Monetary and Fiscal Policy Interactions
A Model with Time-varying Policy Rule Coefficients

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“Fiscal policy is more than just ‘G minus T’ and an associated ‘multiplier.’ There are potentially dozens of instruments, each with their own dynamic effects that depend on the state of the economy and other policies”

Olivier Blanchard, Economic Counsellor and Director of the Research Department, IMF Research conference on “Macro and Growth Policies in the Wake of the Crisis” (March 8, 2011)
Fiscal Policy Modeling

“A central bank that is seriously considering the full range of impacts of its actions, and the actions of fiscal authorities, on future output growth and inflation should be using a quantitative model that treats explicitly and realistically the potential impacts of fiscal policy on the price level.”

Sims (2011)
This paper... 

... incorporates interactions between monetary and fiscal policies in a conventional new Keynesian DSGE setup to model the behavior of inflation and output. To achieve this goal, the paper

- Specifies interactions between monetary and fiscal policies through correlated time-varying monetary and fiscal policy rule coefficients.
- Estimates the model to uncover the evolution of monetary and fiscal policies, and their interactions, consistent with inflation and output over the last 6 decades.
Monetary and Fiscal Policy Interactions in Equilibrium

A simple model

Two equations characterize the equilibrium of most models of money and inflation:

\[ M_t V = P_t Y \]
\[ \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j} \]
Monetary and Fiscal Policy Interactions in Equilibrium

Regime M

\[ M_t V = P_t Y \]  \hspace{1cm} (1)

\[ \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j} \]  \hspace{1cm} (2)

- The CB sets \( M_t \), thus determines \( P_t \) through (1).
- The FA adjusts expected \( \{s_t\} \) such that (2) holds for \( P_t \).
Monetary and Fiscal Policy Interactions in Equilibrium

Regime F

\[ M_t V = P_t Y \]  \hspace{1cm} (1)

\[ \frac{B_{t-1}}{P_t} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j s_{t+j} \]  \hspace{1cm} (2)

- The FA sets expected \( \{s_t\} \), thus determines \( P_t \) through (2).
- The CB adjusts \( M_t \) such that (1) holds for \( P_t \).
Monetary and Fiscal Policy Interactions in Equilibrium
an endowment DSGE model

\[ R_t = \alpha \pi_t + u_t^R \]
\[ \tau_t = \gamma b_{t-1} + u_t^\tau \]

\[ \gamma < \frac{b}{\tau} \left( \frac{1}{\beta} - 1 \right) \quad \gamma > \frac{b}{\tau} \left( \frac{1}{\beta} - 1 \right) \]

\[ \alpha < 1 \quad \text{PM/AF} \quad \text{PM/PF} \]
\[ \alpha > 1 \quad \text{AM/AF} \quad \text{AM/PF} \]

**Determinacy:** AM/PF, PM/AF

**Indeterminacy:** PM/PF

**No solution:** AM/AF
DSGE Models with Time-Varying Coefficients

in MP rule only

- **Eo (2009):** Estimates a NK model with regime switching using the solution proposed by Davig and Leeper (2006).
- **Davig and Doh (2009):** Estimate a Markov-Switching NNK model using the solution proposed by Davig and Leeper (2006).
- **Fernandez-Villaverde et al. (2010):** Estimate a NK model with smoothly time-varying structural parameters using a second-order approximation for the solution.
- **Bianchi (2011):** Estimates a Markov-Switching NK model using the solution proposed by Farmer, Waggoner and Zha (2010).
DSGE Models with Time-Varying Coefficients in MP rule only
DSGE Models with Time-Varying Coefficients
in MP & FP rules

- **Davig and Leeper (2006):** Estimate the parameters of Markov-switching monetary and fiscal policy rules and embed them in a Markov-switching NK model that is solved with the monotone map method.

![](Fig. 1. Estimated U.S. monetary and fiscal regimes.)
Outline

1 Model setup
2 Model solution
3 Model estimation
4 Results
Model Setup

Specification

- Representative household derives utility from consumption relative to a habit stock, leisure and real money balances.
- There is a final good producer behaving competitively.
- There is a continuum of intermediate good producers behaving as monopolistic competitors.
- There are price adjustment costs à la Rottemberg (1982).
- The CB conducts MP with a Taylor interest rate rule.
- The FA conducts FP with lump-sum taxes that adjust to nominal debt and output.
Policy Rules

Monetary Policy Rule

- Interest rate feedback rule

\[
R_t = R_{t-1}^{\rho_R} \bar{R}_t^{(1-\rho_R)} \exp(\varepsilon_t^R),
\]

\[
\rho_R \in (0, 1), \quad \varepsilon_t^R \sim \text{iid} \mathcal{N}(0, \sigma_R^2)
\]

- \(\bar{R}_t\): target short-term nominal interest rate

\[
\bar{R}_t = R \left( \frac{\Pi_t}{\bar{\Pi}} \right)^{\alpha_t^\pi} \left( \frac{Y_t}{Y_t^*} \right)^{\alpha_t^y}
\]

\(R\): steady state nominal interest rate
\(\bar{\Pi}\): target gross inflation rate
\(\Pi_t = P_t/P_{t-1}\): gross inflation rate in period \(t\)
\(Y_t\): output in period \(t\)
\(Y_t^*\): output in the absence of price rigidities in period \(t\)
Policy Rules

Fiscal Policy Rule

- Ratio of taxes (net of transfers) to output, $\tau_t = T_t/(P_tY_t)$, feedback rule
  
  $\tau_t = \tau_{t-1}^{\rho_\tau} \bar{\tau}_t^{1-\rho_\tau} \exp(\varepsilon_t)$

  $\rho_\tau \in (0, 1), \varepsilon_t \sim \text{iid}\mathcal{N}(0, \sigma^2_\tau)$

- $\bar{\tau}_t$: target level of taxes to output

  $\bar{\tau}_t = \tau \left( \frac{b_{t-1}}{\bar{b}} \right)^{\gamma^b_t} \left( \frac{Y_t}{Y^*_t} \right)^{\gamma^y_t}$

  $b_t = B_t/(P_tY_t)$: debt-to-output ratio
  $\tau$: steady state tax-to-output ratio
  $\bar{b}$: target debt-to-output ratio
$\varrho(z_t) = \varrho_0 + \frac{\varrho_1}{1 + \exp(-\varrho_2(z_t - \varrho_3))}$
Model Setup

Policy Rules

Policy Rules Coefficients

\[ \bar{R}_t = R \left( \frac{\Pi_t}{\Pi} \right)^{\alpha_t^\pi} \left( \frac{Y_t}{Y^*_t} \right)^{\alpha_t^y} \Rightarrow \]

\[ \alpha^\pi(z^m_t) = \alpha^\pi_0 + \frac{\alpha^\pi_1}{1 + \exp(-\alpha^\pi_2 z^m_t)} \]

\[ \alpha^y(z^m_t) = \alpha^y_0 + \frac{\alpha^y_1}{1 + \exp(-\alpha^y_2 z^m_t)} \]

\[ \bar{\tau}_t = \tau \left( \frac{b_t-1}{b} \right)^{\gamma_t^b} \left( \frac{Y_t}{Y^*_t} \right)^{\gamma_t^y} \Rightarrow \]

\[ \gamma^b(z^f_t) = \gamma^b_0 + \frac{\gamma^b_1}{1 + \exp(-\gamma^b_2 z^f_t)} \]

\[ \gamma^y(z^f_t) = \gamma^y_0 + \frac{\gamma^y_1}{1 + \exp(-\gamma^y_2 z^f_t)} \]
Policy Rules

Policy interactions (1)

\[
\begin{align*}
    z_t^m &= \rho_{zm} z_{t-1}^m + \xi_t^m \\
    z_t^f &= \rho_{zf} z_{t-1}^f + \xi_t^f,
\end{align*}
\]

\(\rho_{zm} \in (0, 1)\)
\(\rho_{zf} \in (0, 1)\)

\[
\begin{bmatrix}
    \xi_t^m \\
    \xi_t^f
\end{bmatrix}
\sim \text{iid } \mathcal{N}
\begin{pmatrix}
    0_{2 \times 1} \\
    \kappa
\end{pmatrix}
\begin{bmatrix}
    1 & \kappa
\end{bmatrix}
\]

\(\kappa \neq 0 \Rightarrow \text{correlation between the coefficients of the policy rules}\)

\(\Rightarrow \text{(interactions between policy rules)}\)
Policy Rules

Policy interactions (2)

\[ z_t^m = +\infty, z_t^f = +\infty: \text{AM/PF} \]

\[ \alpha^\pi(\cdot) = \alpha_0^\pi + \alpha_1^\pi \quad \alpha^\gamma(\cdot) = \alpha_0^\gamma + \alpha_1^\gamma \]

\[ \gamma^b(\cdot) = \gamma_0^b + \gamma_1^b \quad \gamma^\gamma(\cdot) = \gamma_0^\gamma + \gamma_1^\gamma \]
Policy Rules

Policy interactions (2)

\[ z^m_t = -\infty, z^f_t = -\infty : \text{PM/AF} \]
Policy Rules

Policy interactions (2)

\[ z^m_t = -\infty, \ z^f_t = +\infty : \text{PM/PF} \]

\[ a^\pi(\cdot) = a^\pi_0 \quad a^y(\cdot) = a^y_0 \]

\[ \gamma^b(\cdot) = \gamma^b_0 + \gamma^b_1 \quad \gamma^y(\cdot) = \gamma^y_0 + \gamma^y_1 \]
Policy Rules

Policy interactions (2)

\[ z^m_t = +\infty, \quad z^f_t = -\infty: \text{ AM/AF} \]
Log-linearized Model

\[ 0 = A(z_t)k_t + B(z_t^f)k_{t-1} + C(z_t)\omega_t + Du_t \]
\[ 0 = Gk_t + JE_t\omega_{t+1} + K\omega_t + Mu_t \]
\[ u_{t+1} = Nu_t + \varepsilon_{t+1} \]

- \( \omega_t = [\hat{y}_t, \hat{\pi}_t]' \)
- \( k_t = [\hat{v}_t, \hat{b}_t, \hat{R}_t, \hat{\tau}_t, \Delta \hat{y}_t, y_{t-1}, \hat{y}_t^*]' \)
- \( u_t = [\hat{g}_t, \hat{\theta}_t, \hat{\nu}_t, \varepsilon_t^R, \varepsilon_t^\tau]' \)
- \( \varepsilon_t = [\varepsilon_t^g, \varepsilon_t^\theta, \varepsilon_t^\nu, \varepsilon_t^R, \varepsilon_t^\tau]' \)
- \( z_t = [z_t^m, z_t^f]' \)
Solution (1)

\[
\begin{align*}
    k_t &= P(z_t) k_{t-1} + Q(z_t) u_t \\
    \omega_t &= R(z_t) k_{t-1} + S(z_t) u_t \\
    F_{ij}(z_t) &= F_{ij}^0 + \frac{F_{ij}^1 m^1}{1 + F_{ij}^{1m} e^{-F_{ij}^{2m} z_t^m}} + \frac{F_{ij}^1 f}{1 + F_{ij}^{1f} e^{-F_{ij}^{2f} z_t^f}} + \\
    &+ \frac{F_{ij}^{m_f}}{1 + F_{ij}^{1m} e^{-F_{ij}^{2m} z_t^m} + F_{ij}^{1f} e^{-F_{ij}^{2f} z_t^f} + F_{ij}^{1m} F_{ij}^{1f} (1 - F_{ij}^{4}) e^{-F_{ij}^{2m} z_t^m - F_{ij}^{2f} z_t^f}}
\end{align*}
\]
Solution (2)

\[ F(z_t) = F_0 + \frac{F_{1m}}{1 + F_{3m}e^{-F_{2m}z_t^m}} + \frac{F_{1f}}{1 + F_{3f}e^{-F_{2f}z_t^f}} + \]

\[ + \frac{F_{mf}}{1 + F_{3m}e^{-F_{2m}z_t^m} + F_{3f}e^{-F_{2f}z_t^f} + F_{3m}F_{3f}(1 - F_4)e^{-F_{2m}z_t^m - F_{2f}z_t^f}} \]
Estimation of Policy Rules

Nonlinear State Space

\[ R_t = \rho_R R_{t-1} + (1 - \rho_R) (\alpha^\pi(z^m_t)\pi_t + \alpha^y(z^m_t)y_t) + \varepsilon^R_t \]
\[ \tau_t = \rho_\tau \tau_{t-1} + (1 - \rho_\tau) \left( \gamma^b(z^f_t)b_{t-1} + \gamma^y(z^f_t)y_t \right) + \varepsilon^\tau_t \]
\[ z^m_t = \rho_{z^m} z^m_{t-1} + \xi^m_t \]
\[ z^f_t = \rho_{z^f} z^f_{t-1} + \xi^f_t \]
Data and Parameter Assumptions

- **Period:** 1949:1 - 2010:3
- **Interest rate:** Quarterly average of the 3-month T-Bill rate in the secondary market.
- **Output gap:** Log difference between real GDP and the Congressional Budget Office’s measure of potential real GDP.
- **Inflation:** Percentage change over the last four quarters of the GDP price deflator.
- **Taxes net of transfers to output:** Seasonally adjusted quarterly current receipts of the federal government net of current transfer payments as a fraction of output.
- **Lagged debt to output:** Average debt-output ratio over the previous four quarters of the market value of gross marketable federal debt held by the public.
# Parameter Estimates

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Prior</th>
<th>Posterior</th>
<th>Convergence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
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<tr>
<td>$\alpha_2^\pi$</td>
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<tr>
<td>$\rho_R$</td>
<td>Beta</td>
<td>0.85</td>
<td>0.1</td>
</tr>
<tr>
<td>$\rho_\tau$</td>
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<tr>
<td>$\rho_{zm}$</td>
<td>Beta</td>
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<td>0.15</td>
</tr>
<tr>
<td>$\rho_{zf}$</td>
<td>Beta</td>
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<tr>
<td>$z^f_0$</td>
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</table>

p-values correspond to Geweke (1991) convergence test.

30,000 draws
Fixed Model Parameters

Table: Parameters

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<tr>
<th>$\beta$</th>
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<th>$\theta$</th>
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<th>$\Pi$</th>
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<td>$g$</td>
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<td>$\rho_\theta$</td>
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<td>0.95</td>
<td>0.97</td>
<td>0.65</td>
<td>0.0771</td>
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</table>

$\tau = 1 - \frac{1}{g} - \left(1 - \frac{1}{\Pi \delta}\right) \frac{1}{\nu} + \left(\frac{1}{\beta} - 1\right)b$
Impulse-Response Functions

Contractionary Monetary Policy
Impulse-Response Functions

Contractionary Fiscal Policy
Full Estimation

An Endowment Flexible-Price Economy (1)

\[ \mathbb{E}_t \hat{\pi}_{t+1} = \hat{R}_t \]
\[ \hat{v}_t = \hat{R}_t \]
\[ \hat{b}_t = -\frac{\tau}{b} \hat{\tau}_t + \frac{\hat{v}_t}{vb} - \frac{\hat{v}_{t-1}}{v\Pi b} - \left( \frac{1}{v\Pi b} + \frac{1}{\beta} \right) \hat{\pi}_t + \frac{1}{\beta} (\hat{R}_{t-1} + \hat{b}_{t-1}) \]
\[ \hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) \alpha(z^m_t) \hat{\pi}_t + \varepsilon^R_t \]
\[ \hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + (1 - \rho_\tau) \gamma(z^f_t) \hat{b}_{t-1} + \varepsilon^\tau_t \]
Full Estimation

An Endowment Flexible-Price Economy (2)

\[0 = A k_t + B (z_t^f) k_{t-1} + C (z_t^m) \omega_t + D \varepsilon_t\]

\[0 = E_t \omega_{t+1} + G k_t\]

\[\omega_t = \hat{\pi}_t\]

\[k_t = [\hat{v}_t, \hat{b}_t, \hat{R}_t, \hat{\tau}_t]'\]

\[\varepsilon_t = [\varepsilon_t^R, \varepsilon_t^\tau]'\]

Solution:

\[k_t = P(z_t) k_{t-1} + Q(z_t) \varepsilon_t\]

\[\omega_t = R(z_t) k_{t-1} + S(z_t) \varepsilon_t\]
Full Estimation
An Endowment Flexible-Price Economy (3)

\[
y_t = \Lambda x_t \\
x_t = \Upsilon(z_t)x_{t-1} + \Phi(z_t)\varepsilon_t \\
z_t = \Gamma z_{t-1} + \xi_t
\]

- \(y_t = [\text{INT}_t, \text{TAX}_t]'\)
- \(x_t = [k_t, \omega_t]'\)
- \(\xi_t = [\xi^m_t, \xi^f_t]'\)
## Parameter Estimates

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<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>90% Conf. Set</td>
<td>p-value</td>
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<td>( \alpha_0 )</td>
<td>Gamma 0.5 0.25</td>
<td>0.6969 [0.4455, 0.9556]</td>
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<tr>
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<td>0.7613 [0.1005, 1.3807]</td>
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<tr>
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<td>0.5197 [0.1104, 1.2172]</td>
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<td>( \gamma_0 )</td>
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<td>( \rho_\tau )</td>
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<tr>
<td>( 100\sigma_R )</td>
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<tr>
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<tr>
<td>( \kappa )</td>
<td>TruncatedNormal 0.25 0.25</td>
<td>0.0000 [0.000, 0.000]</td>
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<td>( z_m^0 )</td>
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<td>0.0084 [-1.6659, 1.6665]</td>
<td>0.79</td>
</tr>
</tbody>
</table>

p-values correspond to Geweke (1991) convergence test.

20,000 draws

- AM/AF: 30.50%
- PM/AF: 19.81%
- AM/PF: 82.33%
- PM/PF: 14.61%
- PM/PF: 16.47%
- AM/PF: 0.4%
- AM/AF: 0.8%
Prevailing Regimes

![Graph showing the timeline of prevailing regimes from 1950 to 2010 with different colored lines indicating different regimes: PM/AF, PM/PF, AM/AF, and AM/AF with a line labeled $\pi_t$.]