Chapter 2. Fiscal Policy, the Variety Effect, and the Real Exchange Rate*

Joo Yong Lee†
Indiana University
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Abstract
This paper explores the effects of government consumption spending in a small open economy both under flexible and fixed exchange rate regimes. Varying price elasticity and the variety effect due to firm entry, are introduced to match the empirical evidences on the small open economy. The main mechanism is that firm entry leads to a higher price elasticity (hence variable markup), and increasing returns to specialization (or the variety effect). For the flexible exchange rate regime, I adopt varying price elasticity and the variety effect to resolve the puzzling empirical evidence: consumption crowding in and a positive co-movement between private consumption and the real exchange rate in response to a government spending shock. For the fixed exchange rate regime, I develop two types of models to replicate heterogeneous country dynamics (as in Chapter 1) in response to an increase in government spending. By properly combining the rule-of-thumb consumers and the variety effect (including varying price elasticity) assumptions, I construct two distinct models. For one model, I employ the variety effect as its main dynamics, for the other, the rule-of-thumb consumers. The main result from three models is that in terms of the stimulus effect, the models under the fixed exchange rate regime represent equivalent or inferior policy effect compared to the variety effect model under the flexible exchange rate regime. This result is different from the conventional wisdom. Another featuring result is that three models display somewhat different ordering in terms of the welfare implication, when compared to the ordering in terms of the stimulus effect.

Keywords: Government Consumption Spending, Small Open Economy, Monetary Policy, Fiscal Policy, Variety Effect, Increasing Returns to Specialization

JEL Classification: E52; E62; F41

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†Department of Economics, 105 Wylie Hall, 100 S. Woodlawn, Bloomington, IN 47405, U.S.A.; lee448@indiana.edu
1 Introduction

Since Keynes (1936), the responses of macroeconomic variables to a government consumption spending shock have been explored intensively. For the closed economy, various transmission mechanisms have been developed to explain the empirical evidences in the VAR literature. Extending the closed economy models to the open economy models, especially to the flexible exchange rate regime models, is challenging. In this paper, I develop models which are capable of replicating the empirical evidences on the small open economy both under the fixed and flexible exchange rate regimes.

One of the topics which open economy theorists attempt to explain, is the small open economy empirical evidences. For the flexible exchange rate regime, the finding of a positive co-movement between private consumption and the real exchange rate in response to a government spending shock,\(^1\) is puzzling for the theoretical models. Intuitively, under the flexible exchange rate regime, a real exchange rate depreciation (or real depreciation) implies that the economy’s long-term real interest rate (or the discounted sum of future real interest rates) falls,\(^2\) which is challenging for the theoretical models to generate.\(^3\) To the fixed exchange rate regime, Chapter 1 adds new findings, i.e., the co-movements between private consumption and the real exchange rate are not unilateral across countries in response to an increase in government spending. This implies that adding the rule-of-thumb consumers to the standard New Keynesian model, is not sufficient to replicate the empirical evidences.

To resolve the existing puzzle and match new empirical findings, I introduce varying price elasticity and the variety effect to the otherwise standard small open economy model. The main mechanism is that firm entry leads to varying price elasticity and increasing returns to specialization (or the variety effect). Varying price elasticity results from the market structure. The economy’s structure is constructed such that an increase in the number of firms within an industry induces each producer to face a more elastic demand curve. This varying price elasticity of demand causes markup to decline and labor demand to increase. Increasing returns to specialization (or the variety effect) comes from firm entry and a specific aggregator (i.e., a Benassy aggregator). Increased firm entry results in a rise in employment, which raises the productivity of labor. At the same time, increased firm entry lowers the price of the final goods relative to the intermediate goods. Hence labor demand increases.

By employing varying price elasticity and variety effect, the model can generate a decrease in the long-term real interest rate. This is done by strengthening the intrasubstitution effect,\(^4\)

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\(^1\)A positive co-movement implies that private consumption increases and the real exchange rate depreciates (or increases), when there is an increase in government spending.

\(^2\)With simple complete markets assumption, the real exchange rate can be represented by the difference between the foreign and domestic long-term real interest rates.

\(^3\)Monacelli and Perotti (2010) call this co-movement a puzzle, since the rule-of-thumb consumers assumption, the non-separable preference assumption, and other assumptions used in the closed economy models, fail to generate this positive co-movement.

\(^4\)As Davig and Leeper (2009) analyze, in the standard New Keynesian model of the closed economy, government spending has an effect on the economy through three channels, i.e., the intratemporal substitution effect, the intertemporal substitution effect, and the negative wealth effect.
which can be considered as an efficiency enhancing mechanism.

Adopting varying price elasticity and the variety effect contributes to resolve the puzzle under the flexible exchange rate regime and to replicate the heterogeneous dynamics under the fixed exchange rate regime. Under the flexible exchange rate regime, even though the short term inflation expectations are rising in response to a government spending shock, the sum of expected future inflations falls due to the efficiency enhancing mechanism. The long term real interest rate decreases, hence a real depreciation takes place. Under the fixed exchange rate regime, the nominal interest rate is fixed by the assumption. Some combinations of the rule-of-thumb consumers and the variety effect (including varying price elasticity) can generate different degrees of the sum of expected future inflations, hence various movements of the real exchange rate.

By setting up the small open economy models both under the flexible and fixed exchange rate regimes, a comparison of policy effectiveness across exchange rate regimes is possible.

Section 2 contains related literature. Section 3 describes a simple closed economy model and dynamics. Section 4 analyzes the inflation and the real exchange rate determination in a simple small open economy model, and develops the model. In addition, policy implications of the exchange rate regime, openness, and the trade elasticity, are explored. Section 5 presents concluding remarks.

2 Related Literature

Under the flexible and fixed exchange rate regimes, theoretical models need to be developed to match the empirical evidences. Under the flexible exchange rate regime, empirical evidence poses a puzzle. It is challenging for the rule-of-thumb consumers (Gali et al. (2007)), the non-separable preference model (Linnemann (2006), and Monacelli and Perotti (2008)), and other models in the closed economy, when applied to the small open economy. These models generate an increase in private consumption, but fail to replicate a depreciation in the real exchange rate (or a real depreciation).\(^5\) For the detailed explanation, one can refer to Monacelli and Perotti (2010). Under the fixed exchange rate regime, simply adopting the rule-of-thumb consumers (Gali et al. (2007)) assumption is incapable of generating heterogeneous dynamics as shown in Chapter 1. In Chapter 1, it is argued that a strong real appreciation may or may not take place, when the Bayesian pooling method is applied.

In developing models, this paper stands in line with firm entry and the variety effect (including varying price elasticity) literature. My work is similar to Jaimovich and Floetotto (2008), and Etro and Colciago (2010), in that I adopt varying price elasticity\(^6\) due to the market structure. The main difference from Jaimovich and Floetotto (2008), is that I employ the variety effect in addition to varying price elasticity. My work is different from that of

\(^5\)Note that a real depreciation is conceptually linked to a decrease in the long term real interest rate under the flexible exchange rate regime.

\(^6\)Varying price elasticity of demand generates countercyclical markup in the flexible prices model. Bilbiie et al. (2007) and Chugh and Ghironi (2011) also introduce the variable elasticity assumption, but their models rely on trans-log preferences to generate a variable elasticity of demand.
Etro and Colciago (2010), in that I separate the degree of the variety effect from the elasticity of substitution across goods,\(^7\) and introduce price stickiness.

This paper differs from Etro and Colciago (2010), and Bilbiie et al. (2007), in that I simplify the firm entry decision to minimally perturb the model from the standard open economy model. Etro and Colciago (2010), and Bilbiie et al. (2007), introduce sunk costs through which a realistic description of firm entry is possible. Introducing sunk costs makes their models generate procyclical firm profits. Instead of developing a detailed entry decision as theirs, I employ a simplifying assumption of constant elasticity firm entry. This simplification makes it convenient to compare the model to the small open economy literature.

This paper extends the closed economy model to a small open economy model following the existing literature. For the general model settings, I follow Galí and Monacelli (2005). Instead of the complete market assumption as in Galí and Monacelli (2005), I introduce the debt elastic interest rate premium following Schmitt-Grohé and Uribe (2003), and Justiniano and Preston (2010).\(^8\) In place of staggered pricing in Calvo (1983), I employ adjustment costs following Rotemberg (1982), and Bilbiie et al. (2007). I adopt Taylor type rules for the flexible exchange rate regime. The policy rule for the fixed exchange rate regime is identical to those in Benigno et al. (2007), Justiniano and Preston (2010), and Corsetti et al. (2011).

3 Simple Closed Economy Models

To explain the mechanism of the variety effect (including varying price elasticity) in a simple and intuitive way, I develop closed economy models by adding assumptions one by one. First, I present a simple real business cycle model with the variety effect, and compare this model to the existing models in the literature. Next, I replace the zero profit firm entry assumption with the constant elasticity entry assumption, which facilitates the introduction of price stickiness. Finally, I introduce price stickiness.

3.1 A Simple Real Business Cycle Model with the Variety Effect

The model is based on the firm entry models of Jaimovich and Floetotto (2008) and Etro and Colciago (2010), where varying price elasticity and increasing returns to specialization (or the variety effect)\(^9\) are key ingredients.\(^10\)

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\(^7\)This separation is essential for generating positive private consumption movement in response to a government spending shock under the plausible parameter values. This separation follows the work of Benassy (1996).

\(^8\)The complete market assumption is good for the intuitive explanation, but it is inferior to other models of Schmitt-Grohé and Uribe (2003) in matching business cycle volatilities.

\(^9\)In a rigorous sense, the variety effect and varying price elasticity are different. In this paper, nonetheless, I sometimes use the variety effect in a general sense to include varying price elasticity.

\(^10\)Jaimovich and Floetotto (2008) distinguish the variety effect parameter (or the parameter which governs the degree of increasing returns to specialization) from the elasticity of substitution, but they do not introduce actual variety effect.
The economy is characterized by a continuum of industries of measure one. In each industry, there is a finite number of intermediate firms which produce a differentiated good. More specifically, in each of the industry, there are \( N_t > 1 \) firms producing differentiated goods that are aggregated into an industrial good by a CES aggregating function. Entry and exit of intermediate producers into existing industries occurs such that a zero profit condition is satisfied in each period in every industry.

### 3.1.1 Household’s Problem

The life time utility of the representative household is given by

\[
E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, H_t),
\]

where \( C_t \) represents private consumption goods and \( H_t \) denotes hours of work. The composite consumption good is composed of \([0, 1]\) measure of different industrial goods, \( C_t(i) \). In each industry \( i \), there is \( N_t \) number of consumption goods, \( C_t(i, j) \). It follows that

\[
C_t \equiv \left( \int_0^1 C_t(i)^{1-\frac{1}{\theta}} \, di \right)^{-\frac{\omega}{\theta}}, \quad C_t(i) \equiv N_t^{\frac{\eta-1}{\theta}} \left[ \sum_{j=1}^{N_t} C_t(i, j)^{1-\frac{1}{\theta}} \right]^{-\frac{\theta}{\eta-1}},
\]

where \( \omega \) and \( \theta \) denote the elasticities of intratemporal substitution. \((\omega < \theta)\)

The parameter \( \eta (\eta > 1) \) governs the degree of increasing returns to specialization. \(\text{The consumption index in an industry, } C_t(i), \text{ represents the Benassy aggregator.}\)

The household’s budget constraint is given by

\[
\int_0^1 \left( \sum_{j=1}^{N_t} C_t(i, j) P_t(i, j) \right) \, di + E_t Q_{t,t+1} B_t \leq B_{t-1} + W_t H_t - P_t T_t,
\]

where \( P_t(i, j) \) denotes the price of good \((i, j)\). \( W_t \) is the nominal wage and \( T_t \) is (real) lump-sum taxes. \( B_t \) indicates purchases of one-period bonds. \( E_t Q_{t,t+1} \) is the price of a one-period discount bond paying one unit of domestic currency in period \( t + 1 \).

Optimal allocation of consumption expenditures gives rise to

\[
C_t(i, j) = \left( \frac{P_t(i, j)}{P_t(i)} \right)^{\frac{\omega}{\theta}} \left( \frac{P_t(i)}{P_t} \right)^{-\omega} \frac{C_t}{N_t^{1-(\theta-1)(\eta-1)}}.
\]

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11. Contrary to the constant measure of industries, the number of firms may vary across periods. Within each industry there is monopolistic competition. As in Rotemberg and Woodford (1992), within industry, goods are assumed to be close substitutes, and across industries, less close substitutes.

12. It is assumed that the elasticity of substitution between any two goods within an industry is higher than the elasticity of substitution across industries, \( \omega < \theta \).

13. Note that when \( \eta = 1 \), \( C_t(i) = C_t(i, j) \) in a symmetric equilibrium.

14. Benassy (1996) breaks the link between the variety effect parameter and the elasticity of substitution. In this paper, this separation plays an important role in generating a positive movement of consumption in response to a government spending shock.
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and

\[ \int_0^1 \left( \sum_{j=1}^{N_t} C_t(i, j) P_t(i, j) \right) \, di = P_tC_t. \]

With optimal allocation condition, the household budget constraint can be rewritten as

\[ P_tC_t + E_tQ_{t,t+1}B_t \leq B_{t-1} + W_tH_t - T_t. \tag{5} \]

Under the specific functional form, the household’s life time utility in the present value terms is given by

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} - \frac{1}{1 + \frac{1}{\varphi}} \right], \tag{6} \]

where \( \sigma > 0 \) denotes the elasticity of intertemporal substitution and \( \varphi \) is the Frisch elasticity. Maximization of the household utility (6) subject to the budget constraint (5) yields the optimality conditions,

\[ \frac{W_t}{P_t} = C_t^{\frac{1}{\sigma}} H_t^{\frac{1}{\varphi}}, \tag{7} \]

\[ Q_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right) \left( \frac{P_t}{P_{t+1}} \right)^{-\frac{1}{\sigma}} \frac{P_t}{P_{t+1}}, \tag{8} \]

\[ E_tQ_{t,t+1} = \frac{1}{R_t}. \tag{9} \]

3.1.2 Government Spending

In this paper, government spending implies only consumption spending, i.e. government investment spending is omitted. In addition, it is assumed that government spending does not create the utility. I assume that public goods have the same structure as private consumption goods, which makes aggregation simple. Hence, demand for government consumption spending is given by

\[ G_t(i, j) = \left( \frac{P_t(i, j)}{P_t(i)} \right)^{-\theta} \left( \frac{P_t(i)}{P_t} \right)^{-\omega} \frac{G_t}{N_t^{1-(\theta-1)(\eta-1)}}, \tag{10} \]

3.1.3 The Elasticity of Demand

In the model, the economy’s structure is such that an increase in the number of firms within an industry, induces \( Y_t(i, j) \) producer to face a more elastic demand curve. There is a continuum of industries, \( i \in [0, 1] \), but within each industry there is a finite number of operating firms, \( (j = 1, \cdots, N_t) \). The implication is that each \( Y_t(i, j) \) producer does not affect the general price level \( P_t \), but it does affect the industrial price level, \( P_t(i) \). In this market structure, the price elasticity of demand faced by a single firm is a function of the
number of firms within an industry, \( N_t \). In a symmetric equilibrium, the price elasticity of demand is given by \(^{15}\)

\[
\varepsilon_{Y(i,j)P(i,j)}(N_t) = \theta + [\omega - \theta] \frac{1}{N_t^\theta}.
\] (11)

In contrast to Dixit and Stiglitz (1977), the price elasticity is not constant. Dixit and Stiglitz (1977) assume that the single firm is small relative to the economy, and therefore does not consider its effect on competing firms. This assumption implies that a \( Y_t(i,j) \) producer has no effect on the price level of an industry, \( P_t(i,j) \), or on the aggregate price level, \( P_t \).\(^{16}\) However, as Yang and Heijdra (1993) emphasize, the assumption in Dixit and Stiglitz (1977) is merely an approximation when a finite number of goods exists. In this case, the price elasticity of demand faced by an individual firm is not constant, but rather a function of the number of competing firms. This implies that each monopolistic producer considers its effect on the price level.

### 3.1.4 Monopolistic Firm’s Problem

Adding up individual demand of good \((i,j)\) yields firm’s demand of

\[
Y_t(i,j) = \left( \frac{P_t(i,j)}{P_t(i)} \right)^{-\theta} \left( \frac{P_t(i)}{P_t} \right)^{-\omega} \frac{Y_t}{N_t^{1-(\theta-1)(\omega-1)}}.
\] (12)

Overhead costs \( \phi \) in each intermediate good producer is introduced to the simple model.\(^{17}\) Overhead costs ensure zero profit even though firms have monopolistic power. Individual producer’s production technology is given by

\[
Y_t(i,j) = z_t h_t(i,j) - \phi.
\] (13)

A solution to the monopolistic firm’s problem has to satisfy the condition that the marginal revenue (with respect to the price) equals the marginal cost (with respect to the price),

\[
\frac{P_t(i,j)}{MC_t(i,j)} = \mu(N_t) = \frac{\theta N_t^\theta + (\omega - \theta)}{(\theta - 1)N_t^\theta + (\omega - \theta)} > 1.
\] (14)

For the detailed derivation, refer to the Appendices. Note that the markup function is monotonically decreasing in the number of firms.

With time varying markup, the monopolistic firm’s conditional demand for hours worked\(^{18}\) is given by

\[
\frac{W_t}{P_t(i,j)} = \frac{z_t}{\mu(N_t)}.
\]

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\(^{15}\)For the detailed derivation, refer to the Appendices.

\(^{16}\)A \( Y_t(i,j) \) producer faces a constant price elasticity of demand, \( \varepsilon_{Y(i,j)P(i,j)} = \theta \). A \( Y_t(i,j) \) producer exploits a constant markup rule \( P_t(i,j)/MC_t(i,j) = \mu = \theta/(\theta - 1) \).

\(^{17}\)Later the overhead costs assumption will be dropped out.

\(^{18}\)The marginal product of \( h_t \) is \( dY_t/dh_t = z_t \). The marginal cost is given by \( MC_t = \frac{dT C/dh_t}{dY_t/dh_t} = W_t/z_t \). Utilizing the definition of the markup, \( \mu(N_t) = P_t(i,j)/MC_t \), produces the conditional demand equation.
3.1.5 ARC and Symmetric Equilibrium

The aggregate resource constraint is given by

\[ Y_t = C_t + G_t. \]

This aggregate relation comes from adding up individual demand of good \((i, j)\).

In a symmetric rational expectation equilibrium, \(\forall (i, j) \in [0, 1] \times [1, N_t], Y_t(i, j) = Y_t^\dagger, h_t(i, j) = h_t, \) and \(P_t(i, j) = P_t^\dagger.\) Aggregate labor \(H_t\) satisfies \(H_t = N_t h_t,\) aggregate output, \(Y_t = Y_t(i) = N_t^\eta Y_t^\dagger,\) and aggregate price index, \(P_t = P_t(i) = N_t^{1-\eta} P_t^\dagger.\) Firm entry ensures that the zero profit condition holds always.

The zero profit assumption implies that \(P_t^\dagger Y_t^\dagger - W_t h_t = 0.\) Combining producer’s production technology, firm’s demand for hours worked, and the zero profit assumption yields

\[ [\mu(N_t) - 1]Y_t^\dagger = \phi. \tag{15} \]

From this zero profit condition, the number of firms per industry and aggregate final output equations are derived. The number of firms per industry is

\[ N_t = z_t H_t \left[ \frac{\mu(N_t) - 1}{\mu(N_t)\phi} \right]. \tag{16} \]

Aggregate final output is given by

\[ Y_t = \frac{1}{\mu(N_t)} (z_t N_t^{\eta-1}) H_t. \tag{17} \]

One can see that \(N_t\) is procyclical and markups are countercyclical from the equilibrium relation,

\[ N_t^\eta = \left[ \frac{\mu(N_t) - 1}{\phi} \right] Y_t, \tag{18} \]

which is derived from (16) and (17).

3.1.6 Numerical Experiment

Figure 1 represents numerical experiments of the simple model with respect to various values of \(\eta.\) When the degree of increasing returns to specialization, \(\eta,\) is greater than 2.3, a positive movement of consumption takes place.

3.1.7 Analytical Solution and Intuition

In this simple model, an analytical solution is available, and derived in the following steps. This analytical solution will clarify the intuition.
Table 1: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calibrated to</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Elasticity of intertemporal substitution</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Frisch elasticity</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Elasticity of substitution within industry</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Elasticity of substitution across industries</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Markup</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Degree of increasing returns to specialization</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of firms in each industry</td>
</tr>
<tr>
<td>$g$</td>
<td>Share of government spending to GDP</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Degree of persistence in AR (1) process</td>
</tr>
</tbody>
</table>

* All parameter values except $\eta$, are widely used in the literature. $\eta$ is not directly comparable to the existing literature, but in terms of degree of increasing returns to specialization, $\eta$ does not deviate much from the widely used values. $\mu$ is calculated from the solution of the firm’s problem.

From the labor supply and demand equations, one can have

$$C^\frac{1}{\sigma} H_t^\frac{1}{\sigma} = \frac{z_t}{\mu(N_t)^{\eta-1}}. \quad (19)$$

Log linearizing, substituting for $\hat{h}_t$, using the ARC, and substituting for $\hat{\mu}_t$ and $\hat{n}_t$, yield

$$\hat{c}_t = -\frac{\Lambda_1}{\sigma + (1 - g)\Lambda_1} \hat{g}_t + \frac{1 + \frac{1}{\varphi}}{\sigma + (1 - g)\Lambda_1} \hat{r}_t,$$

$$\Lambda_1 \equiv \frac{1}{\varphi} - \left(1 + \frac{1}{\varphi}\right) \frac{(\theta - 1)\mu - \theta}{(\theta - 1)\mu} - \left(1 + \frac{1}{\varphi}\right) \frac{\eta - 1}{\eta} \frac{1}{\theta - 1} \frac{1}{\mu - 1}.$$  

For the complete equations and detailed derivation, refer to the Appendices. The solution for $\hat{c}_t$ implies that under the plausible parameter values ($\varphi = 4$ as in Etro and Colciago (2010)), private consumption rises in response to a government spending shock.

In the analytical solution, a rise in private consumption takes place due to the price elasticity effect and increasing returns to specialization (or the variety effect). An intuitive explanation of both effects follows as below.

The price elasticity effect, i.e., more price elastic demand,\(^1\) occurs due to the economy’s structure, and this price elasticity effect leads to the markup decline and increased labor

\(^1\)While each $Y_t(i,j)$ producer does not affect the general price level $P_t$, it does affect the price level of an industry, $P_t(i)$. The price elasticity of demand faced by the single firm is therefore a function of the number of firms within an industry, $N_t$.\[^1\]
Figure 1: Responses to the government spending shock in the real business cycle model. Zero profit entry is assumed with different increasing returns parameter $\eta$. ◊: $\eta = 1.0$, ×: $\eta = 1.1$, □: $\eta = 1.2$, ○: $\eta = 1.3$, *: $\eta = 1.4$

demand. Each $Y_t(i, j)$ firm is a monopolist in the production of its own differentiated product and faces a downward sloping demand curve. The economy’s structure is such that an increase in $N_t$ endogenously increases the price elasticity of demand that each producer faces. More elastic demand implies that the size of the price reduction required for selling an additional unit is lower. (More elastic demand increases the marginal revenue productivity of labor, hence the markup declines. A decline in the markup leads to increased labor demand.)

Increasing returns to specialization (or the variety effect) shifts the labor demand curve outward by changing the relative price between intermediate goods and final goods producers. For the industry as a whole, increased employment through entry raises the productivity of labor. (Refer to equation (17).) This raises the relative price of good facing each firm $(i, j)$ through variety effect. In other words, increased firm entry lowers the price of final goods relative to that of intermediate goods. Hence, labor demand increases. Note that in a symmetric equilibrium, $P_t = P_t(i) = N_t^{1-\eta}P_t^\dagger$, which can be rewritten as $P_t^\dagger/P_t(i) = N_t^{\eta-1}$.

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20In the model, the markup is the marginal product of labor over the real wage. ($\mu_t = MPN_t/(W_t/P_t(i,j))$. The production function is assumed to be linear in labor, i.e., $Y_t(i,j) = z_tH_t(i,j)$. If there is no technology shock, the markup and the real wage show an inverse relationship through labor demand.

21Note that $N_t$ is procyclical. From the zero profit assumption, an increase in demand leads to firm entry.

22For the increasing returns literature, one can refer to Devereux et al. (1996) and Heijdra and Ligthart (2007).
Table 2: Comparison of varying price elasticity models

<table>
<thead>
<tr>
<th>Elasticity</th>
<th>CES</th>
<th>Trans-Log</th>
<th>Benassy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon(N_t) = \theta + [\omega - \theta] \frac{1}{SN_t}$</td>
<td>$\varepsilon(N_t) = \frac{1}{2\sigma N_t}$</td>
<td>$\varepsilon(N_t) = \frac{1}{N_t}$</td>
<td></td>
</tr>
<tr>
<td>Variety effect</td>
<td>$\rho_t = \frac{P_t(i,j)^1}{P_t(i)^1} = N_t^\varepsilon$</td>
<td>$\rho_t = \frac{P_t(i)}{P_t(i)^1} = \exp\left(-\frac{N_t-N_i}{\sigma N_t}\right)$</td>
<td>$\rho_t = \frac{P_t(i,j)^1}{P_t(i)} = N_t^{\eta-1}$</td>
</tr>
<tr>
<td>Markup</td>
<td>$\mu(N_t) = \frac{\theta N_t^{\varepsilon} + (\omega - \theta)}{\theta N_t^{\varepsilon} + (\omega - \theta)}$</td>
<td>$\mu(N_t) = 1 + \frac{1}{\sigma N_t^{\varepsilon}}$</td>
<td>$\mu(N_t) = \frac{\theta N_t^{\varepsilon} + (\omega - \theta)}{(\theta - 1) N_t^{\varepsilon} + (\omega - \theta)}$</td>
</tr>
</tbody>
</table>

3.1.8 Comparison to Existing Models

Separation of the degree of increasing returns to specialization, $\eta$, from elasticity of substitution, $\theta$, is essential for generating the positive response of consumption under the plausible parameter values. Existing models, like Bilbiie et al. (2007), Etro and Colciago (2010), and Chugh and Ghironi (2011), fail to produce a positive movement of consumption under the widely used parameter values, e.g., $\varphi = 4$ and $\theta = 6$.

Separation of $\eta$ from $\theta$ differentiates my model from that of Etro and Colciago (2010). They employ the same economy's structure as in this paper. Even though they adopt varying price elasticity of demand and increasing returns to specialization, the parameter $\eta$ is closely tied to the elasticity of substitution, $\theta$. This binding constraint suppresses the effect of increasing returns to specialization. For the comparison, I solve the simplified version$^{24}$ of Etro and Colciago (2010). When $\eta = \theta/(1 - \theta)$, my model is equivalent to the simplified version of Etro and Colciago (2010). The solution is given by

$$\hat{c}_t = -\frac{\Lambda_1}{\sigma + (1 - g)\Lambda_1} \hat{g}_t + \frac{1 + \frac{1}{\varphi}}{\sigma + (1 - g)\Lambda_1} \hat{z}_t,$$

$$\Lambda_1 = \frac{1}{\varphi} \left[ 1 + \frac{1}{\varphi} \right] \left( \frac{(\theta - 1)\mu - \theta}{(\theta - 1)\mu} - \frac{1 + \frac{1}{\varphi}}{\theta \mu - 1 - \mu - 1} \right).$$

In their model, total effect is not enough to generate a positive movement of consumption under the plausible Frisch elasticity and widely used elasticity of substitution values.

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$^{23}$Etro and Colciago (2010) use the CES aggregator,

$$C_t(i) = \left[ \sum_{j=1}^{N_t} C_t(i,j)^{1 - \frac{1}{\varphi}} \right]^{\frac{1}{1 - \frac{1}{\varphi}}}.$$

Bilbiie et al. (2007), and Chugh and Ghironi (2011), introduce the trans-log preference, which has no explicit form of aggregation. In this paper, the Benassy aggregator (equation (2)) is adopted. The Benassy aggregator isolates the increasing returns parameter from the elasticity of substitution.

$^{24}$I replace the sunk costs assumption in Etro and Colciago (2010) with the zero profit entry assumption.
Adopting a CES type aggregator differentiates my model from that of Bilbiie et al. (2007). Bilbiie et al. (2007) introduce the trans-log preference to generate varying price elasticity of demand. I modify their model to be comparable to my model. The solution is

\[ \hat{c}_t = -\frac{\Lambda_1}{\sigma + (1 - g)\Lambda_1} \hat{y}_t + \frac{1 + \frac{1}{\varphi}}{\sigma + (1 - g)\Lambda_1} \hat{z}_t, \]

\[ \Lambda_1 = \frac{1}{\varphi} \left( 1 + \frac{1}{\varphi} \right) \frac{\mu - 1}{\mu} \frac{1}{2 + \frac{1}{2\sigma N}} - \left( 1 + \frac{1}{\varphi} \right) \frac{1}{4\sigma N + 1}. \]

Their model exhibits relatively larger price elasticity effect and smaller increasing returns effect than in my model. Total effect in their model is not strong enough to generate a positive movement of consumption under the plausible Frisch elasticity value.

3.2 A Variety Effect Model with Constant Elasticity Firm Entry

Next, I replace the zero profit entry assumption with the constant elasticity of firm entry. Introducing an elasticity of firm entry with respect to GDP, can be considered as a simplified version of the existing literature which employs sunk costs and develops the detailed firm entry decision.

3.2.1 Constant Elasticity Firm Entry Setup

The elasticity of firm entry with respect to output is given by

\[ \varepsilon_{N,Y} \equiv \frac{dN_t}{N_t} \frac{Y_t}{dY_t}. \]

In a log linearized form,

\[ \hat{n}_t = \lambda_1 \hat{y}_t \]

where \( \lambda_1 = 0.5 \). In a more general form, the constant elasticity of firm entry is given by

\[ \hat{n}_t = \nu \hat{n}_{t-1} + (1 - \nu) \lambda_1 \hat{y}_t. \]

The result of adopting the constant elasticity is in Figure 2. Various degrees of persistence, \( \nu \), can generate diverse dynamics in firm entry and others. I choose \( \nu = 0.8 \).

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25I replace the sunk costs assumption in Bilbiie et al. (2007) with the zero profit entry assumption.
26Widely used set up in the sunk costs model is as follows. If the production technology is given by \( y_t(i, j) = Z_t h_t(i, j) \), unit cost of production in units of consumption goods is \( W_t/(P_t Z_t) \). Sunk entry costs are defined by \( f_{E,t} W_t/(P_t Z_t) \) where \( f_{E,t} \) denotes effective labor units. Firm entry occurs until the firm value is equalized with sunk entry costs, that is \( v_t = f_{E,t} W_t/(P_t Z_t) \), where \( v_t \) represent the (real) value of the firm at time \( t \) (in consumption units). The firm value is defined by the present value of future profits.
27Refer to Bilbiie et al. (2007), Chugh and Ghironi (2011), and Etro and Colciago (2010).
28Constant elasticity firm entry equations are exactly same as those of the zero profit entry equations by construction. The elasticity is slightly higher that of Broda and Weinstein (2010). Broda and Weinstein (2010) find that a one percentage point increase in sales growth is associated with a rise in net creation (or entry) of 0.35 percentage points.
Figure 2: Responses to the government spending shock in the real business cycle model. Constant elasticity firm entry assumption is introduced with different values of persistence, $\nu$. ○: $\nu = 0.0$, ×: $\nu = 0.2$, □: $\nu = 0.4$, ○: $\nu = 0.6$, ∗: $\nu = 0.8$

3.2.2 The Implication of the Simplifying Assumption

The constant elasticity of firm entry can be considered as a simplification of the sunk costs models in Bilbiie et al. (2007), Chugh and Ghironi (2011), and Etro and Colciago (2010). In the sunk costs model, government spending raises the present (real) value of stock price. Firm entry takes place until the value of a product (the present discounted value of profits) equals to the sunk costs. The constant elasticity of firm entry simplifies these steps, and assumes that firm entry takes place following a constant elasticity with respect to GDP. In spite of this simplifying assumption, I take the interpretation of the sunk costs model as the underlying mechanism of the constant elasticity entry model.²⁹

To ensure similar dynamics as in Bilbiie et al. (2007), Chugh and Ghironi (2011), and Etro and Colciago (2010), a conservative value of the persistence parameter (i.e., $\nu = 0.8$) is chosen. In the sunk costs model, the household should postpone consumption into the future to introduce new varieties.³⁰ Hence firm entry takes time in their models, which is mirrored in the persistence parameter.

The constant elasticity of firm entry assumption brings many advantages, but requires additional explanation or justification. One advantage is that this simplified firm entry decision facilitates the introduction of price stickiness, without perturbing the model from

²⁹Note that the simple constant elasticity model can be extended to the sunk costs model easily if necessary. I take this simplifying assumption just to minimally perturb from the existing small open economy literature and to solve the open economy dynamics analytically.

³⁰The number of firms is interpreted as the capital stock of the economy.
the standard New Keynesian model. One justification for adopting this simplified entry decision, is that the essential ingredients in generating an increase in private consumption, are varying price elasticity and increasing returns, not sunk costs.\textsuperscript{31} Another justification is that the main focus of this paper is on the small open economy. To explain the complicated open economy model, it is useful and productive to make the model as simple as possible. Minimal perturbation from the existing small open economy literature will make a comparison easier.\textsuperscript{32}

### 3.3 A Sticky Price Model with the Variety Effect: Constant Elasticity Firm Entry

Following Rotemberg (1982) and Bilbiie et al. (2008),\textsuperscript{33} I introduce price adjustment costs (i.e., price stickiness) to the previous constant elasticity firm entry model. Adopting adjustment costs brings a few changes to the standard New Keynesian Phillips curve.

#### 3.3.1 Introduction of Price Stickiness

Firstly, the firm level inflation equation is given by

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa (\bar{m}c_t^\dagger + \hat{\mu}_{des}^t). \]

For the detailed derivation, refer to the Appendices for the small open economy model. From the aggregate relation, \( p_t = p_t + (\eta - 1)\hat{n}_t \), one can have the economy level inflation equation,

\[ \pi_t + (\eta - 1)(\hat{n}_t - \hat{n}_{t-1}) = \beta E_t [\pi_{t+1} + (\eta - 1)(\hat{n}_{t+1} - \hat{n}_t)] + \kappa (\bar{m}c_t^\dagger + \hat{\mu}_{des}^t). \]

#### 3.3.2 The Implication of Price Stickiness

Introduction of sticky price generates a few characteristics which are different from the standard New Keynesian Phillips curve. Firstly, even though the prices become flexible enough, the flexible price result is different from that of the real business cycle model. Secondly, the economy’s markup is affected by the various factors other than price stickiness.

In Figure 3, one can find out that there is a gap (or a bias in CPI) between the real business cycle model and the monetary model with flexible prices (i.e., smaller degree of stickiness). In general, when price stickiness is introduced, the real business cycle model and the monetary model differ only in price stickiness. That implies that as price stickiness decreases, the monetary model approaches the real business cycle model. However, in the firm entry model,

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\textsuperscript{31}Detailed firm entry decision with sunk cost is mainly introduced to explain procyclical profits.

\textsuperscript{32}When a detailed firm entry specification is introduced, the effect is similar to introducing capital. It will be another subject to investigate the effect of capital flow in a small open economy. This is beyond the scope of this paper.

\textsuperscript{33}One difference from Bilbiie et al. (2008) is that I do not develop a detailed firm entry decision. Instead I continue to employ a simple constant elasticity entry assumption.
there is another gap between the real business cycle model and the monetary model. One can find the gap from the rewritten Phillips curve,

\[ \pi_t + (\eta - 1)(\hat{n}_t - \hat{n}_{t-1}) = \kappa \sum_{i=1}^{\infty} \beta^{t+i}E_t(\hat{m}_c^t + \hat{\mu}_{des}). \]

Firm entry directly decreases inflation through the bias in the CPI measurement. This bias arises because the CPI measurement does not fully include the variety effect.

To find the implication of countercyclical markup, I rewrite the CPI inflation equation\(^34\) as

\[ \hat{\mu}_t = -\hat{m}_c_t = \frac{1}{\kappa} \beta [E_t(\pi_{t+1} - \pi_t)] : \text{Standard New Keynesian Phillips Curve} \]

\[ - (\eta - 1)\hat{n}_t + \hat{\mu}_{des} : \text{Additional Countercyclical markup} \]

\[ + \frac{\beta}{\kappa}(\eta - 1)(E_t\hat{n}_{t+1} - \hat{n}_t) - \frac{1}{\kappa}(\eta - 1)(\hat{n}_t - \hat{n}_{t-1}) : \text{Bias of the CPI}. \]

In the (standard) New Keynesian Phillips curve (NKPC) with sticky prices, the markup falls as the economy’s demand increases. Each firm has an incentive to raise the price while expanding production to meet additional demand from the public sector. With sticky prices, firms are unable to raise the price as in the flexible prices. Hence, the markup falls.

\(^{34}\)I also use the relation \(\hat{m}_c^t = \hat{m}_c + (\eta - 1)\hat{n}_t\).
In my model, there is an additional effect which strengthens countercyclical markup. Fiscal expansion raises the price elasticity through firm entry (i.e., the desired markup, $\hat{\mu}_{t}^{des}$, declines) and generates the variety effect (i.e., $(\eta - 1)\hat{n}_{t}$ increases). Due to the rise in the price elasticity and the occurrence of the variety effect, firms have incentives to lower the prices when they expand production.

Still another term, called the bias of the CPI (Bilbiie et al. (2008)), strengthens countercyclical markup. Increasing returns to specialization makes the economy level inflation deviate from the firm level inflation. Aggregate inflation is directly affected by firm entry.

4 A Small Open Economy Model with the Variety Effect and the Rule-of-Thumb Consumers

In this section, I extend the variety effect model to an open economy model. The small open economy model is featured by the debt elastic risk premium. A small-large country pair interpretation is adopted. There exist two kinds of goods in the economy, i.e. domestically produced goods (henceforth domestic goods) and imported goods. The law of one price (LOP) is assumed to hold for both exported and imported goods. In addition to the incomplete market assumption, non asset holding households (i.e., rule-of-thumb consumers) are introduced.

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35 Introducing a debt elastic risk premium (or a debt elastic interest rate premium) is one way to avoid the non-stationarity of consumption and foreign debt in a small open economy. (Schmitt-Grohé and Uribe (2003))

36 Justiniano and Preston (2010) point out that the model in Galí and Monacelli (2005) is technically a semi-small open economy model since domestic good producers have some market power. Nonetheless, Justiniano and Preston (2010) interpret the model as a small open economy. This kind of treatment can be found in the international economics literature. Even, the Mundell-Flemming-Dornbusch model implicitly employs this market power. (Obstfeld and Rogoff (1996))

37 Even though this specification is quite common in policy analysis, more explanation is needed for this setup. Monacelli and Perotti (2010) decompose the movement of the real exchange rate into two parts. One part is affected by tradable goods, and the other, non-tradable goods. They find that two decomposed real exchange rates display similar results, but they emphasize that the role of non-tradable is non-negligible. One justification of considering tradable goods only, is that tradable goods explain well the behavior of total movement of the real exchange rate. Another justification is that, if researchers are interested in the role of non-tradable goods, the same dynamics can be applied to the non-tradable and tradable goods environment easily.

38 In spite that the economy is considered to be a semi small open economy, the portion of the small open economy is assumed to be negligible. Hence the law of one price (LOP) is assumed to hold for the exported goods as well.
4.1 Description of Model

4.1.1 Households’ Problem

Under the rule-of-thumb consumers assumption, households’ problems are separated into asset holding and non asset holding households problems.

A representative asset holding household seeks to maximize the present value utility

\[ E_t \sum_{i=0}^{\infty} \beta^{t+i} \left[ \frac{(C_{A,t}^{H})^{1-\frac{1}{\sigma}} \varphi \left( H_{t+i}^{A} \right)^{1+\frac{1}{\varphi}}}{1-\frac{1}{\sigma}} - \nu \left( H_{t+i}^{A} \right)^{1+\frac{1}{\varphi}} - 1 \right] \],

where \( \beta \) represents a discount factor. \( \sigma > 0 \) is the elasticity of intertemporal substitution and \( \varphi \) is the Frisch elasticity. \( C_{A,t}^{H} \) denotes private consumption goods of asset holding household. \( H_{t}^{A} \) is hours of work of asset holding household.

Consumption, \( C_{A,t}^{H} \), is a composite consumption index of domestic goods and foreign goods (or imported goods):

\[ C_{A,t}^{H} = \left[ (1 - \alpha)^{\frac{1}{\gamma}} (C_{H,t}^{A})^{\frac{1}{\omega}} + \alpha^{\frac{1}{\gamma}} (C_{F,t}^{A})^{\frac{1}{\omega}} \right]^{\frac{\omega}{\omega - 1}}, \]

where \( \alpha \in [0,1] \) is the import ratio and can be interpreted as the degree of openness. \( \gamma \) denotes the elasticity of substitution between domestic goods and imported goods.

The domestic (or foreign) composite consumption good is composed of \([0,1]\) measure of differentiated industrial goods, \( C_{H,t}^{A}(i) \) (or \( C_{F,t}^{A}(i) \)). In each industry \( i \), there is \( N_{t} \) (or \( N_{t}^{*} \)) number of consumption goods, \( C_{H,t}^{A}(i,j) \) (or \( C_{F,t}^{A}(i,j) \)). The consumption indexes are given by

\[ C_{H,t}^{A}(i) = \left( \int_{0}^{1} C_{H,t}^{A}(i)^{1-\frac{1}{\omega}} di \right)^{\frac{\omega}{\omega - 1}}, \]

\[ C_{F,t}^{A}(i) = \left( \int_{0}^{1} C_{F,t}^{A}(i)^{1-\frac{1}{\omega}} di \right)^{\frac{\omega}{\omega - 1}}, \]

\[ C_{H,t}^{A}(i,j) = N_{t}^{\eta-\frac{\omega}{\omega - 1}} \left[ \sum_{j=1}^{N_{t}} C_{H,t}^{A}(i,j)^{1-\frac{1}{\omega}} \right]^{\frac{\omega}{\omega - 1}}, \]

\[ C_{F,t}^{A}(i,j) = \left( \sum_{j=1}^{N_{t}^{*}} C_{F,t}^{A}(i,j)^{1-\frac{1}{\omega}} \right)^{\frac{\omega}{\omega - 1}}, \]

where \( \omega \) and \( \theta \) represent the elasticities of intertemporal substitution. \( \omega < \theta \) The parameter \( \eta \) \( (\eta > 1) \) governs the degree of increasing returns to specialization.\(^{39}\) The consumption index in an industry, \( C_{H,t}^{A}(i) \), (or, \( C_{F,t}^{A}(i) \)), represents the Benassy aggregator.

Assuming that the available assets are domestic (government) bonds and foreign bonds,\(^{40}\)

\(^{39}\)Increasing returns to specialization comes from love for variety. (Ethier (1982)) The basic Ethier (1982) effect is: more diversity in the differentiated goods sector enables final goods producers to use a more roundabout production process, which lowers unit cost.

\(^{40}\)Hence I introduce incomplete market assumption.
the household’s budget constraint can be written by

\[
\int_{0}^{1} \left( \sum_{j=1}^{N_t} C_{H,t}(i,j)P_{H,t}(i,j) \right) di + \int_{0}^{1} \left( \sum_{j=1}^{N_t} C_{F,t}(i,j)P_{F,t}(i,j) \right) di + \varepsilon_t D_t^F + D_t^G
\]

\[
\leq \varepsilon_t R^*_{t-1} D_t^F \phi_t(A_{t-1}) + R_{t-1} D_{t-1}^G + W_t H_t^A + \Pi_t^A - P_t T_t^A, \tag{22}
\]

where \( D_t^G \) represents one period domestic (government) bonds, and \( D_t^F \), one period foreign bonds. \( R_{t-1} \) and \( R^*_{t-1} \) denote the interest rates corresponding to domestic bonds and foreign bonds respectively. \( \varepsilon_t \) is the nominal exchange rate. \( P_{H,t}(i,j) \) denotes the price of domestic good \((i,j)\), and \( P_{F,t}(i,j)\), the price of imported good \((i,j)\). \( H_t^A \) represents hours of work, and \( W_t \) is the nominal wage. \( \Pi_t^A \) is profits from holding shares. \( T_t^A \) denotes (real) lump-sum taxes. \( \phi_t \) represents a risk premium for holding foreign bonds.

Optimal allocation of consumption expenditure yields the conditional demand curves and price indexes:

\[
C_{H,t}(i) = \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\omega} C_{H,t}^A, \quad C_{F,t}(i) = \left( \frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\omega} C_{F,t}^A,
\]

\[
P_{H,t} = \left[ \int_{0}^{1} P_{H,t}(i)^{1-\omega} di \right]^{\frac{1}{1-\omega}}, \quad P_{F,t} = \left[ \int_{0}^{1} P_{F,t}(i)^{1-\omega} di \right]^{\frac{1}{1-\omega}},
\]

and

\[
C_{H,t}(i,j) = \left( \frac{P_{H,t}(i,j)}{P_{H,t}(i)} \right)^{-\theta} \frac{C_{H,t}^A(i)}{N_t^{1-(\theta-1)(\eta-1)}}, \quad C_{F,t}(i,j) = \left( \frac{P_{F,t}(i,j)}{P_{F,t}(i)} \right)^{-\theta} \frac{C_{F,t}^A(i)}{N_t^{1-(\theta-1)(\eta-1)}},
\]

\[
P_{H,t}(i) = N_t^{\frac{\theta}{\eta} - \eta} \left[ \sum_{j=1}^{N_t} P_{H,t}(i,j)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \quad P_{F,t}(i) = (N_t^*)^{\frac{\theta}{\eta} - \eta} \left[ \sum_{j=1}^{N_t} P_{F,t}(i,j)^{1-\theta} \right]^{\frac{1}{1-\theta}},
\]

where \( 1 - (\theta-1)(\eta-1) < 1 \). Combining the conditional demand curves gives

\[
C_{H,t}(i,j) = \left( \frac{P_{H,t}(i,j)}{P_{H,t}(i)} \right)^{-\theta} \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\omega} \frac{C_{H,t}^A(i)}{N_t^{1-(\theta-1)(\eta-1)}}, \quad C_{F,t}(i,j) = \left( \frac{P_{F,t}(i,j)}{P_{F,t}(i)} \right)^{-\theta} \left( \frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\omega} \frac{C_{F,t}^A(i)}{N_t^{1-(\theta-1)(\eta-1)}}.
\]

Optimality condition with the definition of price indexes, \( P_{H,t} \) and \( P_{F,t} \), and quantity indexes, \( C_{H,t}^A \) and \( C_{F,t}^A \), yields

\[
\int_{0}^{1} \left( \sum_{j=1}^{N_t} C_{H,t}^A(i,j)P_{H,t}(i,j) \right) di = C_{H,t}^A P_{H,t}, \quad \int_{0}^{1} \left( \sum_{j=1}^{N_t} C_{F,t}^A(i,j)P_{F,t}(i,j) \right) di = C_{F,t}^A P_{F,t}.
\]
By assuming symmetry across goods \((i, j)\)’s and numbers of firms \((N_t \text{ and } N_t^*)\), optimal allocation of expenditure between domestic and imported goods is given by

\[
C^A_{H,t} = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\gamma} C^A_t, \quad C^A_{F,t} = \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\gamma} C^A_t. \tag{23}
\]

\(P_t\) denotes the CPI which can be written as

\[
P_t \equiv \left[ (1 - \alpha) P_{H,t}^{1-\gamma} + \alpha P_{F,t}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}. \tag{24}
\]

Hence, total consumption expenditure is given by

\[
P_t C^A_t = R_t C^A_{H,t} + R_t^* C^A_{F,t} = P_t C^A_t.
\]

The representative asset holding household maximizes the present value utility, \((20)\), subject to the budget constraint

\[
P_tC^A_t + \varepsilon_t D^F_t + D^G_t \leq \varepsilon_t R_{t-1} D^F_{t-1} \phi_t(A_{t-1}) + R_{t-1} D^G_{t-1} + W_t H_t^A + \Pi_t^A - P_t T_t^A. \tag{25}
\]

A debt elastic risk premium assumption is introduced. The function \(\phi_t(\cdot)\) is interpretable as the debt elastic risk premium and is given by\(^41\)

\[
\phi_t = \exp\{-\chi(A_{t-1} - A)\},
\]

where

\[
A_{t-1} = \frac{\varepsilon_{t-1} B^F_{t-1}}{Y P_{t-1}}.
\]

Optimality conditions are

\[
\frac{1}{R_t} = E_t \left[ \beta \left( \frac{(C^A_{t+1})^{1-\sigma}}{P_{t+1}^{1-\sigma}} P_t \right) \right],
\]

\[
\frac{1}{R_t} \varepsilon_t = E_t \left[ \beta \left( \frac{(C^A_{t+1})^{1-\sigma}}{P_{t+1}^{1-\sigma}} P_t \right) \varepsilon_{t+1} \phi_{t+1}(A_{t+1}) \right].
\]

A representative non asset holding household (or, rule-of-thumb consumer) maximizes the utility

\[
\left( C^N_t \right)^{1-\frac{1}{\sigma}} - \frac{1}{1 - \frac{1}{\sigma}} - v \left( H^N_t \right)^{1+\frac{1}{\varphi}} - \frac{1}{1 + \frac{1}{\varphi}},
\]

subject to the budget constraint

\[
P_tC^N_t \leq W_t H_t^N - P_t T_t^N.
\]

\(^41\)This set up is slightly different from that of Schmitt-Grohé and Uribe (2003). I follow the description of Justiniano and Preston (2010).
Aggregate variables are given by
\[ C_t = \delta C_t^N + (1 - \delta) C_t^A, \]
\[ H_t = \delta H_t^N + (1 - \delta) H_t^A, \]
\[ (1 - \delta) D_t^G = B_t^G, \]
\[ (1 - \delta) D_t^F = B_t^F, \]
where \( \delta \) denotes the share of the non asset holding households.

In this economy, it is assumed that the wage is set by unions. In the non competitive labor market, the wage setting is given by
\[ \frac{W_t}{P_t} = C_t^\frac{1}{\sigma} H_t^\frac{1}{\phi}. \]

For the detailed derivation refer to the Appendices.

### 4.1.2 Government Consumption Spending

In this paper, government spending implies only consumption spending, i.e. government investment spending is omitted. Public goods are assumed to be composed only of domestic goods. These public goods are measured in terms of the domestic price index. It is assumed that public goods have the same structure as private consumption goods.

Demand for government consumption spending is given by
\[ G_t \equiv \left( \int_0^1 G_t(i)^{1 - \frac{1}{\theta}} di \right)^{\frac{1}{1 - \theta}} \]
\[ G_t(i) \equiv N_t^{\eta - \theta} \left[ \sum_{j=1}^{N_t} G_t(i, j)^{1 - \frac{1}{\theta}} \right]^{\frac{\theta}{\phi - \theta}}. \]

Combining conditional demands yields
\[ G_t(i, j) = \left( \frac{P_{H,t}(i, j)}{P_{H,t}(i)} \right)^{-\theta} \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\omega} \frac{G_t}{N_t^{1 - (\theta - 1) (\eta - 1)}}. \quad (26) \]

### 4.1.3 Prices and the Uncovered Interest Rate Parity Condition

Unlike the closed economy, various prices exist, and these prices need to be defined. The CPI is a weighted average of the domestic price level and the import price level (in terms of domestic currency) as in equation (24). In addition, the terms of trade (TOT) is defined by the ratio of the import price level and the domestic price level in domestic currency terms, that is,
\[ S_t \equiv \frac{P_{F,t}}{P_{H,t}}. \quad (27) \]

In the traditional trade theory, the reverse of this definition has been widely used. Compared to the traditional definition, the definition in this paper has the advantage that the real exchange rate (REER) and the terms of trade (TOT) show the same direction.
I assume that the law of one price (LOP) holds in the import and export sectors. The real (effective) exchange rate (REER), $Q_t$, is defined as

$$Q_t \equiv \frac{\varepsilon_t P_t^*}{P_t},$$

(28)

where $P_t^*$ is the world (or foreign) price of world (or foreign) goods. $\varepsilon_t$ denotes the nominal exchange rate. The real exchange rate (REER) measures the CPIs between home and world (or foreign) in same currency units.\(^{43}\)

The uncovered interest rate parity (UIP) condition\(^ {44}\) is given by

$$R_t = E_t \left[ R_{t+1} \frac{\varepsilon_{t+1}}{\varepsilon_t} \phi_{t+1}(A_t) \right].$$

The uncovered interest rate parity (UIP) condition implies that in equilibrium there is no arbitrage between two assets, domestic (government) bonds and foreign bonds.

### 4.1.4 Domestic Firm’s Problem

Following Rotemberg (1982),\(^ {45}\) I introduce adjustment costs (in pricing) to firm’s maximization problem. Intermediate firm’s real profit (in terms of the domestic price level) is given by

$$\Pi_t(i,j) = P_{H,t}(i,j) Y_t(i,j) - MC_t P_{H,t} Y_t(i,j) - \left( \frac{P_{H,t}(i,j)}{P_{H,t}(i,j)} - 1 \right)^2 Y_t.$$

The last term represents adjustment costs in changing prices. Each firm maximizes the present value of real profit. Firm’s problem is

$$\max_{P_{H,t}} E_t \sum_{k=0}^{\infty} Q_{t+k} \left[ \frac{P_{H,t+k}(i,j)}{P_{H,t+k}} \left( \frac{P_{H,t+k}(i,j)}{P_{H,t+k}(i)} \right)^{-\theta} \left( \frac{P_{H,t+k}(i)}{P_{H,t+k}} \right)^{-\omega} \frac{Y_{t+k}}{N_{t+k}^{1-(\theta-1)(\eta-1)}} - \frac{MC_{t+k}}{P_{H,t+k}} \left( \frac{P_{H,t+k}(i,j)}{P_{H,t+k-1}(i,j)} - 1 \right)^2 Y_{t+k} \right].$$

The first order condition yields

$$\frac{Y_t(i,j)}{P_{H,t}} + \frac{P_{H,t}(i,j)}{P_{H,t}} \frac{dY_t(i,j)}{dP_{H,t}(i,j)} - \frac{MC_t}{P_{H,t}} \frac{dY_t(i,j)}{dP_{H,t}(i,j)} - K \left( \frac{P_{H,t}(i,j)}{P_{H,t-1}(i,j)} - 1 \right) \frac{Y_t}{P_{H,t-1}(i,j)} = 0.$$

\(^{43}\)Since the portion of the small open economy (SOE) is negligible, the world economy has only the CPI, $P_t^*$.\(^ {44}\)This condition comes from the optimality condition of the asset holding household.\(^ {45}\)For the firm entry model, adjustment costs in Rotemberg (1982) are more plausible than the staggered price adjustment.
Note that in a symmetric equilibrium,
\[ Y_t(i, j) = Y_t^\dagger, \quad Y_t = N_t^\eta Y_t^\dagger, \]
\[ P_{H,t}(i, j) = P_{H,t}^\dagger, \quad P_{H,t} = N_t^{1-\eta} P_{H,t}^\dagger. \]

4.1.5 Firm Entry Dynamics

Firm entry is described by
\[ N_t = (N_{t-1})^\nu (N_t^Y)^{1-\nu}. \]
Inertia (or persistence) in firm entry decision is introduced. \( N_t^Y \) represents firm entry resulting from the output fluctuation. The constant elasticity firm entry assumption is applied to \( N_t^Y \).

4.1.6 Policy Rules, ARC, and Identities

The monetary policy rule\(^{46}\) is given by
\[ r_t = \left( \frac{\pi_t}{\pi} \right)^{\phi_e}, \quad \text{or} \quad r_t = \left( \frac{\varepsilon_t}{\varepsilon_{t-1}} \right)^{\phi_e}. \] (29)

The fiscal policy rule is
\[ \frac{\tau_t - \tau}{Y} = \phi_b \left( \frac{b_{t-1} - b}{Y} \right). \] (30)

The government budget constraint is given by
\[ \frac{P_{H,t} G_t}{P_t} = \tau_t + B_{t-1} G_{t-1} - R_{t-1} B_{t-1} - C_{H,t} + C^*_t + G_t. \] (31)

The aggregate resource constraint (ARC) implies\(^{47}\)
\[ Y_t = C_{H,t} + C^*_t + G_t. \] (32)

The foreign aggregate resource constraint (ARC) is given by
\[ Y^*_t = C^*_t + G^*_t. \]

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\(^{46}\)The policy rule for the fixed exchange rate regime is adopted following Benigno et al. (2007), Justiniano and Preston (2010), and Corsetti et al. (2011). This rule is consistent with the rational expectation model.

\(^{47}\)From the individual resource constraint, I derive
\[ Y_t(i, j) = C_{H,t}(i, j) + C^*_t(i, j) + G_t(i, j) = \left( \frac{P_{H,t}(i, j)}{P_{H,t}(i)} \right)^{-\theta \omega} \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\omega} \frac{1}{N_t^{1-(\theta-1)(\eta-1)}} \left( C_{H,t} + C^*_t + G_t \right). \]
The trade balance (or net exports) is represented by

\[ NX_t = \left( Y_t - \frac{P_t}{P_{H,t}} C_t - G_t \right) \]  

(33)

Note that the trade balance is in domestic price. The current account identity is given by

\[ \varepsilon_t B^F_t - \varepsilon_{t-1} B^F_{t-1} R^*_t - R^*_t = P_{H,t} N X_t. \]

### 4.1.7 Competitive Equilibrium

A stationary competitive equilibrium in this economy consists of a sequence of prices, \( \{P_t, P^*_t, P_{H,t}, P_{F,t}, W_t, \varepsilon_t, S_t, Q_t\} \), and sequences of allocations, \( \{C^A_t, C^N_t, H^A_t, H^N_t, B^G_t, B^F_t\} \) for domestic households, \( \{C_{H,t}, C_{F,t}\} \) for domestic final goods producers, \( \{Y^*_t, N_t\} \) for domestic intermediate good producers, \( \{Y^*_t, C^*_t, G^*_t\} \) for foreign economy, and \( \{NX_t\} \) for current account, a sequence of nominal interest rates, \( \{R_t, R^*_t\} \), determined by the monetary policy rules and a sequence of fiscal variables, \( \{\tau_t, G_t\} \), determined by the fiscal authority, such that (i) taking prices and the monetary and fiscal policy rules as given, the households’ allocations solve their utility maximization problems; (ii) taking prices and the monetary and fiscal policy rules as given, the producers’ allocations solve their profit maximization problems; (iii) markets for final goods, intermediate goods, labor, and bonds all clear.

### 4.2 Calibration

For numerical experiments, I set up the parameter values. A period corresponds to a quarter. Discount factor \( \beta \) is set to 0.99. I assume the elasticity of intertemporal substitution, \( \sigma \), takes the value of unity. The Frisch elasticity, \( \varphi \), is set to 4 as in Etro and Colciago (2010). The elasticity of substitution within industry is set to 6 which is the widely used value as in Rotemberg and Woodford (1992). The markup takes the value of 1.3, as in Jaimovich and Floetotto (2008).\(^{48}\) The elasticity of substitution across industries, is set to 1. The degree of price stickiness is set to 0.17 which is equivalent to the degree of price stickiness in Calvo-type pricing. I assume \( \eta = 1.3 \) for the variety effect model, and \( \eta = 1.0 \) for the model without the variety effect. The number of firm in each industry is set to 2.3, which is derived from the steady state relation. I set \( \lambda_1 = 0.5 \) which is slightly above empirical evidence in Broda and Weinstein (2010).\(^{49}\) I assume \( \nu = 0.8 \), such that the degree of persistence in firm entry is chosen conservatively. The degree of openness is set to 0.4, the widely used value in the small open economy literature. The ratio of the rule-of-thumb consumers, \( \delta \), is set to 0.4, 0.2, or nil. The Taylor rule coefficient for inflation, \( \phi_\pi \), is set to 1.5. I assume \( \phi_e = 500 \) to ensure the nominal exchange rate fixed. I set the tax policy coefficient, \( \phi_b \), to 0.1. I set the

\(^{48}\)In Jaimovich and Floetotto (2008), the variety effect is not introduced. The difference of markups in Jaimovich and Floetotto (2008) and this paper, is negligible.

\(^{49}\)In Broda and Weinstein (2010), a one percentage point increase in sales growth is associated with a rise in net creation (or entry) of 0.35 percentage points.
adjustment speed parameter, $\chi$ to 0.01, which is used in Justiniano and Preston (2010). The share of government spending to GDP is set to 0.2. I set the share of government spending to debt, to 0.07. I set the persistence in AR (1) process to 0.9.

4.3 Price Determination in a Simple Small Open Economy Model

I investigate the flexible exchange rate regime dynamics and the fixed exchange rate regime dynamics separately, since dynamics under the flexible exchange rate regime differ from those under the fixed exchange rate regime.

To investigate core price dynamics of a small open economy, I make the model as simple as possible. I assume an endowment economy.\(^5\) I abstract from the rule-of-thumb consumers, and the variety effect. The small open economy model is featured by a debt elastic risk premium. The model is the small open economy with infinitely lived agents. A small-large country pair interpretation is adopted. There exist two kinds of goods in the economy, i.e. domestic goods and imported goods. In each period, the small open economy and the foreign economy are endowed with outputs, $y_t$ and $y^*_t$. The uncovered interest rate parity (UIP) condition holds. The law of one price (LOP) is assumed to hold in both domestic and imported goods.

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5 Even in the endowment economy, trade takes place.
4.3.1 Price Dynamics under Flexible Exchange Rate Regime

Under the flexible exchange rate regime, domestic inflation, $\pi_{H,t}$, and the terms of trade, $\hat{s}_t$, include the forecast error terms. Specific parameter conditions are required for the difference equations to be solved. The nominal exchange rate, $\hat{e}_t$, adjusts to ensure the equilibrium.\footnote{Note that the terms of trade (or the real exchange rate) contains the entire path of the internal short-term real interest rate, $\hat{r}_t - E_t \pi_{H,t+1}$, and the risk premium. This is evident from the uncovered interest parity equation, $\hat{s}_t = E_t \sum_{i=0}^{\infty} \left[ \hat{r}^*_{t+i} - \pi^*_{t+i+1} - (\hat{r}_{t+i} - \pi_{H,t+i+1}) \right] - E_t \sum_{i=0}^{\infty} \chi \hat{a}_{t+i}$.}

The risk premium, $-\chi \hat{a}_t$, can be determined by the terms of trade. (Refer to the below equations.)

The dynamics of the simple model can be represented by 3 equations,

$$\hat{s}_t = E_t \sum_{i=0}^{\infty} \left[ \hat{r}^*_{t+i} - \pi^*_{t+i+1} - (\hat{r}_{t+i} - \pi_{H,t+i+1}) \right] - E_t \sum_{i=0}^{\infty} \chi \hat{a}_{t+i}.$$  

The risk premium, $-\chi \hat{a}_t$, can be determined by the terms of trade. (Refer to the below equations.)

The dynamics of the simple model can be represented by 3 equations,

$$\sigma(\phi_x \pi_{H,t} - E_t \pi_{H,t+1}) = (\alpha \sigma - A) E_t \Delta \hat{s}_{t+1} - \frac{1}{1 - \alpha} \frac{1}{(1 - \rho_y) \hat{y}_t} + \frac{\alpha}{1 - \alpha} (1 - \rho_y^*) \hat{y}^*_t,$$

$$\hat{r}_t - E_t \pi_{H,t+1} = \hat{r}^*_t - \rho \pi^*_t \pi^*_{t+1} + E_t \Delta \hat{s}_{t+1} - \chi \hat{a}_t,$$

$$\hat{a}_t = \frac{1}{\beta} \hat{a}_{t-1} = \left( A - 2 \alpha + \frac{1}{\beta} \right) \hat{s}_t - \frac{1}{\beta} (1 - \alpha) \hat{s}_{t-1} - \frac{\alpha}{1 - \alpha} \hat{y}_t + \frac{\alpha}{1 - \alpha} \hat{y}^*_t - \frac{1}{\beta} \pi^*_t + \frac{1}{\beta} \hat{r}^*_{t-1},$$

where $A \equiv \{(1 - \alpha) \alpha \gamma + \alpha \gamma\}/(1 - \alpha)$. For the complete set of equations and detailed derivations, refer to the Appendices. For the first equation, I substituted the ARC, the price relations, and the policy function into the consumption Euler equation. The second equation is derived from the uncovered interest rate parity condition and the price relations. For the third equation, I started from substituting the ARC into the net exports equation. Then I substitute the net exports representation into the current account identity. Rearranging the equation by using price relations, yields the third equation.

If $\phi_x > 1$, the system has two eigenvalues greater than 1. (\(\lambda_1 = \phi_x, 0 < \lambda_2 < 1, \) and $\lambda_3 > 1$.) Hence, the policy rule, $\hat{r}_t = \phi_x \pi_{H,t}$, and the condition, $\phi_x > 1$, ensure equilibrium determinacy of the simple model.\footnote{If the difference equations for domestic inflation can be solved, so can the difference equations for terms of trade.}

Next, to solve the model conveniently, I abstract from the foreign exogenous shocks.\footnote{This kind of price determination arises in all types of model in Schmitt-Grohé and Uribe (2003).}
Three equations are rewritten as
\[
\begin{align*}
\sigma(\phi_\pi \pi_{H,t} - E_t \pi_{H,t+1}) &= (\alpha \sigma - A)E_t \Delta \hat{s}_{t+1} - \frac{1}{1 - \alpha}(1 - \rho_y)\hat{y}_t, \\
\phi_\pi \pi_{H,t} - E_t \pi_{H,t+1} &= E_t \Delta \hat{s}_{t+1} - \chi \hat{a}_t, \\
\hat{a}_t - \frac{1}{\beta} \hat{a}_{t-1} &= \left( A - 2\alpha + \frac{1}{\beta} \right) \hat{s}_t - \frac{1}{\beta}(1 - \alpha)\hat{s}_{t-1} - \frac{\alpha}{1 - \alpha} \hat{y}_t.
\end{align*}
\]
When \( \chi \neq 0 \), the terms of trade, \( \hat{s}_t \), which includes the forecast error term, can be solved. The solution to this equation is given in the Appendices.

In this simplest model, the terms of trade is determined in the following way. The real net foreign asset (or the risk premium) is affected by net exports and the terms of trade. Hence, the uncovered interest rate parity condition implies that the difference equations for the terms of trade can be solved as long as the difference equations for the internal real interest rate, \( \hat{r}_t - E_t \pi_{H,t+1} \), are solved.

Note that in the flexible price model, both \( \hat{r}_t = \phi_\pi \pi_t \) and \( \hat{r}_t = \phi_\pi \pi_{H,t} \) are consistent with the standard determinacy condition. When stickiness is introduced to \( \pi_t \), \( \hat{r}_t = \phi_\pi \pi_t \) gives the same determinacy condition as the Taylor type policy rule in the closed economy. When stickiness is introduced to \( \pi_{H,t} \), \( \hat{r}_t = \phi_\pi \pi_{H,t} \) yields the same determinacy condition as the Taylor type rule in the closed economy.

### 4.3.2 Price Dynamics under the Fixed Exchange Rate Regime

Under the fixed exchange rate regime, nominal exchange rate, \( \hat{e}_t \), and terms of trade, \( \hat{s}_t \), include the forecast error terms. Specific parameter conditions are required for the difference equations to be solved. Domestic inflation, \( \pi_{H,t} \), adjusts to ensure the equilibrium.

There are the same 3 equations under the fixed exchange rate regime. The only difference is the policy rule. By using the price relations, one can rewrite 3 equations as

\[
\begin{align*}
\sigma(\phi_e \Delta \hat{e}_t - E_t \Delta \hat{e}_{t+1}) &= -[\sigma(1 - \alpha) + A]E_t \Delta \hat{s}_{t+1} - \frac{1}{1 - \alpha}(1 - \rho_y)\hat{y}_t + \frac{\alpha}{1 - \alpha}(1 - \rho_y^*)\hat{y}_t^*, \\
\phi_e \Delta \hat{e}_t - E_t \Delta \hat{e}_{t+1} &= -\chi \hat{a}_t + \hat{r}_t^*, \\
\hat{a}_t - \frac{1}{\beta} \hat{a}_{t-1} &= \left( A - 2\alpha + \frac{1}{\beta} \right) \hat{s}_t - \frac{1}{\beta}(1 - \alpha)\hat{s}_{t-1} - \frac{\alpha}{1 - \alpha} \hat{y}_t + \frac{\alpha}{1 - \alpha} \hat{y}_t^* - \frac{1}{\beta} \hat{\pi}_t^* + \frac{1}{\beta} \hat{\pi}_{t-1}^*.
\end{align*}
\]

where \( A \equiv [(1 - \alpha)\alpha \gamma + \alpha \gamma]/(1 - \alpha) \).

If \( \phi_e > 1 \), the system has two eigenvalues greater than 1. (\( \lambda_1 = \phi_e, \ 0 < \lambda_2 < 1, \) and \( \lambda_3 > 1 \).) Hence, the policy rule, \( \hat{r}_t = \phi_e \Delta \hat{e}_t \), and the condition, \( \phi_e > 1 \), ensure equilibrium determinacy of the simple model.

In this simplest model, the terms of trade is determined in the following way. The real net foreign asset (or the risk premium) is affected by net exports and the terms of trade. Since net exports depend on the terms of trade, the risk premium is determined by the terms
of trade. Hence, the uncovered interest rate parity condition implies that the difference equations for the terms of trade can be solved as long as the difference equations for the external real interest rate, \( \hat{r}_t - E_t \Delta \hat{e}_{t+1} \), are solved.

### 4.3.3 Determinacy Condition

Adding the variety effect feature and the rule-of-thumb consumers to the simple model, does not change the determinacy condition. Not included in this paper, numerical experiments confirms that the determinacy condition with the variety effect under sticky prices shows the same condition as in the simple model. Note that unless the ratio of the rule-of-thumb consumers are extraordinary high, the determinacy condition is not affected by the assumption. This aspect is well recorded in the literature.

### 4.4 Transmission Mechanism

I start with the closed economy and explain how the closed economy transmission mechanism changes with openness. I explore the transmission mechanism for three basic models, that is, the New Keynesian model with the variety effect (including varying price elasticity) under the flexible exchange rate regime, the New Keynesian model with the variety effect (including varying price elasticity) under the fixed exchange rate regime, and the New Keynesian model with the rule-of-thumb consumers under the fixed exchange rate regime.

#### 4.4.1 Transmission Mechanism of the Variety Effect Model under the Flexible Exchange Rate Regime

To explain the transmission mechanism in response to a government spending shock, I use closed economy as a benchmark, since the closed economy mechanism can be described easily.\(^{55}\) Considering the imaginary real exchange rate in a closed economy, I first investigate the effect of openness on the real exchange rate. Next, I turn to three channels affecting private consumption in the closed economy, and explore how the introduction of the real exchange rate changes these channels. Finally, I investigate the effect on GDP by considering private consumption and net exports together.

**The Effect on Domestic Inflation and the Real Exchange Rate**

I focus on the determination of domestic inflation and the real exchange rate in a small open economy. Basic dynamics of the model imply that output (or GDP), domestic inflation, and real exchange rate, are determined together.\(^{56}\) Hence I investigate first how domestic inflation and the real exchange rate are determined when openness is introduced.

\(^{55}\)Note that \( Y_t = C_t + G_t \) in a closed economy. By describing the effect of government spending on private consumption, one can describe the whole mechanism.

\(^{56}\)Note that in a more general set up of section 3.4, GDP is treated as an endogenous variable and the Phillips curve is added to the simple model. The dynamics include three forecasting errors. GDP, domestic inflation, and the real exchange rate, are three variables which need to be determined together.
The first column of Figure 4 presents that an increase in government spending leads to a less increase in GDP, and a less depreciation (and a less appreciation for some initial period) in the real exchange rate, compared to the closed economy (or less open economy). Note that even in a closed economy, still one can think of the real exchange rate as a pricing tool between two consumption baskets.\(^{57}\)

An intuitive explanation would be that in a more open economy, consumption baskets contain a larger portion of imported goods, hence the difference in the values of baskets becomes smaller. The real exchange rate measures the difference in the values of two consumption baskets, and works as a pricing tool between two consumption baskets. More openness implies a larger portion of imported goods in home consumption basket. Larger portion in consumption basket gives rise to a relatively smaller difference between the values of two consumption baskets. Hence, the real exchange rate displays less a depreciation.

Note that with the simple Taylor type policy rule \(\hat{r}_t = \phi_t \pi_t\), domestic inflation is not affected by openness. This is a feature different from that of the fixed exchange rate regime.\(^{58}\)

### The Effect on Private Consumption

Based on the movements of domestic inflation and the real exchange rate, I consider how private consumption is affected by openness. A direct extension of the closed economy channels is employed. In the closed economy model with the variety effect, an increase in government spending invokes private consumption, mainly due to the price elasticity effect and the variety effect, hence, increased intratemporal substitution effect.\(^{59}\)

In the variety effect model under the flexible exchange rate regime, when a real depreciation occurs, private consumption increases less as openness increases. This is mainly because of the strengthened intertemporal substitution effect compared to the closed economy. The impact of openness on the intratemporal substitution effect is ambiguous. The negative wealth effect contributes to increase consumption further, compared to the closed economy. In contrast, intertemporal substitution effect works to increase consumption less, compared to the closed economy (or less open economy).

The intratemporal substitution effect becomes ambiguous when compared to the closed economy.

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\(^{57}\)Even though the degree of openness, \(\alpha\), is zero, and net exports are nil, still the real exchange rate can be defined by the ratio of two consumption baskets, i.e., \(Q_t = \varepsilon_t P_t^*/P_t\).

\(^{58}\)This result comes from the monetary policy rule which does not include GDP. If GDP is included in the policy rule, domestic inflation may change. Even in this case, the movement of domestic inflation will be smaller than that of the fixed exchange rate regime.

\(^{59}\)In a New Keynesian closed economy model with Ricardian equivalence, an increase in unproductive government spending, affects private consumption through the intratemporal substitution effect, the intertemporal substitution effect, and the negative wealth effect. (See Davig and Leeper (2009).) The intratemporal Substitution effect arises as the government spending invokes over all demand. Firms increase labor demand and the real wage rises. A rise in the real wage increases private consumption. The intertemporal substitution effect works through expected inflation. When the prices are sticky, an increase in government spending gradually raises the price level and hence expected inflation. The monetary authority raises the interest rate sharply. The short-term real interest rate rises and households postpone consumption. The negative wealth effect kicks in, when agents expect higher present value of taxes in response to an increase in government spending. The negative wealth effect makes households postpone consumption.
FISCAL POLICY, THE VARIETY EFFECT, AND THE REAL EXCHANGE RATE

A real depreciation leads to an increase in demand even when \( \gamma = 1 \), i.e., when there is no expenditure switching effect towards domestic goods. (Later it will be discussed further.) Hence, firm’s demand for labor increases more than the closed economy. The labor supply decision is affected by a negative income shock (i.e., a real depreciation). Hence, openness has an unclear effect on the real wage in the domestic price, \( W_t/P_{H,t} \). The real depreciation together with the real wage in the domestic price, determines the movement of the real wage in the CPI. Over all, the movement of the real wage in the CPI is ambiguous. The effect of openness on private consumption is unclear as well.

The intertemporal substitution effect is strengthened compared to the closed economy. This channel works via the inflation expectation and the policy reaction. Given monetary policy, the consumption Euler equation can be rewritten as

\[
\hat{c}_t = \hat{E}_t\hat{c}_{t+1} - \sigma [\hat{r}_t - \hat{E}_t\pi_{H,t+1}] - \alpha \hat{E}_t\Delta\hat{s}_{t+1}.
\]

Note that the economy’s short-term real interest rate is affected by the internal short-term real interest rate, \( \hat{r}_t = \hat{E}_t\pi_{H,t+1} \), and the expected change in the terms of trade, \( \hat{E}_t\Delta\hat{s}_{t+1} \). When a real depreciation occurs, the expected change in the terms of trade, \( \hat{E}_t\Delta\hat{s}_{t+1} \), becomes negative. Hence, the expected change in the terms of trade, \( \hat{E}_t\Delta\hat{s}_{t+1} \), raises the economy’s short-term real interest rate. Over all, in a variety effect model, the existence of the terms of trade depreciation strengthens the intertemporal substitution effect.

The negative wealth effect is weakened in a more open economy, when a real depreciation occurs. Combining the household budget constraint and the aggregate resource constraint yields the intertemporal equilibrium condition,

\[
\frac{B^G_{-1}}{P_t} = \hat{E}_t \sum_{i=1}^{\infty} \frac{1}{\Pi_{j=1}^{i-1} R_{t+j-2}} \frac{P_{t+i-1}}{P_t} \left( \tau_{t+i-1} - \frac{P_{H,t+i-1}}{P_{t+i-1}} G_{t+i-1} \right).
\]

In the variety effect model, a real depreciation shrinks the value of government spending in consumption units. At the same time, the value of real debt, \( B^G_{-1}/P_t \), falls, since the

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60In a small open economy, labor demand is determined with respect to the real wage in the domestic price level, \( W_t/P_{H,t} \). In contrast, labor supply decision is related to the real wage in the CPI, \( W_t/P_t \). One way to consider labor demand and supply in a same price space (i.e., \( W_t/P_{H,t} \)), is to treat the terms of trade which is the difference between the domestic price level and the CPI, as an income shock to the households. Note that \( P_t = P_{H,t} + \alpha \hat{s}_t \).

61Agents expect the response of the terms of trade will be weakened. More open economy displays a less depreciation (as in Figure 4), and a smaller negative expectation of the change in the terms of trade. At the same time, increased openness raises over all effect, which is verified from the numerical experiments.

62To derive the intertemporal equilibrium condition, the definition of current account is used as well. For the detailed derivation, refer to the Appendices.

63Rewriting the above condition in a linearized form will make things easier to interpret. The terms in the parenthesis can be written in a linearized form up to a constant. That is, \( \hat{r}_{t+i-1} - (1-g)(p_{H,t+i-1} - p_{t+i-1}) - \hat{g}_{t+i-1} = \hat{r}_{t+i-1} - (1-g)\alpha \hat{s}_{t+i-1} - \hat{g}_{t+i-1} \), where \( \hat{g}_t = (G_t - G)/Y \) and \( \hat{r}_t = (\tau_t - \tau)/Y \).
CPI includes the terms of trade, i.e., \( P_t = P_{H,t} + \alpha \hat{s}_t \). When the Ricardian equivalence holds, agent’s expectation of the present value taxes becomes smaller than in the less open economy.

**The Effect on GDP** Next, for the effect of government spending on output in the variety effect model, I consider the movement of net exports. The movements of net exports and private consumption together describe the movement of GDP. To highlight the role of openness, I assume that the trade elasticity is unity, \( \gamma = 1 \). This is not a realistic assumption. In practice, the trade elasticity plays an important role in determining net exports. In the subsequent discussion, the role of different trade elasticities will be analyzed separately.

As openness increases, less increased private consumption and less depreciation in the terms of trade lead to a larger trade deficit. To see the implication, one can consider the net exports equation,

\[
\hat{nx}_t = (1 - g) \alpha [\hat{c}^*_t - \hat{c}_t + (1 - \alpha) \hat{s}_t] = (1 - g) \alpha [\hat{c}^*_t - \hat{c}_t + \hat{q}_t].
\]

This equation comes from the net exports equation.\(^{64}\) In the equation (35), I call the effect of the gap, \( \hat{c}^*_t - \hat{c}_t \), as the REER effect,\(^{65}\) implying that net exports take place due to the gap in the measurements of two consumption baskets and the real exchange rate.\(^{66}\) As openness increases, net exports which include this gap, is amplified by the increased openness.

### 4.4.2 Transmission Mechanism of the Variety Effect Model under the Fixed Exchange Rate Regime

The main difference between the flexible exchange rate regime and the fixed exchange rate regime lies in specifying the monetary policy rule. Different monetary policy makes agents’

\(^{64}\)Net exports can be rewritten as the difference between export and import in the domestic price terms,

\[ NX_t = C^*_H - C^*_F \left( \frac{P_{H,t}}{P_{F,t}} \right) - \gamma C^*_t - \frac{P_{F,t}}{P_{H,t}} \left( \frac{P_{F,t}}{P_t} \right)^{-\gamma} C_t. \]

By defining \( \hat{nx}_t \equiv (NX_t - NX)/Y \), one can have

\[
\hat{nx}_t = \frac{C^*_H}{Y} \hat{c}^*_H + \frac{C_F}{Y} (\hat{c}_{F,t} + \hat{s}_t) = (1 - g) \alpha [\hat{c}^*_H - \hat{c}_F + \hat{s}_t] = (1 - g) \alpha [\hat{c}^*_t + \gamma \hat{s}_t - (\hat{c}_t - \gamma (1 - \alpha) \hat{s}_t + \hat{s}_t)].
\]

It is assumed that the steady state net exports are balanced. That implies that the share of export to GDP is also \( (1 - g) \alpha \). Note that both \( \hat{c}^*_t + \gamma \hat{s}_t \) and \( \hat{c}_t - \gamma (1 - \alpha) \hat{s}_t + \hat{s}_t \) are in domestic price terms \( p_{H,t} \). \( \hat{c}_t - \gamma (1 - \alpha) \) is in the import price \( p_{F,t} \), and \( \hat{s}_t \) converts the unit into the domestic price \( p_{H,t} \).

\(^{65}\)Since \( \gamma = 1 \), there is no expenditure switching effect.

\(^{66}\)Note that the real exchange rate measures two consumption baskets. When private consumption increases, this can be interpreted as a fall in the long term real interest rate. The real exchange rate reduces the gap between the domestic and foreign long-term real interest rates through the risk premium. Hence, when private consumption increases, the REER effect works negatively, i.e., increases the trade deficit. When private consumption decreases, the REER effect increases the trade surplus.
Figure 4: Responses to a government spending shock. First column: the variety effect model in the flexible exchange rate regime with $\gamma = 1$, second column: the variety effect model in the fixed exchange rate regime with $\gamma = 1$, third column: the rule-of-thumb consumers model in the fixed exchange rate regime with $\gamma = 1$. Solid line: $\alpha = 0.01$, dashed line: $\alpha = 0.2$, dash-dotted line: $\alpha = 0.4$. 
expectations of the real exchange rate under the fixed exchange rate regime different from those under the flexible exchange rate regime. Under the fixed exchange rate regime, the domestic price level and the real exchange rate revert to its initial level as time passes.

To explore the transmission mechanism, I start with the (nearly) closed economy counterpart, and add openness. This will reveal how the transmission mechanism changes when openness is introduced.

The Effect on the Nominal Exchange Rate and the Real Exchange Rate  Basic dynamics of the model imply that GDP, the nominal exchange rate,\textsuperscript{67} and the real exchange rate (or the terms of trade), are determined together.

The second column of Figure 4 implies that an increase in government spending leads to a less increase in GDP, and a more depreciation (or less appreciation) in the real exchange rate, compared to the closed economy (or less open economy). The real exchange rate can be characterized by a smaller fluctuation with increased openness.

Intuitively, more openness works on the real exchange rate to the direction of decreasing the structural difference in the values of two consumption baskets. In a closed economy (where net exports are nil),\textsuperscript{68} the real exchange rate deviates large from the values of differences in two consumption baskets. As the economy becomes more open, the ratio of imported goods in the home consumption basket rises. Hence openness works to decrease the structural gap in pricing two consumption baskets.

Note that one needs not care about the movement of the nominal exchange rate. In equilibrium, the nominal exchange rate is fixed by the policy rule.

The Effect on Private Consumption  I turn to the description of three channels through which a government spending shock affects private consumption. When the economy becomes more open, intratemporal substitution effect and negative wealth effect work as under flexible exchange rate regime.\textsuperscript{69} The intertemporal substitution effect works in a different way from the flexible exchange rate regime model, but produces same results (i.e., strengthened intertemporal substitution effect). Numerical experiments show that private consumption increases less mainly because of the strengthened intertemporal substitution effect. (Refer to second column of Figure 4.)

\textsuperscript{67}In equilibrium, the nominal exchange rate, $\hat{e}_t$, remains constant by the policy reaction. The foreign price level is assumed to be exogenous, by the small open economy assumption. In response to the shocks, the real exchange rate reverts to its initial level and this implies that the domestic price level reverts as well.

\textsuperscript{68}The fixed exchange rate regime and the closed economy are conceptually incompatible. Monetary policy is determined by the external factor, and separation of the domestic economy from the external economy does not determine the domestic variables conceptually. I use very small openness ($\alpha = 0.01$) and treat it as a closed economy. With small $\alpha$, the economy is open, and net exports change.

\textsuperscript{69}Unlike the flexible exchange rate regime case, domestic inflation shows a smaller increase (or a larger decrease) with increased openness. Hence, a less increase in domestic price level occurs. This movement makes the intratemporal substitution effect more complicated. Still, over all effect does not clearly indicate a direction. A smaller increase in the domestic price level, change the size of real debt, $B^{G}_{t-1}/P_t$. Over all, the negative wealth effect will be weakened less than under the flexible exchange rate regime.
Under the fixed exchange rate regime, the intertemporal Substitution effect works through the short-term real interest rate and is strengthened compared to less open economy. When the prices are sticky, an increase in government purchases gradually raises the domestic price level. As expected domestic inflation rises, domestic consumers compare purchasing power of the consumption basket to purchasing power of the foreign consumption basket. Consumers expect a positive increase in the nominal exchange rate (i.e., an increase in \( E_t \Delta \hat{e}_{t+1} \)), since the real exchange rate is defined by \( \text{REER}_t = \varepsilon_t P^*_t / P_t \). The monetary authority raises the interest rate sharply in response to the expected changes in the nominal exchange rate. Hence, the external short-term real interest rate, \( \hat{r}_t - E_t \Delta \hat{e}_{t+1} \), rises. In the fixed exchange rate regime, the external short-term real interest rate, i.e., \( \hat{r}_t - E_t \Delta \hat{e}_{t+1} \), and the terms of trade, \( (1 - \alpha)E_t \Delta \hat{s}_{t+1} \), together determine the movement of short-term real interest rate. One can see the implication from the Euler equation,

\[
\hat{c}_t = E_t \hat{c}_{t+1} - \sigma \left[ (\hat{r}_t - E_t \Delta \hat{e}_{t+1}) - (1 - \alpha)E_t \Delta \hat{s}_{t+1} \right].
\] (36)

Note that even in a small openness case (\( \alpha = 0.01 \)), the terms of trade affects the short-term real interest rate directly. A smaller appreciation (or larger depreciation) which results from increased openness, leads to a larger increase in the short term real interest rate. As openness increases, the direct effect of the terms of trade (or the real exchange rate) on the short-term real interest, i.e. \((1 - \alpha)E_t \Delta \hat{s}_{t+1}\), becomes smaller.\(^72\)

**The Effect on GDP** To explore the effect of government spending on output, I consider the movement of net exports in addition to the movement of consumption. As in the flexible exchange rate model, I assume that the trade elasticity is unity, \( \gamma = 1 \).

As openness increases, less increased private consumption and a less appreciation (or a more depreciation) in the terms of trade lead to a larger trade deficit. The same explanation as in the flexible exchange rate regime model can be adopted. The only difference is the size of the REER effect. Since the nominal exchange rate is fixed under the fixed exchange rate regime, the real exchange rate reflects a larger gap between the measurements of two consumption baskets. Hence, compared to the flexible exchange rate regime, a larger trade deficit occurs as openness increase.

\(^70\)In equilibrium, the nominal exchange rate does not change. However, from the rational expectation perspective, this kind of expectation is consistent with the equilibrium results. Refer to Benigno et al. (2007), Justiniano and Preston (2010), and Corsetti et al. (2011).

\(^71\)Rewriting the consumption Euler equation and using price relations yield this equation.

\(^72\)When a real appreciation takes place, the expected increment in the terms of trade, \( E_t \Delta \hat{s}_{t+1} \), shows a smaller increase. At the same time, openness, \( \alpha \), increases. Total effect represents a smaller increase in \((1 - \alpha)E_t \Delta \hat{s}_{t+1}\) when \( \alpha \) increases. The increase in openness weakens the total size of a decrease in \((1 - \alpha)E_t \Delta \hat{s}_{t+1}\) when \( \alpha \) increases. When a real depreciation takes place, the expected increment in the terms of trade, \( E_t \Delta \hat{s}_{t+1} \), shows a little bit larger decrease. The increase in openness weakens total effect of decrease in \((1 - \alpha)E_t \Delta \hat{s}_{t+1}\).
4.4.3 Transmission Mechanism of the Rule-of-Thumb Consumers Model under the Fixed Exchange Rate Regime

The explanation will be similar to previous fixed exchange rate regime model, except the existence of the rule-of-thumb consumers and exclusion of the variety effect. For the numerical result, refer to the third column of Figure 4.

The Effect on Private Consumption To describe three channels through which the terms of trade affect private consumption, I consider asset holding consumers and non asset holding consumers separately. For the asset holding consumers, the intratemporal substitution effect, the intertemporal substitution effect and the negative wealth effect are the same as the variety effect model under the fixed exchange rate regime, except the size of the intratemporal substitution effect. For the rule of thumb consumers, only intratemporal substitution effect exist.

Even though the size of the intratemporal substitution effect is smaller than that in the variety effect model, the existence of the rule-of-thumb consumers leads to an increase in private consumption during a few periods after the government spending shock. As in the rule-of-thumb consumers literature, the ratio of the rule-of-thumb consumers is sufficiently large enough to increase over all private consumption. Note that private consumption of asset holding consumers decreases which is the outstanding feature of the standard New Keynesian model.

The Effect on GDP To explore the effect of government spending on output, I consider the movement of net exports in addition to private consumption. As in the previous models, I assume that the trade elasticity is unity, $\gamma = 1$.

The effect on GDP is not clearly shown, since conflicting forces work together. When private consumption increases due to the rule-of-thumb consumers, a larger trade deficit takes place with increased openness. As in the previous fixed exchange rate regime model, a large gap between measurements of two consumption baskets and the real exchange rate, leads to a large trade deficit. As time passes, a reversal occurs. This is because of the relatively short-lived increase in private consumption.

\footnote{In the variety effect model, the intratemporal substitution effect is strengthened more than the standard New Keynesian model. The variety effect leads to more demand of firms on labor, and this enlarges the intratemporal substitution effect. This demand boosting effect becomes smaller in the rule-of-thumb consumers model than in the variety model.}

\footnote{As described before, when private consumption decreases, the trade surplus increases due to the REER effect.}
4.5 Analysis of Government Spending Effect

I construct one flexible exchange rate regime model and two fixed exchange rate regime models to replicate the empirical evidences of small open economies. By using three models, I first consider how different exchange rate regimes affect the effectiveness of fiscal policy. Then, I investigate the implication of openness. Finally, I explore the role of the trade elasticity.

4.5.1 The Effect of Exchange Rate Regimes

To match the empirical evidences, I construct one model for the flexible exchange rate regime, and two models for the fixed exchange rate regime. For the flexible exchange rate regime, I introduce the variety effect to the otherwise standard New Keynesian model (Flexible ER model). The ratio of the rule-of-thumb consumers is nil in this model. For the fixed exchange rate regime, I construct two types of models. The first type of the fixed exchange rate regime

75 Under the flexible exchange rate regime, a government spending shock leads to consumption crowding in. There is a positive co-movement between consumption and the real exchange rate. A trade deficit follows. (Monacelli and Perotti (2010)) Under the fixed exchange rate regime, government spending crowds in private consumption, but consumption and the real exchange rate show a negative co-movement. A trade deficit is prevalent. (Bénetrix and Lane (2009), and Beetsma and Giuliodori (2010)) For the fixed exchange rate regime, the panel of VARs (full pooling) is widely used due to the short data series in Euro area. In Chapter 1, I evaluate the plausibility of adopting the panel of VARs and find that country heterogeneity exists. Bayesian pooling reveals that assuming different types of movements in the real exchange rate is more plausible for the EMU. Significant real appreciation and an insignificant real appreciation co-exist.
model (Fixed ER 1 model) features the variety effect and 20 percent of the rule-of-thumb consumers. The main transmission mechanism of generating a positive movement in private consumption in the Fixed ER 1 is the variety effect. The second type of the fixed exchange rate regime model (Fixed ER 2 model) has 40 percent of the rule-of-thumb consumers, and no variety effect \((\eta = 1)\). The rule-of-thumb consumers play a main role in the Fixed ER 2.

Solid line in Figure 5 displays the responses of the Flexible ER model to a government spending shock. In this model, only the variety effect works and there are no rule-of-thumb consumers. There is a positive co-movement between consumption and the real exchange rate, which is consistent with the empirical evidences. For the movement of net exports, the trade elasticity is assumed to be less than unity, i.e., \(\gamma = \frac{2}{3}\).

Dashed line denotes the responses of the Fixed ER 1 model and it features a smaller real appreciation. Even though the rule-of-thumb consumers exist in the model, the main mechanism of generating the positive movement in private consumption is the variety effect. The real exchange rate initially appreciates, but soon reverts to a depreciation.

Dotted line is the responses of the Fixed ER 2 model and it presents a larger real appreciation. The main mechanism works through the rule-of-thumb consumers. In this model, with \(\eta = 1\), the variety effect disappears. Only counter cyclical markup effect works through varying price elasticity. Due to the rule-of-thumb consumers, a real appreciation occurs in the presence of the increase in private consumption.

Note that in both Fixed ER 1 and Fixed ER 2 models, inflation and the real exchange rate represent mean reverting tendency, which is consistent with exchange rate policy under the fixed exchange rate regime. Under the fixed exchange rate regime, agents expect the domestic price level and the real exchange rate revert to its long-run level.

**Welfare Implication** The Fixed ER 1 model is superior to the Flexible ER model in the welfare terms. Between the Flexible ER model and the Fixed ER 2 model, it is difficult to draw a conclusion clearly from the numerical experiments. Roughly, the Flexible ER model is superior to the Fixed ER 2 model.

The main difference occurs from the different transmission mechanisms. Since the welfare analysis is not the main focus of this paper, only rough conjecture is given. The rule-of-thumb consumers assumption has a short-lived effect. The variety effect assumption gives rise to a slow but persistent effect on private consumption. An efficiency enhancing mechanism operates in the variety effect models.

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76In the Fixed ER 2, no variety effect (or no increasing returns to specialization) is assumed, but still countercyclical markup through variable price elasticity is assumed to hold.

77In a small open economy, the nominal exchange rate is fixed by the monetary authority and the foreign price is assumed to be exogenous. Under the purchasing power parity (PPP) assumption, the domestic price level and the real exchange rate revert to its initial level in the long run.

78This result is not robust. If the rule-of-thumb consumers assumption is added to the Flexible ER, the difference between the Flexible ER and the Fixed ER 1 will be smaller.
Output Stimulus Effect In terms of output stimulus, the main result in Figure 5 is different from the conventional wisdom and similar to the result of Corsetti et al. (2011). The conventional wisdom implies that fiscal policy under the fixed exchange rate regime, is more effective than fiscal policy under the flexible exchange rate regime. Corsetti et al. (2011) stand in the opposite side of the conventional wisdom, i.e., fiscal policy in the flexible exchange rate regime is more effective than in the fixed exchange rate regime.\(^{79}\) In Figure 5, if the main mechanism under fixed exchange rate regime is the variety effect, fiscal policy under the fixed exchange rate regime (Fixed ER 1 model), is as effective as fiscal policy under the flexible exchange rate regime (Flexible ER model). When the main mechanism under the fixed exchange rate regime, is the rule-of-thumb consumers, fiscal policy under the fixed exchange rate regime (Fixed ER 2 model), is inferior to fiscal policy under the flexible exchange rate regime (Flexible ER model).

The effect on GDP can be explained by the movement of net exports which is affected by the exchange rate regime, together with private consumption. The movement of private consumption is compensated by net exports, but the degree of compensation differs depending on the exchange rate regime. To see the implication, one can rewrite the equation (35) as

\[
\hat{n}\hat{x}_t = (1 - g)\alpha[\hat{c}_t - \hat{c}_t + \hat{q}_t - (1 - \gamma)(2 - \alpha)\hat{s}_t].
\]

REER effect expenditure switching effect

In determining net exports, there exist two effects, i.e., the REER effect and the expenditure switching effect. When a real depreciation takes place, the REER effect shows smaller negative value in the Flexible ER model. This is because the real exchange rate serves well as a pricing tool between two consumption baskets. Under the fixed exchange rate regime models, the REER effect is large. Hence, the positive expenditure switching effect due to the real appreciation is not enough to decrease the trade deficit. This structural REER effect under the fixed exchange rate regime causes larger trade deficit than under the flexible exchange rate regime. Combined with private consumption, the movement of net exports indicates the effect of the government spending on GDP.

4.5.2 The Effect of Degrees of Openness

Degrees of openness\(^{80}\) work differently depending on the exchange rate regimes. In the welfare terms, the flexible exchange rate regime model is inferior to others, that is, openness undermines private consumption more under the flexible exchange rate regime. In stimulating GDP, more openness implies less increase in GDP under the flexible exchange rate regime. Under the fixed exchange rate regime, this may or may not hold.

Welfare Implication In the three models, openness plays different roles in terms of the welfare implication. The Fixed ER 2 model is superior to other two models regarding the role

\(^{79}\)Corsetti et al. (2011) rely on government spending reversal. They conclude that spending reversal works under the flexible exchange rate regime, but it does not, under the fixed exchange rate regime.

\(^{80}\)The trade elasticity is set to 2/3 which is the conventional value for advanced countries.
Figure 6: Responses to the government spending shock. First column: Flexible ER, second column: Fixed ER 1, third column: Fixed ER2. Solid line: $\alpha = 0$ or $\alpha = 0.01$, dashed line: $\alpha = 0.2$, dash-dotted line: $\alpha = 0.4$. To three models, same trade elasticity, $\gamma = 2/3$, is applied.
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of openness. The Flexible ER model is inferior to others in terms of the welfare implication.\textsuperscript{81}

In the Flexible ER model, openness always causes less increased consumption, hence a less increase in welfare. (Refer to the first column of Figure 6.) The main mechanism works through the strengthened intertemporal substitution effect in the presence of the real depreciation.

In the Fixed ER 1 model, as openness increases, the effect of government spending on consumption is not so much clear. (Refer to the second column of Figure 6.) During the initial real appreciation, consumption increases more than in the closed economy. Initially the small portion of the rule-of-thumb consumers works, but the variety effect becomes stronger. The variety effect and the small portion of the rule-of-thumb consumers, together generate the behavior of private consumption.

In the Fixed ER 2 model, as openness increases, consumption increases more than in the closed economy in response to a government spending shock. (Refer to the third column of Figure 6.) Since a real appreciation works through asset holding consumers, asset holding consumers’ consumption decrease more. The real wage income of the rule-of-thumb consumers (and asset holding consumers as well) in the CPI units, increases more than in the closed economy. Even though firm’s demand on labor displays a smaller increase than in the closed economy, the real wage in the domestic price, $W_t/P_{H,t}$, rises due to the terms of trade depreciation.\textsuperscript{82} The real wage in the domestic price together with a real appreciation describes the real wage in the CPI, $W_t/P_t$. Over all real wage in CPI rises and private consumption increases more.

**Output Stimulus Effect**  By explaining the movement of net exports together with private consumption, one can track the output stimulus effect. In the Flexible ER and Fixed ER 1 models, openness reduces the output stimulus effect by causing private consumption to increase less and the trade deficit to increase more. In the Fixed ER 2 model, openness has an obscure effect on output, because of the ambiguous effect on private consumption.

In the Flexible ER model, openness increases the trade deficit. There are two conflicting forces in the effect of openness on net exports, that is, the REER effect and the expenditure switching effect. (Refer to the equation (37).) When private consumption increases, a smaller real depreciation leads to a weakened expenditure switching effect toward domestic goods. The sum of two effects is amplified by increased openness. Hence, a larger trade deficit occurs, when compared to less open economy.

In the Fixed ER 1 model, openness deepens the trade deficit. The (negative) size of the REER effect is larger than in the Flexible ER model, due to the weak pricing role of the real exchange rate. A smaller real appreciation (hence, a smaller expenditure switching effect toward foreign goods) compensates the trade deficit. Since the size of the REER effect is large, total effect on trade deficit is amplified by increased openness.

\textsuperscript{81}If the rule-of-thumb consumers assumption is added to the Flexible ER model, the result may be different.

\textsuperscript{82}The real appreciation can be considered as a positive income shock in the determination of labor demand and supply in the real wage in the domestic price.
The Fixed ER 2 model exhibits similar result as the Fixed ER 1 model, but one difference is that the trade surplus takes place for some periods. When private consumption decreases, the REER effect works in the opposite direction, i.e., increases the trade surplus. Hence a reversal in net exports appear.

4.5.3 The Effect of Trade Elasticities

The trade elasticity affects GDP mainly through net exports and plays different roles depending on the exchange rate regimes. Under the flexible exchange rate regime, a higher trade elasticity \( \gamma = 1.5 \) can generate a more stimulating effect, by inducing a trade surplus. In contrast, under the fixed exchange rate regime, a higher trade elasticity does not change the direction of net exports in the short run, which implies that the trade deficit is structural.

Numerical experiments shows that in the Flexible ER model, the effect of government spending on output, depends on the trade elasticity, \( \gamma \). (Refer to the first column of Figure 7.) When \( \gamma < 1 \), a trade deficit occurs in response to a government spending shock. The trade deficit has a downward pressure on output. When \( \gamma > 1 \), a trade surplus takes place. The trade surplus leads to an upward pressure on output.

In the Fixed ER 1 model, the trade elasticity, \( \gamma \), has little effect on the effectiveness of government spending shock on output. (Refer to the second column of Figure 7.) Whether the trade elasticity is greater than unity or not, there is always a downward pressure on output. When \( \gamma < 1 \), a trade deficit appears. The trade deficit leads to a downward pressure on output. When \( \gamma > 1 \), initially the same downward pressure appears. After 10 periods or later, the trade surplus compensates this downward pressure on GDP through the expenditure switching effect toward domestic goods.

In the Fixed ER 2 model, similar result as in the Fixed ER 1 model appears. (Refer to the third column of Figure 7.) Only the duration differs from the Fixed ER 1 model.

These differences come from the relative size of the REER effect. Depending on the size of the REER effect, the role of the expenditure switching effect differs from regime to regime. In the Flexible ER model, the size of the REER effect is smaller than in Fixed ER 1 model or Fixed ER 2 model. Hence, the expenditure switching effect reverses the direction of net exports depending on the values of \( \gamma \). In the Fixed ER 1 model or Fixed ER 2 model, this phenomenon does not happen because of the larger size of the REER effect.

Intuition follows from the movements of consumption and the real exchange rate. Under the flexible exchange rate regime, consumption and the real exchange rate closely co-moves in response to a government spending shock even though not exact. This implies that the real exchange rate serves well as a pricing mechanism between two consumption baskets. Hence, net exports are affected by the change in the size of the trade elasticity. Under the fixed exchange rate regime, consumption and the real exchange rate moves in the opposite

\(^{83}\)This movement of the REER effect comes from core dynamics of the model. Refer to the previous discussion.

\(^{84}\)Under the flexible exchange rate regime, the gap between the relative price in two consumption baskets and real exchange rate, is smaller. The nominal exchange rate freely adjusts, and the real exchange rate represents the relative price between two consumption baskets well.
Figure 7: Responses to the government spending shock. First column: Flexible ER, second column: Fixed ER 1, third column: Fixed ER2. Solid line: $\gamma = 0.67$, dashed line: $\gamma = 1.5$. To three models, same openness, $\alpha = 0.4$, is applied.
way. This implies that the pricing role of the real exchange rate is weak. Different trade elasticity plays a smaller role for net exports, hence for output.

One important application to the small open economy modeling is that models with the complete market assumption may cause a large distortion for the fixed exchange rate regime. Models with the complete market assumption, make the dynamics simple and tractable. Nevertheless, as Schmitt-Grohé and Uribe (2003) point out, the complete market assumption is worse than other small open economy models in that models with the complete market assumption do not capture the volatility of consumption data well. Here, for the fixed exchange rate regime, models with the complete market assumption abstract from some important feature, i.e., the real exchange rate movement deviates from the movement of private consumption.

5 Concluding Remarks

In this paper, I introduce the variety effect (including varying price elasticity) to the otherwise standard small open economy model. In the variety effect model, the economy’s structure is such that firm entry leads to a higher price elasticity, hence the markup decline. Firm entry also generates increasing returns to specialization (or the variety effect). The latter effect is modeled by employing the Benassy aggregator. Both effects raise labor demand and serve to explain consumption crowding-in with respect to a government spending shock.

To replicate empirical results of the small open economy, I construct the variety effect model for the flexible exchange rate regime, and develop two models for the fixed exchange rate regime. In the flexible exchange rate regime, the empirical evidences indicate that private consumption and the real exchange rate co-moves in response to a government spending shock. The trade deficit follows. The variety effect model is capable of replicating the flexible exchange rate regime evidences which are puzzling to the small open economy theories. For the fixed exchange rate regime, empirical findings in chapter 1, reveal that it is plausible to assume that heterogeneous country dynamics exist in response to an increase in government spending. For the fixed exchange rate regime, I set up two models, one features the variety effect as its main dynamics, the other, the rule-of-thumb consumers. These models generate heterogeneous movements respectively.

Based on three models, I first consider the effect of the exchange rate regime on stimulation and welfare. With respect to the effectiveness of fiscal policy (especially of government spending) on output, the model in the fixed exchange rate regime exhibits equivalent or inferior to the model in the flexible exchange rate regime whose main dynamics are the variety effect. This result is the opposite to the conventional wisdom that fiscal policy under the fixed exchange rate regime is more powerful than fiscal policy under the flexible exchange rate regime. In terms of the welfare implication, slightly different ordering appears. The fixed exchange rate regime model whose main dynamics are the variety effect, is expected to be superior to other models. This finding implies that in the small open economy, the output stimulus effect and the welfare implication need to be investigated separately.

Next, openness depresses the stimulus effect with the conventional trade elasticity, but has
an ambiguous effect on welfare. Regarding the stimulus effect, openness affects the economy through net exports, together through private consumption, in response to the government spending shock. Models both under the fixed and flexible exchange rate regimes, show a downward pressure on output with less than unity trade elasticity. With respect to welfare, the size of consumption increase in response to the government spending shock, matters. The model under the fixed exchange rate regime whose main dynamics are the variety effect, is superior (i.e., do less harm with increased openness) to the other model under the fixed exchange rate regime or the model with the variety effect under the flexible exchange rate regime.

Finally, the trade elasticity plays a different role under the flexible and fixed exchange rate regimes. Under the flexible exchange rate regime, a higher trade elasticity ($\gamma = 1.5$) can generate more a stimulating effect, by inducing a trade surplus. In contrast, under the fixed exchange rate regime, a higher trade elasticity does not change the direction of net exports in the short run, which implies that the trade deficit is structural. The explanation follows that under the flexible exchange rate regime, private consumption and the real exchange rate movements are closely related in response to a government spending shock, but under the fixed exchange rate regime, they are not. This is because the real exchange rate under the flexible exchange rate regime, serves well as a pricing tool between two consumption baskets. This result comes from the fact that under the flexible exchange rate regime, the nominal exchange rate can freely adjust. When private consumption and the real exchange rate co-move closely in response to the government spending shock, the direction of net exports is affected much more by the size of the trade elasticity. The relative difference in the co-movement suggests further implication for the evaluation of the terms of trade effect or trade policy.

One modeling implication is that models with the complete market assumption abstract some important feature of the fixed exchange rate regime. One need to be cautious to introduce the complete market assumption for modeling the fixed exchange rate regime, since models with the complete market assumption do not reflect the aspect that the real exchange rate may deviated from the movement of private consumption under the fixed exchange rate regime.
Appendices

A Simple Closed Economy Models

A.1 Real Business Cycle Model with Zero Profit Firm Entry and the Variety Effect

A.1.1 Derivation of Price Elasticity

The price elasticity of demand is defined by

\[ \varepsilon_{Y(i,j)P(i,j)}(N_t) = -\frac{dY(i,j) \cdot P(i,j)}{dP(i,j) \cdot Y(i,j)} \]

where

\[ \frac{dY(i,j)}{dP(i,j)} = \frac{\partial Y(i,j)}{\partial P(i,j)} + \frac{\partial Y(i,j)}{\partial P(i)} \cdot \frac{\partial P(i)}{\partial P(i,j)} \]

The first term in the RHS is given by

\[ \frac{\partial Y(i,j)}{\partial P(i,j)} = -\theta \left[ \frac{P(i,j)}{P(i)} \right]^{-\theta} \frac{Y}{N_t^{1-(\eta-1)(\theta-1)}} \cdot \frac{1}{P(i,j)} = -\theta \frac{Y(i,j)}{P(i,j)} \]

The second term in the RHS is represented by

\[ \frac{\partial Y(i,j)}{\partial P(i)} \cdot \frac{\partial P(i)}{\partial P(i,j)} = \left[ \theta \frac{Y(i,j)}{P(i)} - \omega \frac{Y(i,j)}{P(i,j)} \right] \cdot \frac{1}{N_t^{\eta}} \]

I utilized the relation

\[ \sum_{j=1}^{N_t} P(i,j)^{1-\theta} = N_t P(i,j)^{1-\theta} \]

which holds in a symmetric equilibrium. The price elasticity of demand can be rewritten as

\[ \varepsilon_{Y(i,j)P(i,j)}(N_t) = \theta + [\omega - \theta] \cdot \frac{1}{N_t^{\eta}} \]

A.1.2 Derivation of Firm’s Markup

The profit function of firm \( j \) in industry \( i \) is given by

\[ \Pi_t(i,j) = (P_t(i,j) - MC_t) \left[ \frac{P_t(i,j)}{P_t(i)} \right]^{-\theta} \left[ \frac{P_t(i)}{P_t} \right]^{-\omega} \frac{Y_t}{N_t^{1-(\theta-1)(\eta-1)}} \]

\[ = Y_t(i,j) \]
The first order condition is

\[
\frac{d\Pi_t(i, j)}{dP_t(i, j)} = Y_t(i, j) + P_t(i, j) \frac{dY_t(i, j)}{dP_t(i, j)} - MC_t \frac{dY_t(i, j)}{dP_t(i, j)} = 0.
\]

Note that

\[
\frac{dY_t(i, j)}{dP_t(i, j)} = -\left[\theta + (\omega - \theta) \frac{1}{N_t}\right] \frac{Y_t(i, j)}{P_t(i, j)}.
\]

Rearranging yields

\[
\frac{d\Pi_t(i, j)}{dP_t(i, j)} = -\varepsilon(N_t)Y_t(i, j) \left[ -\frac{1}{\varepsilon(N_t)} + 1 - \frac{MC_t}{P_t(j, i)} \right] = 0.
\]

This condition implies that

\[
\frac{P_t(i, j)}{MC_t} \equiv \mu(N_t) = \frac{\varepsilon(N_t)}{\varepsilon(N_t) - 1} = \frac{\theta N_t^\eta + (\omega - \theta)}{(\theta - 1)N_t^\eta + (\omega - \theta)} > 1.
\]

A.1.3 Derivation of the Zero Profit Condition

The zero profit condition implies

\[
P_t(i, j)Y_t(i, j) - W_t h_t = 0.
\]

Rewriting gives

\[
Y_t(i, j) - \frac{W_t}{P_t(i, j)} h_t = 0.
\]

From the firm’s conditional demand for hours worked, it follows

\[
Y_t(i, j) - \frac{z_t}{\mu(N_t)} h_t = 0.
\]

By using the production technology equation to substitute for \(z_t h_t\), one can have

\[
Y_t(i, j) - \frac{1}{\mu(N_t)} [Y_t(i, j) + \phi] = 0.
\]

Imposing the symmetric equilibrium condition yields the zero profit condition,

\[
[\mu(N_t) - 1]Y_t^* = \phi.
\]
A.1.4 Derivation of the Number of Firms

From the individual firm’s production technology, it follows

\[ Y_t^\dagger = z_t h_t - \phi = z_t \frac{H_t}{N_t} - \phi. \]

Applying the zero profit condition to substitute for \( Y_t^\dagger \), produces

\[ \frac{\phi}{\mu(N_t) - 1} = \frac{1}{N_t} z_t H_t - \phi. \]

Rearranging yields the number of firms per industry,

\[ N_t = z_t H_t \left[ \frac{\mu(N_t) - 1}{\mu(N_t)\phi} \right]. \]

A.1.5 Derivation of Aggregate Final Output

Multiplying the number of firms equation by \( Y_t^\dagger N_t^{\eta-1} \) and utilizing the zero profit condition yield

\[ Y_t^\dagger N_t^{\eta-1} = z_t H_t \left( \frac{\mu(N_t) - 1}{\mu(N_t)\phi} \right) N_t^{\eta-1} = \frac{1}{\mu(N_t)} (z_t N_t^{\eta-1}) H_t. \]

Rearranging gives the aggregate final output equation,

\[ Y_t = \frac{1}{\mu(N_t)} (z_t N_t^{\eta-1}) H_t. \]

A.1.6 Equations

The labor supply equation is

\[ \frac{W_t}{P_t} = C_t^\frac{\phi}{\beta} H_t^\frac{1}{\beta}. \]

The consumption Euler equation is given by

\[ \frac{1}{1 + i_t} = E_t Q_{t,t+1} = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{\frac{\beta}{1-\beta}} \frac{P_t}{P_{t+1}} \right\}. \]

The markup equation is

\[ \mu(N_t) = \frac{\theta N_t^\eta + (\omega - \theta)}{(\theta - 1) N_t^\eta + (\omega - \theta)}. \]

The labor demand equation can be written as

\[ \frac{W_t}{P_t} = \frac{z_t}{\mu(N_t)} N_t^{\eta-1}. \]
The aggregate resource constraint (ARC) reads
\[ Y_t = C_t + G_t. \]

The production technology is given by
\[ Y_t = \frac{1}{\mu(N_t)} (z_t N_t^{\eta-1}) H_t. \]

The number of firms (or diversity) is
\[ N_t^{\eta} = \left[ \frac{\mu(N_t) - 1}{\phi} \right] Y_t. \]

The government spending shock and technology shock are given by
\[ \log G_t - \log G = \rho_g (\log G_{t-1} - \log G) + \epsilon^g_t, \]
\[ \log z_t - \log z = \rho_z (\log z_{t-1} - \log z) + \epsilon^z_t. \]

A.1.7 Log Linearized Equations

Log linearized equations of the simple closed economy model are

\[ \hat{w}_t - \hat{p}_t = \frac{1}{\sigma} \hat{c}_t + \frac{1}{\varphi} \hat{h}_t, \]
\[ \hat{c}_t = E_t \hat{c}_{t+1} - \sigma (\hat{r}_t - E_t \pi_{t+1}), \]
\[ \hat{n}_t = -\frac{1}{\eta \mu - 1} \frac{1}{(\theta - 1) \mu - \theta} \hat{\mu}_t, \]
\[ \hat{w}_t - \hat{p}_t = \hat{z}_t - \hat{\mu}_t + (\eta - 1) \hat{n}_t, \]
\[ \hat{y}_t = (1 - g) \hat{c}_t + \hat{\gamma}_t, \]
\[ \hat{y}_t = -\hat{\mu}_t + \hat{z}_t + (\eta - 1) \hat{n}_t + \hat{h}_t, \]
\[ \hat{\mu}_t = -\frac{(\theta - 1) \mu - \theta}{(\theta - 1) \mu - \theta} \hat{y}_t, \]
\[ \hat{\gamma}_t = \rho_g \hat{\gamma}_{t-1} + \epsilon^g_t, \]
\[ \hat{z}_t = \rho_z \hat{z}_{t-1} + \epsilon^z_t. \]

Note that
\[ \hat{n}_t = -\frac{1}{\eta \mu - 1} \frac{1}{(\theta - 1) \mu - \theta} (-1)^{\mu - \theta} \hat{y}_t = \frac{1}{\eta \mu - 1} \frac{1}{(\theta - 1) \mu - \theta} \hat{y}_t \equiv \lambda_t. \]
A.1.8 Solution of the Model

From the labor demand and supply equations, it follows
\[ \frac{1}{\sigma} \dot{c}_t + \frac{1}{\varphi} \dot{h}_t = \dot{z}_t - \dot{\mu}_t + (\eta - 1)\dot{n}_t. \]

Substituting for \( \dot{h}_t \) gives
\[ \frac{1}{\sigma} \dot{c}_t + \frac{1}{\varphi} (\dot{y}_t + \dot{\mu}_t - \dot{z}_t - (\eta - 1)\dot{n}_t) = \dot{z}_t - \dot{\mu}_t + (\eta - 1)\dot{n}_t. \]

Substituting the ARC for \( \dot{y}_t \), yields
\[ \frac{1}{\sigma} \dot{c}_t + \frac{1}{\varphi} (1 - g)\dot{c}_t + \frac{1}{\varphi} \dot{g}_t + \frac{1}{\varphi} \dot{\mu}_t - \frac{1}{\varphi} \dot{z}_t - \frac{1}{\varphi} (\eta - 1)\dot{n}_t = \dot{z}_t - \dot{\mu}_t + (\eta - 1)\dot{n}_t. \]

Substituting for \( \dot{\mu}_t \) and \( \dot{n}_t \) produces
\[
\begin{align*}
\frac{1}{\sigma} \dot{c}_t + \frac{1}{\varphi} (1 - g)\dot{c}_t + \frac{1}{\varphi} \dot{g}_t - \frac{1}{\varphi} \frac{(\theta - 1)\mu - \theta}{(\theta - 1)\mu} [(1 - g)\dot{c}_t + \dot{g}_t] \\
- \frac{1}{\varphi} \dot{z}_t - \frac{1}{\varphi} \frac{\eta - 1}{\eta} \frac{1}{\theta - 1 \mu - 1} [(1 - g)\dot{c}_t + \dot{g}_t] \\
= \dot{z}_t + \frac{(\theta - 1)\mu - \theta}{(\theta - 1)\mu} [(1 - g)\dot{c}_t + \dot{g}_t] + \frac{\eta - 1}{\eta} \frac{1}{\theta - 1 \mu - 1} [(1 - g)\dot{c}_t + \dot{g}_t].
\end{align*}
\]

Rearranging yields the equation (19).

A.2 Real Business Cycle Model with Constant Elasticity Entry

The production function equation is replaced by
\[ \dot{y}_t = \dot{z}_t + (\eta - 1)\dot{n}_t + \dot{h}_t. \]

The relation between output and markup is replaced by
\[ \dot{n}_t = \nu \hat{n}_{t-1} + (1 - \nu)\lambda_1 \hat{y}_t. \]

A.3 Sticky Prices Model with Constant Elasticity Entry

In addition to the modifications in constant elasticity entry model, the labor demand equation is replaced by
\[
\begin{align*}
\pi_t + (\eta - 1)(\hat{n}_t - \hat{n}_{t-1}) &= \beta E_t[(\eta - 1)(\hat{n}_{t+1} - \hat{n}_t)] \\
+ \kappa \left( \frac{1}{\sigma} \dot{c}_t + \frac{1}{\varphi} \dot{h}_t - (\eta - 1)\dot{n}_t - \dot{z}_t + \dot{\mu}_t \right) \\
&= \beta E_t[(\eta - 1)(\hat{n}_{t+1} - \hat{n}_t)] + \kappa \left( \frac{1}{\sigma} \dot{c}_t + \frac{1}{\varphi} \dot{h}_t - (\eta - 1)\dot{n}_t - \dot{z}_t + \dot{\mu}_t \right).
\end{align*}
\]
B A Small Open Economy Model with the Variety Effect and the Rule-of-Thumb Consumers

B.1 Price Determination under the Flexible Exchange Rate Regime

B.1.1 Log Linearized Equations in the Simple Small Open Economy Model

Log linearized consumption Euler equation is given by

\[ \hat{c}_t = E_t \hat{c}_{t+1} - \sigma (\hat{r}_t - E_t \pi_{t+1}). \]

The price relations are

\[ \pi_t = (1 - \alpha) \pi_{H,t} + \alpha \pi_{F,t}, \]
\[ \Delta \hat{s}_t = \pi_{F,t} - \pi_{H,t}, \]
\[ \hat{q}_t = (1 - \alpha) \hat{s}_t. \]

From the uncovered interest rate parity (UIP) condition, one can have

\[ \hat{r}_t - E_t \pi_{H,t+1} = \hat{r}_t^* - E_t \pi_{H,t+1}^* + E_t \Delta \hat{s}_{t+1} - \chi \hat{a}_t. \]

The monetary policy rule is given by

\[ \hat{r}_t = \phi \pi_t \quad \text{or} \quad \hat{r}_t = \phi e \Delta \hat{e}_t. \]

The aggregate resource constraint (ARC) is given by

\[ \hat{y}_t = (1 - \alpha) \hat{c}_t + \alpha \hat{c}_t^* + [(1 - \alpha) \alpha \gamma + \alpha \gamma] \hat{s}_t. \]

Substitution yields

\[ \hat{y}_t = (1 - \alpha) \hat{c}_t + \alpha \hat{c}_t^* + [(1 - \alpha) \alpha \gamma + \alpha \gamma] \hat{s}_t. \]

The foreign aggregate resource constraint is

\[ \hat{y}_t^* = (1 - \alpha) \hat{c}_t + \alpha \hat{c}_t^* + g \hat{g}_t^*. \]

The net exports equation is

\[ \hat{nx} = \hat{y}_t - \hat{c}_t - \alpha \hat{s}_t, \]

where \( \hat{nx} \equiv (NX_t - NX)/Y. \) From the current account identity it follows

\[ \hat{a}_t - \frac{1}{\beta} \hat{a}_{t-1} - \frac{1}{\beta}(\hat{e}_t - \hat{e}_{t-1}) + \frac{1}{\beta} \pi_t - \frac{1}{\beta} \hat{r}_{t-1} = \hat{nx}_t + \left( \frac{1}{\beta} - 1 \right) \alpha \hat{s}_t. \]

\(^{85}\)The last relation comes from \( \hat{q}_t = \hat{e}_t + p_t^* - p_t = p_{F,t} - p_t = (1 - \alpha) \hat{s}_t. \)

\(^{86}\)The uncovered interest rate parity (UIP) is \( \hat{r}_t = \hat{r}_t^* + E_t \Delta \hat{e}_{t+1} - \chi \hat{a}_t. \) In real terms, \( \hat{r}_t - E_t \pi_{t+1} = \hat{r}_t - E_t \pi_{t+1}^* + E_t \Delta \hat{q}_{t+1} - \chi \hat{a}_t. \) By using the price relation, one can get the equation.
B.1.2 3 Equation System

To explore the dynamics of the simple small open economy (SOE) model, one needs to combine and simplify equations. To derive the first equation, firstly one can start with the ARC and the foreign ARC. Combining the ARC and the foreign ARC produces

\[
\dot{c}_t = \frac{1}{1-\alpha} \dot{y}_t - \frac{\alpha}{1-\alpha} \dot{y}_t^* - \frac{(1-\alpha)\alpha \gamma + \alpha \gamma}{1-\alpha} \dot{s}_t. \tag{38}
\]

Substituting for \( \dot{c}_t \) into the consumption Euler equation produces

\[
\frac{1}{1-\alpha} E_t \Delta \dot{y}_{t+1} - \frac{\alpha}{1-\alpha} E_t \Delta \dot{y}_{t+1}^* - AE_t \Delta \dot{s}_{t+1} - \sigma(\phi_{\pi} \pi_{H,t} - E_t \pi_{H,t+1}) + \alpha \sigma E_t \Delta \dot{s}_{t+1} = 0.
\]

Rearranging gives

\[
\sigma(\phi_{\pi} \pi_{H,t} - E_t \pi_{H,t+1}) = (\alpha \sigma - A)E_t \Delta \dot{s}_{t+1} - \frac{1}{1-\alpha}(1-\rho_y)\dot{y}_t + \frac{\alpha}{1-\alpha}(1-\rho_{y^*})\dot{y}_t^*.
\]

The second equation comes from the uncovered interest rate parity condition,

\[
\hat{r}_t - E_t \pi_{H,t+1} = \hat{r}_t^* - \rho_{\pi^*} \pi_{t+1}^* + E_t \Delta \dot{s}_{t+1} - \chi \dot{a}_t.
\]

The third equation comes from combining the ARC, the net exports equation and the current account identity. From the ARC and the foreign ARC, one can have the equation (38). Substituting for \( \dot{c}_t \) into the net exports equation yields

\[
\hat{n}_x = \dot{y}_t - \frac{1}{1-\alpha} \dot{y}_t + \frac{\alpha}{1-\alpha} \dot{y}_t^* + A \Delta \dot{s}_t - \alpha \Delta \dot{s}_t = -\frac{\alpha}{1-\alpha} \dot{y}_t + \frac{\alpha}{1-\alpha} \dot{y}_t^* + (A - \alpha) \Delta \dot{s}_t.
\]

Substituting for \( \hat{n}_x \) in the current account identity gives

\[
\dot{a}_t - \frac{1}{\beta} \dot{a}_{t-1} - \frac{1}{\beta}(\dot{e}_t - \dot{e}_{t-1}) + \frac{1}{\beta} \pi_t - \frac{1}{\beta} \pi_{t-1} = -\frac{\alpha}{1-\alpha} \dot{y}_t + \frac{\alpha}{1-\alpha} \dot{y}_t^* + (A - \alpha) \Delta \dot{s}_t + \left(\frac{1}{\beta} - 1\right) \alpha \Delta \dot{s}_t.
\]

Applying the price relations and rearranging the equation produce

\[
\dot{a}_t - \frac{1}{\beta} \dot{a}_{t-1} = \left(A - 2\alpha + \frac{1}{\beta}\right) \Delta \dot{s}_t - \frac{1}{\beta}(1-\alpha) \Delta \dot{s}_{t-1} - \frac{\alpha}{1-\alpha} \dot{y}_t + \frac{\alpha}{1-\alpha} \dot{y}_t^* - \frac{1}{\beta} \pi_t^* + \frac{1}{\beta} \pi_{t-1}^*.
\]

Dynamics of the simple model can be represented by these 3 equations:

\[
\sigma(\phi_{\pi} \pi_{H,t} - E_t \pi_{H,t+1}) = (\alpha \sigma - A)E_t \Delta \dot{s}_{t+1} - \frac{1}{1-\alpha}(1-\rho_y)\dot{y}_t + \frac{\alpha}{1-\alpha}(1-\rho_{y^*})\dot{y}_t^*,
\]
\[
\hat{r}_t - E_t \pi_{H,t+1} = \hat{r}_t^* - \rho_{\pi^*} \pi_{t+1}^* + E_t \Delta \dot{s}_{t+1} - \chi \dot{a}_t,
\]
\[
\dot{a}_t - \frac{1}{\beta} \dot{a}_{t-1} = \left(A - 2\alpha + \frac{1}{\beta}\right) \Delta \dot{s}_t - \frac{1}{\beta}(1-\alpha) \Delta \dot{s}_{t-1} - \frac{\alpha}{1-\alpha} \dot{y}_t + \frac{\alpha}{1-\alpha} \dot{y}_t^* - \frac{1}{\beta} \pi_t^* + \frac{1}{\beta} \pi_{t-1}^*.
\]
In a matrix form, the above system can be rewritten as

\[
\begin{bmatrix}
\pi_{H,t+1} \\
\hat{s}_{t+1} \\
\hat{a}_{t+1}
\end{bmatrix} = \begin{bmatrix}
\phi_\pi & 0 & -\chi \frac{a\sigma - A}{\lambda} \\
0 & 1 & \sigma \chi \frac{1}{(1-\sigma) + A} \\
0 & A - 2\alpha + \frac{1}{\beta} & \sigma \chi \frac{1-2\alpha + \frac{1}{\beta}}{(1-\sigma) + A}
\end{bmatrix}
\begin{bmatrix}
\pi_{H,t} \\
\hat{s}_t \\
\hat{a}_t
\end{bmatrix} + \text{other terms.}
\]

The eigenvalues of the system can be calculated from the polynomial

\[
(\phi_\pi - \lambda) \left[ \lambda^2 - \left( 1 + \frac{1}{\beta} + \sigma \chi \frac{A - 2\alpha + \frac{1}{\beta}}{(1-\sigma) + A} \right) \lambda + \frac{1}{\beta} + \sigma \chi \frac{A - 2\alpha + \frac{1}{\beta}}{(1-\sigma) + A} - \sigma \chi \frac{A - 2\alpha + \frac{1}{\beta}}{(1-\sigma) + A} \right] = 0.
\]

If \( \phi_\pi > 1 \), the system has two eigenvalues greater than 1. (\( \lambda_1 = \phi_\pi \), 0 < \( \lambda_2 < 1 \), and \( \lambda_3 > 1 \).)

### B.1.3 Dynamics

To focus on the essential dynamics, I abstract from the foreign exogenous shocks. Three equations are rewritten as

\[
\sigma (\phi_\pi \pi_{H,t} - E_t \pi_{H,t+1}) = (\alpha \sigma - A) E_t \Delta \hat{s}_{t+1} - \frac{1}{1 - \alpha} (1 - \rho_y) \hat{y}_t,
\]

\[
\hat{r}_t - E_t \pi_{H,t+1} = E_t \Delta \hat{s}_{t+1} - \chi \hat{a}_t,
\]

\[
\hat{a}_t - \frac{1}{\beta} \hat{a}_{t-1} = \left( A - 2\alpha + \frac{1}{\beta} \right) \hat{s}_t - \frac{1}{\beta} (1 - \alpha) \hat{s}_{t-1} - \frac{\alpha}{1 - \alpha} \hat{y}_t.
\]

In the third equation, recursive substitution yields

\[
\hat{a}_t = \left( \frac{1}{\beta} \right)^t \hat{a}_0 + \left( A - 2\alpha + \frac{1}{\beta} \right) \hat{s}_t + \left( A - 2\alpha + \frac{1}{\beta} \right) \sum_{j=1}^{t-1} \left( \frac{1}{\beta} \right)^j \hat{s}_{t-j} - \frac{1}{\beta} (1 - \alpha) \left( \frac{1}{\beta} \right)^t \hat{s}_0 - \frac{1}{\beta} (1 - \alpha) \sum_{j=1}^{t-1} \left( \frac{1}{\beta} \right)^{j-1} \hat{s}_{t-j} - \frac{\alpha}{1 - \alpha} \sum_{j=1}^t \left( \frac{1}{\beta} \right)^{j-1} \hat{y}_{t-j+1}.
\]

Without loss of generality, one can let \( \hat{a}_0 = 0 \) and \( \hat{s}_0 = 0 \). The equation becomes

\[
\begin{aligned}
\hat{a}_t &= \left( A - 2\alpha + \frac{1}{\beta} \right) \hat{s}_t + \left( A - 2\alpha + \frac{1}{\beta} - (1 - \alpha) \right) \sum_{j=1}^{t-1} \left( \frac{1}{\beta} \right)^j \hat{s}_{t-j} - \frac{\alpha}{1 - \alpha} \sum_{j=1}^t \left( \frac{1}{\beta} \right)^{j-1} \hat{y}_{t-j+1}.
\end{aligned}
\]

From the first and second equations in the system, one can have

\[
\begin{aligned}
\left( 1 - \frac{\alpha \sigma - A}{\sigma} \right) E_t \hat{s}_{t+1} &= \left( 1 - \frac{\alpha \sigma - A}{\sigma} \right) \hat{s}_t + \chi \hat{a}_t - \frac{1}{\sigma} \frac{1}{1 - \alpha} (1 - \rho_y) \hat{y}_t.
\end{aligned}
\]
Substituting for \( \hat{a}_t \) produces

\[
\left( 1 - \frac{\alpha \sigma - A}{\sigma} \right) E_t \hat{s}_{t+1} = \left[ \left( 1 - \frac{\alpha \sigma - A}{\sigma} \right) + \chi \left( A - 2\alpha + \frac{1}{\beta} \right) \right] \hat{s}_t \\
+ \chi \left( A - \alpha + \frac{1}{\beta} + 1 \right) \sum_{j=1}^{t-1} \left( \frac{1}{\beta} \right)^j \hat{s}_{t-j} - \chi \frac{\alpha}{1 - \alpha} \sum_{j=1}^{t} \left( \frac{1}{\beta} \right)^{j-1} \hat{y}_{t-j+1} - \frac{1}{\sigma} \frac{1}{1 - \alpha} (1 - \rho_y) \hat{y}_t.
\]

Rearranging gives

\[
\hat{s}_t = \Gamma_1 E_t \hat{s}_{t+1} - \Gamma_2 \sum_{j=1}^{t-1} \left( \frac{1}{\beta} \right)^j \hat{s}_{t-j} + \Gamma_3 \sum_{j=1}^{t} \left( \frac{1}{\beta} \right)^{j-1} \hat{y}_{t-j+1} + \Gamma_4 (1 - \rho_y) \hat{y}_t.
\]

where

\[
\Gamma_1 = \frac{1 - \frac{\alpha \sigma - A}{\sigma}}{1 - \frac{\alpha \sigma - A}{\sigma} + \chi \left( A - 2\alpha + \frac{1}{\beta} \right)}, \quad \Gamma_2 = \frac{\chi \left( A - \alpha + \frac{1}{\beta} + 1 \right)}{1 - \frac{\alpha \sigma - A}{\sigma} + \chi \left( A - 2\alpha + \frac{1}{\beta} \right)},
\]

\[
\Gamma_3 = \frac{\chi \frac{\alpha}{1 - \alpha}}{1 - \frac{\alpha \sigma - A}{\sigma} + \chi \left( A - 2\alpha + \frac{1}{\beta} \right)}, \quad \Gamma_4 = \frac{\frac{1}{\sigma} \frac{1}{1 - \alpha}}{1 - \frac{\alpha \sigma - A}{\sigma} + \chi \left( A - 2\alpha + \frac{1}{\beta} \right)}.
\]

Note that

\[
S_t = \frac{1}{\beta} S_{t-1} + \frac{1}{\beta} \hat{s}_t, \quad E_t Y_{t+1} = \frac{1}{\beta} Y_t + E_t \hat{y}_{t+1}.
\]

Rewriting the equation by using undetermined coefficients yields

\[
F_1 (\hat{s}_t - F_2 S_{t-1}) = \Gamma_1 (E_t \hat{s}_{t+1} - F_2 S_t) + \Gamma_3 Y_t + \Gamma_4 (1 - \rho_y) \hat{y}_t.
\]

Comparing coefficients results in

\[
F_2 = \frac{\beta}{\Gamma_1} - \frac{\beta}{\Gamma_1} F_1, \quad F_1 = \frac{\beta + \Gamma_1 + \sqrt{(\beta + \Gamma_1)^2 - 4\beta \Gamma_1 (1 - \Gamma_2)}}{2\beta}.
\]

Hence, the difference equation can be rewritten as

\[
\hat{s}_t - F_2 S_{t-1} = \frac{\Gamma_1}{F_1} (E_t \hat{s}_{t+1} - F_2 S_t) + \frac{\Gamma_3}{F_1} Y_t + \frac{\Gamma_4}{F_1} (1 - \rho_y) \hat{y}_t,
\]

where \( \Gamma_1 / F_1 < 1 \). The solution to this equation is given by

\[
\hat{s}_t = F_2 S_{t-1} + \frac{\Gamma_3}{F_1} \frac{1}{1 - \frac{\rho_y}{\Gamma_1}} Y_t + \frac{\Gamma_1}{F_1} \frac{\Gamma_3}{\rho_y} \frac{1}{1 - \frac{\Gamma_1}{F_1} (1 + \rho_y)} \hat{y}_t + \frac{\Gamma_1}{F_1} (1 - \rho_y) \frac{1}{1 - \frac{\Gamma_1}{F_1} \rho_y} \hat{y}_t.
\]
B.2 Price Determination under the Fixed Exchange Rate Regime

Under the fixed exchange rate regime, there are the same 3 equations except the policy rule. By using the price relations, one can get 3 rewritten equations,

\[ \sigma(\hat{\phi}_t \Delta \hat{e}_t - E_t \Delta \hat{e}_{t+1}) = -[\sigma(1 - \alpha) + A]E_t \Delta \hat{s}_{t+1} - \frac{1}{1 - \alpha}(1 - \rho_y)\hat{y}_t + \frac{\alpha}{1 - \alpha}(1 - \rho_y^*)\hat{y}_t^*, \]

\[ \phi_c \Delta \hat{e}_t - E_t \Delta \hat{e}_{t+1} = -\chi \hat{a}_t + \hat{r}_t^*, \]

\[ \hat{a}_t - \frac{1}{\beta} \hat{a}_{t-1} = \left( A - 2\alpha + \frac{1}{\beta} \right) \hat{s}_t - \frac{1}{\beta}(1 - \alpha)\hat{s}_{t-1} - \frac{\alpha}{1 - \alpha} \hat{y}_t + \frac{\alpha}{1 - \alpha} \hat{y}_t^* - \frac{1}{\beta} \hat{n}_t^* + \frac{1}{\beta} \hat{r}_t^{*}. \]

In a matrix form, the above system can be rewritten as

\[
\begin{bmatrix}
\Delta \hat{e}_{t+1} \\
\hat{s}_{t+1} \\
\hat{a}_{t+1}
\end{bmatrix}
= \begin{bmatrix}
\phi_c & 0 & \frac{1}{A} \\
0 & A - 2\alpha + \frac{1}{\beta} & 0 \\
0 & -1 & \frac{1}{\beta}
\end{bmatrix}
\begin{bmatrix}
\Delta \hat{e}_t \\
\hat{s}_t \\
\hat{a}_t
\end{bmatrix}
+ \text{other terms}.
\]

The eigenvalues of the system can be calculated from the equation

\[
(\phi_c - \lambda) \left[ \lambda^2 - \left( 1 + \frac{1}{\beta} + \sigma \chi \frac{A - 2\alpha + \frac{1}{\beta}}{(1 - \alpha)\sigma + A} \right) \lambda + \frac{1}{\beta} + \sigma \chi \frac{A - 2\alpha + \frac{1}{\beta}}{(1 - \alpha)\sigma + A} - \sigma \chi \frac{A - 2\alpha + \frac{1}{\beta}}{(1 - \alpha)\sigma + A} \right] = 0.
\]

If \( \phi_c > 1 \), the system has two eigenvalues greater than 1. \( (\lambda_1 = \phi_c, 0 < \lambda_2 < 1, \text{ and } \lambda_3 > 1) \)

B.3 Log Linearized Equations

Log linearized equations for the flexible exchange rate regime are as below.\(^{87}\) The consumption Euler equation is given by

\[
\dot{c}_t = E_t \Delta \hat{c}_{t+1} - \hat{\sigma}(\hat{r}_t - E_t \pi_{t+1}) - \Theta_h E_t \Delta \hat{h}_{t+1} + \Theta_r E_t \Delta \hat{r}_{N,t+1},
\]

\[
\hat{\sigma} \equiv (1 - \delta)\sigma \frac{(1 - g)\mu\sigma}{(1 - g)\mu\sigma -\delta}, \quad \Theta_h \equiv \frac{\delta\sigma}{(1 - g)\mu\sigma - \delta} \left( 1 + \frac{1}{\varphi} \right),
\]

\[
\Theta_r \equiv \frac{\delta\sigma\mu}{(1 - g)\mu\sigma - \delta}.
\]

The New Keynesian Phillips equation is

\[
\pi_{H,t} + (\eta - 1)\hat{n}_t - \hat{n}_{t-1} = \beta E_t[\pi_{H,t+1} + (\eta - 1)(\hat{n}_{t+1} - \hat{n}_t)] + \kappa \left( \frac{1}{\sigma} \hat{c}_t + \frac{1}{\varphi} \hat{h}_t - (\eta - 1)\hat{n}_t + \alpha \hat{s}_t - \hat{z}_t + \hat{\mu}_t^{des} \right).
\]

\(^{87}\)For the fixed exchange rate regime, some modifications are needed. Firstly, one needs to replace the policy rule \( \hat{r}_t = \phi_c \hat{n}_t \) with \( \hat{r}_t = \phi_c \Delta \hat{n}_t \). Secondly, for the consumption Euler equation and New Keynesian Phillips curve, the forward looking dynamics for \( \hat{y}_t, \pi_t, \) and \( \hat{s}_t \) should be replaced by the dynamics for \( \hat{y}_t, \Delta \hat{c}_t, \) and \( \hat{s}_t \). This replacement can be done by utilizing the price relation, \( \pi_{H,t} = \pi_{F,t} - \Delta \hat{c}_t = \pi_t^* + \Delta \hat{c}_t - \Delta \hat{c}_t. \)
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The UIP condition is given by
\[ \hat{r}_t - E_t \pi_{H,t+1} = \hat{r}^*_t - E_t \pi^*_{t+1} + E_t \Delta \hat{s}_{t+1} - \chi \hat{a}_t. \]

The current account identity reads
\[ \hat{a}_t - 1 - \frac{1}{\beta} \hat{a}_{t-1} - \frac{1}{\beta} (\hat{e}_t - \hat{e}_{t-1}) + \frac{1}{\beta} \pi_t - \frac{1}{\beta} \hat{r}^*_{t-1} = \hat{nx}_t + \left( \frac{1}{\beta} - 1 \right) \alpha \hat{s}_t. \]

The net exports equation\(^{88}\) is defined by
\[ \hat{nx}_t = \hat{y}_t - (1 - g) \hat{c}_t - \hat{g}_t - (1 - g) \alpha \hat{s}_t. \]

The aggregate resource constraint (ARC) is given by
\[ \hat{y}_t = (1 - g) [(1 - \alpha) \hat{c}_t + \alpha c^*_t + ((1 - \alpha) \alpha \gamma + \alpha \gamma) \hat{s}_t] + \hat{g}_t. \]

The foreign aggregate resource constraint (ARC) is
\[ \hat{y}^*_t = (1 - g) \hat{c}^*_t + \hat{g}^*_t. \]

The monetary policy rule is given by
\[ \hat{r}_t = \phi_\pi \pi_{H,t}. \]

The fiscal rule\(^{89}\) is
\[ \hat{r}_t = \phi_b \hat{b}_{t-1}. \]

The real (effective) exchange rate is related to terms of trade according to
\[ \hat{q}_t = (1 - \alpha) \hat{s}_t. \]

The terms of trade change is defined by
\[ \Delta \hat{s}_t = \pi_{F,t} - \pi_{H,t}. \]

The CPI inflation can be rewritten by
\[ \pi_t = \pi_{H,t} + \alpha \Delta \hat{s}_t. \]

Import price inflation is defined by
\[ \pi_{F,t} = \pi_t^* + \Delta \hat{e}_t. \]

\(^{88}\)Net exports are defined by \(\hat{nx}_t = \frac{NX_t - NX}{Y_{t-1}}\) and government spending, \(\hat{g}_t = \frac{G_t - G}{Y_{t-1}}\).

\(^{89}\)Fiscal variables are defined by \(\hat{r}_{N,t} = \frac{r_{N,t} - r_{N,Y}}{Y_{t-1}}\) and \(\hat{b}_{N,t} = \frac{B_{t-1} - B/P}{Y_{t-1}}\).
The labor supply decision is given by
\[ \hat{w}_t - \hat{p}_t = \frac{1}{\sigma} \hat{c}_t + \frac{1}{\varphi} \hat{h}_t. \]

Firm entry is related to the markup according to
\[ \hat{n}_t = -\frac{1}{\eta} \frac{\mu}{(\theta - 1)\mu - \theta} \hat{\mu}^{des}. \]

The aggregate production technology is
\[ \hat{y}_t = \hat{z}_t + (\eta - 1) \hat{n}_t + \hat{h}_t. \]

Firm entry is described as
\[ \hat{n}_t = \nu \hat{n}_{t-1} + (1 - \nu) \lambda_1 \hat{y}_t. \]

The intertemporal equilibrium condition is
\[ \hat{b}_t = \left( \frac{1}{\beta} - \phi_b \right) \hat{b}_{t-1} - \frac{1}{\beta} \frac{b}{G} \pi_t + \frac{1}{\beta} \frac{b}{G} \hat{r}_{t-1} + \hat{y}_t - \alpha \frac{G}{Y} \hat{s}_t. \]

Exogenous variables are
\[ \hat{g}_t = \rho g \hat{g}_{t-1} + \epsilon g, \quad \hat{z}_t = \rho z \hat{z}_{t-1} + \epsilon z, \quad \hat{y}^* = \rho y^* \hat{y}^*_{t-1} + \epsilon y^* \]
\[ \hat{g}^* = \rho g^* \hat{g}^*_{t-1} + \epsilon g^*, \quad \hat{r}^* = \rho r^* \hat{r}^*_{t-1} + \epsilon r^*. \]

**B.4 Optimality Conditions in the Non Competitive Labor Market**

It is assumed that the wage is set by unions. Effective labor input hired by the firm \((i, j)\) is a CES function of the quantities of the different labor input types employed. It follows
\[ h_t(i, j) = \left[ \int_0^1 h_t(i, j, z)^{\varepsilon_{\omega} - 1} \frac{d z}{\varepsilon_{\omega}} \right]^{1/\varepsilon_{\omega}} \]
\[ h_t(i) = \sum_{j=1}^{N_i} h_t(i, j), \quad H_t = \int_0^1 h_t(i) d i, \]
where \(\varepsilon_{\omega}\) denotes the elasticity of substitution across different types of households. The fraction of non-asset holding consumers and asset holding consumers is assumed to be uniformly distributed across worker types (and hence across unions). Each period, a typical union sets the (real) wage for its workers in order to maximize the objective function,
\[
\max \left\{ \frac{1}{C_{N,t}(z)^{1/\sigma} N_t(z)^{1/\varphi}} W_t^{R}(i, j, z) h_t(i, j, z) - \frac{h_t(i, j, z)^{1 + \frac{1}{\varphi}} - 1}{1 + \frac{1}{\varphi}} \right\} \\
+ (1 - \delta) \left\{ \frac{1}{C_{A,t}(z)^{1/\varphi} N_t(z)^{1/\varphi}} W_t^{R}(i, j, z) h_t(i, j, z) - \frac{h_t(i, j, z)^{1 + \frac{1}{\varphi}} - 1}{1 + \frac{1}{\varphi}} \right\}, \\
\text{s.t. } h_t(i, j, z) = \left( \frac{W_t^{R}(i, j, z)}{W_t^{R}(i, j)} \right)^{-\varepsilon_{\omega}} h_t(i, j). \]
Imposing symmetricity across \( z \) (and also across \((i, j)\)) yields
\[
\begin{align*}
\frac{1}{\tilde{C}_N(z) \frac{1}{\sigma} N_t(z) \frac{1}{\varphi}} (-\varepsilon) h_t + \frac{1}{\tilde{C}_A(z) \frac{1}{\sigma} N_t(z) \frac{1}{\varphi}} h_t & - \frac{1}{W_t^R} - \frac{1}{W_t^R} \varepsilon \omega h_t^{1+\frac{1}{\varphi}} \\
+ (1 - \delta) \frac{1}{\tilde{C}_A(z) \frac{1}{\sigma} N_t(z) \frac{1}{\varphi}} (-\varepsilon) h_t + (1 - \delta) \frac{1}{\tilde{C}_A(z) \frac{1}{\sigma} N_t(z) \frac{1}{\varphi}} h_t - (1 - \delta)(-\varepsilon) h_t^{1+\frac{1}{\varphi}} \frac{1}{W_t^R} = 0.
\end{align*}
\]

Note that in a symmetric equilibrium,
\[
H_{N,t} = H_{A,t} = H_t,
H_{N,t} = N_t h_{N,t}, \quad H_{A,t} = N_t h_{N,t}, \quad H_t = N_t h_t.
\]

Rearranging gives
\[
\left( \frac{\delta}{\tilde{C}_N(z) \frac{1}{\sigma} (N_t h_t) \frac{1}{\varphi}} + \frac{1 - \delta}{\tilde{C}_A(z) \frac{1}{\sigma} (N_t h_t) \frac{1}{\varphi}} \right) W_t^R = \mu^W.
\]

Using \( N_t h_t = H_t \), one can rewrite the above equation as
\[
\left( \frac{\delta}{MRS_{N,t}} + \frac{1 - \delta}{MRS_{A,t}} \right) \frac{W_t}{P_t} = \mu^W.
\]

Log linearized equation becomes
\[
\hat{w}_t - \hat{p}_t = \chi_N \tilde{m} \tilde{r}_N + \chi_A \tilde{m} \tilde{r}_A = \frac{1}{\sigma} \hat{c}_t + \frac{1}{\varphi} (\chi_N + \chi_A) \hat{h}_t,
\]
\[
\chi_N \equiv \frac{\delta W^R}{MRS_{N,t} \mu^W}, \quad \chi_A \equiv \frac{(1 - \delta) W^R}{MRS_{A,t} \mu^W}, \quad \hat{c}_t \equiv \chi_N \hat{c}_t + \chi_A \hat{c}_t.
\]

It is assumed that tax policy equates steady state consumption across household types (i.e., \( C_N = C_A \)). Then \( MRS_N = MRS_A \), \( \chi_N = \delta \), and \( \chi_A = 1 - \delta \). The above equation can be rewritten as
\[
\hat{w}_t - \hat{p}_t = \frac{1}{\sigma} \hat{c}_t + \frac{1}{\varphi} \hat{h}_t. \tag{39}
\]

Here the assumption that the wage markup, \( \mu^W \), is sufficiently large, is maintained. This assumption guarantees that the conditions,
\[
\frac{W_t}{P_t} > MRS_{N,t} \quad \text{and} \quad \frac{W_t}{P_t} > MRS_{A,t},
\]
are satisfied. Both conditions imply that both types of households are willing to meet firms’ labor demand at the prevailing wage.
B.5 Derivation of the Consumption Euler Equation

The consumption Euler equation of asset holding household is given by

$$\hat{c}_{A,t} = E_t \hat{c}_{A,t+1} - \sigma(r_t - E_t \pi_{t+1}).$$

To derive the consumption Euler equation of non asset holding household (or rule-of-thumb consumer), consider first the budget constraint of non asset holding household:

$$P_t C_{N,t} = W_t H_{N,t} - P_t T_{N,t}.$$

Rearranging yields

$$C_{N,t} = \frac{W_t}{P_t} H_{N,t} - T_{N,t}.$$

Note that in a steady state,

$$T_N = \frac{W}{P} H_N - C_N.$$

Log linearization gives

$$C_N \hat{c}_{N,t} = \frac{W}{P} H_N (\hat{w}_t - \hat{p}_t + \hat{h}_{N,t}) - T_N \hat{\tau}_{N,t}.$$

The consumption Euler equation of non asset holding household is given by

$$\hat{c}_{N,t} = H_N \cdot \frac{W}{P} C_N (\hat{w}_t - \hat{p}_t + \hat{h}_{N,t}) - \frac{Y}{C_N} \hat{\tau}_{N,t},$$

where $$\hat{\tau}_{N,t} \equiv \frac{\tau_{N,t} - \tau_N}{C_N}$$. By choosing an appropriate $$T_N$$ and $$T_A$$, $$C_N = C_A = C$$ can be obtained. This implies that $$H_N = H_A = H$$.

For the aggregate consumption Euler equation, substituting (39) into expression (41) produces

$$\hat{c}_{N,t} = \frac{1}{(1-g)\mu} \frac{1}{\sigma} \hat{c}_t + \frac{1}{(1-g)\mu} \left(1 + \frac{1}{\varphi}\right) \hat{h}_t - \frac{1}{1-g} \hat{\tau}_{N,t}$$

$$\left( 1 - g = \frac{C}{Y}, \quad \mu = \frac{W/P}{Z} = \frac{W}{P}, \quad Y = ZH_N = H_N \right).$$

Using the operator $$(1 - L^{-1})$$ gives

$$\hat{c}_{N,t} - E_t \hat{c}_{N,t+1} = \frac{1}{(1-g)\mu} \frac{1}{\sigma} [\hat{c}_t - E_t \hat{c}_{t+1}] + \frac{1}{(1-g)\mu} \left(1 + \frac{1}{\varphi}\right) [\hat{h}_t - E_t \hat{h}_{t+1}] - \frac{1}{1-g} [\hat{\tau}_{N,t} - E_t \hat{\tau}_{N,t+1}].$$

Applying the operator $$(1 - L^{-1})$$ to the aggregate consumption expression, gives

$$\hat{c}_t - E_t \hat{c}_{t+1} = \delta [\hat{c}_{N,t} - E_t \hat{c}_{N,t+1}] + (1 - \delta) [\hat{c}_{A,t} - E_t \hat{c}_{A,t+1}].$$
By substituting (40) and (42) into (43), one can have the aggregate consumption Euler equation

\[ \dot{c}_t = E_t \dot{c}_{t+1} - \dot{\sigma}(r_t - E_t \pi_{t+1}) - \Theta_h E_t \Delta \dot{h}_{t+1} + \Theta_r E_t \Delta \dot{r}_{N_t,t+1}, \]

\[ \dot{\sigma} = (1 - \delta)\sigma \frac{(1 - g)\mu \sigma}{(1 - g)\mu \sigma - \delta}, \quad \Theta_h = \frac{\delta \sigma}{(1 - g)\mu \sigma - \delta} \left(1 + \frac{1}{\varphi} \right), \]

\[ \Theta_r = \frac{\delta \sigma \mu}{(1 - g)\mu \sigma - \delta}. \]

### B.6 Derivation of the Inflation Equation

Following Rotemberg (1982), I introduce adjustment costs to firm’s maximization problem. Intermediate firm’s real profit is given by

\[ \frac{\Pi_t(i, j)}{P_{H,t}} = \frac{P_{H,t}(i, j)}{P_{H,t}} Y_t(i, j) - \frac{MC_t}{P_{H,t}} Y_t(i, j) - K \left( \frac{P_{H,t}(i, j)}{P_{H,t-1}(i, j)} - 1 \right)^2 Y_t. \]

Firm maximizes the present value of real profit. Firm’s problem is

\[ \max_{P_{H,t}} \sum_{k=0}^{\infty} Q_{t,t+1} \left[ \frac{P_{H,t+k}(i, j)}{P_{H,t+k}} \right] \left( \frac{P_{H,t+k}(i, j)}{P_{H,t+k}(i)} \right)^{-\theta} \left( \frac{P_{H,t+k}(i)}{P_{H,t+k}} \right)^{-\omega} \frac{Y_{t+k}}{N_{t+k}^{1-(\theta-1)(\eta-1)}} - \frac{MC_{t+k}}{P_{H,t+k}} \left( \frac{P_{H,t+k}(i, j)}{P_{H,t+k-1}(i, j)} - 1 \right)^2 Y_{t+k} \right]. \]

The first order condition reads

\[ \frac{Y_t(i, j)}{P_{H,t}} + \frac{P_{H,t}(i, j)}{P_{H,t}} dY_t(i, j) - \frac{MC_t}{P_{H,t}} dY_t(i, j) - K \left( \frac{P_{H,t}(i, j)}{P_{H,t-1}(i, j)} - 1 \right) \frac{Y_t}{P_{H,t-1}(i, j)} = 0. \]

Rewriting the above condition yields

\[ \frac{Y_t(i, j)}{P_{H,t}} - \frac{Y_t(i, j)}{P_{H,t}} \varepsilon(N_t) + \frac{MC_t}{P_{H,t}} \frac{Y_t(i, j)}{P_{H,t}} \varepsilon(N_t) - K \left( \frac{P_{H,t}(i, j)}{P_{H,t-1}(i, j)} - 1 \right) \frac{Y_t}{P_{H,t-1}(i, j)} = 0. \]

In a symmetric equilibrium,

\[ Y_t(i, j) = Y_t^\dagger, \quad Y_t = N_t^{\eta}Y_t^\dagger, \quad P_{H,t}(i, j) = P_{H,t}^\dagger, \quad P_{H,t} = N_t^{1-\eta}P_{H,t}^\dagger. \]
In terms of the firm level prices and the economy level demand, the optimality condition can be rewritten as

$$\frac{Y_t}{N_t P^!_{H,t}} - \frac{Y_t}{N_t P^!_{H,t}} \varepsilon(N_t) + \frac{MC_t}{N_t P^!_{H,t}} \frac{Y_t}{P^!_{H,t}} \varepsilon(N_t) - K \left( \pi^!_{H,t} - 1 \right) \frac{Y_t}{P^!_{H,t}} \pi^!_{H,t}$$

$$+ E_t Q_{t,t+1} K \left( \pi^!_{H,t+1} - 1 \right) \pi^!_{H,t+1} \frac{Y_{t+1}}{P^!_{H,t}} = 0.$$ 

Note that in a steady state

$$m_c^! = 1 = \frac{\varepsilon - 1}{\varepsilon}.$$ 

Log linearized equation is

$$\frac{1}{N} \bar{\varepsilon} \cdot m_c^! (\hat{m}c_t^! + \hat{\mu}_t^{des}) - K \pi^!_{H,t} + E_t \beta K \pi^!_{H,t+1} = 0.$$ 

Firm level inflation is given by

$$\pi^!_{H,t} = \beta E_t \pi^!_{H,t+1} + \frac{1}{N} \bar{\varepsilon} \cdot m_c^! \frac{1}{K} (\hat{m}c_t^! + \hat{\mu}_t^{des}).$$

PPI inflation is

$$\pi^*_{H,t} + (\eta - 1)(\hat{n}_t - \hat{n}_{t-1}) = \beta E_t [\pi^!_{H,t+1} + (\eta - 1)(\hat{n}_{t+1} - \hat{n}_t)] + \kappa (\hat{m}c_t^! + \hat{\mu}_t^{des}).$$

### B.7 The Intertemporal Equilibrium Condition

The household budget constraint is given by

$$P_t C_t + \varepsilon_t B_{F,t} + B^G_t = \varepsilon_t B_{F,t-1} R^*_{t-1} + B^G_{t-1} R_{t-1} + W_t H_t + \Pi^*_t - P_t \tau_t.$$ 

The aggregate resource constraint is

$$Y_t = C_{H,t} + C^*_{H,t} + G_t.$$ 

From the definition of net exports in the equation (33), one can have

$$P_{H,t} Y_t - P_t C_t - P_{H,t} G_t = P_{H,t} N X_t.$$ 

Rearranging gives

$$P_t C_t = P_{H,t} Y_t - P_{H,t} G_t - P_{H,t} N X_t.$$ 

Combining the household budget constraint and the aggregate resource constraint yields

$$P_{H,t} Y_t - P_{H,t} G_t - P_{H,t} N X_t + \varepsilon_t B_{F,t} + B^G_t = \varepsilon_t B_{F,t-1} R^*_{t-1} + B^G_{t-1} R_{t-1} + W_t H_t + \Pi^*_t - P_t \tau_t.$$
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By using the current account identity,
\[ \epsilon_t B_{F,t} - \epsilon_{t-1} B_{F,t-1}^* R_{t-1}^* = P_{H,t} N X_t, \]
and the income identity,
\[ P_{H,t} Y_t = W_t H_t + \Pi_t^H, \]
one can have the intertemporal equilibrium equation in nominal terms,
\[ P_{H,t} G_t = P_t \tau_t + B_t^G - B_{t-1}^G R_{t-1}. \]

Linearized intertemporal equilibrium condition is given by
\[ \hat{b}_t = \left( \frac{1}{\beta} - \phi_b \right) \hat{b}_{t-1} - \frac{1}{\beta} \frac{b}{G} \pi_t + \frac{1}{\beta} \frac{b}{H} \phi_x \pi_{t-1} + \hat{g}_t - \alpha g s_t. \]

where the policy function \( \tau_t = \phi_b \hat{b}_{t-1} \) and the steady state relation \( P_H/P = 1 \) are utilized.

Note that
\[ \hat{g}_t \equiv \frac{G_t - G}{Y} = \frac{G}{Y} (\log G_t - \log G), \quad \hat{\tau}_t \equiv \frac{\tau_t - \tau}{Y} = \frac{\tau}{Y} (\log \tau_t - \log \tau), \]
\[ \hat{b}_t \equiv \frac{B_t/Y_t - B/P}{Y}. \]

B.8 The Current Account Identity

The definition of current account reads
\[ \epsilon_t B_{F,t} - \epsilon_{t-1} B_{F,t-1}^* R_{t-1}^* = P_{H,t} N X_t. \]

Dividing by \( Y \cdot P_t \) and rearranging the equation yield
\[ \frac{\epsilon_t B_{F,t}}{Y P_t} - \frac{\epsilon_{t-1} B_{F,t-1}}{Y P_t} - \frac{\epsilon_t}{P_t} P_{t-1} \frac{R_{t-1}}{P_t} = \frac{P_{H,t} N X_t}{P_t} \cdot \frac{1}{Y}. \]

From the definition of (real) foreign debt, \( A_t \equiv \epsilon_t B_{F,t}/(Y P_t) \), the above equation is simplified as
\[ A_t - A_{t-1} \frac{1}{\epsilon_t \pi_t} R_{t-1}^* = \frac{P_{H,t} N X_t}{P_t} \cdot \frac{1}{Y}. \]

Note that in a steady state,
\[ 1 - \frac{1}{\beta} = \frac{N X}{Y}. \]

Linearization gives
\[ \hat{a}_t - \frac{1}{\beta} \hat{a}_{t-1} - \frac{1}{\beta} (\hat{\epsilon}_t - \hat{\epsilon}_{t-1}) + \frac{1}{\beta} \hat{\pi}_t - \frac{1}{\beta} \hat{\pi}_{t-1}^* = \hat{\alpha} x_t + \left( \frac{1}{\beta} - 1 \right) \alpha \hat{s}_t, \]

where
\[ \hat{\alpha} x_t \equiv \frac{N X_t - N X}{Y} = \frac{N X}{Y} (\log N X_t - \log N X). \]
References


