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"Online Sales with Commissions"

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Online Sales with Commissions*

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Abstract

This paper investigates price dispersion in the online duopoly market with commission rates. Compared to previous literature, we add an intermediate role, a cashback site, into the model, and allow that retailers strategically use commissions to obtain informed buyers. We find that the retailer who offers a lower commission rate has a non-continuous density function and obtains the same profits as those it makes when it charges reservation price to its loyal buyers. The retailer who gives a higher commission rate achieves greater profits than its rival. It seems reasonable that the way retailers promote their products on the cashback site would increase more competition between online retailers and possibly drive prices down. Losing part of profits by offering commissions, retailers, however, raise prices to compensate for the costs. At the end, this sale pattern does not increase competition, as we expected, but is anti-competitive. All buyers have to face higher average prices.

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1 Introduction

In a traditional sale pattern, manufacturers pay retailers fixed fees called slotting allowances to buy shelf space so that manufacturers can obtain customer patronage. In a two-part tariff model, manufacturers must determine the slotting allowances and the wholesale price to maximize their profits. This sale pattern is the most common in "brick and mortar" business. In the online market, we can observe a similar relationship that exists between online retailers and some special sites. These special sites are typically online forums or search engines.

One of the distinct characteristics of such sites is having a large number of frequent users. Many users on the online forums post their reviews for products or recommend where others can find a good merchandise from online retailers. Thanks to the users' reviews, other users refer this information when they want to buy similar products. This kind of forum on the internet is very popular. The most famous are fatwallet.com and slickdeals.net, each of which have a large number of frequent loyal users who share information with each other.

On the other hand, product search engines are a popular tool for online shoppers due to the free search services provided. In addition to price information of products, they list other information, such as store reviews and shipping information about the retailers. Many online shoppers would like to search products and compare prices first before making purchases. As a direct result, search engines possess a lot of regular users. Online retailers, especially those entrants, want internet shoppers to know where they are selling products and what they are selling. For this reason, they could buy advertising space from search engines or popular websites, or pay websites fees if they receive purchases through links on other websites. Like slotting allowances, these fees are used to buy link spaces to reach more online shoppers. Such fees can be categorized into two types, click-through fees and commissions.

Click-through fees are the cost if shoppers click the link on the websites regardless of real purchases. Some examples are Pricegrabber.com and Nextag.com. They
receive referral fees even though buyers may just click the link to read more product and seller information. Commissions, however, are based on real purchases. If shoppers successfully make purchases from links provided by other sites, retailers will pay a proportion of total purchases to these sites. Examples are bing.com and ebates.com. "These savings sites aren't retailers themselves. Rather, they link you to online merchants and collect commissions when you buy things." (Wall Street Journal November 26, 2005). But how do these sites let online shoppers buy products from links on their own websites instead of buying directly from online retailers? As a purchase incentive to online shoppers, some of these websites return a part of commissions received from retailers to online shoppers. These websites are called "cashback sites" in this paper. Online shoppers have to register an account on cashback sites. When they log on to the account, they have information about prices from many retailers and the percentage different sites will return them. In this paper, we call these online shopper "informed buyers". Rand (2005, Forbes) gives a vivid description of the shopping behavior of informed buyers. He says that "one good way to find deals is to find the cheapest price on a comparison-shopping site and then check at Ebates.com to see if there is cashback offered for the merchant you found."

More and more cashback sites are emerging on the internet nowadays. Bing.com, an online search engine by Microsoft, provides price comparisons and cashback at the same time. From cashback sites, online shoppers not only obtain information of price comparisons, but also know how much the site will return. As an example, suppose one goes to bing.com and choose the tag "shopping" on the left column, a search for a Nikon D40 camera will generate the page shown in the figure 1. The information on the page includes where to buy the camera, cashback percentages and final prices after cashback. Price-oriented shoppers would choose the lowest prices on this site if the retailer is a trusted merchant. The figure 1 indicates Photo For Less is offering the lowest price after cashback of 2%. Informed buyers would choose this retailer to buy the camera. Due to the positive reward, the commission delivered by Photo For Less to bing.com is believed to be higher than 2%. For example, if one buys Nikon D40 from
Buy.com, the cashback is 3%. The highest cashback is from Officedepot, reaching 10%.

This paper focuses on a duopoly model where online retailers sell products directly and through cashback sites. The model adds commission rates as a factor that influences price dispersion. Similar to Varian (1980), Narasimhan (1988) and Baye and Mogan (2001), online buyers are divided into two parts, informed buyers (shoppers) and uninformed buyers (loyal buyers). Without commissions and cashbacks involved, Varian (1980) argues that informed buyers would enjoy the lower average price when each firm has an identical segment of the uninformed buyers. Narasimhan (1988) finds that the size of uninformed buyers determines the equilibrium price distribution. The firm with a larger size of loyal customers tends to charge a higher price. Greg Shaffer (1991) report that retailer prices are higher when manufacturers are able to use slotting allowances to obtain shelf space than when they are not. Baye and Morgan (2001) employ a symmetric model with a one-time advertising fee. Baye and Morgan (2002) further extend the model to allow price discrimination. The fees charged by comparison sites play a key role in determining the price dispersion and equilibrium.
profits. They find that when firms pay a positive fee to list their price at a gatekeeper’s site, prices listed at the gatekeeper’s site are lower than prices at firms’ own websites. As we regularly observed, however, price discrimination is not a common phenomenon in the online market. Li, Chenguang, et al. (2008) revisit Narasimhan’s (1988) model. However firms in their asymmetric model have two decisions to make, advertising and prices. They report that in equilibrium the firm with the smaller size of loyal buyers advertises more aggressively but prices less competitively than the firm with the larger loyal market.

This paper re-investigates price dispersion with commissions involved. Online retailers in the model are assumed to have two policies to decide, commission rate and price. Since commission rates observed from cashback sites change much less frequently than the prices and are determined by other factors, we further assume nature first determines commission rates and retailers then decide price. The paper focuses on the retailers’ stage and examines the price dispersion, assuming the commission rates are given. The model used in the paper is a special case of Varian (1980), Narasimhan (1988) and Baye and Mogan (2001). We find the commission rate affects not only price limits, but also the equilibrium price distribution. In the equilibrium, a retailer that offers a higher commission rate charges higher prices more frequently than a retailer who gives a lower commission rate. On average, the retailer that gives higher commission rates tends to price higher. We further compare the model with commission rates to the model without. The way that online retailers use cashback sites to promote their products seems to increase competition between retailers. However, we find that both retailers would price higher on average at equilibrium. In other words, buyers have to bear the higher final prices even though a part of them enjoy some cash back. Considering the commission, retailers increase their profits by raising prices. Besides the retailers, the cashback site is another beneficiary in this anti-competition sale pattern.

In the section 2 we present a symmetric model in which two online retailers have an identical segment of loyal buyers. Using backward induction, we lay out the optimal decision of the cashback site in section 3. Retailers’s pricing decision, average price
comparison and related comparative statics are given in section 4. We then conclude in section 5.

2 Model

In this section, we lay out assumptions that the model is built on. To examine the cashback site's strategy, we would consider a case in which two retailers have an identical commission rate first and then extend to a model with asymmetric commission rates.

We start with a model with symmetric segment of informed buyers. Considering an online market, we have the assumptions as follows:

1. There are two online retailers, RT1 and RT2, competing in a homogeneous market. They use a same cashback site as an affiliate in the market. They advertise their products by listing products associated with prices on the cashback site. If buyers click the link provided on the cashback site and make purchases successfully, retailers give commissions to the cashback site as advertising fees. Let $\alpha_i \ (\alpha_i \in (0, 1] \ (i = 1, 2))$ denote commission rates and $\alpha_i$ is a percentage of total purchases made by buyers. We assume $\alpha_i$ is pre-determined by nature $^1$. If retailers know they will lose the entire segment of informed buyers, retailers would charge buyers $r$, which is buyers’ reservation price. We assume that $p$ determined by retailers $i$ is drawn from a common distribution $F_i(p), p \in [p^*, r]^2$. The marginal costs of selling the homogeneous products is normalized to 0.

2. We assume that there is only one online site (CB) that provides cashback to buyers in the online market. Cashback is also a percentage of buyers’ purchased and is denoted $\beta_i \in [0, \alpha_i]$.

3. There are $N$ buyers with 1 unit of consumption in the market. Without loss of generality, the number of buyers is normalized to 1. We assume that there are

$^1$ Commission rates are usually determined by the market power of the retailer, the agreement between retailers and manufacturers as well as operational costs of retailers. We assume they are determined by the nature. And the nature makes the first move.

$^2$ The lower bound of price will be defined later.
two types of buyers, informed buyers (shoppers) and uninformed buyers (loyal buyers), with a unit of consumption. The informed buyers consult the cashback site first and purchase goods at the lowest price after cashbacks or say net price, \( p_i(1 - \beta_i) \), through the cashback site, while the uninformed buyers only purchase from their favorite online retailer. Let the number of the uninformed buyers for each retailer be \( U_i \). For simplicity, we assume \( U_1 = U_2 = U \). We denote \( I \) as the remaining of buyers, informed buyers. \( 2U + I = 1 \) and \( I < U \) are held. A few more specific assumptions about informed buyers follow. Informed buyers first go to the cashback site due to no costs of visiting the cashback site and purchase at the lowest net price on the site. If the net price \( p_1(1 - \beta_1) \) is equal to \( p_2(1 - \beta_2) \), informed buyers would randomly pick a retailer. If none of retailers would like to advertise on the cashback, informed buyers would not make any purchase.

The timing and decisions made by buyers, retailers and the cashback site are as follows. First, the nature makes the first move of setting commissions. Given the commission rates, retailers advertise their products and prices on the cashback site. Observing the prices \( p_i \) and commission rates \( \alpha_i \), the cashback site announces cashback rates \( \beta_i \) on its own website. Informed buyers finally shop in this online market. Their decision depends on commission rates between retailers and the cashback site, whether retailers list prices on the cashback site, retailers' pricing decisions and the cashback site's cashback decisions. As we know, uninformed buyers will buy directly from the site of their favorite retailer.

In the assumptions above, buyers’ optimal decisions are already given. To analyze equilibrium behaviors of all other participants in this three-stage game, we first examine CB’s strategy using the backward induction.

3 Cashback Site’s Decisions

The goal of CB is how to allocate informed buyers using cashback to maximize its profits. Its profits function is followed by
\[ \pi_{CB} = \begin{cases} p_i(\alpha_i - \beta_i) & p_i(1 - \beta_i) < p_j(1 - \beta_j) \\ \frac{1}{2}p_i(\alpha_i - \beta_i) + \frac{1}{2}p_j(\alpha_j - \beta_j) & p_i(1 - \beta_i) = p_j(1 - \beta_j) \end{cases} \]

where \( i \neq j \).

CB’s decisions depend on prices and commission rates while informed buyers decisions are determined by prices and cashback rates. We break the model down to two cases, symmetric commission rates and asymmetric commission rates, to explore CB’s behaviors.

## 3.1 Symmetric Commission Rates

We consider a simplified case with an assumption of an identical commission rate offered by two retailers, i.e. \( \alpha_1 = \alpha_2 = \alpha \). We further assume \( p_1 > p_2 \) without loss of generality. The maximum commission that CB can obtain from RT1 is \( \alpha p_1 \) and from RT2 is \( \alpha p_2 \). CB obviously cannot have the profits of \( \alpha p_1 \) since informed buyers choose the retailer who prices lower. In fact, CB has the power to change informed buyers’ selection. CB may offer cashback to informed buyers if they choose RT1. To achieve this, CB must offer \( \beta_1 \) as defined below to informed buyers and set \( \beta_2 = 0 \).

\[ \beta_1 = (1 - \frac{p_2}{p_1}) + \varepsilon \]

Where \( \varepsilon \) is a very small value. We can see that \( p_1(1 - \beta_1) = p_1 \frac{p_2}{p_1} - p_1 \varepsilon = p_2 - p_1 \varepsilon < p_2 \) and informed buyers go with RT1. However, it is not worth doing that for CB because that \( p_1(\alpha - \beta_1) \) is strictly less than \( p_2 \alpha \). Therefore, CB’s optimal strategy is listing prices for retailers without any cashback. The maximum profits that CB obtains are \( \alpha p_2 \). It is easy to see CB’s optimal strategy is same if \( p_1 < p_2 \). We notice that when \( \alpha_1 = \alpha_2 \) CB essentially plays no role in the game. We focus on asymmetric commission rates to find how asymmetric commission rates affect the price policy in the equilibrium.

\({\textsuperscript{3}}\) We can show that \( p_2 \alpha - p_1(\alpha - \beta_1) = (p_1 - p_2)(1 - \alpha) + \varepsilon > 0 \)
3.2 Asymmetric Commission Rates

When \( \alpha_1 \) is not equal to \( \alpha_2 \), we first use the following two examples to see how the commission rates and cashback rates change informed buyers' decisions.

**Example 1** Assume RT1 lists a price of $100 and give a commission of 5% to CB, while RT2's price is $99 and it provides a commission of 2%. If CB returns nothing to buyers, CB would have maximum commissions from retailers. If a buyer chose RT1, CB will receive $5. Otherwise, CB will obtain $1.98 only. However, informed buyers would not choose RT2 due to the higher price. Can CB receive a commission higher than $1.98? Yes, it can. Actually, CB could give informed buyers more cashbacks to realize this. More specifically, if CB returns 2% of the total to buyers, informed buyers then will choose RT1 due to the lower final price. CB still can enjoy $3 commission after subtracting $2 from the initial $5.

This example is illustrated in figure 2 shown below:

```
100 × 5% = 5
95
RT1

100

99 × 2% = 1.98
97.02
RT2

100 × 5% - 99 × 2% > 100 - 99
```

**Figure 2:** Illustration of Example 1.

When the commission difference is greater then the price difference, CB has an incentive to return part of the commissions to buyers. In this example the commission difference, \( 100 \times 5\% - 99 \times 2\% \), is greater than the price difference, \( 100 - 99 \). By the effect of CB's optimal strategy, RT1 will have all the informed buyers. Using a little algebra, we have \( 100(1 - 5\%) < 99(1 - 2\%) \). We notice that \( 100(1 - 5\%) \) are simply the profits from one unit of product sold by RT1 and \( 99(1 - 2\%) \) are those from one unit.
of product sold by RT2. What will happen if the commission difference is less than the price difference? CB’s optimal strategy under this situation is demonstrated in example 2.

**Example 2** Assume RT1 lists a price of $100 and gives a commission of 2.5% to CB, while RT2’s price is $99 and it provides a commission of 2%. Informed buyers would choose RT2 if the cashback site offers nothing to buyers. In fact, CB would have profits less than $99 × 2% if it induces buyers to choose RT1 by returning some commissions. Informed buyers choose RT2.

When the commission difference is less than the price difference, i.e. $100 × 2.5% − 99 × 2% < 100 − 99$, CB has no incentives to offer cashback so that informed buyers choose higher price products. Returning nothing to buyers is its best strategy. Informed buyers again choose RT2 who makes fewer profits from one unit of product sold.

From the two examples, we know that the cashback site does not always return cashback to buyers when the commission rates offered by two retailers are different. We are now ready to give the detailed analysis about CB’s optimal strategy. Since we know that CB may not return cashback to buyers, we divide its strategies into two possible actions, returning cashback and doing nothing.

When will the cashback site return nothing to internet buyers? Given the fact that informed buyers compare the net price to make purchase decision, they are indifferent to the products sold on two retailer’s websites if $p_i(1 − α_i) = p_j(1 − α_j)$. In this case, the cashback site is indifferent in two actions. It can either do nothing or return $1 − \frac{p_i}{p_n}$ to those buyers who choose retailer $j$. Both strategies result in the same profits of $p_iα_i$. Doing nothing is strictly better if there is some cost associated with returning cashback to buyers. As a result of the costs incurred in the cashback process, the cashback site would return zero commissions to any buyers. When $p_i(1 − α_i)$ is not equal to $p_j(1 − α_j)$, we have proposition 3 to characterize CB’s decisions in the equilibrium.

**Proposition 3** At equilibrium, CB’s optimal strategies of cashback rates are as follows when $p_i(1 − α_i) < p_j(1 − α_j)$, $i \neq j$
1. When $p_i > p_j$,
\[
\beta_i^* = (1 - \frac{p_j}{p_i}) + \varepsilon \\
\beta_j^* = 0
\]

Where $\varepsilon$ is a very small positive value. The profits of CB then are $p_i(\alpha_i - \beta_i)$.

2. When $p_i < p_j$,
\[
\beta_i^* = \beta_j^* = 0
\]

The profits obtained by the cashback site are $p_i\alpha_i$.

In case 1, retailer $i$ lists a higher price and offers a higher commission at the same time. In order to obtain a higher commission from retailer $i$, CB would choose an optimal strategy of cashback rate $\beta_i$ that is close to but greater than $1 - \frac{p_j}{p_i}$ and $\beta_j$ that is equal to 0. Informed buyers would choose retailer $i$ since the net price, $p_i(1 - \beta_i)$ is less than $p_j$. The cashback site, thus, has profits of $p_i(\alpha_i - \beta_i)$ which is strictly more than $p_j\alpha_j$.

In case 2, CB has no incentive to change informed buyers' decision. All informed customers will go for retailer $j$. Any commission return to informed buyers results in lower profits than $p_i\alpha_i$. Thus, the cashback site simply returns nothing to informed buyers.

**Lemma 4** Given the optimal cashback rates determined by CB, the probabiility that informed buyers choose retailer $i$ depends on prices and commission rates only.

**Proof.** The probability that informed buyers choose retailer $i$ is defined by $Pr(p_i(1 - \beta_i^*) < p_j(1 - \beta_j^*))$. From Porposition (3), we should have CB’s optimal policy as follows,
\[
\beta_i^* = \begin{cases} 
(1 - \frac{p_j}{p_i}) + \varepsilon & p_i(1 - \alpha_i) < p_j(1 - \alpha_j) \text{ and } p_i > p_j \\
0 & \text{otherwise}
\end{cases}
\]

We show that when $p_i(1 - \alpha_i) < p_j(1 - \alpha_j)$, $p_i(1 - \beta_i^*) < p_j(1 - \beta_j^*)$ is true whenever $p_i > p_j$ and $p_i < p_j$. Similarly we have $p_i(1 - \beta_i^*) > p_j(1 - \beta_j^*)$ when $p_i(1 - \alpha_i) > p_j(1 - \alpha_j)$.
\( \alpha_j \). With consideration of CB’s optimal policy, therefore, the prices and commission rates determine informed buyers’ selection. That is that informed buyers would choose whoever has a lower value of \( p_i(1 - \alpha_i) \). As a result, \( \Pr(p_i(1 - \alpha_i) < p_j(1 - \alpha_j)) = \Pr(p_i(1 - \beta^*_i) < p_i(1 - \beta^*_j)) \).

Lemma 4 implies that with the involvement of the cashback site retailers’ prices and commission rates are two factors to nail down informed buyers’ decision. The probability to choose retailer \( i \) is \( \Pr(p_i(1 - \alpha_i) < p_j(1 - \alpha_j)) \). Having the optimal CB’s decision, we are in the position to check retailers’ pricing decisions.

4 Retailers’ Pricing Decisions

Due to no listing fee and no marginal costs, it is strictly better off for retailers to list prices on the cashback site. Therefore each retailer in the market advertises on the cashback site. Informed buyers definitely purchase from CB but take prices and cashback into their consideration when they make decision. The decision of informed buyers depends on \( p_i(1 - \beta^*_i) \). The expected profits function of retailer \( i \) is defined as the following.

\[
E\pi_i = p_i[(1 - \alpha_i) \Pr(p_i(1 - \beta^*_i) < p_j(1 - \beta^*_j))I + U].
\] (1)

By lemma 4, however, the probability to choose retailer \( i \) for informed buyers is \( \Pr(p_i(1 - \alpha_i) < p_j(1 - \alpha_j)) \) due to involvement of CB, which is unrelated to cashback rates anymore. The expected profits then can be written as

\[
E\pi_i = p_i[(1 - \alpha_i) \Pr(p_i(1 - \alpha_i) < p_j(1 - \alpha_j))I + U]
\] (2)

Since commission rates are exogenous, retailers need determine prices to maximize their profits. We rewrite the equation above as the following.

\[
E\pi_i = p_i[(1 - \alpha_i)(1 - F_j(p_i \theta_i))I + U]
\] (3)

where \( \theta_i = \frac{1 - \alpha_i}{1 - \alpha_i} \), which is a constant.
Similar to Narasimhan (1988) and Li, Chenguang, et al. (2008), pricing equilibrium does not exist in pure strategies. The main reason is that with fixed commission rates one retailer can always change its price by a little amount to increase its profits no matter what its rival’s price is.

**Lemma 5** For any $\alpha_i \in [0, 1], i = 1, 2$, both RTs have no pricing Nash equilibrium in pure strategies.

**Proof.** We need discuss two cases to prove it. In case 1, we assume both RTs list an identical price on CB, i.e. $p_i = p_j = p$. The profit for each retailer then is

$$E\pi_i = p((1 - \alpha_i)(1 - F_j(p\theta_i))I + U), \quad i = 1, 2$$

If $\alpha_i < \alpha_j$, RT$_j$ has the entire segment of informed buyers. However, retailer $j$ still can increase profits by raising the price by a small amount as long as $p_j(1 - \alpha_j) < p_i(1 - \alpha_i)$.

If $\alpha_1 = \alpha_2 = \alpha$, the profit for each retailer then is

$$E\pi_i = p((1 - \alpha)\frac{1}{2}I + U), \quad i = 1, 2$$

However, one retailer would always undercut the price by a very small amount, $\delta$, to attract more informed buyers. The profit for the retailer becomes

$$E\pi' = (p - \delta)((1 - \alpha)I + U)$$

When $\delta < \frac{(1-\alpha)pI}{2(1-\alpha)I+U}$, $E\pi'$ is guaranteed to be greater than $E\pi_i$.

In case 2, we assume that two RTs list different prices on CB. Without loss of generality, we assume that $p_i > p_j$.

If $p_i(1 - \alpha_i) > p_j(1 - \alpha_j)$, retailer $j$ has all the informed buyers. It’s profit can always go up by increasing $p_j$ by a small amount as long as $p_j < \min(p_i, p_t\frac{(1-\alpha_i)}{(1-\alpha_j)}).

If $p_i(1 - \alpha_i) = p_j(1 - \alpha_j)$, retailers split the market of informed buyers evenly. Similarly, one retailer can raise profits by decreasing its own price by a small amount equal to $\delta < \frac{(1-\alpha_i)pI}{2(1-\alpha_i)I+U_i}$. 

13
In addition, we can further prove that there is no possibility that one RT has a pure strategy and another RT has a mixed strategy. Without loss of generality, we assume RT1 has a pure strategy by pricing at $p_1$. We notice that when $p_1(1 - \alpha_1) > p_2(1 - \alpha_2)$, the revenue for RT2 is $p_2((1 - \alpha_2)I + U)$; when $p_1(1 - \alpha_1) < p_2(1 - \alpha_2)$, the revenue is just $p_2U$. Under both conditions, the revenue for RT2 is strictly increasing in $p_2$. It is impossible for RT2 to be indifferent between several pure strategies when RT1 has a pure strategy of $p_1$.

Given no pricing equilibrium in pure strategies, we explore pricing equilibrium in mixed strategies in the next section. To simplify our analysis, we assume that $\alpha_1 < \alpha_2$ without loss of generality in the rest of the paper.

### 4.1 The Range of Prices

The range of prices plays a key role to determine equilibrium price distributions and equilibrium profits. It is worthwhile to visit the range of prices for two retailers before deriving price equilibrium in mixed strategies. The upper bound of both price ranges is apparently the reservation price $r$. The lower bound of the price is determined as follows.

$$p_1[(1 - \alpha_1)(1 - F_j(p_1 \theta_1))I + U] = rU$$

The lowest possible price is the price that guarantees the bottom line of profits equal to $rU$. Treating $\alpha_1$ as a fixed parameter, the lower bound of $p_1$ is

$$p_1 = \frac{rU}{(1 - \alpha_1)I + U}$$

Similarly, the lower bound of $p_2$ with the fixed commission rate is

$$p_2 = \frac{rU}{(1 - \alpha_2)I + U}$$

Retailers would not charge any price lower than the lower bound of prices just determined. If retailers charge a lower price than $p_1$, their profits would be strictly less
than if they charge \( r \) to their loyal customers only. As we can see, the lower bound of prices is affected by commission rates. It is easy to show that \( p_1 < p_2 \) when \( \alpha_1 < \alpha_2 \).

From equation ??, the probability that informed buyers select one retailer on the cashback site depends on the price and the commission rate, i.e. \( p_i(1 - \alpha_i) \). Given the existence of commission rate, the lowest price that the retailers are willing to charge is different from the lower bound we just derived. The lowest price essentially determines retailers' price equilibrium. In fact, \( \alpha_i \) affects the price equilibrium through changing the price range. The following lemma shows how the commission rate changes the retailer's optimal behaviors.

**Lemma 6** If \( \{ \alpha_i, \alpha_j \} \in \{ \alpha_i, \alpha_j \mid \alpha_i < \alpha_j, i \neq j \text{ and } \alpha_i, \alpha_j \in (0, 1) \} \), The price density function in the mixed strategy for retailer \( i \) is not continuous over its price range.

Proof. As defined in the expected profits function, the probability to have the segment of informed buyers for retailer \( i \) is \( 1 - F_i(p_j^{1-\alpha_j}) \). The highest price retailer \( j \) could charge is \( r \). Given that \( \alpha_i < \alpha_j \), retailer \( i \) only has its own loyal customers when it charges any price higher than or equal to \( r^{1-\alpha_j} \). Charging any price on \( [r^{1-\alpha_j}, r] \) would result profits less than \( rU \) which are profits when retailer \( i \) charged \( r \) to its loyal customers alone. In retailer \( i \)'s mixed strategy, therefore, there is no positive density over this interval. However retailer \( i \) has to charge \( r \) when retailer \( j \) charges \( p_j^{1-\alpha_j} \). In this case, retailer \( i \) cannot do better than charging \( r \) to guarantee the bottom line of \( rU \). The price range for retailer \( i \) then is \( \{ p | p \in [p_j^{1-\alpha_j}, r] \cup \{ r \} \} \). The dot line on the following figure shows the price range that retailers can but won't charge. Similarly, retailer \( j \) would not charge any price lower than \( p_j^{1-\alpha_j} \).

Assuming \( \alpha_1 < \alpha_2 \), we have \( p_1 < p_2 \) and \( p_1(1-\alpha_1) > p_2(1-\alpha_2) \). When two retailers compete against each other, retailer 2 has the ability to obtain the entire segment of informed buyers by choosing a price lower than \( p_1 \theta_1 \). Therefore the lowest price that retailer 2 is willing to charge is \( p_1 \theta_1 \), which is \( \frac{rU}{(1-\alpha_1) + U} \). On the other hand, we have that \( r(1 - \alpha_1) > r(1 - \alpha_2) \). RT1 would not charge a price on \( [r^{1-\alpha_2}, r] \). RT1 could do strictly better by charging \( r \) rather than charging any price on this interval. Let us
define $p_1^*$ as the lowest price of $p_i$. Then we should have $p_1^* = p_1$ and $p_2^* = p_1 \theta_1$. The price ranges for RT1 and RT2 are $[p_1^*, r^{1-\alpha_2}_{1-\alpha_1}] \cup \{r\}$ and $[p_2^*, r]$ respectively. We notice that $p_1^*$ is less than $p_2^*$ and both retailers have the same highest price that they are willing to charge.

### 4.2 Price Equilibrium

In this section, we study price equilibrium with asymmetric commission rates. Since $\alpha_1 < \alpha_2$, RT2 is able to reach a value of $p_2(1 - \alpha_2)$ that is lower than the lowest value of $p_1(1 - \alpha_1)$ that RT1 can achieve. In other words, RT2 can have the entire segment of informed buyers if it charges $p_2 \frac{(1-\alpha_1)}{(1-\alpha_2)}$. On the other hand, there is a positive density at $r$ in RT1’s mixed strategy. In the RT1 price equilibrium, we should have

$$p[(1 - \alpha_1)(1 - F_2(p\theta_1))I + U] = rU$$

RT2 would have the best equilibrium profit by charging $p_2 \frac{(1-\alpha_1)}{(1-\alpha_2)}$. At this price, RT2 not only has its own loyal buyers, but also obtains the entire segment of informed buyers.

$$p[(1 - \alpha_2)(1 - F_1(p\theta_2))I + U] = ((1 - \alpha_2)I + U)p_2 \theta_1$$

**Figure 3: Price Ranges**

$\alpha_i < \alpha_j, \frac{1-\alpha_i}{1-\alpha_j} > 1$
Let \( p' = p\theta_1 \). From 4,

\[
F_2(p') = 1 - \frac{(r - p)U}{pI(1 - \alpha_1)}
\]

Thus,

\[
F_2(p) = 1 - \frac{(r\theta_1 - p)U}{pI(1 - \alpha_1)}
\]

Note that \( \theta_i > 1 \) and is a constant term in the distribution. We can see that \( F_2(\frac{rU}{(1 - \alpha_1)I + U}\theta_i) = 0 \) and \( F_2(r) = 1 - \frac{(\theta_i - 1)U}{I(1 - \alpha_1)} \). Therefore retailer \( j \) has a mass point at \( r \) equal to \( \frac{(\theta_i - 1)U}{I(1 - \alpha_1)} \).

From 5, we have

\[
F_1(p) = 1 + \frac{U}{I(1 - \alpha_2)} - \frac{((1 - \alpha_2)I + U)rU}{(1 - \alpha_2)pI[(1 - \alpha_1)I + U]}
\]

We have that \( F_1(p_1) = 0 \), and \( F_1(r\frac{1 - \alpha_2}{1 - \alpha_1}) = 1 - \frac{U^2(\theta_i - 1)}{I(1 - \alpha_2)(1 - \alpha_1)I + U} \). Since \( RT1 \) would not charge any price higher than \( r\frac{1 - \alpha_2}{1 - \alpha_1} \) except \( r \), the distribution function shows that there is a mass point at \( r \) equal to \( \frac{U^2(\theta_i - 1)}{I(1 - \alpha_2)(1 - \alpha_1)I + U} \).

To summarize, two retailers adopt mixed strategies given by the following distribution functions

\[
F_1(p) = \begin{cases} 
0, & p < p_1^* \\
1 + \frac{U}{I(1 - \alpha_2)} - \frac{((1 - \alpha_2)I + U)rU}{(1 - \alpha_2)pI[(1 - \alpha_1)I + U]}, & p_1^* \leq p < r\frac{1 - \alpha_2}{1 - \alpha_1} \\
1 - \frac{U^2(\theta_i - 1)}{I(1 - \alpha_2)(1 - \alpha_1)I + U}, & r\frac{1 - \alpha_2}{1 - \alpha_1} \leq p < r \\
1, & p \geq r
\end{cases}
\]

and

\[
F_2(p) = \begin{cases} 
0, & p < p_2^* \\
1 - \frac{(r\theta_1 - p)U}{pI(1 - \alpha_1)}, & p_2^* \leq p < r \\
1, & p \geq r
\end{cases}
\]

Note further that the interval \( [r\frac{1 - \alpha_2}{1 - \alpha_1}, r) \) shrinks to an empty set as \( \alpha_1 \) approaches to \( \alpha_2 \). As a result, the density function of retailer 1 is continuous and equal to that of retailer 2 when \( \alpha_1 = \alpha_2 \). Asymmetric commission rates result in asymmetric price
distribution functions. Considering two distribution functions, we have the following proposition.

**Proposition 7** Given the equilibrium distribution functions, the following properties characterize two retailers’ price dispersions:

1) \( F_1(p) > F_2(p) \) for \( p \in [p_1^*, r] \)
2) \( E(p_1) = \int_{p_1^*}^{r} pdF_1(p) < E(p_2) = \int_{p_2^*}^{r} pdF_2(p) \).
3) \( E\pi_1 < E\pi_2 \).

**Proof.** 1) Since \( p_2^* > p_1^* \), we can prove \( F_1(p) > F_2(p) \) for \( p \in [p_1^*, r] \) if we show that \( F_1(p) - F_2(p) > 0 \) for \( p \in [p_2^*, r] \). We first show \( F_1(p) - F_2(p) > 0 \) for \( p \in [p_2^*, r^{1-\alpha_2/(1-\alpha_1)}] \). Let \( \Delta = \frac{(1-\alpha_2)I+U}{(1-\alpha_1)I+U} \), we have

\[
F_1(p) - F_2(p) = \frac{(r\theta_1 - p)U}{pI(1-\alpha_1)} - \frac{(r\Delta - p)U}{pI(1-\alpha_2)}
\]

\[
= \frac{(1-\alpha_2)(r\theta_1 - p)U - (1-\alpha_1)(r\Delta - p)U}{pI(1-\alpha_2)(1-\alpha_1)}
\]

\[
= \frac{(1-\Delta)(1-\alpha_1)r + (\alpha_2 - \alpha_1)p}{pI(1-\alpha_2)(1-\alpha_1)}
\]

since \( \Delta < 1 \), \( F_1(p) - F_2(p) > 0 \) for \( p \in [p_2^*, r^{1-\alpha_2/(1-\alpha_1)}] \). Next we show that \( F_1(p) - F_2(p) > 0 \) for \( p \in [r^{1-\alpha_2/(1-\alpha_1)}, r] \). Since \( F_1(p) \) is constant and \( F_2(p) \) is increasing in \( p \) on this interval, we only need show that \( F_1(r) - F_2(r) > 0 \).

\[
F_1(r) - F_2(r) = \frac{(\theta_1 - 1)U}{I(1-\alpha_1)} - \frac{U^2(\theta_1 - 1)}{I(1-\alpha_2)[(1-\alpha_1)I + U]}
\]

\[
= \frac{(\theta_1 - 1)U}{\Gamma} [(1-\alpha_2)(1-\alpha_1)I + U] - U(1-\alpha_1)]
\]

\[
= \frac{(\theta_1 - 1)U}{\Gamma} [(1-\alpha_2)(1-\alpha_1) - U(\alpha_2 - \alpha_1)]
\]

Where \( \Gamma = I(1-\alpha_2)(1-\alpha_1)[(1-\alpha_1)I + U] \).

Since \( \frac{(1-\alpha_1)(1-\alpha_2)}{(\alpha_2 - \alpha_1)} > \frac{U}{I}, I(1-\alpha_2)(1-\alpha_1) - U(\alpha_2 - \alpha_1) > 0 \). Therefore \( F_1(p) > F_2(p) \) for \( p \in [p_1^*, r] \).

2) 1) shows that \( P_2 \) is dominant over \( P_1 \). We have \( E(p_1) < E(p_2) \).
3) By the equilibrium, we have

\[ E\pi_1 = rU \]

and

\[ E\pi_2 = \left( (1 - \alpha_2)I + U \right) \theta_1 = \frac{(1 - \alpha_1)(1 - \alpha_2)I + (1 - \alpha_1)U}{(1 - \alpha_1)(1 - \alpha_2)I + (1 - \alpha_2)U} rU \]

\[ E\pi_1 < E\pi_2 \] directly follows that \( \frac{(1 - \alpha_1)(1 - \alpha_2)I + (1 - \alpha_1)U}{(1 - \alpha_1)(1 - \alpha_2)I + (1 - \alpha_2)U} > 1. \]

1) implies that the retailer providing higher commissions more likely charges higher prices than its competitor. This also explains 2). On average, retailer 2 tends to price higher. 3) states that retailer 1 earns the same profit as the most profit it would earn from its loyal buyers. But retailer 2 makes \( \frac{(1 - \alpha_1)(1 - \alpha_2)I + (1 - \alpha_1)U}{(1 - \alpha_1)(1 - \alpha_2)I + (1 - \alpha_2)U} rU > rU \), where \( rU \) is the profit it would make from its own loyal segment. As \( (\alpha_2 - \alpha_1) \to 0 \), the equilibrium is symmetric and the profit made by retailer 2 approaches \( rU \).

We notice that there is no mass point in the equilibrium pricing strategies when the commission rates offered by the two retailers are identical or 0. The equilibrium price distribution functions are identical for these two cases. More specifically, \( F_1(p) = F_2(p) = 1 - \frac{(r-p)U}{p} \) if \( \alpha_1 = \alpha_2 = 0 \).

To find the relationship between the equilibrium strategy and exogenous parameters, we analyze the comparative statics of the equilibrium price distribution with respect to fixed parameters. In addition to this, we include the lowest prices in the following table. In the table, \( p_i^* \) is the lowest price retailer \( i \) is willing to charge.

<table>
<thead>
<tr>
<th>( p_1^* )</th>
<th>( p_2^* )</th>
<th>( F_1 )</th>
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</table>

As we see from the table, the lowest prices are positively correlated with the reservation price \( r \). As the segment of loyal customers increases, two retailers lowest prices increase. \( \alpha_2 \) only affects RT2's lowest price. The more commissions given out, the higher the lowest price RT2 is willing to charge. In contrast to \( \alpha_2 \), \( \alpha_1 \) influences on the
lowest prices for both retailers. On the one hand, the lowest price for RT1 increases as $\alpha_1$ increases. On the other hand, RT2 would decrease the lowest price when RT1 raises the commission rate $\alpha_1$. It is plausible with economic intuition. When its rival uses higher commissions to win more informed buyers, the retailer would include more lower prices in its strategies.

The parameters' effects on the price distribution is intuitive, too. When $r$ and $U$ increase, $F_2$ decreases and the average price increases. Retailers have less incentive to compete for informed buyers if $U$ becomes large. An increase in the reservation price also would drive the average price up. The relation between $\alpha_2$ and $F_2$ has the same pattern. A higher commission rate offered by RT2 leads to a decrease in $F_2$. RT2 uses a higher average price to compensate for the commissions given to the cashback site. However, $F_2$ is positively correlated with the rival's commission rate. Facing an increase in $\alpha_1$, RT2 cuts the average price by increasing the likelihood it will charge lower prices in order to increase its competitive strength.

Since $F_1$ has two different functional forms on $\left[\frac{rU}{(1-\alpha_1)F+U}, r\right)$, we use two signs in the $F_1$ column. The first sign indicates comparative statics on $\left[\frac{rU}{(1-\alpha_1)F+U}, \frac{1-\alpha_2}{1-\alpha_1}\right)$ and the second sign implies the same on $\left[r \frac{1-\alpha_2}{1-\alpha_1}, r\right)$. Generally, $r$ and $U$ have the same effects on $F_1$ as on $F_2$. Given that 0 density appears on $\left[r \frac{1-\alpha_2}{1-\alpha_1}, r\right)$ and the functional form is independent of $r$ on this interval, $r$ has no effect on $F_1$ over this first interval. Interestingly, commission rates have different effects on the two intervals. A change in $\alpha_1$ impacts $F_1$ directly and indirectly. As $\alpha_1$ increases, $F_1$ decreases on the first interval directly. In the meantime, an increase in $\alpha_1$ makes the first interval wider. Due to the increasing value of $r \frac{1-\alpha_2}{1-\alpha_1}$, $F_1$ increases on the second interval in spite of decreasing on the first. In other words, retailer 2 decreases the likelihood of charging extreme prices as $\alpha_1$ increases. In contrast to $\alpha_1$, $\alpha_2$ has positive effects on $F_1$ over the first interval, and negative effects on $F_1$ over the second interval. When $\alpha_2$ increases, RT1 tends to increase the likelihood of charging lower prices as the way to decrease its own average price.
4.3 Equilibrium Profits

Before we examine equilibrium profits made by the two retailers, we take a look at the profits when two retailers give no commission to the cashback site. We assume that the cashback site could list their prices for free. Informed buyers will pick a retailer who offers a lower price. In this case, the equilibrium of the price distribution is a special case of Varian (1983), Narasimhan (1988) and Baye and Mogan (2001). Both retailers would not do better than when they ignore the segment of informed buyers and charge \( r \) to their loyal customers. Compared to the no commission case, there is no virtual difference when they provide an identical commission rate to the cashback site. They also sacrifice a part of profits to attract informed buyers. They end up with the same profits, \( rU \), as the profits when they don't offer commissions.

We turn to the case of different commission rates offered by two retailers. When \( \alpha_1 > \alpha_2 \), RT2 only obtains equilibrium profits which are equal to its profits when it charges \( r \) on its loyal customers only. When \( \alpha_1 < \alpha_2 \), RT2 enjoys higher equilibrium profits than \( rU \). The equilibrium profit function for two retailers as shown below is determined by the retailer’s commission rate and its competitor’s commission rate.

\[
E\pi_i = \begin{cases} 
  rU, & \alpha_i \leq \alpha_j \\
  \frac{(1-\alpha_i)(1-\alpha_j)}{(1-\alpha_i)(1-\alpha_j) + (1-\alpha_j)U} rU, & \alpha_i > \alpha_j
\end{cases}
\]

The equilibrium profits are decreasing as \( \alpha_2 \) is approaching \( \alpha_1 \). We notice that both retailers have the same equilibrium profits of \( rU \) when they don’t offer commissions. Compared to the strategy of no commissions, RT1 is not able to gain more profits if it’s commission rate is lower than its competitor. Offering higher commissions brings RT2 more profits than \( rU \). But both retailers are not worse off by providing commissions. How can they have no less profits even when they give a part of the revenue back to the cashback site? The only way to achieve this is by raising price. Retailers try to use the cashback site to induce competition among informed buyers because more competition drives prices lower. However, it appears that the strategy of giving commission may be anti-competitive because of the higher prices that online buyers have to face. In the
next section, we give a detailed analysis which compares the average prices between when a commission is offered and when no commission is given.

4.4 Average Price Comparison

In this section, we will compare the average price when retailers offer positive commission with the case without commissions. \( \bar{p}_1 \) and \( \bar{p}_1^N \) are denoted the average prices when retailers offer the commission and that when they don’t. The equilibrium distribution functions for two retailers when there is no commission involved are

\[
F_1^N(p) = F_2^N(p) = 1 - \frac{(r - p)U}{pI}
\]  

(9)

Since \( \theta_1 > 1 \), we have the following result From 6

\[
F_2(p) = 1 - \frac{(r\theta_1 - p)U}{pI(1 - \theta_1)} < 1 - \frac{(r - p)U}{pI} = F_2^N(p) \quad \text{for} \quad p \in [p_1, \theta_1, r)
\]

We know that \( p_2^* = p_1\theta_1 > p_1^N = \frac{rU}{I+U} \). Therefore, \( \bar{p}_2 > \bar{p}_2^N \). As RT2 offers commissions to the cashback site, it tends to charge higher prices more frequently. As a result, the equilibrium average price is higher than when it sells products without any commission.

There is no implicit solution when we directly compare the average prices of retailer 1 in both conditions using the distribution functions. We derive average prices using the equilibrium density functions. The density functions from the probability distributions as given by the following equations.

\[
f_1^N(p) = \frac{rU}{I} p^{-2}, \quad \frac{rU}{I + U} \leq p \leq r
\]

and

\[
f_1(p) = \begin{cases} 
\frac{(1-\alpha_2)(I+U)}{I(I-\alpha_1)(I-\alpha_1)(I+U)} \frac{rU}{I} p^{-2}, & \frac{rU}{I+U} \leq p \leq \frac{r}{1-\alpha_1} \\
\frac{rU}{I(I-\alpha_1)(I+U)} \frac{U^\alpha(\theta_1-1)}{[I(1-\alpha_2)(1-\alpha_1)+U]}, & p = r
\end{cases}
\]

22
As we can see in figure 4, there is a slight difference between the two density functions. In fact, the two density functions are simply \( p^{-2} \) but in different scales and supports. Notice that \( f_1(p) \) is not continuous and has a mass point at \( r \) that is not shown in the figure.

![Figure 4: Density with commission offered and that without commission.](image)

The average prices are

\[
\tilde{p}_1^N = \int_{rU}^{rU(1 - \alpha_2)} \frac{rU}{I} p^{-1} dp = \frac{rU}{I} (\log(I + U) - \log(U))
\]

\[
\tilde{p}_1 = \int_{rU}^{rU(1 - \alpha_2)} \frac{((1 - \alpha_2)I + U)rU}{(1 - \alpha_2)[(1 - \alpha_1)I + U]} p^{-1} dp + U^2(\frac{1 - \alpha_1}{1 - \alpha_2} - 1)r \]

There seems to be no simple answer to the difference between the two average prices. In order to explore the relationship between commission rate, loyal customer and average price difference, we use simulated data to show the price differences, \( \tilde{p}_1 - \tilde{p}_1^N \), in various levels of \( \alpha_2 \) and \( U \). In figure, \( r \) and \( \alpha_1 \) is fixed at 5 and 0.02 respectively in the simulation. \( U \) is randomly draw 500 times from a uniform distribution on \([0.34, 0.5] \). Given the fact that \( r \frac{1 - \alpha_2}{1 - \alpha_1} \geq \tilde{p}_1 \), values of \( \alpha_2 \) are randomly selected from \([0, 0.5] \) and satisfy these two conditions at the same time: 1) \( \alpha_2 > \alpha_1 \); 2) \( \frac{(1 - \alpha_2)(1 - \alpha_3)}{(\alpha_2 - \alpha_1)} \geq U^5 \).

\(^5\)Condition 2 guarantees that \( \frac{1 - \alpha_2}{1 - \alpha_1} \leq \frac{r}{U} \).
Figure 5: Price Difference with Varied Loyal Customers and $\alpha_2$.

From figure 5, we can see that the average equilibrium price with commissions is greater than that without commissions. However, the difference between average prices is shrinking overall as the segment of loyal customers increases or $\alpha_2$ increases. This seems intuitively plausible. When the proportion of loyal buyers is greater than that of informed buyers, retailers have no incentive to attract informed buyers, and the effects of the commission rate on the average price diminishes. On the other hand, when its rival's commission increases, RT1 would have lower prices on average in the equilibrium. Both strategies would drive the two average prices closer. We notice that the two average prices are approaching to $r$ when $U$ approaches to 0.5. Further study of this figure, the difference between average prices increases substantially when $\alpha_2$ and $U$ reach threshold levels. Specifically, when $U$ stays close to 0.5 and $\alpha_2$ is between 0.022 and 0.023 this difference increases sharply. The reason is that RT1 charges prices closer to $r$ more frequently when the proportion of informed buyers become negligible, and when its competitor gives a much higher commission that it does itself. Thus the average price increases substantially to $r$ and deviates from the average price when there are no commissions.
5 Conclusion

This paper investigates price dispersion in the online duopoly market with commission rates. We use a symmetric model in which two online retailers have an identical segment of loyal buyers. Compared to previous literature, we add an intermediate role, a cashback site, into the model, and allow that retailers strategically use commissions to obtain informed buyers. When the commission rates are asymmetric, price distributions at equilibrium are different from those obtained when two retailers list only their prices but offer no commissions. We find that at equilibrium both retailers have a mass point at \( r \) in their price distributions. The retailer who offers a lower commission rate has a non-continuous density function and obtains the same profits as those it makes when it charges \( r \) to its loyal buyers. The retailer who gives a higher commission rate, on the other hand, would have more market power and achieve the same profits as those it makes when it charges its preferred minimum price. Equilibrium profits are higher than the maximum profits it makes from its loyal buyers.

It seems reasonable that the way retailers use commissions to promote their products on the cashback site would increase more competition between online retailers and possibly drive prices down. At equilibrium, however, buyers, including uninformed and informed buyers, have to face higher average prices. Losing part of profits by offering commissions, retailers raise prices to compensate for the costs. At the end, this sale pattern does not increase competition, as we expected, but is somewhat anti-competitive. Both the retailer who gives higher commissions and the cashback site benefit from this online market. The final buyers have to bear the increase in prices and sacrifice consumer surplus.

The model in this paper is worth further discussion. First, it may extend to an asymmetric model in which retailers have different segment of informed buyers. Second, the optimal policy of commission rates is hard to evaluate due to many issues. If we solve \( \alpha_i \) in the first stage of the sequential game, the optimal strategy for two retailers is setting \( \alpha_i \) equa. to the maximum level, 1. This result is far from the observed
practices. We believe that the optimal value of $\alpha_i$ may depend on many other factors, such as the asymmetric segment of informed buyers, different wholesale prices given by manufacturers, different operational costs, etc. Finally we find that the cashback site does not always give cash back to customers even though the retailer gives the rebate site commissions. This discovery is not consistent with the evidence. The main reason for this anomaly is that there is no competition between cashback sites in the model. We only have one cashback site which is a monopoly in this online market.
References


