right) + M^x \exp\left(\frac{v-p-\gamma c}{\mu_2}\right) + (M - M^x - 1) \exp\left(\frac{v-p-\gamma c - \delta}{\mu_2}\right)}$$

Let $\pi_a$ indicate this probability. The expected value of the chrysler nest is given by

$$ECV_i(M^x) = \mu_2 \ln \left( \exp \left( \frac{v_p}{\mu_2} \right) + M^x \exp \left( \frac{v_p - \gamma c}{\mu_2} \right) + (M - M^x - 1) \exp \left( \frac{v_p - \gamma c - \delta}{\mu_2} \right) \right)$$

Finally, given the expected value of the Chrysler nest at location $i$, the probability that the buyer chooses a Chrysler at $i$ is given by

$$\sigma_c(M^x) = \frac{\exp\left( ECV_i(M^x) \right)}{\exp\left( ECV_i(M^x) \right) + \exp\left( \lambda M^x_i \right)}$$

5.2 Existence of Symmetric Pairwise Stable Equilibria

With the above notation in place, I am ready to prove the main existence result.

**Theorem 5.1.** A symmetric pairwise stable equilibrium exists.

**Proof.** Suppose that each dealer currently has $M^x$ links and dealer $i$ is contemplating severing a link. His expected profit with the link is

$$\sigma_c(M^x)(\pi_{ii}(M^x)(p - I) + M^x \pi_h(M^x)(p - I - \gamma c)) + (M - M^x - 1) \sigma_c(M^x) \pi_a(M^x)(p - I - \gamma c)$$

(1)

The dealers expected profit after severing the link is

$$\sigma_c(M^x - 1)(\pi_{ii}(M^x - 1)(p - I) + (M^x - 1) \pi_h(M^x - 1)(p - I - \gamma c)) + (M - M^x - 1) \sigma_c(M^x) \pi_a(M^x)(p - I - \gamma c) + \sigma_a(M^x - 1) \pi_a(M^x - 1)(p - I - \gamma c)$$

(2)

The dealer will prefer to sever the link if

$$\sigma_c(M^x - 1)(\pi_{ii}(M^x - 1)(p - I) + (M^x - 1) \pi_h(M^x - 1)(p - I - \gamma c)) + \sigma_a(M^x - 1) \pi_a(M^x - 1)(p - I - \gamma c) > \sigma_c(M^x)(\pi_{ii}(M^x)(p - I) + M^x \pi_h(M^x)(p - I - \gamma c))$$

(3)

Now suppose that all dealers have $M^x - 1$ links and a pair of dealers is contemplating forming a trade relationship. Expected profit with $M^x - 1$ links is

$$\sigma_c(M^x - 1)(\pi_{ii}(M^x - 1)(p - I) + (M^x - 1) \pi_h(M^x - 1)(p - I - \gamma c)) + (M - M^x) \sigma_c(M^x - 1) \pi_a(M^x - 1)(p - I - \gamma c)$$

(4)
If the link is added, expected profit is
\[ \sigma_c(M^*) (\pi_i(M^x)(p - I) + (M^x)\pi_h(M^x)(p - I - \gamma c)) + (M - M^x - 1)\sigma_a(M^x - 1)\pi_a(M^x - 1)(p - I - \gamma c) \] (5)

The dealer doesn’t want to form the relationship provided
\[ \sigma_c(M^x - 1) (\pi_i(M^x - 1)(p - I) + (M^x - 1)\pi_h(M^x - 1)(p - I - \gamma c)) + \sigma_c(M^x - 1)\pi_a(M^x - 1)(p - I - \gamma c) > \sigma_c(M^x)(\pi_i(M^x)(p - I) + (M^x)\pi_h(M^x)(p - I - \gamma c)) \] (6)

Therefore, if all dealers have \( M^x \) relationships, a dealer severing a link will never add a link if all dealers have \( M^x - 1 \) relationships. The symmetric equilibrium is then obtained by iterating through the possible \( M - 1 \) candidates.

Proof of the following lemma is sufficient to prove the uniqueness of the symmetric pairwise stable equilibrium.

**Conjecture 5.2.** The expression
\[ \pi_i(M^x - 1)(p - I) + (M^x - 1)\pi_h(M^x - 1)(p - I - \gamma c) + \pi_a(M^x - 1)(p - I - \gamma c) - (\pi_i(M^x)(p - I) + (M^x)\pi_h(M^x)(p - I - \gamma c)) \] (7)
crosses zero at most once.

**Theorem 5.3.** If conjecture 7 holds, then the symmetric pairwise stable equilibrium is unique.

### 5.3 Ruling out Non-Symmetric Pairwise Stable Equilibria

**Theorem 5.4.** Suppose that conjecture 7 holds. If network \( g \in G \) is not symmetric, then \( g \) is not pairwise stable.

**Proof.** Suppose that network \( g \in G \) is not symmetric. Then there exists nodes \( i \) and \( j \) with different numbers of trade partners. Let \( M_k \) be the number of partners of each, \( k = i, j \). Further, let \( M^x \) be the symmetric equilibrium number of links. There are three cases to consider.

1. Suppose \( M_i = M_j \neq M^x \). Since \( M_i \) doesn’t constitute a symmetric pairwise stable equilibrium and deviations only depend upon activity at nodes \( i \) and \( j \), network \( g \) is not pairwise stable.

2. Suppose \( M_i > M^x \) and \( M_i > M_j \). Since \( \pi_a(M_i) < \pi_a(M_j) \) and \( i \) prefers to sever a link with any partner with \( M_i \) links, \( i \) prefers to sever the link with \( j \).

3. Suppose \( M^x > M_j \) and \( M_i > M_j \). Since \( \pi_a(M_i) < \pi_a(M_j) \) and \( j \) prefers to add a link with any partner with \( M_j \) links, \( j \) prefers to add a link with \( i \).

**Theorem 5.5.** Suppose that conjecture 7 holds. Then there exists a unique pairwise stable equilibrium. Further, it is symmetric.

**Proof.** By theorem 5.1, a symmetric pairwise stable equilibrium exists. Since conjecture 7 holds, the symmetric pairwise stable equilibrium is unique. By conjecture 5.4, any non symmetric network is not pairwise stable. Therefore, the symmetric pairwise stable equilibrium is the unique pairwise stable equilibrium.

### 5.4 The Role of \( \lambda \)

Recall that \( \lambda \) is the share of buyer search cost that the dealer is able to extract by trading for the buyers most desired model. The motivation for this is that by trading, the dealer alleviates the buyer from having to search for his most desired model. As noted above, because of asymmetric information about \( \delta \) in favor of the buyer, it is unlikely that \( \lambda = 1 \). However, since the selection of \( \lambda \) is crucial to the functioning model, this section briefly describes model results with \( \lambda = 1 \).
Suppose that $\lambda = 1$. If the dealer trades for model $j$,

$$V_{ij} = v - p - (1 - \gamma)c - \lambda \delta = v - p - (1 - \gamma)c - \delta$$

If the buyer must travel for model $j$ then

$$V_{ij} = v - p - (1 - \gamma)c - \delta$$

Therefore, $\pi_{ij}$ and $\sigma_{ci}$ are constant over the number of dealer links. In this case, the symmetric pairwise stable equilibrium $M^x$ satisfies

$$\max_{M^x} \{\pi_i(p - I) + M^x \pi_h(p - I - \gamma c + \delta) + (M - M^x - 1)\pi_a(p - I - \gamma c)\}$$

After simplifying the maximand, the optimal $M^x$ satisfies

$$\max_{M^x} \{M^x(\pi_h(p - I - \gamma c + \delta) - \pi_a(v - p - \gamma c))\}$$

Since the buyer gets the same value whether he travels or gets the distant vehicle through the local dealer $\pi_a = \pi_h$. Therefore, the unique symmetric pairwise stable network is the complete network. This result is stated in the following theorem.

**Theorem 5.6.** If $\lambda = 1$, then the complete network is the unique symmetric pairwise stable equilibrium. Since conjecture 7 holds, the symmetric equilibrium is the only pairwise stable equilibrium.

The intuition for this result is clear, since the dealer is able to extract all of the buyer search cost through trade, probabilities of sale are unaffected by the number of trade partners. Therefore, the dealer would much rather trade for the buyers most desired vehicle since he is able to extract the search cost. For the remainder of the paper, $\lambda$ will be set to zero to emphasize the role of substitution buyer substitution through asymmetric information about buyer search cost.

## 6 Optimal Access to Global Inventory

This section explains the incompleteness of the inter dealer trade network as being a property of each dealer’s preferred access to global inventory. Too many trade partners implies too many trades. Severing a link effectively raises the buyers cost of obtaining that model. The buyer can then more likely be sold on other models in stock. Models on the lot typically generate higher profit margins since they don’t incur transportation costs. At the other end of the spectrum, too few trade partners result in an overwhelming amount of lost sales to other Chrysler dealerships and to the alternative make. This dynamic creates a preference for intermediate access to global inventory. The incompleteness of the inter dealer trade network is seen explicitly in Figure 5 and was revealed through discussions with sales managers in charge of dealer trades.

What follows analyzes the optimal number of trade partners in the symmetric equilibrium; that is, where all dealers have the same number of trade partners. By assumption, $i$ doesn’t care with whom he trades, only the total number of options available through trade. The focus here will be the intuition for the optimality of intermediate access to global inventory. Dealer $i$’s expected profit with access to $M^x$ trade partners is

$$\sigma_{C_i}(M^x) [\pi_i(M^x)(p - I) + M^x \pi_h(M^x)(p - I - \gamma c)] + (M - M^x - 1)\sigma_C(M^x)\pi_a(M^x)(p - I - \gamma c)$$

Dealer 1 is able to sell model $i$ from his lot, and trade for $M^x$ models from his partners. $M - M^x - 1$ buyers travel to his location to purchase model $i$. Suppose that $M^x = M - 1$ and that dealer $i$ is contemplating severing a link. If he does, he can no longer sell one model to his domestic buyer but he might be able to sell him the model he has on the lot, at a higher profit margin. Since he has access to many models, if he can’t sell him on the model he has in stock he still might be able to sell him another model he must trade for. Worst case scenario the buyer travels or buys the alternative make but this is unlikely given the dealers wide access to global Chrysler inventory. Further, dealer 1 can now sell model 1 to the buyer that travels from his old trade partner. If dealer $i$ severs the link with dealer $j$, then his expected profit is

$$\sigma_C(M - 2) [\pi_i(M - 2)(p - I) + (M - 2)\pi_h(M - 2)(p - I - \gamma c)] + \sigma_C(M - 2)\pi_a(M - 2)(p - I - \gamma c)$$
If ECV doesn’t change much, then \( \sigma_C \) doesn’t change much. In words, the buyer still likes the Chrysler nest just as much as before. This is likely since the buyer at \( i \) still has wide access to global Chrysler inventory. Since dealer \( i \) can no longer trade for model \( j \), the probability of his buyer choosing this model reduces to \( \pi_a(M-2) \). This reduction in probability is transferred over to the higher margin model and remaining \( M-2 \) models the dealer can trade for. In words, the dealer can more likely sell the buyer off on his higher margin model. Given his wide access to global Chrysler inventory, even if he can’t sell him on the model he owns, its still likely he can trade for the model the buyer wants. Further, since the dealer no longer trades with one dealership, he is able to sell model \( i \) to the buyer traveling from location \( j \). The positive substitution effect from deleting links decreases after some threshold since its more likely the buyer will buy the alternative make or travel to purchase a Chrysler from another dealership. Deleting a link with \( M^x < M-1 \) is a slightly more complicated expression given by

\[
\sigma_C(M^x-1)\pi_a(M^x-1)\pi_c(M^x-1)(p-I) + \pi_a(M^x-1)\pi_c(M^x-1)(p-I) + \pi_a(M^x-1)\pi_c(M^x-1)(p-I) - \gamma c
\]

Next, consider a deviation by adding a link. Start with the empty network, so that \( M^x = 0 \). The expected profit of dealer \( i \) is

\[
\sigma_c(0)\pi_c(0)(p-I) + (M-1)\pi_a(0)\pi_c(0)(p-I) - \gamma c
\]

If dealer \( i \) adds a link with dealer \( j \), then his expected profit is

\[
\sigma_c(1)\pi_a(1)(p-I) + \pi_c(0)(p-I) + (M-2)\pi_a(0)\pi_c(0)(p-I) - \gamma c
\]

That is, he is able to trade for one model for his domestic buyer but loses one sale from the buyer at \( j \). The intuition for why this should be beneficial is that lowering the cost of the buyer obtaining one of the models keeps that buyer from substituting away to the alternative make and other Chrysler dealerships, all of which the dealer gets no profit. The dealer loses some profit from the substitution away from the higher margin model, but this isn’t large since the probability is switched from all the other models he can’t obtain as well. Therefore, adding a link with too few links can be beneficial.

To gain further intuition for this result, I will first consider symmetric equilibria where \( \sigma_c = 1 \) for every \( M^x \). This focuses attention on the channel for interior equilibria that has to do with substitution within the Chrysler network. Recall that higher \( \mu_1 \) indicates greater heterogeneity across nests. Therefore, a constant sigma \( (\sigma_c = 1/2) \) could be obtained by setting \( \mu_1 \) very high.

### 6.1 Symmetric Equilibria: The Role of \( \delta \)

Consider the parametrization with \( p = 50, v = 100, \lambda = 0, I = 10, M = 100, \gamma = .5, c = 20 \) and \( \mu_2 = 10 \). Figure 9 shows the effect of increasing delta on the symmetric equilibrium number of links. The figure shows a clear tendency for the symmetric equilibrium number of links to decrease as search/inconvenience costs increase. The intuition for this result is that higher levels of \( \delta \) imply that the gains from severing links last longer. Higher levels of \( \delta \) imply higher substitution when severing a link. Since the model dealers model in stock has higher net value for the buyer, this additional substitution goes relatively more towards the dealers most profitable model; the model in stock. Further, models the buyer must travel for have very low net value with higher search/inconvenience cost. Therefore, this increased substitution doesn’t increase the probability the buyer prefers to travel by much. This variation in substitution is shown explicitly in section 6.4. Therefore, deleting links increases profit for longer with increased search/inconvenience cost.

### 6.2 Symmetric Equilibria: The Role of \( \lambda \)

This section analyzes the role of \( \lambda \) in determining the pairwise stable symmetric equilibrium. Recall that \( \lambda \) is the amount of buyer search/inconvenience cost the dealer is able to extract by trading. As before, the parametrization is \( p = 50, v = 100, I = 10, M = 100, \gamma = .5, c = 20 \) and \( \mu_2 = 10 \). Figure 7 show the effect of increasing \( \lambda \) from .02 to .016 on the symmetric equilibrium number of links. Increasing \( \lambda \) has the same effect for every \( \lambda \). Figure 7 shows a clear positive relationship between \( \lambda \) and the symmetric equilibrium number of links. Note that \( \lambda \) affects two channels. Since it decreases the buyers net value of trading for a vehicle, it decreases the amount of substitution toward the newly traded model when adding a link. Therefore, adding a link takes less probability away from the dealers more profitable model. However, trading now also increases dealer profit by more since the dealer is able to extract more of buyer search/inconvenience cost. These two channels make it more profitable to add links for longer with higher \( \lambda \).
6.3 Symmetric Equilibrium: The Role of $\gamma$

Recall that $\gamma$ is the fraction of transportation cost the dealer pays when trading for the buyers most desired vehicle. The buyer pays fraction $(1 - \gamma)$. Figure 7 shows a clear negative relationship between $\lambda$ and the symmetric equilibrium number of links for every level of $\delta$. Again $\gamma$ affects the equilibrium through two channels. Increased $\gamma$ increases buyer net value when both traveling and trading. Therefore we should expect similar substitution by severing links. However, the dealer now gets less profit when trading and selling to buyers that travel. Therefore, increasing $\gamma$ implies that adding links doesn’t increase profit for as long as before.

6.4 Symmetric Equilibria: The Role of $\mu_1$ and $\mu_2$

This section relaxes the assumption that $\sigma_c = 1$ for all $M^x$. Recall that higher values of $\mu_2$ indicate lower levels of substitution but also increase the expected Chrysler value. To anchor the value of $AM$ then, it is set to be the average of $ECV$ over $M^x$. Without doing this it would be difficult to differentiate the direct effect of changing the substitutability across Chrysler models from the effect it also has on the relationship between $ECV$ and $AM$.

<table>
<thead>
<tr>
<th>$M^x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ECV$</td>
<td>228.16</td>
<td>233.98</td>
<td>239.48</td>
<td>244.7</td>
<td>249.65</td>
<td>254.37</td>
<td>258.88</td>
<td>263.2</td>
<td>267.33</td>
<td>271.31</td>
</tr>
</tbody>
</table>

Figures 1 and 3 show the effect of $\mu_2$ on $ECV$. Higher values of $\mu_2$ indicate lower levels of substitutability and increase the spread in $ECV$. Intuitively this occurs because higher $\mu_2$ indicate the buyer has wider access to more heterogeneous models. This increases the overall nest value. Since $AM$ is anchored at the midpoint of $ECV$, higher values of $\mu_2$ create a large jump in $\sigma_C$. For example, from Table 2 $\sigma_C$ jumps down from 1 to
Figure 7: Symmetric Equilibrium Number of Links: Vary Delta and Lambda

<table>
<thead>
<tr>
<th>M^x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECV</td>
<td>50</td>
<td>50</td>
<td>50.0001</td>
<td>50.0001</td>
<td>50.0002</td>
<td>50.0002</td>
<td>50.0003</td>
<td>50.0003</td>
<td>50.0004</td>
<td>50.0004</td>
</tr>
</tbody>
</table>

.1894 from the 6th to 5th link. The probability that the buyer wants a Chrysler below the 5th link is zero. This represents the idea that after some threshold the buyer becomes so dissatisfied with the Chrysler dealers access to global inventory that he doesn’t even consider a purchase. Playing around with the substitutability across Chrysler models changes this threshold point of dissatisfaction. As \( \mu_2 \) decreases this threshold property evaporates and the buyer always wants to purchase a Chrysler with 50 percent probability. This is shown in Table 4. Intuitively, as \( \mu_2 \) decreases, the Chrysler models become perfect substitutes so that raising the cost of obtaining one of the models doesn’t change the overall Chrysler dealership value since the buyer simply substitutes to another model. Higher levels of \( \mu_2 \) indicate lower levels of substitutability so that raising the costs of obtaining one model decreases the Chrysler dealership value substantially. Therefore, one way to produce an equilibrium with intermediate access is to increase \( \mu_2 \) so that at some point deleting a link decreases profits substantially because of buyer dissatisfaction.

Figure 9 displays the symmetric equilibrium number of links for \( \mu_2 = 100 \) for \( \delta \) between 0 and 500. The tendency is clear, increasing delta decreases the optimal number of links up until the point where the buyer becomes so dissatisfied that he buys the alternative make with probability one. The intuition for this result is clear. Higher values of delta indicate greater levels of substitution to the other Chrysler models. This can be seen in Figure 10 for the Chrysler model in stock. The \( x \) axis is the change between \( x \) and \( x - 1 \) links. Because the model in stock has a higher expected value than any of the other models that must be traded or traveled for, substitution toward the more profitable model is relatively larger than substitution towards the other models. Figure 11 shows this property. The chart measures the difference in substitution toward the in stock model to the difference in substitution towards models the buyer must travel for. That is,

\[
[\pi_{ii}(M^x - 1) - \pi_{ii}(M^x)] - [\pi_a(M^x - 1) - \pi_a(M^x)]
\]

This difference increases with higher search/inconvenience costs. Therefore, the arbitrage gain from deleting links lasts longer with higher levels of delta. This produces symmetric equilibria with lower levels of global inventory access.
7 Inefficiency of incomplete Symmetric Pairwise Stable Equilibria

For simplicity, this section only considers Chrysler sales. That is \( \sigma_c(M^x) = 1 \) for all \( M^x \). The main result is stated in theorem one. Define the value of network \( g \in G \) to be

\[
V(g) = \sum_i (\text{Profit}_i + CS_i)
\]

That is, the value of network \( g \in G \) is just the sum of expected profit and buyer expected utility. The main result is stated in theorem one.

**Theorem 7.1.** Any interior symmetric equilibrium is inefficient. That is, if \( g \in G \) is a symmetric interior pairwise stable equilibrium then there exists \( g' \in G \) such that

\[
V(g') > V(g)
\]

**Proof.** Suppose that network \( g \in G \) is an interior symmetric pairwise stable equilibria. The value of the symmetric network with \( M^x \) links per node is given by

\[
V(M^x) = \sum_i (\text{Profit}_i + CS_i)
\]

It will be shown that \( V(M^x) \) is maximized at either \( M^x = 0 \) or \( M^x = M - 1 \), thus any interior symmetric equilibria is dominated by either the empty or complete network. Consider then the problem of maximizing \( M^x \). This is the same as

\[
\max_{M^x} \{ \pi_i(M^x)(p - I) + M^x \pi_b(M^x)(p - I - \gamma c) + (M - M^x - 1)\pi_a(M^x)(p - I - \gamma c) \\
+ \pi_i(M^x)(v - p) + M^x \pi_b(M^x)(v - p - (1 - \gamma)c) + (M - M^x - 1)\pi_a(M^x)(v - p - (1 - \gamma)c - \delta) \}
\]
Simplifying the maximand gives
\[
\max_{M^x} \{ \pi_i(M^x)(v - I) + M^x \pi_h(M^x)(v - I - c) + (M - M^x - 1)\pi_a(M^x)(v - I - c - \delta) \}
\]

Note that for \( k = i, h, a \)
\[
\frac{\partial^2 \pi_k}{\partial (M^x)^2} = \frac{\pi_k(M^x)D}{[B(M^x)]^2} > 0
\]
where
\[
D = \exp\left\{ \frac{v - p - \gamma c}{\mu_2} \right\} - \exp\left\{ \frac{v - p - \gamma c - \delta}{\mu_2} \right\}
\]
\[
B(M^x) = \exp\{ \frac{v - p}{\mu_2} \} + M^x \exp\{ \frac{v - p - \gamma c}{\mu_2} \} + (M - M^x - 1)\exp\{ \frac{v - p - \gamma c - \delta}{\mu_2} \}
\]
Define \( \bar{M} = [0, M] \). Since maximizing the convex maximand over compact set \( \bar{M} \) yields extreme point solutions, the optimal \( M^x \) is either 0 or \( M - 1 \).

The next result establishes the efficiency of the symmetric pairwise stable equilibrium when \( \lambda = 1 \).

**Theorem 7.2.** If \( \lambda = 1 \), then the unique efficient network is the complete network.

**Proof.** Suppose that \( \lambda = 1 \). Then, the number of links that maximizes any locations welfare is
\[
\max_{M^x} \{ \pi_i(M^x)(v - I) + M^x \pi_h(M^x)(v - I - c + \delta) + (M - M^x - 1)\pi_a(M^x)(v - I - c - \delta) \}
\]
Since the probabilities \( \pi_j \) for \( j = i, h, a \) are unaffected by \( M^x \), the solution to this problem is \( M^x = M - 1 \). Now suppose there are two unconnected dealers \( j \) and \( k \) in network \( g \in G \). This implies each has fewer than \( M - 1 \) links. Let \( g' = g \cup (j, k) \). By the reasoning just stated, the value at locations \( j \) and \( k \) increases from network \( g \) to \( g' \). Let \( i \) be any other location. If the buyers at locations \( j \) and \( k \) don’t travel to \( i \) to purchase a vehicle in network \( g \), then the value at \( i \) is unchanged. Suppose the buyer at \( j \) travels to location \( i \) to purchase a vehicle in network \( g \). Because the probability \( \pi_a \) from location \( j \) is unaffected by the change in buyer costs, the value at \( i \) is left unchanged. Therefore, the value at each location either increases or stays the same from \( g \) to \( g' \). Therefore, the complete network is the unique efficient network.

This result is standard in microeconomic theory. If each dealer is able to perfectly extract buyer hassle cost, then the unique symmetric equilibrium is efficient. Further, it is the only efficient network. The next result states the result formally.

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Theorem 7.3. If $\lambda = 1$, the unique efficient and unique pairwise stable network is the complete network.

Proof. Combine the results from 5.6 and 7.2

8 Conclusion

This paper analyzed the incompleteness and inefficiency in automobile dealer trade networks. In the model, asymmetric information about buyer search cost is the main source of network incompleteness. Since dealers cannot extract the entire search/inconvenience cost when trading, resisting a dealer trade causes the buyer to substitute away to other models. This substitution can help the dealer when he is able to sell the buyer off on the model in stock. Absence of transportation costs gives this model a higher profit margin. The buyer may also substitute toward another model the dealer can trade for, in which case there is no net change in profit. However, the buyer may just as well travel to purchase his most desired vehicle in which case the dealer loses profit. When the number of links is high, the substitution toward the higher profit margin outweighs loss if the buyer travels, since the buyer attached higher value to model in stock (no transportation cost) and there are relatively few models he must travel for. Its more likely the buyer substitutes toward another model the dealer must trade for than travel, again because the number of links is high. Eventually this net gain in profit by deleting links evaporates as the buyer must travel for more models. Severing an additional link with fewer links has greater substitution toward these distant models. Further, as links are severed the expected value of buying a model at the Chrysler dealership decreases, making it more likely the buyer will purchase the alternative make.

Computational comparative statics suggests that the symmetric equilibrium number of links moves negatively with search/inconvenience costs, positively with the extraction parameter $\lambda$, and negatively with the dealer transportation cost share $\gamma$. In the absence of asymmetric information, the dealer is able to extract the entire search/inconvenience cost when trading. In this case, the unique pairwise stable equilibrium is the complete network. The reason is simple, severing the link doesn’t cause any substitution because the buyer attaches the same value to a traded model and the model he must now travel for. Therefore, the dealer would much rather trade for a model than wait for a distant buyer to travel to him because the traded model has a higher profit margin.

Finally, any symmetric interior pairwise stable equilibrium is shown to be inefficient. Therefore, the incompleteness seen in the Chrysler dealer trade network causes economic inefficiency. Alternatively, if dealers were able to extract the entire search cost, the unique efficient network is the complete network.
Therefore, if $\lambda = 1$ the unique pairwise stable and unique efficient network is the complete network.

This paper sheds light one important characteristic of dealer trade networks; that of network incompleteness. Section gives several other important factors that might be important for network structure. Their implications are saved for future research. The dealer trade network is crucial to understanding the functioning of retail automobile sales (as dealer trade business accounts for a large percentage of transactions), and thus a careful understanding of the network is required for studying several interesting questions. Dealers both compete through competitive pricing and cooperate through trades. This paper shows that network theory can shed some light on this interesting dynamic.

9 References


