An Information Theory of Worker Flows and Wage Dispersion

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Abstract

Type of employer transition impacts wages differently: direct employer-to-employer average an 8% wage gain and employer-to-unemployment-to-employer average a 19% persistent wage loss. To understand the joint process determining wages and job turnover, I provide an unified information theory of labor markets consistent with both types of transition. The two key ingredients are: (1) incumbent employers learn workers' abilities over a match and (2) this information diffuses slowly to outside potential employers. I formalize this theory in a dynamic general equilibrium model where behavior of workers and plants satisfies a Markov Perfect Equilibrium. I find skill accumulation to the model and identify parameters using worker flows, particularly multiple job losses of displaced workers. I find employer learning accounts for 28% of lifetime wage growth, most of which is accrued by switching employers. I estimate skill loss affects 22% of the unemployed. Skills are reacquired in less than two years and information accounts for long-term wage cuts following unemployment. I predict policy restricting discrimination by employment status, as in the American Jobs Act, increases output by 1% and lowers unemployment by 10%.

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1 Introduction

A job change permanently affects a worker’s future wages and employment. A quit directly to another employer is followed by job stability and higher wages. An unemployment spell, no matter how short, is followed by a wage cut of about 20%. Lower wages persist even 15 years later and are accompanied by repeat unemployment spells. In this paper, I provide a unified theory of quits and firings to better understand the role of employer learning about workers’ abilities in job turnover and wage dispersion.

To formalize this theory I add asymmetric employer learning to an environment similar to Postel-Vinay and Robin (2002). There are single plant-worker matches. Workers receive wage offers from their current employer and outside employers. All employers are identical, but workers differ in their ability. Low ability workers are less productive in any job.¹ My twist is two key assumptions about information employers have about workers’ abilities. First, an employer cannot observe a worker’s ability before she is hired and instead learns it slowly over the match. Second, outside potential employers learn a worker’s ability more slowly than their current employer and the current employer does not observe when outside employers learn.

This theory generates wage and employment patterns of unemployed workers by the logic of Farber and Gibbons (1994). If an employer hires a worker and discovers she is low ability, she will be fired. All workers may be displaced by a firm shut down, regardless of their ability. Potential employers cannot observe which unemployed workers have been fired and which have been displaced at random. Since the pool of unemployed contains workers of all abilities, firms continue to hire unemployed workers. Since the unemployed are of lower average ability than the population, they are hired at lower wages. Overtime new employers learn workers’ abilities and the low ability workers are fired again. This produces the cycle of wage losses and multiple displacements that is consistent with data.

Asymmetry in current and potential employers information gives wage gains of the job-to-job switchers. If an employer learns a worker is of high ability but outside employers do

¹This is different than learning about an employer specific factor as in pure matching models like Jovanovic (1979).
not receive this information, then they do not need to increase the worker’s wage to keep her. She cannot get a higher wage by quitting. The employer enjoys high profits from employing the worker at a low wage until outside employers learn that she is high ability. They then can offer her a higher wage and poach her. The result is similar to the data: permanent wage increases following voluntary quits and low likelihood of future unemployment.

The theoretical contribution is a unified general equilibrium model\textsuperscript{2} where behavior of plants and workers satisfy a Markov Perfect Equilibrium. Plants and workers behave strategically. They consider their information, the information of other players, and how their actions affect the beliefs of other players. The environment is carefully constructed such that an unique pure strategy equilibrium exists, the equilibrium is pooling over worker types and the evolution of beliefs is easily characterized. This is what allows me to put this game theoretic mechanism in a dynamic macro model.\textsuperscript{3}

The second contribution is quantitative. I provide identification to separately measure the effects of asymmetric employer learning and skill growth on wages and job turnover. I model skills following Ljungqvist and Sargent (1998). Employed workers become skilled with an exogenous arrival rate and lose skills with exogenous probability in unemployment. For measurement, I use panel data from the PSID which gives the reason for unemployment: fired or establishment closure. I follow an identification strategy of Katz and Gibbons, mapping workers displaced by establishment closure to workers displaced exogenously in the model. I use tenure and turnover to identify parameters. This avoids relying on wages which have high measurement error. I find that skill loss affects about 20% of the unemployed and skills are acquired in 1.5 years. This is much faster than the estimates in models without employer-learning of worker ability. They predict skill losses from unemployment take 12 years, on average, to recover. I find employer learning accounts for a little less than 1/3 of life-cycle wage growth, but accounts for almost all of the long term wage losses of the unemployed.

I use the calibrated model to assess a natural policy question for this framework: making it illegal to discriminate by employment status. This is part of the American Jobs Act:

\textsuperscript{2}There is an entry cost for job creation and an equilibrium is defined as requiring vacancies to satisfy zero profit condition

\textsuperscript{3}The reader can find a more thorough exposition in the literature review section.
designed to lower US unemployment. This policy is relevant in a standard search environment where search frictions cause equilibrium unemployment. The goal of this policy is to overcome the inefficiency caused by employers poaching already employed workers, at no increase in total surplus, while other workers remain idle in unemployment. This inefficiency arises because of lack of commitment and asymmetric information that induces a worker to quit. This inefficiency is amplified by matching frictions. In the model without the policy, markets are segmented into vacancies for employed and for unemployed. An unemployed worker produces lower expected profits, so market tightness is lower in the unemployed market. This is an equilibrium outcome. To satisfy zero profits across the two markets, lower expected profits from hiring an unemployed worker are offset by having to wait less time to fill a vacancy. Therefore, there are fewer vacancies for the unemployed than the employed. When I restrict vacancy posting to one market for both employed and unemployed, vacancies adjust for the expected profit from hiring an arbitrary worker. This profit is decreased compared to posting just towards the employed because there is a higher likelihood of a low ability worker. There is a second effect that a worker is less likely to be poached by an outside employer. This makes the profits from hiring a good unemployed worker even larger. The net is that the law increases employment by 3% and output by 2%.

Related Literature This work is related to two strands of literature: microeconomic models of employer learning and macroeconomic models of employment flows.4

Most econometric estimates employer learning use partial equilibrium wages to avoid difficulties of dynamic learning and strategic behavior. The advantage of these models is that they are able to include rich learning processes. The disadvantage is that measurement of learning parameters using wage data may not be well identified. Also, these models do not include general equilibrium effects of policy or parameter changes. Still, these models have

4There is a large body of microeconomic theory literature focused on contract design and job assignment with asymmetric employer learning (Waldman (1984), Harris and Holstrom (1982), and many others). The goal of this paper is quite separate from this literature. I focus on instances where employers have no control over the speed of learning or spread of information. Further the environment is constructed such that there is pooling in workers' strategies; contracts cannot separate the workers.
made much progress in testing hypotheses of employer learning. Altonji and Pierret (2001) find empirical support for employer learning by comparing aptitude test scores not observed by employers to wage patterns. This identification strategy has been expanded to measure the signaling value of education (Lange (2007)) and the asymmetry in employer learning (Kahn (2009)). In this paper I provide a new measurement of speed and asymmetry in employer learning. My approach is general equilibrium, but the particular learning process is less rich. I find the estimates of speed of learning are similar to this literature, but the effects on wages are very different.

A subset of the general equilibrium literature prohibits strategic behavior of the worker to avoid issues dynamic learning of employers from observing workers’ behaviors. These models provide a general equilibrium notion of wages, but at a cost of abstracting from the decisions a worker may make to maximize her future earnings. Examples include environments where the worker is auctioned to employers (Pinkston (2009)) and where the worker is myopic, always choosing to take the higher wage (Eeckhout (2004)). This paper, instead, models the worker as fully forward looking and active in choosing whether to switch employers. The worker in the model of this paper will sometimes reject a higher wage contract in favor of remaining with an incumbent at a lower wage contract without any other additional factors besides information. The key to tractability in my model is I chose a learning process where worker’s actions cannot speed up or slow down the rate of employer learning.

Another technique used by the theory literature to make dynamic learning tractable is a finite horizon; typically two to three periods. The advantage is that strategic behavior and dynamic learning is very tractable over one or two iterations. The disadvantage is that modeling learning in this way restricts the value of employer learning by placing a rigid assumption on the speed of employer learning. If an employer gains information rents from learning a worker’s type, then the length of time during which it will enjoy these rents determines their overall value. It also determines the extent of an employer’s monopsony power over these rents. If learning is very fast, the lost rents of switching to an uninformed employer are quickly recovered. I provide a many period model where the speed of employer learning is flexible. This allows the model to be used to measure the speed of employer learning and its quantitative importance on both the aggregate labor markets and individuals’ outcomes.
Asymmetric learning models, where outside employers and current employers have different information, face dynamic learning and potential for lemon markets failure. One way to avoid these problems is to model learning as fully private or fully public. However, this puts an implicit assumption on the size of information rents: when learning is fully public there are no information rents and when learning is fully private private there are full information rents. I choose something in between. I model asymmetry as a single parameter that determines the likelihood information is private or public each period. Varying this parameter determines a current employer's monopsony power over information rents and subsequently affects the share of these rents paid to the worker as wages. It is also key to generating non-trivial employer changes without an additional mechanism. In past literature, private learning has lead to a "lemons" problem of market failure. The incumbent employer has better information about that worker and sets wages accordingly, so any outside offer that is accepted has paid more than the worker is worth. On the other hand, when learning is completely public the worker is completely indifferent between employers and a continuum of equilibria may exist. Models of this type usually assume an additional non-pecuniary shock to study switches (such as in Acemoglu and Pischki (1996)). By modeling asymmetry in the way I do, switching is an equilibrium outcome and has meaningful implications for future switches.

In this paper, I handle the issues of dynamic learning, strategic behavior, and lemons with the following simplifications. Instead of gradual learning about a worker, full information arrives to incumbent employers and outside potential employers with different poisson arrival rates. The first implication of this assumption is that the worker's actions can do nothing to change the speed of learning. As a result, the unique equilibrium is a pooling equilibrium in workers' strategies which shuts down dynamic learning of the incumbent and strategic action of workers. The next assumption is that the incumbent plant cannot verify the wage offer of outside plants. If the incumbent chooses a wage to beat all outside offers, it will earn zero profits. Instead, the incumbent will choose a wage below the worker's value and earn a profit, even though it knows it will lose the worker eventually to a better outside offer.
This avoids lemon market shut down. The poachers continue to make offers because there is opportunity to get a worker at a profit.

This paper is a first step towards incorporating information frictions in a general macroeconomic model of employment flows. It is related to models of upwards mobility through on-the-job search such as Jovanovic (1979) and Burdett and Mortensen (1989). The distinction between this paper and Jovanovic (1979) is that employers in my model are learning about a worker's general productivity while in Jovanovic (1979) employers learned about specific productivity. This is important in comparing the worker's inside and outside option when determining wages. I consider take-it-or-leave it wage contracts are offered by incumbent plants and poaching plants each period similarly to Postel-Vinay and Robin (2002). This method of wage determination has been prescribed by critiques of standard search models such as Shin (2005) and Hall (2005). The value added of incorporating this type of information friction in the macroeconomic literature is that the theory can quantitatively match the large and persistent wage losses of displaced workers above those implied by skill or match quality loss. As such, it is important to understand the implications for the entire economy.

2 Evidence of Asymmetric Employer Learning

Information cannot be directly measured. The identification strategy of this paper follows Farber and Gibbons (1994). They offer a three period model in which a current employer privately learns a worker's productivity. Low productivity workers are fired endogenously and outside firms correctly infer that the pool of unemployed are of lower productivity. Workers displaced exogenously suffer wage losses from this signal, but new employers learn they are of high productivity and their wages recover. They document the wage patterns of individually fired workers are consistent with endogenous selection while workers unemployed by plan: shut down are constant with exogenous displacement.\(^5\) I additionally exploit the wage and

\(^5\) This empirical fact has subsequently been documented in an array of data sets spanning different countries and time periods.
tenure patterns of these groups of workers to identify the speed of employer learning and the composition of the unemployment pool. I build upon this work by generating non-trivial employer-to-employer changes by modeling employer learning to be only partially private. This allows me to incorporate additional information on direct employer-to-employer switchers to measure the speed of outside employer learning.

2.1 Facts on Wage Changes at Job Change

The standard regression for quantifying long-term wage changes associated with an employer change was developed by Jacobson et al. (1992). For individual $i$, the effect of employer change at time $t-n$ on current wages $w_t$ is estimated by the following linear regression:

$$ln(w_t) = \gamma_1 exp_{it} + \gamma_2 exp_{it}^2 + \sum_{n=-1}^{15} \beta_n D_{nit} + \delta_t + \alpha_i + \epsilon_{it}$$

The dependent variable is the natural logarithm of hourly earnings. The independent variables include a quadratic of experience, $D_{it}$ a dummy variable that captures employer change in year $t-n$, year dummy $\delta_t$ and individual fixed effects dummies $\alpha_i$. Note that the employer change variable $D_{it}$ includes dummies for one year prior to employer change, the year of the change, and each of the first through sixteenth years following the change (ie: $n \in \{-1, 0, 1, 2, ..., 15, 16\}$).

I follow sample selection in Farber and Gibbons (1994) to construct a panel from the PSID covering years 1976-1998. As they do, I distinguished workers by reason for job change (1) "shut down": and involuntary employment to unemployment transition accompanied by plant or firm closure and (2) "lay-off": an involuntary employment to unemployment transition not accompanied by firm closure. I add (3) voluntary quit without unemployment. The estimated coefficient on displacement dummies from these regressions are displayed in the graphs below. These results indicate that all involuntary displaced workers, regardless of reason for displacement, experience initial wages losses, but the depth and duration of

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See the appendix for summary statistics

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these losses vary across the two groups. Laid-off workers face short term losses of about 15%, while shut down workers face short term losses of about 11%. The wages of shut down workers recover such that the effects of displacement are no longer statistically significant at the 5% level in the 9th year following displacement. Wage losses for fired workers are persistent, lasting at least 15 years following displacement and even worsening over time.\(^7\)

\(^7\)Long-term wage losses following lay-off have been estimated using this regression in a variety of datasets. Studies using different samples from the PSID estimate persistent wage losses within the range of 7-13%. Estimates from the Displaced Workers Survey fall within the range of 8-15%. See Couch and Placzek (2010) for a review of these results.
Wage Dynamics of Workers Displaced by Shut Down
PSID 1976-1998 Estimates

It is important to interpret these regressions correctly to inform a theory. Since these regressions include individual fixed effects, the depth of the loss is in comparison to the population experience-wage profile adjust for the mean of the individual worker’s average wage. Therefore, long-term losses can arise only if a worker’s average wages are higher than post-displacement wages. This means these workers must have a combination of much higher wages prior to displacement and sufficiently long tenure at higher wages prior to displacement. I interpret this as suggesting that an employer does not learn immediately if a worker is of low productivity. Instead learning takes place over many years. Second, I follow Farber and Gibbons (1994) and consider the shut down workers to be randomly displaced and the fired workers to be negatively selected. As they do, I consider the initial losses of the shut-down workers to be evidence that employer learning is, to some extent, private. Outside employers cannot distinguish between a shut down and a fire, pay all unemployed an initially low wage, and raise the wages of shut down workers when they learn their true ability.
For comparability, I include similarly estimated wage changes of workers who make direct employer to employer changes without intermediate unemployment. The key stylized fact is wage changes at the time of switch are small, but long run wage growth is significant and positive.

2.2 Facts on Multiple Displacements

The second piece of evidence, used to discipline the first, is the employment patterns of displaced workers. The chart below shows the distribution of total observed lay-offs for ever laid-off workers compared to shut-down workers. It shows that over 90\% of laid-off workers are laid-off more than once, while the majority of shut-down worker are never laid off. My interpretation is that if all information about a worker's productivity were public, outside employers would hire employees under wages or contracts appropriate for the worker's type. This means that when a worker is hired after lay-off, the new employer has done so only if they believe they will profit from the match. Instead, it appears that employers continue to make "mistakes" in hiring these workers, discover they have hired a low productive worker, and then fire the worker. This happens repeatedly for laid-off workers with many future
Shut down workers, on the other hand, experience most of their wage growth within the first match they take from unemployment. I find only 21% switch employers after reemployment. This contradicts other theories of wage growth through mobility including match specific or firm specific wage effects.

3 Model with Asymmetric Employer Learning

The environment I consider is similar to Postel-Vinay and Robin (2002). Each period an incumbent and many poaching plants offer wage-employment contracts to workers. I add information frictions. An incumbent has private information about their worker’s ability and does not know whether poaching plants have been informed of this information. Unlike Postel-Vinay and Robin (2002), I assume incumbent plants cannot counter the offers of poaching plants. The worker chooses which offer to take and that plant becomes the incumbent. At the end of the period the new incumbent learns the worker’s ability perfectly.
with an exogenous probability and may choose to fire the worker.

These assumptions on wage determination are supported by observational evidence documented in Hall and Krueger (2010). They survey over 2,000 employed and unemployed workers. They find bargaining took place less than a third of the time and less than half of new employers knew the worker's last wage. I cite the second finding as evidence for both asymmetric information between employers and that current employers do not bargain over a worker with outside potential employers.\footnote{It should be noted that these findings differ across some demographic groups.}

3.1 Players, Actions and Payoffs

Time is infinite and discrete. Production takes place between single worker-plant matches.

**Players** The economy is populated by a unit measure of workers and a continuum of plants with identical technologies. Worker are characterized by time-invariant ability \( i \in \{l, h\} \) drawn independently across workers: \( i = h \) with probability \( \pi \) and \( i = l \) with probability \( 1 - \pi \). Ability determines the amount of the consumption good a worker can produce in a single period: \( y_h > 0 > y_l \). A measure \( 1 - d \) of plants die each period and are replaced by vacant plants. A measure \( 1 - \delta \) of workers die every period and are replaced by new entrants.

**Actions** Workers begin each period matched with an incumbent plant with a standing contract \( w \) and two poaching plants.\footnote{Denote poaching plants with a star.} The action choice of a plant is a downward rigid wage contract \( w' \geq w, w^* \geq w \) to offer the worker. It specifies a wage to be paid every period until either (1) the worker quits; (2) the plant lays off the worker; or (3) the plant raises the wage to a new contract. They may not reduce the wage. Incumbents additionally choose whether to fire \( F = 1 \) the worker. The action space of a worker is whether to accept the poacher \( Q = 1 \) or stay with the incumbent \( Q = 0 \). They may not quit to unemployment.
Payoffs If \( Q = 0 \) the incumbent plant remains incumbent: the worker receives payoff \( w' \) and the plant receives payoff \( y_i - w' \). If \( Q = 1 \) a poaching plant becomes incumbent: the worker receives payoff \( w^* \) and the plant receives payoff \( y_i - w^* \). If \( F = 0 \), the incumbent enters the next period as incumbent with appropriate standing contract. If the worker is fired \( F = 1 \) all plants collect zero and die; the worker collects zero and is assigned to a new plant that observes she is unemployed. If the incumbent dies, it receives a payoff of zero and the worker is matched with a new plant with standing contract \( w_i \). If the worker dies, the plant dies as well and both receive a payoff of zero. All plants that are not the new incumbent receive a payoff of zero and die. Workers have with time-separable linear utility of consumption. There is no storage technology. Workers consume current period wages. Workers and plants share common future discount factor \( \rho \geq \max\{1 - \delta, 1 - d\} \). Therefore, a worker’s objective is:

\[
E[\sum_{t=0}^{\infty} \rho^t w_{it}]
\]

The objective of a plant is to maximize discounted expected future profits defined as:

\[
E_i \sum_{t=0}^{\infty} \rho^t [y_i - w_{it}]
\]

3.2 Information

A worker’s ability is her private information. An incumbent newly matched with a worker from non-employment observes if the worker is a new entrant or if she has been displaced. Each period during a match, the incumbent learns the worker’s ability with probability \( \mu \). Poaching plants that meet the worker are informed of all information of the incumbent with probability \( \nu \) and with probability \( 1 - \nu \) they have no information on the worker or the action of the incumbent.\(^{10}\) Note that all poaching plants have the same information about a worker in a given period. The incumbent does not observe the action of the poaching plants or whether they are informed. The worker observes all actions, information, and states.

\(^{10}\)Since poachers only live a single period, there is no "memory" of poachers
3.3 Timing

I will explain the timing for a worker with incumbent prior \( p \) and contract \( w \).

1. Incumbent wage offers: Each incumbent chooses whether to raise the worker's wage \( w' > w \) or remain at contract \( w' = w \).

2. Poachers' wage offers: Poachers offer the worker they meet a contract \( w^*(p^*) \).

3. Worker Chooses Offer: Employed workers choose which contract to accept.

4. Employer Learning With probability \( \mu \) the incumbent learns the worker's type.

5. Displacement

   - Fires: Each incumbent chooses whether to fire their worker \( \{ F(p', w') = 1 \} \).
   - Shut Down: The worker is displaced with exogenous probability \( d \).

6. Worker Death, Birth: Measure \( \delta \) random workers die, replaced by new entrants

7. Direct Matches: New Entrants & Displaced matched to vacant plants

<table>
<thead>
<tr>
<th>Incumbent ((p, w))</th>
<th>Poacher (p^*)</th>
<th>Worker ( Q \in {0, 1} )</th>
<th>Displacements</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w' \in [w, \bar{w}] )</td>
<td>( w^* \in [\bar{w}, \bar{w}] )</td>
<td>( Q = 0 ): Incumbent ( w' = w'(p, w) ), ( p' = B(p, w'(p, w)) )</td>
<td>( Q = 1 ): Poacher ( w' = w^<em>(p^</em>) ), ( p' = B^<em>(p^</em>, w^*) )</td>
</tr>
</tbody>
</table>

Figure 1: Timing
3.4 Beliefs and Strategies

Incumbents and poachers have priors $p$ and $p^*$, respectively, the probability the worker is of the high type. If the poachers are informed $p^* = p$. If the poachers are uninformed, $p^* = \pi$, the population average. The incumbent’s prior over the poachers’ type and action is a joint probability distribution $q_p(p^*, w^*)$. The poachers’ prior over the incumbent’s type, standing wage contract, and action is a joint probability distribution $q_p^*(p, w; w')$. This object depends on the poachers’ own type ($p^*$): whether it is informed or uninformed of the incumbent’s type.

An incumbent newly matched with a new entrant has rational prior $p = \pi$ according to the population distribution. An incumbent newly matched with a displaced worker has prior $p^*$, the proportion of unemployed that are high type.

The plant whose offer the worker accepts has a posterior belief about the worker’s type based on their prior and the worker’s action. Denote the Bayesian posterior as $B(p, w)$ for the incumbent if the worker stays and $B^*(p^*, w^*)$ for the poaching plant if the worker quits. Plants the worker does not choose die, forget all information about a worker, and never meet her again.

Strategies are a mapping of priors $P$ and the state $W$ to actions $A$. Eliminating redundant state variables, pure strategies can be defined as:

- **Incumbent** $w'(p, w) \in [w, w_{\text{max}}]$  
  $F(p, w) \in \{0, 1\}$
- **Poachers** $w^*(p^*, w) \in [w, w_{\text{max}}]$
- **Worker** $Q(p, w, p^*) \in \{0, 1\}$

3.5 Value Functions and Strategies

Here I write the dynamic problems of players.

**Employed worker** Consider worker $i$ with contract $w$, incumbent belief $p$, and poachers’ belief $p^*$. Consider wage offer policies $w(p)$ and $w^*(p^*)$; and firing policy $F(p, w) \in \{0, 1\}$.
functions, her value of this current match is:\footnote{By considering pure strategies I implicitly guess an equilibrium exists where agents play pure strategies and later verify.}

\[
V_i(p, w, p^*) = \max_{Q \in [0,1]} (1 - Q)[w'(p, w) + \rho(1 - \mu)(E_{p^*}[V_i(B(p, w), w', p^*)]B(p, w) + \rho \mu \tilde{V}_i(w'))] + Q[w^*(p^*) + \rho(1 - \mu)(E_{p^*}[V_i(B^*(p^*, w^*), w^*(p^*), p^*)]B^*(p^*, w^*) + \rho \mu \tilde{V}_i(w^*(p^*))]
\]
\[
+ (1 - d)\rho E_{p^*}[V_i(p^u, w(p^u), p^*)|p^u]
\]

\[p^*|p \sim p \text{ with pr } \nu; \quad p^{\text{minf}} \text{ with pr } 1 - \nu\]

In the first two lines the worker is choosing between the incumbent \((p, w')\) and the poacher \(p^*, w^*(p^*)\), where \(\tilde{V}_i(w)\) is the value if the plant learns her type. The third line is an exogenous shut-down, at which her state becomes \(p^u\). Her value the period and incumbent learns depends on whether she is fired \(F(\delta_i) = 1\) or not:

\[
\tilde{V}_i(w) = (1 - F(\delta_i))E_{p^*}[V_i(\delta_i, w, p^*)|\delta_i]
\]
\[
+ F(\delta_i)E_{p^*}[V_i(p^u, w(p^u), p^*)|p^u]
\]

Her choice between the incumbent and poacher gives a quit policy, \(Q_i(p, w', w^*)\).

\[
Q_i(p, w', w^*) = \begin{cases} 1, & \text{if } w^* + \rho(1 - \mu)E_{p^*}[V_i(B^*(p^*, w^*), w^*, p^*)|B^*(p^*, w^*)] + \rho \mu \tilde{V}_i(w^*) \\ > w' + \rho(1 - \mu)E_{p^*}[V_i(B(p, w'), w', p^*)|B(p, w')] + \rho \mu \tilde{V}_i(w') & \text{otherwise.} \end{cases}
\]

**Incumbent** Denote \(p_i = (1 - p)\) and \(p_h = p\). Given worker policy \(Q_i(p, w', w^*)\) and poachers' policy \(w^*(p^*)\) the problem of an incumbent is:

\[
\Omega(p, w) = \max_{w \geq w} \sum_{i=l,h} p_i[\nu(1 - Q_i(p, w, w^*(p)) + (1 - \nu)(1 - Q_i(p, w, w^*(p_c)))] \times
\]
\[
[y(B(p, w')) - w' + \rho \mu \tilde{V}(B(p, w'), w') + \rho(1 - \mu)\Omega(B(p, w'), w')]
\]

\footnote{By considering pure strategies I implicitly guess an equilibrium exists where agents play pure strategies and later verify.}
The plant is choosing a wage offer to balance the probability the worker stays and expected future profits conditional on the worker staying, where \( \Omega(B(p), w') = (1 - F(\delta_i, w))B(p, w')\Omega(\delta_i, w) + (1 - F(\delta_i, w))(1 - B(p, w'))\Omega(\delta_i, w) \) is the continuation value in the period when the incumbent learns the worker's type. Trivially,

\[
F(\delta_i, w) = \begin{cases} 
1, & \Omega(\delta_i, w) < 0; \\
0, & \text{otherwise}.
\end{cases}
\]

**Poachers** Bertrand competition among the poachers yields the worker the total expected surplus. This implies \( w^*(p^*) \) solves:

\[
\Omega(p^*, w^*(p^*)) = 0
\]

### 3.6 Definition of a Symmetric Perfect Bayesian Equilibrium

An equilibrium \( \Phi \) is a set of pure strategies \( \{w, w^*, F, Q\} \), posterior beliefs \( \{B(p, w), B^*(p^*, w^*)\} \), and distribution \( \lambda(p, w) \) with support \( \Psi \) such that, given exogenous objects \( \Theta = \{y_i, \pi, \mu, \nu, \delta, \delta_i, \delta_p\} \):

- Strategies are sequentially rational given \( \Psi \):
  
  - \( w'(p, w) = \text{argmax}_w \Omega(B(p, w), w) \)
  
  - \( w^*(p^*) \) such that \( \Omega(B(p^*, w^*), w^*(p^*)) = 0 \)
  
  - \( F(\delta_i, w) = \begin{cases} 
1, & \Omega(\delta_i, w) < 0; \\
0, & \text{otherwise.}
\end{cases} \)
  
  - \( Q_i(p, w', w^*) = \begin{cases} 
1, & w^* + \rho(1 - \mu)E_{p^*}[V_i(B^*(p^*, w^*), w^*, p^*)|B^*(p^*, w^*)] + \rho \mu \tilde{V}_i(w^*) \\
> w' + \rho(1 - \mu)E_{q'}[V_i(B(p, w'), w', q')|B(p, w')] + \rho \mu \tilde{V}_i(w'); \\
0, & \text{otherwise.}
\end{cases} \)

- Priors are consistent with equilibrium distribution \( \lambda(p, w) \)
  
  - \( p^a = \sum_{\langle p, w \rangle \in \Psi} \lambda(p, w) = \frac{p(\delta(1 - \delta) \mu F(\delta_i, w))}{p(\delta(1 - \delta) \mu F(\delta_i, w)) + (1 - p)(\delta(1 - \delta) \mu F(\delta_i, w))} \)
  
  - \( q^a_p(p, w) = \begin{cases} 
\lambda(p, w), & \text{if } p^* = p_a; \\
\lambda(p, w|p = p^*), & \text{if } p^* \neq p_a.
\end{cases} \)
• Posterioris are Bayesian where possible

\[
B(p, w) = \frac{p \cdot \nu(1 - Q_h(p, w, w^*(p)) + (1 - \nu)(1 - Q_l(p, w, w^*(p)))}{(1 - p) \cdot \nu(1 - Q_l(p, w, w^*(p)) + (1 - \nu)(1 - Q_h(p, w, w^*(p))) + p \cdot \nu(1 - Q_h(p, w, w^*(p)) + (1 - \nu)(1 - Q_l(p, w, w^*(p)))}
\]

\[
B^*(p^*, w^*) = \sum_{(p, w) \in \Psi} q^*_p(p, w) \frac{p \cdot Q_h(p, w, w^*)}{p \cdot Q_h(p, w, w^*) + (1 - p) \cdot Q_l(p, w, w^*)}
\]

Off path: \(B(p, w) = \lim_{\epsilon \to 0} Q_{1\epsilon}\) where \(Q_{1\epsilon} \in [\epsilon, 1 - \epsilon]\)

• Distributions are stationary\(^{12}\)

\[T \lambda(p, w) = \lambda(p, w)\]

I define off-path beliefs as the limit of the workers' forced mixed strategies. However, this equilibrium can be supported by many off-path beliefs. Further it satisfies many equilibrium refinements including Cho-Kreps Intuitive Criterion, Sequential Equilibrium as Kreps and Wilson, and is the limit of Control Cost equilibria as Selton.

4 Characterization of the Equilibrium

In this simple version of the model, the equilibrium is that a plant matched with a worker from unemployment pays her the minimum wage in the economy and loses her in the first period to either informed or uninformed poachers. Subsequently, the uninformed poachers know the only worker who accepts has been unemployed and pays a wage that gives zero profits for the average worker from unemployment. All other plants pay the minimum wage required to beat an uninformed poacher and keep her until an informed poacher arrives.

Instead of guess and verify, I will provide general propositions that can be applied to extended versions of the game. There are two key propositions: (1) workers' strategies are independent of type; and (2) the uninformed poacher wage equals the minimum informed poacher wage in the economy. I will begin with the first proposition. The only equilibrium is a pooling equilibrium; i.e., high types cannot signal their type through their actions. The intuition is that it is costless for low types to mimic high types. This is because the only

\(^{12}\)see appendix for explicit form
pay-off that depends on their type is when the incumbent learns their type and they are fired. The rate at which this learning happens is exogenous and the pay-off is the same regardless of the worker's past actions.

**Proposition 4.1.** The worker's policy is independent of her type: \( Q_h(p, w, w^*) = Q_l(p, w, w^*) \) for all \((p, w, w^*) \in \bar{\Psi} \times \bar{\Psi}\)

*Proof.* See appendix.

Since \( Q(p, w, w^*) \) is the same for both types of workers, incumbent plants and informed poachers learn nothing from the worker’s actions. However, uninformed poachers form posterior beliefs about what the incumbent’s prior must have been if the worker accepts. Since they are Bayesian, the incumbent’s prior is informative about the worker’s type. The result is the only beliefs in the economy are: \( \mathcal{P} = \{B(q^{\text{uninf}}, w^*(q^{\text{uninf}})), \pi, p^*, \delta_h\}. \)

Although the environment is dynamic, the problem for an incumbent is static before and after the poisson arrival of full information. Sequential rationality then implies wage contracts are only changed when full information arrives. Define \( w(p) \) as the minimum incumbent wage such that the worker rejects the uninformed poacher.\(^{14}\) The following proposition and corollary show that for any state \( p \not\in \{p^{\text{uninf}}, \delta_h\} \), the worker will be paid one of three wages: the informed poacher wage \( w^*(p) \), the minimum wage \( w \), or \( w(p) \).

**Proposition 4.2.** \( w'(p, w) < w^*(p) \) for all \( w < w^*(p) \) and any \( p \in \mathcal{P} \).

**Corollary 4.3.** If \( w^*(p) < w^*(p^{\text{uninf}}) \) then \( w'(p, w) = w \)

**Corollary 4.4.** If \( w^*(p) > w^*(p^{\text{uninf}}) \) then \( w'(p, w) = \max\{w, w(p)\} \)

By the proposition, incumbent offers are always strictly less than the informed poacher offer, unless the standing contract prevents it. This is because informed poachers Bertrand compete, so their wage contracts leave zero surplus. The first corollary states when the informed poacher wage is less than the uninformed poacher wage, then the incumbent

---

\(^{13}\) To avoid the uninteresting case with no lay-offs, assume that \( y_h < 0 \). This implies low workers are fired when the employer learns their type \( F(\delta_l) = 1 \). Also, let \( y_h \) be sufficiently large such that \( F(\delta_h) = 0 \).

\(^{14}\) Below I show \( w(p) \neq w^*(p^{\text{uninf}}) \) unless \( p = p^{\text{uninf}} \).
pays the minimum wage, if possible. The second corollary states that if the informed poacher wage is greater than the uninformed, the incumbent offers the wage such that the worker rejects the uninformed poacher if it is above the minimum wage and the minimum wage otherwise. The result is that the only potential wages in the economy are: \( \mathcal{W} = \{w, w^*(p^{\min f}), w^*(\pi), w^*(\delta_h), w(p^v), w(\pi), w^H\} \)\(^{15}\) The last set of wages with underlines are offers that marginally beat an uninformed incumbent. Algebra on the worker’s problem show that these wages are not equal to the uninformed poacher wage:

Before Learning  
\[
    w(p) = \frac{(1 - (1 - \nu)p)w^*(p^{\min f}) - \nu pw^*(p)}{1 - \rho} \quad p \in \{p^v, \pi\}
\]

After Learning  
\[
    w^H = \frac{(1 - (1 - \mu)(1 - \nu)p)w^*(p^{\min f}) - (1 - \mu)\rho w^h}{1 - (1 - \mu)p}
\]

These wages are less than the uninformed poacher wage. This is because the worker’s offer from an informed poacher depends on the information of the incumbent. If the information in the current match implies she will receive a higher wage from informed poachers than the uninformed, the worker may forgo an offer from an uninformed poacher, even if it is higher than her current wage, in order to wait for the informed poachers. This strength of this effect is tempered by the likelihood of meeting an informed poacher \( \nu \). If there are no informed poachers, \( \nu = 0 \), it is the private learning case: the incumbent pays the uninformed wage and captures full rents. If there are only informed poachers, \( \nu = 1 \), it is the public learning case: the incumbent pays the worker her marginal product. For the high type with and incumbent who has learned her type, \( w^H \) also takes into consideration the speed of learning \( \mu \). If \( \mu = 0 \), the worker’s type is always private and all plants are the same. If \( \mu = 1 \) learning is immediate and the incumbent pays the uninformed wage.\(^{16}\) For interior values of \( \mu \), the incumbent pays less than the uninformed poacher wage.

The wage offer of the uninformed poacher can be affected by the offer of the incumbent. However, sequential rationality implies the incumbent will not mix between beating and

\(^{15}\)To avoid uninteresting cases, let the minimum wage be sufficiently low such that wages from Bertrand competition are strictly positive and do not result in lay-off.

\(^{16}\)Many two period models in the theory literature consider the employer learns after the first period. The equivalent assumption in this environment is \( \mu = 1 \).
failing to beat an uninformed poacher because one strategy will always dominate the other in the continuation at a given node. Since $w^*(p^{\text{min}f}) > w^H$, the strategy of a learned incumbent is:

$$w^*(\delta_h, w) = \max\{w, w^H\}$$

Given the strategy of a learned incumbent, the offer of the informed poachers that satisfies zero profit under Bertrand Competition is:

$$w^*(p) = \begin{cases} y_h - \frac{1-(1-\nu)\rho}{1-\rho(1-\nu)-\mu l}(1 - p)(y_h - y_l), & \text{if } w^*(p) > w^H; \\ p(y_h + \frac{1-\nu}{1+\mu l}y_l), & \text{otherwise}. \end{cases}$$ \hspace{1cm} (4.1)

Note that all $p \geq p^{\text{min}f}$, fall into the first case: $w^*(p) = y_h - \frac{1-(1-\nu)\rho}{1-\rho(1-\nu)-\mu l}(1 - p)(y_h - y_l)$.

Given $p^{\text{min}f}$, the offer strategy of a new incumbent is:\footnote{At $p = p^{\text{min}f}$ the incumbent is indifferent between the minimum wage and $w^*(p^{\text{min}f})$. However, only strategy $w(p) = w$ satisfies the notion of sequential equilibrium with respect to the workers’ quit strategy.}

$$w(p) = \begin{cases} \max\{w(p), w\}, & \text{if } p > p^{\text{min}f}; \\ \max\{w^*(p), w\}, & \text{if } p < p^{\text{min}f}; \\ w, & \text{if } p = p^{\text{min}f}. \end{cases}$$ \hspace{1cm} (4.2)

By Proposition 4.1, the workers’ strategy is the same for both types. The following proposition shows a worker maximizes her offers by staying with an incumbent when offered an identical wage from a poacher with identical beliefs.

**Proposition 4.5.** $Q(p, w^*(p), w^*(p)) = 0$.

**Proof.** See appendix. \qed

The remaining quit strategies follow immediately from the incumbent strategy:

$$Q(p, w, p^*) = \begin{cases} 0, & \text{if } p \geq p^* \text{ and } p \neq p^{\text{min}f}; \\ 1, & \text{if } p < p^* \text{ or } p = p^{\text{min}f}. \end{cases}$$

The final strategy to solve for is the wage offer by the uninformed poacher. All other strategies above can be written as functions of this variable and exogenous parameters. It is
defined by Equation (4.1) given the posterior of the uninformed poacher. Given worker quit policy and equilibrium distribution \( \lambda(p, w) \), the posterior of the uninformed poachers is:

\[
B^*(p^{\text{uninf}}) = \sum_{(p, w) < p^*} \lambda(p, w) \log Q(p, w, p^{\text{uninf}})
\]

This gives:

\[
w^*(p^{\text{uninf}}) = y_h - \frac{1 - (1 - \nu)p}{1 - \rho((1 - \nu) - B^*(p^{\text{uninf}}, w^*(p^{\text{uninf}}))) \mu} (1 - B^*(p^{\text{uninf}}, w^*(p^{\text{uninf}}))))(y_h - y_l)
\]

What is left is to characterize the equilibrium distribution. This object is large. Please see the appendix.

The proposition below shows the equilibrium satisfying the above equations exists and is unique, given exogenous parameters \( \Theta = \{y_h, \pi, \mu, \nu, \delta, d, \rho\} \). Further, it details the key result that the uninformed poacher belief equals the minimum informed poacher belief in the economy. In this case \( p^{\text{uninf}} = p^* \)

**Proposition 4.6.** If \( \mu \in (0, 1) \) and \( \nu \in (0, 1) \), there exists a unique \( \Psi(\Theta) \) that satisfies the equilibrium conditions and refinement and it is characterized by \( p^{\text{uninf}} = p^* \) such that \( p \geq p^* \) for all \( p \in \mathcal{P} \)

**Proof.** Given \( \mu \in (0, 1) \) and \( \nu \in (0, 1) \), \( p \leq p^* \) for all \( p \in \mathcal{P} \). Then proof by contradiction need only consider the case where \( p^{\text{uninf}} > p^* \). Suppose \( p^{\text{uninf}} > p^* \). Let \( \tilde{\mathcal{P}} \subset \mathcal{P} \) such that \( w'(\tilde{p}, w) < w(\tilde{p}) \) for all \( \tilde{p} \in \tilde{\mathcal{P}} \). Then, by definition of \( B(p^{\text{uninf}}, w'(p^{\text{uninf}})) \), \( p^{\text{uninf}} > p^* \) implies \( \tilde{\mathcal{P}} \setminus p^* \neq \emptyset \). Define \( \tilde{p}_{\text{max}} \in \tilde{\mathcal{P}} \) such that \( \tilde{p}_{\text{max}} \geq \tilde{p} \) for all \( \tilde{p} \in \tilde{\mathcal{P}} \). By equation 4.2, if \( p \in \mathcal{P} \) and \( p < \tilde{p}_{\text{max}} \), then \( p \in \tilde{\mathcal{P}} \). Then, by definition of \( B(p^{\text{uninf}}, w'(p^{\text{uninf}})) \), \( \tilde{p}_{\text{max}} > p^{\text{uninf}} \). But also by equation 4.2, \( \tilde{p}_{\text{max}} > p^{\text{uninf}} \Rightarrow w'(\tilde{p}_{\text{max}}, w) > w(\tilde{p}_{\text{max}}) \Rightarrow \tilde{p}_{\text{max}} \notin \tilde{\mathcal{P}} \rightarrow \rightarrow. \)

The two key results of the equilibrium strategies that can be extended to models with many types are as follows. First, the uninformed poacher wage is always equal to the minimum informed poacher wage in the economy. This pins down the uninformed poacher wage in a general setting. Second, there is a unique pooling equilibrium in worker's strategy. This implies there is no dynamic learning of incumbents and no strategic actions of workers. The result is a finite set of equilibrium beliefs.
5 Comparative Statics

5.1 Employer Learning

Figures on page 43 of the Appendix illustrate how the wages in the economy are affected by the speed of employer learning. *The speed of employer learning determines the value of the incumbent's private information and the composition of the pool of unemployed.* As employer learning becomes fast (μ approaches 1), the plant’s monopsony power over information falls. Therefore, wages after learning and the offers of informed plants increase. The second effect of fast employer learning is that the pool of the unemployed worsens. This means the wages of re-hired unemployed workers fall and, as a result, the wage offer of the uninformed incumbent falls. This lowers the “outside option” of workers. These two effects work in the same direction to lower the starting wage of a new entrant: she is worth less to her employer and collects a smaller share of the surplus. The cumulative effects of faster employer learning are a steeper wage profile of new entrants (lower starting wages and faster growth) and greater initial wage losses for displaced workers, but faster recoveries.

In summary, when employer learning is fast, a worker’s wages start lower and grow by a larger amount and more quickly. However, plants’ profits may be non-monotone in speed of learning.

5.2 Diffusion of Information to Potential Employers

Figures on page 45 of the Appendix illustrate how the wages in the economy are affected by the likelihood a poacher is informed. *Greater chance of an informed poacher raises the worker’s reservation wage to accept an uninformed poachers and lowers the value of the incumbent’s private information.* Workers always accept the offer of an informed poacher. Therefore, the more likely an informed offer is, the shorter the match will be and the less value a worker is to a plant. When informed poachers dominate the pool of offers, workers accept very low incumbent offers over higher offers of uninformed poachers in favor of waiting for the informed poacher offer. It is only when a large portion
of the offers are from uninformed poachers that the incumbent raises her wage above the minimum. After an the employer learns the worker’s type, her wage remains the same until she meets an informed poacher. The more likely a poacher is to be informed, the faster wage growth is in this economy. Also note that jobs early in life are paid faster when diffusion of information to poachers is more likely.

*In summary, when poachers are more likely to be informed, a worker’s wages start lower and grow by a larger amount and more quickly after employer learning. The more likely informed poachers are, the lower plants’ profits are.*

6 Extensions

The basic framework above can be extended in a variety of ways. Extensions where the workers’ relative inside and outside values are either independent of her actions or her type will maintain the result of the pooling equilibrium in workers’ strategies and no dynamic learning of the incumbent. An example of where this would not be true would be cases of stochastic match-specific productivity that include states where the low types of workers would be fired by the high types would not. In this situation the low type of worker would have higher payoff from leaving the incumbent if the match quality falls and this action (or inaction of a high type) would reveal something about her type to both incumbent and poaching plants. Workers’ strategies would then have to consider this revelation and could become quite complex. Extensions such as these can be done, but care must be taken in verifying the strategies of workers and posteriors of plants. Much more tractable applications would add static decisions of the worker or plant. Some examples are a choice between different occupations with different speeds of learning or speeds of diffusion, a one-time choice of investment in general or specific human capital at the beginning of life, or incorporating a cost of employers to obtain a signal about workers before the start of the match.

The framework can also be added to equilibrium models of unemployment such as Mortensen and Pissarides (1994). This extension allows the measurement of how much this mechanism adds to unemployment duration and reduces aggregate output. Interesting
questions to be considered in this environment include measuring the welfare gains and losses of policy such as preventing employers from discriminating by employment status in making new hires or imposing firing costs.

I will provide two extensions: (1) adding skill accumulation through learning by doing and a probability of skill depreciation at unemployment; and (2) a basic model of equilibrium unemployment.

6.1 Adding Human Capital

I make the following changes to the basic environment. First, I add skill accumulation and loss in the style of Ljungqvist and Sargent (1998). Workers’ skills are public information. Workers start life unskilled \((s = 0)\). During each period of employment they become skilled \((s = 1)\) with probability \(\gamma\). If a skilled worker becomes unemployed, either by endogenous firing or exogenous shut down, she becomes unskilled with probability \(\tau\). A worker’s output is a function of their type and their skill level. Second, By adding skills to the basic model, I can now consider both types to always be productive. I assume an unskilled low type worker’s productivity is below the opportunity cost of the firm of hiring a new worker. Thus the low type is profitable if she is skilled, but unprofitable if she is unskilled and will be fired. Denote the output of worker of ability \(i \in \{l, h\}\) and skills \(s \in \{0, 1\}\) as \(y_{is}\) with the following ordering assumed to hold: \(y_{l0} < y_{h0} < y_{l1} < y_{h1}\). The third component I add is that the worker’s wage can be lowered if both parties agree. This prevents plants from offering workers wage contracts above the skilled low worker’s value that would trigger a firing if she is discovered to be the low type. Instead, she will offer the plant to be paid a wage such that the plant earns zero profits. This makes the plant indifferent and I assume the offer is accepted.

I will now write the value functions of workers. I begin with skilled workers and impose the parameters are chosen such that neither worker is fired. The theorems above trivially extend to this environment and I will save notation by applying the results of the pooling
equilibrium that the Bayesian posterior of the incumbent is equal to the prior.

\[
V_{11}(p, w, p^*) = \max_{\sigma \in \{0, 1\}} (1 - Q)[w'(p, w) + \rho(1 - \mu)(E_{p^*}[V_{11}(p, w', p^*)|p] + \rho \mu \tilde{V}_{11}(w'))],
\]

\[
Q[w^*(p^*) + \rho(1 - \mu)[E_{p^*}[V_{11}(p^*, w^*(p^*), p^*)|p^*] + \rho \mu \tilde{V}_{11}(w^*(p^*))] + (1 - d).\rho E_{p^*}[(1 - \gamma)V_{11}(p^u, w(p^u), p^*) + \gamma V_{11}(p, w(p^u), p^*)|p^*]
\]

\[
p^*|p \sim p \text{ with pr } \nu; \quad p^{\text{uninf}} \text{ with pr } 1 - \nu
\]

Where the value after her type is learned is:

\[
\tilde{V}_{11}(w) = E_{p^*}[V_{11}(\delta_i, w, p^*)|\delta_i]
\]

The value function of a worker that has not acquired skills is as follows. It features the probability she becomes skilled and if her type is learned before she gains skills, the low type will be fired.

\[
V_{10}(p, w, p^*) = \max_{\sigma \in \{0, 1\}} (1 - Q)[w'(p, w) + \rho(1 - \mu)E_{p^*}[(1 - \gamma)\tilde{V}_{11}(p, w', p^*) + \gamma V_{11}(p, w', p^*)|p]] + \rho \mu [(1 - \gamma)\tilde{V}_{10}(w') + \gamma \tilde{V}_{11}(w')]] + \rho \mu [(1 - \gamma)\tilde{V}_{10}(w^*) + \gamma \tilde{V}_{11}(w^*)]] + (1 - d)E_{\sigma^*}[V_{10}(p^u, w(p^u), p^*)|p^*]
\]

\[
p^*|p \sim p \text{ with pr } \nu; \quad p^{\text{uninf}} \text{ with pr } 1 - \nu
\]

Where the value after her type is learned is:

\[
\tilde{V}_{10}(w) = (1 - F(\delta_i))E_{p^*}[V_{10}(\delta_i, w, p^*)|\delta_i]
\]

\[
+ F(\delta_i)E_{p^*}[V_{10}(p^u, w(p^u), p^*)|p^u]
\]

The value function for an incumbent with a skilled worker believed to be the high type
with probability $0 < p < 1$ is as follows.

$$\Omega_1(p, w) = \max_{w' \geq w} \sum_{i=1}^{h} p_i[\nu(1 - Q(p, w', w_0^*(p))) + (1 - \nu)(1 - Q(p, w', w_0^*(p^{uninf}))) \times
\left[y_{h1} - w' + \rho \mu \tilde{\Omega}_{h1}(w') + \rho(1 - \mu)\Omega_1(p, w')\right]$$

The value after the type $i$ is revealed is:

$$\tilde{\Omega}_{h1}(w) = \max_{w' \geq w}[\nu(1 - Q(p, w', w_1^*(\delta_i))) + (1 - \nu)(1 - Q(p, w', w_1^*(p^{uninf}))) \times
\left[y_{h1} - w' + \rho \tilde{\Omega}_{h1}(w')\right]$$

The value function for an incumbent with an unskilled worker believed to be the high type with probability $0 < p < 1$ is as follows.

$$\Omega_0(p, w) = \max_{w' \geq w} \sum_{i=1}^{h} p_i[\nu(1 - Q(p, w', w_0^*(p))) + (1 - \nu)(1 - Q(p, w', w_0^*(p^{uninf}))) \times
\left[y_{h0} - w' + \rho \mu ((1 - \gamma)\tilde{\Omega}_{h0}(w') + \gamma \tilde{\Omega}_{h1}(w')) + \rho(1 - \mu)((1 - \gamma)\Omega_0(w') + \gamma \Omega_1(w'))\right]$$

If the type is revealed before skills are acquired, the low type will be fired leaving the plant with value of a skilled high type worker:

$$\tilde{\Omega}_{h0}(w) = \max_{w' \geq w}[\nu(1 - Q(p, w', w_0^*(\delta_h))) + (1 - \nu)(1 - Q(p, w', w_0^*(p^{uninf}))) \times
\left[y_{h0} - w' + \rho ((1 - \gamma)\tilde{\Omega}_{h0}(w') + \gamma \tilde{\Omega}_{h1}(w'))\right]$$

The result holds that the incumbent always chooses the lowest wage to beat an uninformed poacher. To characterize the equilibrium, I will start with the wages of the skilled
workers because the functional forms are identical to the basic framework without skills.\footnote{The poachers pay workers hired by the incumbent from unemployment or as new entrants their expected output in this case because they will not fire the low type worker.}

\[
w_{h1}^* = y_{h1} \quad \text{Skilled, known high type; informed poachers}
\]

\[
w_{l1}^* = y_{l1} \quad \text{Skilled, known low type; informed poachers}
\]

\[
w_1^*(p_u^v) = p_u^v y_{h1} + (1 - p_u^v) y_{l1} \quad \text{Skilled, hired from unemployment as skilled; informed poachers}
\]

\[
w_1^*(p_u^v) = p_u^v y_{h1} + (1 - p_u^v) y_{l1} \quad \text{Skilled, hired from unemployment as unskilled; informed poachers}
\]

\[
w_1^*(p_{ne}^v) = p_{ne}^v y_{h1} + (1 - p_{ne}^v) y_{l1} \quad \text{Skilled, was hired as newentrant; informed poachers}
\]

\[
w_1^*(p_{uninf}^v) = w_1^*(p_0^v) \quad \text{Skilled, uninformo poachers}
\]

\[
w_{h1} = \frac{(1 - (1 - \mu)(1 - \nu)\rho)w_1^*(p_{uninf}^v) - (1 - \mu)\rho y_{h1}}{1 - (1 - \mu)\rho} \quad \text{Skilled, known high type; incumbent}
\]

\[
w_{l1} = \frac{(1 - (1 - \mu)(1 - \nu)\rho)w_1^*(p_{uninf}^v) - (1 - \mu)\rho y_{l1}}{1 - (1 - \mu)\rho} \quad \text{Skilled, known low type; incumbent}
\]

\[
w_1(p_u^v) = \frac{(1 - (1 - \nu)\rho)w_1^*(p_{uninf}^v) - \nu \rho w_1^*(p_{ne}^v)}{1 - \rho} \quad \text{Skilled, was hired as skilled unemployed; incumbent}
\]

\[
w_1(p_u^v) = w_1^*(p_0^v) \quad \text{Skilled, was hired as unskilled unemployed; incumbent}
\]

\[
w_1(p_{ne}^v) = \frac{(1 - (1 - \nu)\rho)w_1^*(p_{uninf}^v) - \nu \rho w_1^*(p_{ne}^v)}{1 - \rho} \quad \text{Skilled, was hired as newentrant; incumbent}
\]

Now, to solve the problem of plants with a worker that has not acquired skills takes additional algebra. Note there are no wages for unskilled workers of the low type because
they are fired.

\[ w_{h0}^* = \frac{(1 - \rho(1 - \nu))y_{h0} + \rho \gamma y_{h1}}{1 - \rho(1 - \nu) + \rho \gamma} \quad \text{Unskilled, known high type; informed poachers} \]

\[ w_0^*(p_{h0}^{\mu}) = \left[ 1 + \frac{\rho(1 - \gamma)\mu p_{h0}^{\mu}}{1 - (1 - \nu)(1 - \gamma)\rho} \right]^{-1} \times \quad \text{Unskilled, was hired from unemployment; informed poachers} \]

\[ \frac{\rho(1 - \gamma)\mu p_{h0}^{\mu}}{1 - (1 - \nu)(1 - \gamma)\rho} \left( y_{h1} - w_{1h} \right) + \frac{\rho(1 - \gamma)\mu p_{h0}^{\mu}}{1 - (1 - \nu)(1 - \gamma)\rho} \left( y_{h0} - w_{1h} \right) \]

\[ w_0^*(p_{h0}^{ne}) = \left[ 1 + \frac{\rho(1 - \gamma)\mu p_{h0}^{ne}}{1 - (1 - \nu)(1 - \gamma)\rho} \right]^{-1} \times \quad \text{Unskilled, was hired as new entrant; informed poachers} \]

\[ \frac{\rho(1 - \gamma)\mu p_{h0}^{ne}}{1 - (1 - \nu)(1 - \gamma)\rho} \left( y_{h1} - w_{1h} \right) + \frac{\rho(1 - \gamma)\mu p_{h0}^{ne}}{1 - (1 - \nu)(1 - \gamma)\rho} \left( y_{h0} - w_{1h} \right) \]

\[ w_0^*(p_{h0}^{\mu}) = w_0^*(p_{h0}^{\mu}) \quad \text{Unskilled, known high type; incumbent} \]

\[ \frac{\rho(1 - \gamma)\mu p_{h0}^{ne}}{1 - (1 - \nu)(1 - \gamma)\rho} \left( y_{h1} - w_{1h} \right) + \frac{\rho(1 - \gamma)\mu p_{h0}^{ne}}{1 - (1 - \nu)(1 - \gamma)\rho} \left( y_{h0} - w_{1h} \right) \]

In this environment, there are two separate beliefs for unemployed workers that need to be determined: (1) the probability an unskilled unemployed worker is the high type; and (2) the probability a skilled unemployed worker is the high type. This requires first solving for the stationary distributions of skill-ability combinations in the economy. They are as follows:

\[ \lambda_{h0} = \frac{\pi (\delta + (1 - \delta)\tau)}{\delta + (1 - \delta)\tau + \gamma} \]

\[ \lambda_{h1} = \pi - \lambda_{h0} \]

\[ \lambda_{0} = \frac{\pi (\delta + (1 - \delta)\tau)}{\delta + (1 - \delta)\tau + \gamma} \]

\[ \lambda_{1} = 1 - \pi - \lambda_{0} \]
Then the proportion of high types in unemployment at each skill level are:

\[
P_0^k = \frac{d(\tau \lambda_{h1} + \lambda_{h0})}{d(\tau \lambda_{h1} + \lambda_{h0}) + d\tau \lambda_{l1} + (d + \mu)\lambda_{l0}}
\]

\[
P_1^k = \frac{\lambda_{h1}}{\lambda_{h1} + \lambda_{l1}}
\]

7 Quantitative Analysis

**Calibration** To measure the contribution of lost skills and information to the wage losses of displaced workers, I calibrate model parameters to moments dealing with worker tenure, multiple displacements, and wages. Algebraic closed form solutions for distributions of workers makes the model calibration direct for parameters not chosen to match wages. Three parameters are chosen directly. I normalize the output of the low type worker \( y_l = 1 \). I choose the worker death rate to give an average working life of 40 years \( 1 - \delta = 0.0025 \). I choose discount rate \( \rho = 0.95 \) within the standard range. Since learning follows a poisson arrival rate, it is iid over workers and across experience groups and can be identified with single equations. I choose \( \mu \), the speed of employer learning, to match the tenure at second fire of 4.2 years observed in the data. This gives \( \mu = 0.24 \). I choose \( \nu \), the speed of information diffusion to outside employers, to match the tenure of direct employer to employer quitters of 5.1 years observed in the data. This gives \( \nu = 0.18 \).\(^{19}\)

**Estimation** The next set of parameters can be directly solved for a solution of a system of equations, given a chosen value of \( d \), the exogenous displacement rate. The problem reduces to solving a problem of the form \( \phi X_d = a \). The vector \( \phi \) is the model parameters \( \phi = [y_n \ s \ \pi \ \gamma \ \tau] \) and the vector \( a \) is the data moments in the table below.\(^{20}\)

---

\(^{19}\) I choose tenure at second firing and later in life to avoid un-modeled idiosyncracy in the employment experiences of young workers.

\(^{20}\) I do not consider all of the worker's reporting a lay-off to be endogenously selected. Instead, I estimate the probability a worker is exogenously displaced compared to the endogenous selection in the model to match
matrix $X$ is the system of model equations giving equivalents of the data equations. This is given in the appendix.\textsuperscript{21}

<table>
<thead>
<tr>
<th><strong>Calibrated Parameters: Joint System of Equations Given $d$</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters to match: $\delta, \gamma, \pi, \tau, y_h, s$</td>
</tr>
<tr>
<td>Data Moments</td>
</tr>
<tr>
<td>StDev ln(wage) workers with 25 years exp 0.58</td>
</tr>
<tr>
<td>Proportion fired in year after a firing 0.38</td>
</tr>
<tr>
<td>Yearly Agg Layoffs- JOLTS 7%</td>
</tr>
<tr>
<td>Av ln(wage) growth after 25 years exp 0.219</td>
</tr>
<tr>
<td>Proportion fired in year after a Shut Down 0.167</td>
</tr>
</tbody>
</table>

All of the above parameters were solved from systems of linear equations, or chosen directly, and so they are unique. What remains is to identify the probability of exogenous displacement $d$. To do so, the target is the initial wage loss of all displaced workers. I proceed as follows. I begin with a guess of $d$, compute $\phi$ such that $\phi X_d = \alpha$. Then simulate the model and perform the same regression on the model generated data as I did above on the PSID sample, except with no time fixed effects because the model is time-stationary:

$$\ln(w_{it}) = \gamma_1 exp_{it} + \gamma_2 exp^2_{it} + \sum_{n=1}^{15} \beta_n D^n_{it} + \alpha_i + \epsilon_{it}$$

The target is $\hat{\beta}_1^{model} = \hat{\beta}_1^{data}$. The update rule for $\pi$ follows bisection method on $(0,1)$ since $d$ is a probability. Define the function generated by this procedure as $L(d)$ and the resulting mapping as $T\pi = L(\pi)$. The following theorem shows this method defines a unique value of $d$.

**Theorem 7.1.** $T = L(d)$ defines a contraction mapping on $[0,1]$.

**Proof.** Since $d$ is bounded, it suffices to show $\hat{\beta}_1^{model}$ is monotone in $d$. Let $1 > \tilde{d} > \gamma 0$ and the corresponding sets of equilibrium wages $\tilde{W}$ and $W$ be given. To show $\hat{\beta}_1^{model}$ is monotone in $d$ I will show $\frac{\partial \ln}{\partial \tilde{d}} < \frac{\partial \lambda_0}{\partial \tilde{d}}$. Calculus gives $\frac{\partial \ln}{\partial \tilde{d}} < \mu \frac{\partial \ln}{\partial \tilde{d}} < \frac{\partial \lambda_0}{\partial \tilde{d}}$. \hfill $\square$

these data moments. This means when I match parameters to data, I do not consider all of the worker’s reporting a lay-off to be endogenously selected. I maintain all of the shut down workers are exogenously displaced.

\textsuperscript{21}I partition this system into wage and non-wage equations for computational ease.
Estimation Results  The results of the estimation are given in the table below.

<table>
<thead>
<tr>
<th>Parameter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Calibrated Parameters</strong></td>
</tr>
<tr>
<td>Speed of Learning $\mu$</td>
</tr>
<tr>
<td>Prob. Poacher Informed $\nu$</td>
</tr>
<tr>
<td>Worker Death $1-\delta$</td>
</tr>
<tr>
<td>Discount Rate $\rho$</td>
</tr>
<tr>
<td>Low Ability Output $y_L$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Estimated Parameters</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parameter</strong></td>
</tr>
<tr>
<td>High Ability Output</td>
</tr>
<tr>
<td>Proportion High Ability $\pi$</td>
</tr>
<tr>
<td>Exogenous Displacement Probability $d$</td>
</tr>
</tbody>
</table>

The highlights of these results are as follows. Skill accumulation probability is very high compared to standard models. The average worker takes one year to acquire skills in this model compared to 10 years in the calibration of Ljungqvist and Sargent (1998). This is because in models with just skill accumulation, it must take a long time to accumulate skills in order for the model to match the life-cycle wage growth of workers. In this model, wages also grow through the channel of employer learning and information diffusion. The estimated parameters imply it takes seven years on average for an employer to learn about a worker's ability and this information to spread to outside employers. Therefore, for slow life-cycle wage growth, the information channel dominates the skill accumulation channel.

With respect to unemployed workers, exogenous displacement occurs with probability 0.0248 or once every 40 years. This is less than in the skill accumulation only models where exogenous displacement occurs once every 25 years. The difference is made up by endogenous firings of unskilled low ability workers. The probability a low ability worker will be fired in her life time is 33.9%. Fired workers account for 66% of all unemployed workers in this
model. The probability an unemployed worker becomes unskilled is also slightly higher in this model, 22%, than a span of parameters considered for standard models of skill loss: 8-15%. This contributes to the higher likelihood of future displacement for fired workers that other models do not target.

Figure A.5 on page 49 shows the different wage paths of displaced workers generated by the model. High ability workers have the smallest wage losses and fastest recoveries. Low ability workers who are exogenously displaced have wage losses similar to high ability workers, but recover less quickly. This is because some of the exogenously displaced low ability workers will lose skills and face repeat displacement in the future where as high ability workers will have stable employment. Finally, only low ability workers without skills are fired (endogenously displaced). These workers face the largest wage losses and slowest recoveries. However, most of them will recover to wage greater than their pre-displacement wage because the fired workers are disproportionately young workers who have yet to acquire skills.

Figure A.5 on page 51 shows the different life-cycle wage paths of workers generated by the model. Low ability workers have slower wage growth than high ability workers because they have greater employment instability. If an employer learns a worker is of low ability before she acquires skills, she will be fired. This both delays her skill accumulation and generates worse beliefs about her type. Both lower the worker’s wages and slow down wage growth.

**Model Fit** Model simulations match several statistics on the wage and employment paths of displaced workers well. For comparison, I estimate the same regression on model generated data as in the PSID sample in the evidence section. Graphical depictions are shown in Figure A.5 on page 52 and the chart below gives summary statistics. The depth of the initial losses are not targeted but amount to -0.13 and -0.17 log points in the model for shut down and laid off workers, respectively, compared to -0.15 - 0.18 in the data. The model exactly matches the time to recover of the shut down (exogenous) displaced workers, but over states the depth of losses of fired workers by 0.06 log points. It additionally over predicts the total lifetime displacements of workers ever fired and under predicts the aggregate lay-off rate,
2.3\% in the model versus 6\% in data\textsuperscript{22}.

With regards to direct employer-to-employer turnover, the model yields mixed results. It does well to match the extent of employer-to-employer transitions. It over-predicts initial wage changes of job switchers and under-predicts the permanent increases associated with these changes. Two additions would improve the model's fit for employer-to-employer turnover. First adding some idiosyncratic match shock to generate higher turnover of the average ability workers. Second, modeling heterogeneity in ability as a growth rate instead of a fixed effect.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Wage Loss Fired (Endog)</td>
<td>-0.18</td>
<td>-0.17</td>
</tr>
<tr>
<td>Initial Wage Loss Shut Down (Exog)</td>
<td>-0.15</td>
<td>-0.13</td>
</tr>
<tr>
<td>Long Term Loss Fired</td>
<td>-0.24</td>
<td>-0.3</td>
</tr>
<tr>
<td>Time To Recovery Shut Down</td>
<td>9 years</td>
<td>9 years</td>
</tr>
<tr>
<td>Average Life Fires if ever Fired</td>
<td>4.63</td>
<td>3.05</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Wage Change (Endog)</td>
<td>0.02</td>
<td>0.05</td>
</tr>
<tr>
<td>Long Term Wage Change</td>
<td>0.08</td>
<td>0.01</td>
</tr>
<tr>
<td>Average Life Switches</td>
<td>10.3</td>
<td>5.8</td>
</tr>
<tr>
<td>Aggregate Yearly Turnover Rate</td>
<td>10-13%</td>
<td>10%</td>
</tr>
</tbody>
</table>

**Contribution of Skills vs Ability** Figure A.5 on page 50 shows the average wage paths of all unemployed workers (both endogenously and exogenously displaced) after shutting down each separate channel in the model. To generate the path given for skills alone, I set $\pi = 1$ which implies that all workers are of the high type and all displacements are exogenous. To consider ability alone, I set the probability of losing skills to zero $\tau = 0$. These graphs illustrate two results of the calibration. First is that reputation has a large

\textsuperscript{22}Time period 1990-2007 Davis et al. (2011)
negative effect on wages than skill loss. Second is that skills are acquired more quickly than employer learning about the worker's ability. These effects combine to imply wage losses of skill alone are smaller and more quickly recovered than wage losses from ability alone. The important take away is that a worker's productivity is rebuild more quickly than their wages recover.

8 Policy: No Discrimination by Employment Status

To assess the effect of employer information on aggregate unemployment, I now embed the model in a standard matching framework of equilibrium unemployment. In this environment, lack of commitment and asymmetric information amplify the inefficiency caused by search frictions. Asymmetric information allows plants to earn a profit from poaching an employed worker and lack of commitment makes worker quits possible. Since workers produce the same output in any match, this poaching does not increase aggregate output and uses up resources in the form of vacancy posting costs. Additionally, poaching lowers the expected profit of hiring a worker because the hiring plant only collects profits from the match until the worker quits. Since low ability workers are fired before they quit, poaching does not affect the expected loss from hiring a low ability worker. The net effect is that poaching lowers the expected profit from hiring an unemployed worker. This increases unemployment because market clearing in the search market requires entry of vacancies until the marginal entry earns zero profits. When expected profits are lower, there is less entry and higher unemployment.\footnote{Note the inflows to unemployment are rigid from the exogenous destruction rate and the firing rate being independent of the value of unemployment to a worker.}

Segmented search markets by employment status lead to greater inefficiency than a single search market. Two channels lead to this effect. First, the expected value of hiring an unemployed worker is less than hiring an employed worker. This lowers the probability an unemployed worker meets a vacant plant when markets are segmented compared to when there is a single search market. Second, the same mechanism increases the probability an
employed worker meets a vacant plant when markets are segmented. This increases the probability a worker is poached when there are segmented markets than when there is a single search market. The second channel lowers the value of hiring an unemployed worker and makes the first channel even worse.

Legislation ending discrimination by employment status forces a single search market and increases efficiency. In April 2011, the state of New Jersey enacted this legislation. On July 12, 2011, the Fair Employment Opportunity Act of 2011 was introduced in the US House of Representatives with the same stipulation. At the time of this writing, legislation with this aim is at the subcommittee review stage in both the House and the Senate. I now use the model to predict the effect of this policy.

I consider a standard matching function \( m(u, v) = \chi v^n u^{1-n} \). Define market tightness as the ratio of vacancies to unemployed workers \( \theta = \frac{v}{u} \). Then the probability a vacant plant meets a worker is \( q(\theta) = \chi \frac{m(u,v)}{v} = \chi \theta^{n-1} \) and that a worker meets a plant is \( r(\theta) = \chi \frac{m(u,v)}{u} = \chi \theta^n \). Posting a vacancy requires a cost of \( \kappa \). An equilibrium is defined as above with the additional general equilibrium zero expected profit condition for vacancy posting in all markets. I will index search markets by \( l \in \{e, u\} \) for the segmented case.

\[
E[\Omega_l] = \frac{1}{1 - \rho(1 - q(\theta))} (\Omega(p', 0) - \kappa) = 0
\]

Quantitatively, all of the parameters of the model are unchanged by the introduction of search frictions except for one: the speed of outside employer learning \((\nu)\). The outflow rate of workers from unemployment is the same for both types and, in a stationary equilibrium, must equal the inflow rate. The result is that the ratio of low ability to high ability workers in unemployment is the same as the ratio of workers entering unemployment and then the same as the environment with instantaneous reemployment. However, I will adjust parameters to fit a period length of 8 weeks, or 15 periods per year. This is chosen such that one period equals the median duration of unemployment from the Bureau of Labor Statistics for 1976-1998 of 7.8 weeks.

For a first analysis, I consider the same matching function for the employed and unemployed search markets. The elasticity, \( \eta \) is chosen in the standard range \( 0.25 - 0.55 \). I set \( \theta = 1 \) for a pre-policy benchmark. This choice will generate a lower bound of the policy
effect. I calibrate the constant $\chi$ to match the median monthly unemployment rate reported by the Bureau of Labor Statistics for 1976-1998: 6.6%. Stationarity given an inflow rate of 7.1% per annum requires $1 - r(\theta) = 0.08$. I calculate $\kappa$ such that the equilibrium conditions of zero-profits for new vacancies hold.

Outside employer learning, $\nu$, must be adjusted into two components, learning and arrival rate of outside poachers as affected by the matching function. Then $\nu = \hat{\nu} + r(\theta^c)$, with $r(\theta^c)$ a function of parameters given by the previous calibration equations.

<table>
<thead>
<tr>
<th>Non-discrimination by Employment Status Policy Predictions</th>
<th>Pre-Policy</th>
<th>Post-Policy</th>
<th>Percentage Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate Output</td>
<td>1.362</td>
<td>1.377</td>
<td>+1.0790%</td>
</tr>
<tr>
<td>Unemployment Level</td>
<td>6.6%</td>
<td>6.0%</td>
<td>-8.9693%</td>
</tr>
<tr>
<td>Lifetime Earnings- High Type</td>
<td>697.05</td>
<td>696.90</td>
<td>-0.022%</td>
</tr>
<tr>
<td>Lifetime Earnings- Low Type</td>
<td>403.27</td>
<td>405.83</td>
<td>+0.6356%</td>
</tr>
</tbody>
</table>

The table above presents model predictions of the policy effect for a matching function with elasticity (0.4). Figures on page 53 of the Appendix illustrate the effect of the policy for re-calibrations of the model under a range of specified matching function elasticities. As shown, the policy raises aggregate output by about 1% and lowers unemployment by about 10%. The policy raises the present discounted value of lifetime earnings for low types by 0.6356%. This increase comes from shorter duration of unemployment. The policy lowers present discounted lifetime earnings of high types by -0.022%. This decrease comes from slower wage growth through lower job mobility. Overall, the policy increases aggregate welfare as measured in output. This analysis abstracts from non-monetary costs of unemployment which would also be reduced following the decline in unemployment and unemployment duration.

---

24 All other equilibrium objects depend only on the cumulative effect which has been taken into consideration by the original calibration.

25 These statistics are calculated for a newborn. Effects at different stages in life will be greater or smaller
9 Conclusion

I have provided a unified general equilibrium framework to consider the impact of information available about workers on wages and employment transitions. First, the model is useful to understand how lay-offs, direct quits, and wages are jointly determined by the same underlying process. Second, I show this mechanism can replicate the diverging wage and employment patterns of unemployed and never unemployed workers. In particular, it can account for the large and persistent wage losses of unemployed workers that other models vastly under-predict. I use the model to measure the contribution of information and skill loss to wage losses of displaced workers. I find estimate that skill loss affects about a quarter of all displaced workers, but that skills are reacquired much more quickly than has been estimated in prior literature. Instead, employer learning about a worker’s ability drives the slow growth of workers’ wages after unemployment and over the life-cycle. In summary, both lost reputation and lost skills are equally as important for the magnitude of displaced workers wage losses, but reputation is the dominant reason for slow wage recovery.

The model I have constructed is a tool to measure the effect of policy. In this paper, I have considered policy designed to lower unemployment. I predict the provision of the Fair Employment Opportunity Act of 2011 to end hiring discrimination by employment status would lower unemployment by about 10% and raise aggregate output by about 1%. The model could also be used to predict the effects of other policies, such as firing restrictions and retraining programs, on unemployment and wage inequality. Lastly, analysis of changes in the speed of employer learning may help understand evolving trends of worker turnover, life-cycle wage profiles, and inequality.
A Appendix

A.1 Data

<table>
<thead>
<tr>
<th>Characteristic at time of change</th>
<th>Statistic</th>
<th>Firm Shut Down</th>
<th>Individual Lay-off</th>
<th>E-to-E Quit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tenure</td>
<td>Mean</td>
<td>6.51</td>
<td>4.33</td>
<td>2.03</td>
</tr>
<tr>
<td></td>
<td>stdev</td>
<td>8.01</td>
<td>5.57</td>
<td>4.17</td>
</tr>
<tr>
<td>Experience</td>
<td>Mean</td>
<td>15.24</td>
<td>13.55</td>
<td>12.4</td>
</tr>
<tr>
<td></td>
<td>stdev</td>
<td>11.59</td>
<td>13.52</td>
<td>10.95</td>
</tr>
<tr>
<td>Hourly Earnings</td>
<td>Mean</td>
<td>9.31</td>
<td>9.24</td>
<td>8.90</td>
</tr>
<tr>
<td></td>
<td>stdev</td>
<td>0.96</td>
<td>0.91</td>
<td>0.72</td>
</tr>
<tr>
<td>Persons</td>
<td>number</td>
<td>416</td>
<td>1611</td>
<td>2958</td>
</tr>
</tbody>
</table>

A.2 Proofs

**Proposition A.1.** The worker's policy is independent of her type i: \( Q_h(p, w, w^*) = Q_l(p, w, w^*) \) for all \((w, w^*) \in \bar{\Psi} \times \bar{\Psi}\)

**Proof.** Case 1) The quit policy a worker with an incumbent that was an informed poacher of state \( p \) is independent of type: \( Q_h(p, w^*(p), w^*(p_b)) = Q_l(p, w^*(p), w^*(p_b)) \)

The worker strategy that maximized the wage of the informed poacher specifies \( Q_i(p, w^*(p), w^*(p)) = 0 \), given defined off path beliefs. The non-trivial case is the quit policy for wage offer from uninformed poacher \( w^*(p_b) \). Let \( \bar{V}^H = V(\delta_h, w^*(\delta_h)) \), the value of a high type poached by an informed poacher from a learned incumbent. By Bertrand competition it is fixed and
independent of all choices. The choices are:

High Type: \[
\max\{[\rho(1 - \mu)]^{-1}[(1 - \mu)w^*(p_c) + \mu \frac{(1 - \nu)w^*(p_e) + \nu \rho \tilde{V}^H}{\rho(1 - \nu)}],
\]

\[
[\rho(1 - \mu)]^{-1}[(1 - \mu)w^*(p) + \mu \frac{(1 - \nu)w^*(p) + \nu \rho \tilde{V}^H}{\rho(1 - \nu)}]\}
\]

Low Type: \[
\max\{[\rho(1 - \mu)]^{-1}[(1 - \mu)w^*(p_c)],
\]

\[
[\rho(1 - \mu)]^{-1}[(1 - \mu)w^*(p)]\}
\]

Both take the highest wage: \(Q_h(w^*(p), w^*(p_c)) = Q_l(w^*(p), w^*(p_c)) = 1\) if \(w^*(p_c) > w^*(p)\).

Case 2) The quit policy of two workers never poached with incumbent state \(p\) is independent of type: \(Q_h(p, w(p), w^*(p_c)) = Q_l(p, w(p), w^*(p_c))\)

If \(p^* < p_c\), the workers choose both; the non-wage continuation value is the same as staying with the incumbent and they earn a higher wage. If: \(p_c < p\). Denote

\[
c_1 = [1 - \rho(1 - \mu)]^{-1}, \quad c_2 = [1 - \rho(1 - \nu)(1 - \mu)]^{-1}
\]

The choice is:

High Type: \[
\max\{c_1[(1 - \mu)w^*(p_c) + \mu \frac{(1 - \nu)w^*(p_e) + \nu \rho \tilde{V}^H}{\rho(1 - \nu)}],
\]

\[
c_1 c_2 \nu[(1 - \mu)w^*(p) + \mu \frac{(1 - \nu)w^*(p) + \nu \rho \tilde{V}^H}{\rho(1 - \nu)}]
\]

\[
c_2 (1 - \nu)[(1 - \mu)w + \mu \frac{(1 - \nu)w + \nu \rho \tilde{V}^H}{\rho(1 - \nu)}]\}
\]

Low Type: \[
\max\{c_1(1 - \mu)w^*(p_c),
\]

\[
c_1 c_2 \nu(1 - \mu)w^*(p)
\]

\[
c_2 (1 - \nu)(1 - \mu)w\}
\]

Then \(Q_l(p, \min w, w^*(p_c)) = 1 \iff c_1(1 - \mu)w^*(p_c) > c_1 c_2 \nu(1 - \mu)w^*(p) + c_2(1 - \nu)(1 - \mu)w\)

\[
\iff c_1[(1 - \mu)w^*(p_c) + \mu \frac{(1 - \nu)w^*(p_c) + \nu \rho \tilde{V}^H}{\rho(1 - \nu)}] > c_1 c_2 \nu[(1 - \mu)w^*(p) + \mu \frac{(1 - \nu)w^*(p) + \nu \rho \tilde{V}^H}{\rho(1 - \nu)}] +
\]

\[
c_2 (1 - \nu)((1 - \mu)w + \mu \frac{(1 - \nu)w + \nu \rho \tilde{V}^H}{\rho(1 - \nu)})\]

\(\iff Q_h(p, w, w^*(p_c)) = 1\). Further, the cost of deviation is greater for the high type. Thus, this equilibrium satisfied the control costs refinement. \(\Box\)
Proposition A.2. \( w'(p, w) < w^*(p) \) for all \( w < w^*(p) \) and any \( p \in \mathcal{P} \).

Proof. Let \( p, w^*(p) \) be given. Suppose \( w'(p, w) = w^*(p) = \arg \max_{w'} \Omega(p, w) \). By Bertrand competition \( \Omega(p, w^*(p)) = 0 \). Consider alternative policy \( w' = w, \Omega(p, w) \geq py_h + (1-p)y_t - w > 0 \), the case where \( Q(p, w, w^*) = 1 \) for all \( w^* \). Then \( \Omega(p, w) > 0 = \Omega(p, w^*(p)) \). \( \rightarrow \leftarrow \quad \square \)

Corollary A.3. If \( w^*(p) < w(p) \) then \( w'(p, w) = w \)

Proof. Suppose \( w^*(p) < w(p) \). By Bertrand competition \( \Omega(p, w^*(p)) = 0 \). Then \( w^*(p) < w(p) \) implies \( \Omega(p, w(p)) < 0 \). By monotonicity of \( \Omega \), it must be \( w < w^*(p) \). Then \( \min \omega = \arg \max_{w \geq w < w^*(p)} \Omega(p, w) = py_h + (1-p)y_t - w \). Therefore \( w' = w \) \( \square \)

Corollary A.4. If \( w^*(p) < w^*(p_e) \) then \( w'(p, w) = w \)

Proof. Suppose \( w^*(p) < w(p) \). By Bertrand competition \( \Omega(p, w^*(p)) = 0 \). Then \( w^*(p) < w(p) \) implies \( \Omega(p, w(p)) < 0 \). By monotonicity of \( \Omega \), it must be \( w < w^*(p) \). Then \( \min \omega = \arg \max_{w \geq w < w^*(p)} \Omega(p, w) = py_h + (1-p)y_t - w \). Therefore \( w' = w \) \( \square \)

Proposition A.5. \( Q(p, w^*(p), w^*(p)) = 0 \).

Proof. Poacher wage \( w^*(p) \) is defined by \( \Omega(p, w^*(p)) = 0 \). Consider two worker policies: \( Q(p, w^*(p), w^*(p)) = \phi \) and \( Q(p, w^*(p), w^*(p)) = \phi', \ 0 \leq \phi \leq \phi' \leq 1 \). By definition of \( \Omega \), \( w^*(p) > w^*(p) \), which implies \( V^\prime(p, w^*(p), q) > V^\prime(p, w^*(p), q) \) for all \( q \). Therefore \( \phi' > 0 \) does not maximize the worker's problem. The unique strategy that does is \( Q(p, w^*(p), w^*(p)) = 0 \). \( \square \)
A.3 Comparative Statics

Figure 2: Faster employer learning lowers the quality of the unemployed which lowers the offer uninformed poachers (a worker's outside option). Therefore, starting wages are declining in speed of learning.
Figure 3: Faster employer learning raises a worker’s value and a new entrants’ poaching wage. The lower quality of the unemployed pool dominates to lower displaced workers’ poaching wage.
Figure 4: If informed poachers are more likely, the worker will accept a lower wage to wait for the informed poacher.
Figure 5: The incumbent’s monopsony over information declines when poachers are more likely to be informed.
A.4 Calibration Equations

First, I solve the non-wage equations to get parameters $\pi, \tau, \gamma$.

(1) Percent of Workers Displaced Yearly:

$$\lambda_{0}(\mu + d) + d(1 - \lambda_{0})$$

I will maintain that in the data, an establishment closure is always equivalent to an exogenous shut down in the model, but a lay-off may be either exogenous or endogenous. 26 The Business Employment Dynamics published by the Bureau of Labor statistics separates separations into those where establishment closed and those where it did not. Over 2000-2007 average gross job loss equaled 7.1% and job loss from closings equaled 1.2%. This implies the percent of workers reporting lay-off that should be mapped or exogenous is $\hat{d} = (d - .012)/(.071 - .012)$. I will use this in the next equation.

(2) Proportion of all workers fired and then fired again within two years after reemployment:

$$\hat{d}[(1 - \gamma)\mu(\lambda_{0} + \tau \lambda_{1})] + (1 - \hat{d})[(1 - \gamma)\mu \lambda_{10}]$$

(3) Proportion of all workers displaced exogenously and then fired within two years after reemployment:

$$\hat{d}(1 - \gamma)\mu(\lambda_{0} + \tau \lambda_{1})$$

The distribution $\lambda$ is as above:

$$\lambda_{0} = \frac{(1 - \pi)(\delta + (1 - \delta)d\tau)}{\delta + (1 - \delta)d\tau + \gamma}$$

$$\lambda_{11} = 1 - \pi - \lambda_{10}$$

---

26You can think of this as her position became obsolete and she was let go regardless of her ability. There are many more plausible scenarios and since respondents to the PSID do not report if they were selected based on their ability, it is best to estimate this.
Algebra reduces this system to the following functions of \(d\) and data moments:

\[
\hat{d} = \frac{(EU\text{FlowRate} - d)}{(EU\text{FlowRate} - SD\text{Rate})}
\]

\[
\gamma = 1 - \frac{F2(EU\text{FlowRate} - \hat{d}) - \hat{d}SDF}{(1 - \hat{d})(EU\text{FlowRate} - d)}
\]

\[
0 = \tau^2(\gamma d) + \tau[\gamma^2 - \frac{\mu^2SDF[d(\delta + d + \gamma) + \gamma d]}{(1 + d)\gamma(EU\text{FlowRate} - d)^2}] - \frac{\mu^2SDF[\delta(\delta + d + \gamma)]}{(1 + d)\gamma(EU\text{FlowRate} - d)^2}, \quad \tau \in [0, 1]
\]

\[
1 - \pi = \frac{(d + \delta + \gamma)SDF\mu(\delta + d\tau)}{\delta(1 + d)\gamma(EU\text{FlowRate} - d)}
\]

Where the data moments are

<table>
<thead>
<tr>
<th>Moment</th>
<th>Variable Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Aggregate Employment to Unemployment Yearly Flow Rate</td>
<td>(EU\text{FlowRate})</td>
<td>7.1%</td>
</tr>
<tr>
<td>Aggregate EU Yearly Flow Rate from Establishment Closure</td>
<td>(SD\text{Rate})</td>
<td>1.2%</td>
</tr>
<tr>
<td>Percentage of fired workers fired in year after reemployment</td>
<td>(F2)</td>
<td>38%</td>
</tr>
<tr>
<td>Percentage of Shut Down workers fired in year after reemployment</td>
<td>(SDF)</td>
<td>16.7%</td>
</tr>
</tbody>
</table>

Next, I solve for the parameters involving wage equations. I restrict my sample to never displaced workers for easy expressions of the following: standard deviation of log(wages) at 25 years of experience and average log(wage) change after 25 years of experience. I can simulate worker flows in the model since the actual timing and distribution of the flows is independent of the actual wages. Then I can calculate the distribution of the above wage changes to calculate the desired model moments.
A.5 Model Fit & Comparisons

![Wage Paths of Displaced Workers](image)

**Figure 6:** Model simulated wage changes of displaced workers by ability (low vs high) and reason (fired=endogenous, shut down = exogenous).
Figure 7: Model simulated wage changes of displaced workers for skills only (all workers high ability: \(p_{i}=1\)), ability only (probability of losing skills = 0), and both (see calibration).
Model Simulated Age-Wage Profiles

Figure 8: Model simulated age wage profiles for workers of each ability. Low type workers have slower wage growth because they have more unstable employment relationships.
Figure 9: Regression analysis on model generated data compared to PSID sample. Controls = quadratic in age and individual fixed effects.
A.6 Policy Results- Sensitivity to Matching Elasticity

Figure 10: Effect of Non-discrimination by employment status policy.
References


