Information Asymmetry and Equilibrium Monitoring in Education

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Abstract

We develop a theoretical and computational model of equilibrium school choice and achievement that embeds information asymmetries in the production of education. School effort is unobservable to households and the policymaker, leading to moral hazard. Although household monitoring of schools can mitigate this problem, some households may free-ride on the monitoring of others. Moral hazard affects both public and private schools, yet public schools are subject to an additional distortion because their funding is fixed. Using our calibrated model we simulate two policies aimed at raising achievement: public monitoring of public schools and private school vouchers. In our simulations, public monitoring raises public school effort but can crowd out private monitoring, thus undermining its own effectiveness. Vouchers may not be able to help households in the low-income, low-ability segment because of these households’ high monitoring costs; furthermore, vouchers may hurt the public school by causing the loss of high-ability households who provide monitoring. These results indicate that in large-scale settings, no single tool may suffice, but a combination of them may succeed. Our results also indicate that setting the policy parameters for public schools at the appropriate level may mitigate the effort and funding distortions. This level is closer to that preferred by the majority of households than to that preferred by the public schools themselves. Moreover, the evidence suggests that the current values of these parameters are quite close to those preferred by the public schools rather than the households.

Keywords: Information asymmetry, monitoring, equilibrium, school effort, accountability, vouchers
JEL codes: H42, H44, I21, I28

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1. Introduction

An educated population is a fundamental ingredient for a well-functioning democracy and a crucial driver of growth in the modern economy. Thus, education has both private returns that accrue to the individual, and public returns that accrue to society. For this reason, the policymaker often has a minimum goal of basic academic proficiency for every student in the economy. Many students, however, do not meet this goal, even after substantial public spending in the marketplace for education.

In this paper we focus on an information-based explanation for the lack of academic achievement, namely the information asymmetries among the policymaker, households, and schools. In particular, school effort (from a school’s administration or teachers) is not fully observable to parents or policymakers, and this creates a potential moral hazard problem as the school has an incentive to under-provide effort. Parental involvement in schools can function as a monitoring device that mitigates moral hazard. However, since monitoring is a public good, it may itself introduce an additional distortion if households free-ride on the monitoring of others. This externality can in turn lead to the under-provision of monitoring relative to socially optimal levels.

Concerns about underachievement among K-12 students in the United States has prompted an aggressive federal response over the last few years, starting with No Child Left Behind in 2002 and continuing with programs such as the recent Race to the Top. Moreover, individual states and districts have implemented similar programs. These programs contain incentives linking outcomes for schools and teachers to student achievement. These incentives would not be needed if school and teacher effort were perfectly observable, in which case the policymaker would establish the socially desired effort and would reimburse the cost of effort to schools and teachers. In contrast, the policymaker is currently relying on indirect measures of school and teacher effort such as student achievement. Thus, the very existence of these programs points to the information asymmetry between schools, parents, and the policymaker as one possible explanation for underachievement.

Furthering the concerns about underachievement is the disappointing performance of U.S. students in international assessments relative to other countries in the OECD (see, for instance, Fleishman et al 2010). At the same time, international evidence shows that student
achievement is higher in countries with more competition and public assistance for school choice, external and/or exit exams, and greater parental interest in education (Wößmann 2007). Hence, policies such as these, which mitigate moral hazard through greater monitoring and competition might motivate greater school effort and raise the standing of the U.S. in international assessments.

Although information asymmetry is at the root of other economic problems facing policymakers and market participants, such as the regulation of natural monopolies and banking (Laffont and Tirole 1983, Freixas and Rochet 2008), to our knowledge we are the first to model the moral hazard associated with school effort in an equilibrium setting of education provision. Moreover, large-scale policies that address underachievement, such as public school accountability and private school vouchers, have been the subject of extensive empirical research (e.g., Figlio and Ladd 2008 and Zimmer and Bettinuer 2008). Nonetheless, researchers have rarely modeled the information asymmetries underlying many such policies.

Thus, we develop a theoretical equilibrium model of household school and monitoring choice in the presence of information asymmetry. We calibrate the computational version of the model to 2000 data from the United States and use it to conduct policy analysis. In our model, the production of educational achievement requires three inputs: school effort, household learning effort, and peer quality. School effort is not observable to households or the policymaker in public or private schools and is hence under-provided (as in Holmstrom 1979). This hurts achievement directly and also indirectly by making other inputs less productive.

Faced with moral hazard, households have the option to exert costly effort to monitor the school; monitoring mitigates but does not eliminate moral hazard. However, households vary in their costs and benefits from monitoring. In addition, they have incentives to free-ride on the monitoring of others. The underlying agency or hidden action problem, along with the concomitant free-riding associated with household monitoring, is one friction in our model.

The second friction is the limited competition faced by public schools. Public schools are chosen by households who do not have the ability or the willingness to pay for private schools. Yet while free entry and exit discipline private schools and eliminate their rents, the policymaker restricts entry and exit of public schools. Furthermore, the policymaker sets a fixed per-pupil funding for public schools. This allows the school to potentially reap a rent, which unnecessarily raises the fiscal cost of public education.
The effect of these frictions can in turn be aggravated by the endogenous sorting of households across schools. For instance, some high-income, high-ability households may choose private schools and monitor them because they anticipate a low public school effort due to free-riding in monitoring at public schools. In other words, household sorting across schools may prevent the public school from attracting the high-income, high-ability households that would improve peer quality and provide monitoring. Hence, the public school may provide less effort than many private schools while earning a rent, the size of which grows as school effort falls.

Using our calibrated model, we have computed the equilibrium in a variety of scenarios. Our analysis highlights the distortions introduced by these frictions in the equilibrium behavior of households and schools, and how these distortions vary along with policy parameters. For instance, moral hazard leads to lower effort and achievement in all schools but especially in public school, where limited competition and fixed funding aggravate the problem.

One policy parameter in our model is the effort standard for the public school, from which the school may deviate when choosing its actual effort. This deviation measures the distortion due to moral hazard. A higher standard forces the school to exert more effort and allows it to attract high-income, high-ability students; these, in turn, monitor the school and force it to exert further effort. The effort standard implied by our data is quite close to that which would maximize public school’s profit given the current per-student funding. This contrasts with the standard that the majority of households would prefer –one that maximizes achievement and minimizes public school rent- perhaps indicating public schools’ influence on setting the current standard. Moreover, given that entry of public schools is limited and their funding is fixed, eliminating the effort distortion depends critically on the choice of the effort standard.

In the policy arena, discussions about addressing underachievement are often centered around two alternative approaches. The first is regulation-based mechanisms, which attach consequences to academic outcomes. The second is market-based mechanisms, which provide households with additional school choices. Schools are disciplined by the regulator in the first case and by the market in the second. The frictions highlighted in our model suggest a role for policies that increase public school monitoring, give households the means to attend higher-effort schools, or provide more competition to public schools. Hence, we have simulated two policies: public monitoring of public schools (a regulation-based mechanism, inspired by actual public school accountability), and private school vouchers (a market-based mechanism).
According to our simulations, while public monitoring can raise school effort and attract high-ability households, it can also crowd out private monitoring on the part of lower-income, lower-ability households. Private school vouchers increase achievement among voucher users, although high monitoring costs among the lowest income and ability households prevents them from using the voucher. These households are further hurt by the loss of high-ability households to private schools, as this lowers peer quality and monitoring rate in public schools.

Thus, both regulation- and market-based mechanisms may have unintended effects that limit their effectiveness, and neither is a complete solution. The reason is that the information asymmetry is embedded in an equilibrium setting. Attempts to solve the information problem may affect household and school choices in equilibrium, hence modifying the original distortion or creating new ones. While these unintended consequences may render a single policy tool less effective and even counterproductive, they suggest a role for thoughtful combinations of tools.

A theme of our findings is that the choice of policy parameters for public schools affects achievement and welfare. Although we do not model the determination of these parameters, we have analyzed household preferences over them. Households that prefer private schools would select parameters that minimize public school attendance and hence fiscal burden, whereas households that prefer public schools would select either a higher effort standard than the current one, or lower funding. These findings suggest that public schools may currently have more influence over policy parameters than the households themselves. Furthermore, support for public schools is sensitive to the demographic balance in the population.

Our work contributes to two distinct literatures. First, we contribute to the education literature by modeling moral hazard regarding school effort and household monitoring as an equilibrium response. Whereas equilibrium models in education have been used to analyze policies such as private school vouchers and public school finance reform (e.g. Epple and Romano 1998, Ferreyra 2007, Nechyba 1999), it is novel to use them to study school effort and household learning and monitoring efforts. Although some researchers have modeled student learning effort in the production of achievement (Blankenau and Camera 2009, De Fraja and Landeras 2006, MacLeod and Urquiola 2009, Albornoz et al 2010), and others have studied parental involvement from an empirical perspective (McMillan 2000, Walsh 2010), to our knowledge we are the first to model household monitoring effort.
Other researchers have explored information-driven distortions in education provision. McMillan (2005) studies a rent-seeking public school. He assumes that school effort is observable but not contractible either by the state or the household (i.e., although the policymaker can observe school effort, he cannot attach any consequences to its under-provision). Thus, information asymmetries are absent in his model, as is monitoring. Chakrabarti (2008) uses a similar model to study different voucher systems. In Acemoglu et al (2008), schools provide a multi-dimensional effort (“good” effort which increases students’ human capital and “bad” effort which increases outsiders’ perception of the amount of good effort exerted), which leads schools to under provide good effort. Neither household monitoring, which could presumably mitigate the misallocation of school efforts, nor household sorting are present in this model. Ahn (2009) and Hansen (2010) study teacher effort in light of accountability or career incentives respectively, yet do not model household monitoring or school choice either.

The second literature to which we contribute is the agency literature, by embedding a micro-based bilateral agency model into an equilibrium framework for education. Well-known agency problems (such as Holmstrom 1979 and Sappington 1983) have been studied in bilateral, partial equilibrium settings. Monitoring and its associated free-riding have been studied in professional partnerships (Miller 1997, Huddart and Liang 2003, 2005) where monitoring must be performed by the very partners whose productive effort is subject to moral hazard. Moreover, our work relates to the literature on incentives problems in government procurements (Laffont and Tirole 1993), which focuses on optimal contracts between contractors that have an information advantage and the government. In contrast, we do not model the determination of funding or policy mandates, nor do we search for the optimal contract between the public school and the policymaker (which is likely to be extremely complex). Rather, we focus on policies that may not be optimal but are commonly discussed to address underachievement.

The remainder of this paper is organized as follows: section 2 presents the model, section 3 describes the computational version of the model, section 4 analyzes the equilibrium of the model, section 5 discusses policy simulations, and section 6 concludes.

2. The Model
We embed information asymmetry about school effort into an equilibrium model of school choice. There are three categories of players in our model: households, public and private schools, and the policymaker. In this one-period model, the timeline of events is as follows:

1. The policymaker exogenously establishes two policy parameters for public schools -- funding level and school effort standard (more on the effort standard below);
2. Households choose a school for their children. Conditional on this choice, they choose their monitoring effort, learning effort and consumption;
3. Schools choose school efforts.

To capture the agency conflict between households (and the policymaker) and schools, we distinguish between the school’s effort standard (or promised effort) and the actual school effort. The effort standard is the school input promised by the school to the parents, whereas actual effort is the effort delivered by the school. Our assumption of unobservable school effort is motivated by the fact that in reality, neither the parents nor the policymaker are present in the school to observe school effort all the time. As a consequence, the school can deliver less effort than it promises even though households anticipate this behavior, in accordance with the basic insight from the moral hazard literature (e.g., Holmstrom 1979). Although parental monitoring can mitigate moral hazard, it cannot completely eliminate it. Hence, the effort distortion persists.

In addition to the effort distortion, moral hazard introduces a cost distortion. In our model, the household’s cost to procure a given school effort from a private school is higher than the actual cost of effort to the school. It includes a tuition premium - an agency cost- which cannot be competed away. Although they charge zero tuition, public schools reap a similar rent, further enhanced by their fixed funding.

2.1. Households

The economy is populated by a finite number of households. Each household has one child who must go to school. Households are heterogeneous in income, $y$, and ability, $\mu$. There

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2 In reality, education occurs over an extended period of time, only at the end of which achievement may be measured perfectly. We equate this period to our model’s one period. Hence, our model does not capture the interim actions that may actually take place over that period. For instance, households may use sequential enrollment choices to discipline schools and mitigate moral hazard. This interesting extension is beyond the scope of this paper.

3 For instance, a school may claim to offer a highly stimulating learning environment, a novel and rigorous curricula, highly qualified teachers, profound intellectual engagement, individualized instruction, state-of-the-art teaching methods and facilities, etc. Most of these claims are not observable if parents are not at the school all the time. Even if they were there all the time, they might lack the pedagogical knowledge necessary to verify some of these claims.
are a finite number of income types, $I$, and a finite number of ability types, $J$. Thus, there are $H = I \times J$ household types, each representing an (income, ability) combination. In the computational version of the model we assume one household per type, in which case the total number of households in the economy equals $H$. The model can be extended to more than one household per type without loss of generality.

Household preferences are described by the following utility function:

$$U = c^\beta s - \rho_a \frac{a^2}{2\mu} - \rho_m \frac{m^2}{2\mu}$$

(1)

where $c$ is numeraire consumption, $s$ is school achievement, $a$ is household learning effort, $m$ is household monitoring effort (the roles of $a$ and $m$ in the production of achievement are described below), and $\rho_m, \rho_a, \beta > 0$. Households incur disutility from exerting school and monitoring efforts, and this disutility represents the cost of effort. Importantly, the marginal cost of effort varies among households, and is higher for lower-ability households. In the computational version of the model, we assume that monitoring is a binary choice: $m \in \{0,1\}$.

Households maximize utility (1) subject to the following budget constraint:

$$(1 - t)y = c + T$$

(2)

where $t$ is income tax rate and $T$ is private school tuition ($T=0$ in public school). Although the household procures consumption and school effort in the market, learning and monitoring efforts are privately produced at a direct utility cost. They cannot be outsourced and are thus “off-budget,” as we assume that education requires some inputs that only the agent can provide.

The production of child achievement, $s$, is as follows:

$$s = e^{\eta_1 q^{\eta_2} a}$$

(3)

where $e$ is school effort, $q$ is the school’s peer quality (defined as the school’s average ability), and $\eta_1, \eta_2 > 0$.

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4 We normalize the coefficient on school achievement in the utility function to one in order to facilitate the calculations. Changing this coefficient simply amounts to re-scaling the other parameters.

5 De Fraja and Landeras (2006) model the cost of effort in a similar fashion. It could be argued that the household might outsource its learning or monitoring effort to a third party. Since this party’s effort would be subject to moral hazard and require monitoring, we simplify by assuming that learning and monitoring efforts cannot be outsourced.

6 Due to lack of data, very few studies (if any) estimate achievement functions incorporating all these inputs. Stinebrickner and Stinebrickner (2008) document that extra study time has large, positive effects on achievement. In a structural framework, De Fraja et al (2008) find that both school and household effort affect achievement positively, higher ability children exert higher effort, and parent effort is positively correlated with household income and SES. Several of these findings are echoed in Bonesrønning (2004), Datar and Mason (2008), and Houtenville and Conway (2008). Moreover, Bonesrønning (2004) and Datar and Mason (2008) find evidence that
2.2. Private schools

School effort can be provided by private or public schools that incur a production cost equal to \( Ae^\lambda \) (with \( A > 0 \) and \( \lambda > 1 \)), which can be viewed as the teaching and administrative cost of running a school. We model private schools as competitive firms that can select their students and charge them a uniform tuition (e.g., Ferreyra 2007, McMillan 2005, Nechyba 1999). Since there are no fixed costs to providing school effort, and households have incentives to join a school with households of equal or higher ability and income according to (3), in equilibrium each private school is attended by households of a single type. Thus, the peer quality \( q \) for a private school attended by a household of ability \( \mu \) is equal to \( \mu \).

Consider a household of a particular type, and the competitive market for schools that cater to it. If there were no moral hazard, each school in this market would offer its profit-maximizing effort given the market tuition. Since competition would drive profits to zero, in equilibrium the tuition would be equal to the cost of effort, and the household would choose a school whose effort maximizes household utility given the effort’s production cost.

However, in the presence of moral hazard the household cannot observe the effort provided by the schools. Instead, the schools catering to this particular household type promise an effort equal to \( e^{pri} \), which denotes the effort standard or promised effort. Taking as given the market tuition \( T \) for this \( e^{pri} \), each school in this market chooses its actual effort \( e \), which may be different from \( e^{pri} \). Such deviation is costly to the school. A school’s profit is given by:

\[
\pi^{pri} = T(e^{pri}) - Ae^\lambda - \frac{\alpha m}{2} (e^{pri} - e)^2
\]

where \( \alpha > 0 \) and \( m \) denotes the monitoring effort exerted by the household.\(^7\) Equation (4) captures the tradeoff facing the school when choosing actual effort for a given effort standard.

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\(^7\) As explained before, households choose monitoring \( m \) before schools choose effort \( e \). This timing is critical for monitoring to play a role in the model. If the school chooses its effort first, the household has no incentive to choose positive monitoring because it will not affect the level of school effort, which has already been chosen. If school effort and household monitoring are chosen simultaneously, there is no pure strategy equilibrium with positive monitoring. To see why, consider a strategy pair of positive effort and positive monitoring. Given that the household monitors, the school does not benefit from deviating to zero effort. Yet given positive school effort, the household can only benefit by deviating to zero monitoring. Hence, only our timing choice yields a role for monitoring.
The school has an incentive to choose an actual effort below its promise in order to lower its production cost, hence exploiting the information asymmetry to its advantage. However, doing so imposes the cost of deviating from the promise, a cost which rises with greater household monitoring. In other words, household monitoring disciplines the school’s incentive to under provide effort. The quadratic cost for the effort deviation implies that small deviations from \( e^{pri} \) are costless to the school, leading to a non-zero (downward) effort distortion in equilibrium.\(^8\) Thus, household monitoring raises school effort, but not up to the promised level.

The optimal effort chosen by the school is:

\[
e^* = f(e^{pri}) \in \text{argmax} \left[ T(e^{pri}) - A e^\lambda - \frac{a m}{2} (e^{pri} - e)^2 \right] (5)
\]

which, in turn, yields \( e^{pri} = f^1(e) \). In other words, for each standard \( e^{pri} \) set by the school (or requested by the consumers) there is a corresponding effort level \( e \) effectively provided by the school. Competition drives each private school’s equilibrium profit to zero: \( \pi^{pri} = 0 \). Thus, after substituting the first-order condition from equation (5), the equilibrium tuition \( T^* \) for each \( e^{pri} \) (and the corresponding actual \( e \)) becomes:

\[
T^*(e^{pri}) = A[f(e^{pri})]^\lambda + \frac{a m}{2} (e^{pri} - f(e^{pri}))^2 = A(e^*)^\lambda + \frac{A^2 \lambda^2}{2 a m} (e^*)^2(\lambda - 1) (6)
\]

As a result, the equilibrium tuition covers the production cost of effort as well as an agency cost, even though the private school market is competitive. This is consistent with the standard intuition of agency theory (Holmstrom 1979, Grossman and Hart 1986) by which the price of any given effort is higher than its actual production cost in the presence of moral hazard.

In our model, moral hazard is introduced in a reduced-form fashion. As (4) shows, household monitoring directly enters into school’s objective function, thus reducing effort distortion.\(^9\) In a fully specified agency model, monitoring produces output measures which are informative about the agent’s effort and are used explicitly in an optimal pay-for-performance contract. This contract, in turn, affects input provision. Our model captures the key role of

\(^8\) This cost may include monetary losses such as fines for failing to follow regulations or loss of future income due to damaged reputation, and non-monetary losses such as psychological aversion to breaking promises. The key to our cost assumptions is that the marginal cost is zero at zero deviation, and is increasing in monitoring.

\(^9\) Input-based monitoring also has empirical support. Duflo et al (2009) provide evidence that parental involvement in school management (a proxy for parental monitoring) improves teacher effort. Banerjee and Duflo (2006) conclude that if the goal of an intervention is to boost the provision of an input (such as teacher attendance), then the incentives must target the input. Input-based monitoring, in turn, does boost output (Duflo et al 2007). In the developed world, the Ofsted Reports in England exemplify a public monitoring of schools that evaluates inputs and output. For an example for an example of the evaluator’s attention to inputs such as teaching quality and practices, see http://www.ofsted.gov.uk/oxedu_reports/download/%28id%29/116266/%28as%29/134943_345339.pdf.
monitoring but does not capture insights such as the use of performance metrics in contract design. In Appendix A we present an extension that incorporates output measures omitted from this reduced form model.

2.3. Public school

In addition to private schools, a public school exists in this economy. All households are eligible to attend public school. Public school effort is also subject to moral hazard problem. The public school effort standard, \( e^{pub} \), is set exogenously by the policymaker. As in McMillan (2005) and (Chakrabarti 2008), the policymaker also sets per-student funding, \( X \). The public school chooses effort \( e \) to maximize its profit:

\[
\pi^{pub} = (X - Ae^k)N - \frac{\alpha M}{2} (e^{pub} - e)^2
\]

where \( N \) is total enrollment and \( M \) is the sum of monitoring efforts from households attending the public school. In contrast with private schools, monitoring at the public school is a public good. As long as some households monitor, it may be optimal for another household to free-ride on others’ effort and not provide its own. This potential free-riding leads to the under provision of household monitoring in public school and adds a distortion relative to private schools.

The limited competition faced by public schools adds yet another friction. Free entry of private schools ensures that each private school’s tuition \( T \) is tied to the school’s effort standard \( e^{pri} \) (and, indirectly, to actual school effort \( e \)), and leads to zero private school profit. In contrast, public school funding \( X \) is determined exogenously and is not necessarily tied to the public school effort standard \( e^{pub} \) or actual effort \( e \). Hence, for a given \( e^{pub} \) a sufficiently high \( X \) may lead to public school rents \( (\pi^{pub} > 0) \) in equilibrium. These rents could be eliminated by the entry of another public school that receives the same funding but sets a higher effort standard (and hence offers higher effort). Since there is no free entry of public schools, those rents persist. The only competition faced by public school comes from private schools, and is limited because not every household is able or willing to attend private schools.

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10 Assuming one public school is equivalent to assuming one public school district with multiple public schools and open enrollment. The Chicago Public Schools district, with its extensive public school choice program (Cullen et al 2006), is a good example of this setting. A multi-district setting would be an interesting extension yet outside the scope of this paper, as it would necessitate the treatment of housing markets and voting over policy parameters.
2.4. Model summary and equilibrium

To summarize the model, consider the problem facing a household with income $y$ and ability $\mu$. The household must choose a school (public or private) as well as its consumption, learning effort and monitoring effort while taking the tax rate, private schools’ tuition and promised efforts, public school funding and promised effort, public school peer quality, and other households’ school and monitoring effort choices as given. Importantly, the household correctly anticipates the level of effort that each school will optimally provide in response to a given monitoring level, and makes its choices accordingly.

If attending a private school, the household chooses $c, e^{pri}, a, m$ to maximize utility (1) subject to the budget constraint (2), the anticipation of school effort (5), the school effort pricing function (6), and the achievement function (3). Note that in the absence of household monitoring the school would choose to provide $e=0$, leading to zero household achievement and utility. Thus, a household that attends a private school always chooses a positive level of monitoring (equal to 1 in the case of binary monitoring).\(^{11}\) Given that we assume one household per type and hence per private school, monitoring in private schools is a private good.\(^{12}\) In other words, we focus on the monitoring gap between public and private schools rather than the absolute level of monitoring in each school.

If attending the public school, the household chooses $c, a, m$ to maximize utility (1) subject to the budget constraint (2), the achievement function (3), and the anticipation of the school’s effort choice: $e \in \arg\max \pi^{pub} = (X - Ae^\lambda)N - \frac{aM}{2} (e^{pub} - e)^2$. This constraint reflects the household’s recognition that its choice for $m$ affects $M$ and hence the public school’s optimal $e$. The household chooses the school (public or private) that maximizes its utility (in case of a tie, it chooses the public school).

\(^{11}\) The assumption of binary monitoring greatly simplifies computations and facilitates the interpretation of results. While the marginal cost of monitoring is inversely related to household ability and is straightforward to calculate, the marginal benefit is directly related to ability and income yet is highly nonlinear. Since school and household learning effort are complements, monitoring benefits a household because it raises achievement directly (by raising school effort) and indirectly (by leading to an increase in learning effort). The increase in learning effort is higher for higher ability households, for whom the cost of learning effort is lower. Hence, they benefit more from monitoring. Since achievement is a normal good, the demands for school and student effort are higher among higher-income households. Hence, they also benefit more from monitoring.

\(^{12}\) If there is more than one household per type, a given private school may contain multiple households (of the same type). In this case, a household’s monitoring effort may depend on other households’ monitoring, and free-riding may arise in private schools. Thus, some households may leave their current private school and start a new, smaller private school to mitigate free-riding. Without fixed costs, this leads to one household per private school in the limit.
An equilibrium consists of a set of household and school choices satisfying the following: 
(a) household rationality: conditional on other households’ choices, no household has an 
incentive to deviate from its own optimal choices; (b) school rationality: each school chooses 
school effort to maximize its own profit, and the school is open only if its profits are non-
negative; (c) market clearing: each household attends one and only one school and total tax 
revenue equals total public school funding: $t \sum_i y_i = XN$.

Since the model does not have a closed-form solution, we rely on computations to study 
and apply the model. We have established conditions sufficient to determine whether an 
allocation is an equilibrium and have developed an algorithm that relies on them in order to 
compute the equilibrium.\textsuperscript{13} In Appendix B, we prove that if an equilibrium exists, it satisfies 
standard stratification properties (across income and ability).

\textbf{2.5. Policymaker and policy alternatives}

In our policy analysis, we consider two alternative programs: public monitoring of public 
schools and private school vouchers. Public monitoring is inspired by public school 
accountability programs that provide incentives for public schools to raise achievement while 
attaching consequences to school outcomes. We operationalize this alternative by introducing a 
public monitoring effort, $M_0$, which changes the public school profit function as follows:

$$\pi^{\text{pub}} = (X - Ae^a)N - \frac{\alpha (M + M_0)}{2} (e^{\text{pub}} - e)^2$$

\text{(8)}

Since we assume that public monitoring is costly, the state budget constraint changes to

$$t \sum_i y_i = XN + \kappa M_0$$

\text{(9)}

where $\kappa$ is the unit cost of public monitoring.\textsuperscript{14}

\textsuperscript{13} We conjecture that our equilibrium is unique, and this conjecture is supported by the fact that we have never 
found multiple equilibria in our computational application although our algorithm is capable of finding all the 
equilibria for a given parameter point. See Appendix C for a description of the algorithm.

\textsuperscript{14} Modeling $M_0$ as an additive term is more general than it might seem. The key is that public monitoring changes 
the marginal benefit of private monitoring – which, in turn, differs among households. Hence, aggregate private 
monitoring $M$ can go up or down in response to $M_0$ depending on its level before the introduction of public 
monitoring, the cost of public monitoring, and other parameters of the problem. Thus, our formulation allows for 
public monitoring to behave either as a substitute or a complement of aggregate private monitoring, as illustrated by 
the simulations presented in Section 5. Empirically, parents who face barriers to private monitoring seem to view 
public and private monitoring as substitutes (Hassrick and Schneider 2009 and Figlio and Kenny 2009).
Vouchers are tuition subsidies for private schools. We consider universal and income-targeted vouchers. We assume that they are funded by the state through income taxes, and that the voucher dollar amount can depend on household income as denoted by the voucher function \( v(y) \). With universal vouchers, \( v(y) = v \) for all \( y \). A household may supplement the voucher with additional payments toward tuition but cannot retain the difference when the tuition is lower than the voucher level. Hence, the tuition is never set below the voucher level. Under vouchers, the household attending a private school faces the following budget constraint:

\[
(1 - t)y = c + \max (T - v(y), 0)
\]  

To summarize, in this section we have described our theoretical model and stated properties of the equilibrium. Since the model does not have a closed-form solution, we compute the equilibrium numerically. Thus, the next section provides some computational details.

3. Computational Version of the Model

To analyze the model, we must first choose adequate values for the parameter vector \( \theta = (\beta, \eta_1, \eta_2, \lambda, A, e^{pub}, \alpha, \rho_a, \rho_m) \). Hence, we calibrate our model to 2000 data for K-12 education in the United States. The calibration strategy is to compute the equilibrium at alternative parameter points in order to find the point that minimizes a well-defined distance between the predicted equilibrium and the observed data. Since the equilibrium does not have a closed-form solution, we solve it for a tractable representation of the economy using a numerical algorithm. In this section we describe this representation, our calibration strategy, and the fit of our model to the data. Appendix C provides further details on these matters.

Our computational representation of the economy includes five income types, whose incomes equal the 10th, 30th, 50th, 70th and 90th percentile of the 2000 national income distribution for households with children in grades K through 12. This distribution comes from the 2000 School District Data Book. All dollar amounts are expressed in dollars of 2000. We include five ability levels, equal to the 10th, 30th, 50th, 70th and 90th percentile of the IQ distribution (a normal distribution with a mean of 100 and a standard deviation of 15). We
assume that income and ability are independently distributed.\textsuperscript{15} Our setting of income and ability types yields twenty-five household types and one household per type, yet our results are robust to the inclusion of more types. We set per-pupil spending in public schools, $X$, equal to the observed national average of $7,000. Since our computations assume binary monitoring effort $m$, total monitoring in public school $M$ equals the number of public school households that monitor.

To calibrate the model, we choose the parameter point that best matches the observed values of nine variables of interest. Appendix C offers further details on the construction of these variables. The first is fraction of households with children in private schools (equal to 0.16 according to the 2000 School District Data Book). The second is average income for households with children in private schools (equal to $82,800 according to the 2000 School District Data Book). The third is average private school tuition (equal to $5,000 according to US Department of Education 2002). The fourth is proportional difference between average public and private school teacher salaries (equal to 0.44 according to the 1999-2000 Schools and Staffing Survey). When we compute predicted salaries we work with teacher compensation rather than salaries, as we assume that public school profits are re-distributed among teachers.

The fifth variable is difference between average effort among private v. public school teachers (equal to zero standard deviations according to the 1999-2000 Schools and Staffing Survey). In the absence of perfect measures for school effort, we use number of hours worked by teachers. The sixth variable is difference in average achievement between private and public school students (equal to 0.45 standard deviations according to the 2000 National Assessment for Educational Progress). The seventh variable is difference in average ability between private and public school students (equal to 0.76 standard deviations according to Epple et al (2004)’s calculations based on the National Education Longitudinal Survey).

The eighth variable is difference in average student effort between private and public schools (equal to 0.5 standard deviations according to the 2004 Digest of Education Statistics). In the absence of good empirical measures for student effort, we use average number of hours spent doing homework per week among high school students in 2004. The ninth variable is the fraction of households who monitor in public schools. Normalizing the private school average to 1 since

\textsuperscript{15} We have experimented with positive, low correlations between income and ability in light of recent evidence from the UK that suggests that this correlation might be on the order of 0.2 (Gregg et al, 2007). Since such low correlations do not alter our results, we have retained a zero correlation for computational simplicity.
our model views private schools as a benchmark of full parental monitoring, we arrive at a public school monitoring rate of 0.76 based on the 1999 Digest of Education Statistics.

We use $y_j$ to denote the observed values of the variables we are matching, $j=1,...,9$. As we search over the parameter space, for each value of the parameter point $\theta$ we compute the equilibrium, from which we extract the predicted values $\hat{y}_j(\theta)$, $j=1,...,9$, for the variables listed above. Thus, we choose the value for $\theta$ that minimizes the following distance between the data and the model’s predictions:

$$L(\theta) = \sum_{j=1}^{9} w_j(y_j - \hat{y}_j(\theta))^2$$

(11)

where the distance for variable $j$ is weighed by a factor which is inversely related to the precision in the variable’s measurement. In particular, the first four variables are measured with greater precision than the others in the sense that their empirical counterparts are more adequate, and for the fifth through eighth variable we are likely to observe a lower bound for the actual construct of interest. Note that the non-linearity of the model and the coarseness of our household representation prevent us from matching the data exactly.

Table 1 shows the parameter values delivered by our calibration. Table 2 lists the observed and predicted values for the matched variables. As expected given their measurement, the first four variables are matched better than the following four, and the fifth through eight variables are over predicted. Overall, however, we are encouraged by the model’s fit to the data. Now we turn to the analysis of the model’s equilibrium.16

4. Analyzing the equilibrium

In this section we first analyze the equilibrium of our model computed at our calibrated parameter values (henceforth called “benchmark” or “baseline” equilibrium). A central contribution of our paper is modeling informational frictions in education. To highlight their role, we analyze the equilibrium that would prevail if there were perfect observability in the

16 In equilibrium models, changes in one parameter trigger changes in several endogenous variables. However, it is still possible to identify computationally the first-order effects of parameter changes on the variables matched in the calibration. See Appendix C for further details.
economy. In this case, school effort would be observable (with $e^{pub}=e$ and $e^{pri}=e$ in public and private schools, respectively) and monitoring would be unnecessary. Since the difference between promised and actual effort captures the agency conflict in our model, in this section we also investigate the equilibrium response to changes in the public school effort standard.

4.1. Benchmark Equilibrium

Column 1 of Table 3 displays the benchmark equilibrium, in which 84 percent of households attend public school. As the top panel of Figure 3 shows, high-ability, high-income households attend private schools. All private school households monitor, yet the lowest-income households in public school do not monitor. On average across schools, the model predicts greater monitoring among higher income households. This prediction has empirical support (see US Department of Education 2002, Table 25), which lends further validity to our model since the positive correlation between monitoring and income was not used in the calibration.

Our model enables us to study how public and private schools spend their revenue. The public school spends 59 percent to cover its total cost (44 percent pays for teacher effort, and 15 percent pays for agency costs), and captures the remaining 41 percent as a rent. Private schools, in contrast, enjoy zero profits and spend almost 80 percent of their revenue in teacher effort, hence using their funding more efficiently. Note that the persistence of the effort distortion gives rise, in equilibrium, to agency costs both in public and private schools.

Column 2 of Table 3 describes the equilibrium under perfect observability. As standard agency intuition would indicate, school effort under perfect observability is higher (by approximately 30 percent) than in the presence of moral hazard. Under perfect observability monitoring is not necessary, which makes private schools more attractive since private schooling requires monitoring. However, the higher effort exerted by the public school under perfect observability makes public schools more attractive. The net outcome of these forces is that only the highest-income, highest-ability type remains in private school. Greater school effort leads to greater household learning effort, and both to higher achievement in the economy.

From a policy perspective, the superiority of the perfect observability outcomes means that bringing the economy closer to those outcomes may enhance welfare and achievement. Since parental monitoring mitigates moral hazard, policies that lower private monitoring costs
can help. For instance, many argue that the greatest contribution from No Child Left Behind has been creating and disseminating vast amounts of information on public schools, which have presumably helped parents’ monitoring.\footnote{We have conducted simulations for $\rho_m=0$, i.e. zero private monitoring costs. Relative to the benchmark equilibrium, public school effort, attendance and achievement are higher. Details are available upon request.} Moreover, the literature documents that parents are indeed responsive to that kind of information (Figlio and Lucas 2004), including low-income parents (Hastings et al 2008). In the next section we take private monitoring costs as given and study additional policies that seek to mitigate moral hazard.

4.2. The Role of Public School Effort Standard in Equilibrium Choices

While we assume that public school funding is fixed, perhaps for political or institutional reasons, the public school effort standard ($e^{pub}$) is likely more flexible. For instance, a district’s board of education may promise more engaging teaching while keeping funding constant. From (7) it follows that a change in the public school effort standard alters public school effort and hence household choices. Thus, the top panel of Figure 1 depicts the equilibrium value of public school profit for alternative values of $e^{pub}$ (recall that our calibrated $e^{pub}$ is equal to 0.66). The bottom panel depicts the equilibrium actual public school effort, fraction of households attending public schools, and public school monitoring rate as a function of $e^{pub}$.

For low values of effort standard, profits are positive but flat. Only 20 percent of households attend public school. These low-income households do not benefit from monitoring, so they choose to attend public school and not monitor. This, in turn, allows the school to deliver zero effort and enjoy a rent of $7,000 per student.

As the value of the effort standard rises, profits first rise and then fall. However, public school attendance, effort and monitoring rate rise steadily. The non-monotonic path of profits is explained as follows. For a given actual effort, a higher effort standard raises the cost of effort deviation for the school and motivates greater school effort. This, in turn, attracts more students into the school – students from higher-income, higher-ability households. As these households join the school they also engage in monitoring, which in turn forces greater school effort. While higher attendance increases revenue and rents for a given school effort, higher effort and monitoring reduces rents and profits. As long as the first effect dominates, profit is increasing in
effort standard; the reverse happens when the second effect dominates, eventually leading to negative profits (a situation not displayed in Figure 1, as it is not an equilibrium).

The top panel of Figure 1 also suggests that in an environment where funding is not flexible, the policymaker can in principle eliminate or at least minimize rents by choosing the appropriate effort standard, equal to 0.85 in our setting. As we will see later, an effort standard of 0.85 is also what households would choose if they were able to do so. This standard is certainly higher than that implied by the data, equal to 0.66.

Similarly, the top panel of Figure 1 suggests that if the public school were able to choose its optimal effort standard, it would maximize profit at $e_{\text{pub}}=0.60$. The school’s optimal standard is quite close to that implied by the data, indicating that public schools might play a strong role in the actual determination of effort standards. The profit-maximizing effort standard is high enough to attract a sufficiently large number of students, yet low enough to attract relatively few high-ability, high-income households who monitor the school.

The choice of effort standard has clear achievement and distributional impacts. Column 2 of Table 4 shows the equilibrium when the effort standard minimizes public school rent. For comparison, column 1 shows the benchmark equilibrium (which is very close to the equilibrium under the public school’s optimal effort standard). As column 2 shows, in the zero-profit equilibrium, public school effort, attendance and monitoring are higher than in the baseline, and public schools use their revenues more efficiently. Greater school effort raises student effort and hence achievement. Relative to other households, achievement gains accrue at a higher rate to the low-income, low-ability segment which constitutes the captive demand for the public school. Although taxes are higher to pay for more public school students, welfare is also higher.\(^\text{18}\)

4.3. **Effort Standard, Fixed Funding and Observability**

As Figure 2 shows, the behavior of public school profit with respect to the effort standard is qualitatively the same with or without perfect observability (recall that under perfect observability, the effort standard equals the actual effort). This is because of the tradeoff induced by the effort standard – on the one hand, a higher effort standard raises enrollment and hence

\(^{18}\) Costrell (1994) models the determination of output-based (graduation) standards and analyzes their achievement and distributional consequences. His setting is different from ours in that he does not model school effort, information asymmetries or equilibrium effects.
total revenue; on the other hand, it raises effort cost. Moral hazard preserves this pattern but raises school profit relative to perfect observability when the standard is very low or very high. When the standard is very low, the extra profit comes from the public school’s captive audience of low-income, low-ability households that do not monitor (under perfect observability, these households would attend private schools because of their greater effort and would not need to incur costly monitoring). When the standard is very high, the extra profit comes from the lower effort cost. For intermediate values of the effort standard, public school profit is lower than under perfect observability mostly because of the cost of the effort deviation.

In an environment with perfect observability and inflexible funding, consider the two zero-profit cases presented in Table 4. First, a low effort standard (less than or equal to 0.15; see column 4), which induces zero public school attendance and thus eliminates public school profit. Second, a high effort standard (equal to 0.7; see column 5), which induces high public school attendance yet minimum profits. If one views column 4 of Table 4 as the first best because neither informational nor funding distortions exist, then it follows that it can be attained without public schools. Not surprisingly, of all the scenarios presented in this paper, this one commands the highest aggregate welfare. Yet relative to the other perfect observability scenarios (columns 3 and 5), in the first best the lowest-income, lowest-ability households obtain lower achievement and utility given the absence of public schools with mandated effort standards - left to their own devices, these households choose a lower school effort (and achievement) than the policymaker would choose for them. Thus, effort standards play a role when the policymaker has a minimum-proficiency goal for every student even if effort is observable.

If, in contrast, one views the perfect observability, minimum profit equilibrium with high $e^{pub}$ (column 5 of Table 4) as the first best, then the baseline average effort is about 30 percent lower than in the first best (compare the actual effort of 0.51 in column 1 to 0.70 in column 5). Assuming that funding is fixed, most of this effort distortion would disappear if effort were observable (compare the actual effort of 0.51 in column 1 to 0.67 in column 3). This might not seem useful from a policy perspective because observability is hardly a policy parameter. However, almost the same reduction in the effort distortion would be accomplished by raising the effort standard up to the level needed to eliminate public school profit (compare the actual effort of 0.51 in column 1 to 0.64 in column 2). In other words, given that school effort is not observable and public schools are not subject to free entry yet their funding is fixed, the choice
of public school effort standard is critical, as it can lead almost to the same level of effort that would be accomplished if effort were indeed observable.\textsuperscript{19} Moreover, a higher effort standard has the additional effect of attracting high-income, high-ability households into public schools. Since these households provide monitoring, that further increases public school effort and reinforces the effect of the higher standard.

To summarize, in this section we have quantified the effort and achievement distortions due to the lack of observability. We have also studied the equilibrium response of households and schools to changes in the public school effort standard. Now we turn to specific policies.

5. Policy Analysis

In this section we study the effects of public monitoring of public schools and private schools vouchers, when applied separately or in conjunction. Then we explore the distribution of household preferences over policy parameters for public schools.

5.1. Public Monitoring

In order to simulate public monitoring, we need to choose values for its level ($M_0$) and unit cost ($\kappa$). Hoxby (2002a) argues that accountability is a low-cost policy. While this may be true for a mere testing system, we consider a kind of monitoring with actual impact on school effort, which might entail detailed evaluations of public school performance, direct observation of classroom and administrative practices, etc. Appendix C describes how we calibrate $M_0$ and $\kappa$ in the absence of direct information about the cost of this type of monitoring, and how we calculate the monitoring intensity $m_0$ based on $M_0$.

Columns 2 through 9 of Table 5 describe the equilibrium for several combinations of public monitoring intensity and unit cost. Not surprisingly, the more intense the public

\textsuperscript{19} We assume that changing the public school effort standard is costless and does not affect $\alpha$, a parameter related to deviating from the standard. In reality, changing the standard may be politically or administratively costly. In addition, it may be associated with a change $\alpha$. For instance, a low standard (such as a requirement that teachers merely come to the school) is easy to verify and may be thus associated with a severe punishment.
monitoring, the greater the effort for any given unit cost. Other researchers have documented that public monitoring increases school effort (Rouse et al 2007, Chiang 2009).

5.1.1. The Crowd-Out of Private Monitoring

Public monitoring affects household school and monitoring choices, as shown in Figure 3. Relative to the benchmark equilibrium, public monitoring raises public school attendance by raising public school effort, as only the highest-ability, highest-income households remain in private school. However, the impact of public monitoring on household monitoring depends on several forces. On the one hand, public monitoring raises public school effort, hence attracting high-ability, high-income households away from private schools. The fact that these households monitor the public school can raise private monitoring, further increasing public school effort. On the other hand, public monitoring can crowd out private monitoring and thus lower it. The entry of high-income, high-ability households into the public school can induce households for whom monitoring is more costly to free-ride on the newly arrived households and no longer monitor, also leading to a decrease in private monitoring.

The net outcome of these effects depends on the fiscal cost of public monitoring. When this cost is not too high the first effect prevails, yet the second and third effects dominate otherwise. For instance, Figure 3 shows that while public monitoring increases public school attendance and the number of monitoring households by attracting high-income, high-ability households into the public school, it also causes low-income households to stop monitoring when the fiscal cost of monitoring rises.20

To our knowledge, there have been no empirical studies of the response of private to public monitoring. The closest evidence comes from Figlio and Kenny (2009), who document that when a school receives a low grade in the Florida accountability system, parents reduce their donations to the school because they perceive it as poorly run. Importantly, this is more pronounced among schools serving low-income or minority families. The reason is that disadvantaged parents tend to be less involved in the school and have less direct, first-hand

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20 To understand this response in monitoring, recall that the benefit of monitoring is directly related to income. The taxes levied to pay for public monitoring lower households’ disposable income and hence their benefits from monitoring. This effect might lead low-income households to stop monitoring. Although one could argue that public monitoring in reality might not be costly enough to double or triple the benchmark fiscal burden as in the last columns of Table 5, we emphasize that effective public monitoring might actually be quite costly.
information about the school’s effectiveness. Thus, they rely more heavily on school grades and respond more strongly to them. In other words, public monitoring seems to crowd-out private monitoring more among disadvantaged parents, which is consistent with our model’s predictions.

5.1.2. Public Monitoring and Achievement

An important question is whether public monitoring raises public school achievement. The answer to this question depends again on the net effect of two forces. On the one hand, public monitoring raises public school effort and peer quality, both of which enhance achievement. In addition, greater peer quality and school effort induce greater household learning effort. On the other hand, the fiscal cost of public monitoring lowers disposable income and hence the demand for household learning effort. Only when the unit cost of accountability is low does the first effect prevail. Hence, low-income households – often the intended beneficiaries of these policies – only gain when these policies cost little.21

Households that remain in private schools lose achievement because they pay higher taxes to fund public monitoring and hence purchase less school effort and exert less household learning effort. Households that switch from private into public schools induced by public monitoring also lose achievement because of the loss of disposable income (that leads to lower household effort) and peer quality. Though the effect of public monitoring on average achievement is negative, most households gain achievement if the cost of public monitoring is sufficiently low. On balance, the effects of public monitoring on achievement illustrate the complexity of the outcomes induced by a policy tool when applied in a large-scale setting.

5.2. Private School Vouchers

Table 6 shows the effects of universal and income-targeted vouchers (columns 2-3 and 4-5, respectively). All households are eligible for the former, yet only households with an income below the threshold (equal to median household income in these simulations) are eligible for the latter. We consider $3,500- and $7,000-vouchers (“low” and “high” vouchers, respectively).

21 The empirical evidence on the effect of accountability on achievement is quite mixed (see, for instance Figlio and Ladd 2008). Due to the lack of data, the literature does not disentangle the role of school and household (or student) effort in school-based accountability, although Jacob (2005) provides evidence that part of the achievement gains due to accountability in Chicago Public Schools were driven by greater student effort at the test.
Although in reality income-targeted vouchers are more feasible given their lower eligibility rate, universal voucher simulations are of interest because they show the effects of an unrestricted voucher. Figure 4 depicts voucher effects on household school choice and monitoring.

We begin by analyzing universal vouchers. Not surprisingly, they increase private school attendance. However, low-income, low-ability households remain in public school because monitoring a private school would be prohibitively costly for them. With the departure of higher ability households from public school they lose good peers and monitors. Thus, public school effort\(^{22}\) and achievement fall.

Many voucher users gain school effort, peer quality or achievement. The higher their income or ability, the more likely they are to gain. Those who switch into private schools have access to higher school effort and better peers than they had in public school. Overall, more than half of the population gains achievement in these simulations. While private schools use funding more efficiently than public schools by being subject to free entry, public schools become more inefficient – under a high voucher they are no longer monitored and thus offer zero effort, which means that their full funding constitutes rent.

Some of the losses inflicted by universal vouchers upon the low-income, low-ability segmented are tempered by income-targeted vouchers. In our simulations, 40 percent of the households are eligible for these vouchers, yet only the highest-ability, highest-income eligible households use the voucher because the monitoring required in private schools is too costly for the others. Public school monitoring rate falls with income-targeted vouchers though not as much as with universal vouchers because fewer households leave public school, which means that public school effort does not fall as much either. All voucher users gain achievement.

A theme of these voucher simulations is the inability of vouchers to improve outcomes for the lowest income and ability segment because of informational frictions, as having to monitor in private schools (while losing the benefits of free-riding on public school monitoring) is too costly for those households. This raises the question of whether vouchers would be more effective in the absence of informational frictions. As Table 1 in Appendix D shows, under perfect information voucher use is higher, and more low-income households gain achievement.

\(^{22}\) This result contrasts the standard argument that by creating competition, vouchers would raise public school effort. The reason is that in our model, the public school takes attendance and monitoring as given when choosing effort. Hence, a policy that reduces household monitoring (without compensating with greater public monitoring) also reduces school effort. Nonetheless, if the public school were able to choose \(\epsilon^\text{pub}\) in our model, then the school would optimally raise \(\epsilon^\text{pub}\) in order to mitigate enrollment losses due to the voucher.
This suggests that under informational frictions vouchers may need to be supplemented by some form of public monitoring, as discussed below.

5.3. Regulation or Markets?

In public debates, regulation and markets are often presented as substitute approaches to discipline public schools and raise achievement. In what follows we argue against this - both mitigate moral hazard, yet their achievement and distributional implications are quite different.

Public monitoring relies on the logic that monitoring mitigates moral hazard. As the public school raises its effort it attracts higher-ability households, whose monitoring further enhance the school’s competitiveness. Vouchers also mitigate moral hazard, as they allow households dissatisfied with public school effort to choose their preferred promised effort in a school subject to full private monitoring. Yet the critical feature of vouchers is that they rely directly on competition, as they give households the means to choose other schools. Moreover, vouchers lead to greater efficiency because competition eliminates rents from private schools. This, in turn, benefits all households by lowering fiscal burdens.

Both public monitoring and vouchers have the potential of raising achievement for the majority of the population. However, (low-cost) public monitoring generally benefits low-income students while vouchers hurt them. In other words, public monitoring and vouchers are not substitutes for some households. Since vouchers rely on private rather than public monitoring, households with high monitoring costs are better served by public monitoring.

While this may suggest that public monitoring is superior, we emphasize that public monitoring can be fiscally costly for two reasons. The first is the direct cost of monitoring. Only when this cost is very low does public monitoring yield achievement and welfare gains for more households than vouchers, particularly in the low-income segment. The second is that as long as public schools are not subject to free entry and receive a fixed payment per student, they have incentives to reap rents. While public monitoring induces greater public school effort, it does not completely eliminate such rents without further manipulation of policy parameters such as effort standard or the intensity of public monitoring.

Moreover, neither regulation- nor market-based mechanisms provide a complete solution to underachievement. Moral hazard in schools interacts with other forces, such as household and school choices, in an equilibrium context and is hence endogenous to policy parameters. Any
large-scale attempt to mitigate the problem generates unintended equilibrium effects that alter the intensity of the original distortion. For instance, when public monitoring raises public school effort and attracts new, monitoring households into the public school, the original households in the school may free-ride on the newcomers. In other words, the composition of the private monitoring force in the public school is endogenous, as is the effort distortion.

Two lessons emerge from here. The first is that given the unintended effects of any single tool, underachievement may require a thoughtful combination of tools, such as private school vouchers supplemented with public monitoring for private schools, or private school vouchers combined with public monitoring for public schools. Column 7 of Table 6 and Figure 5 illustrate one such combination – vouchers for $3,500 coupled with very intense public monitoring for public schools (we assume the unit cost of monitoring is very low). To facilitate comparisons with other policies, this policy requires an income tax rate of 0.12, same as for universal high vouchers (column 3) and very low cost-, high-intensity monitoring (column 6).

At the same fiscal cost as these alternatives, the policy combination delivers achievement gains for a greater proportion of the population. Although the public school still loses households to private schools because of the voucher, effort in public school rises because of public monitoring. This, in turn, keeps more households in public schools and prevents the sudden decline of peer quality. Moreover, on average students receive greater school effort than in any of the voucher or public monitoring policies presented in the paper, all of which have at least as high a fiscal cost. Nonetheless, the intense public monitoring crowds out private monitoring in public schools relative to the benchmark equilibrium.

The inferiority of single tools relative to a combination is illustrated by the voucher program in Chile (Hsieh and Urquiola, 2006). Despite being decades-old, this program has failed to lift achievement significantly. In Chile, private schools have not been subject to any kind of public monitoring, and some voucher users may not have provided private monitoring either. Current efforts to revamp the program in Chile include tighter public monitoring of private schools (Lara et al, 2009), thus recognizing the need to combine policy tools.

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23 Neal (2009) argues in favor of policy combinations on the grounds that when households can exercise school choice, they have greater incentives to monitor the school.

24 Since most voucher programs draw upon excess capacity in existing private schools, voucher users join schools where parents are active monitors. The Milwaukee voucher program is the only program in the US that has spurred substantial entry of new schools to serve voucher students, most of whom presumably have high monitoring costs.
Another combination of markets and regulations is currently illustrated in the U.S. by charter schools, which provide households with market-based school choices yet are regulated by chartering agencies. These agencies oversee charter operations and occasionally close charters for academic or financial reasons. The issue, of course, is who monitors the monitor. This takes us to our second lesson: a particularly effective answer to moral hazard may be given by policies that lower monitoring costs for households, particularly those at the bottom of the distribution.

Finally, it could be argued that some households might prefer market-based mechanisms because they seek certain elements that conventional public schools do not provide, such as a religious education or a particular curricular focus. Our model does not address such horizontal differentiation (treated, for instance, in Ferreyra 2007 for private schools and Epple et al 2010 for charter schools). However, as long as public funding is used for education the policymaker may uphold the goal of furthering academic achievement in the population - in addition, perhaps, to providing horizontal variety - in which case the considerations offered above continue to apply.

5.3. Household Preferences Over Policy Parameters

Although so far we have viewed \( \varepsilon^{pub} \) (effort standard), \( X \) (funding per student) and \( m_0 \) (public monitoring intensity) as exogenous, it is conceivable that they are ultimately chosen by the households, perhaps through voting. Hence, it is interesting to study household preferences over these parameters. Since some parameters may be harder to alter than others, at least in the short run, we have explored preferences over \( \varepsilon^{pub} \), \( (\varepsilon^{pub}, X) \) pairs, \( m_0 \), \( (\varepsilon^{pub}, m_0) \) pairs, and \( (\varepsilon^{pub}, X, m_0) \) triplets. To study preferences over \( \varepsilon^{pub} \), we computed the equilibrium for values of \( \varepsilon^{pub} \) between 0 and 1.2. For each household, we found the value of \( \varepsilon^{pub} \) corresponding to the equilibrium in which the household attains its highest utility – that is the household’s preferred \( \varepsilon^{pub} \). In a similar fashion we studied preferences over the other policy parameters or combinations thereof.\(^{25}\)

Table 7 shows the outcome of this exercise. To facilitate comparisons, row 1 presents the benchmark equilibrium, computed for the current \( \varepsilon^{pub} \) of 0.66, the observed \( X \) and \( m_0=0 \). A theme in this exercise is the presence of two most preferred bundles – one preferred by households who

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The case of a few unscrupulous new entrants has contributed to the recent implementation of extensive public regulation for the program (see http://www.schoolinfofsystem.org/pdf/2010/02/2010VoucherBrief.pdf).

25 When studying preferences over \( X \), we computed the equilibrium for values of \( X \) between $1,000 and $1,200, thus elimination zero funding for public schools as an option for households.
choose public schools, and the other preferred by households who choose private schools. Hence, columns 2-4 and 5-7 characterize the bundle preferred by public and private school households, respectively, and column 8 shows the fraction of households who prefer public schools. In other words, columns 2-4 and 5-7 contain information on how households might vote in a poll over policy parameters, and column 8 shows the fraction that would support the bundle preferred by public school households. In this spirit, in what follows we refer to bundles as “chosen” or “preferred” in that hypothetical poll. In addition, Figure 6 displays the distribution of preferences among households that prevails for most instances of this exercise, as indicated in column 9.

Preferences over Effort Standard. As row 2 shows, public school households prefer $e_{pub}^{\text{pub}}=0.85$, whereas private school households prefer $e_{pub}^{\text{pub}}\leq 0.20$. Illustrating a theme of this analysis, the $e_{pub}^{\text{pub}}$ preferred by public school households is that which minimizes public school profits and maximizes public school effort, whereas the $e_{pub}^{\text{pub}}$ favored by private school households minimize public school attendance and hence the tax rate (see Figure 1). In our calibrated model, the majority of households prefer an $e_{pub}^{\text{pub}}$ of 0.85 (see Figure 6). Note that is the same $e_{pub}^{\text{pub}}$ that a policymaker interested in minimizing rents would establish (see section 4.2).

Preferences over Effort Standard and Funding. When households are allowed to choose both $X$ and $e_{pub}^{\text{pub}}$ (row 3), two preferred bundles emerge: $(e_{pub}^{\text{pub}}=0.65, X=$4,000) and $(e_{pub}^{\text{pub}}\leq 0.2, X=$1,000), preferred by the same households that prefer $e_{pub}^{\text{pub}}=0.85$ and $e_{pub}^{\text{pub}}\leq 0.20$ in the previous instance, respectively. Thus, when allowed to choose funding as well as effort standard, households that prefer public schools choose a (slightly) lower effort standard yet also a concomitantly lower funding relative to when they can only choose the effort standard. In both cases the outcome is greater public school effort and lower public school rent.

These preferences convey an interesting message. If one believes that in reality funding can be hardly altered by households, then the fact that the current effort standard is lower than households’ preferred standard suggests that public schools may bias the standard downward. If, on the other hand, one believes that in reality households can affect both funding and effort standard, then the fact that the current effort standard is almost the same as households’ preferred level yet funding is higher suggests that public schools may bias funding upward. Both stories point to potential bargaining power on the part of public schools.

Preferences over Public Monitoring. Row 4 summarizes household preferences for $m_0$ when its cost is very low. Most households prefer intense public monitoring in this case; only
private school households prefer no monitoring. Although preferences over \( m_0 \) are similar for slightly higher costs, the monitoring intensity preferred by the majority falls rapidly and becomes zero (see rows 5-7). This finding persists even when households can choose other parameters in addition to \( m_0 \) and is consistent with Table 5, which shows that no household gains welfare by having public monitoring unless its cost is very low.

Preferences over Effort Standard and Public Monitoring Since raising the effort standard and introducing public monitoring are two options that raise public school effort, households may view them as substitutes. Hence, we studied preferences for \( (e^{\text{pub}}, m_0) \) combinations for alternative costs of public monitoring (rows 8-11). When households can choose \( m_0 \) in addition to \( e^{\text{pub}} \) and monitoring costs are very low, they choose lower \( e^{\text{pub}} \) and compensate with high \( m_0 \) (see rows 8 and 2). Similarly, when they can choose \( e^{\text{pub}} \) in addition to \( m_0 \) and monitoring costs are very low, they support higher \( e^{\text{pub}} \) but lower \( m_0 \) (see rows 8-10 and 4-6). In other words, households are indeed willing to trade effort standard for public monitoring. Private school households prefer \( m_0 = 0 \) regardless of its cost.

Preferences over Effort Standard, Public Monitoring and Funding Rows 12-15 summarize preferences when households are allowed to choose all policy parameters – perhaps in the long run, when short-term political and/or contractual rigidities disappear. When households can choose \( X \) in addition to \( e^{\text{pub}} \) and \( m_0 \), they choose lower \( e^{\text{pub}} \), lower \( X \) and higher \( m_0 \). Once again they prefer a lower standard in exchange for greater consumption, and they compensate for the lower standard with higher public monitoring.

To summarize, household preferences over policy parameters are split along the lines of school choice. Households that prefer private schools always choose the policy parameters that minimize public school attendance and hence the fiscal burden, whereas households that prefer public schools choose bundles that maximize public school effort and minimize public school rents. These households view public monitoring and effort standard as substitutes and are willing to optimally lower one while raising the other. Only when the cost of public monitoring is low are these households willing to use it. When they can choose all policy parameters, these households select an effort standard quite close to the current one, but lower funding and positive (low-cost) public monitoring. A consistent outcome is that the current funding is above the level that households would choose, which may point to public schools’ influence in the determination of their funding. Moreover, these simulations indicate that if the demographic balance in the
population were to change such that the majority of households preferred private education, support for public schools (for instance, through funding or standards) would drop substantially.

6. Concluding Remarks

In this paper we have focused on the information asymmetry among the policymaker, households, and schools and its role on academic underachievement. We have built a simple moral hazard model of school effort and have embedded it within an equilibrium model of education choice in which households sort across schools and exert learning and monitoring efforts. From a policy perspective, we have focused on reforms to raise achievement, and on whether they address the distortions created by the underlying information frictions. Our analysis highlights the fact that since school moral hazard interacts with equilibrium choices, neither market-based nor regulation-based mechanisms alone will solve the underachievement problem. In the horse race between regulation- and market-based mechanisms, the winner seems to be a thoughtful combination of such mechanisms. Setting policy parameters at the appropriate levels can also mitigate moral hazard; these levels are closer to those preferred by the majority of households than by the public schools themselves.

Since for a segment of the population private monitoring is costly, policies that lower private monitoring costs (for instance, by providing more information on schools) or that provide public monitoring of schools (even though this may crowd out private monitoring) are likely to help. Public monitoring, however, raises the issue of who monitors the monitor.

We view our model as a foundation for analyzing information-related problems in education. While the present paper deals with broad institutional arrangements, designing and evaluating more specific policies would require extending the model to accommodate for the measurement of school and teacher value added, teacher heterogeneity and teacher sorting, and incentives induced by measurement problems. While many of these problem have been analyzed in managerial settings (e.g. Holmstrom and Milgrom 1991, Liang 2004, Dutta and Reichelstein 2005), the education context is quite unique because of the interaction between household and school choices, the nature of the achievement production, and the unintended implications of large-scale policies. Through our work we hope to shed light over these issues and inform the policy discussion.
References


### TABLE 1
*Parameter Values*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Coefficient of consumption in utility</td>
<td>6.351</td>
</tr>
<tr>
<td>$\eta_1$</td>
<td>Elasticity of achievement with respect to school effort</td>
<td>0.843</td>
</tr>
<tr>
<td>$\eta_2$</td>
<td>Elasticity of achievement with respect to peer quality</td>
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<tr>
<td>$\lambda$</td>
<td>Elasticity of teacher salary with respect to teacher effort</td>
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<tr>
<td>$A$</td>
<td>Monotonic transformation of teachers’ reservation utility</td>
<td>1.280</td>
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<tr>
<td>$e^{pub}$</td>
<td>Public school’s promised effort</td>
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</tr>
<tr>
<td>$\alpha$</td>
<td>Agency cost</td>
<td>9.939</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Disutility of household learning effort</td>
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</tr>
<tr>
<td>$\rho_m$</td>
<td>Disutility of household monitoring</td>
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### TABLE 2
*Predicted and Observed Values*

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<th>Variable</th>
<th>Observed Value</th>
<th>Predicted Value</th>
</tr>
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<tbody>
<tr>
<td>Fraction of Households with Children in Private Schools</td>
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<td>0.16</td>
</tr>
<tr>
<td>Average Income for Households with Children in Private Schools</td>
<td>$82,800</td>
<td>$90,400</td>
</tr>
<tr>
<td>Average Private School Tuition</td>
<td>$5,000</td>
<td>$4,900</td>
</tr>
<tr>
<td>Difference in Teacher Salary between Public and Private School</td>
<td>0.44</td>
<td>0.53</td>
</tr>
<tr>
<td>Difference in Teacher Effort between Private and Public School</td>
<td>0</td>
<td>1.26</td>
</tr>
<tr>
<td>Difference in Achievement between Private and Public School</td>
<td>0.45</td>
<td>1.56</td>
</tr>
<tr>
<td>Difference in Ability between Private and Public School</td>
<td>0.76</td>
<td>1.45</td>
</tr>
<tr>
<td>Difference in Student Effort between Private and Public School</td>
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</tr>
<tr>
<td>Monitoring Rate in Public School</td>
<td>0.76</td>
<td>0.76</td>
</tr>
</tbody>
</table>

Note: Measurement of each variable is described in the text. Dollar amounts rounded to the nearest hundred.
### TABLE 3

*Equilibrium with Imperfect and Perfect Observability*

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<tr>
<th></th>
<th>Imperfect Observability</th>
<th>Perfect Observability</th>
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<tbody>
<tr>
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<td>(2)</td>
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<tr>
<td>Fraction Hhs. In Public School</td>
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<td>0.96</td>
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<tr>
<td>Average Income</td>
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<td>$57,600</td>
</tr>
<tr>
<td>Public School</td>
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<td>$55,000</td>
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<tr>
<td>Private School</td>
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<td>$119,400</td>
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<tr>
<td>Average Ability</td>
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<td>100</td>
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<td>99</td>
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<tr>
<td>Private School</td>
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<td>119</td>
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<td>Monitoring Rate</td>
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<td>0.00</td>
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<tr>
<td>Public School</td>
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<td>0.00</td>
</tr>
<tr>
<td>Private School</td>
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<td>0.00</td>
</tr>
<tr>
<td>Average Spending per Student</td>
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<td>$7,000</td>
</tr>
<tr>
<td>Public School</td>
<td>$7,000</td>
<td>$7,000</td>
</tr>
<tr>
<td>Private School (tuition)</td>
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<td>$6,400</td>
</tr>
<tr>
<td>Average Promised School Effort</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public School</td>
<td>0.66</td>
<td>0.66</td>
</tr>
<tr>
<td>Private School</td>
<td>0.69</td>
<td>0.71</td>
</tr>
<tr>
<td>Average Actual School Effort</td>
<td>0.51</td>
<td>0.67</td>
</tr>
<tr>
<td>Public School</td>
<td>0.50</td>
<td>0.66</td>
</tr>
<tr>
<td>Private School</td>
<td>0.55</td>
<td>0.71</td>
</tr>
<tr>
<td>Public School Profit</td>
<td>$60,800</td>
<td>$35,600</td>
</tr>
<tr>
<td>Average Teacher Compensation</td>
<td></td>
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<tr>
<td>Public School</td>
<td>$6,000</td>
<td>$7,000</td>
</tr>
<tr>
<td>Private School</td>
<td>$3,900</td>
<td>$6,400</td>
</tr>
<tr>
<td>Average Use of School Revenues</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public School</td>
<td>Salaries: 0.44, Agency cost: 0.15, Rent: 0.41</td>
<td>Salaries: 0.79, Agency cost: 0.21, Rent: 0.21</td>
</tr>
<tr>
<td>Private School</td>
<td>Salaries: 0.79, Agency cost: 0.21, Rent: 0.0</td>
<td>Salaries: 1.0, Agency cost: 0, Rent: 0</td>
</tr>
<tr>
<td>Avg. Household Learning Effort</td>
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<td></td>
</tr>
<tr>
<td>Public School</td>
<td>-0.20</td>
<td>-0.06</td>
</tr>
<tr>
<td>Private School</td>
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<td>3.08</td>
</tr>
<tr>
<td>Average Achievement</td>
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</tr>
<tr>
<td>Public School</td>
<td>-0.25</td>
<td>-0.09</td>
</tr>
<tr>
<td>Private School</td>
<td>1.31</td>
<td>4.37</td>
</tr>
<tr>
<td>Income Tax Rate</td>
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</tr>
<tr>
<td>Aggregate Welfare</td>
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<td>1.03E+13</td>
</tr>
</tbody>
</table>

Note: Dollar amounts are rounded to the nearest hundred. For “Use of School Revenues”, we display the fraction of revenues that pays for salaries, agency cost or rent. In the imperfect observability (benchmark) equilibrium, achievement and learning effort are normalized to have zero mean and unit standard deviation. In all columns and tables, achievement and learning effort are measured in units of standard deviation of the benchmark equilibrium distributions.
TABLE 4
Equilibrium with Imperfect and Perfect Observability for Alternative Public School Effort Standards

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
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<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
</tr>
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<td>Fraction Public School</td>
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<td>0.96</td>
<td>0</td>
<td>0.96</td>
</tr>
<tr>
<td>Avg. Income Public School</td>
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<td>$55,000</td>
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<td>$55,000</td>
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<tr>
<td>Avg. Ability Public School</td>
<td>97</td>
<td>99</td>
<td>99</td>
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<td>99</td>
</tr>
<tr>
<td>Monitoring Rate Pub. School</td>
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<td>0.00</td>
<td>n/a</td>
<td>0.00</td>
</tr>
<tr>
<td>Avg. Spending per Student</td>
<td>$6,700</td>
<td>$7,000</td>
<td>$7,000</td>
<td>$3,500</td>
<td>$7,000</td>
</tr>
<tr>
<td>Public School</td>
<td>$7,000</td>
<td>$7,000</td>
<td>$7,000</td>
<td>n/a</td>
<td>$7,000</td>
</tr>
<tr>
<td>Private School</td>
<td>$4,900</td>
<td>$6,400</td>
<td>$6,400</td>
<td>$3,500</td>
<td>$6,400</td>
</tr>
<tr>
<td>Avg. Promised School Effort</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public School</td>
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<td>0.85</td>
<td>0.66</td>
<td>Up to 0.15</td>
<td>0.70</td>
</tr>
<tr>
<td>Private School</td>
<td>0.69</td>
<td>0.80</td>
<td>0.71</td>
<td>0.50</td>
<td>0.71</td>
</tr>
<tr>
<td>Avg. Actual School Effort</td>
<td>0.51</td>
<td>0.64</td>
<td>0.67</td>
<td>0.50</td>
<td>0.70</td>
</tr>
<tr>
<td>Public School</td>
<td>0.50</td>
<td>0.64</td>
<td>0.66</td>
<td>n/a</td>
<td>0.70</td>
</tr>
<tr>
<td>Private School</td>
<td>0.55</td>
<td>0.64</td>
<td>0.71</td>
<td>0.50</td>
<td>0.71</td>
</tr>
<tr>
<td>Public School Profit</td>
<td>$60,800</td>
<td>$3,000</td>
<td>$35,600</td>
<td>n/a</td>
<td>$19,900</td>
</tr>
<tr>
<td>Avg. Use of School Revenues</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public School</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Salaries</td>
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<td>0.74</td>
<td>0.79</td>
<td>n/a</td>
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</tr>
<tr>
<td>Agency Cost</td>
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</tr>
<tr>
<td>Rent</td>
<td>0.41</td>
<td>0.02</td>
<td>0.21</td>
<td>n/a</td>
<td>0.12</td>
</tr>
<tr>
<td>Private School</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Salaries</td>
<td>0.79</td>
<td>0.79</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Agency Cost</td>
<td>0.21</td>
<td>0.21</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Rent</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Avg. Learning Effort</td>
<td>0</td>
<td>0.04</td>
<td>0.06</td>
<td>0.48</td>
<td>0.08</td>
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<tr>
<td>Avg. Achievement</td>
<td>0</td>
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<td>0.09</td>
<td>0.60</td>
<td>0.12</td>
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<tr>
<td>Public School</td>
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<td>-0.10</td>
<td>-0.09</td>
<td>n/a</td>
<td>-0.05</td>
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<td>Private School</td>
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<td>3.52</td>
<td>4.37</td>
<td>0.60</td>
<td>4.37</td>
</tr>
<tr>
<td>Proportion of Hhs. with Higher Achievement</td>
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<td>0.92</td>
<td>0.64</td>
<td>0.92</td>
<td></td>
</tr>
<tr>
<td>Among Low-Income Hhs.</td>
<td>1</td>
<td>1</td>
<td>0.20</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Income Tax Rate</td>
<td>0.1</td>
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<td>0.12</td>
<td>0</td>
<td>0.12</td>
</tr>
<tr>
<td>Aggregate Welfare</td>
<td>8.34E+12</td>
<td>9.35E+12</td>
<td>1.03E+13</td>
<td>3.55E+13</td>
<td>1.10E+13</td>
</tr>
</tbody>
</table>

Note: Column (1) and (3) are the same as columns (1) and (2) from Table 3, respectively. Column (5) corresponds to the value of $e_{pub}$ that yields the lowest non-negative public school profit under perfect observability. “Hh.” is short for “household”; “low-Income” means income = 10th percentile. “Proportion of households with higher achievement” is calculated relative to the benchmark equilibrium.
### TABLE 5
Public Monitoring of Public School

<table>
<thead>
<tr>
<th>Fraction Public School</th>
<th>Monitoring Rate Public School</th>
<th>Average Ability Public School</th>
<th>Actual Public School Effort</th>
<th>Public School Profit</th>
<th>Use of Revenues Public School</th>
<th>Proportion of Households Who Gain Achievement</th>
<th>Proportion of Households Who Gain Welfare</th>
<th>Income Tax Rate</th>
<th>Aggregate Welfare</th>
<th>Proportion of Households Who Gain Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.84</td>
<td>0.76</td>
<td>97</td>
<td>0.50</td>
<td>$60,800</td>
<td>Salaries</td>
<td>0.84</td>
<td>0.84</td>
<td>0.10</td>
<td>8.34E+12</td>
<td>0.88</td>
</tr>
<tr>
<td>0.92</td>
<td>0.78</td>
<td>98.33</td>
<td>0.55</td>
<td>$55,700</td>
<td>Agency Cost</td>
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<td>0.84</td>
<td>0.11</td>
<td>7.90E+12</td>
<td>0.84</td>
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<tr>
<td>0.96</td>
<td>0.79</td>
<td>99.2</td>
<td>0.57</td>
<td>$54,900</td>
<td>Rent</td>
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<td>0.84</td>
<td>0.11</td>
<td>7.73E+12</td>
<td>0.84</td>
</tr>
<tr>
<td>0.92</td>
<td>0.78</td>
<td>98.33</td>
<td>0.55</td>
<td>$55,700</td>
<td>Avg. Achievement</td>
<td>0.84</td>
<td>0.84</td>
<td>0.12</td>
<td>5.93E+12</td>
<td>0.84</td>
</tr>
<tr>
<td>0.96</td>
<td>0.79</td>
<td>99.2</td>
<td>0.57</td>
<td>$54,900</td>
<td>Public School</td>
<td>0.84</td>
<td>0.84</td>
<td>0.13</td>
<td>5.01E+12</td>
<td>0.84</td>
</tr>
<tr>
<td>0.96</td>
<td>0.79</td>
<td>99.2</td>
<td>0.57</td>
<td>$54,900</td>
<td>Private School</td>
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<td>0.84</td>
<td>0.15</td>
<td>2.95E+12</td>
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<tr>
<td>0.96</td>
<td>0.79</td>
<td>99.2</td>
<td>0.57</td>
<td>$54,900</td>
<td>Among Low-Income Hhs.</td>
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<td>0.84</td>
<td>0.17</td>
<td>1.80E+12</td>
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<tr>
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<td>0.79</td>
<td>99.2</td>
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<td>$54,900</td>
<td>Among Public-Public Hhs.</td>
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<td>0.84</td>
<td>0.20</td>
<td>1.02E+12</td>
<td>0.84</td>
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<tr>
<td>0.96</td>
<td>0.79</td>
<td>99.2</td>
<td>0.57</td>
<td>$54,900</td>
<td>Among Private-Private. Hhs.</td>
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<td>0.84</td>
<td>0.21</td>
<td>3.24E+11</td>
<td>0.84</td>
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<tr>
<td>0.96</td>
<td>0.79</td>
<td>99.2</td>
<td>0.57</td>
<td>$54,900</td>
<td>Among Private-Public Hhs.</td>
<td>0.84</td>
<td>0.84</td>
<td>0.25</td>
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<tr>
<td>0.96</td>
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<td>99.2</td>
<td>0.57</td>
<td>$54,900</td>
<td>Income Tax Rate</td>
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<td>0.84</td>
<td>0.31</td>
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<td>0.84</td>
</tr>
<tr>
<td>0.96</td>
<td>0.79</td>
<td>99.2</td>
<td>0.57</td>
<td>$54,900</td>
<td>Aggregate Welfare</td>
<td>0.84</td>
<td>0.84</td>
<td>0.88</td>
<td></td>
<td>0.84</td>
</tr>
<tr>
<td>0.96</td>
<td>0.79</td>
<td>99.2</td>
<td>0.57</td>
<td>$54,900</td>
<td>Proportion of Households Who Gain Achievement</td>
<td>0.84</td>
<td>0.84</td>
<td>0.88</td>
<td></td>
<td>0.84</td>
</tr>
<tr>
<td>0.96</td>
<td>0.79</td>
<td>99.2</td>
<td>0.57</td>
<td>$54,900</td>
<td>Proportion of Households Who Gain Welfare</td>
<td>0.84</td>
<td>0.84</td>
<td>0.88</td>
<td></td>
<td>0.84</td>
</tr>
<tr>
<td>0.96</td>
<td>0.79</td>
<td>99.2</td>
<td>0.57</td>
<td>$54,900</td>
<td></td>
<td></td>
<td></td>
<td>0.88</td>
<td></td>
<td>0.84</td>
</tr>
<tr>
<td>0.96</td>
<td>0.79</td>
<td>99.2</td>
<td>0.57</td>
<td>$54,900</td>
<td></td>
<td></td>
<td></td>
<td>0.88</td>
<td></td>
<td>0.84</td>
</tr>
</tbody>
</table>

Note: Column (1) is the same as column (1) in Table 3 – the benchmark equilibrium for imperfect observability, with no public monitoring. Very low-, low-, moderate- and high-cost monitoring correspond to values of $\gamma$ equal to 0.01, 0.10, 0.3 and 0.6 respectively. Medium and high-intensity monitoring correspond to values of $m_0$ equal to 0.5 and 0.75, respectively. For an explanation on the calibration of $\gamma$ and $m_0$, see Appendix C. “Proportion of Households Who Gain Achievement” and “Proportion of Households Who Gain Welfare” are calculated relative to the benchmark equilibrium. “Public-Public Hhs.” are the households who attend public school in the benchmark equilibrium and under public monitoring; “Private-Private Hhs.” attend private schools in both cases, and “Private-Public Hhs” attend private schools in the benchmark equilibrium but switch into public school under public monitoring.
<table>
<thead>
<tr>
<th></th>
<th>Benchmark Eqbrm. (1)</th>
<th>Universal Low Voucher (2)</th>
<th>Universal High Voucher (3)</th>
<th>Income-targeted Low Voucher (4)</th>
<th>Income-targeted High Voucher (5)</th>
<th>Very Low Cost, High Monitoring (6)</th>
<th>Vouchers and Public Monitoring (7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction Public School</td>
<td>0.84</td>
<td>0.36</td>
<td>0.2</td>
<td>0.64</td>
<td>0.68</td>
<td>0.96</td>
<td>0.64</td>
</tr>
<tr>
<td>Fraction of Eligible Hhs.</td>
<td>0.64</td>
<td>0.80</td>
<td>0.30</td>
<td>0.40</td>
<td>0.40</td>
<td>n/a</td>
<td>0.36</td>
</tr>
<tr>
<td>Using Voucher</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Avg. Income Public School</td>
<td>$51,300</td>
<td>$38,000</td>
<td>$13,400</td>
<td>$49,300</td>
<td>$55,800</td>
<td>$57,600</td>
<td>$48,200</td>
</tr>
<tr>
<td>Avg. Ability Public School</td>
<td>97</td>
<td>91</td>
<td>100</td>
<td>94</td>
<td>95</td>
<td>99.2</td>
<td>93.22</td>
</tr>
<tr>
<td>Monitoring Rate Public School</td>
<td>0.76</td>
<td>0.44</td>
<td>0.00</td>
<td>0.69</td>
<td>0.71</td>
<td>0.79</td>
<td>0.50</td>
</tr>
<tr>
<td>Avg. Actual School Effort</td>
<td>0.51</td>
<td>0.49</td>
<td>0.53</td>
<td>0.50</td>
<td>0.53</td>
<td>0.57</td>
<td>0.60</td>
</tr>
<tr>
<td>Public School</td>
<td>0.50</td>
<td>0.42</td>
<td>0.00</td>
<td>0.48</td>
<td>0.49</td>
<td>0.57</td>
<td>0.63</td>
</tr>
<tr>
<td>Private School</td>
<td>0.55</td>
<td>0.53</td>
<td>0.66</td>
<td>0.53</td>
<td>0.61</td>
<td>0.63</td>
<td>0.53</td>
</tr>
<tr>
<td>Public School Profit</td>
<td>$60,800</td>
<td>$31,800</td>
<td>$35,000</td>
<td>$48,100</td>
<td>$50,600</td>
<td>$54,900</td>
<td>$27,700</td>
</tr>
<tr>
<td>Public School Use of Revenues</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Salaries</td>
<td>0.44</td>
<td>0.31</td>
<td>0.41</td>
<td>0.42</td>
<td>0.54</td>
<td>0.25</td>
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<tr>
<td>Agency Cost</td>
<td>0.15</td>
<td>0.18</td>
<td>0.16</td>
<td>0.15</td>
<td>0.11</td>
<td>0.72</td>
<td></td>
</tr>
<tr>
<td>Rent</td>
<td>0.41</td>
<td>0.5</td>
<td>0.43</td>
<td>0.43</td>
<td>0.35</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>Private School Avg. Use of Revenues</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Salaries</td>
<td>0.79</td>
<td>0.79</td>
<td>0.79</td>
<td>0.79</td>
<td>0.79</td>
<td>0.79</td>
<td></td>
</tr>
<tr>
<td>Agency Cost</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td>Rent</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Avg. Achievement</td>
<td>0</td>
<td>0.16</td>
<td>0.13</td>
<td>0.05</td>
<td>-0.02</td>
<td>-0.02</td>
<td>0.06</td>
</tr>
<tr>
<td>Prop. who Gain Achievement</td>
<td>0.52</td>
<td>0.64</td>
<td>0.36</td>
<td>0.16</td>
<td>0.84</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Among Low-Income Hhs.</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Among Public-Hh.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Among Private-Hh.</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Among Private-Public Hhs.</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>0</td>
<td>n/a</td>
</tr>
<tr>
<td>Among Public-Private Hhs.</td>
<td>0.75</td>
<td>0.75</td>
<td>1</td>
<td>1</td>
<td>n/a</td>
<td>1</td>
<td>1</td>
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<tr>
<td>Income Tax Rate</td>
<td>0.1</td>
<td>0.08</td>
<td>0.12</td>
<td>0.09</td>
<td>0.1</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>Aggregate Welfare</td>
<td>8.34E+12</td>
<td>1.41E+13</td>
<td>1.21E+13</td>
<td>9.85E+12</td>
<td>7.88E+12</td>
<td>7.73E+12</td>
<td>9.53E+12</td>
</tr>
<tr>
<td>Prop. Who Gain Welfare</td>
<td>0.52</td>
<td>0.48</td>
<td>1</td>
<td>0.16</td>
<td>0.88</td>
<td>0.36</td>
<td></td>
</tr>
</tbody>
</table>

Note: Column (1) is the benchmark equilibrium for imperfect observability. Low and high vouchers are for $3,500 and $7,000, respectively. Column (6) is the same as column (3) in Table 5 ($\gamma=0.01$, $m_0=0.75$). Proportion of households who gain achievement and welfare are computed relative to the benchmark equilibrium. Column (7) features universal vouchers for $3,500 and very low cost, very high intensity public monitoring ($\gamma=0.01$, $m_0=4$).
### Table 7

**Household Preferences over Policy Parameters**

<table>
<thead>
<tr>
<th>Parameters of Choice</th>
<th>Monitoring cost ((\gamma))</th>
<th>Public School Households</th>
<th>Private School Households</th>
<th>Fraction Public School</th>
<th>Distribution depicted in Figure 6?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. None</td>
<td>n/a</td>
<td>0.66 $7,000 0</td>
<td>0.66 $7,000 0</td>
<td>0.84</td>
<td></td>
</tr>
<tr>
<td>2. (e^{pub})</td>
<td>n/a</td>
<td>0.85 $7,000 0</td>
<td>&lt;=0.20 $7,000 0</td>
<td>0.60</td>
<td>Yes</td>
</tr>
<tr>
<td>3. (e^{pub}, X)</td>
<td>n/a</td>
<td>0.65 $4,000 1.7</td>
<td>&lt;=0.20 $1,000 0</td>
<td>0.60</td>
<td>Yes</td>
</tr>
<tr>
<td>4. (m_0)</td>
<td>(\gamma = 0.01)</td>
<td>0.66 $7,000 0.4</td>
<td>0.66 $7,000 0</td>
<td>0.88</td>
<td></td>
</tr>
<tr>
<td>5. (m_0)</td>
<td>(\gamma = 0.03)</td>
<td>0.66 $7,000 0.3</td>
<td>0.66 $7,000 0</td>
<td>0.84</td>
<td></td>
</tr>
<tr>
<td>6. (m_0)</td>
<td>(\gamma &gt; 0.10)</td>
<td>0.66 $7,000 0</td>
<td>0.66 $7,000 0</td>
<td>0.84</td>
<td></td>
</tr>
<tr>
<td>7. (e^{pub}, m_0)</td>
<td>(\gamma = 0.01)</td>
<td>0.80 $7,000 0.85</td>
<td>&lt;=0.20 $7,000 0</td>
<td>0.60</td>
<td>Yes</td>
</tr>
<tr>
<td>8. (e^{pub}, m_0)</td>
<td>(\gamma = 0.03)</td>
<td>0.85 $7,000 0.05</td>
<td>&lt;=0.20 $7,000 0</td>
<td>0.60</td>
<td>Yes</td>
</tr>
<tr>
<td>9. (e^{pub}, m_0)</td>
<td>(\gamma = 0.05)</td>
<td>0.85 $7,000 0.05</td>
<td>&lt;=0.20 $7,000 0</td>
<td>0.60</td>
<td>Yes</td>
</tr>
<tr>
<td>10. (e^{pub}, X, m_0)</td>
<td>(\gamma &gt; 0.10)</td>
<td>0.85 $7,000 0</td>
<td>&lt;=0.20 $7,000 0</td>
<td>0.60</td>
<td>Yes</td>
</tr>
<tr>
<td>11. (e^{pub}, X, m_0)</td>
<td>(\gamma = 0.01)</td>
<td>0.60 $4,000 1.20</td>
<td>&lt;=0.20 $1,000 0</td>
<td>0.60</td>
<td>Yes</td>
</tr>
<tr>
<td>12. (e^{pub}, X, m_0)</td>
<td>(\gamma = 0.03)</td>
<td>0.60 $3,500 0.15</td>
<td>&lt;=0.20 $1,000 0</td>
<td>0.60</td>
<td>Yes</td>
</tr>
<tr>
<td>13. (e^{pub}, X, m_0)</td>
<td>(\gamma &gt;= 0.10)</td>
<td>0.65 $4,000 0</td>
<td>&lt;=0.20 $1,000 0</td>
<td>0.60</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Note: “Fraction Public School” is the fraction of households that prefers public schools under its preferred parameter combination. Row 1 corresponds to the benchmark equilibrium. In each row, values in bold and italics correspond to cases in which households are allowed to choose the corresponding policy parameter, and the remaining values are from the benchmark equilibrium. For instance, row 2 corresponds to the case in which households are allowed to choose \(e^{pub}\) only. In this case, \(e^{pub} = 0.85\) is preferred by public school households, \(e^{pub} <= 0.2\) is preferred by private school households, and \(X\) and \(m\) are equal to $7,000 and 0 respectively. \(\gamma < 0.10\) represents very low monitoring costs; \(\gamma >= 0.10\) represents low, moderate or high costs.
FIGURE 1
Public School Profit, Effort, Attendance and Monitoring under Imperfect Observability

FIGURE 2
Comparing Public School Profit under Perfect and Imperfect Observability
Note: each (income, ability) combination represents a household. Benchmark Equilibrium is the equilibrium for imperfect observability. “Public, Monitoring” means that the household attends public school and monitors; “Public, No Monitoring” means that the household attends public school and does not monitor; “Private” means that the household attends private school (and hence monitors). Moderate- and high-cost monitoring corresponds to values of \( \gamma \) equal to 0.30 and 0.60 respectively. Medium and high-intensity monitoring corresponds to values of \( m_o \) equal to 0.5 and 0.75, respectively.
FIGURE 4
Household School Choice and Monitoring under Private School Vouchers

Note: Benchmark Equilibrium is the equilibrium for imperfect observability.
Note: “Very low-cost, high-intensity public monitoring” corresponds to $\gamma=0.01$, $m_0=0.75$; “Universal voucher and public monitoring” corresponds to a $3,500$-voucher and $\gamma=0.01$, $m_0=4$.

Note: all the household types depicted as “public” prefer one set of policy parameters, and all the household types depicted as “private” prefer another. See the text and Table 7 for the specific sets they prefer.
Appendix A: Output- versus Input-Based Agency Model

In section 2 of the paper, we laid out an agency setting where households exert monitoring effort $m$ to mitigate agency costs, thus increasing school effort $e$. This model is a reduced-form version of a standard agency model with hidden effort. In particular, it specifies directly the school’s payoff as determined by its deviation from the effort (input) standard. This contrasts with the standard agency model in which the school’s payoff would be determined by the deviation from the school’s output with respect to the output standard. Nonetheless, these approaches are similar in that greater monitoring induces greater effort. To show this similarity, in this appendix we provide a brief extension of our model in which monitoring (exerted by the households and/or the policymaker) produces output-based signals that are informative of the school’s effort. These signals are used optimally in an explicit compensation contract between the school and the household, and the school’s payoff depends on this contract.

In this extension, the household is the principal and the school is the agent. Let signal $y$ be informative about school effort $e$ such that $y = (e^{pub} - e) + \varepsilon$, where $\varepsilon$ is normally distributed with variance $\sigma^2/m$. For instance, $y$ can consist of a test score, which is an imperfect measure of achievement (output) and reflects the school’s effort $e$, the other inputs in the achievement production, and random factors. Delivering effort $e$ has a cost equal to $C(e)$ for the school. Assume that compensation $w$ is linear in $y$: $w = a - by$, where $b > 0$ denotes the bonus rate. The household seeks to find the optimal $a$ and $b$ in order to minimize expected compensation for any given level of actual school effort.

This compensation scheme creates risk for the agent, as her compensation depends on the realization of the random signal $y$. Assuming CARA utility, the agent’s expected utility can be represented by its certainty equivalent equal to $a - b \ E[y] - .5 \ b^2 r \sigma^2/m - C(e^{pub}) + \Delta C(e^{pub} - e)$. Here, $r$ measures the agent’s risk aversion and $\Delta C(e^{pub} - e)$ is the cost saving from exerting $e$ as opposed to $e^{pub}$. The term $b^2 r \sigma^2/m$ is the agent’s risk premium, or agency cost — i.e., it represents the compensation needed by the agent in order to enter the risky contract. The risk premium is higher the higher the bonus rate $b$ and the lower the monitoring effort $m$. This is because higher monitoring provides a more precise measure of effort $e$. In the case of test scores,
higher parental monitoring means that the household is able to obtain more precise information about school effort based on test scores. For instance, parents who interact very frequently with teachers and other parents can assess teacher effort better than those who only rely on test scores.

The agent chooses optimal effort $e$ to maximize her expected utility (or equivalently, its certainty equivalent), trading off cost savings (decreasing in $e$) and expected bonus (increasing in $e$). The first order condition of the agent’s problem implies $b = -\Delta C_e > 0$, where $\Delta C_e < 0$ denotes the partial derivative with respect to school effort $e$. The higher the bonus rate $b$, the higher the effort that the agent exerts.

Competition among identical agents means that the agent will only earn her reservation (or minimum) certainty equivalent, denoted by $CE$. Hence, for a given $b$, the principal will set $a$ to be equal to $CE + b E[y] + .5 b^2 r \sigma^2/m + C(e_{pub}^{e}) - \Delta C(e_{pub}^{e} - e)$. As for the optimal choice of $b$, the principal will seek to minimize the expected cost of procuring a given level of effort, which is equal to $E[w] = a - bE[y] = CE + .5 b^2 r \sigma^2/m + C(e_{pub}^{e}) - \Delta C(e_{pub}^{e} - e)$. This expression highlights the classical trade-off between inducing effort and insuring the agent that the principal faces in agency models: a high $b$ induces high effort but raises the agent’s risk premium and hence the principal’s agency cost; a low $b$ induces low effort but lowers the agent’s risk premium and hence the principal’s agency cost.

The principal’s optimal $b$ is a function of the monitoring effort $m$. Raising $m$ lowers the signal’s variance and hence the agency cost, leading to a marginal benefit for the principal equal to $ .5 b^2 r \sigma^2/m^2$. Thus, the principal will choose the level of $m$ whose marginal benefit equals marginal cost. Since higher monitoring $m$ reduces the variance of the signal, it allows the principal to pay a higher bonus rate $b$ (see the $E[w]$ expression above). The agent responds by optimally raising its effort $e$. As a result, higher $m$ leads to higher $e$, just as in the reduced-form model in the text.

This analysis will carry through if $y$ is a vector instead of a scalar, though the reduced-form model will not capture some features of the extended models such as possible different weights on each signal in $y$. If $e$ is a vector instead of a scalar, the model becomes considerably more involved. See Holmstrom and Milgrom (1991) and Feltham and Xie (1994) for details and insights on multi-task, multi-signal agency models.
Appendix B: Characterization of the Equilibrium

This appendix proves the properties that characterize the benchmark equilibrium. Here we assume binary monitoring – namely, $m$ is either 0 or 1. If monitoring is continuous, we cannot characterize the equilibrium analytically, and the computations become significantly more complex.

We show that the equilibrium, if it exists, satisfies the following:

1. Income and ability stratification over school choice. If a household attends a private (public) school in equilibrium, any household with the same ability but higher (lower) income will also attend a private (public) school. Similarly, if a household attends a private (public) school in equilibrium, any household with the same income but higher (lower) ability will also attend a private (public) school.

2. Income and ability stratification over parental monitoring in public school. If a household monitors the public school in equilibrium, any public school household with the same ability but higher income (or with the same income but higher ability) will also monitor it. Similarly, if a household does not monitor the public school in equilibrium, any public school household with the same ability but lower income (or with the same income but lower ability) will not monitor it either.

3. Greater effort distortion in public school for a given school effort standard. The school effort distortion is larger in public than private schools as long as not all public school households monitor.

After substituting the achievement production function (3) and the budget constraint (2) into the household utility function (1), we write the latter as follows

$$U = ((1 - t)y - T)^{\beta} e^{\eta_1} q^{\eta_2} a - \rho_a \frac{a^2}{2\mu} - \rho_m \frac{m^2}{2\mu}$$  \hspace{1cm} (B1)

The optimal $a$ is

$$a = ((1 - t)y - T)^{\beta} e^{\eta_1} q^{\eta_2} \frac{\mu}{\rho_a}$$  \hspace{1cm} (B2)

Substituting (A2) back into (A1), we have
\[
U = \left[\left((1 - t)g - r\right) e^{\eta_1} q^{\eta_2}\right]^2 \frac{\mu}{2\rho_a} - \frac{m^2}{2\mu} - \rho_m \frac{m^2}{2\mu} \tag{B3}
\]

Recall that at public school, the household receives the bundle \{e(e_{\text{pub}}, M, T=0)\}. If the household attends a private school, it receives its optimal (conditional on attending private school) bundle \{e, T^*(e)\}. Define the utilities of the private and public school options as

\[
U_{\text{PRI}}(e|y, \mu) = \left[\left((1 - t)g - T^*(e)\right) e^{\eta_1} \mu^{\eta_2}\right]^2 \frac{\mu}{2\rho_a} - \frac{\rho_m m^2}{2\mu} \tag{B4}
\]

\[
U_{\text{PUB}}(m|y, \mu, q, e(e_{\text{pub}}, M, m)) = \left[\left((1 - t)g\right) e^{\eta_1} q^{\eta_2}\right]^2 \frac{\mu}{2\rho_a} - \frac{\rho_m m^2}{2\mu} \tag{B5}
\]

where \(U_{\text{PRI}}(e|y, \mu)\) denotes the utility of a household with income \(y\) and ability \(\mu\) that procures an actual school effort of \(e\) from a private school, and \(U_{\text{PUB}}(m|y, \mu, q, e(e_{\text{pub}}, M, m))\) denotes the utility of a household with income \(y\) and ability \(\mu\) that attends a public school and chooses monitoring \(m\). The actual school effort offered by this school, \(e(e_{\text{pub}}, M, m)\), is the school’s optimal effort as a function of \(e_{\text{pub}}\), total monitoring from other households \(M\), and monitoring \(m\) by the household under consideration.

Define school quality \(Q\) as \(Q = e^{\eta_1} q^{\eta_2}\). Then, our household utility function satisfies single-crossing in income (SCI), or

\[
\partial \left(\frac{\partial U/\partial Q}{\partial U/\partial y}\right)/\partial y > 0
\]

as in Epple and Romano (1998). In other words, the household demand for educational quality has positive income elasticity.

Now we prove the following properties of the equilibrium:

1) Income and ability stratification over school choice:
   a) If a household attends a private (public) school in equilibrium, any household with the same ability but higher (lower) income will also attend a private (public) school.
   b) If a household attends a private (public) school in equilibrium, any household with the same income but higher (lower) ability will also attend a private (public) school.

2) Income and ability stratification over parental monitoring in public school:
a) If a household monitors the public school in equilibrium, any public school household with the same ability but higher income (or with the same income but higher ability) will also monitor it.
b) If a household does not monitor the public school in equilibrium, any public school household with the same ability but lower income (or with the same income but lower ability) will not monitor it either.

3) Greater effort distortion in public school for a given school effort standard. The school effort distortion is larger in public than private schools as long as not all public school households monitor.

To prove claim 1a, consider household 1 with \( \{y_1, \mu\} \) that prefers to attend private school instead of attending public school and monitoring:

\[
U^{PRI}(e_1^* | y_1, \mu_1) > U^{PUB}(m = 1 | y_1, \mu_1, q, e^{pub})
\]  (B6)

We need to show that if \( y_1 < y_2 \), then we have

\[
U^{PRI}(e_2^* | y_2, \mu_1) > U^{PUB}(m = 1 | y_2, \mu_1, q, e^{pub})
\]  (B7)

From (B6), we derive

\[
((1 - t)y_1 - T^*(e_1^*))^\beta (e_1^*)^\eta_1 \mu_1^{\eta_2} > ((1 - t)y_1)^\beta e(e^{pub}, M_{-}, 1)^\eta_1 q^{\eta_2}
\]  (B8)

Multiplying both sides of (B8) by \( (y_2/y_1)^\beta \) yields

\[
\left((1 - t)y_2 - T^*(e_1^*)\frac{y_2}{y_1}\right)^\beta (e_1^*)^\eta_1 \mu_1^{\eta_2} > \left((1 - t)y_2\right)^\beta e(e^{pub}, M_{-}, 1)^\eta_1 q^{\eta_2}
\]  (B9)

Because \( y_2/y_1 > 1 \), we have

\[
\left[\left((1 - t)y_2 - T^*(e_1^*)\right)^\beta (e_1^*)^\eta_1 \mu_1^{\eta_2}\right] > \left[\left((1 - t)y_2\right)^\beta e(e^{pub}, M_{-}, 1)^\eta_1 q^{\eta_2}\right]
\]  (B10)

This implies

\[
U^{PRI}(e_1^* | y_2, \mu_1) > U^{PUB}(m = 1 | y_2, \mu_1, q, e^{pub})
\]  (B11)

which means that household 2, characterized by \( \{y_2, \mu_1\} \) would also prefer a private school procuring the school effort that is optimal to household 1 \( (e_1^*) \) instead of attending public school and monitoring it. This result is driven by SCI. Hence, at the optimal school effort for household 2 \( (e_2^*) \), household 2’s utility would be even higher:

\[
U^{PRI}(e_2^* | y_2, \mu_1) \geq U^{PRI}(e_1^* | y_2, \mu_1) > U^{PUB}(m = 1 | y_2, \mu_1, q, e^{pub})
\]  (B12)
which implies (B7). So a household with \( \{ y_2, \mu_1 \} \) must also choose private school. The reverse argument applies for public school choice, with households having a weak preference for public school.

To prove 1b, consider household 2 with \( \{ y_1, \mu_2 \} \), and \( \mu_1 < \mu_2 \). (A8) implies

\[
(1 - t)y_1 - T^*(e_1^*) (e_1^*)^{\eta_1} \mu_2^{\eta_2} > (1 - t)y_1 (e^{pub}, M_-, 1)^{\eta_1} q^{\eta_2}
\]  

This, in turn, implies

\[
U^{PRI}(e_1^* | y_1, \mu_2) > U^{PUB}(m = 1 | y_1, \mu_2, q, e^{pub})
\]  

In other words, household 2 would prefer to attend the private school that is optimal for household 1 rather than attending public school and monitoring it. This means that household 2 would attain an even higher utility from attending a private school that provides household 2’s optimal effort (\( e_2^* \)):

\[
U^{PRI}(e_2^* | y_1, \mu_2) \geq U^{PRI}(e_1^* | y_1, \mu_2) > U^{PUB}(m = 1 | y_1, \mu_2, q, e^{pub})
\]  

The reverse argument applies for public school choice, with households having a weak preference for public schools. Sufficient conditions for property 1b are that the marginal benefit of household learning effort \( a \) is linear, that the cost of \( a \) is quadratic and enters additively in the utility function, and that the cost of monitoring enters additively in the utility function with monitoring being a binary choice.

To prove claim 2a, consider household 1 with \( \{ y_1, \mu_1 \} \) that chooses to monitor, or:

\[
U^{PUB}(m = 1 | y_1, \mu_1, q, e^{pub}) \geq U^{PUB}(m = 0 | y_1, \mu_1, q, e^{pub})
\]  

We need to show that if \( y_1 < y_2 \), then we have

\[
U^{PUB}(m = 1 | y_2, \mu_1, q, e^{pub}) \geq U^{PUB}(m = 0 | y_2, \mu_1, q, e^{pub})
\]  

From (B13), we derive

\[
[e(e^{pub}, M_-, 1)^{2\eta_1} - e(e^{pub}, M_-, 0)^{2\eta_1}] [((1 - t)y_1)^{\beta} q^{\eta_2}]^2 \frac{\mu_1}{2\rho_a} \geq \rho_m \frac{1}{2\mu_1}
\]  

If \( y_1 < y_2 \), then we have

\[
[e(e^{pub}, M_-, 1)^{2\eta_1} - e(e^{pub}, M_-, 0)^{2\eta_1}] [((1 - t)y_2)^{\beta} q^{\eta_2}]^2 \frac{\mu_1}{2\rho_a} > \rho_m \frac{1}{2\mu_1}
\]
which implies (B14). In other words, household 2 with \( \{y_2, \mu_1\} \) that attends public school must also choose to monitor. This result is driven by SCI. Similarly, (B15) also implies that if \( \mu_1 < \mu_2 \), then

\[
\frac{e(e^{pub}, M, 1)\eta_1 - e(e^{pub}, M, 0)\eta_1}{2\eta_1} \left[ \left( (1 - t)y_1 \right)^\beta q^{\eta_2} \right]^2 \frac{\mu_2}{2\alpha} > \rho m \frac{1}{2\mu_2} \quad (B17)
\]

Hence, household 2 with \( \{y_1, \mu_2\} \) that attends public school must also choose to monitor. The following conditions are sufficient for this result: the marginal benefit of household learning effort \( a \) is linear, the cost of \( a \) is quadratic and enters additively in the utility function, and the cost of monitoring enters additively in the utility function with monitoring being a binary choice. 2b is proved by reversing the arguments and replacing the weak inequalities with strict inequalities.

To prove claim 3, we need to show that if \( e^{PUB} = e^{PRI} = e^{STD} \), then \( e^{STD} - e(e^{STD}, M) > e^{STD} - e(e^{STD}) \) if and only if \( M < N \), where \( e(e^{STD}, M) \) denotes the optimal effort chosen by the public school in response to \( e^{PUB} = e^{STD} \) and total monitoring \( M \), and \( e(e^{STD}) \) denotes the optimal effort chosen by the private school in response to \( e^{PRI} = e^{STD} \). In other words, we need to show that if the effort standard (or promised effort) is the same for public and private schools but the monitoring rate is higher in the private school, then the effort distortion is higher in the public school.

Recall from the text that taking \( m \) as given, the private school’s objective function (equation 4 in the text) is

\[
\pi^{pri} = T - Ae^\lambda - \frac{\alpha m}{2} (e^{pri} - e)^2
\]

And taking \( N \) and \( M \) as given, the public school’s objective function (equation 7 in the text) is

\[
\pi^{pub} = (X - Ae^\lambda)N - \frac{\alpha M}{2} (e^{pub} - e)^2
\]

Since \( N \) is taken as given by the public school when choosing \( e \), (B19) can be transformed to

\[
(X - Ae^\lambda) - \frac{\alpha M/N}{2} (e^{pub} - e)^2
\]

With respect to the school’s control variable \( e \), (B18) and (B20) are equivalent. Both public and private schools choose effort according to the same function, taking two parameters as given - the effort standard \((e^{PUB} \text{ or } e^{PRI}) \) for public and private schools, respectively) and the monitoring
rate \((m\ or\ M/N\ for\ public\ and\ private\ schools,\ respectively)\). Below, we show that along the optimal school effort function, \(e\) is increasing in \(m\) and in \(M/N\). This suffices to prove claim 3.

Consider a public and private school subject to the same effort standard, \(e^{STD}\). In this case, the first order conditions for (A18) and (A20) have the following form:

\[
-\lambda A e^{\lambda - 1} + \alpha \theta (e^{STD} - e) = 0
\]  

(B21)

where \(\theta\) is either \(m\) or \(M/N\) for private and public school, respectively. Totally differentiating (B21) yields

\[
\frac{de}{d\theta} = \frac{\alpha (e^{STD} - e)}{\lambda (\lambda - 1) A e^{\lambda - 2} + \alpha m} > 0
\]  

(B22)

The inequality comes from the fact that \(e^{STD} - e > 0, \lambda > 1,\) and \(A, a, m > 0\). Hence, along the optimal school effort function, \(e\) is increasing in \(\theta\).

Since households always monitor in private schools, we have \(\theta=m=1\). For public school, \(\theta=M/N\). Given (B22), the necessary and sufficient condition for \(e^{STD} - e(e^{STD}, M) > e^{STD} - e(e^{STD})\) (i.e., a larger school effort distortion in public school) is \(M/N < m=1\) or \(M < N\) (i.e., not all public households monitor in equilibrium).
Appendix C: Computational Considerations

In this appendix we offer details regarding the construction of the variables matched in the calibration, the algorithm used to compute the equilibrium, and the first-order effect of each parameter in the model.

Measurement of the Variables Matched in the Calibration

The first and second variables we match (fraction of households with children in private schools, and average income for household with children in private schools) are straightforward to construct. The third variable is average private school tuition. The actual average is $4,700 according to US Department of Education (2002). We match an average of $5,000 to account for the fact that Catholic schools comprise almost half of the private school enrollment in 2000, and their tuition is often subsidized (Guerra and Donahue 1990), a feature that our model does not capture. For a model that incorporates this subsidy, see Ferreyra (2007).

The fourth variable matched in the calibration is proportional difference between average public and private school teacher salaries. According to the 1999-2000 Schools and Staffing Survey, the average salary for public and private school teachers is $42,900 and $29,800 respectively – namely, a 44% premium for public over private school teachers.

The fifth variable matched in the calibration is difference between average effort among private v. public school teachers. According to the 1999-2000 Schools and Staffing Survey, teachers in public and private schools work virtually the same number of weekly hours (about 38 hours required at school, and 50 hours including all school-related activities). Hence, we consider the observed value for the proportional difference in teacher effort to be zero.

The sixth variable matched is difference in average achievement between private and public school students. According to the 2000 National Assessment for Educational Progress, private school students score between 0.40 and 0.50 standard deviations higher than public school students depending on the grade (4th, 8th or 12th) and the subject (math or reading). Hence, we match a value of 0.45 standard deviations for this variable.

The seventh variable matched is difference in average ability between private and public school students. Based on the National Education Longitudinal Survey, Epple et al (2004) report that 8th grade scores among private high school students are 0.76 standard deviations higher, on
average, than among public high school students. While 8th grade scores are not an ideal measure of ability, we do not know of other evidence on ability sorting across public and private schools.

The eighth variable matched is difference in average student effort between private and public schools, measured in units of standard deviation. We use data from the 2004 Digest of Education Statistics, as data for the variable is not available for 2000. This average is equal to 8.5 and 5.9 hours for private and public school students, respectively.

The ninth variable matched is the fraction of households who monitor in public schools, or public school monitoring rate. The Digest of Education Statistics reports the percent of children whose parents participate in general school meetings, parent-teacher conferences, class events, and volunteering activities. For 1999, these percents were equal to 76.8, 71.4, 63.5, and 33.8 respectively for public schools, and 91.4, 85, 81.7 and 63.8 for private schools. Constructing a simple average of the four activities, and normalizing the private school average to 100 since our model views private schools as a benchmark of full parental monitoring, we arrive at a public school monitoring rate of 0.76.

We believe that for the fifth through ninth variables, our measures are either a lower or an upper bound on the actual constructs of interest. For instance, an hour of effort by a private school teacher may yield a higher educational input (i.e., a higher value of $e$) than an hour of effort by a public school teacher if the former is more qualified than the latter. Some empirical literature suggests that this may indeed be the case (Hoxby 2002b, Ballou and Podgursky 1998). Similarly, our measure of student effort does not include other activities that require student effort, such as attending class or behaving in class, though attendance is indeed higher among private than public schools students (US Department of Education 2002). Both ability and achievement differences are likely to be biased downwards because they are based on test scores, which are truncated at the top. The monitoring rate is also likely to be biased, as the survey on which it is constructed does not specify the frequency with which parents exert their monitoring activities. For instance, while parents may participate in the monitoring activities listed in the survey, they may not participate in other activities such as communicating regularly with the teacher and other parents. As Hassrick and Schneider (2009) note, teachers are difficult to monitor because they work in relatively closed classroom spaces, yet parents differ widely in their ability to “open” the closed classroom door and exert everyday teacher surveillance in the
classroom. For these reasons, we attach to the first four variables a weight ten times as large as that of the remaining variables.

**Computation of the Equilibrium**

Our algorithm to compute the equilibrium proceeds as follows: for a given $\theta$, we consider all possible $(N, M, q)$ combinations by drawing an extremely fine grid for public school peer quality $q$ (recall that $N$ and $M$ are integers in our representation, with $N$ ranging between 0 and 25 and $M \leq N$). We compute household and school choices for each of these combinations. This yields a predicted $(N', M', q')$ for each $(N, M, q)$. If $(N', M', q') = (N, M, q)$ and public school profits are non-negative, then we have an equilibrium. Thus, our algorithm is capable of finding all equilibria for a given $\theta$. We have never found multiple equilibria.

**First-Order Effect of Parameter Changes**

A higher coefficient of consumption in the utility function ($\beta$) raises the share of consumption allocated to income and lowers private school tuition. A higher elasticity of achievement with respect to school effort ($\eta_1$) raises the demand for school effort, hence raising private relative to public school teacher effort and increasing private school attendance. A higher elasticity of achievement with respect to peer quality ($\eta_2$) raises demand for private schools on the part of high-ability households and increases private school attendance. A lower disutility of household learning effort ($\rho_a$) makes every household exert higher more effort. However, since the optimal effort is increasing in household income and ability and school peer quality, students in private schools raise their effort to a larger extent, hence widening the gap between private and public school student effort. A lower disutility of monitoring effort ($\rho_m$) raises the public school monitoring rate and the public school effort, hence raising public school attendance.

A higher reservation utility for teachers ($A$) raises the cost of any given level of teacher effort. It lowers the demand for teacher effort in all schools, though more so in private schools because teacher compensation in private schools is more sensitive to changes in the market value of teacher effort. Hence, a higher $A$ lowers the difference in teacher effort between private and public schools. Since teacher effort is usually less than one, a lower elasticity of teacher wages with respect to effort ($\lambda$) also raises the cost of teacher effort but can lower the agency cost,
particularly at the high-effort private schools attended by the highest-income households. This can lead to an increase in the demand of teacher effort in these schools and hence to a greater effort gap between private and public school teachers.

A higher agency cost ($\alpha$) raises private and public school effort. In addition, it raises the payoff to household monitoring, hence raising the public school monitoring rate. However, when $\alpha$ is very high, it leads to lower monitoring rate. The reason is that the higher effort attracts some high-ability, monitoring households into public schools, hence leading lower-ability households to no longer monitor. A higher effort standard in public schools ($e_{pub}$) leads to higher effort in public schools, lower gap between public and private school efforts, and higher public school attendance and monitoring rate. However, very high levels of $e_{pub}$ create negative profits for the public school and hence drive it out of business.

**Public Monitoring of Public Schools**

We parameterize the level of monitoring as follows: $M_0 = m_0 N$, and use $N=21$ in our calculations since this is the benchmark equilibrium public school attendance. Henceforth we refer to $m_0$ as the intensity of public monitoring, and use $m_0$ equal to 0.25, 0.5 and 0.75 to denote low-intensity, medium-intensity and high-intensity public monitoring respectively.

Lacking direct empirical evidence on the cost of this type of policy, we calibrate the unit cost of $M_0$, $\kappa$, as follows. We assume that the cost of public monitoring is proportional to total public school funding, i.e., $\kappa M_0 = \gamma XN$, where $\gamma$ is a factor of proportionality. Hence, $\kappa = \gamma X / m_0$. We calculate $\kappa$ as the unit cost of low-intensity monitoring in the baseline equilibrium, or $\kappa = \gamma *$7,000/0.25.
# Appendix D

## TABLE 1 - Private School Vouchers under Perfect Observability

<table>
<thead>
<tr>
<th>No Voucher</th>
<th>Universal Low Voucher</th>
<th>Universal High Voucher</th>
<th>Income-targeted Low Voucher</th>
<th>Income-targeted High Voucher</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>Fraction Public School</td>
<td>0.96</td>
<td>0.2</td>
<td>0</td>
<td>0.72</td>
</tr>
<tr>
<td>Avg. Income Public School</td>
<td>$55,000</td>
<td>$57,600</td>
<td>$57,600</td>
<td>$57,500</td>
</tr>
<tr>
<td>Avg. Income Private School</td>
<td>$119,400</td>
<td>$57,600</td>
<td>$57,600</td>
<td>$57,500</td>
</tr>
<tr>
<td>Avg. Ability Public School</td>
<td>99</td>
<td>81</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Avg. Ability Private School</td>
<td>119</td>
<td>105</td>
<td>100</td>
<td>114</td>
</tr>
<tr>
<td>Monitoring Rate</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Public School</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Private School</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Avg. Tuition Private School</td>
<td>$6,400</td>
<td>$4,400</td>
<td>$7,000</td>
<td>$4,400</td>
</tr>
<tr>
<td>Avg. Promised School Effort</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public School</td>
<td>0.66</td>
<td>0.66</td>
<td>0.66</td>
<td>0.66</td>
</tr>
<tr>
<td>Private School</td>
<td>0.71</td>
<td>0.58</td>
<td>0.74</td>
<td>0.59</td>
</tr>
<tr>
<td>Avg. Actual School Effort</td>
<td>0.67</td>
<td>0.60</td>
<td>0.74</td>
<td>0.64</td>
</tr>
<tr>
<td>Public School</td>
<td>0.66</td>
<td>0.66</td>
<td></td>
<td>0.66</td>
</tr>
<tr>
<td>Private School</td>
<td>0.71</td>
<td>0.58</td>
<td>0.74</td>
<td>0.59</td>
</tr>
<tr>
<td>Public School Profit</td>
<td>$35,600</td>
<td>$7,400</td>
<td>$26,700</td>
<td>$19,300</td>
</tr>
<tr>
<td>Avg. Teacher Compensation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public School</td>
<td>$7,000</td>
<td>$7,000</td>
<td>$7,000</td>
<td>$7,000</td>
</tr>
<tr>
<td>Private School</td>
<td>$6,400</td>
<td>$4,400</td>
<td>$7,000</td>
<td>$4,400</td>
</tr>
<tr>
<td>Avg. Use of School Revenues</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public School</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Salaries</td>
<td>0.79</td>
<td>0.79</td>
<td>0.79</td>
<td>0.79</td>
</tr>
<tr>
<td>Agency Cost</td>
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<td>0</td>
<td>0</td>
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</tr>
<tr>
<td>Rent</td>
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<td>0.21</td>
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<td>0.21</td>
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<tr>
<td>Private School</td>
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<td></td>
</tr>
<tr>
<td>Salaries</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Agency Cost</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Rent</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
</tr>
<tr>
<td>Avg. Hh. Learning Effort</td>
<td>0.06</td>
<td>-0.24</td>
<td>0.14</td>
<td>0.09</td>
</tr>
<tr>
<td>Public School</td>
<td>-0.06</td>
<td>-0.16</td>
<td></td>
<td>-0.08</td>
</tr>
<tr>
<td>Private School</td>
<td>3.08</td>
<td>0.33</td>
<td>0.14</td>
<td>0.53</td>
</tr>
<tr>
<td>Avg. Achievement</td>
<td>0.09</td>
<td>0.33</td>
<td>0.24</td>
<td>0.13</td>
</tr>
<tr>
<td>Public School</td>
<td>-0.09</td>
<td>-0.27</td>
<td>0</td>
<td>-0.14</td>
</tr>
<tr>
<td>Private School</td>
<td>4.37</td>
<td>0.48</td>
<td>0.24</td>
<td>0.83</td>
</tr>
<tr>
<td>Prop. Who Gain Achievement</td>
<td>0.48</td>
<td>0.60</td>
<td>0.28</td>
<td>0.40</td>
</tr>
<tr>
<td>Among Low-Income Hhs</td>
<td>0.40</td>
<td>0.60</td>
<td>0.40</td>
<td>0.60</td>
</tr>
<tr>
<td>Income Tax Rate</td>
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<td>0.07</td>
<td>0.12</td>
<td>0.1</td>
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<td>Aggregate Welfare</td>
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<td>1.96E+13</td>
<td>1.47E+13</td>
<td>1.16E+13</td>
</tr>
<tr>
<td>Prop. Who Gain Welfare</td>
<td>0.60</td>
<td>0.60</td>
<td>0.60</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: Column (1) is the benchmark equilibrium for perfect observability. Income-targeted vouchers are for households with incomes below $50,000. “Low-Income” means income = 10th percentile.