Time-Varying Oil Price Volatility and Macroeconomic Aggregates

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Abstract

We illustrate the theoretical relation among output, consumption, investment, and oil price volatility in a New Keynesian model. The model incorporates demand for oil by a firm, as an intermediate input, and by a household, used in conjunction with a durable good. We estimate a stochastic volatility process for the real price of oil over the period 1986-2010 and utilize the estimated process in a non-linear approximation of the model. Two key findings emerge. First, following an exogenous increase in oil price volatility, GDP, non-durable and durable consumption, and capital investment decline. We find support for the decline in real GDP from a SVAR. Second, the quantitative responses to an oil price volatility shock are trivial. Despite the low quantitative significance of oil volatility shocks, we show that historically oil price volatility in the U.S. has had important amplification effects following large oil price level changes.

Keywords: DSGE model, energy, oil price, stochastic volatility
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1 Introduction

Policymakers, Wall Street, consumers, and firms all share a day-to-day concern over the volatility of oil prices.\(^1\) Oil price uncertainty is thought to increase during conflicts in the Middle East and political events involving OPEC members. These types of uncertainty stem from events exogenous to economic activity, but potentially have real effects on the business cycle. Rational expectations imply that increased volatility will change expectations and the behavior of households and firms, as it raises uncertainty surrounding the future path of oil prices. In this paper, we investigate the qualitative and quantitative impacts that changes in oil price volatility at the business cycle frequency have on the macroeconomy using a general equilibrium framework.

Our key contribution is to illustrate the theoretical relation among output, consumption, investment, and oil price volatility in a standard New Keynesian (NK) model. The model incorporates both firm demand for oil as an intermediate input and household demand for oil that is used in conjunction with a durable good. We estimate a stochastic volatility process for the real U.S. price of oil using Bayesian methods, similar to Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez, and Uribe (2011), and use the estimated process in the model. The model is calibrated to match features of U.S. oil consumption and solved to a third order approximation.

By specifically modeling household and firm demand for oil along with oil price uncertainty, we distinguish how the various transmission mechanisms apply distinctively to oil price volatility. In addition, the general equilibrium approach of our analysis provides a coherent framework to study simultaneously the different transmission mechanisms of oil price volatility shocks and to determine which effects dominate overall.

Two key findings emerge. First, following an exogenous increase in oil price volatility, economic activity is dampened. GDP, non-durable and durable consumption, investment, and hours worked decline. This recessionary effect is due to the presence of countercyclical markups due to nominal rigidities, as highlighted by Basu and Bundick (2012). Second, the quantitative responses to an oil price volatility shock are trivial. However, we show that volatility shocks can serve as an important amplification mechanism for oil price level shocks. We feed our estimates of the historical oil price level and volatility shocks into the model and show that oil price volatility had an important influence on macroaggregates in three recent periods: 1) the first Gulf War in 1991, 2) in 1999-2000, and 3) following the recent global financial crisis.

\(^1\)See appendix D for example quotations from FOMC meeting transcripts explicitly discussing oil price uncertainty.
In addition, we provide new empirical support for the dampening effect on real GDP due to increased oil price volatility. We construct an oil price uncertainty measure in a similar spirit as Baker, Bloom, and Davis (2013) based on the frequency of article coverage of oil supply and OPEC uncertainty and estimate a structural vector autoregression (SVAR). Unlike previous works, our analysis attempts to control for general economic uncertainty and provides a measure of oil supply uncertainty. We find that an increase in our oil price uncertainty measure decreases the growth rate of real U.S. GDP.

Most past research on oil prices has centered on the effects of price level changes, but our work is related to a recent, growing literature on the consequences of oil price volatility. The results from the empirical literature are mixed, depending upon the particular model specification and macroeconomic variables considered (see Jo (2013) for a review of the literature). Using a structural VAR, Federer (1996) finds that increased uncertainty leads to declines in industrial production. Guo and Kliesen (2005) finds that increased oil price volatility predicts lower industrial production and nonresidential business fixed investment but no significant effect on durables. However, both of these studies treat the oil price as exogenous, which has been contended (see, for example Kilian (2008) and Alquist, Kilian, and Vigfusson (2013)). Kellogg (2010) finds that uncertainty has an important impact on drilling in the oil industry. Elder and Serletis (2010) finds that uncertainty dampens the mining component of private fixed investment and has a negative effect on durable spending and real GDP, but has no statistically significant impact on private fixed investment net of mining. However, Kilian and Vigfusson (2011b) discuss issues with their econometric approach and inconsistencies between their results and theory.

Most closely related to our modeling framework, Jo (2013) estimates a VAR allowing for stochastic volatility in the oil price. She finds that a doubling of oil price uncertainty leads to a statistically significant decline in world industrial production. In addition, she finds that a doubling of oil price uncertainty lowers real U.S. GDP by 0.14 annualized percentage points. Our empirical approach compliments her analysis and provides further evidence of the negative effect of increased oil price volatility.

Theoretical studies have focused mainly on partial equilibrium analysis (e.g. Bernanke (1983) and Pindyck (1991)). Two recent papers have examined the effects of oil price volatility in general equilibrium models [Baskaya, Hulagu, and Kucuk (2013) and Castillo, Montoro, and Tuesta (2010)]. Our paper complements these works, but differs in important ways. Baskaya, Hulagu, and Kucuk (2013) abstracts from the role of durables and household energy usage and uses a small open economy framework. Castillo, Montoro, and Tuesta (2010) show analytically how a change in the unconditional oil price volatility affects average inflation in a simple New Keynesian model. However, their model abstracts from investment.
decisions and does not consider the dynamic implications of changes in oil price volatility. Our focus is on the dynamic responses of firms and households to changes in oil price volatility, particularly on the effects to both durable and capital investment decisions. Our results suggest the importance of jointly considering firm and household energy usage and quantify the theoretical dynamics implied by an estimated stochastic volatility process for the real oil price.

The paper is organized as follows. Section 2 presents empirical evidence of oil price volatility and its effects on real GDP. Section 3 introduces the model and our estimation of time-varying volatility for the U.S. real oil price. Section 4 examines the dynamic effects of changes in oil price volatility, and section 5 concludes.

2 Empirical Evidence

Before turning to the theoretical model, we present some evidence of the presence of oil price uncertainty and its impact on real GDP. Figure 1 presents various measures of oil price volatility. The dashed, circle line plots the quarterly average of the Chicago Board Options Exchange Crude Oil ETF Volatility Index (OVX), measuring the market’s expectation of 30-day volatility of crude oil prices. The dashed and solid lines present the implied volatility of the 1-month and 3-month ahead futures contracts on West Texas Intermediate crude oil. All three measures display variation across time, suggesting a time-varying component to oil price volatility.

The four measures show increases in the volatility following terrorist attacks and conflicts in the Middle East. Guo and Kliesen (2005) suggest that most of oil price volatility is due to conflicts in the Middle East and political events involving OPEC members. These types of uncertainty stem from events exogenous from economic activity. Our interest is to characterize how uncertainty from such exogenous activities affects macroeconomic activity, particularly GDP. However, the measures in 1 also suggest that oil price uncertainty is correlated with general economic uncertainty, as oil price volatility rose substantially during the 2008 recession. How much of oil price volatility is due to general macroeconomic uncertainty versus oil specific uncertainty, such as political strife in the Middle East? To gauge the affects of exogenous oil specific uncertainty, we first must characterize its prevalence in the overall oil price volatility.

To present a rough guide to answer this question, we construct an oil price uncertainty measure in a similar spirit as Baker, Bloom, and Davis (2013) and estimate a SVAR to

\[\text{OVX} = \text{OVX}_t + \alpha \cdot \text{OVX}_{t-1} + \epsilon_t\]

\[\text{Futures}_1 = \text{Futures}_1 + \alpha \cdot \text{Futures}_{1,t-1} + \epsilon_{1,t}\]

\[\text{Futures}_3 = \text{Futures}_3 + \alpha \cdot \text{Futures}_{3,t-1} + \epsilon_{3,t}\]

\[\text{The futures contracts data are only available since the third quarter of 1993, while the OVX measure started only in 2007.}\]
consider the effects of an identified shock from our constructed oil price uncertainty measure. We hypothesize that uncertainty about the supply of oil, particularly uncertainty related to OPEC members, may be an important component of the variations in the oil price volatility. Because of this, we construct an oil price uncertainty measure based on the frequency of article coverage of oil supply and OPEC uncertainty. Specifically, we count the number of articles posted to the Bloomberg terminal within a quarter that match with the keywords “OPEC” and “uncertainty.” Figure 2 plots this measure from 1993Q3 to 2013Q2, along with some alternative measures with varying keyword combinations with “OPEC.” The measure increases with events in OPEC countries that increase the uncertainty of near-term oil supply. For instance, there are increases following the 9/11 attacks on the World Trade Center and in the first part of 2003 following the U.S. led invasion of Iraq and strikes in Venezuela. We use this measure as a proxy for exogenous OPEC and oil supply uncertainty and estimate a VAR on quarterly data from 1993Q3 to 2013Q2.

We recover orthogonal shocks using a Cholesky decomposition of the following variables: our OPEC uncertainty measure, the Chicago Board Options Exchange Volatility Index (VIX) to control for broader economic uncertainty, the log of real GDP growth, and the implied volatility of the 1-month ahead crude oil futures contract. In the baseline specification, we run the VAR with two lags. Including the OPEC uncertainty measure identifies shocks related specifically to OPEC supply. Including the VIX measure ordered before the implied oil price volatility ensures the impact of general macroeconomic uncertainty is already controlled for when looking at oil price volatility.

Figure 3 plots the responses of the variables to a shock to the OPEC uncertainty measure. Confidence intervals are computed from 5000 Monte-Carlo draws assuming normal innovations. Following an increase in the OPEC uncertainty shock, both the VIX and implied oil price volatility measures increase while GDP growth declines by approximately 0.2 percent. All results are significant as the bands do not encompass zero. Figure 9 in appendix A shows that the results are significant at the 95 percentile as well. Table 1 presents variance decompositions for oil price volatility. Although general economic uncertainty (captured by the VIX shock) explains a significant portion of oil price volatility, more than half of the volatility is explained by the OPEC uncertainty and residual oil price volatility shocks. The OPEC uncertainty shock alone explains about a quarter of the variation.

Appendix A confirms the robustness of these results to a range of alternative approaches over variable ordering, variable definition, lag length, and estimation period. In particular,

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3The Schwartz Criterion suggests 1 lag while the AIC suggests 4. We choose 2 lags as a benchmark since it falls within the range and document in the appendix the robustness of the results to less and more lag lengths.
these results are robust to alternative measures of the OPEC uncertainty article count, alternative measures of oil price volatility, including dummies for the Great Recession, ordering VIX first, including more (and less) lag lengths, and estimating with monthly instead of quarterly data. In addition the appendix documents that responses to a VIX and oil price volatility shock additionally produce declines in GDP growth.

The results of this section suggest that time varying oil price volatility is present in the data, with a sizeable portion of the oil price volatility being attributed to oil specific uncertainty. In addition, the results suggest that increases in oil price uncertainty dampen economic activity. Motivated by this evidence, we turn now to the main discussion of the paper, the theoretical effects of increased oil price volatility.

3  Modeling Oil Price Uncertainty

The model economy is a New Keynesian business cycle model that allows for capital accumulation, firm and household demand for oil products, and durable goods. The economy consists of a representative household, a final goods producer, and intermediate monopolistically competitive goods producers. Oil is imported from abroad at an exogenous world price.\textsuperscript{4} Trade is balanced each period, as oil imports are paid for with exports of domestic output.

The real world price of oil evolves according to a stochastic volatility model. Let $\hat{P}_t^o$ represent log deviations of the real price of oil from steady state. Then the oil price evolves according to

\begin{align}
\hat{P}_t^o &= \rho_p \hat{P}_{t-1}^o + \exp\{\eta_t^p\} \epsilon_t, \\
\eta_t^p &= (1 - \rho_p)\eta_t^p + \rho_p \eta_{t-1}^p + \phi \zeta_t,
\end{align}

where both $\epsilon_t$ and $\zeta_t$ are normally distributed, uncorrelated shocks with mean zero and unit variance. $\epsilon_t$ shocks directly change the price level, while $\zeta_t$ shocks affect the spread of possible price changes and uncertainty about the oil price level.

\textsuperscript{4}We model the oil price as exogenous to focus on oil price uncertainty related to world oil supply or global demand apart from US demand. The results of section 2 suggest these account for substantial variation of the overall oil price volatility. We have confirmed the results presented in this section remain if instead we assume the country imports all oil products and world oil supply evolves according to a stochastic volatility model. Results are available from the authors.
3.1 Firms and Price Setting

The production sector consists of intermediate and final goods producing firms. A perfectly competitive final goods producer uses a continuum of intermediate goods \( y_t(i) \), where \( i \in [0, 1] \), to produce the final goods, \( Y_t \), according to the constant-return-to-scale technology,

\[
Y_t \leq \left[ \int_0^1 y_t(i) \frac{\theta_p - 1}{\theta_p} \, di \right]^{\frac{\theta_p}{\theta_p + p}}. \tag{3}
\]

where \( \theta_p > 1 \) is the price elasticity of demand. We denote the price of good \( i \) as \( p_t(i) \) and the price of final goods \( Y_t \) as \( P_t \).

The final goods producing firm chooses \( Y_t \) and \( y_t(i) \) to maximize profits subject to the technology (3). The demand for \( y_t(i) \) is given by

\[
y_t(i) = \left[ \frac{p_t(i)}{P_t} \right]^{-\theta_p} Y_t. \tag{4}
\]

Maximization implies an index for core CPI as

\[
P_t = \left[ \int_0^1 p_t(i)^{1-\theta} \, di \right]^{\frac{1}{1-\theta}}. \tag{5}
\]

By definition, core inflation is then

\[
\Pi_t = \frac{P_t}{P_{t-1}}. \tag{6}
\]

Intermediate goods producers are monopolistic competitors in their product market. Firm \( i \) produces using the technology

\[
y_t(i) = A_t L_t(i)^{\alpha} [u_t(i) K_{t-1}(i)]^{1-\alpha} - \Omega, \tag{7}
\]

where \( \alpha \in [0, 1] \) and \( \Omega > 0 \) represents fixed costs to production, calibrated so that profits are zero in steady state. Labor demand is given by \( L_t(i) \), and capital services \( K_u \) are the product of the capital stock \( K_{t-1}(i) \), and the utilization rate of capital \( u_t(i) \). As discussed below, oil usage affects the utilization rate of capital, as in Finn (2000). \( A_t \) denotes exogenous technological productivity that follows the stationary AR(1) process \( \hat{A}_t = \rho_a \hat{A}_{t-1} + \sigma^a \epsilon_t, \quad \epsilon_t \sim N(0, 1) \).

Each firm \( i \) chooses labor, capital services, and its price so as to maximize the expected sum of discounted profits given the level of aggregate output and the aggregate price level.
Price adjustments are subject to quadratic adjustment costs relative to the individual firm’s output, similar to Rotemberg (1982):

\[
\frac{\Phi_p}{2} \left( \frac{p_t(i)}{\Pi_{p_{t-1}(i)}} - 1 \right)^2 y_t(i).
\]

where \(\Pi\) is steady state inflation. The maximization problem is given by

\[
\max E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left( \frac{p_t(i)}{P_t} \right)^{1-\theta_p} \left( Y_t - w_t L_t(i) - r_t u_t(i) K_{t-1}(i) - \frac{\Phi}{2} \left( \frac{p_t(i)}{\Pi_{p_{t-1}(i)}} - 1 \right)^2 Y_t \right), \quad (9)
\]

subject to

\[
\left( \frac{p_t(i)}{P_t} \right)^{-\theta_p} Y_t \leq A_t L_t(i)^\alpha [u_t(i)K_{t-1}(i)]^{1-\alpha} - \Omega. \quad (10)
\]

### 3.2 Households

A representative household derives utility from non-durable consumption \(C_t\) and the service flow \(S_t(D, u^d)\) of a pre-determined stock of durable consumption \(D_{t-1}\) with a utilization rate of \(u_t^d\). In addition, the household receives disutility from working. Preferences are given by

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left[ \frac{\sigma_c^{-1} C_t^{\sigma_c^{-1}} + \kappa_1 S_t(D, u^d) \sigma_c^{-1}}{1 - \frac{1}{\tau}} \right]^{\frac{\tau}{\sigma_c}} - \kappa_2 \int_0^1 \frac{L_t(j)^{1+\mu}}{1 + \mu} dj \right\}
\]

where

\[
S_t(D, u^d) = D_{t-1} u^d_t. \quad (11)
\]

The household consists of a unit mass of members who supply differentiated labor services \(L_t(j)\), as in Erceg, Henderson, and Levin (2000). Hours worked are demand-driven. The household works sufficient hours to meet the market demand for the chosen monopolistic wage rates \(W_t(j)\) for each type of labor service \(L_t(j)\). The parameter \(\beta \in (0, 1)\) is the discount factor, \(\tau > 0\) is the intertemporal elasticity of substitution, \(\mu > 0\) the inverse Frisch elasticity of labor supply, and \(\kappa_1\) and \(\kappa_2\) are distribution parameters. The elasticity of substitution between non-durable consumption and the service flow is \(\sigma_c > 0\).
The durable good and the capital stock evolve according to
\[ K_t = [1 - \delta^k_t] K_{t-1} + I_t^k - \frac{\phi_k}{2} \left( \frac{I_t^k}{K_{t-1}} - \bar{\delta}^k \right)^2 K_{t-1}, \]
\[ D_t = (1 - \delta^d_t) D_{t-1} + I_t^d - \frac{\phi_d}{2} \left( \frac{I_t^d}{D_{t-1}} - \bar{\delta}^d \right)^2 D_{t-1}, \]

where \( \omega^k_2, \omega^d_2 > 0 \) imply convex adjustment costs to adjusting the stocks of capital and durables, as in Pindyck and Rotemberg (1983). \( \delta^d \) and \( \delta^k \) are the depreciation rates for the durable good and the capital stock and are a function of the utilization rates:
\[ \delta^k_t = \frac{\omega^k_0}{\omega^k_1} u_t^{k}, \]
\[ \delta^d_t = \frac{\omega^d_0}{\omega^d_1} (u_t^d)^{d}, \]

here \( \omega_0^k, \omega_0^d > 0, \omega_1^k, \omega_1^d > 1 \). The steady state values of the depreciation rates are given by \( \bar{\delta}^k \) and \( \bar{\delta}^d \).

We assume that the utilization rates of the capital and durable goods depend on oil usage \( O^t_l \). Following Finn (2000), energy is essential to the utilizations of capital and durable goods, with increases in utilization requiring more energy usage:
\[ \frac{O^f_t}{K_{t-1}} = \frac{\nu^k_0 \nu^k_1}{\nu^k_1} u_t^{k}, \]
\[ \frac{O^h_t}{D_{t-1}} = \frac{\nu^d_0 (u_t^d)^{d}}{\nu^d_1} \]

where \( \nu^k_0, \nu^d_0 > 0, \nu^k_1, \nu^d_1 > 1 \). Thus, increasing the durable service flow or the amount of capital services provided to firms requires more energy usage. Because of this, changes to the oil price have a direct effect on production and the durable service flow.

The household receives wage income, capital rental income, and dividend payments (\( \Gamma \)) from firms each period. In addition, the household has access to a private nominal one-period risk-free bond that has the gross interest rate \( R_t \) for bonds held from periods \( t \) to \( t + 1 \). Total household expenditures consist of non-durable and oil consumption and capital and durable

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5Loosely speaking, one can view household oil usage as general energy usage from oil inputs, such as gasoline, diesel, and heating oil.
investment. The agent’s real budget constraint is

\[
\int_0^1 W_t(j) L_t(j) dj + r_t u_t K_{t-1} + (1 - R_{t-1}) \frac{B_{t-1}}{\Pi_t} + \Gamma_t = C_t + P_o^k O^k_t + P_o^l O^l_t + I^k_t + I^d_t + B_t + \int_0^1 \frac{\Phi_w}{2} \left( \frac{W_t(j)}{W_{t-1}(j)} - 1 \right)^2 Y_t dj.
\] (18)

Individual real wage adjustments are subject to quadratic adjustment costs relative to the final output good.

A market competitive labor packer combines the differentiated labor services into an aggregated labor product \( L_t \) supplied to firms. The labor packer maximizes

\[
L_t = \left[ \int_0^1 L_t(j)^{\frac{\theta_l - 1}{\theta_l}} dj \right]^{\frac{\theta_l}{\theta_l - 1}}
\]

subject to the

\[
\int_0^1 W_t(j) l_t(j) di = \bar{E}
\]

where \( \bar{E} \) is a given wage bill. Maximization implies demand for individual labor services:

\[
L_t(j) = \left( \frac{W_t(j)}{W_t} \right)^{-\theta_l} L_t
\]

where the aggregate real wage \( W_t \) is equal to

\[
W_t = \left[ \int_0^1 W_t(j)^{1-\theta_l} dj \right]^{\frac{1}{1-\theta_l}}
\]

3.3 Monetary Policy

The monetary authority follows a Taylor-type rule, in which the nominal interest rate \( R_t \) responds to its lagged value, output growth, and the current consumer inflation rate. Specifically, the interest rate is set according to

\[
\frac{R_t}{\bar{R}} = \left( \frac{R_{t-1}}{\bar{R}} \right)^{\rho_r} \left( \frac{\Pi_t}{\bar{\Pi}} \right)^{(1-\rho_r)\rho_{\Pi}} \left( \frac{Y_t}{Y_{t-1}} \right)^{(1-\rho_r)\rho_y} \exp(\sigma_r e^r_t).
\] (19)

where \( \bar{R} \) and \( \bar{\Pi} \) denote the steady state values of the nominal interest rate and gross inflation and \( e^r_t \) is an i.i.d. standard normal shock.
3.4 Aggregation

We focus on a symmetric equilibrium in the goods and labor markets. After imposing the equilibrium, the aggregate resource constraint for the economy is

$$Y_t = C_t + I_t^h + I_t^d + P_o^eO_t^h + P_o^eO_t^l + \frac{\Phi_p}{2} \left( \frac{\Pi_t}{\Pi} - 1 \right)^2 Y_t + \frac{\Phi_w}{2} \left( \frac{W_t}{W_{t-1}} - 1 \right)^2 Y_t.$$ 

(20)

Value-added (GDP) $Y_t^g$ is equal to gross output minus expenditure on intermediate inputs

$$Y_t^g = Y_t - P_o^eO_t^l.$$ 

(21)

3.5 Calibration

Table 2 lists the calibrated values assigned to parameters. We calibrate the model to a quarterly frequency. We assign values to some variables at their steady state, parameters, and the oil price process. The remaining parameters and steady state relationships are either implied by the model’s steady state equations or calculated to match business cycle statistics.

We normalize the real oil price at steady state to one, $\bar{P}_o = 1$. The depreciation rate of capital ($\bar{\delta}_k$) is set to 0.0249, implying an annual rate of 10 percent. The depreciation rate of durables ($\bar{\delta}_d$) is set to 0.0513, implying an annual rate of 20.5 percent. These rates of depreciation match those of the annual Historical Cost Depreciation data from the BEA over 1986-2010. The discount factor, $\beta$, is set to 0.99. We fix $\alpha$ so that labor’s share of output is 70 percent. We fix the utilization rates of steady state capital and durables to 0.832, as measured by Finn (2000).

We calibrate $\tau$ so that the intertemporal elasticity of substitution is 0.5, well within the wide range of estimates in the literature (see Guvenen (2006)). We set $\mu$ so that the Frisch elasticity of labor supply is 0.5. The price elasticity of demand ($\theta_p$) is set to 8, which implies a steady-state product markup of 14 percent, consistent with evidence that the average price markup of U.S. firms is 10-15 percent [Basu and Fernald (1995)]. For symmetry, we also set $\theta_l$ to 8, implying 14 percent average wage markups. We fix the price adjustment fixed cost parameter to be consistent with the slope of a Calvo-type Phillips curve when prices last on average for a year, see Keen and Wang (2007) for the exact relationship. Likewise, we fix the wage adjustment fixed cost parameter to be consistent with the slope of a Calvo-type wage equation when wages last on average one year.

We calibrate the Taylor rule parameters in line with estimates in the DSGE literature over similar sample periods (examples include Smets and Wouters (2007) and Del Negro, Schorfheide, Smets, and Wouters (2007)). Steady state inflation is equal to average inflation
over the period 1986-2010 and implies an annualized rate of approximately 2 percent.

We calibrate $\kappa_2$ so that in the deterministic steady state households devote a third of their time to labor, $\bar{L} = 0.33$. In addition, we fix the durable investment to GDP ratio and household and firm oil usage to GDP ratios to match the average values over the period 1986-2010 in the BEA’s NIPA accounts and data in table 3.6 of the Energy Information Administration’s Annual Energy Review. Following Finn (2000), we measure GDP as model consistent gross domestic product plus energy usage by firms. This makes the output measure consistent with our theory’s gross output $Y$.

The standard deviation of technology, as well as $\phi_d$, $\phi_k$, and the elasticity between consumption and durable services $\sigma_c$, are pinned down to match the volatilities of output, fixed capital investment, durables investment, the nominal interest rate, and consumption to those values observed in the data. To do so, we simulate 1,000 time series each for 500 quarters discarding the first 400 observations to have samples of 100 length, which is the same length as our estimated oil price series. We HP filter each series with $\lambda = 1600$ and compare the average standard deviations implied by the simulations to that in the data.

### 3.6 Measuring Oil Price Volatility

We estimate the law of motion for the real price of oil assuming the oil price follows the stochastic volatility model given by equations (1) and (2).

We use U.S. quarterly data ranging from 1986Q1 to 2010Q4 and calculate the real oil price by dividing the spot price of West Texas Intermediate oil by the core CPI.\(^6\)

Given the nonlinear structure of the stochastic volatility model, we use the sequential importance resampling particle filter to evaluate the likelihood function.\(^7\) In macroeconomics, a particle filter increasingly is used to estimate a stochastic volatility model (recent examples include Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez, and Uribe (2011), Fernandez-Villaverde, Guerron-Quintana, Kuester, and Rubio-Ramirez (2011), and Born and Pfeifer (2011)). We use Bayesian methods and construct the posterior distribution of the oil process parameters using the random walk Metropolis-Hastings algorithm.\(^8\) See appendices

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\(^6\)We start the sample from 1986 for two reasons. First, there is a well documented change in U.S. monetary policy in the 1980s compared to the 1970s. Second, OPEC market power was significantly reduced starting in the mid-1980s due to production decisions by Saudi Arabia. Since then, the data generating process for changes in the quarterly oil price look fairly different from what occurred from 1973 to the mid-1980s. This can be seen in Figure 1 of Guo and Kliesen (2005). For the theoretical exercises we consider, the retail price of energy is the most relevant energy cost measure. We treat the WTI as a proxy for this variable. Kilian (2010) notes some exceptions in this approximation.

\(^7\)See Doucet, de Freitas, and Gordon (2001) and Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez, and Uribe (2011) for more details on the method. We use 5,000 particles for each evaluation of the likelihood.

\(^8\)We sample 350,000 draws from the posterior distribution, discard the first 50,000 draws, thin every 5
Given the lack of guidance for the oil price process parameter values, we employ uniform priors that are a priori independent. The serial correlation parameters $\rho_p$ and $\rho_{\eta p}$ are drawn from uniform priors on the unit interval. We let the average value of stochastic volatility $(\bar{\eta}^p)$ vary uniformly from -20 to 20. The standard deviation of the volatility shock $\phi$ varies uniformly from 0 to 6. The upper bound implies that, on average, the standard deviation of oil price innovations increases at most by an implausible factor of 400 following a positive stochastic volatility shock of one standard deviation.

Table 3 reports the priors along with the mean, median, and 5 and 95 credible intervals from the posterior distributions. The shocks to the level and standard deviation of the oil price process are persistent. The median estimated value of $\bar{\eta}^p$ implies that the oil price innovation has an average standard deviation of 0.087. The median estimated value of $\phi$ implies that a positive stochastic volatility shock of one standard deviation increases the standard deviation of the oil price innovation by a factor of 1.51.

The first panel of figure 4 plots smoothed estimates of the time-varying volatility of the oil price ($\exp(\bar{\eta}^p_t)$), constructed from the sequential monte carlo approximation of the forward-backward smoothing recursion at the posterior median. For most of the sample, the quarterly volatility of the oil price process is low (within one standard deviation of the mean, as seen in the black dashed lines).\(^9\) Larger increases in the volatility occurred in 1990-91 during the Gulf War, in 1999-2000, and following the recent global financial crisis. These three time frames were associated with increases in volatility above 2 standard deviations. In addition, the second panel of figure 4 plots the difference between the actual oil price series and a counterfactual series constructed without the smoothed estimates of time-varying volatility (ie, with the standard deviation held at its mean $\exp(\bar{\eta}^p)$). The differences are noticeable for periods of high volatility.

The estimates suggest that time-varying volatility is a relevant component of the historical oil price process. In what follows, we use the posterior medians from our estimated oil price process to calibrate the oil price process in our model.

### 3.7 Model Solution

Due to our interest in the effects of stochastic volatility, we solve a third-order approximation of the model around the non-stochastic steady state. In the analysis in section 4, we examine the effects of the volatility shock when the price level is held constant. Starting with third-order approximations there are non-zero coefficients attached to the stochastic volatility draws, and perform diagnostic tests to ensure the convergence of the MCMC chain.

\(^9\)Plante and Traum (2012) show that the volatility is higher at a monthly frequency.
term independent of other shocks and variables in the model. The third-order solution, therefore, allows us to consider how a shock to the standard deviation independently affects macro aggregates. Since this can be done independently of the shock to the price level, the volatility shock demonstrates how pure uncertainty about the future oil price affects economic activity.

We solve the model to a third order approximation. We calculate the ergodic mean of model variables and obtain impulse responses by perturbing the ergodic mean by $n\%$ to shock $s$ and subtracting the ergodic mean from the implied path of variables. See Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez, and Uribe (2011) for further details of the approach.

### 3.8 Model Properties

Table 4 lists standard deviations, autocorrelations, and correlations with output of HP-filtered series for both the data and model simulations. The data and the benchmark model properties are listed in the second and third columns, respectively. The model columns display simulated moments in the presence of the technology shock, monetary policy shock, and oil price level and volatility shocks. The top panel in the table gives the percent standard deviations while the second panel lists the volatilities relative to the volatility of value added output. The benchmark model performs fairly well in matching the relative volatilities of most of these variables, even though our calibration did not attempt to match them all. Households do a better job of smoothing consumption in the model than in the data, as the model’s relative consumption volatility is lower than the data’s. This result has been documented previously in the literature for this type of model (see for example Cooley and Prescott (1995) and King and Rebelo (1999)). The calibration also implies responses to an oil price level, monetary policy, or technological productivity shock that are in line with the literature. Section 4 discusses the responses to oil price level and volatility shocks.

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10For discussion of this issue, see Fernandez-Villaverde, Guerron-Quintana, and Rubio-Ramirez (2010), which provides a theorem highlighting the role of higher-order expansions for studying time-varying volatility shocks.

11We solve the model in Dynare. Additionally, we have solved the model using the method of Swanson, Anderson, and Levin (2005) and the authors’ program PerturbationAIM and found the same results. We have also considered a fourth order approximation and found the additional terms to have negligible impacts on the responses to an oil price volatility shock.

12We use an HP parameter of $\lambda = 1600$ for quarterly data. For the model simulations, simulate 1,000 time series each for 500 quarters discarding the first 400 observations to have samples of 100 length, which is the same length as our estimated oil price series.

13Results available in an online appendix.
4 Theoretical Effects of Oil Price Uncertainty

Since our focus is on the effects of oil price uncertainty, namely time-varying conditional heteroskedasticity, we first consider a scenario where $\zeta_t$, the shock to the volatility of the oil price, is increased by two percent while holding fixed the level of the price of oil. A two percent shock captures the large, infrequent increases in the oil price volatility documented in section 3.6 that can have potentially large impact on the economy. In addition, two standard percent volatility shocks are often considered in the literature (see for example Bloom (2009) and Fernandez-Villaverde, Guerron-Quintana, Kuester, and Rubio-Ramirez (2012)). In these experiments, agents continue to pay the same amount for oil but understand that future shocks to the oil price have a larger spread.

Figure 5 presents the results for the baseline calibration given in table 2 with the dashed lines. Following increased oil price uncertainty, economic activity dampens as capital investment, non-durable and durable consumption, hours worked, and real GDP decline. The qualitative features of the responses are driven to a large extent by the precautionary savings motive present in the model. On impact, agents know that the future spread of the oil price has increased, while the price remains unchanged today. This induces a wealth effect leading households to want to increase precautionary savings. Thus, agents are willing to work more on impact to increase income, and in turn savings, to provide higher future income given the heightened future uncertainty. However, the increase in the labor supply decreases marginal costs and causes labor demand to decline, and, in turn, equilibrium labor and output to fall. As GDP/income falls, households lower expenditures. Households have access to two durable assets: the durable good ($D$) and the capital good ($K$). Increased energy cost uncertainty implies that both of these are potentially more costly, leading households to disinvest in both assets.

In contrast, a specification of the model abstracting from monopolistic competition and nominal frictions produces a boom in investment and GDP (see solid lines of figure 5). Without adjustment costs to wages and prices, a willingness to increase labor supply translates into higher equilibrium hours worked and output. Households choose to use the increased income to increase their precautionary savings, increasing investment in physical capital and durable goods while decreasing non-durable spending. These results provide an example of

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14Since at least Sandmo (1970), it has been known that higher uncertainty can lead agents to consume less and save more, i.e. the precautionary savings motive (see Carroll and Kimball (2008) for a survey of the literature). In addition, precautionary savings have been discussed in the context of energy prices in Edelstein and Killian (2009) and Killian and Vigfusson (2011a).

15See Basu and Bundick (2012) for more discussion on the comovement of hours worked and volatility shocks in real business cycle models.
the mechanism in Basu and Bundick (2012), highlighting the importance of nominal rigidities for recessionary uncertainty shocks.

To examine how sensitive the results are to the baseline calibration, we compare the impact responses of output over a range of parameter values. Figure 6 shows the results. Two key results emerge. First, in all cases, output decreases on impact. Since each panel varies one parameter at a time, in all cases there are nominal rigidities present, which leads output to fall on impact. Second, the quantitative effects of oil price volatility shocks are generally small. When the steady state ratio of firm oil usage to GDP \( \frac{Q_O}{Y} \) is high, the impact response of output is largest. However, such large GDP shares of firm oil usage are inconsistent with historical U.S. data. Even with a liberal classification of energy usage including the combination of oil, natural gas, and electricity, the largest ratio of firm energy consumption to GDP in 1970-2011 is 0.09.\(^{16}\) In addition, the monetary authority’s response to inflation and the average length of price contracts (used to calibrate price adjustment costs) have larger impacts on the output response.

Quantitatively, the effects of oil price volatility shocks alone are trivial. However, volatility shocks can serve as an important amplification mechanism of price level shocks. To highlight this issue, we examine how time varying volatility matters in conjunction with oil price level shocks. To explore the joint significance of oil price level and volatility shocks, we first compute smoothed estimates for the two estimated shocks from the stochastic volatility model (equations 1-2) using the sequential monte carlo approximation of the forward-backward smoothing recursion at the posterior median. We then feed the sequence of estimated shocks into the model’s policy functions to determine how model variables evolve following the historical sequence of oil price shocks.\(^{17}\) Figure 7 displays the model-implied paths of several macroaggregates implied by the simulation. The dashed lines demonstrate the paths conditional only on the historical oil price level shocks. The solid lines show the paths conditional on both oil price level and volatility shocks. For most of the time series, the two paths match closely, suggesting that variations in oil price volatility are not a major force behind business cycle fluctuations on average. In addition, the small, quantitative effects suggest that energy price shocks alone do not play a significant role in driving the business cycle.

Figure 7 indicates three noticeable periods in recent history when oil price volatility mattered quantitatively for macroaggregates: 1) the first Gulf War in 1991, 2) in 1999-2000, 3) in 2006-2009.}

\(^{16}\)See the appendix for a description of the data used to make these calculations. In addition, for many industries, the oil price is a secondary component of the investment cost channel, which also can diminish the quantitative importance of oil price volatility shocks (see, for example, Kilian and Park (2009)).

\(^{17}\)We keep technology and monetary policy shocks at zero for the simulation and initialize the model variables at their ergodic steady state.
and 3) following the recent global financial crisis. For instance, in 1999-2000 the real oil price level rose (see dashed lines), while at the same time an increase in oil price volatility further increased the oil price level (see solid lines). Following these increases from 1999Q1 to 2000Q1 (the peak of the increase), capital and durable investment declined by 0.05 and 0.02 percent respectively due to the oil price level shock, and by 0.08 and 0.05 percent respectively due to the combined increase in the oil price level and volatility shocks. The results suggest that during periods of high oil price uncertainty, variations in expected oil price volatility can have important effects on the dynamics of macroaggregates.

To further highlight the amplification mechanism of volatility shocks, we examine the joint impulse responses of price level and volatility shocks. To understand the effects, we first examine how the economy responds to an oil price level shock while keeping the volatility constant. The dashed lines of figure 8 plot the responses following a one percent increase in the real oil price. We then compare the responses when combined with an additional one percent increase in the oil price standard deviation, as seen by comparing the solid and dashed lines of figure 8.

An increase in the oil price level causes households and firms to cut back on oil usage, causing both capital and durable utilization rates to decline. This, in turn, leads investment spending on both capital and durable goods to fall. Prices increase, decreasing the real value of income and inducing households to work more. Finally, higher energy costs and lower real income additionally cause households to decrease non-oil consumption.\footnote{These responses are similar in nature to those found in other works on oil price shocks including Leduc and Sill (2004), Plante (2012), Bodenstein, Erceg, and Guerrieri (2011), Nakov and Pescatori (2010) and Bodenstein, Guerrieri, and Kilian (2012).}

Following an increase in the oil price level, the initial responses of durable and capital investment, output, hours worked, inflation, and consumption are more than doubled when there is a simultaneous increase in oil price volatility (solid lines of figure 8). The oil price more than doubles on impact. The decline in capital investment is 2.5 times higher than the decline from an oil price level shock alone, while the decline in durable investment is 3 times higher. The results are due to the interactions of oil price level and volatility shocks through cross-product terms in higher order solutions of the model, particularly in the higher order approximations of equation 1.

5 Conclusion

There is considerable interest in the uncertainty of oil prices and how it can impact the macroeconomy. In this paper we investigate the relationships among output, consumption,
investment, and oil price volatility in a New Keynesian model that incorporates household and firm demand for oil. We calibrate the model to match business cycle statistics using U.S. data from the 1986 - 2010 period.

Two key findings emerge from our analysis. First, following an exogenous increase in oil price volatility, GDP and other macroaggregates decline. We find support for this result in our SVAR analysis of an oil uncertainty shock. We also find that the quantitative responses to the volatility shock are trivial in size. This result holds for a variety of calibrations considered. However, we show that volatility shocks can serve as an important amplification mechanism for oil price shocks at certain points in time, particularly when there is a large shock to the price level. We feed our estimates of the historical oil price level and volatility shocks into our model and show that it had an important influence on macroaggregates in three recent periods: 1) the first Gulf War in 1991, 2) in 1999-2000, and 3) following the recent financial crisis.

While this paper gives insight into the role of oil price uncertainty, important issues remain. For example, many empirical papers have asked whether uncertainty could cause asymmetric responses in variables to changes in oil prices (for example, Kilian and Vigfusson (2011a), Kilian and Vigfusson (2011b), and Elder and Serletis (2010)). Although we do not explicitly consider this issue, our analysis suggests that understanding the interactions between oil prices and oil price uncertainty is not as simple as adding up the responses from both types of shocks, and that interactions between the two shocks may only be important in specific, historical episodes. In addition, considering the underlying sources that cause stochastic volatility could be important. As shown in Kilian (2009), the responses of macroeconomic variables to an increase in the price of oil does vary depending upon what causes the oil price to increase. It is plausible that a similar story may hold for oil price uncertainty as well. Thus, formally modeling the underlying causes of oil price volatility and the linkages between oil price level and uncertainty effects are potential important avenues for future research.
Table 1: Variance decompositions for implied oil price volatility from VAR.

<table>
<thead>
<tr>
<th></th>
<th>impact</th>
<th>10 year</th>
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<tr>
<td>article count shock</td>
<td>17</td>
<td>25.24</td>
</tr>
<tr>
<td>VIX shock</td>
<td>36.5</td>
<td>45.5</td>
</tr>
<tr>
<td>GDP growth shock</td>
<td>0.07</td>
<td>0.86</td>
</tr>
<tr>
<td>oil price volatility shock</td>
<td>46.3</td>
<td>28.4</td>
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Table 2: Calibration for Benchmark Model.

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
<th>Calibration</th>
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<tbody>
<tr>
<td>τ</td>
<td>Intertemporal Elasticity of Substitution</td>
<td>0.5</td>
</tr>
<tr>
<td>μ</td>
<td>Inverse Frisch Elasticity of Labor Supply</td>
<td>2</td>
</tr>
<tr>
<td>β</td>
<td>Discount Factor</td>
<td>0.99</td>
</tr>
<tr>
<td>α</td>
<td>Share of labor income in gross output</td>
<td>0.70</td>
</tr>
<tr>
<td>L</td>
<td>Steady state Time to Labor</td>
<td>0.33</td>
</tr>
<tr>
<td>𝑢̅</td>
<td>Steady state capital utilization</td>
<td>0.832</td>
</tr>
<tr>
<td>𝑢̅𝑑</td>
<td>Steady state durable utilization</td>
<td>0.832</td>
</tr>
<tr>
<td>δ̅𝑖</td>
<td>Depreciation of durables</td>
<td>0.0513</td>
</tr>
<tr>
<td>δ̅𝑘</td>
<td>Depreciation of capital</td>
<td>0.0249</td>
</tr>
<tr>
<td>θ̅𝑝</td>
<td>Price elasticity of demand</td>
<td>8</td>
</tr>
<tr>
<td>θ̅𝑤</td>
<td>Price elasticity of demand</td>
<td>8</td>
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<tr>
<td>Φ̅𝑝</td>
<td>Price adjustment cost</td>
<td>81.55</td>
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<tr>
<td>Φ̅𝑤</td>
<td>Wage adjustment cost</td>
<td>5852</td>
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<tr>
<td>𝐼̅𝑑</td>
<td>Durable Investment to Output ratio</td>
<td>0.107</td>
</tr>
<tr>
<td>1−𝑝̅𝑜̅</td>
<td>Household Oil Consumption to Output ratio</td>
<td>0.030</td>
</tr>
<tr>
<td>1−𝑝̅𝑜̅</td>
<td>Firm Oil Consumption to Output ratio</td>
<td>0.029</td>
</tr>
<tr>
<td>ρ𝑖</td>
<td>Lagged interest rate response</td>
<td>0.7</td>
</tr>
<tr>
<td>ρπ</td>
<td>Response to inflation</td>
<td>1.25</td>
</tr>
<tr>
<td>ρ𝑦</td>
<td>Response to output growth</td>
<td>0.1</td>
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<tr>
<td>σ𝑐</td>
<td>Elasticity of Substitution between C and S</td>
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<tr>
<td>ω𝑑</td>
<td>Durable adjustment cost</td>
<td>13.3</td>
</tr>
<tr>
<td>ω𝑘</td>
<td>Capital adjustment cost</td>
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</tr>
<tr>
<td>σ𝐴</td>
<td>Standard deviation of technology</td>
<td>0.005</td>
</tr>
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<td>ρ𝐴</td>
<td>Technology AR(1)</td>
<td>0.97</td>
</tr>
<tr>
<td>σ𝑟</td>
<td>Standard deviation of MP shock</td>
<td>0.003</td>
</tr>
<tr>
<td>𝑀𝑜</td>
<td>Steady State Oil Price</td>
<td>1</td>
</tr>
<tr>
<td>𝔇</td>
<td>Steady State Core Inflation</td>
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</tr>
<tr>
<td>𝐴̅</td>
<td>Steady State Technology</td>
<td>1</td>
</tr>
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Table 3: Prior and Posterior Distributions for the Estimated Parameters. Reports the posterior mean, median, and 90% credible interval.

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<th>Parameter</th>
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<th>Prior</th>
<th>Posterior</th>
<th>Posterior</th>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Mean</td>
<td>Median</td>
<td>95% CI</td>
<td>95% CI</td>
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<tr>
<td>ρη𝑝</td>
<td>ρη𝑝</td>
<td>R</td>
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<td>0.57</td>
<td>(0.17, 0.87)</td>
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</tr>
<tr>
<td>ρ𝑝</td>
<td>ρ𝑝</td>
<td>R</td>
<td>0.76</td>
<td>0.77</td>
<td>(0.64, 0.88)</td>
<td></td>
</tr>
<tr>
<td>𝜏̅ 𝑝</td>
<td>𝜏̅ 𝑝</td>
<td>R</td>
<td>-2.44</td>
<td>-2.44</td>
<td>(-2.70, -2.20)</td>
<td></td>
</tr>
<tr>
<td>ϕ</td>
<td>ϕ</td>
<td>R</td>
<td>0.42</td>
<td>0.41</td>
<td>(0.24, 0.62)</td>
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</tr>
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Table 4: Percent standard deviations in the data and models.

<table>
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<tr>
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<th>Data</th>
<th>Model</th>
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<tr>
<td>$Y^g$</td>
<td>1.05</td>
<td>1.74</td>
</tr>
<tr>
<td>$C$</td>
<td>0.88</td>
<td>1.17</td>
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<td>$P^o Q^h$</td>
<td>8.87</td>
<td>10.06</td>
</tr>
<tr>
<td>$I^d$</td>
<td>3.66</td>
<td>2.83</td>
</tr>
<tr>
<td>$I^k$</td>
<td>4.20</td>
<td>4.09</td>
</tr>
<tr>
<td>$R$</td>
<td>0.61</td>
<td>0.37</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>0.24</td>
<td>0.31</td>
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<table>
<thead>
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<th>Model</th>
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</thead>
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<tr>
<td>$Y^g$</td>
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<td>1</td>
</tr>
<tr>
<td>$C$</td>
<td>0.83</td>
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<td>$P^o Q^h$</td>
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<td>5.78</td>
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<td>$I^d$</td>
<td>3.48</td>
<td>1.63</td>
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<tr>
<td>$I^k$</td>
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<tr>
<td>$R$</td>
<td>0.58</td>
<td>0.21</td>
</tr>
<tr>
<td>$\Pi$</td>
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<td>0.18</td>
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<thead>
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<tr>
<td>$Y^g$</td>
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<td>0.81</td>
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<td>$C$</td>
<td>0.92</td>
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<tr>
<td>$P^o Q^h$</td>
<td>0.73</td>
<td>0.51</td>
</tr>
<tr>
<td>$I^d$</td>
<td>0.86</td>
<td>0.8</td>
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<tr>
<td>$I^k$</td>
<td>0.95</td>
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<tr>
<td>$R$</td>
<td>0.98</td>
<td>0.70</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>0.67</td>
<td>0.51</td>
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<table>
<thead>
<tr>
<th>Variable</th>
<th>Data</th>
<th>Model</th>
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<tr>
<td>$Y^g$</td>
<td>1</td>
<td>1</td>
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<tr>
<td>$C$</td>
<td>0.80</td>
<td>0.90</td>
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<tr>
<td>$P^o Q^h$</td>
<td>0.40</td>
<td>0.52</td>
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<tr>
<td>$I^k$</td>
<td>0.37</td>
<td>0.85</td>
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<tr>
<td>$R$</td>
<td>0.21</td>
<td>-0.71</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>-0.15</td>
<td>-0.27</td>
</tr>
</tbody>
</table>
Figure 1: Various measures of oil price volatility. y-axis measures annualized percentage points.

Figure 2: Number of articles in Bloomberg terminal over the quarter with various keywords.
Figure 3: Impulse response and 68 percent bands to OPEC article count shock.

Figure 4: Panel 1: Smoothed estimates of the time-varying volatility of the oil price (exp $\eta_p$), constructed from the posterior median. Solid line: mean estimate of $\eta_p$; Dashed lines: 1 standard deviations of $\eta_p$; Dashed dotted line: 2 standard deviation of $\eta_p$. Panel 2: the difference between the actual log oil price and a counterfactual series without time varying volatility.
Figure 5: Impulse responses to a two percent (normalized standard deviation) increase in the oil price volatility. Dashed lines: benchmark, New Keynesian model. Solid lines: model without nominal rigidities.

Figure 6: Parameter sensitivity analysis. Initial response of output following a two percent (normalized standard deviation) increase in the oil price volatility for the benchmark model. In each panel, all parameters except the one indicated are held at the benchmark calibration values.
Figure 7: Dashed lines: paths of model variables conditional on historical estimates of oil price level shocks; solid lines: paths of model variables conditional on historical estimates of both oil price level and volatility shocks.

Figure 8: Solid lines: impulse responses to a simultaneous one percent (normalized standard deviation) increase in the oil price level and oil price volatility for the benchmark model. Dashed lines: impulse responses to one percent increase in oil price level.
References


A VAR Robustness

Figure 9 shows the results following the OPEC article count shock of the VAR analysis of section 2 are significant at the 95 percentile. In addition, figures 10 and 11 show that increases in the VIX and oil price uncertainty shocks also dampen economic activity. We have documented the robustness of the VAR OPEC article count results to a range of features:

- Ordering the VIX variable first, see figure 12
- Estimating the VAR with 1 lag (given by the Schwartz Criterion), see figure 13
- Estimating the VAR with 4 lags, see figure 13
- Using the historical volatility for the active futures contract instead of the implied volatility.
- Using the historical volatility for the WTI spot price instead of the futures contract.
- Using the implied volatility for the 3-month ahead futures contract instead of the 1-month ahead contract.
- Using GDP level data instead of growth rate.
- Estimating the VAR from 2000 onwards.
- Dummying out the Great Recession (including a dummy in 2008Q4 and 2009Q1).
- Using the combination OPEC article count, which includes article counts for “OPEC” and “uncertainty” or “OEPC” and “unpredictable” or “oil supply” and “uncertainty” or “oil supply” and “unpredictable” or “oil price” and “uncertainty” or “oil price” and “unpredictable”
- Estimating with monthly data
Figure 9: Impulse response and 95 percent bands to OPEC article count shock.

Figure 10: Impulse response and 68 percent bands to oil price volatility shock.
Figure 11: Impulse response and 68 percent bands to the VIX shock.

Figure 12: Impulse response and 68 percent bands to OPEC article count shock. Alternate ordering in VAR: VIX, OPEC uncertainty measure, GDP growth, oil price volatility.
Figure 13: Impulse response and 68 percent bands to OPEC article count shock. 1 lag in VAR.

Figure 14: Impulse response and 68 percent bands to OPEC article count shock. 4 lags in VAR.
B Model Details

2.1 Benchmark Model in Equilibrium

Production:

\[ Y_t = A_t L_t^\alpha [u_t K_{t-1}]^{1-\alpha} - \Omega \]

Firm optimization leads to the expressions

\[ mc_t = \left( \frac{r_t}{1-\alpha} \right)^{1-\alpha} \frac{W_t^\alpha}{\alpha A_t} \]

\[ \frac{W_t L_t}{r_t K_{t-1} u_t} = \frac{\alpha}{1-\alpha} \]

\[ \left[(1-\theta_p) + \theta_p mc_t - \Phi_p \Pi_t \left( \frac{\Pi_t}{\Pi} - 1 \right) + \frac{\theta_p \Phi_p}{2} \left( \frac{\Pi_t}{\Pi} - 1 \right)^2 \right] + \Phi_p \beta E_t \lambda_{t+1} Y_{t+1} \Pi_{t+1} \left( \frac{\Pi_{t+1}}{\Pi} - 1 \right) = 0, \]

where \( mc_t \) represents marginal costs. Technological productivity process:

\[ \log A_t = (1-\rho) \log \bar{A} + \rho \log A_{t-1} + \eta_a \epsilon_t^a \]

where \( \bar{A} = 1 \). Laws of motion for capital and durables:

\[ K_t = (1-\delta_t^k) K_{t-1} + I_t^k - \frac{\phi_k}{2} \left( \frac{I_t^k}{K_{t-1}} - \bar{\delta}^k \right)^2 K_{t-1}, \]

\[ D_t = (1-\delta_t^d) D_{t-1} + I_t^d - \frac{\phi_d}{2} \left( \frac{I_t^d}{D_{t-1}} - \bar{\delta}^d \right)^2 D_{t-1} \]

Depreciation rates:

\[ \delta_t^k = \frac{\omega_0^k}{\omega_k^k} u_t^k \]

\[ \delta_t^d = \frac{\omega_0^d}{\omega_d^d} (u_t^d)^{\omega_d^d} \]

Energy usage connection to utilization:

\[ \frac{O_t^f}{K_{t-1}} = \frac{\nu_0^k u_t^k}{\nu_1^k} \]

\[ \frac{O_t^h}{D_{t-1}} = \frac{\nu_0^d (u_t^d)^{\nu_d^d}}{\nu_1^d} \]
Service flow of durables:

\[ S_t = D_{t-1} u_t^d \]

Let \( \lambda_t \) be the multiplier on the budget constraint, \( \lambda_t^k \) and \( \lambda_t^d \lambda_t \) be the multipliers for the constraints in equations (16) and (17) in the main paper, and \( \chi_t \lambda_t \) and \( \chi_t^d \lambda_t \) be the multipliers on the constraints in equations (18) and (19) in the main paper. Then the first order conditions for the household can be written as

\[
C_t^{\frac{\sigma_c}{\sigma_c - 1}} \left[ C_t^{\frac{\sigma_c - 1}{\sigma_c}} + \kappa_1 S_t(D, u^d) \right]^{\frac{1}{\sigma_c - 1}} = \lambda_t,
\]

\[
\Phi_w \left( \frac{W_t}{W_{t-1}} - 1 \right) \frac{W_t}{W_{t-1}} Y_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \Phi_w \left( \frac{W_{t+1}}{W_t} - 1 \right) \frac{W_{t+1}}{W_t} Y_{t+1}
\]

\[
+ \theta_t \kappa_2 \frac{L_{t+\mu}}{\lambda_t} + (1 - \theta_t) L_t W_t
\]

\[
\lambda_t^d = \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right) \left[ (1 - \delta^d_t) - \frac{\phi_d}{2} \left( \frac{I_{t+1}^d}{D_t} - \delta^d \right)^2 + \phi_d \left( \frac{I_{t+1}^d}{D_t} - \delta^d \right) \frac{I_{t+1}^d}{D_t} \right] - \chi_t^d \mu_0^d (u_{t+1})^{\nu^d_d}
\]

\[
1 = \lambda_t^d \left[ 1 - \phi_d \left( \frac{I_t^d}{D_{t-1}} - \delta^d \right) \right],
\]

\[
P_t^o = \chi_t^d,
\]

\[
\chi_t^d \mu_0^d (u_t^d)^{\nu^d_d-1} D_{t-1} = C_t^{\frac{1}{\sigma_c}} \kappa_1 S_t^{\frac{\sigma_c - 1}{\sigma_c}} (u_t^d)^{-1} - \lambda_t^d \mu_0^d (u_t^d)^{\omega^d_d-1} D_{t-1},
\]

\[
\lambda_t^k = \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right) \left[ (1 - \delta^k_t) - \frac{\phi_k}{2} \left( \frac{I_{t+1}^k}{K_t} - \delta^k \right)^2 + \phi_k \left( \frac{I_{t+1}^k}{K_t} - \delta^k \right) \frac{I_{t+1}^k}{K_t} \right]
\]

\[
- \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right) \chi_{t+1}^{\frac{\nu_1}{\nu_1}} \frac{u_{t+1}}{u_t}
\]

\[
1 = \lambda_t^k \left[ 1 - \phi_k \left( \frac{I_t^k}{K_{t-1}} - \delta^k \right) \right],
\]

\[
P_t^o = \chi_t,
\]

\[
\chi_t \nu_0 u_t^{\nu_1-1} K_{t-1} = r_t K_{t-1} - \lambda_t^k \omega_0^k u_t^{\omega^k_d-1} K_{t-1},
\]

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Monetary Policy:
\[
\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{R}\right)^{\rho_r} \left(\frac{\Pi_t}{\Pi}\right)^{(1-\rho_n)} \left(\frac{Y_t}{Y_{t-1}}\right)^{(1-\rho_y)}
\]

Aggregate resource constraint:
\[
Y_t = C_t + I^k_t + I^d_t + P^o_t\bar{O}^h + P^o_t\bar{O}^f + \frac{\Phi_p}{2} \left(\frac{\Pi_t}{\Pi} - 1\right)^2 Y_t + \frac{\Phi_w}{2} \left(\frac{W_t}{W_{t-1}} - 1\right)^2 Y_t
\]

### 2.2 Implied Steady State Values and Parameters

We normalize the real oil price at steady state to one, \(\bar{P}^o = 1\). In steady state, \(\chi = \chi^d = \lambda^k = \lambda^d = 1\). The capital rental rate, capital stock, wages, output, and fixed costs are solved simultaneously from the system of equations:

\[
\bar{r} = \frac{1}{\beta} - 1 + \bar{\delta}^k + \bar{\delta}^f \bar{K}
\]
\[
\frac{\alpha}{1-\alpha} = \frac{\bar{W}\bar{L}}{\bar{r}\bar{K}\bar{u}}
\]
\[
\Omega = \left(\frac{\bar{r}}{1-\alpha}\right)^{1-\alpha} \left(\frac{\bar{W}}{\alpha}\right)^\alpha
\]
\[
\frac{\Omega}{\bar{L}} = \left(\frac{\bar{K}\bar{u}}{\bar{L}}\right)^{1-\alpha} + \bar{r}\frac{\bar{K}\bar{u}}{\bar{L}} + \bar{W}
\]
\[
\bar{Y} = \bar{L}^\alpha (\bar{K}\bar{u})^{1-\alpha} - \Omega
\]

Given these values, we solve for capital investment:
\[
\bar{I}^k = \bar{\delta}^k \bar{K};
\]

durables:
\[
\bar{D} = \frac{\bar{I}^d}{\bar{\delta}^d}
\]

non-durable consumption:
\[
\bar{C} = \bar{Y} - \bar{I}^k - \bar{I}^d - \bar{O}^h - \bar{O}^f;
\]

services:
\[
\bar{S} = \bar{D}\bar{u}
\]

\(\kappa_1\) distribution parameter:
\[
\kappa_1 = \left(\frac{1}{\beta} - 1 + \bar{\delta}^d + \frac{\bar{O}^h}{\bar{D}}\right) \left(\frac{\bar{C}}{\bar{S}}\right)^{-1/\sigma_c} \left(\frac{\bar{D}}{\bar{S}}\right);
\]
and the $\kappa_2$ distribution parameter:

$$\kappa_2 = \bar{W}L^{-\mu_{\theta_l}} - \frac{1}{\theta_l}$$

### C Data & Estimation

#### 3.1 Data

All data is from the Bureau of Economic Analysis’ NIPA unless otherwise noted. The data is in levels and are nominal values. All real values were calculated by dividing the nominal value with the core CPI. Quarterly data was used to calculate volatilities, while annual data was used to calculate GDP ratios.

The following series were used but not discussed in great detail in the body of the paper:

**Depreciation Rates.** Calculated using BEA data on historical cost net stocks and depreciation.

**Durable Goods Spending.** Personal consumption expenditures (PCE) on “Durable Goods.”

**Investment Spending.** “Fixed Investment” in table 1.1.5 of the NIPA.

**Household Spending on Oil Products.** PCE on “Motor Vehicle Fuels, Lubricants, and Fluids” plus PCE on “Fuel Oil and Other Fuels.”

**(Non-oil) Consumption Spending** - Total PCE minus PCE on “Durable Goods” minus PCE on “Motor Vehicle Fuels, Lubricants, and Fluids,” minus PCE on “Fuel Oil and Other Fuels.”

**Firm Spending on Oil Products.** Table 3.6 of the Energy Information Administration’s (EIA) Annual Energy Review (AER) provides nominal expenditures on petroleum products by the residential, commercial, industrial, and transportation sectors. The transportation sector data includes spending by both households and firms. We subtract PCE on “Motor Vehicle Fuels, Lubricants, and Fluids” from the EIA’s transportation sector data. The remainder is spending in the transportation sector by firms. Total firm spending on oil products is therefore the sum of commercial and industrial spending on petroleum products, plus spending in the transportation sector on petroleum products due to firms.

**Expanded Measures of Energy Spending.** In section 3 we mentioned that energy spending to GDP ratios are higher if one includes spending on natural gas and electricity. Table 3.6 of the EIA’s AER provides nominal expenditure by the residential, industrial, and commercial sectors on both natural gas and electricity. The broader measures of household and firm spending are calculated by adding the totals for spending on oil products with the
3.2 Particle Filter Algorithm

Let \( p^T \) denote \( \{ \hat{P}_t^o \}_{t=1}^T \), which evolves according to equations 1 and 2 in the text. To evaluate the likelihood function \( L(p^T) \), we use a sequential Monte Carlo filter. The algorithm is as follows:

- **Step 1.** Initialize the state variable \( \eta_0^p \) by generating 5,000 values for \( \eta_0^p \) from \( \eta^p \)'s unconditional distribution, \( N(\bar{\eta}^p, (\bar{\eta}^p)^2) \). Denote these particles by \( \eta_{0i}^p \). Draw 5,000 values from \( N(0,1) \) and call each \( \zeta_{1|0i} \). By induction, in period \( t \) these are particles \( \zeta_{t|t-1,i} \).

- **Step 2.** Construct \( \eta_{t|t-1,i} \) using equation 2 in the text. Assign to each draw \( (\zeta_{t|t-1,i}, \eta_{t|t-1,i}) \) a weight defined as:

\[
w_i^t = \frac{1}{(2\pi)^{0.5} \exp(\eta_{t|t-1,i})} \exp \left[ -\frac{1}{2} \left( \frac{\hat{P}_t^o - \rho_p \hat{P}_{t-1}^o}{\exp(\eta_{t|t-1,i})} \right)^2 \right]
\]

- **Step 3.** Normalize the weights:

\[\tilde{w}_i^t = \frac{w_i^t}{\sum_{i=1}^N w_i^t}\]

Update the values of \( \eta_{t|t-1,i} \) by sampling with replacement 5,000 values of \( \eta_{t|t-1,i} \) using the relative weights \( \tilde{w}_i^t \) and the stratified resampling algorithm.

- **Repeat steps 2-3 for** \( t \leq T \).

The likelihood function is approximated by

\[
L(p^T) \simeq \left( \frac{1}{N} \sum_{i=1}^N \frac{1}{(2\pi)^{0.5} \exp(\eta_0^p)} \exp \left[ -\frac{1}{2} \left( \frac{\hat{P}_1^o - \rho_p \hat{P}_0^o}{\exp(\eta_0^p)} \right)^2 \right] \right) \left( \prod_{i=2}^T \frac{1}{N} \sum_{i=1}^N \frac{1}{(2\pi)^{0.5} \exp(\eta_{t|t-1,i})} \exp \left[ -\frac{1}{2} \left( \frac{\hat{P}_t^o - \rho_p \hat{P}_{t-1}^o}{\exp(\eta_{t|t-1,i})} \right)^2 \right] \right)
\]

3.3 MCMC Algorithm

The random walk Metropolis-Hastings algorithm used for the estimation works as follows:
• Step 1. Compute the posterior log-likelihood for 500 draws from the priors. Call the draw with the highest posterior log-likelihood value \( \theta^* \).

• Step 2. Starting from \( \theta^* \), generate a MCMC chain using the following random-walk proposal density

\[
\theta_{j+1}^{\text{prop}} = \theta_j^{\text{prop}} + cN(0, \Sigma), \quad j = 1, \ldots, 100,000
\]

where \( \Sigma \) is the covariance matrix of 5,000 draws from the priors and \( c > 0 \) is a tuning parameter set to determine the acceptance ratio.

• Step 3. Compute the acceptance ratio \( \varphi = \min \left\{ \frac{p(\theta_{j+1}^{\text{prop}} | \theta^T)}{p(\theta_j | \theta^T)}, 1 \right\} \). Let \( u \) be a drawn from a uniform distribution over the unit interval. Then \( \theta_{j+1} = \theta_{j+1}^{\text{prop}} \) if \( u < \varphi \) and \( \theta_{j+1} = \theta_j \) otherwise. Repeat for \( j = 1, \ldots, 100,000 \).

• Step 4. Update the random walk proposal density in the following way. Update \( \Sigma \) to be the covariance matrix from the previous draws \( \{ \theta_j \}_{1}^{100,000} \). Update \( \theta^* \) to be the mean of previous draws \( \{ \theta_j \}_{1}^{100,000} \). Starting from the new \( \theta^* \), proceed through steps 2 and 3 for 350,000 draws from the new MCMC chain.

We burn the first 50,000 draws from the final MCMC chain and thin every 5 draws. The final acceptance rate is 0.33.

D Oil Uncertainty and Policy

The following are direct quotes taken from Federal Reserve FOMC transcripts.

• There are a lot of uncertainties in the outlook for oil that, depending on their resolution, can fundamentally alter the overall course of the economy. Those uncertainties have to be changing the structure of our capital stock, making it somewhat less efficient. They must be creating negatives that we really don’t recognize and don’t get picked up or have never been up by our econometric models. – Chairman Greenspan, pg. 81, Oct. 3, 2000 meeting.

• But I am curious about how sensitive your inflation outlook is to (oil price sensitivity), because there are a lot of volatile personalities who still have powerful positions in places in the world that produce oil. So, who can predict what will happen? – Mr. Jordan, pg. 13, Nov. 16, 1999 meeting.

• We recently had a briefing by a friend of mine who is an oil market operator and he pointed out that we risk an oil supply shock due to unrest in the USSR. That certainly has not been factored into the market. – Mr. Lindsey, pg. 34, August 19, 1992 meeting.
• The oil price volatility and uncertainty about the outlook for oil prices is slowing activity in the District’s oil patch. – Mr. Guffey, pg. 28, February 5-6, 1991 meeting.