Evaluating Macroprudential Policy from Operational Perspectives

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Abstract

This paper examines the effects of macroprudential policy and its interaction with monetary policy in a New Keynesian DSGE model with financial friction. Macroprudential policy can stabilize credit cycle. However, if credit market consists of household and business credit, a macroprudential instrument that only aims the adjustment of one market can create regulatory arbitrage. A welfare-maximizing optimal combination of monetary and macroprudential policy is the monetary policy focusing only on stabilizing inflation and the macroprudential policy focusing only on stabilizing credit. This policy combination improves welfare by effectively reducing inflation and credit volatility. Credit stabilization is welfare improving because lower credit volatility is rewarded by higher mean credit and capital. Meanwhile, allowing monetary policy to react to credit, is neither optimal nor effective for credit stabilization.

JEL Classification: E44, E50, E61

Keywords: Macroprudential policy, Capital requirement ratio, Loan-to-Value (LTV) ratio, Optimal macroprudential policy, Interaction between monetary and macroprudential policy

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1. **Introduction**

After recent financial crisis, the role of credit, leverage, and macroprudential policy instruments which can effectively control those variables has become an area of active research. However, there are still issues about macroprudential policy that need further exploration, concerning its effects and coordination with other macro policies. Among those, this paper aims to examine operational issues of macroprudential policy, that is, about how the policy should be implemented. In particular, main research questions in this paper are as follows: What is the effect from macroprudential policy? Can macroprudential policy be welfare improving? Is there any optimal rule to operate monetary and macroprudential policy together? If there is, why is that particular rule optimal?

To analyze such issues about macroprudential policy, I construct a calibrated DSGE model to assess macroprudential policy. The model is based on New Keynesian framework with price rigidity, so that the joint assessment of monetary and macroprudential policy can be done using the methodologies in the New Keynesian literature. The financial contract features Bernanke, Gertler, and Gilchrist (1999) (BGG) financial accelerator mechanism in business and household lending. This allows the interaction between the possibility of default, entrepreneurs’ net worth (in business lending) and housing value (in household lending) in credit market. Financial intermediaries share aggregate risk as the forecast error between expected aggregate return and realized return is reflected by changes in bank capital. The regulation on bank capital ratio or loan-to-value ratio (LTV) affects the real economic activity through credit markets. Macroprudential policy is those regulations which are operated by rules countercyclically react to credit cycle. In particular, macroprudential instruments are distinguished between bank capital requirement ratio regulation, an universal regulation affecting both household and business credit market, and LTV regulation, a household credit market-specific instrument. The welfare-maximizing optimal combination of monetary and macroprudential policies is constructed conditional on this DSGE model. The optimal rule is simple and implementable in the sense used by Schmitt-Grohe and Uribe (2004a) [1]. Based on this optimal policy result, I analyze the interaction between monetary and macroprudential policy. Specifically, I examine how macroprudential policy affects the performance of monetary policy, and vice versa. Finally, I discuss whether it is desirable to allow monetary policy to react counter-cyclically to credit movements.

The main findings of this paper suggest that rule-based countercyclical macroprudential

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[1] They define a policy being simple when rules are set as a function of a small number of easily observable macroeconomic indicators and being implementable when it delivers the uniqueness of the rational expectations equilibrium.
policy can play a role as a built-in stabilizer regarding business and credit cycle. Dynamic capital requirement ratio (DCRR) rule is effective in stabilizing credit, output and consumption. Dynamic LTV (DLTV) ratio rule is effective in stabilizing housing price, which it targets, but it can result in amplifying the volatility of business sector by creating regulatory arbitrage. In a welfare perspective, macroprudential policy can be welfare improving. Most of the welfare gain comes from DCRR, and the gain from DLTV is marginal. In the optimal policy combination, monetary policy commits to stabilize inflation and DCRR commits to stabilize credit. Credit stabilization brings welfare gain because reduced credit volatility is rewarded by higher mean credit and capital in the economy. In terms of credit stabilization, Macroprudential policy is better tool than monetary policy. The former can effectively control credit without conflicting inflation stability given monetary policy response against inflation is aggressive enough. The monetary policy rule reacting to credit, on the other hand, is neither optimal nor effective for credit stabilization.

Literatures about macroprudential policy can be categorized by their main research interest. One group of researches focus on why adopting macroprudential policy is necessary (Lorenzoni (2008), Bianchi, Boz, and Mendoza (2011), Jean and Korinek (2011)). They emphasize that the externality from credit boom-bust cycle provides a role for countercyclical macroprudential policy. The other group focuses on how macroprudential policy would effect the economy (Angeloni and Faia (2009), Kannan, Rabanal, and Scott (2009), Angelini, Neri, and Panetta (2011)). They demonstrate how dynamics of the economy change with macroprudential policy, using medium-size DSGE model. This paper can be considered to belong in the latter group, but it also contains welfare analysis result showing there is a welfare gain from macroprudential policy. Several new attempts in the paper can be regarded as a contribution to the literature. First, the model differentiates market-specific policy instrument and market-universal policy instrument, enabling comparisons between them. Next, this paper is the first to find simple optimal combination of monetary and macroprudential policy, based on household welfare measure. In addition, I discuss how macroprudential policy and monetary policy should be implemented together, which is a subject that has not been extensively researched yet.

2Decentralized agents do not internalize systemic loss when credit boom turns to bust, thus they tend to take excessive leverage. Macroprudential policy can prevent those excessive risk taking.
2. MACROPRUDENTIAL POLICY: CONCEPT, PRACTICE AND ISSUE

Macroprudential policy refers to a set of regulatory policies imposed mainly on financial institutions for macroeconomic purposes. According to BIS (2010a), there are two, not mutually exclusive, objectives of macroprudential policy - to strengthen the financial system’s resilience against adverse shocks in the economy and to actively limit the buildup of financial systemic risk\(^3\). Being preventive in nature, it aims ex-ante stabilization and should be distinguished from ex-post crisis management policy. Macroprudential policy instruments include a wide range of financial regulation measures. Table 1 summarizes the examples of macroprudential policy that are already in practice or proposed. Instruments are classified by the channels they intervene, such as financial intermediaries’ balance sheet, terms and conditions of credit contract and transactions, and market structure.

Although some of these macroprudential policy measures are already being implemented in practice, there are still several questions about macroprudential policy in need of further study. In particular, following issues are especially of interest in this paper. First is the mechanism how each macroprudential policy tool affects credit and real activity. To figure this, it is necessary to use models with financial friction that can identify the role of credit and leverage in the economy and the consequence of regulating them. Next, policymakers should take into account the general equilibrium effect from regulatory arbitrage when implementing market / sector specific policy instrument. Some of macroprudential instruments are designed as sector / market-specific, aiming the adjustment of imbalance within a particular sector or market. Examples of such instruments are sectoral capital requirement, LTV or DTI cap for household lending. However, those instruments can create a regulatory arbitrage, a movement of funds from heavily regulated market to less regulated market. Another issue which has become the one of the primary policy concerns is how to achieve monetary and macroprudential policy objectives together. This concern is well addressed in recent discussion between Woodford (2012) and Svensson (2012). They both agree on the perspective that the financial stability should be one of the prior policy concern, but have somewhat different views on the effectiveness of monetary and macroprudential policy instrument to achieve it. Woodford argues that using monetary policy to maintain financial stability can be justified even with macroprudential instruments, as long as those macroprudential policy instruments cannot provide a complete solution to the welfare loss from financial instability. On the other hand, Svensson advocates ‘one policy target for one policy instrument’ view,

\(^3\)BIS (2010a) defines the systemic risk as “a risk of disruption to financial services that is caused by an impairment of all or parts of the financial system and has the potential to have serious negative consequences for the real economy.”.
using monetary policy for the traditional price stabilization purpose and macroprudential policy for the financial stabilization purpose. He argues that macroprudential policy instruments have more direct effects on leverage than monetary policy, thus it is more efficient to use them separately.

Next section, I suggest a model that can assess above issues about macroprudential policy.\footnote{There are also several other issues about macroprudential policy that is not explicitly addressed in this paper. One example is the rule vs. discretion in policy action. In most countries, the financial regulatory instruments mentioned above are conducted largely at the policymaker’s discretion, except for Spain’s dynamic provisioning rule (IMF (2010)), which requires banks to set aside additional provisions according to a formula during phases of rapid credit expansion. Despite these practices, rule-based approach can still be an appealing option. It can better anchor agents’ expectations about future regulatory policies, and overcome the bias for the regulator’s inaction facing strong political and market resistance. This paper assumes rule-based macroprudential policy.}

Table 1: Macroprudential policy instruments: examples

<table>
<thead>
<tr>
<th>Category</th>
<th>Instruments</th>
<th>Description</th>
<th>Adopted by</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balance sheet instruments</td>
<td>Countercyclical capital buffer</td>
<td>Increase capital buffer in expansion, release it in downturn</td>
<td>Basel III, Spain(2000, provisioning)</td>
</tr>
<tr>
<td></td>
<td>Sectoral capital requirements</td>
<td>Requires additional capital on lendings to specific sectors</td>
<td>Australia(2004), India (2005)</td>
</tr>
<tr>
<td></td>
<td>Maximum leverage ratio</td>
<td>Cap on the ratio of total assets to bank equity</td>
<td>Basel III, Canada (1980s), Swiss (2011)</td>
</tr>
<tr>
<td></td>
<td>Time-varying liquidity buffer</td>
<td>Increase liquidity ratio in expansion, decrease it in downturn</td>
<td>Croatia (2003), New Zealand (2008)</td>
</tr>
<tr>
<td>Terms and conditions of transactions</td>
<td>Loan to value ratio</td>
<td>Cap on the ratio of loan value to collateral value</td>
<td>Many countries</td>
</tr>
<tr>
<td></td>
<td>Debt service to income ratio</td>
<td>Cap on the ratio of debt service to borrower’s income</td>
<td>Many countries</td>
</tr>
<tr>
<td>Market structure</td>
<td>Use of central counterparty</td>
<td>Financial trade using centralized clearing center rather than OTC</td>
<td>Many countries</td>
</tr>
</tbody>
</table>

3. Model

The model in this paper is based on the BGG financial accelerator mechanism in a New Keynesian framework. The financial intermediation in this paper mainly follows that of Zhang (2009), which supplements traditional BGG mechanism by introducing risk sharing banking sector. Bank capital then functions as a buffer stock to absorb forecast error between expected and realized return. In addition, I distinguish between saving households and borrowing households, so that borrowing households and entrepreneurs are dual borrowers in the economy (household and business credit). Housing goods are not only in the utility function but also used as collateral for the borrowing households. Monetary policy influences saving households’ real deposit rate by choosing nominal interest rate, and macroprudential policy affects bank real lending rates by imposing regulations on the bank. Figure 1 summarizes the basic framework of the model. A summary of variables used in the model is provided in appendix A.1.

![Figure 1: Overview of the model](image)

3.1. Household

There exist two types of households, savers \((s)\) and borrowers \((b)\), who are distinguished by time preference parameters. Borrowing households are less patient about future consumption than saving households \((\beta_b < \beta)\). This distinction by time preference parameters is often used in credit friction models, as in Iacoviello (2005). Because of different time preferences, saving households will always save and borrowing households will always borrow in the steady state and its neighborhood. I assume that both types have the same population, as if there is one saving member and one borrowing member in a single household. Agents do not move between the two groups.
3.1.1. Saving Household

Saving households (denoted by $s$) with future discount rate $\beta$ solve

$$\max_{C_s, H_s, N_s, I_{hs}^H, B, D, e} \mathbb{E}_\omega \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \gamma \log C_{t,s} + (1 - \gamma \epsilon^H_t) \log H_{t,s} + \varphi \log(1 - N_{t,s}) \right] \right\}$$

subject to

$$C_{t,s} + P_{t, s}^H I_{t,s}^H + B_t + D_t + e_t + T_{t,s} \leq R_N^t \frac{B_{t-1}}{P_t} + R_{t-1}^D D_{t-1} + R_{t-1}^e e_{t-1} + w_t N_{t,s} + Div_t.$$  \hspace{1cm} (2)

$C$, $H$, $N$ denotes consumption goods, housing goods and labor supply, respectively. $\epsilon^H$ is a preference shock on housing goods, such that a positive $\epsilon^H$ shock can cause housing demand to drop. In the budget constraint, $P^H$ is the relative price of housing goods in terms of final consumption goods and $I^H$ is the investment in housing goods. Saving households can invest in an asset portfolio that consists of nominal assets ($B$), real bank deposits ($D$) and real bank equity capital ($e$). Each asset yields return $R^N$, $R^D$ and $R^e$. For the rest of the paper I assume all credits in the economy are real credits, by assuming equilibrium quantity of nominal asset is zero. $w$ is real wage, $Div$ is dividends from entrepreneurs, and $T_s$ is a lump-sum tax equally imposed on both savers and borrowers. Appendix A.2 provides the details of the saving households’ optimization problem.

3.1.2. Borrowing Household

Borrowing households (denoted by $b$) with future discount rate $\beta_b$ maximize

$$\max_{C_b, H_b, N_b, I_{b}^H, L^H} \mathbb{E}_\omega \left\{ \sum_{t=0}^{\infty} \beta_b^t \left[ \gamma \log C_{t,b} + (1 - \gamma \epsilon^H_t) \log H_{t,b} + \varphi \log(1 - N_{t,b}) \right] \right\}$$

subject to

$$C_{t,b} + P_{t, b}^H I_{t,b}^H + \int_0^{\omega_{t,b}^H} \omega_t P_t^H H_{t,b}(\omega)d\omega + [1 - F(\omega_{t,b}^H)] R_{t-1}^{LH} L_{t-1}^H + T_{t,b} \leq w_t N_{t,b} + L_t^H.$$  \hspace{1cm} (4)

$L^H$, $R^{LH}$ denote real household borrowing and its interest rate. Other terms are defined
similarly to the saving households’ problem. $\omega$ is an idiosyncratic shock in housing price, with $E(\omega) = 1$ and hits each household after aggregate variables are determined. Borrowing households use housing goods as collateral, and default occurs when $\omega$ falls below the default threshold $\bar{\omega}_{H,b}$ which is set according to the debt repayment value. When default occurs, lender claims the remaining value $= \int_{0}^{\omega_{H,b}} \omega P_{t}^{H,H} H_{t,b} f(\omega) d\omega$ of housing stock. For computational convenience, I assume that defaulting households pay the cash value of their housing goods to the lender rather than losing actual housing stock. This way they keep their housing stock and resume economic activity. A more detailed description of the household borrowing contract is given in section 3.3.2. Finally, $[1 - F(\bar{\omega}_{t}^{H,b})]$ fraction of households avoid default and redeem $[R_{L,H}^{t}]$ amount of the debt obligations. In appendix A.2, the borrowing household’s optimization problem are provided in detail. In steady state, borrowing households have to provide more labor and consume less than saving households to pay interest on their debt.

3.2. Entrepreneurs

Entrepreneurs produce intermediate goods using capital ($K$), labor ($N$), entrepreneurs’ labor ($N_{e}$) and bankers’ labor ($N_{f}$). Production technology includes entrepreneurs’ and bankers’ labor, and those labor incomes are added to entrepreneur’s net worth and bank capital. However, their contribution in aggregate output is assumed to be very small ($\alpha_{ne} = \alpha_{nf} = 0.01$).

$$Y_{t} = A_{t}(K_{t-1}^{\alpha_{k}})(N_{t}^{\alpha_{n}})(N_{t,e}^{\alpha_{ne}})(N_{t,f}^{\alpha_{nf}}), \quad \alpha_{k} + \alpha_{n} + \alpha_{ne} + \alpha_{nf} = 1. \quad (5)$$

Gross return from one unit of capital is defined by:

$$R_{t}^{K} = \frac{z_{t} + (1 - \delta)q_{t}}{q_{t-1}}, \quad (6)$$

where $q$ is the price of capital in terms of consumption goods, and $z_{t}$ is the value of marginal product of capital ($z_{t} = mc_{t} \cdot \alpha_{k} Y_{t}/K_{t-1}$). Note $R_{t}^{K}$ is the return on capital chosen last period ($K_{t-1}$), determined only when this period’s aggregate shocks are realized. It reflects price changes ($q_{t}/q_{t-1}$) of capital as well as $z_{t}$. Given real marginal cost $mc_{t}$, labor demand for
each type of labor is given by

\[ w_t = m c_t \cdot \alpha_n \frac{Y_t}{N_t} \quad \text{\( w_{t,e} = m c_t \cdot \alpha_{ne} \frac{Y_t}{N_{t,e}} \)} \quad \text{\( w_{t,f} = m c_t \cdot \alpha_{nf} \frac{Y_t}{N_{t,f}} \)}. \]  

(7)

Here labor supply of the entrepreneurs and the bankers is fixed at 1.

Entrepreneur’s net worth, denoted by \( W_t \), is retained earnings and similar concept with owner’s equity in firm’s balance sheet. Using aggregated terms, we have

\[ W_t = \nu V_t + w_{t,e} \]  

(8)

where \((1 - \nu)\) is the fraction that is paid off to saving households as dividends (\( Div_t = (1 - \nu)V_t \)), and \( V \) is the return from each period’s project net of the borrowing cost.

\[ V_t = \int_{\omega_t}^{\infty} \omega R_t^K q_{t-1} K_{t-1} f(\omega)d\omega - (1 - F(\bar{\omega}_b^t)) R_{t-1}^{LB} L_{t-1}^B. \]  

(9)

The first term in the right hand side is the gross payoff for entrepreneurs, and the second term is the debt repayment obligation to the lender. \( \omega \) is an idiosyncratic shock that hits each entrepreneur and \( \bar{\omega}_b \) is the default threshold determined by the debt repayment obligation. \( L^B \) is the amount of borrowing and \( R^{LB} \) is its interest rate. If an entrepreneur defaults, \((\omega < \bar{\omega}_b)\), he will end up with nothing and the remaining value of the project will be accrued to the bank. Lending contract between between entrepreneurs and the financial intermediary is further discussed below.

3.3. Financial Contract and Banking Sector : BGG Mechanism

Financial contracts in this paper is based on Zhang (2009), which refines the BGG model by introducing risk sharing banking sector and bank capital. In original BGG model, there is no need for the financial intermediary (‘bank’ hereafter) to set aside buffer capital to perform the intermediation, since it can diversify the idiosyncratic risk and is fully insured against aggregate risk. The bank is insured against aggregate risk as risk-neutral entrepreneurs offer aggregate-state contingent default threshold (and debt repayment value). This guarantees the intermediary zero profit condition at all states. However, in real world, it would be general to believe that the bank is also exposed to aggregate risk, and its profit and capital are affected. Zhang’s model has a financial contract that debt repayment value is not state-contingent,
and is set according to the next period’s expected aggregate return. Thus there are ex-ante default threshold that determines debt repayment value, and the ex-post default threshold that determines the actual default. The forecast error in the next period’s return on capital or housing value creates a discrepancy between expected default rate and actual default rate, causing profit or loss to the bank that is reflected by the changes in bank capital.

3.3.1. Financial Contract: Business Loan

The size of a business loan for an individual entrepreneur is defined by the difference between the size of an investment project and the entrepreneur’s net worth. That is, \( L_{t}^{B,i} = q_t K_t^i - W_t^i \). In standard BGG model, entrepreneurs take all aggregate risk and offer state-contingent default threshold and debt repayment value. On the other hand, here debt repayment value is not state-contingent. Entrepreneurs offer ex-ante default threshold, chosen from the distribution of idiosyncratic shock (\( \omega \)), given next period’s expected aggregate return on capital. Denoting this ex-ante default threshold by \( \bar{\omega}_t^{i,a} \), the relationship between gross loan repayment value and the expected project return is

\[
R_{t}^{LB,i} L_{t}^{B,i} = \bar{\omega}_t^{i,a} E_t R_{t+1}^K q_t K_t^i .
\]

(10)

Entrepreneurs’ optimization problem becomes

\[
\max_{K_t^i, \bar{\omega}_t^{i,a}} \int_{-\infty}^{\infty} \omega E_t R_{t+1}^K q_t K_t^i f(\omega) d\omega - (1 - F(\bar{\omega}_t^{i,a})) R_{t}^{LB,i} L_{t}^{B,i} .
\]

(11)

subject to the bank’s ex-ante participation incentive constraint

\[
R_{t}^{f}(q_t K_t^i - W_t^i) = (1 - F(\bar{\omega}_t^{i,a})) R_{t}^{LB,i} L_{t}^{B,i} + (1 - \mu) \int_{0}^{\bar{\omega}_t^{i,a}} \omega E_t R_{t+1}^K q_t K_t^i f(\omega) d\omega .
\]

(12)

\( R_f \) is the funding rate of the bank. \( \mu \) represents the monitoring cost that the bank has to pay when it claims post-default investment project. This ‘costly state verification’ problem by Townsend (1979) is the core friction in BGG model. Note that competition among banks will lead them to accept this zero profit participation constraint. However, zero profit condition holds only ex-ante, and profit or loss can occur once aggregate return \( R_{t+1}^K \) is realized and different from \( E_t R_{t+1}^K \). In period \( t+1 \), after \( R_{t+1}^K \) is realized, ex-post actual default threshold
\( \bar{\omega}_{t+1} \) is defined as

\[
\bar{\omega}_{t+1}^b = \frac{R_t^L L_t^B}{R_{t+1}^K q_t K_t} = \bar{\omega}_t^a \frac{E_t R_{t+1}^K}{R_{t+1}^K}.
\] (13)

Details of the entrepreneurs’ optimization problem, given lognormal assumption for \( \omega \), is provided in appendix A.3.

3.3.2. Financial Contract: Household Loan

Similar to the BGG mechanism applied to the business sector, we can derive a mechanism so that household lending is subject to default. Suppose an idiosyncratic housing price shock hits each borrowing household after aggregate variables are determined. The \( i \)th borrowing household will face foreclosure if the value of the idiosyncratic price shock \( (\omega_{H,i}^t) \) is less than some threshold level \( (\bar{\omega}_{H,i}^a) \). Since every borrowing household is homogenous before idiosyncratic shock, one can drop \( i \) superscript and the household lending contract can be written as

\[
R_t^L H_t^t = \bar{\omega}_t^H a E_t P_{t+1}^H H_{t+1,b}.
\] (14)

Here \( E_t P_{t+1}^H H_{t+1,b} \) is the expected value of the collateral owned by borrowing households, and \( \bar{\omega}_t^H a \) is ex-ante default threshold. The zero profit condition for the bank becomes

\[
(R_t^f + \nu_c)L_t^H = (1 - F(\bar{\omega}_t^H a))R_t^L H_t^t + (1 - \mu^H) \int_0^{\bar{\omega}_t^H a} \omega E_t P_{t+1}^H H_{t+1,b} f(\omega) d\omega.
\] (15)

\( \nu_c \) is the markup in household lending so that the steady-state risk premium in household lending rate matches the historical U.S. observation. It is assumed that the profit from this markup is redistributed to saving households as dividends. After aggregate shocks are realized, the ex-post default threshold is defined by \( \bar{\omega}_{t+1}^{H,b} = (R_t^L H_t^t) / (P_{t+1}^H H_{t+1,b}) \).

3.3.3. Financial Intermediary

Similar to the original BGG model, the bank has the ability to diversify idiosyncratic risk, and can insure lenders against it. Contrast to BGG model, as mentioned, the bank shares aggregate risk. The difference between expected value and realized value in aggregate return is reflected by the changes in bank capital. The bank has two means of financing, deposits
(D_t) and equity capital (e_t), and its asset side consists of business (L^B_t) and household lending (L^H_t). Bank capital ratio is defined by \( \kappa_t \equiv e_t / L_t \), where \( L_t \) is total lending \( (L_t = L^B_t + L^H_t) \). Also, the asset side of the bank must be balanced with its liability and equity side, that is, \( D_t + e_t = L_t \). It is assumed that the bank funding cost is affected by bank capital structure in a reduced form. Bank funding rate \( (R^f_t) \) is determined by adding a markup to the actual funding rate (weighted average of the deposit rate and the return on bank capital).

\[
R^f_t = \kappa_t R^e_t + (1 - \kappa_t) R^D_t + s(\kappa_t, \bar{\kappa}_t). \tag{16}
\]

\( s \) is a markup function that captures the effect of bank capital structure on bank funding rate. It is decreasing in \( \kappa \), implying that if the bank is badly capitalized, it will face higher cost of funding. \( s \) is also a function of \( \bar{\kappa} \), capital requirement ratio, to capture the effect of bank capital regulation imposed by the regulatory authority. The violation of capital requirement regulation will lead bank to face a corrective measure affecting its reputation adversely or constraining its managerial decision. This, in turn, worsens the bank’s funding cost as higher markup is charged to the bank. Thereby I call \( s \) as ‘regulatory markup function’.

Bank capital is required by the regulator, owned by households, and functions as a buffer stock which absorbs the forecast error in aggregate return. The law of motion for the bank capital is given as below.

\[
e_t = (1 - \phi)e_{t-1} + w_{t,f} - R^f_{t-1}(L^B_{t-1} + L^H_{t-1}) + R^L_{t-1}B^B_{t-1}(1 - F(\bar{\omega}^b_t)) + (1 - \mu) \int_0^{\bar{\omega}^b_t} \omega R^K_t q_{t-1} K_{t-1} f(\omega) d\omega + R^L_{t-1}H^H_{t-1}(1 - F(\bar{\omega}^{H,b}_t)) + (1 - \mu^H) \int_0^{\bar{\omega}^{H,b}_t} \omega P^H_t H_{t,b} f(\omega) d\omega + \epsilon^e_t. \tag{17}
\]

Net profit from lending activity (inflows from loans minus outflows to depositors and shareholders) affects bank capital. Bank capital increases when this net profit is positive, or written off otherwise. This net profit is nonzero when there exists forecast error in return on capital \( (R^K_t - E_{t-1} R^K_t) \) or housing value \( (P^H_t H_{t,b} - E_{t-1} P^H_t H_{t,b}) \). Bankers labor income is also added to the capital. Note since the profit from lending activity is zero in steady state, the steady state bank capital level is entirely dependent on bankers’ labor income. To prevent capital overaccumulation, it is assumed that bankers spend \( \phi \) fraction of bank capital every period. It should be noted that the adjustment process of bank capital is slow when \( \phi \) and \( w_{t,f} \) is small. Therefore, the bank complies capital requirement ratio obligation which will be
shown later mainly by adjusting credits rather than bank capital. \( \epsilon^c_t \) is exogenous shock in bank capital which captures the type of shocks originating from the financial sector. As an example, one can imagine the changes in bank capital due to the changes in asset quality, or due to the differences between ex-ante and ex-post variance of idiosyncratic shocks.

3.4. Capital Producer

At the beginning of each period, the capital producer purchases \( I_t \) amounts of consumption goods at a price of one and turns them into the same amount of new capital. Transformation costs arise during the process and at the end of the period she resells new capital to entrepreneurs at price \( q_t \). The law of motion for capital stock is given by

\[
K_t = I_t + (1 - \delta)K_{t-1}. \tag{18}
\]

Appendix A.4 discusses further details about capital producer’s optimization problem.

3.5. Retail Goods Producers

Each retail goods producer \((i)\) purchases intermediate goods and turns them into retail goods \((Y_t(i))\) in a monopolistically competitive market. Total final usable goods \(Y_t\) are the following composite of retail goods.

\[
Y_t = \left[ \int_0^1 Y_t(i)^{1-\theta} \, di \right]^{\frac{1}{1-\theta}} \tag{19}
\]

Only \((1-\theta)\) fraction of retail goods producers are allowed to change price, à la Calvo (1983). Appendix A.5 discusses the details of the retail goods producers’ optimization problem, using the method used by Schmitt-Grohe and Uribe (2004a).

3.6. Exogenous Processes and Market Clearing

I assume shocks in productivity, housing preference and government spending follow stationary AR(1) processes in log-linearized form:

\[
\dot{A}_t = \rho_A \dot{A}_{t-1} + \epsilon^A_t, \quad \dot{\gamma}_t = \rho_\gamma \dot{\gamma}_t + \xi^\gamma_t, \quad \dot{G}_t = \rho_G \dot{G}_{t-1} + \epsilon^G_t. \tag{20}
\]
Housing stock is determined by the following law of motion:

\[ H_{t,s} = (1 - \delta_H)H_{t-1,s} + I^H_{t,s}, \quad H_{t,b} = (1 - \delta_H)H_{t-1,b} + I^H_{t,b}. \] (21)

Aggregate housing investment demand is the sum of the housing investments of both types of households \((I^H_t(D) = I^H_{t,s} + I^H_{t,b})\). It is possible to invent production technology for housing goods. However, for simplicity, I assume the housing investment supply is given exogenously, fluctuating around the certain fraction of steady state output \((I^H_t(S) = \varrho Y + \epsilon^H_t)\). In equilibrium, \(I^H_t(D) = I^H_t(S)\).

The government budget is balanced every period as government expenditure is financed by lump-sum taxes from households.

\[ G_t = T_{t,s} + T_{t,b}. \] (22)

Finally, we have market clearing conditions for goods and labor markets. Note that in the aggregate resource constraint we have terms for monitoring cost and regulatory markup.

\[ \frac{Y_t}{s_t} = C_{t,s} + C_{t,b} + q_tI_t + I^H_t + G_t + \phi e_{t-1} \]
\[ + \mu \int_0^{\omega^H} \omega R^K_i q_{t-1}K_{t-1}dF(\omega) + \mu^H \int_0^{\omega^H} P^H_i H_{t,b}dF(\omega) + \varsigma_t. \] (23)

\((\varsigma_t\) denotes the terms representing the resource usage by regulatory markups.)

and \(N_{t,s} + N_{t,b} = N_t(S), \quad N_t(S) = N_t(D)\).

3.7. Monetary Policy

Monetary policy is set to follow an extended Taylor rule in log-linearized form. The central bank sets the nominal interest rate according to the deviation in inflation, output, and credit from their steady state values.

\[ \hat{R}^N_t = \rho_r \hat{R}^N_{t-1} + (1 - \rho_r)[\phi_y \hat{\pi}_t + \phi Y \hat{Y}_t + \phi L \hat{L}_t] + \epsilon^R_t. \] (24)
3.8. Macropurdenrial Policy

The regulatory authority sets the target capital requirement ratio \( \bar{\kappa}_t \) and target Loan-to-Value ratio \( \bar{ltv}_t \) dynamically according to rules that systemically react to observable macro variables such as output, credit, or housing prices. The set of variables that the macroprudential policy reacts to are chosen with practical considerations, from among those policymakers are most likely to be concerned about. As mentioned in 3.3.3, the bank has to pay higher funding costs when regulatory markup increases. I assume the functional form of this capital requirement regulatory markup in equation (16) to be

\[
s(\bar{\kappa}_t, \kappa_t) = \nu_a \exp[\nu_b (\bar{\kappa}_t - \kappa_t)/\kappa].
\]  

(25)

Here \( \nu_a \) determines the level of intervention and \( \nu_b \) determines the responsiveness of regulation. Required capital ratio is set as a simple function of output and gross credit \( \bar{\kappa}_t = \zeta_\alpha(Y_t, L_t) \). This idea of dynamic capital requirement ratio is close to countercyclical capital buffer suggested in Basel III. \(^5\) Specifically, the requirement ratio is set in a similar way as Taylor-rule type interest rate determination.

\[
\hat{\bar{\kappa}}_t = \rho_a \hat{\bar{\kappa}}_{t-1} + \phi_Y \hat{Y}_t + \phi_L \hat{L}_t.
\]

(26)

The LTV ratio regulation is a market-specific regulation pertaining only to the household lending market. I assume this LTV regulation operates as a markup over the bank funding rate that applies to household lending. With this regulatory markup, bank's ex-ante zero-profit condition in the household loan contract (15) should be modified as below.

\[
[R_t + \nu_c + Q(ltv_t, ltv_{\bar{t}})]L_{H,i}^{H,i} = (1 - F(\bar{\omega}^{H,i,a}_t))R_t^{LH,i}L_{H,i}^{H,i} + (1 - \mu^H) \int_0^\infty \omega E_t F_{t+1}^{H,i} \omega^b f(\omega) d\omega.
\]

(27)

Here \( Q \) is the regulatory markup function, a function of actual LTV ratio and target LTV ratio. The borrowing household has to pay a regulatory penalty for taking a higher LTV ratio than the target LTV ratio set by the financial regulator. The functional form of this

\(^5\)See BIS (2011)
regulatory markup is assumed to be

\[ Q(ltv_t, \overline{ltv}_t) = \nu_a^H \exp[\nu_b^H (ltv_t - \overline{ltv}_t)/ltv]. \] (28)

Again, the target LTV ratio is set using a simple rule where the policymaker cares only about housing prices \((\overline{ltv}_t = \zeta_{ltv}(P_t^H))\). This specification of the LTV rule corresponds with policy practices in Asian countries such as Hong Kong and Korea, which use LTV regulation in household credit as a tool to stabilize housing prices. \(^6\) Specifically, in log-linear form,

\[ \hat{ltv}_t = \rho_{ltv}\hat{ltv}_{t-1} - \phi_{ltv}^t \hat{P}^H_t. \] (29)

It should be emphasized that each government policy affects different agents and interest rates differently. Monetary policy, by determining nominal interest rate, influences real deposit rate through Fisher equation. Capital requirement ratio regulation directly affects bank funding rate, and thus have direct effect on both business and household borrowing condition. LTV regulation is imposed on household lending, and have direct effect on household borrowing condition.

4. Calibration

The parameters in the utility function, housing and the production sector are chosen in the range that accords with the standard values found in the literature. They are adjusted so that the variables are presented as their quarterly values. The future discount factors are chosen to be 0.99 for saving households and 0.9885 for borrowing households. \(\gamma\), which determines the weight of consumption goods and housing goods in the utility function, is calibrated as 0.875. The weight of labor in the utility function \(\varphi\) is assumed to be 2. The rate of depreciation for capital goods \(\delta\) is chosen to be 0.025, implying that it takes 10 years to completely depreciate. The rate of depreciation for housing goods \(\delta^H\) is 0.0125. In the production sector, the share of capital, labor, entrepreneur’s labor, and banker’s labor in a Cobb-Douglas production function is chosen to be 0.31, 0.67, 0.01, and 0.01, respectively. Dividend ratio of entrepreneurial sector \((1 - \nu)\) is 0.027 and the capital adjustment cost parameter \(\chi_K\) is 4. The fraction of retail goods producers who can reset the sale price each period \(\theta\) is 0.25, and retailers’ degree of monopolistic power \(\epsilon\) is chosen so that

\(^6\)see Gerlach and Peng (2005).
steady state real markup is 1.1. For the exogenous processes, autoregressive coefficients are $\rho_A = 0.85$, and $\rho_G = 0.8$, $\rho_\gamma = 0.95$. The degree of inertia in monetary policy ($\rho_r$) is 0.85. The standard deviations of the technology shock, the government spending shock, and the housing demand shock are chosen so that they can match the historical volatility of output, government expenditure and housing price [7] given that the monetary and macroprudential policy follows the baseline specification that is shown in section 5. The standard deviation of the bank capital shock is chosen so that the quarterly volatility of bank capital is 5% of its steady state value, and of the monetary shock is chosen to be the 25bps when annualized. In addition, the parameters for the financial contract and banking sector are calibrated so that they imply certain spread levels that match the historical data. [8]. Table 6 in the appendix A.1 provides a summary of these parameters and their calibrated values.

In steady state, the borrowing households/saving households ratio of consumption ($C^B/C^S$) is 0.78, and that of labor supply is (1.78). Regarding credit variables, business credit/household credit ratio ($L^B/L^H$) is 3.27, entrepreneurs’ capital-net worth ratio ($K/W$) in steady state is 1.66, and household debt/annualized GDP ratio ($L^H/Y$) is 0.19. The probability of default for entrepreneurs and households is 3.0% and 0.8%, respectively. For prudential variables, the steady state bank capital ratio ($\kappa$) is 0.080 and the LTV ratio is 0.708. Steady state interest rate and variable ratios is provided in Table 7 in the appendix A.1.

5. Effects of Macroprudential Policy

In this section, I provide simulation results to evaluate the effects of macroprudential policy on the economy. Three policy specifications are considered. They differ by the different parametrization of regulatory markup functions. The first is the baseline setup with static capital regulation and no LTV regulation (‘BL’), which has $\nu_a^I = \nu_a^H = 0.0025$ for the degree of intervention, and $\nu_b^I = 25$ and $\nu_b^H = 0$ for the sensitivity of regulation to the movement of the target variable. This particular setup mimics the current regulatory stance in many countries. The second specification is a dynamic capital requirement ratio rule reacting to gross credit (‘DCRR’). Again, it has $\nu_a^I = \nu_a^H = 0.0025$, $\nu_b^I = 25$ and $\nu_b^H = 0$, but now capital requirement ratio moves over time in response to gross credit. The third is a dynamic LTV

---

7During 1982-2011, the standard deviation of quarterly change in output, government expenditure and housing price is measured to be 0.7%, 0.9% and 1.7%, respectively. (Source : National account, U.S. Bureau of Economic Analysis, house price index, new single-family houses sold, U.S. Census Bureau.)

8 As proxies of interbank spread, business lending spread and household lending spread, I choose federal funds rate - financial CP spread, 6 month treasury bill - prime loan spread, and 6 month treasury bill - 30 year fixed mortgage spread. During 1982-2011, the average of each spread is 0.1%p, 2.9%p and 3.6 %p, respectively. (Source : Federal Reserve Board)
ratio rule reacting to housing price ('DLTV'). It has $\nu_a^I = \nu_a^H = 0.0025$, $\nu_b^I = \nu_b^H = 25$ and target LTV ratio moves over time in response to housing price. It is assumed that monetary policy responds to inflation and output ($\phi_\pi = 1.5, \phi_Y = 0.1$) in all three cases.

5.1. **Effects of The Capital Requirement Ratio Rule**

In figure 2 and 3, I compare impulse response functions from the baseline model (BL) and the dynamic capital requirement ratio model (DCRR). Impulse responses are measured in percentage changes from steady state values and the magnitude of the shocks is scaled as one standard deviation for each. Here parameters in the capital requirement ratio rule (26) are chosen so that the capital ratio reacts countercyclically to gross credit ($\phi_{L}^\kappa = 1.5$) and does not react to output ($\phi_{Y}^\kappa = 0$). The capital requirement ratio has a high degree of inertia ($\rho_{\kappa} = 0.9$).

Figure 2 shows the impulse response functions from a positive productivity shock. In the baseline model, as is seen in typical RBC model, consumption, investment and output increase and inflation falls. Housing price rises because housing supply is given exogenously in this model and housing price is completely determined by its demand. With respect to the household optimization problem, it is straightforward that housing demand must increase when consumption increases and the marginal utility of consumption decreases. In banks’ balance sheets, both business and household lending increase. On the other hand, in DCRR macroprudential policy model, the capital requirement ratio rises as it responds to the increase in gross credit. Because of the regulation, credit expansion is dampened in the DCRR model compared to the baseline model. Consequently, we have less persistent movements in consumption, investment, output and housing price. Since the regulation requires a higher capital ratio, bank capital is higher with the DCRR policy.

Figure 3 shows the response of the economy to a negative shock in bank capital, which represents a shock originating from the financial intermediary sector. Given static capital requirement regulation and slow recovery of bank capital, both business lending and household lending shrink. It generates recessionary pressure as consumption, investment, output decrease and inflation falls. Here DCRR policy helps faster recovery this time by lowering capital requirement ratio. Declines in lending are not as great as in the baseline model and the recessionary effects in investment, output and inflation is less severe, although the effect is not dramatic.
5.2. Effects of LTV Ratio Rule

Figure 4 and 5 present impulse responses from the model with DLTV macroprudential policy. Here the DLTV rule is assumed to react to housing price ($\phi_{ph}^{LTV} = 1.5$) with a high
degree of inertia ($\rho_{LTV} = 0.9$). Figure 4 shows that this DLTV rule is successful in dampening housing price upon a productivity shock. A positive productivity shock raises housing price and the target LTV ratio falls in response. Stronger regulation dampens household lending expansion and the rise in housing price. It is noticeable that regulatory arbitrage - a substitution of credit from the heavily regulated (household) market to the less heavily regulated (business) market - occurs here. As a result, business lending, investment are more volatile than in the baseline model. Lower growth in household lending due to LTV regulation means higher bank capital ratio, and it gives the bank more room to increase business lending which is less regulated.

In figure 5, positive housing demand shock, a housing market-specific shock, is imposed. The effect of this housing demand shock is to raise housing price and increase credit to borrowing households. With DLTV policy, target LTV ratio goes down with higher housing price. This in turn curbs the lending to borrowing households and eventually housing price. Again, regulatory arbitrage occurs from household credit to business credit, in this case offsetting the original credit shift from business to household credit market. Both figure 4 and 5 indicate that while DLTV policy is effective in stabilizing housing price which it targets, policymakers should be aware that it can trigger regulatory arbitrage because of its market-specific nature.

![Figure 4: Impulse response functions given productivity shock, from baseline (‘BL’) model and dynamic LTV (‘DLTV’) model](image)

Figure 4: Impulse response functions given productivity shock, from baseline (‘BL’) model and dynamic LTV (‘DLTV’) model
5.3. Effects on Economic Volatility

As is expected from the above impulse response functions, macroprudential policy can function as an automatic stabilizer. Table 2 identifies the effect of macroprudential policy on the volatility of major variables. The numbers in the table are unconditional standard deviations of consumption ($\sigma_C$), output ($\sigma_Y$), investment ($\sigma_I$), inflation ($\sigma_\pi$), housing price ($\sigma_{PH}$), business lending ($\sigma_{LB}$), household lending ($\sigma_{LH}$). It is observed that DCRR policy reduces the volatility of consumption, output, investment, housing price, credit and raises the volatility of inflation. In particular, the standard deviation of business and household lending significantly decreases. However, the fact that the volatility of inflation increases with DCRR policy suggests a possible conflict with the monetary policy objective. A detailed discussion about this conflict will be presented in section 5 and 6. On the other hand, the effect of the DLTV macroprudential policy is rather mixed. Clearly, this type of policy can stabilize credit to household hence housing price, as those variables show significant volatility reduction. However, it can amplify the volatility of the real sector, as the volatility of output, investment and business lending increases. This result strongly suggests that when implementing a regulatory measure that only influences a partial segment of the economy, policymakers need to be careful about the possibility of regulatory arbitrages.
Table 2: Effects of macroprudential policy on economic volatility

<table>
<thead>
<tr>
<th></th>
<th>(\sigma_C)</th>
<th>(\sigma_Y)</th>
<th>(\sigma_I)</th>
<th>(\sigma_p)</th>
<th>(\sigma_{L_B})</th>
<th>(\sigma_{L_H})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline (A)</td>
<td>0.008</td>
<td>0.012</td>
<td>0.005</td>
<td>0.003</td>
<td>0.016</td>
<td>0.048</td>
</tr>
<tr>
<td>DCRR Macroprudential (B)</td>
<td>0.007</td>
<td>0.011</td>
<td>0.003</td>
<td>0.003</td>
<td>0.015</td>
<td>0.033</td>
</tr>
<tr>
<td>Change ((B-A)/A))</td>
<td>-10.09%</td>
<td>-9.44%</td>
<td>-24.92%</td>
<td>9.02%</td>
<td>-7.41%</td>
<td>-30.59%</td>
</tr>
<tr>
<td>DLTV Macroprudential (C)</td>
<td>0.008</td>
<td>0.013</td>
<td>0.005</td>
<td>0.003</td>
<td>0.013</td>
<td>0.052</td>
</tr>
<tr>
<td>Change ((C-A)/A))</td>
<td>-4.05%</td>
<td>1.11%</td>
<td>5.75%</td>
<td>4.62%</td>
<td>-21.58%</td>
<td>7.72%</td>
</tr>
</tbody>
</table>

* Figures are unconditional standard deviations of consumption (\(\sigma_C\)), output (\(\sigma_Y\)), investment (\(\sigma_I\)), inflation (\(\sigma_p\)), housing price (\(\sigma_{P_H}\)), business lending (\(\sigma_{L_B}\)), and household lending (\(\sigma_{L_H}\)).

6. WELFARE ANALYSIS AND OPTIMAL POLICY

In this section, I analyze the welfare-maximizing optimal combination of monetary and macroprudential policy. The macroprudential policy rule is designed to be simple and implementable, as in Schmitt-Grohe and Uribe (2004a). They define a policy being simple when rules are set as a function of a small number of easily observable macroeconomic indicators, and being implementable when it delivers the uniqueness of the rational expectations equilibrium. Here macroprudential policy is simple because it is a function of observable macro variables (output, credit and housing price), and implementable because policy coefficients are restricted to a range that guarantees an unique bounded solution. For the solution method, I used the second order perturbation method using Dynare. Details of the advantage that the second order approximation method offers can be found in Schmitt-Grohe and Uribe (2004b).

6.1. OPTIMAL MONETARY AND MACROPRUDENTIAL POLICY COMBINATION

As a welfare measure, the unconditional expectation of average household utility in period zero is calculated. That is,

\[
E_0V = E_0\left\{\sum_{t=0}^{\infty} \beta^t [\gamma \log \tilde{C}_t + (1 - \gamma \epsilon_t^2) \log \tilde{H}_t + \varphi \log (1 - \tilde{N}_t)] \right\}
\]

(30)

where \(\tilde{X}\) denotes the average household variable ((\(X_s + X_h\))/2). This average household concept is consistent with the model assumption that there is one saving member and one
borrowing member in a single household. The policy gain is measured as the fraction of consumption goods that households have to give up with an inferior policy, that is, the value of $\lambda$ in the equation below.

$$E_0V(A) = E_0V((1 - \lambda)\tilde{C}(B)).$$ \hfill (31)

Here A and B represent two different government policies such that B being superior in terms of welfare measure, and $\tilde{C}(B)$ represents the consumption stream associated with the policy B. The range of macroprudential policy parameters $\phi_Y^c$, $\phi_L^c$, $\phi_{\kappa}^{ltv}$, reaction of capital requirement ratio and target LTV ratio in response to output, gross credit and housing price, are restricted to the interval between 0 and 2. \footnote{For example, $\phi_L^c = 2$ means that capital requirement ratio increases by 2 percent of its steady state value 0.08 in response to 1 percent increase in credit.} The coefficient larger than 2 is ruled out on the practical assumption that the policymaker is unlikely to favor such extreme response of the regulation policy. This range is then partitioned with grids of size 0.2. For monetary policy parameters, a range between 1.2 and 3 of grid size 0.05 for $\phi_\pi$ and a range between 0 and 1 with grid size 0.05 for $\phi_Y$ and $\phi_L$ are examined. For each combination of monetary and macroprudential policy, I calculate $E_0V$ and define the optimal policy as the policy combination that maximizes $E_0V$. Macroprudential policies are assumed to have inertia ($\rho_\kappa = \rho_{ltv} = 0.9$).

Table 3 suggests optimal policy combinations and welfare gains. There are five different panels according to regulation regime. The ‘baseline’ panel features the baseline policy calibration in the previous section. Monetary policy parameters are given as $\phi_\pi = 1.5$, $\phi_Y = 0.1$, and regulatory regime is given by static capital requirement regulation and no LTV regulation. ($\nu_a^d = \nu_a^H = 0.0025$, $\nu_b^d = 25$, $\nu_b^H = 0$, $\phi_Y^c = \phi_L^c = 0$). In ‘Monetary policy only’ panel, optimal monetary policy based on the same regulatory regime is found. Thus policies in this panel is the optimal monetary policy without macroprudential policy. In three lower panels, macroprudential policy is in action. To see how much each macroprudential policy instrument contributes to welfare, I separately calculate the optimal policy and welfare gains when only DLTV or DCRR policy is in action, and when both policies operate together. Note that the nonstochastic steady state associated with each panel is the same. Welfare gains in each panel is the increase in welfare compared to the welfare in the ‘baseline’ panel. The measure of welfare gain ($\lambda$) is the fraction of the consumption stream defined in (31). For each panel, three different scenarios regarding the volatility of housing and financial market are suggested. In ‘Stable’ environment, housing demand and bank capital shocks are assumed to
be nonexistent, providing stable condition for both markets. ‘Normal environment’ follows the basic calibration for the standard deviation of housing demand and bank capital shocks ($\sigma^\gamma = 0.001$ and $\sigma^\epsilon = 0.0048$). In ‘Volatile’ scenario, both $\sigma^\gamma$ and $\sigma^\epsilon$ are doubled to 0.002 and 0.0096, respectively.

Table 3: Optimal monetary (MOP) and macroprudential policy (MPP)

<table>
<thead>
<tr>
<th>Regulation Regime</th>
<th>Volatility</th>
<th>Policy Parameters</th>
<th>Welfare Gains: $\lambda(%)^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>Stable</td>
<td>$\phi_\pi$ 1.5</td>
<td>$\phi_Y$ 0.1 $\phi_L$ 0.0</td>
</tr>
<tr>
<td></td>
<td>Normal</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Volatile</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Monetary policy only</td>
<td>Stable</td>
<td>3.0 $\phi_\pi$ 0.1 $\phi_Y$ 0.0</td>
<td>- $\phi_L$ 0.0</td>
</tr>
<tr>
<td></td>
<td>Normal</td>
<td>3.0 $\phi_\pi$ 0.1 $\phi_Y$ 0.0</td>
<td>- $\phi_L$ 0.15</td>
</tr>
<tr>
<td></td>
<td>Volatile</td>
<td>3.0 $\phi_\pi$ 0.1 $\phi_Y$ 0.0</td>
<td>- $\phi_L$ 0.15</td>
</tr>
<tr>
<td>Monetary Policy + DLTV</td>
<td>Stable</td>
<td>3.0 $\phi_\pi$ 0.1 $\phi_Y$ 0.0</td>
<td>$\phi_{ltv}^{PH}$ 0.1 $\phi_{pt}^{PH}$ 0.0</td>
</tr>
<tr>
<td></td>
<td>Normal</td>
<td>3.0 $\phi_\pi$ 0.1 $\phi_Y$ 0.0</td>
<td>$\phi_{ltv}^{PH}$ 0.15</td>
</tr>
<tr>
<td></td>
<td>Volatile</td>
<td>3.0 $\phi_\pi$ 0.1 $\phi_Y$ 0.0</td>
<td>$\phi_{ltv}^{PH}$ 0.2</td>
</tr>
<tr>
<td>Monetary Policy + DCRR</td>
<td>Stable</td>
<td>3.0 $\phi_\pi$ 0.1 $\phi_Y$ 0.0</td>
<td>- $\phi_L$ 0.0 $\phi_{DCRR}$ 0.0</td>
</tr>
<tr>
<td></td>
<td>Normal</td>
<td>3.0 $\phi_\pi$ 0.1 $\phi_Y$ 0.0</td>
<td>- $\phi_L$ 0.0 $\phi_{DCRR}$ 2.0</td>
</tr>
<tr>
<td></td>
<td>Volatile</td>
<td>3.0 $\phi_\pi$ 0.1 $\phi_Y$ 0.0</td>
<td>- $\phi_L$ 0.0 $\phi_{DCRR}$ 2.0</td>
</tr>
<tr>
<td>Monetary Policy + DLTV, DCRR</td>
<td>Stable</td>
<td>3.0 $\phi_\pi$ 0.1 $\phi_Y$ 0.0</td>
<td>$\phi_{ltv}^{PH}$ 0.95 $\phi_{pt}^{PH}$ 0.0</td>
</tr>
</tbody>
</table>

* Welfare gains compared with the baseline policy regime

In the ‘Monetary policy only’ panel when macroprudential policy is nonexistent, the optimal monetary policy has an aggressive reaction of monetary policy to inflation ($\phi_\pi = 3$) and a positive reaction to output ($\phi_Y = 0.1 \sim 0.15$). Note the optimal response of monetary policy to output is nonzero but the reaction to gross credit ($\phi_L$) is zero in the optimal rule. The welfare gain from optimal monetary policy in terms of $\lambda$, compared to the ‘baseline’ policy specification is around 0.1% regardless of the volatility in housing and financial market. In the ‘Monetary policy+DLTV’ panel, DLTV is the only macroprudential instrument. Optimal monetary policy remains similarly and the optimal DLTV policy response to housing price ($\phi_{ltv}^{PH}$) is quite weak, around 0.1 $\sim$ 0.2. Additional welfare gain from DLTV over optimal monetary policy is marginal, at most 0.01%. In the ‘Monetary policy+DCRR’ panel, DCRR is the only macroprudential instrument. In ‘normal’ and ‘volatile’ scenarios, optimal monetary policy does not react to output ($\phi_Y = 0.0$) and capital requirement ratio to credit
reaches its ceiling ($\phi^c_s = 2.0$). Additional welfare gain DCRR brings over optimal monetary policy increases as the volatility in the financial market and housing price increases, and is quite significant (0.33%p) in the ‘volatile’ scenario. This optimal combination of monetary and DCRR policy remains the same when both DCRR and DLTV are allowed to be in action (‘Monetary policy+DLTV+DCRR’). In that case, the optimal DLTV coefficient varies and positive. It is observed that the welfare gains in the ‘Monetary policy+DLTV+DCRR’ do not substantially larger than those in ‘Monetary policy+DCRR’, indicating most gains are from DCRR policy. In section 6.2, I explain how credit-stabilizing DCRR policy brings welfare gain in this model. After all, the result in table 3 can support the argument for using macroprudential policy against asset price and financial market disturbances, by showing that welfare gains from macroprudential policy become greater when housing price and bank capital are highly volatile.

6.2. INTERPRETING THE GAINS FROM DCRR POLICY

The optimal policy result shows that when there are disturbances in housing price and bank capital, monetary policy focusing on inflation stabilization and DCRR policy focusing on credit stabilization can improve welfare. Inflation stabilization reduces welfare loss from relative price dispersion when price rigidity is present. But how does credit stabilization using DCRR policy improve welfare? This question is especially intriguing because DCRR policy to stabilize credit may increase inflation volatility, a tradeoff shown in the previous section. To further analyze this question I examine how expected values and standard deviations of major macro variables change with the aggressiveness of DCRR policy against credit. Those moments are calculated assuming that the volatility of housing and financial market is in ‘normal’ times.

In figure 6, I display the mean and standard deviation of key variables, varying the response of capital requirement ratio to credit ($\phi^c_s$) from 0 to 50. As DCRR response increases, welfare measure, and the expected value of consumption, capital, credit, and default threshold monotonically increase and credit volatility monotonically decreases. One possible interpretation of this result is that the credit stability due to the DCRR policy yields less conservative financial contract, represented by higher expected value of default threshold and credit. In other words, there is a compensation mechanism that policy-induced risk reduction leads to a larger expected credit. Larger credit in the economy results in welfare gain through high investment, capital and consumption. In fact, this linkage between credit volatility and the size of credit stems from BGG financial contract in the model, as is shown in the partial equilibrium analysis in the next paragraph. Meanwhile, inflation volatility shows monotonic
increase as \( \phi^c_L \) increases, implying that this is a policy region where welfare gain from credit stability outweighs the loss from larger inflation volatility. More analysis about this tradeoff follows in Section 7 where the interaction between monetary and macroprudential policy is discussed. Finally, since the expected values of entrepreneur’s net worth or bank credit is not monotonic, it is not clear whether and how they affect the welfare. Note the welfare gain comes from capital accumulation, therefore saving households are borrowing households are both better off with macroprudential policy.

Figure 6: Reaction of capital requirement ratio to credit \( (\phi^c_L) \), economy and welfare

How can policy-induced volatility reduction result in larger credit in BGG financial contract? For better interpretation, I slightly modify the setup so that a composite random variable \( \omega K \) instead of \( \omega \) determines the default. That is, the default occurs if \( \omega K < \bar{\omega} K \) and does not occur otherwise. Although the distribution of idiosyncratic shock \( \omega \) is independent from government policy, the macroprudential policy can still reduce the volatility of \( \omega K \)
through $K$. Then the bank zero-profit condition (12) can be rewritten as

$$(1 - F(\omega^t_{i,a} K^t_i)) E_t R^K_{t+1} q_t \omega^t_{i,a} K^t_i + (1 - \mu) E_t R^K_{t+1} q_t \int_0^{\omega^t_{i,a} K^t_i} \omega K f(\omega K) d\omega K = \frac{R^f_t}{R^L_t} (E_t R^K_{t+1} q_t \omega^t_{i,a} K^t_i).$$

Suppose $E_t R^K_{t+1}, q_t, R^f_t$ and $R^L_t$ are given. An increase in default threshold $\omega^t_{i,a} K^t_i$ changes the value of the left hand side of (32) with two opposite directions. While it positively contributes by increasing non-default payoff for the bank $(E_t R^K_{t+1} q_t \omega^t_{i,a} K^t_i)$, it also negatively contributes by increasing the probability of default $F(\omega^t_{i,a} K^t_i)$. Following the lines of Bernanke, Gertler, and Gilchrist (1999), given parameter values and with a general regularity condition about the distribution of $\omega K$, it is possible to show that the value of the left hand side is increasing in $\omega^t_{i,a} K^t_i$ and concave.

![Diagram](image)

(a) Bank zero profit condition  
(b) Default threshold

Figure 7: The effect of policy-induced volatility reduction. Panel (a) shows the relationship in equation (32). Panel (B) shows the default threshold when there is mean-preserving volatility reduction in composite random variable $\omega K$.

The panel (a) in Figure 7 illustrates the relationship in equation (32). The left hand side of (32) is increasing and concave in $\omega K$, and the right hand side is linear in $\omega K$. The initial equilibrium level of default threshold is given at $\omega K^*$. Suppose there is a macroprudential policy-induced, mean-preserving volatility reduction in $\omega K$. The distribution of $\omega K$ now changes to a one with thinner left tale, as in the panel (b). Since the probability of non-default $(1 - F(\omega K))$ is higher given $\omega K$, it pushes LHS upward to LHS', resulting in higher level of default threshold $\omega K^* \rightarrow \omega K'$. From the relationship $R^L L = \omega K \cdot ER^K q_t$, it follows
that the equilibrium level of lending is higher, and assuming entrepreneurs’ net worth is constant, the equilibrium capital level is also higher. To summarize, policy-induced credit stabilization can be welfare improving to the economy, because volatility reduction can be rewarded with less conservative financial intermediation.

7. How Should Monetary and Macroprudential Policy Be Implemented Together?

This section examines the interaction between monetary and macroprudential policy. To make the analysis more tractable, I rule out DLTV policy which has little welfare implication and assume DCRR is the only macroprudential policy tool. The optimal policy analysis in the previous section shows that it is optimal to assign monetary policy to exclusively target inflation stabilization and macroprudential policy to exclusively target credit stabilization. Based on this separation optimality result, I discuss how policies should be operated together, and the consequences from the monetary policy reacting to credit. Results show that the strategy to assign each policy a single objective seems to be successful: Given that each policy is aggressive enough to its assigned target variable, it can achieve the stabilization of the target variable with only a limited effect on the stability of the other variable. Monetary policy rule reacting to credit is not only non-optimal, but also ineffective as a credit stabilization instrument.

In order to see how DCRR policy affects inflation stability, I calculate unconditional standard deviation of inflation \( \sigma_\pi \) while varying the response of capital requirement ratio to gross credit \( \phi_\kappa L \). Panel (a) in figure 8 shows the relationship for four different monetary policy specifications. Suggested monetary policy specifications are strict and aggressive inflation targeting \( (\phi_\pi = 3, \phi_Y = 0) \), weaker response to inflation \( (\phi_\pi = 2, \phi_Y = 0) \), \( (\phi_\pi = 1.5, \phi_Y = 0) \), weaker response to inflation and positive response to output \( (\phi_\pi = 1.5, \phi_Y = 0.1) \). As expected, it is observed that the inflation volatility is lower when monetary policy response to inflation is stronger, and response to output is weaker. Stronger DCRR comes with higher inflation volatility, as the graph shows that the inflation volatility is monotonically increasing in \( \phi_\kappa L \). However, it also shows that when monetary policy has large enough reaction to inflation, it can still effectively stabilize inflation even with DCRR policy. For example, inflation volatility for \( (\phi_\pi = 3, \phi_Y = 0, \phi_\kappa L = 2) \) is still smaller than \( (\phi_\pi = 2, \phi_Y = 0, \phi_\kappa L = 0) \). This result indicates that macroprudential policy does not necessarily impede inflation stability as long as monetary policy has strong enough commitment to inflation stability.

In addition, I examine how different monetary policy affects the credit stabilization effect of macroprudential policy. Panel (b) in figure 8 shows the unconditional standard deviation
of credit ($\sigma_L$) while varying the response of capital requirement ratio to gross credit ($\phi^c_L$). Credit volatility monotonically decreases as $\phi^c_L$ increases. Interestingly, credit volatility is lower as monetary policy is more aggressive against inflation.

Figure 8: Welfare, inflation volatility, credit volatility, given different monetary policy parameters ($\phi_\pi, \phi_Y$)

Finally, I analyze the performance of monetary policy rule reacting to credit, to see whether it can better function in terms of inflation and credit stabilization. Figure 9 and 10 suggest how this type of monetary policy affects welfare, inflation and credit volatility. In figure 9, monetary policy reaction to inflation and output are set at $\phi_\pi = 3$, $\phi_Y = 0$ and the reaction to credit $\phi_L$ is allowed to vary between 0 and 0.5 (on the horizontal axis). In panel (a) macroprudential policy exists and in panel (b) it does not. As $\phi_L$ increases, credit volatility can either increase or decrease depending on the presence of macroprudential policy. However, welfare measure decreases, inflation volatility increases in both panels. Although monetary policy can decrease credit volatility without macroprudential policy, the effect is smaller than what macroprudential policy can bring (credit volatility in panel (a) with macroprudential policy is overall about 0.5 %p lower than in panel (b) without it). To summarize, monetary policy reacting to credit is less efficient tool in credit stabilization than macroprudential policy, and it comes with the cost of greater inflation instability and welfare loss.

Figure 10 shows the same relationship with different monetary policy reaction to inflation and output ($\phi_\pi = 1.5$, $\phi_Y = 0.1$). In both panel (a) (with macroprudential policy) and (b) (without macroprudential policy), welfare measure decreases, inflation and credit volatility increases as $\phi_L$ increases. A panel-by-panel comparison between figure 9 and 10
shows that welfare measure is lower, inflation and credit volatility is higher in figure 10.

![Figure 9](image1)

(a) MOP ($\phi_\pi = 3, \phi_Y = 0.2$), MPP ($\phi^*_L = 2$)

(b) MOP ($\phi_\pi = 3, \phi_Y = 0.2$), MPP ($\phi^*_L = 0$)

Figure 9: Reaction of monetary policy to credit ($\phi_L$), inflation volatility, credit volatility and welfare

![Figure 10](image2)

(a) MOP ($\phi_\pi = 1.5, \phi_Y = 0.2$), MPP ($\phi^*_L = 2$)

(b) MOP ($\phi_\pi = 1.5, \phi_Y = 0.2$), MPP ($\phi^*_L = 0$)

Figure 10: Reaction of monetary policy to credit ($\phi_L$), inflation volatility, credit volatility and welfare

Results in figure 9 and 10 supports that macroprudential policy is more efficient tool to control credit. To explain this more intuitively, I present the role of each policy instrument in a simple supply-demand model for private credit market. Figure 11 compares the credit
stabilization using monetary and macroprudential policy. The main difference between two is that while the former affects both supply and demand of the credit, the latter only influences the demand. In panel (a), the credit stabilization process using monetary policy is illustrated. Private credit supply ($S$) is savers’ willingness to deposit that is increasing in interest rate ($R$). Private credit demand ($D$) is borrowers’ willingness to draw credit that is decreasing in interest rate. Initial equilibrium is given by ($L^*, R^*$). Suppose for some reason the policymaker decides to reduce private credit in the economy. Suppose she chooses monetary policy as the policy instrument, and raises target interest rate from $R^* \rightarrow R'$. It causes excessive supply of private credit amount to $L^b - L^a$, and the central bank action should be to reduce real money supply to absorb this excessive supply. However, this monetary contraction affects both supply and demand for private credit; Savers, with reduced money available, will save less given interest rate, shifting the credit supply curve to the left ($S \rightarrow S'$). Borrowers, also with less money available, will need more borrowing given interest rate, shifting the credit demand curve to the right ($D \rightarrow D'$). Consequently, the equilibrium private credit ($L'$) can be anywhere between $L^a$ and $L^b$, depending on the extent of the shift of supply and (or) demand curve upon monetary contraction. In other words, monetary contraction can either increase or decrease equilibrium private credit. On the other hand, panel (b) illustrates the credit control using macroprudential policy maintaining the target interest rate at $R^*$. Since macroprudential policy works exclusively on credit demand (borrowers), it moves credit demand inward ($D \rightarrow D'$). There should be a monetary contraction to pin down the interest rate at $R^*$, but the equilibrium credit is still somewhere between $L^c$ and $L^*$. Macroprudential policy results in the reduction of the equilibrium credit for sure.

(a) Credit control using monetary policy  
(b) Credit control using macroprudential policy

Figure 11: Equilibrium for private credit
8. CONCLUSION

Results in this paper show that countercyclical macroprudential policy can function as an automatic stabilizer in overall. However, the result that the LTV ratio reacting to housing price, which is a market-specific instrument targeting market-specific asset price, can create regulatory arbitrages causes a challenge for regulators. In fact, besides the dual credit market case presented in this paper, regulatory arbitrage can be ubiquitous. For example, it can be found in an economy with non-negligible secondary credit markets that are, in general, subject to weaker regulation. Likewise, it can be found in international financial market where countries adopt different regulatory standards. Thus the cooperation between regulatory authorities, including that of international dimension, becomes crucial for the macroprudential policy to effectively deal with this regulatory arbitrage.

Welfare analysis shows that credit stabilization using macroprudential policy is welfare improving. It should be noted that this welfare gain is obtained from BGG model where agents internalize default risk and there is no externality in credit boom-bust cycle. While many researches in the literature find the need for macroprudential policy from that externality, this paper shows that macroprudential policy can be beneficial even without such externality. Macroprudential policy helps capital accumulation by providing expectation that future financial crisis will be less likely to occur.

Another important issue highlighted here is the interaction between monetary and macroprudential policy. It is important not only for its effect on the economy, but also for its implication on the institutional structure of macroprudential regulation. For example, it can reestablish the relationship between the central bank and the regulatory authority, as well as the delegation of the regulatory duties between them. In the model framework laid in this paper, the policy recommendation is to assign monetary policy solely to inflation stabilization and macroprudential policy solely to credit stabilization. This policy specification is welfare improving mainly because each policy instrument is assigned to the target which it is the most effective.

One question not discussed in this paper but worth a further investigation is the possible conflict between macroprudential objectives and microprudential objectives. The nature of this conflict originates from the additional constraint that macroprudential policy imposes on each individual financial intermediary’s optimization behavior. Although it automatically keeps financial institutions from becoming too leveraged and increases protection against systemic risk, it may also worsen the profitability of each individual bank because of that additional constraint. This might pose a dilemma to the regulator, calling for the need to evaluate how both dimensions of regulatory policies interact.
REFERENCEs


CURLIDA, V., AND M. WOODFORD (2010): “Credit Spreads and Monetary Policy,” Journal of Money, Credit and Banking, 42(s1), 3–35.


### Table 4: Notations

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Variable</th>
<th>Description</th>
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<td>$Y$</td>
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<td>Marginal return of capital</td>
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<tr>
<td>$I$</td>
<td>Investment</td>
<td>$q$</td>
<td>Capital price</td>
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<td>$K$</td>
<td>Capital</td>
<td>$V$</td>
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<td>Aggregate Labor</td>
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<td>Bank deposit</td>
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<td>Labor, entrepreneurs</td>
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<td>Household lending</td>
</tr>
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<td>$N_f$</td>
<td>Labor, bankers</td>
<td>$L^B$</td>
<td>Business lending</td>
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<td>Consumption, savers</td>
<td>$L$</td>
<td>Aggregate lending (credit)</td>
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<td>Bank capital</td>
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<td>Housing price</td>
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<td>$ω^{H,a}$</td>
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<td>$ω^{H,b}$</td>
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<td>– household</td>
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<td>$I_b^H$</td>
<td>Housing investment, borrowers</td>
<td>$ω^a$</td>
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<td>Aggregate housing investment</td>
<td>– business</td>
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<td>$T_s$</td>
<td>Tax, savers</td>
<td>$ω^b$</td>
<td>Ex-post default threshold</td>
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<td>$T_b$</td>
<td>Tax, borrowers</td>
<td>– business</td>
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<td>Wage, households</td>
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<td>Productivity shock</td>
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<td>Wage, entrepreneurs</td>
<td>$G$</td>
<td>Government spending shock</td>
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<td>$e^γ$</td>
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<td>$R^c$</td>
<td>Return on bank capital</td>
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<td></td>
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<td>Household lending rate</td>
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<td></td>
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<tr>
<td>$R^{LB}$</td>
<td>Business lending rate</td>
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<td></td>
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<tr>
<td>$R^f$</td>
<td>Interbank rate</td>
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<tr>
<td>$R^K$</td>
<td>Return on capital</td>
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Table 5: Parameter calibrations in the baseline model: static CRR, no LTV

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<th>Description</th>
<th>Value</th>
<th>Description</th>
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<td>$\beta$</td>
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<td>Discount factor, savers</td>
<td>$\rho_A$</td>
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<tr>
<td>$\beta_b$</td>
<td>0.9885</td>
<td>Discount factor, borrowers</td>
<td>$\rho_A$</td>
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<td>$\gamma$</td>
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<td>Weight of housing in the utility</td>
<td>$\rho_G$</td>
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<td>$\varphi$</td>
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<td>Weight of labor in the utility</td>
<td>$\rho_r$</td>
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<td>$\alpha_k$</td>
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<tr>
<td>$\alpha_n$</td>
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<td>Weight, households’ labor in the production</td>
<td>$\rho_{ltv}$</td>
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<td>$\alpha_e$</td>
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<td>Weight, entrepreneurs’ labor in the production</td>
<td>$\phi_\pi$</td>
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<td>Weight, bankers’ labor in the production</td>
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<td>$\delta$</td>
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<td>$\phi_L$</td>
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<td>$\nu$</td>
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<td>$\sigma$</td>
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<td>SD, idiosyncratic shock, business</td>
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<td>$\lambda_H$</td>
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<td>$\nu_c$</td>
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<td>Markup in household lending</td>
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Table 6: Steady state interest rates and variable ratios

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<td>$R$</td>
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<td>Bank funding rate</td>
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<td>$R^{LH}$</td>
<td>Household borrowing rate</td>
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<td>Entrepreneur borrowing rate</td>
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<td>$C/Y$</td>
<td>Consumption-output ratio</td>
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<tr>
<td>$I/Y$</td>
<td>Investment-output ratio</td>
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<td>$I^H/Y$</td>
<td>Housing investment-output ratio</td>
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<td>$G/Y$</td>
<td>Investment-output ratio</td>
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<td>$K/W$</td>
<td>Entrepreneur’s capital-net worth ratio</td>
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<td>$L^B/L^H$</td>
<td>Business lending-household lending ratio</td>
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<td>$L^H/Y$</td>
<td>Household debt-output ratio</td>
<td>0.19</td>
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Note: all values are in real, annualized terms
A.2. HOUSEHOLDS’ OPTIMIZATION PROBLEM

Saving households (denoted by \(s\)) with future discount rate \(\beta\) maximize

\[
\max_{C_t, H_t, N_t, I_{t,s}^H, B_t, D_t, e_t} E_o \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \gamma \log C_{t,s} + (1 - \gamma e_t) \log H_{t,s} + \varphi \log(1 - N_{t,s}) \right] \right\}
\]  \hspace{1cm} (33)

subject to

\[
C_{t,s} + \frac{I_{t,s}^H}{P_t} + \frac{B_t}{P_t} + D_t + \epsilon_t + T_{t,s} \leq R_t^N \frac{B_t}{P_t} + R_t^D D_{t-1} + R_{t-1}^e \epsilon_{t-1} + w_t N_{t,s} + Div_t
\]  \hspace{1cm} (34)

We can setup the Lagrangian as

\[
\mathcal{L} = E_o \sum_{t=0}^{\infty} \beta^t \left[ \gamma \log C_{t,s} + (1 - \gamma e_t) \log H_{t,s} + \varphi \log(1 - N_{t,s}) \right]
\]

\[
+ \lambda_t (R_t^N \frac{B_t}{P_t} + R_t^D D_{t-1} + R_{t-1}^e \epsilon_{t-1} + w_t N_{t,s} + Div_t)
\]

\[
-C_{t,s} - \frac{I_{t,s}^H}{P_t} - \frac{B_t}{P_t} - D_t - \epsilon_t - T_{t,s} + \mu_t ((1 - \delta H) H_{t-1,s} + I_{t,s}^H - H_{t,s})
\]  \hspace{1cm} (35)

where the Lagrange multiplier \(\lambda\) is on the household budget constraint and \(\mu\) is on the law of motion for housing stock.

The FOCs are shown below:

\[
C_{t,s}: \quad \frac{\gamma}{C_{t,s}} = \lambda_t
\]  \hspace{1cm} (36)

\[
N_{t,s}: \quad \frac{\varphi}{1 - N_{t,s}} = \lambda_t w_t \Rightarrow \frac{\varphi}{1 - N_{t,s}} = \frac{\gamma}{C_{t,s}} w_t
\]  \hspace{1cm} (37)

\[
D_t: \quad \lambda_t = \beta E_t \lambda_{t+1} R_t^D \Rightarrow \frac{1}{C_{t,s}} = \beta E_t \frac{1}{C_{t+1,s}} R_t^D
\]  \hspace{1cm} (38)

\[
B_t: \quad \lambda_t \frac{P_t}{P_t} = \beta E_t \lambda_{t+1} \frac{R_t^N}{P_{t+1}} \Rightarrow \frac{1}{C_{t,s}} = \beta E_t \frac{1}{C_{t+1,s}} \frac{R_t^N}{P_{t+1}}
\]  \hspace{1cm} (39)

\[
\epsilon_t: \quad \lambda_t = \beta E_t \lambda_{t+1} R_t^e \Rightarrow \frac{1}{C_{t,s}} = \beta E_t \frac{1}{C_{t+1,s}} R_t^e
\]  \hspace{1cm} (40)

\[
I_{t,s}^H: \quad \lambda_t P_{t}^H = \mu_t
\]  \hspace{1cm} (41)

\[
H_{t,s}: \quad \mu_t = \frac{1 - \gamma e_t}{H_{t,s}} + \beta E_t \mu_{t+1} (1 - \delta H)
\]  \hspace{1cm} (42)

Combining (36) and (37) gives us the labor supply decision. (38) and (39) are Euler equations for real deposits and nominal assets. Combining them gives us the Fisher equation. (40) is the Euler equation with respect to bank capital, which implies that the equilibrium return from bank capital should be the same.
as deposit rate. (41) and (42) show that the shadow price of housing goods in the current period is given by the sum of the current period’s marginal utility from housing goods and the discounted value of the next period’s expected shadow price.

Borrowing households’ Lagrangian is given as below.

\[ L = E_t \sum_{i=0}^{\infty} \beta^i [\gamma \log C_{t,b} + (1 - \gamma \epsilon_t^i) \log H_{t,b} + \varphi \log (1 - N_{t,b}) + \lambda b_t \left( w_t N_{t,b} + \int_0^{\tilde{\omega}_{t,b}} \omega H_{t,b} f(\omega) d\omega \right) - \left[ 1 - F(\tilde{\omega}_{t,b}) \right] R_{t+1} \left( L_{t+1} - T_{t,b} \right)] \]

(43)

The first order condition for housing goods also becomes different from that of the saving household due to the possibility of default.

\[ \frac{1}{C_{t,b}} = \beta_t E_t \left[ \frac{1}{C_{t+1,b}} (1 - F(\tilde{\omega}_{t+1})) R_{t+1}^{b} \right]. \]

(44)

The optimal intertemporal borrowing decision of the borrowing household is derived considering the possibility of default.

\[ \frac{\gamma}{C_{t,b}} P_t^b (1 + \int_0^{\tilde{\omega}_{t,b}} \omega f(\omega) d\omega) = \frac{1 - \gamma \epsilon_t}{H_{t,b}} + \beta E_t \frac{\gamma}{C_{t+1,b}} P_{t+1}^b (1 - \delta_H). \]

(45)

The optimal labor decision of borrowing households is similar to that of saving households.

A.3. Financial Accelerator Mechanism in The Business Sector

For simplification, define the bank’s claim when non-default \((ND^b)\) and default \((D^b)\) as \(ND^b \equiv (1 - F(\tilde{\omega}))\), \(D^b \equiv \int_0^{\tilde{\omega}} \omega f(\omega) d\omega\), and the entrepreneur’s claim when non-default \((ND^e)\) as \(ND^e \equiv \int_\omega^{\infty} \omega f(\omega) d\omega - ND^b(\tilde{\omega})\). If \(\omega\) is assumed to follow lognormal distribution \((\ln(\omega) \sim N(\sigma)\), \(E(\omega) = 1\)), and define an auxiliary variable \(x = (\ln(\tilde{\omega}) + 0.5 \sigma^2) / \sigma\), then it can be shown that

\[ ND^b(\tilde{\omega}) = 1 - \Phi(x), \quad D^b(\tilde{\omega}) = \Phi(x - \sigma), \quad ND^e(\tilde{\omega}) = 1 - D^b(\tilde{\omega}) - ND^b(\tilde{\omega}). \]

(46)

Entrepreneur’s optimization problem is

\[ \max ND^e_t(\tilde{\omega}_{t}^e) E_t \left[ P_{t+1}^K K_t \right] \]

(47)
subject to

\[ R_t^f(q_tK_t - W_t) = \{ND_t^b(\bar{\omega}_t^a) \cdot \bar{\omega}_t^a + (1 - \mu)D_t^b(\bar{\omega}_t^a)\} E_tR_{t+1}^K_q, \]  

(48)

The first order conditions are given by

\[ \partial \bar{\omega}_t^a : \quad ND_t^e(\bar{\omega}_t^a) = -\lambda [ND_t^b(\bar{\omega}_t^a) + ND_t^b(\bar{\omega}_t^a)\bar{\omega}_t^a + (1 - \mu)D_t^b(\bar{\omega}_t^a)], \]  

(49)

\[ \partial K_t : \quad ND_t^e(\bar{\omega}_t^a) \frac{E_tR_{t+1}^K}{R_t^f} = \lambda [1 - \{ND_t^b(\bar{\omega}_t^a) \cdot \bar{\omega}_t^a + (1 - \mu)D_t^b(\bar{\omega}_t^a)\} \frac{E_tR_{t+1}^K}{R_t^f}], \]  

(50)

\[ \partial \lambda : \quad ND_t^b(\bar{\omega}_t^a) \cdot \bar{\omega}_t^a + (1 - \mu)D_t^b(\bar{\omega}_t^a)] \frac{E_tR_{t+1}^K}{R_t^f} q_tK_t = q_tK_t \frac{W_t}{W_t} - 1, \]  

(51)

where

\[ ND_t^b(\bar{\omega}_t^a) = -D^b(\bar{\omega}_t^a) - ND_t^b(\bar{\omega}_t^a) - ND_t^b(\bar{\omega}_t^a)\bar{\omega}_t^a, \]  

(52)

\[ D^b(\bar{\omega}_t^a) = (\bar{\omega}_t^a)f((\bar{\omega}_t^a)) = \frac{f(x_t - \sigma)}{\sigma \omega} = \frac{f(x_t)}{\sigma}, \]  

(53)

\[ ND_t^b(\bar{\omega}_t^a) = -\frac{f(x_t)}{\omega_t^a \sigma}. \]  

(54)

Substituting \( \lambda \) out, we have two equilibrium conditions

\[ \frac{ND_t^b}{ND_t^e - \mu f(x_t)/\sigma} = \frac{ND_t^e}{1 - \{ND_t^b(\bar{\omega}_t^a) \cdot \bar{\omega}_t^a + (1 - \mu)D_t^b(\bar{\omega}_t^a)\} \frac{E_tR_{t+1}^K}{R_t^f}} \]  

(55)

and

\[ [ND_t^b(\bar{\omega}_t^a) + (1 - \mu)D_t^b(\bar{\omega}_t^a)] \frac{E_tR_{t+1}^K}{R_t^f} q_tK_t = q_tK_t \frac{W_t}{W_t} - 1. \]  

(56)
From (55) and (56) it is possible to derive BGG financial accelerator equation.

\[ E_t R_{t+1}^K = S\left(\frac{q_t K_t}{W_t}\right) R_f^t. \]  

(57)

\( S \) is an increasing function in \((q_t K_t/W_t)\), implying that the external finance premium \((E_t R_{t+1}^K/R_f^t)\) is increasing in the asset-net worth (leverage) ratio. This is the reason why the mechanism is called a financial accelerator. If a positive shock which improves the net worth of the entrepreneur is realized, with better balance sheet condition she can further increase investment with lower external finance premium.

**A.4. Capital Goods Producer’s Optimization Problem**

Capital good producer’s optimization problem is given by

\[ \max_I (q_t - 1) I_t - f\left(\frac{I_t}{K_{t-1}}\right) K_{t-1}. \]  

(58)

The first order condition is written as

\[ q_t = 1 + f'\left(\frac{I_t}{K_{t-1}}\right). \]  

(59)

The function \( f \) is assumed to have a simple quadratic form,

\[ f\left(\frac{I_t}{K_{t-1}}\right) = \frac{\chi k}{2} \left(\frac{I_t}{K_{t-1}} - \delta\right)^2 K_{t-1}. \]  

(60)

**A.5. Retailors’ Optimization Problem**

Each retail goods producer \((i)\) purchases intermediate goods and turns them into retail goods \((Y_t(i))\) in a monopolistically competitive market. Total final usable goods \(Y_t\) are the following composite of retail goods:

\[ Y_t = \left[ \int_0^1 Y_t(i) \frac{d\epsilon}{\epsilon} \right]^{\frac{1}{1-\epsilon}} \]  

(61)

\[ P_t = \left[ \int_0^1 P_t(i)^{1-\epsilon} \frac{d\epsilon}{\epsilon} \right]^{\frac{1}{1-\epsilon}} \]  

(62)

Only \((1-\theta)\) fraction of retail goods producers are allowed to change price, à la [Calvo (1983)](#).
Define a measure of the resource cost induced by price dispersion \((s_t)\) as

\[
s_t = \int_0^1 \left( \frac{P_{it}}{P_t} \right)^{-\epsilon} = (1 - \theta)(p^*_t)^{-\epsilon} s_{t-1}
\]

and introduce state variables \(x^1_t\) and \(x^2_t\) such that

\[
x^1_t = (p^*_t)^{-1-\epsilon} \frac{Y_t + mc_t}{s_t} + \theta \beta E_t \frac{\lambda_{t+1,b} Y_t}{\lambda_{t+1} p_t^*} \left( \frac{p_t^*}{p_{t+1}^*} \right)^{-1-\epsilon} x^1_{t+1}
\]

\[
x^2_t = (p^*_t)^{-\epsilon} \frac{Y_t}{s_t} + \theta \beta E_t \frac{\lambda_{t+1,b} Y_t}{\lambda_{t+1} p_t^*} \left( \frac{p_t^*}{p_{t+1}^*} \right)^{-\epsilon} x^2_{t+1}
\]

and

\[
\frac{\epsilon}{\epsilon-1} x^1_t = x^2_t.
\]

The aggregate price will be determined as

\[
1 = \theta \pi_t^{-1+\epsilon} + (1 - \theta)(p^*_t)^{1-\epsilon}.
\]

A.6. Steady State Conditions

\[
\begin{align*}
A &= \pi = p^* = s = q = N_e = N_f = 1, \quad mc = (\epsilon - 1)/\epsilon. \\
x^1 &= mc \cdot Y/(1 - \theta \beta), \quad x^2 = \epsilon/(\epsilon - 1)x^1.
\end{align*}
\]

\[
N = N_b + N_s, \quad C = C_b + C_s, \quad H = H_b + H_s.
\]
\[ C_b = \frac{\gamma w (1 - N_b)}{\varphi}, \quad C_s = \frac{\gamma w (1 - N_s)}{\varphi}. \quad (71) \]

\[ w = mc \cdot \alpha_n \frac{Y}{N}, \quad w_f = mc \cdot \alpha_{nf} \frac{Y}{N_f}, \quad w_e = mc \cdot \alpha_{ne} \frac{Y}{N_e}. \quad (72) \]

\[ R^D = \frac{1}{\beta}, \quad R^{L,H} = \frac{1}{\beta_b \cdot ND_{B,H}}. \quad (73) \]

Ex-ante and Ex-post default thresholds are the same in the steady state.

\[ \bar{\omega} \equiv \bar{\omega}^a = \bar{\omega}^b, \quad \bar{\omega}^H \equiv \bar{\omega}^{H,a} = \bar{\omega}^{H,b}. \quad (74) \]

\[ I_s^H = I^H / \left[ 1 + \frac{C_b}{C_s} \cdot \frac{1 - \beta (1 - \delta_H)}{1 + D_{B,H} - \beta_b (1 - \delta_H)} \right]. \quad (75) \]

\[ I^H = \frac{I^H}{Y}, \quad I^H = I_s^H + I_b^H, \quad H_s = I_s^H / \delta_H, \quad H_b = I_b^H / \delta_H. \quad (76) \]

\[ P^H = \frac{1 - \gamma}{\gamma} \frac{C_s}{H_s} (1 - \beta (1 - \delta_H)). \quad (77) \]

\[ N_b = \left[ \frac{\gamma w}{\varphi} + (R^{L,H} ND_{B,H} - 1)L^H + P^H (I_b^H + D_{B,H} H_b) + \frac{1}{2} G \right] / (w + \frac{\gamma w}{\varphi}). \quad (78) \]

\[ R^{L,H} L^H = \bar{\omega}^H P^H H_b. \quad (79) \]

\[ [R^f + \nu_e + \nu^H_a] L^H = ND_{B,H} \cdot R^{L,H} L^H + (1 - \mu)D_{B,H} \cdot P^H H_b. \quad (80) \]

\[ R_i^f = \kappa_i R_i^c + (1 - \kappa_i) R_i^D + s (\pi_t - \kappa_t). \quad (81) \]
\[ e = w_f/\phi, \quad L = L^B + L^H, \quad D + e = L, \quad \kappa = \frac{e}{L}. \] 

(82)

\[ z = mc \cdot \alpha \frac{Y}{K}, \quad R^K = z + (1 - \delta). \] 

(83)

\[ V = (1 - D^b)R^KK_{t-1} - (ND^b)R^L^BL^B, \quad W = \nu V + w_e, \quad L^B = K - W. \] 

(84)

\[ R^I L^B = (ND^b\bar{\omega} + (1 - \mu)D^b)R^K K. \] 

(85)

\[ \frac{ND^b}{ND^b - \mu f(x)/\sigma} = \frac{ND^b(R^K/R^I)}{1 - (\bar{\omega} \cdot ND^b + (1 - \mu)D^b)(R^K/R^I)}. \] 

(86)

Here \( f(x) \) is the pdf of the standard normal distribution. See the appendix of Bernanke, Gertler, and Gilchrist (1999).

\[ Y = K^{\alpha_k} N^{\alpha_n}. \] 

(87)

\[ K = I/\delta, \quad g = \frac{g}{Y} Y, \quad I^H = \frac{I^H}{Y} Y. \] 

(88)

\[ Y = C_s + C_b + I + G + \mu D^B R^K K + \mu^H D^{B,H} P^H H + I^H + \nu^I L + \nu^H L^H. \] 

(89)