The Effects of Trade Liberalization on the Soft Budget Contrain

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Introduction

Literature Review

Model

I. Domestic market

1. Demand

- CES preferences:

$$\max_{q(x)} U = \left[ \int_0^N q(x)^\rho dx \right]^{\frac{1}{\rho}}, 0 < \rho < 1 \quad (1)$$

s.t. $$\int_0^N p(x)q(x)dx = I \quad (2)$$

- $$q(x):$$ demand of good $$x$$ / $$p(x):$$ price of good $$x$$,
- $$N:$$ The number of available goods, $$x \in [0, N]$$
- $$\sigma:$$ Constant elasticity of substitution
  i) $$\rho = \frac{\sigma-1}{\sigma} \leftrightarrow \sigma = \frac{1}{1-\rho} > 1 \quad : 0 < \rho < 1$$
  ii) $$\rho = 0:$$ Cobb-Douglas function (perfect differentiation) / $$\rho = 1:$$ Perfect substitution
     $$\therefore \sigma \uparrow (or \uparrow):$$ Substitution $\uparrow = \text{Products differentiation} \downarrow$

- $$I = \int_0^N I(x)dx:$$ Aggregate expenditure = total income (Given)

- $$Q = U = \left[ \int_0^N q(x)^\rho dx \right]^{\frac{1}{\rho}}:$$ Aggregate good (Given)

- $$P = \left[ \int_0^N p(x)^{1-\sigma} dx \right]^{\frac{1}{1-\sigma}}:$$ Aggregate price (Given) = Expenditure function of CES function (indirect utility)

  $$\therefore I = PQ$$

- Solve the maximization problem:

  $$\mathcal{L} = \left[ \int_0^N q(x)^{\frac{\sigma-1}{\sigma}} dx \right]^{\frac{\sigma}{\sigma-1}} + \lambda \left[ I - \int_0^N p(x)q(x)dx \right]$$

  i) $$\frac{\partial \mathcal{L}}{\partial q(x)} \rightarrow q(x)^{1/\sigma} = \lambda p(x)Q^{-\frac{1}{\sigma}} \quad (3)$$
ii) \( \frac{\partial c}{\partial \lambda} \to l = \int_0^N p(x)q(x)dx = PQ \) \hspace{1cm} (4)

iii) \( \lambda : \text{Marginal utility of income} \)

\[ \therefore \lambda = \frac{\partial U}{\partial l} = \frac{\partial Q}{\partial l} = \frac{1}{P} \] \hspace{1cm} (5)

- Optimal consumption for a particular variety \( x \):

From (3), (4) and (5),

\[ q(x) = \left( \frac{l}{P} \right) \left( \frac{p(x)}{P} \right)^{-\sigma} = \frac{Q}{p(x)} \left( \frac{p(x)}{P} \right)^{-\sigma} \] \hspace{1cm} (6)

& \( \varepsilon_p = -\frac{dq}{dp} = \sigma \) \hspace{1cm} (7)

- Optimal expenditure over varieties:

\[ i(x) = p(x)q(x) = p(x) \left( \frac{l}{P} \right) \left( \frac{p(x)}{P} \right)^{-\sigma} \]

\[ \therefore i(x) = \left( \frac{p(x)}{P} \right)^{1-\sigma} \] \hspace{1cm} (8)

2. Production

- Monopolistic competitive market
- The total cost function for each firm:

\[ TC(\theta) = f + \frac{1}{\theta} q \] \hspace{1cm} (9)

- \( f (> 0) : \text{Fixed cost (Given)} / \theta (> 0): \text{(heterogeneous) productivity (Given)} \)
- After entering the market, a firm will realize its productivity \( \theta \) and then make decisions.
- Profit maximization:

\[ \max_p \pi_d = pq(p) - \frac{1}{\theta}q(p) - f \] \hspace{1cm} (10)

- Optimal price:

\[ \frac{\partial \pi_d}{\partial p} \text{ & (7) } \to p(\theta) = \frac{1}{\rho \theta} \] \hspace{1cm} (11)

- Optimal quantity:

(6): \( q(\theta) = \left( \frac{l}{P} \right) \left( \frac{p(\theta)}{P} \right)^{-\sigma} = \frac{Q}{p(\theta)} \left( \frac{p(\theta)}{P} \right)^{-\sigma} = 1P^{\sigma-1}(\rho \theta)^{\sigma} \) \hspace{1cm} (12)

- Firm’s revenue:

\[ r_d(\theta) = pq = i(x) = \left( \frac{p(\theta)}{P} \right)^{1-\sigma} = \left[ \frac{1}{\rho \theta P} \right]^{1-\sigma} \]

\[ \therefore r_d(\theta) = \frac{1}{(\rho \theta P)^{\sigma-1}} \] \hspace{1cm} (13)

- Firm’s profit:

\[ \pi_d(\theta) = p(\theta)q - \frac{1}{\theta} q - f = q \left( \frac{1 - \rho}{\rho \theta} \right) - f = \frac{qp(\theta)}{\sigma} - f \]
\[ \pi_d(\theta) = \frac{r(\theta)}{\sigma} - f = \frac{1}{\sigma} (\rho \theta P)^{\sigma-1} - f \quad (14) \]

- Cut-off level of productivity

\[ \pi_d(\theta) = \frac{1}{\sigma} (\rho \theta P)^{\sigma-1} - f = 0 \]

\[ \rightarrow \theta_d = \frac{1}{\rho P} \left( \frac{f \sigma}{I} \right)^{\frac{1}{\sigma-1}} \quad (15) \]

- Before introducing a firm’s effort \( a \) in the next section, a firm with \( \theta < \theta_d \) will exit the market, while a firm with \( \theta \geq \theta_d \) will keep operating.

3. SBC

- To consider the SBC as a commitment problem between the firm and the government (Dewatripont and Maskin, 1995), I will introduce the firm’s effort and the government’s subsidy in the model.

1) Introduction of a firm’s effort \( a \)

- After entry a firm can choose \( a \): Effort \((0 \leq a \leq A) \rightarrow \) Reducing MC, i.e., when \( a \uparrow \), \( MC \rightarrow 0 \)
- However, \( MC \) cannot be zero \( \therefore \) when \( \pi = pq - f \), \( \frac{\partial \pi}{\partial p} \rightarrow \sigma = 1 \), which is inconsistent with the condition on \( \sigma \), i.e. \( \sigma > 1 \).
- Total cost with \( a \)

\[ TC(\theta, a) = f + \left( \frac{1}{\theta} - a \right) q, \quad (16) \quad : A < \frac{1}{\theta} \text{ because } MC \text{ cannot be zero.} \]
- Firm’s objective function with \( a \)

\[ \pi_{dA}(\theta, a) = pq + aq - \frac{1}{\theta} q - f \quad (17) \]

- Price, quantity, revenue and profit with \( a \)

\[ \frac{\partial \pi_{dA}}{\partial p} \rightarrow \]

i) \[ p(\theta, a) = \frac{1}{\rho} \left( \frac{1}{\theta} - a \right) = \frac{1-a\theta}{\rho \theta} = \frac{1-a}{\rho} - \frac{a}{\rho} = p(\theta) - \frac{a}{\rho} \quad (18) \]

ii) \( (6) \): \[ q(\theta, a) = \left( \frac{1}{\rho} \right) \left[ \frac{p(\theta, a)}{\rho} \right]^{-\sigma} = \left( \frac{1}{\rho} \right) \left[ \frac{1-a\theta}{\rho \theta} \right]^{-\sigma} = \frac{1}{\rho} \left[ \frac{1-a}{\rho \theta} \right]^{-\sigma} \]

\[ p(\theta) \left( \frac{1}{1-a} \right)^{\sigma} \quad (19) \]

iii) \[ r_{dA}(\theta, a) = p(\theta, a)q = I \left[ \frac{p(x)}{\rho} \right]^{1-\sigma} = I \left[ \frac{1}{\rho p} \left( \frac{1}{\theta} - a \right) \right]^{1-\sigma} = \]

\[ I \left[ \frac{1}{\rho p} \left( \frac{1}{1-a} \right) \right]^{1-\sigma} = r_{dA}(\theta) \left( \frac{1}{1-a} \right)^{\sigma-1} \quad (20) \]

iv) \[ \pi_{dA}(\theta, a) = pq + aq - \frac{1}{\theta} q - f = q \left[ \frac{(1-\rho)(1-a\theta)}{\rho \theta} \right] - f = \frac{qp(\theta, a)}{\sigma} - f \]
\[ \pi_{dA}(\theta, a) = \frac{r(\theta, a)}{\sigma} - f = \frac{1}{\sigma} \left[ \rho P \left( \frac{\theta}{1-\theta a} \right) \right]^{\sigma-1} - f \]

- \[ \frac{\partial q(\theta, a)}{\partial a} > 0, \quad \frac{\partial r(\theta, a)}{\partial a} > 0 \]

- Since \[ \frac{\partial^2 \pi_{dA}(\theta, a)}{\partial a^2} = \theta \left( \frac{\sigma-1}{\sigma} \right) r(\theta, a) \left( \frac{1}{1-\theta a} \right) = \theta \left( \frac{\sigma-1}{\sigma} \right) r(\theta) \left( \frac{1}{1-\theta a} \right) > 0 \] (\( 0 \leq a \leq A < \frac{1}{\theta} \to (1-\theta a) > 0 \)) and \[ \frac{\partial^2 \pi_{dA}(\theta, a)}{(\partial a)^2} = (\sigma - 1)r(\theta, a) \left( \frac{\theta}{1-\theta a} \right)^2 > 0 \], every firm will make best effort \( A \) to make its profit maximize.

- Cut-off level of productivity with effort

\[ \pi_{dA}(\theta, A) = \frac{1}{\sigma} \left[ \rho P \left( \frac{\theta}{1-\theta A} \right) \right]^{\sigma-1} - f = 0 \]

\[ \Rightarrow \frac{\theta}{1-\theta A} = \frac{1}{\rho P \left( \frac{\theta}{r} \right)^{\frac{1}{\sigma-1}}} \]

\[ \Rightarrow \theta_{dA} = \frac{1}{\rho P \left( \frac{\theta}{r} \right)^{\frac{1}{\sigma-1}}} = \frac{\theta_d}{1 + A \left( \frac{\theta}{r} \right)^{\frac{1}{\sigma-1}}} \]

- For \( \theta \geq \theta_d \),

\[ 0 \leq \pi_{dA}(\theta, 0) < \pi_{dA}(\theta, A) \]

\( \Rightarrow \) Every firm will serve the market regardless of its effort.

\( \Rightarrow \) Since \[ \frac{\partial \pi_{dA}(\theta, a)}{\partial a} > 0 \], a firm’s profit will be \( \pi(\theta, A) \) to maximize its profit.

- For \( \theta_{dA} \leq \theta < \theta_d \),

\[ \pi_{dA}(\theta, 0) < 0 \leq \pi_{dA}(\theta, A) \]

\( \Rightarrow \exists a^0 \text{ s. t. } \pi_{dA}(\theta, a^0) = 0 \)

i) From \[ \pi_{dA}(\theta, a^0) = \frac{1}{\sigma} \pi_{dA}(\theta) \left( \frac{1}{1-\theta a^0} \right)^{\sigma-1} - f = 0 \],

\[ a^0(\theta) = \frac{1}{\sigma} \left( \frac{1}{\rho P} \right)^{\frac{1}{\sigma-1}} = \frac{1}{\theta} - \frac{1}{\theta_d} \]

ii) \[ \frac{\partial a^0}{\partial \theta} < 0 \] : More productive firm is less likely to have \( \pi_{dA}(\theta, a^0) = 0 \)

\( \Rightarrow At \theta = \theta_d, a^0 = \frac{1}{\theta_d} - \frac{1}{\theta_d} = 0, \) i.e. \( \pi(\theta_d, 0) = 0 \)

\( \Rightarrow At \theta = \theta_{dA}, a^0 = \frac{1}{\theta_{dA}} - \frac{1}{\theta_{dA}} = 1 + \theta_{dA} - \frac{1}{\theta_{dA}} = A, \) i.e., \( \pi_{dA}(\theta_{dA}, A) = 0 \)

\( \Rightarrow \) Some firm might exit depending on its effort.

\( \Rightarrow \) However, every firm will serve the market with \( \pi_{dA}(\theta, A) \geq 0 \).
- For $\theta < \theta_{dA}$, $\pi_{dA}(\theta,A) < 0$ : the firm will exit the market.

2) Introduction of the subsidy from the government

- Now the benevolent government might support some firms in the industry with the subsidy because of political issue or employment issue (there are many reasons and examples why the government provides the subsidy to firms).
- Let $s$ be the subsidy from the government.
  
  \[ s = \begin{cases} 
  s & \text{if } \pi < 0 \\
  0 & \text{if } \pi \geq 0 
  \end{cases} \text{ for each firm} \]
- Assume $0 \leq s \leq f$ for the industry

3) A game between each firm and the government : After a firm enters the market and realizes its $\theta$, 

i) Stage 1: The firm chooses $a$.

ii) Stage 2: The firm suggests the government to support $s$.

iii) Stage 3: The government will accept or reject its suggestion.

iv) Stage 4: If accepted, the firm will produce and have $s$, otherwise it should decide whether to produce.

- **Proposition 1**: For a firm with $\theta \geq \theta_d$, 
  The firm chooses its effort $A$ without any subsidy (HBC).

  **Proof**: By backward induction, 
  In stage 4, the firm with $\theta \geq \theta_d$ will produce even without $s$ because $\pi_{dA}(\theta,A) \geq 0, \forall A$. 
  Hence, the firm’s threat not to produce without $s$ is incredible. 
  In stage 3, the government will reject the suggestion. 
  In stage 2, the firm knows that the government will not accept its suggestion so that it will not ask $s$. 
  In stage 1, the firm’s profit is $\pi_{dA}(\theta,A)$ so that it will choose $A$ to maximize its profit because $\frac{\partial \pi_{dA}(\theta,a)}{\partial a} > 0$.

- **Proposition 2 (Segal, 1998)**: For a firm with $\theta_{dA} \leq \theta < \theta_d$ , 
  i) If $\pi_{dA}(\theta,A) < s$, the firm will choose $a^0$, receives a subsidy of $s$ and produces (SBC outcome with moral hazard)
  ii) If $\pi_{dA}(\theta,A) > s$, the firm will choose $A$, receives no subsidy and produces (HBC)
  iii) If $\pi_{dA}(\theta,A) = s$, the result would be either i) or ii).

  **Proof**: By backward induction, 
  If $\pi_{dA}(\theta,a) \leq 0$ which is equivalent to $a \leq a^0$, 
  In stage 4, the firm’s threat not to produce is credible.
In stage 3, the government will accept the firm’s suggestion for $s$.
In stage 2, the firm knows that the government will accept its suggestion so that it will ask for $s$.
In stage 1, the firm’s profit is $\pi_{dA}(\theta, a) + s$.
To maximize its profit, the firm will choose $a^0$ to maximize its profit because $\frac{\partial \pi_{dA}(\theta, a)}{\partial a} > 0$.
Hence, the firm with $a \leq a^0$ will obtain $\pi_{dA}(\theta, a^0) + s = s$.

If $\pi_{dA}(\theta, a) \geq 0$ which is equivalent to $a \geq a^0$,
In stage 4, the firm’s threat not to produce is incredible because $\pi_{dA}(\theta, a) \geq 0$.
In stage 3, the government will reject the firm’s suggestion.
In stage 2, the firm knows that the government will reject its suggestion so that it will not ask any subsidy.
In stage 1, the firm’s profit is $\pi_{dA}(\theta, a)$ so that it will choose $A$ to maximize its profit because $\frac{\partial \pi_{dA}(\theta, a)}{\partial a} > 0$.
Hence, the firm’s profit is $\pi_{dA}(\theta, A)$.

Now, a firm’s choice of effort is depending on whether $s$ is greater than $\pi_{dA}(\theta, A)$. If the firm is indifferent, both are possible.

- **Proposition 3**: For a firm with $\theta < \theta_{dA}$,
  
  i) If $\pi_{dA}(\theta, A) + s \geq 0$, the firm will choose $A$, receives a subsidy of $s$ and produces (SBC outcome without moral hazard).
  
  ii) If $\pi_{dA}(\theta, A) + s < 0$, the firm will exit the market (HBC).

**Proof**: By backward induction,

In stage 4, the firm’s threat not to produce is credible because $\pi_{dA}(\theta, a) < 0, \forall a$.
In stage 3, the government will accept the firm’s suggestion.
In stage 2, the firm knows that the government will accept its suggestion so that it will ask for $s$.
In stage 1, the firm’s profit is $\pi_{dA}(\theta, a) + s$.
Hence, the firm will choose $A$ because it want to reduce deficit (i.e., $\pi_{dA}(\theta, a) < 0$) as possible and obtain $\pi_{dA}(\theta, A) + s$ which should be greater than 0.
If $\pi_{dA}(\theta, A) + s$ is less than zero, then a firm will exit the industry.

4) The relationship between $s$ and $\theta$

- Both $\theta_d$ and $\theta_{dA}$ are independent of $s$

- Cut-off level of productivity for the moral hazard SBC ($\theta_{dM}$)
  
  From $\pi_{dA}(\theta, A) = \pi_{dA}(\theta, a^0) + s = s$, $\theta_{dA} < \theta < \theta_d$. 


\[ \theta_{ds}^M = \frac{1}{\rho P} \left( \frac{\sigma(f + s)}{I} \right)^{\frac{1}{\sigma-1}} \left( 1 + \frac{1}{\rho P} \left[ \frac{\sigma(f + s)}{I} \right]^{\frac{1}{\sigma-1}} A \right) \]

\[ \Leftrightarrow s \uparrow \rightarrow (\theta_{ds}^M - \theta_{dA}) \uparrow \land (\theta_d - \theta_{ds}^M) \downarrow \land \frac{\partial \theta_{ds}^M}{\partial s} > 0 \]

- Cut-off level of productivity for the non-moral hazard SBC \( \theta_{ds}^N \)

From \( \pi_d(\theta, A) + s = 0, 0 \leq \theta < \theta_{dA} \),

\[ \theta_{ds}^N = \frac{1}{\rho P} \left( \frac{\sigma(f - s)}{I} \right)^{\frac{1}{\sigma-1}} \left( 1 + \frac{1}{\rho P} \left[ \frac{\sigma(f - s)}{I} \right]^{\frac{1}{\sigma-1}} A \right) \]

\[ \Leftrightarrow s \uparrow \rightarrow (\theta_{dA} - \theta_{ds}^N) \uparrow \land \frac{\partial \theta_{ds}^N}{\partial s} < 0 \]

- Hence, when the government provides the subsidy, there is non-moral hazard SBC for least productive firms (between \( \theta_{ds}^N \) and \( \theta_{dA} \)) and moral hazard SBC for intermediate productive firms (between \( \theta_{dA} \) and \( \theta_{ds}^M \)).

- If the government increases the subsidy, both moral hazard and non-moral hazard SBC will increase, while the HBC will decrease.

5) Order of \( \theta_d, \theta_{ds}^N, \theta_{dA} \) and \( \theta_{ds}^M \)

Eq.(15): \( \theta_d = \frac{1}{\rho P} \left( \frac{\sigma f}{I} \right)^{\frac{1}{\sigma-1}} \)

Eq.(24): \( \theta_{ds}^M = \frac{1}{\rho P} \left( \frac{\sigma(f + s)}{I} \right)^{\frac{1}{\sigma-1}} \left( 1 + \frac{1}{\rho P} \left[ \frac{\sigma(f + s)}{I} \right]^{\frac{1}{\sigma-1}} A \right) \)

Eq.(22): \( \theta_{dA} = \frac{1}{\rho P} \left( \frac{\sigma f}{I} \right)^{\frac{1}{\sigma-1}} \left( 1 + \frac{1}{\rho P} \left[ \frac{\sigma f}{I} \right]^{\frac{1}{\sigma-1}} A \right) = \frac{\theta_d}{1 + \theta_{dA}} \)

Eq.(25): \( \theta_{ds}^N = \frac{1}{\rho P} \left( \frac{\sigma(f - s)}{I} \right)^{\frac{1}{\sigma-1}} \left( 1 + \frac{1}{\rho P} \left[ \frac{\sigma(f - s)}{I} \right]^{\frac{1}{\sigma-1}} A \right) \)

\[ i) \quad 0 \leq \theta_{ds}^N \leq \theta_{dA} \leq \theta_{ds}^M \quad \Rightarrow s \geq 0 \quad \text{and} \quad \frac{\partial \theta_{ds}^M}{\partial s} > 0 \land \frac{\partial \theta_{ds}^N}{\partial s} < 0 \]

\[ ii) \quad \theta_{dA} \leq \theta_d \]
\[
\theta_d - \theta_d^A = \theta_d - \frac{\theta_d}{1 + \theta_d A} = \frac{\theta_d^2 A}{1 + \theta_d A} \geq 0
\]

iii) \(\theta_d^M\) vs \(\theta_d^M\)?

Let \(\frac{1}{\rho} \left( \frac{\sigma}{l} \right)^{-\frac{1}{\sigma-1}} = K > 0\)

\[
\theta_d - \theta_d^M = \frac{1}{\rho} \left( \frac{\sigma f}{l} \right)^{-\frac{1}{\sigma-1}} - \frac{1}{\rho} \frac{\left[ \sigma (f + s) \right]^{\frac{1}{\sigma-1}}}{l^{\frac{1}{\sigma-1}} A} = f^{\frac{1}{\sigma-1}} K - \frac{(f + s)^{\frac{1}{\sigma-1}} K}{1 + A (f + s)^{\frac{1}{\sigma-1}} K} = \frac{[f^{\frac{1}{\sigma-1}} - (f + s)^{\frac{1}{\sigma-1}}] + A[f(f + s)]^{\frac{1}{\sigma-1}} K}{[1 + A (f + s)^{\frac{1}{\sigma-1}} K]^2}
\]

\[
\frac{A[f(f + s)]^{\frac{1}{\sigma-1}} K^2 - (f + s)^{\frac{1}{\sigma-1}} K + f^{\frac{1}{\sigma-1}} K}{[1 + A (f + s)^{\frac{1}{\sigma-1}} K]^2} = \frac{[A f^{\frac{1}{\sigma-1}} K - 1] (f + s)^{\frac{1}{\sigma-1}} K f^{\frac{1}{\sigma-1}} K}{[1 + A (f + s)^{\frac{1}{\sigma-1}} K]}
\]

- \(\sigma\): The sign of \((\theta_d - \theta_d^M)\) depends on \(s\), given \(A, f\) and \(K\).
- The critical value of \(s\) for \(\theta_d \geq \theta_d^M\):

\[
\text{From} \quad \theta_d - \theta_d^M = \frac{[A f^{\frac{1}{\sigma-1}} K - 1] (f + s)^{\frac{1}{\sigma-1}} K f^{\frac{1}{\sigma-1}} K}{[1 + A (f + s)^{\frac{1}{\sigma-1}} K]^2}
\]

\[
[A f^{\frac{1}{\sigma-1}} K - 1] (f + s)^{\frac{1}{\sigma-1}} K f^{\frac{1}{\sigma-1}} K = 0
\]

\[
(f + s)^{\frac{1}{\sigma-1}} = \frac{f^{\frac{1}{\sigma-1}} K}{[1 - A f^{\frac{1}{\sigma-1}} K]}
\]

\[
\therefore s^0 = \left[ \frac{f^{\frac{1}{\sigma-1}} K}{[1 - A f^{\frac{1}{\sigma-1}} K]} \right]^{\sigma-1} - f = \pi(\theta_d, A) \quad (26)
\]

6) Three cases, depending on \(s\) given \(f, A\) and \(K\).

i) If \(s < s^0\), then \(0 \leq \theta_d^N \leq \theta_d^M < \theta_d\)

- \(\theta \leq \theta_d^N\): A firm will exit the market
- \(\theta_d^N \leq \theta \leq \theta_d\): Non-moral hazard SBC outcome
- \(\theta_d^M \leq \theta \leq \theta_d\): Moral hazard SBC outcome
- \(\theta \leq \theta_d\): HBC outcome

- Hence, there exit both (moral hazard) SBC and HBC for the interval between \(\theta_d^M\) and \(\theta_d\).

ii) If \(s = s^0\), then \(0 \leq \theta_d^N \leq \theta_d^M = \theta_d\)

- \(\theta \leq \theta_d^N\): A firm will exit the market
- \(\theta_d^N \leq \theta \leq \theta_d\): Non-moral hazard SBC outcome
- \(\theta_d^M \leq \theta \leq \theta_d\): Moral hazard SBC outcome
- \(\theta \leq \theta_d\): HBC outcome

- Hence, there exits only (moral hazard) SBC for the interval between \(\theta_d^M\) and \(\theta_d\).

iii) If \(s^0 < s\), then \(0 \leq \theta_d^N \leq \theta_d^M < \theta_d < \theta_d^M\)
• $\theta \leq \theta_{ds}^N$: A firm will exit the market
• $\theta_{ds}^N \leq \theta \leq \theta_{dA}$: Non-moral hazard SBC outcome
• $\theta_{dA} \leq \theta \leq \theta_d$: Moral hazard SBC outcome
• $\theta_d \leq \theta \leq \theta_{ds}^M$: HBC outcome
• $\theta_{ds}^M \leq \theta$: HBC outcome

- For $\theta_d \leq \theta$, $\pi(\theta, a) \geq 0, \forall a$. Hence, the firm’s threat not to produce is incredible so that it will choose $A$ and not have any subsidy, even though it is more profitable to have the subsidy with the effort $a^0$, i.e. $\pi(\theta, a^0) + s$, than the profit with maximum effort, i.e., $\pi(\theta, A)$.

- In the case (iii), the gain ($s$) of some less productive firms between $\theta_{dA}$ and $\theta_d$ is higher than the profits, $\pi(\theta, A)$ of some more productive firms between $\theta_d$ and $\theta_{ds}^M$. Also, the profits of some least productive firms between $\theta_{ds}^N$ and $\theta_{dA}$ are higher than the gain ($s$) of some less productive firms between $\theta_{dA}$ and $\theta_d$. (Hence, the last case is unrealistic).

- Hence, assume that there is the maximum value of $s$, that is,

$$s \leq s^0 = \left[ \frac{f^{1/(\sigma-1)}}{1-A f^{1/(\sigma-1)} K} \right]^{\frac{\sigma-1}{\sigma}} - f = \pi(\theta_d, A) \Leftrightarrow 0 \leq \theta_{ds}^N \leq \theta_{dA} \leq \theta_{ds}^M \leq \theta_d$$

II. Foreign market

- Symmetric assumption: All countries share the same $P, Q$ and $I$.
- A new game between each firm and the government: After a firm enters the market and realizes its $\theta$,
  
  i) Stage 1: The firm chooses $a$.
  ii) Stage 2: The firm suggests the government to support $s$.
  iii) Stage 3: The government will accept or reject its suggestion.
  iv) Stage 4: If accepted, the firm will produce only in the domestic or both in the domestic and the foreign market and have $s$, otherwise it should decide whether to produce both in the domestic and the foreign market (exit) or only in the foreign market.

1. Production in the foreign market

- $\tau > 1$: A per-unit iceberg costs
- $f_x > f$: Fixed costs for exporting
- Total cost of exporting

$$TC_x(\theta) = f_x + \left( \frac{\tau}{\theta} - A \right) q_x, \quad (27) \quad \Rightarrow \left( \frac{\tau}{\theta} - A \right) > 0 \quad \Rightarrow A < \frac{1}{\theta}$$

- Firm’s objective function for exporting

$$\pi_{xA}(\theta) = p_x q_x + A q_x - \frac{\tau}{\theta} q_x - f_x \quad (28)$$

- Price, quantity, revenue and profit for exporting

$$\frac{\partial \pi_{xA}}{\partial p_x} \rightarrow$$
\[ p_x(\theta) = \frac{1}{\rho} \left( \frac{\tau}{\theta} - A \right) = \frac{\tau - A \theta}{\rho \theta} = \frac{\tau}{\rho \theta} - \frac{A}{\rho} \quad (29) \]

\[ q_x(\theta) = \left( \frac{1}{\rho} \right)^\sigma \left[ \frac{p_x(\theta)}{\rho \theta} \right]^{\sigma-1} = \left( \frac{1}{\rho} \right)^\sigma \left[ \frac{\tau - A \theta}{\rho \theta} \right]^{\sigma-1} = \left[ \frac{\rho P(\theta - A \theta)}{\tau - A \theta} \right]^{\sigma-1} \quad (30) \]

\[ r_x A(\theta) = p_x(\theta) q_x = \left( \frac{\tau - A \theta}{\rho \theta} \right) \left[ \frac{\rho P(\theta - A \theta)}{\tau - A \theta} \right]^{\sigma-1} = \left[ \frac{\rho P(\theta - A \theta)}{\tau - A \theta} \right]^{\sigma-1} \quad (31) \]

\[ \pi_x A(\theta) = q_x \left[ \frac{1}{\rho \theta} \right] - f_x = q_x p_x(\theta) - f_x = \frac{r_x(\theta)}{\sigma} - f_x \]

\[ \therefore \pi_x A(\theta) = \frac{1}{\sigma} \left[ \frac{\rho P(\theta - A \theta)}{\tau - A \theta} \right]^{\sigma-1} - f_x \quad (32) \]

- Cut-off level of productivity for exporting

\[ \text{(21)}: \pi_x A(\theta) = \frac{1}{\sigma} \left[ \frac{\rho P(\theta - A \theta)}{\tau - A \theta} \right]^{\sigma-1} - f_x = 0 \]

\[ \Rightarrow \frac{\theta}{\tau - \theta A} = \frac{1}{\rho P(\theta)} \frac{1}{\sigma-1} \]

\[ \therefore \theta_x A = \frac{\tau}{\rho P(\theta)} \frac{1}{\sigma-1} \frac{1}{\tau - \theta A} \quad (33) \]

2. \( \theta_x A > \theta_d A \)

- \( \theta_x A - \theta_d A = \frac{\tau (f_x \sigma \frac{1}{\sigma-1})}{\rho P(\theta)} - \frac{1}{\rho P(\theta)} \frac{1}{\sigma-1} = \frac{\tau f_x^{1/(\sigma-1)}_K}{1 + Af_x^{1/(\sigma-1)}_K} - \frac{f_x^{1/(\sigma-1)}_K}{1 + Af_x^{1/(\sigma-1)}_K} = \frac{\tau f_x^{1/(\sigma-1)}_K - f_x^{1/(\sigma-1)}_K + (\tau - 1)(Af_x^{1/(\sigma-1)}_K)^2}{(1 + A f_x^{1/(\sigma-1)}_K)(1 + A f_x^{1/(\sigma-1)}_K)} \]

\[ \therefore \text{The sign depends on} \ A, f, f_x, \tau \text{and} \ K \text{ (there is no} \ s). \]

3. \( \theta_d \) vs \( \theta_x A \)?

- \( \theta_x A - \theta_d = \frac{\tau (f_x \sigma \frac{1}{\sigma-1})}{\rho P(\theta)} - \frac{1}{\rho P(\theta)} \frac{1}{\sigma-1} = \frac{\tau f_x^{1/(\sigma-1)}_K}{1 + Af_x^{1/(\sigma-1)}_K} - f_x^{1/(\sigma-1)}_K = \frac{\tau f_x^{1/(\sigma-1)}_K - f_x^{1/(\sigma-1)}_K - A(f_x f)^{1/(\sigma-1)}_K}{1 + Af_x^{1/(\sigma-1)}_K} \]

\[ \text{The sign depends on} \ A, f, f_x, \tau \text{and} \ K \text{ (there is no} \ s). \]

- The critical value of \( \tau \) for \( \theta_x A - \theta_d \)

From \( \theta_x A - \theta_d = 0 \),

\[ \frac{\tau f_x^{1/(\sigma-1)}_K - f_x^{1/(\sigma-1)}_K - A(f_x f)^{1/(\sigma-1)}_K}{1 + Af_x^{1/(\sigma-1)}_K} = 0 \]

\[ \Rightarrow \tau f_x^{\sigma-1}_K = A(f_x f)^{\sigma-1} K + f_x^{\sigma-1} \]

\[ \therefore \tau^0 = \frac{Af_x^{\sigma-1} K + (f_x^{1/(\sigma-1)})}{(f_x^{1/(\sigma-1)})} \]
4. $\theta_{\text{ds}}^{M} \text{vs} \theta_{\text{xA}}^{M}$?

$$\theta_{\text{ds}}^{M} - \theta_{\text{xA}}^{M} = \frac{1}{p[f(\sigma+\theta)]} \frac{1}{1-A} \left[ \frac{1}{1-A(f+s)} \right]^{\frac{1}{\sigma-1}} - \frac{1}{1-A(f+s)} \frac{1}{1-A} = \frac{(f+s)^{(1/\sigma-1)}K - \tau f_{x}^{1/(\sigma-1)}K}{1+A(f+s)^{(1/\sigma-1)}K} - \frac{\tau f_{x}^{1/(\sigma-1)}K}{1+A f_{x}^{1/(\sigma-1)}K}$$

$$[\frac{(f+s)^{(1/\sigma-1)}-\tau f_{x}^{1/(\sigma-1)}K}{1+A(f+s)^{(1/\sigma-1)}K}] \frac{1}{1+A f_{x}^{1/(\sigma-1)}K}$$

- The sign of $(\theta_{x} - \theta_{\text{ds}}^{M})$ depends on $s, f, K, f_{x}$ and $\tau$.
- As $\frac{\partial^{2}_{\text{ds}}}{\partial s} = 0$ & $\frac{\partial^{2}_{\text{ds}}}{\partial s} > 0$, $(\theta_{x} - \theta_{\text{ds}}^{M}) \downarrow \Leftrightarrow (\theta_{\text{ds}}^{M} - \theta_{x}) \uparrow$ as $s \uparrow$
- The critical value of $s$ for $\theta_{\text{xA}} = \theta_{\text{ds}}^{M}$:

From $\theta_{\text{ds}}^{M} - \theta_{\text{xA}} = \frac{[1-(\tau + 1)A f_{x}^{1/(\sigma-1)}K] (f + s)^{(1/\sigma-1)} - \tau f_{x}^{1/(\sigma-1)}}{1+\tau f_{x}^{1/(\sigma-1)}}$,  

$$[1-(\tau + 1)A f_{x}^{1/(\sigma-1)}K] (f + s)^{(1/\sigma-1)} - \tau f_{x}^{1/(\sigma-1)} = 0$$

$$\frac{\tau f_{x}^{1/(\sigma-1)}}{[1-(\tau + 1)A f_{x}^{1/(\sigma-1)}K]} = f$$

$$\therefore S_{x}^{0} = \left( \frac{\tau f_{x}^{1/(\sigma-1)}}{[1-(\tau + 1)A f_{x}^{1/(\sigma-1)}K]} \right)^{\sigma-1} - f \quad (34)$$

- Hence, there exist various market situations, depending on the order of $\theta$.
- Assume that $f, K, f_{x}$ and $A$ are moderate (not extreme)
- Set up two stages to determine the order of $\theta$.

1) Stage 1: $\theta_{d} \text{vs} \theta_{\text{xA}}$

- First, the value of $\tau$ determines the sign of $(\theta_{\text{xA}} - \theta_{d})$

i) High $\tau$ ($\tau \geq \tau^{0}$)

$\Rightarrow \theta_{d} \leq \theta_{\text{xA}}$ (i.e., $\pi_{x}(\theta_{d}) \leq 0$)

- There exist only one situation: $0 \leq \theta_{d}^{N} \leq \theta_{d}^{A} \leq \theta_{d}^{M} \leq \theta_{d} \leq \theta_{\text{xA}}$ (Case 1)

- $\theta \leq \theta_{d}^{N}$ : The least productive firm will exit the market
- $\theta_{d}^{N} \leq \theta \leq \theta_{d}^{A}$: The less productive firm has non-moral hazard SBC and serves only the domestic market
- $\theta_{d}^{A} \leq \theta \leq \theta_{d}^{M}$: The intermediate productive firm has moral hazard SBC and serves only the domestic market
- $\theta_{d}^{M} \leq \theta \leq \theta_{\text{xA}}$: The more productive firm has HBC and serves only the domestic market
- $\theta_{\text{xA}} \leq \theta$: The most productive firm has HBC and serves both the domestic and the foreign market

- In the domestic market, there exist both SBC and HBC.
ii) Low $\tau$ ($\tau \leq \tau^0$)

$\rightarrow \theta_d \geq \theta_{xA}$ (i.e., $\pi_{xA}(\theta_d) \geq 0$) $\rightarrow$ go to Stage 2.

2) Stage 2: $\theta_{ds}^M$ vs $\theta_{xA}$

- Second, given $\tau$ is low($\tau \leq \tau^0$), the value of $s$ determines the sign of $(\theta_{xA} - \theta_{ds}^M)$
- There exist two situations
i) Low $s$ ($s \leq s^0_N$):

$\rightarrow 0 \leq \theta_{ds}^N \leq \theta_{dA} \leq \theta_{ds}^M \leq \theta_{xA} \leq \theta_d$ (Case 2)

- $\theta \leq \theta_{ds}^N$: The least productive firm will exit the market
- $\theta_{ds}^N \leq \theta \leq \theta_{dA}$: The less productive firm has non-moral hazard SBC and serves only the domestic market
- $\theta_{dA} \leq \theta \leq \theta_{ds}^M$: The intermediate productive firm has moral hazard SBC and serves only the domestic market
- $\theta_{ds}^M \leq \theta \leq \theta_{xA}$: The more productive firm has HBC and serves only the domestic market
- $\theta_{xA} \leq \theta$: The most productive firm has HBC and serves both the domestic and the foreign market

$\therefore$ In the domestic market, there exist both SBC and HBC (same as Case 1).
$\rightarrow$ In other words, either high $\tau$ or low $s$ will develop the same market allocation of firm productivity.

ii) High $s$ ($s \geq s^0_N$):

$\rightarrow 0 \leq \theta_{ds}^N \leq \theta_{dA} \leq \theta_{xA} \leq \theta_{ds}^M \leq \theta_d$ (Case 3)

- $\theta \leq \theta_{ds}^N$: The least productive firm will exit the market
- $\theta_{ds}^N \leq \theta \leq \theta_{dA}$: The less productive firm has non-moral hazard SBC and serves only the domestic market
- $\theta_{dA} \leq \theta \leq \theta_{xA}$: The intermediate productive firm has moral hazard SBC and serves only the domestic market
- $\theta_{ds}^M \leq \theta$: The most productive firm will have HBC and serves both the domestic and the foreign market.

- Now, for $\theta_{xA} \leq \theta \leq \theta_{ds}^M$, the question is, “Do all firms with $\theta \leq \theta_{ds}^M$ have moral hazard SBC, compared with Case 1 and 2?”, and “Do all firms with $\theta_{xA} \leq \theta$ will export, compared with Melitz (2003)?” The answer is “No”.

$\rightarrow$ For $\theta \leq \theta_{ds}^M$, some firms will have HBC.

$\rightarrow$ For $\theta_{xA} \leq \theta$, some firms will not export and have non-hazard SBC.

- Let $\pi_T$ be total profit in the domestic and the foreign market.
\[ \pi_T(\theta, a) = \begin{cases} 
\pi_{dA}(\theta, a), & \text{if a firm serves only the domestic market} \\
\pi_{dA}(\theta, a) + \pi_{xA}(\theta, a), & \text{if a firm serves both the domestic and the foreign market} 
\end{cases} \]

- For \( \theta \leq \theta \leq \theta_{dM} \),
\[ \pi_T(\theta, 0) = \pi_{dA}(\theta, 0) + \pi_{xA}(\theta, 0) < 0 \text{ because } \pi_{xA}(\theta, 0) < \pi_{dA}(\theta, 0) < 0. \]

Also \( \pi_T(\theta, A) = \pi_{dA}(\theta, A) + \pi_{xA}(\theta, A) > 0 \text{ because } \pi_{dA}(\theta, A) > \pi_{xA}(\theta, A) > 0 \) as
\[ \pi_{xA}(\theta_{xA}^\prime, A) = 0 \text{ and } \frac{\partial \pi_{xA}(\theta, A)}{\partial \theta} > 0 \]

Hence, there is \( a_t^0 \) s. t. \( \pi_T(\theta, a_t^0) = 0 \).

- At \( a^0, \pi_T(\theta, a^0) = \pi_d(\theta, a^0) + \pi_x(\theta, a^0) < 0 \) as \( \pi_d(\theta, a^0) = 0 \) and \( \pi_x(\theta, a^0) < 0 \) with \( \pi_x(\theta, a^0) < \pi_d(\theta, a^0) \).

\[ \therefore a^0 < a_t^0 \]

- **Proposition 4**: Given low \( \tau (\tau \leq \tau_0) \) and high \( s (s \geq s_x^0) \), for a firm with \( \theta_{xA} \leq \theta \leq \theta_{dM}^M \),

i) If \( \pi_T(\theta, A) < s \), some firm will choose \( a^0 \), receives a subsidy of \( s \) and produces only in the domestic market. Other firms will choose \( a_t^0 \), receives a subsidy of \( s \) and produces in both the domestic and the foreign market (SBC outcome with moral hazard)

ii) If \( \pi_T(\theta, A) > s \), the firm will choose \( A \), receives no subsidy and produces in both the domestic and the foreign market (HBC)

iii) If \( \pi_T(\theta, A) = s \), the result would be either i) or ii).

**Proof**: The proof is similar as Proposition 2.

By backward induction,
If \( \pi_T(\theta, a) \leq 0 \) and \( \pi_{dA}(\theta, a) > 0 \) which is equivalent to \( a^0 < a \leq a_t^0 \),

In stage 4, the firm’s threat not to produce in the foreign market is credible. However, the firm’s threat not to produce only in the domestic market is incredible.

In stage 3, If the firm exports, the government will accept the firm’s suggestion for \( s \).

However, if the firm serves only the domestic market, the government will not accept the firm’s suggestion for \( s \).

In stage 2, the firm knows that the government will accept its suggestion if it exports so that it will ask for \( s \)

In stage 1, the firm’s profit is \( \pi_T(\theta, a) + s \).

To maximize its profit, the firm will choose \( a_t^0 \) to maximize its profit because \( \frac{\partial \pi(\theta, a)}{\partial a} > 0 \).

Hence, the firm with \( a \leq a_t^0 \) will obtain \( \pi_T(\theta, a_t^0) + s = s \).

If \( \pi_T(\theta, a) \leq 0 \) and \( \pi_d(\theta, a) \leq 0 \) which is equivalent to \( a \leq a^0 < a_t^0 \),

In stage 4, the firm’s threat not to produce is credible, regardless of exporting.

In stage 3, the government will accept the firm’s suggestion for \( s \).
In stage 2, the firm knows that the government will accept its suggestion so that it will ask for subsidy.
In stage 1, the firm’s profit is \( \pi_T(\theta, a) + s \).
To maximize its profit, the firm will choose \( a^0 \) to maximize its profit because \( \frac{\partial \pi(\theta, a)}{\partial a} > 0 \) and will not export because \( \pi_d(\theta, a^0) \geq \pi_T(\theta, a^0) = \pi_d(\theta, a^0) + \pi_x(\theta, a^0) \) as \( \pi_x(\theta, a^0) < \pi_d(\theta, a^0) = 0 \).
Hence, the firm with \( a \leq a^0 < a^0 \) will obtain \( \pi_d(\theta, a^0) + s = s \).

If \( \pi_T(\theta, a) \geq 0 \) which is equivalent to \( a \geq a^0 \),
In stage 4, the firm’s threat not to produce is incredible.
In stage 3, the government will reject the firm’s suggestion.
In stage 2, the firm knows that the government will reject its suggestion so that it will not ask any subsidy.
In stage 1, the firm’s profit is \( \pi(\theta, a) \) so that it will choose \( A \) to maximize its profit because \( \frac{\partial \pi(\theta, a)}{\partial a} > 0 \)
Hence, the firm’s profit is \( \pi_T(\theta, A) \).

Now, a firm’s choice of effort is depending on whether \( s \) is greater than \( \pi_T(\theta, A) \). If the firm is indifferent, both are possible.

5. New cut-off level of \( \theta \) in Case 3

- From Proposition 4, there is the cut-off level \( \theta \) for determining to export with HBC or export with SBC between \( \theta_{xA} \) and \( \theta^M_{ds} \), that is, \( \theta^M_{ds} \).
- From \( \pi_T(\theta, A) = s \),
  \[ \frac{1}{\sigma} \left[ \rho P \left( \frac{\theta}{1 - \theta A} \right) \right]^{\sigma - 1} - f + \frac{1}{\sigma} \left[ \rho P \left( \frac{\theta}{\tau - \theta A} \right) \right]^{\sigma - 1} - f_x = s \]
  \[ \rightarrow \frac{1}{\sigma} (\rho P)^{\sigma - 1} \left[ \left( \frac{\theta}{1 - \theta A} \right)^{\sigma - 1} + \left( \frac{\theta}{\tau - \theta A} \right)^{\sigma - 1} \right] = s + f + f_x \]
  \[ \rightarrow \left[ \left( \frac{\theta}{1 - \theta A} \right)^{\sigma - 1} + \left( \frac{\theta}{\tau - \theta A} \right)^{\sigma - 1} \right] = \frac{\sigma (s + f + f_x)}{I(\rho P)^{\sigma - 1}} \quad (35) \]
  \[ \rightarrow \text{From Eq.(35), We will get } \theta^M_{ds} = \theta^M_{ds} (\tau, s, f, f_x, K) \] .
- Hence, rewrite the allocation of firms productivity in Case 3:
  - \( \theta \leq \theta^N_{ds} \): The least productive firm will exit the market
  - \( \theta^N_{ds} \leq \theta \leq \theta_{dA} \): The less productive firm has non-moral hazard SBC and serves only the domestic market
  - \( \theta_{dA} \leq \theta \leq \theta_{xA} \): The intermediate productive firm has moral hazard SBC and serves only the domestic market
• $\theta_{XA} \leq \theta \leq \theta_{dS}^{MX}$: The more productive firm has moral hazard SBC. Some firms with $a^0$ will serve only the domestic market and the others with $a^0$ will serve both the domestic and the foreign market. In the latter case, the subsidy would be kind of the export subsidy.
• $\theta_{dS}^{MX} \leq \theta$: The most productive firm will have HBC and serves both the domestic and the foreign market.

- If the transportation cost ($\tau$) is low between two countries and the subsidy ($s$) in the domestic market is high, I conclude that
  
  i) Negative effect of SBC on exporting:
  Compared with Melitz(2003), some firms with $\theta_{XA} \leq \theta \leq \theta_{dS}^{MX}$ do not export any more, even though they can export ($\theta_{XA} \leq \theta$). They serve only the domestic market and have a subsidy.

  ii) Positive effect of exporting on SBC
  Compared with Case 1 and 2, all firms with $\theta_{dS}^{MX} \leq \theta \leq \theta_{dS}^{M}$ will have HBC and export.

III. The impact of trade liberalization

- Rewrite all cutoff values of productivity:

  \[ \text{Eq.(15): } \theta_d = \frac{1}{\rho P} \left( \frac{\sigma f}{I} \right)^{\frac{1}{\sigma - 1}} \]

  \[ \text{Eq.(24): } \theta_{dS}^{M} = \frac{1}{\rho P} \left( \frac{\sigma (f+s)}{I} \right)^{\frac{1}{\sigma - 1}} \frac{1}{1 + \frac{1}{\rho P} \left( \frac{\sigma (f+s)}{I} \right)^{\frac{1}{\sigma - 1}} A} \]

  \[ \text{Eq.(22): } \theta_{dA} = \frac{1}{\rho P} \left( \frac{\sigma f}{I} \right)^{\frac{1}{\sigma - 1}} \frac{1}{1 + \frac{1}{\rho P} \left( \frac{\sigma f}{I} \right)^{\frac{1}{\sigma - 1}} A} = \frac{\theta_d}{1 + \theta_d A} \]

  \[ \text{Eq.(25): } \theta_{dS}^{N} = \frac{1}{\rho P} \left( \frac{\sigma (f-s)}{I} \right)^{\frac{1}{\sigma - 1}} \frac{1}{1 + \frac{1}{\rho P} \left( \frac{\sigma (f-s)}{I} \right)^{\frac{1}{\sigma - 1}} A} \]

  \[ \text{Eq.(33): } \theta_{XA} = \frac{\tau (f \sigma)}{\rho P} \left( \frac{f \sigma}{I} \right)^{\frac{1}{\sigma - 1}} \frac{1}{1 + \frac{1}{\rho P} \left( \frac{f \sigma}{I} \right)^{\frac{1}{\sigma - 1}} A} \]

- I can express an increase in the exposure of an economy to trade by considering decreased $f_x$ or decreased $\tau$. 
1. Case 1 \((0 \leq \theta_{ds}^N \leq \theta_{dA} \leq \theta_{d} \leq \theta_{xA})\) and Case 2 \((0 \leq \theta_{dA} \leq \theta_{ds}^M \leq \theta_{xA} \leq \theta_{d})\)

i) \(f_x\)

\[
\frac{\partial \theta_{ds}^N}{\partial f_x} = 0, \frac{\partial \theta_{dA}}{\partial f_x} = 0, \frac{\partial \theta_{ds}^M}{\partial f_x} = 0, \frac{\partial \theta_{d}}{\partial f_x} = 0
\]

\[
\frac{\partial \theta_{xA}}{\partial f_x} = \frac{\tau}{(\sigma - 1)\rho P\left(\frac{f_x\sigma}{I}\right)^{\frac{2-\sigma}{\sigma-1}}}A \left[1 - \frac{1}{\rho P\left(\frac{f_x\sigma}{I}\right)^{\frac{1}{\sigma-1}}}A \right] > 0
\]

ii) \(\tau\)

\[
\frac{\partial \theta_{ds}^N}{\partial \tau} = 0, \frac{\partial \theta_{dA}}{\partial \tau} = 0, \frac{\partial \theta_{ds}^M}{\partial \tau} = 0, \frac{\partial \theta_{d}}{\partial \tau} = 0
\]

\[
\frac{\partial \theta_{xA}}{\partial \tau} = \frac{1}{\rho P\left(\frac{f_x\sigma}{I}\right)^{\frac{1}{\sigma-1}}}A > 0
\]

As \(f_x\) or \(\tau\) decrease, \(\theta_{xA}\) will also decrease. Hence Case 1 and Case 2 is more likely to be Case 3.

2. Case 3: \(0 \leq \theta_{ds}^N \leq \theta_{dA} \leq \theta_{xA} \leq \theta_{ds}^M \leq \theta_{d}\)

- Rewrite Eq.(35),

\[
G(\theta_{ds}^M, f_x, \tau) = I\left[\left(\frac{\rho P\theta_{ds}^M}{1 - \theta_{ds}^M A}\right)^{\sigma-1} + \left(\frac{\rho P\theta_{ds}^M}{\tau - \theta_{ds}^M A}\right)^{\sigma-1}\right] - \sigma f_x = \sigma(s + f) \quad (36)
\]

- With the implicit function theorem in Eq.(36),

i) \(\frac{\partial \theta_{ds}^M}{\partial f_x} = -\frac{\partial G(\theta_{ds}^M, f_x)}{\partial \theta_{ds}^M(\theta_{ds}^M, f_x)} \quad (37)\)

- \(\frac{\partial G(\theta_{ds}^M, f_x)}{\partial \theta_{ds}^M} = -\sigma < 0\)

- \(\frac{\partial G(\theta_{ds}^M, f_x)}{\partial \theta_{ds}^M} = I(\sigma - 1)\left(\rho P\theta_{ds}^M\right)^{\sigma-1}\left[\left(\frac{1}{1 - \theta_{ds}^M A}\right)^{\sigma-1} \left(\frac{1}{1 - \theta_{ds}^M A} + \frac{A}{\tau - \theta_{ds}^M A}\right) + \left(\frac{1}{\tau - \theta_{ds}^M A}\right)^{\sigma-1} \frac{A}{\tau - \theta_{ds}^M A}\right] > 0\)

\[
\therefore \frac{\partial \theta_{ds}^M}{\partial f_x} > 0
\]
\[ \frac{\partial \theta_{ds}^{MX}}{\partial \tau} = -\frac{\frac{\partial G}{\partial \theta_{ds}^{MX}}(f_x)}{\frac{\partial G}{\partial \theta_{ds}^{MX}}(f_x)} \] (38)

\[ \frac{\partial G}{\partial \theta_{ds}^{MX}}(f_x) = -l(\sigma - 1) \left( \frac{1}{\tau - \theta_{ds}^{MX} A} \right)^{\sigma - 2} \frac{1}{(\tau - \theta_{ds}^{MX} A)^2} < 0 \]

\[ : \frac{\partial \theta_{ds}^{MX}}{\partial \tau} > 0 \]

- Hence, when \( f_x \) or \( \tau \) decreases, \( \theta_{ds}^{MX} \) will also decrease (\( \theta_{ds}^{MX} \rightarrow \theta_{ds}^{MX'} \)) \( \rightarrow \) This means that a firm between \( \theta_{ds}^{sM} \) and \( \theta_{ds}^{sM} \) transfers from SBC to HBC and serves both the domestic and the foreign market in response to trade liberalization.

- More specifically, when the exposure of an economy to trade increases, between \( \theta_{ds}^{sM} \leq \theta \leq \theta_{ds}^{sM} \), some firms whose effort was \( a^0 \) before would still export, but give up the (export) subsidy and make best effort, A. The other firms whose effort was \( a^0 \) before would start to export, give up the subsidy and make best effort, A.

**Conclusion**

- Given low transportation cost (\( \tau \)) for exporting and high the subsidy (\( s \)) in the domestic,
  
  i) **Negative effects of SBC on exporting:**

  Compared with Melitz(2003), not all firms do not export, even though they can export with best effort. They serve only the domestic market and have a subsidy without best effort. The subsidy in the domestic market can hurt foreign consumers’ welfare because the number of variety decreases in the foreign market.

  ii) **Positive effects of trade on SBC:**

  Some more productive firms will have HBC and export with best effort, even though they would have the subsidy potentially in the domestic market.

- An increase in the exposure of an economy to trade encourages a firm to give up moral-hazard SBC.

- This is more likely to happen when the original transportation cost (\( \tau \)) is low between two countries and the subsidy (\( s \)) in the domestic is high.