Resurrecting the Weak Credibility Hypothesis in a Simple Model of Exchange-Rate-Based Stabilization

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We analyze how weak credibility affects the volatility of consumption spending in a model of exchange-rate-based stabilization that allows for both durable and nondurable goods. The inclusion of durables greatly improves the explanatory power of the weak credibility hypothesis. The hypothesis can account for the main qualitative properties of the boom-bust cycle provided the elasticity of durables expenditure with respect to Tobin’s q is greater than the intertemporal elasticity of substitution. Moreover, the quantitative effects are very large. In numerical simulations based on conservative assumptions about the expenditure share of durables (20%) and wealth effects (none), aggregate consumption increases 12-27% during the low-crawl phase. Variants of the model that incorporate habit formation can also explain the hump-shaped paths of durables spending and nondurables consumption.

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The announcement of an exchange-rate-based stabilization (ERBS) program is usually followed by a pronounced surge in consumption spending. This stylized fact is the key to understanding many of the other stylized facts associated with ERBS. It is a small step from a consumption boom to a large current account deficit, large capital inflows, and persistent, strong appreciation of the real exchange rate. If prices are sticky and firms in the nontradables sector produce to demand, the consumption boom also fuels a temporary output boom.

In a pair of seminal papers, Calvo and Vegh (1993, 1994a) focused on weak credibility as the underlying source of the consumption boom. The link between credibility and spending arises when holdings of real money balances affect the cost of consumption. Following Calvo and Vegh, suppose money demand is governed by a cash-in-advance constraint and that the country operates in a perfect world capital market. In this setup, a temporary (i.e., non-credible) reduction in the rate of crawl lowers the price of consumption today relative to the price of consumption in the future. Intertemporal substitution then leads to a consumption boom and a current account deficit financed by private capital inflows. The bill for high spending during the boom phase is paid in perpetuity in the post-ERBS period: when the policy collapses, consumption drops below its previous level and the country runs a trade surplus year in and year out to cover higher interest payments on the external debt.\footnote{While the weak credibility (WC) hypothesis exercises a strong intuitive appeal, its explanatory power is thought to be limited by the fact that the intertemporal elasticity of substitution is low in LDCs. Consider the solution for the peak increase in real consumption ($C_p$) in the Calvo-Vegh model. This is
\[
\frac{C_p - C_o}{C_o} = \frac{\mu}{1 + \mu(r + \pi_o)} \tau e^{-rT}(\pi_o - \pi_1),
\]
where $C_o$ is initial consumption; $r$ is the world market real interest rate; $\mu$ is the ratio of money balances to aggregate consumption (the parameter in the CIA constraint); $\pi_o$ is the initial rate of crawl and $\pi_1$ the rate during ERBS; $T$ is the length of the ERBS program; and $\tau$ is the intertemporal elasticity of substitution. For $T = 3$, $r = .08$, $\mu = .10$, $\pi_o = 1$ and $\pi_1 = .10$, we have $(C_p - C_o)/C_o = .0647\tau$. The empirical evidence places $\tau$ between .20 and .50 in LDCs (Agenor and Montiel, 1996, Table 10.1). But with $\tau = .20 - .50$, the peak}
increase in consumption is only 1-3%. Consistent with this, Reinhart and Vegh (1995) and Mendoza and Uribe (1996) found that the WC hypothesis predicts increases in consumption only 10-20% as large as the increases observed in the southern cone tablitas and other ERBS episodes. Thus both theory and empirical tests seem to argue that the WC cannot deliver strong quantitative effects (Agenor and Montiel, 1996, p.353).

In retrospect, the WC hypothesis was never given a fair hearing. Rebelo and Vegh (1995) and Reinhart and Vegh (1995) were careful to note that the spending boom might be much stronger in models that incorporate durable consumer goods. Certainly there is abundant casual evidence to support this conjecture. According to case studies and Calvo and Vegh’s (1999) stabilization time profiles, the boom-bust cycle is driven by the tremendous expansion and subsequent collapse in durables purchases. But despite the “hints” in the data, durables have not figured in most ERBS models. The sole exception is De Gregorio, Guidotti, and Vegh’s (1998) elegant analysis of the “bunching” pattern in durables spending when purchases follow a S-s rule. Their model, however, abstracts from nondurables consumption and assumes ERBS is permanent and fully credible. It is too stylized therefore to confront with the data. The question of whether weak credibility can account for the consumption boom via its impact on durables spending remains unanswered.

Our objective in this paper is to resolve the issue by investigating the outcome in an ERBS model that accomodates both durable and nondurable goods. Including durables complicates matters but not so much as to preclude the derivation of sharp analytical results. Many of the qualitative properties of the consumption path depend on whether the elasticity of durables spending with respect to Tobin’s q (Ω) is larger or smaller than the intertemporal elasticity of substitution (τ). In the benchmark case of a separable utility function, we demonstrate that Ω > τ is sufficient for: (i) durables spending to increase more than nondurables expenditure on impact; (ii) aggregate consumption to rise more than in the counterfactual scenario where all consumption is nondurable; (iii) durables spending to decrease more than nondurables consumption at the time of the policy reversal and (iv) durables spending to overshoot its lower steady-state level during the ERBS period. These results — especially overshooting — are consistent with durables being the most volatile component of aggregate consumption and with the stylized fact that durables spending leads in both the boom and the bust
phases of the ERBS cycle.

The model and the theoretical results are developed in the first three sections of the paper. In section 4 we calibrate the model and present solutions for the global nonlinear saddle path. The numerical results confirm that weak credibility triggers a huge, double-digit spending boom. Aggregate consumption increases 9-18% in the first year, rising to 12-27% by the end of ERBS. Throughout, most of the heavy lifting is done by the smallest component of expenditure: durables comprise only 20% of consumption but account for 70-90% of the increase in total spending.

Explaining the magnitude of the consumption boom is the main task for theory. It is also important, however, to get the slope of the consumption path right. This has proven difficult. Most ERBS models predict that, after an initial jump, the path of consumption is flat or declining during the low-crawl period. In point of fact, consumption either increases continuously or follows a hump-shaped path, with the downturn coming in the last 6-12 months of the program.

Our model predicts that the post-jump path of consumption is flat for the first half of the ERBS program and positively sloped in the second half. This improves on the existing literature but is still unsatisfactory. Accordingly, in section 5 we investigate whether habit formation eliminates the problematic flat stretch in the consumption path. It does, provided habit formation is rapid. When habit enters the utility function in the normal way, the path of nondurables consumption is hump-shaped and aggregate consumption rises smoothly from the beginning to the end of ERBS. In a less conventional specification where habit affects deliberation costs (i.e., costs associated with new durables purchases), the paths of durables expenditure and aggregate consumption are hump-shaped. The turning points in the different runs come at the right time, 6-9 months before ERBS collapses. Moreover, in contrast to the results in Uribe (2002), there is no marked tradeoff between the slope of the consumption path and its height. Depending on the specification, the peak increase in aggregate consumption is either about the same or twice as high as in the model without habit formation.

The final section contains concluding remarks.
1. The Model

Durables enter the consumption basket. Otherwise, the model is the same as in Reinhart and Vegh (1995). The economy is small and completely open (no nontradables sector). Domestic output is fixed at $Q$ and the inflation rate equals the rate of crawl of the currency $\pi$. The private sector divides its wealth between money $m$ and a tradable bond $b$ that pays the world market interest rate $r$. $C$, $S$, and $D$ denote, respectively, nondurables consumption, gross new durables purchases, and the stock of durables.

We lay out the model in stages, starting with the specification of financial markets and the transactions technology.

Financial Markets and the Transactions Technology

Bonds are bought and sold in a perfect world capital market. The nominal interest rate $i$ is tied down therefore by the interest parity condition:

$$i = r + \pi. \quad (1)$$

Money is held to reduce transactions costs. These costs enter the budget constraint [see equation (4) below] via the term $(C + S)L[m/(C + S)]$, where $L$ is decreasing and strictly convex in the ratio of money balances to total spending ($L' < 0$, $L'' > 0$).

The Private Agent’s Optimization Problem

All economic decisions in the private sector are controlled by a representative agent who possesses an instantaneous utility function of the form $U(C, D) - R(\dot{D}/D)$, where a dot signifies a time derivative and $U(\cdot)$ is increasing and strictly concave in $C$ and $D$. The $R(\cdot)D$ component of the utility function is taken from Bernanke (1985). It introduces a friction that prevents durables purchases from being absurdly volatile. As Bernanke emphasizes, new durables purchases are not easy or automatic: in contrast to spending on nondurables, the decision to buy a durable often involves time-consuming search and careful deliberation. The utility cost of worrying and lost leisure time is assumed to be increasing, symmetric, and convex in net purchases of durable goods: $R(0) = 0$, $R' \geq 0$ as $\dot{D} \geq 0$, and $R'' > 0$.

After imposing interest parity and defining $A \equiv m + b$ to be total wealth, the private
agent’s optimization problem may be written as

\[
\begin{align*}
\max_{(C,S,m,b)} \int_0^\infty [U(C, D) - R(S/D - \delta)D]e^{-\rho t} dt, \\
\text{subject to}
\end{align*}
\]

\[
\begin{align*}
A &= m + b, \\
\dot{A} &= Q + \tilde{g} + rb - (C + S) \left[ 1 + L \left( \frac{m}{C + S} \right) \right] - \pi m, \\
\dot{D} &= S - \delta D,
\end{align*}
\]

where \( \rho \) is the time preference rate; \( \tilde{g} = g + (C + S)L \) is lump-sum transfers; and \( \delta \) is the depreciation rate of the durable good. Transfer payments are split into two components: government transfers, \( g \), and rebated profits of firms that supply transactions services, \((C + S)L\). The artificial component, \((C + S)L\), ensures that transactions costs wash out in the budget constraint. This eliminates a potentially dubious income effect. With the income effect removed, variations in the cost of liquidity influence spending only insofar as they alter the price of current vs. future consumption.

The Maximum Principle furnishes the necessary conditions for an optimum. These consist of

\[
\begin{align*}
U_C(C, D) &= \omega_1(1 + L - L'm/X), \\
-L' &= r + \pi, \\
\omega_2 &= \omega_1(1 + L - L'm/X) + R'(S/D - \delta), \\
\dot{\omega}_1 &= \omega_1(\rho - r) = 0 \quad \text{for} \quad \rho = r, \\
\dot{\omega}_2 &= \omega_2(\rho + \delta) + R - R'S/D - U_D,
\end{align*}
\]

where \( X \equiv C + S \) and \( \omega_1 \) and \( \omega_2 \) are the multipliers attached to the constraints (4) and (5). Equation (6) says that the marginal utility of nondurables consumption equals the shadow price of wealth multiplied by the effective price of consumption \((1 + L - L'm/X)\), while (7) requires money to earn the same return at the margin as bonds. In equation (9) we have assumed \( \rho = r \) in order to abstract from trends in saving. Finally, equations (8) and (10) define a Tobin’s q model of durables purchases in which \( \omega_2/\omega_1(1 + L - L'm/X) = \omega_2/U_C \) is...
the ratio of the demand price (or shadow price) of a durable to its supply price (unity) and \( R' \) captures additional adjustment costs incurred by increasing \( S \) a small amount.

**Path of the Crawl**

The path for the crawl is

\[
\pi = \begin{cases} 
\pi_1 < \pi_o & \text{for } 0 < t < T \\
\pi_o & \text{for } 0 < t < T 
\end{cases}
\]  

(11)

ERBS commences with an announcement that the rate of crawl will be reduced from \( \pi_o \) to \( \pi_1 \) and maintained at the lower level forever more. This proves false. Forever more lasts only until year \( T \), at which time the government aborts the program and raises the crawl to its original level.

**The Public Sector Budget Constraint**

Money is injected into the economy whenever the central bank accumulates foreign exchange reserves \( k \) or runs the printing press to finance the fiscal deficit. Assuming reserves are invested in the tradable bond, the consolidated public sector budget constraint reads

\[
\dot{k} = rk + \pi m + \dot{m} - g.
\]

(12)

Fiscal policy is passive. Crucially, the reduction in the crawl is not supported by a cut in real transfer payments.\(^6\) When the program fails and \( \pi \) returns to its original level, \( g \) adjusts only enough to offset any changes in the sum of interest income and revenue from the inflation tax \((rk + \pi m)\).\(^7\)

**Net Foreign Asset Accumulation and the Current Account Balance**

Summing the public and private budget constraints produces the accounting identity that net foreign asset accumulation equals the current account surplus, viz.:

\[
\dot{Z} = Q + rZ - C - S,
\]

(13)

where \( Z \equiv k + b \).

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\(^6\)\(^7\)
Functional Forms

To obtain concrete analytical results and prepare the model for calibration, we assume

\[
U(C, D) = \left\{ \frac{a_1 C^{(\sigma-1)/\sigma} + a_2 D^{(\sigma-1)/\sigma}}{\sigma^{1/\tau} + \sigma^{1/\tau}} \right\}^{1-1/\tau},
\]

\[
R(S/D - \delta) = x \frac{(S/D - \delta)^2}{2}, \quad x > 0,
\]

\[
L \left( \frac{m}{C+S} \right) = h \left( \frac{m}{C+S} \right)^{1-1/\beta}, \quad h > 0, \quad 0 < \beta < 1,
\]

where \(a_1\) and \(a_2\) are distribution parameters, \(\sigma\) is the elasticity of substitution between durable and nondurable consumer goods, and \(\tau\) is the intertemporal elasticity of substitution. These are familiar functional forms. Nondurable consumption and the service flow from durables combine in a CES-CRRA function, while deliberation costs are a quadratic function of new durables purchases. The specification of transactions costs is the same as in Reinhart and Vegh (1995) and Uribe (2002).

The CES-CRRA utility function is flexible but a bit ungainly. In what follows, it will prove helpful to have some elasticity formulas at hand:

\[
- \frac{U_C}{U_{CC} C} = \frac{\sigma \tau}{\tau \theta_d + \sigma \theta_c}, \quad \frac{U_{CD} D}{U_C} = \frac{\tau - \sigma}{\tau \sigma} \theta_d,
\]

\[
- \frac{U_D}{U_{DD} D} = \frac{\sigma \tau}{\tau \theta_c + \sigma \theta_d}, \quad \frac{U_{DC} C}{U_D} = \frac{\tau - \sigma}{\tau \sigma} \theta_c.
\]

\(\theta_c\) and \(\theta_d\) are the respective shares of nondurables and durables in total consumption. It is easy to show from the first-order conditions that

\[
\theta_c = \frac{a_1 C^{(\sigma-1)/\sigma}}{a_1 C^{(\sigma-1)/\sigma} + a_2 D^{(\sigma-1)/\sigma}} = \frac{U_C C}{U_C C + U_D D} = \frac{C}{C + (r + \delta) D},
\]

\[
\theta_d = \frac{a_2 D^{(\sigma-1)/\sigma}}{a_1 C^{(\sigma-1)/\sigma} + a_2 D^{(\sigma-1)/\sigma}} = 1 - \theta_c,
\]

evaluated at a steady state.
2. Solving the Model for Small Changes

It is possible to derive analytical results when the reduction in the crawl is small. This is worth doing. Although the final word rests with the solution for the global nonlinear saddle path, a lot can be learned about the general nature of the dynamics by solving the model for differential changes.

We start by manipulating the first-order conditions. Differentiate (8) with respect to time and substitute for $\dot{\omega}_2$ from (10). Since $m/X$ and $U_C$ are constant during intervals where $\pi$ is constant, we get

$$\dot{\omega}_2 = R'' \left( \frac{\dot{S}}{D} - \frac{S}{D^2} \frac{\dot{D}}{D} \right),$$

$$\implies \frac{R''}{D} \dot{S} = (\rho + \delta)U_C + (\rho + \delta - S/D)R' + \frac{R''S}{D^2}(S - \delta D) + R - U_D. \quad (14)$$

Equations (5) and (14) are a self-contained sub-system in $S$ and $D$ when the utility function is separable. ($U_C$ is constant in the first term.) In the non-separable case, however, we need to know how $C$ varies on the transition path. This information is supplied by (6):

$$dC = -\frac{U_{CD}}{U_{CC}}dD. \quad (15)$$

Linearizing (14) around a stationary equilibrium $(\bar{S}, \bar{D})$ now produces

$$\frac{R''}{D} \dot{S} = (\rho + \delta)R'' \left[ (S - \bar{S}) - \delta(D - \bar{D}) \right] + \frac{R''S}{D^2}(S - \delta D) + R - U_D,$$

or

$$\frac{R''}{D} \dot{S} = (\rho + \delta)R'' \left[ (S - \bar{S}) - \delta(D - \bar{D}) \right] - \frac{(\rho + \delta)U_C}{(\tau\theta_d + \sigma\theta_c)D}(D - \bar{D}), \quad (16)$$

after using the elasticity formulas.\(^8\) To relate $R''$ to observable magnitudes, write (8) as

$$1 + R'(S/D - \delta)/U_C = q,$$  \hspace{1cm} (17)

where $q \equiv \omega_2/U_C$ is Tobin’s $q$, the ratio of the demand price of a durable to its supply price.
(unity). Differentiating with respect to $S$ and $q$ yields
\[
\frac{R'' S}{U_C D} \frac{dS}{S} = q \frac{dq}{q}.
\]
Define $\Omega \equiv (dS/dq)q/S$ to be the elasticity of durables spending with respect to Tobin’s $q$. Evaluated at a steady state where $S/D = \delta$ and $q = 1$,
\[
R'' = \frac{U_C}{\Omega \delta}.
\]
The linearized system is thus
\[
\begin{bmatrix}
\dot{S} \\
\dot{D}
\end{bmatrix} =
\begin{bmatrix}
\rho + \delta & -(\rho + \delta)\delta - c \\
1 & -\delta
\end{bmatrix}
\begin{bmatrix}
S - \bar{S} \\
D - \bar{D}
\end{bmatrix},
\]
where
\[
c \equiv -\frac{(\rho + \delta)\delta \Omega}{\tau \theta_d + \sigma \theta_c} < 0.
\]
The steady state is a saddle point with eigenvalues
\[
\lambda_{1,2} = \frac{\rho \pm \sqrt{\rho^2 - 4c}}{2}, \quad \lambda_1 > 0, \, \lambda_2 < 0.
\]
During the ERBS phase, the dynamics are governed by a nonconvergent path of the system associated with the low rate of crawl $\pi_1$. For this phase, (18) gives
\[
S(t) - S_1 = (\lambda_1 + \delta)h_1 e^{\lambda_1 t} + (\lambda_2 + \delta)h_2 e^{\lambda_2 t}, \quad t < T, \quad (19)
\]
\[
D(t) - D_1 = h_1 e^{\lambda_1 t} + h_2 e^{\lambda_2 t}, \quad t < T, \quad (20)
\]
where $h_1$ and $h_2$ are constants and $(S_1, D_1)$ is the stationary equilibrium paired with $\pi_1$.

After the policy reversal at time $T$, the economy follows the saddle path that leads to the new long-run equilibrium $(S_2, D_2)$. On the convergent path, the term involving the positive eigenvalue drops out:
\[
S(t) - S_2 = (\lambda_2 + \delta)h_3 e^{\lambda_2 t}, \quad t \geq T, \quad (21)
\]
\[
D(t) - D_2 = h_3 e^{\lambda_2 t}, \quad t \geq T. \quad (22)
\]
$h_3$ is another constant. Also, we have exploited the fact that for small changes the negative eigenvalue is the same as in (19) and (20).

Five boundary conditions are required to pin down $D_1$, $D_2$, and $h_1 - h_3$. ($S_i = \delta D_i$ takes care of durables spending.) We get two conditions right away by evaluating the solution for the state variable at the beginning and the end of the ERBS program. Let $o$ refer to the initial steady state. At $t = 0$, equation (20) then reads

$$D_o - D_1 = h_1 + h_2. \quad (23)$$

Furthermore, (20) and (22) must return the same solution at $T$. Thus

$$h_1 e^{\lambda_1 T} + (h_2 - h_3) e^{\lambda_2 T} = D_2 - D_1. \quad (24)$$

Turn next to the solution paths for $S$ in (19) and (21). These link up through a jump in spending at time $T$. To determine the jump, note that optimizing behavior and perfect foresight imply that $\omega_2$ is constant at time $T$. Since $\omega_1$ is also constant, $C$, $m$ and $S$ jump to preserve the first-order conditions (6)-(8) when $\pi$ abruptly increases to its pre-stabilization level. The jumps in $C$ and $S$ are

$$\hat{C}(T) \equiv \frac{C(T^+) - C(T^-)}{C(T^-)} = -\frac{\sigma \tau \mu}{(\tau \theta_d + \sigma \theta_c) J} (\pi_o - \pi_1), \quad (25)$$

$$\hat{S}(T) \equiv \frac{S(T^+) - S(T^-)}{S(T^-)} = -\frac{\Omega \mu}{J} (\pi_o - \pi_1), \quad (26)$$

where $\mu \equiv m/X$ and $J \equiv 1 + L - L'm/X$. We also have from (19) and (21)

$$S(T^+) - S(T^-) = (\lambda_2 + \delta) e^{\lambda_2 T} (h_3 - h_2) + \delta (D_2 - D_1) - (\lambda_1 + \delta) e^{\lambda_1 T}. \quad (27)$$

Equating the solutions in (26) and (27) provides

$$(\lambda_2 + \delta) e^{\lambda_2 T} (h_3 - h_2) + \delta (D_2 - D_1) + \frac{\Omega \mu \delta D}{J} (\pi_o - \pi_1) - (\lambda_1 + \delta) e^{\lambda_1 T} = 0. \quad (28)$$

The last two boundary conditions fix the stationary equilibria associated with the low
and high rates of crawl. Comparing steady states, we have from equations (6)-(8) and (10)

\[
\frac{a_2}{a_1} \left( \frac{D_i}{C_i} \right)^{-1/\sigma} = \rho + \delta, \quad i = 1, 2
\]  

(29)

\[
\frac{U_C(C_1, D_1)}{U_C(C_2, D_2)} = \frac{1 + (h/\beta)[m_1/(C_1 + \delta D_1)]^{1-1/\beta}}{1 + (h/\beta)[m_2/(C_2 + \delta D_2)]^{1-1/\beta}}.
\]  

(30)

\[
\frac{m_1}{C_1 + \delta D_1} = \left( \frac{r + \pi_1}{r + \pi_o} \right)^{-\beta} \frac{m_2}{C_2 + \delta D_2}.
\]  

(31)

This group of equations can be solved for \(C_1 - C_2, D_1 - D_2\) and \(m_1 - m_2\) as a function of \(\pi_o - \pi_1\). Routine algebra delivers

\[
\frac{C_1 - C_2}{C_1} = \frac{D_1 - D_2}{D_1} = \frac{\tau \mu}{J} (\pi_o - \pi_1).
\]  

(32)

The pull of intertemporal substitution is evident here. The effective price of consumption in the ERBS period is lower relative to the price in the post-ERBS period by the amount \((\pi_o - \pi_1)\mu/J\). Multiplying this by the intertemporal elasticity of substitution \(\tau\) gives the percentage difference between \(C_1(D_1)\) and \(C_2(D_2)\).

Finally, we need to invoke the condition that the path in the post-ERBS period converge to a stationary equilibrium where the current account deficit equals zero and the stock of external debt is constant. Working toward this end, interpret the differentials in (15) as deviations from the stationary equilibrium:

\[
C(t) = C_i - \frac{U_{CD}}{U_{CC}}[D(t) - D_i],
\]

\[
\Rightarrow C(t) = C_i + f[D(t) - D_i],
\]  

(33)

where\(^{10}\)

\[
f \equiv \frac{(\tau - \sigma)\theta_c(\rho + \delta)}{\tau \theta_d + \sigma \theta_e}.
\]

Substitute the solution paths for \(C\) and \(S\) into (13):

\[
\dot{Z} = Q + rZ - C_1 - \delta D_1 - (f + \lambda_1 + \delta)h_1e^{\lambda_1 t} - (f + \lambda_2 + \delta)h_2e^{\lambda_2 t}, \quad t < T, \quad (34a)
\]

\[
\dot{Z} = Q + rZ - C_2 - \delta D_2 - (f + \lambda_2 + \delta)h_3e^{\lambda_2 t}, \quad t \geq T. \quad (34b)
\]
For $Z_o = 0$, equation (34a) yields

$$Z(T) = Z_1(1 - e^{rT}) + h_1 x_1 (e^{rT} - e^{\lambda_1 T}) - h_2 x_2 (e^{\lambda_2 T} - e^{rT}),$$

(35)

where

$$x_i = \frac{\lambda_i + \delta + f}{\lambda_i - r}, \quad i = 1, 2.$$  

In the post-ERBS period,

$$Z(t) = Z_2 - h_3 x_2 e^{\lambda_2 t} + e^{r(t-T)}[Z(T) - Z_2 + h_3 x_2 e^{\lambda_2 T}], \quad t > T.$$  

The term in brackets must vanish if $Z$ is to converge to $Z_2$. Hence

$$Z_2 = Z(T) + h_3 x_2 e^{\lambda_2 T}.$$ 

(36)

We are almost done. Note that $h_1 - h_3$ depend on $D_i$. But the $D_i$ depend on $Z_i$ because variations in the stock of debt affect national income and the sustainable level of spending [see equation (13)]. Across steady states

$$Z_i = \frac{\delta D_i + C_i - Q_i}{r}.$$  

Since changes in $C_i$ are proportional to changes in $D_i$,

$$D_2 - D_o = \frac{r(1 - \gamma)}{\delta} Z_2,$$

(37a)

$$D_1 - D_o = \frac{\tau D_o \mu}{J} (\pi_o - \pi_1) + \frac{r(1 - \gamma)}{\delta} Z_2,$$

(37b)

$$C_i - C_o = \frac{C_o}{D_o} (D_i - D_o), \quad i = 1, 2,$$

(37c - 37d)

$$Z_1 = \frac{\delta}{r(1 - \gamma)} (D_1 - D_o),$$

(37e)

where $\gamma \equiv C/(C + \delta D)$, the share of nondurables in total consumption spending. Equations (23), (24), (28), (35), (36), and (37a)-(37e) determine $h_1 - h_3$, $Z_i$, $D_i$, $Z(T)$ and $C_i$. Working out the solution entails a good deal of tedious algebra, but, in the end, the results are sensible.
and intuitive. In a supplementary appendix (available upon request), we show that

\[ D_1 > D_o > D_2, \]
\[ C_1 > C_o > C_2, \]

and that

\[
h_1 = \frac{e^{-\lambda_1 T}}{\lambda_1 - \lambda_2} \frac{\delta \Omega D\mu}{J} x_2 (\pi_o - \pi_1),
\]
\[
Z_2 = -\frac{\delta \tau D\mu}{J[\delta + r(1 - \gamma)x_2]} (\pi_o - \pi_1)(\frac{1 - e^{-rT}}{r(1 - \gamma)} + (1 - e^{-\lambda_1 T})x_2) < 0,
\]
\[
h_3 = \frac{e^{-rT} D\mu}{(\lambda_1 - \lambda_2) J} (\pi_o - \pi_1) \left[ \delta \Omega (e^{\lambda_1 T} - e^{\lambda_2 T}) + \tau (\lambda_1 e^{\lambda_1 T} - \lambda_2 e^{\lambda_2 T}) \right] + D_o - D_1 > 0.
\]

3. The Path of Spending

We are now in a position to delineate the path of spending. At \( t = 0 \), equations (19), (33) and (37c)-(37d) give

\[
\hat{C}(0) \equiv \frac{C(0) - C_o}{C_o} = \frac{D_1 - D_o}{D_o} \frac{\sigma}{\tau \theta_d + \sigma \theta_c} > 0,
\]
\[
\hat{S}(0) \equiv \frac{S(0) - S_o}{S_o} = \frac{h_1 (\lambda_1 - \lambda_2) - \lambda_2 (D_1 - D_o)}{\delta D_o}.
\]

The percentage jump in \( S \) exceeds that of \( C \) when

\[
h_1 (\lambda_1 - \lambda_2) > (D_1 - D_o) \left( \lambda_2 + \frac{\delta \sigma}{\tau \theta_d + \sigma \theta_c} \right).
\]

It is easy to verify that

\[ h_1 \geq 0 \quad \text{as} \quad x_2 = 1 + \lambda_2 \tau / \delta \Omega \geq 0. \]

Hence

\[ \delta \Omega + \lambda_2 \tau > 0 \quad \text{and} \quad \lambda_2 + \frac{\delta \sigma}{\tau \theta_d + \sigma \theta_c} < 0. \]
ensure that (40) holds. The first condition is equivalent to

$$\Omega > \frac{\tau \rho \theta \sigma + \delta \tau}{\sigma (\tau d + \sigma c)},$$

(41a)

while the second requires

$$\Omega > \sigma \left[ 1 + \frac{\delta (\sigma - \tau) \theta d}{(\rho + \delta)(\tau d + \sigma c)} \right].$$

(41b)

Combining (41a) and (41b) produces

$$\hat{S}(0) > \hat{C}(0) \quad \text{if} \quad \Omega > \max \left\{ \frac{\tau \rho \theta \sigma + \delta \tau}{\sigma (\tau d + \sigma c)}, \sigma \left[ 1 + \frac{\delta (\sigma - \tau) \theta d}{(\rho + \delta)(\tau d + \sigma c)} \right] \right\}. \quad (42)$$

Now consider the path in the post-ERBS period. From (21), (25) and (26), we have

$$\hat{S}(T), \hat{C}(T) < 0 \quad \text{with} \quad |\hat{S}(T)| > |\hat{C}(T)| \quad \text{iff} \quad \Omega > \frac{\sigma \tau}{\tau d + \sigma c},$$

(43)

and

$$S(t) < S_2 \quad \text{iff} \quad \lambda_2 + \delta < 0$$

$$\implies S(t) < S_2 \quad \text{iff} \quad \Omega > \tau d + \sigma c. \quad (44)$$

Both $S$ and $C$ decrease at the time of the policy reversal, but the percentage decrease in $S$ is greater when $\Omega$ satisfies (43). Furthermore, if (44) holds, $S$ overshoots its lower steady-state level on the path to the new long-run equilibrium.

The conditions in (42)-(44) have the same general structure: in each, $\Omega$ stands alone on the left side and a term involving $\sigma$ and $\tau$ occupies the right. These parameters claim the spotlight because they play pivotal roles in conditioning the intertemporal responses of $S$ and $C$. The scope for intertemporal substitution in nondurables consumption is tied to concavity of the utility function in $C$, which depends on both $\sigma$ and $\tau$. Concavity of the utility function also affects durables spending but is much less important as very little of the durable good is consumed at the time of purchase. Intertemporal substitution is limited instead by rising marginal deliberation costs. This friction is subsumed in the elasticity of $S$ with respect to Tobin’s $q$. Consequently, when $\Omega$ is large relative to $\sigma$ and $\tau$, the response of durables to intertemporal variations in the effective price of consumption is stronger than
the response of nondurables. In the subsections that follow, we will be precise about what “large relative to $\sigma$ and $\tau$” means.

3.1 The Benchmark Case of a Separable Utility Function ($\sigma = \tau$)

Econometricians have yet to estimate the value of $\sigma$ for even one LDC. Nevertheless, there is some basis for the view that a separable utility function deserves the status of the benchmark case. Estimates of demand systems with 5-10 goods find that compensated own-price elasticities in both developed and less developed countries are on the order of .15-.65 (Lluch et al., 1977; Deaton and Muellbauer, 1980; Blundell et al., 1993), implying a slightly higher range of .20-.75 for the intratemporal elasticity of substitution. Some of the product categories in the demand systems refer mainly to semi-durables (e.g., clothing and textiles); others mix durable, non-durable, and semi-durable goods. At present, therefore, the best educated guess is that $\sigma$ also lies in the .20-.75 range. Since this closely overlaps the estimated range for $\tau$ in LDCs, we center the analysis around the dynamics for $\sigma = \tau$.

The results in the benchmark case are exceptionally clean. When the utility function is separable, the conditions in (41a),(41b),(43) and (44) all reduce to $\Omega > \tau$. This gives

**Proposition 1** When the utility function is separable between durables and nondurables, $\Omega > \tau$ is necessary and sufficient for durables spending to (i) increase more than nondurables spending at the start of ERBS [$\hat{S}(0) > \hat{C}(0)$], (ii) decrease more than nondurables spending at the time of the policy reversal, and (iii) overshoot and remain below its steady-state level throughout the post-ERBS period [$S(t) < S_2$, $t > T$].

Figure 1 shows the complete transition path. The $\dot{S} = 0$ and $\dot{D} = 0$ schedules determine which way the north-south and east-west directional arrows point during the ERBS phase, while $A_iA_i$ is the saddle path associated with $(S_i, D_i)$. In the lower quadrant, we use equation (33) to track the path of nondurables consumption.

When ERBS is announced, $C$ jumps to $C_1$ and $S$ jumps to a point above $A_1A_1$ (see the appendix). After the initial jumps, $C$ stays at $C_1$ until ERBS collapses while $S$ either rises continuously or follows a shallow U-shaped path. In the latter case, the transition path includes a period of decreasing expenditure. This phase is strictly transitory. Before the program fails, the path crosses the $\dot{S} = 0$ schedule. In the numerical simulations, the crossing point is typically around the middle of the second year (when $T = 3$).
The consumption spree and ERBS end at the same moment. When the crawl abruptly increases at \( T \), spending plummets across-the-board. The diagram suggests and the numerical simulations will shortly confirm that the collapse is concentrated in durables purchases: \( C \) jumps to \( C_2 \) and is constant thereafter, but \( S \) overshoots \( S_2 \).\(^{13}\) Durables lead in the sudden crash just as they lead in the boom. Over the whole cycle, the gyrations in aggregate consumption mirror the volatile dynamics of durables expenditure.

3.2 Edgeworth Substitutes (\( \sigma > \tau \))

In the case where durables and nondurables are Edgeworth substitutes, \( \Omega \geq \sigma \) guarantees the conditions in (41a), (43), and (44), but not the condition in (41b). This can be handled by pushing \( \Omega \) a little above \( \sigma \). Write (41b) as

\[
\Omega > \sigma \left[ 1 + \frac{(\sigma - \tau)(1 - \gamma)}{\tau(1 + \rho/\delta)(1 - \gamma) + \sigma \gamma} \right],
\]

where, to repeat, \( \gamma \equiv C/(C + \delta D) \) is the share of nondurables in total consumption spending. Since the term in square brackets is less than \( 1/\gamma \), we have

**Proposition 2** When durables and nondurables are Edgeworth substitutes, \( \Omega > \sigma/\gamma \) is sufficient for durables spending to (i) increase more than nondurables spending at the start of ERBS \( \hat{S}(0) > \hat{C}(0) \), (ii) decrease more than nondurables spending at the time of the policy reversal, and (iii) overshoot and remain below its steady-state level throughout the post-ERBS period \( S(t) < S_2, t > T \).

**Remark 1** The data place \( \gamma \) around .80 in LDCs; hence \( \Omega \) does not have to be very much above \( \sigma \) to meet the sufficient condition.

Figure 2 portrays the path to the new steady state. The main difference compared to the benchmark case is that both components of consumption overshoot at time \( T \).

3.3 Edgeworth Complements (\( \tau > \sigma \))

To derive the strongest result for the case of Edgeworth complements, return to (40), substitute for \( h_1 \) and \( D_1 - D_o \), and multiply both sides by \( e^{\lambda_1 T}[x_2 + \delta/r(1 - \gamma)] \). After extensive simplification, this produces

\[
\Omega x_2 \left[ x_2 + \frac{\delta}{r(1 - \gamma)} \right] > \left( \frac{\lambda_2 \tau}{\delta \Omega} \Omega + \frac{\sigma \tau}{\tau \theta_d + \sigma \theta_c} \right) \left[ x_2 + \frac{\delta e^{-\lambda_2 T}}{r(1 - \gamma)} \right].
\]
Recall that $\lambda_2 \tau / \delta \Omega = x_2 - 1$. Making this substitution on the right side leads to

$$\Omega_2 (1 - e^{-\lambda_2 T}) > \left( \frac{\sigma \tau}{\tau \theta_d + \sigma \theta_c} - \Omega \right) \left[ x_2 \frac{r(1 - \gamma)}{\delta} + e^{-\lambda_2 T} \right].$$

(40')

The term in square brackets is always positive (see the appendix). Thus the above condition holds for $x_2 < 0$ and $\Omega > \sigma \tau / (\tau \theta_d + \sigma \theta_c)$. On the other hand, if $x_2$ is positive, then (41a) must hold. We need only appeal to (41b) therefore to ensure $\hat{S}(0) > \hat{C}(0)$. But since $\tau > \sigma$ in the case under consideration, any value of $\Omega$ that satisfies (41a) also satisfies the weaker condition (41b). The upshot of all this is

**Proposition 3** When durables and nondurables are Edgeworth complements,

$$\Omega > \max \{ \sigma \tau / (\tau \theta_d + \sigma \theta_c), \tau \theta_d + \sigma \theta_c \}$$

is sufficient for durables spending to (i) increase more than nondurables spending at the start of ERBS [$\hat{S}(0) > \hat{C}(0)$], (ii) decrease more than nondurables spending at the time of the policy reversal, and (iii) overshoot and remain below its steady-state level throughout the post-ERBS period [\(S(t) < S_2, t > T\)].

**Remark 2** The borderline value of $\Omega$ that satisfies the sufficient condition is smaller than $\tau$. ($\tau > \sigma$ and the arguments of $\max \{ \cdot \}$ are the weighted harmonic mean and the weighted arithmetic mean.)

Edgeworth complementarity changes the look of the adjustment process in two ways (Figure 3). First, nondurables consumption rises throughout the ERBS phase and undershoots its steady-state level in the post-ERBS period. Second, $S$ does not necessarily jump to a point above the $A_1A_1$ schedule (see the appendix). Consequently, durables spending may decrease steadily after its initial jump.

### 3.4 The Grand Corollary

Absent information about the sign of $\sigma - \tau$, it is useful to have a general result that applies irrespective of whether durables and nondurables are Edgeworth substitutes, Edgeworth complements, or independent. The union of Propositions 1-3 fills the bill:

**Corollary 1** $\Omega > \max \{ \sigma / \gamma, \tau \}$ is sufficient for durables spending to (i) increase more than nondurables spending at the start of ERBS [$\hat{S}(0) > \hat{C}(0)$], (ii) decrease more than nondurables spending at the time of the policy reversal, and (iii) overshoot and remain below its steady-state level throughout the post-ERBS period [$S(t) < S_2, t > t_1$].
Taking stock, after three propositions and one corollary, where do we stand? Certainly the weak credibility hypothesis has regained some of its swagger. Given the weight of the evidence that \( \sigma \) and \( \tau \) are well below unity and the tremendous volatility of durables spending in ERBS programs, there is not much doubt that \( \Omega \) is considerably larger than both \( \sigma/\gamma \) and \( \tau \) and that the boom-bust cycle in durables expenditure drives the boom-bust cycle in aggregate consumption. On its own, however, this is not enough to rehabilitate the hypothesis. The analytical results were derived for small changes. As such, they are only suggestive of strong quantitative effects. Confirmation is needed from numerical simulations that the predicted paths for durables expenditure and aggregate consumption lie within shouting distance of the big numbers seen in the data.

4. Calibration of the Model and Numerical Results

To calibrate the model, we set

\[
\gamma_o = .80, \quad \mu_o = .10, \quad \beta = .50, \quad \pi_o = 1, \quad \pi_1 = .10, \quad \delta = .10, \quad T = 3, \quad r = .05,
\]

and let \( \sigma, \Omega \) and \( \tau \) assume multiple values:

\[
\sigma = .25, .50, .75, \quad \tau = .25, .50, \quad \Omega = 5, 10.
\]

The ratio of money balances to GDP (\( \mu \)) is an ordinary 10%. With respect to the other choices:

- *Elasticity of money demand with respect to the interest rate* (\( \beta \)). Reinhart and Vegh (1995), Rossi (1989), and Arrau et al. (1995) have estimated money demand functions of the type employed in our model. The value assigned to \( \beta \) is almost the same as the average of their estimates for Argentina, Brazil, Mexico, Chile and Uruguay.

- *Consumption share of durables* (\( 1 - \gamma \)). The share of durables in aggregate spending is \( S/(C+S) \). A figure close to this can be computed from the United Nations National Income Accounts. For a broad definition that includes semi-durables, the share lies in the .18-.22 range: .223 for Mexico (2000), .179 for Colombia (1998), .180 for Bolivia (1992), .203 for the Philippines, and .217 for S. Africa (2001) and S. Korea (2002). The value in the model (.20) is the average of the values for Mexico and Colombia.

- *Depreciation rate for durables* (\( \delta \)). The Central Statistical Office of Great Britain and the U.S. Department of Commerce estimate the service life to be ten years for major
appliances, cars, and other vehicles (Williams, 1998). We used this figure to fix \( \delta \). (There are no data for LDCs.)

- **Length of the ERBS program \((T)\).** The low-crawl period lasts three years. Three is a popular choice in the literature and close to the average value in Calvo and Vegh’s (1999) dataset for major ERBS episodes.

- **Rate of crawl before vs. during ERBS \((\pi_0, \pi_1)\).** The numerical simulations cut the rate of crawl from an initial value of 100% to 10% during ERBS. This is larger than the reductions in the Chilean and Uruguayan tablitas but far smaller than the reductions in the Argentine tablita, Mexico’s Solidarity Pact, or Argentina’s Convertibility Plan.

- **Real interest rate \((r)\).** Reinhart and Vegh (1995) peg the world market real interest rate at 3%. Burstein et al. (2001) and Rebelo and Vegh (1995) opt for 4.1%, while Uribe (2002) prefers 6.5%. We compromise on 5%, a value inbetween the long-run real returns paid by U.S. stocks and treasury bonds.

- **Intertemporal elasticity of substitution \((\tau)\).** The low and high values for the intertemporal elasticity of substitution are in line with estimates for LDCs, most of which place \( \tau \) somewhere between .20 and .50 (Agenor and Montiel, 1996, Table 10.1).

- **Elasticity of substitution between durable and nondurable consumer goods \((\sigma)\).** Although there are no estimates of \( \sigma \) for LDCs, a range of .25-.75 is at least consistent with the empirical evidence that compensated elasticities of demand tend to be small at high levels of aggregation. This range is also wide enough to encompass cases of Edgeworth complementarity and Edgeworth substitutability.

- **Elasticity of durables spending with respect to Tobin’s q \((\Omega)\).** Baxter (1996) sets \( \Omega \) at 200. (Note: 200 is not a typo.) Baxter and Crucini (1993) and Rebelo and Vegh (1995) choose 15 in models where the durable good is physical capital. 200 and 15 appear to be a pure guesses. (Our own literature search for information about \( \Omega \) proved fruitless.) We restrict \( \Omega \) to much lower values but still get tremendous variation in durables expenditure over the ERBS cycle.

### 4.1 Solution Technique

The numerical solutions report the results for the global nonlinear saddle path. This required a technical innovation to deal with the unit root problem generic to models that postulate an exogenous world market interest rate. (In continuous time, the unit root shows up as a zero eigenvalue.) Due to the unit root, it is not possible to solve for the steady state independent of the transition path. But without prior knowledge of the steady state, a conventional shooting program does not know what to shoot for. The existing literature has relied therefore on linear approximations to the true solution. This is troubling, for in the ERBS context there are good reasons to fear that approximation error may be much greater
than normal. Rebelo and Vegh (1995) acknowledge the problem is serious when they write: “The fact that [the] model displays a unit root (associated with the ability to borrow and lend at a fixed interest rate) may lower the accuracy of the numerical approximations, since we linearize around a steady state to which the economy never returns” (footnote 12, p.146, our emphasis).

We solved the unit root problem by employing a different search strategy. Instead of shooting for the path that takes the economy to the unknown steady state, our program shoots for the path that eventually satisfies the conditions for a stationary equilibrium. Guided by this strategy, the program solves for the transition path and the steady state simultaneously. See Atolia and Buffie (2006) for a more detailed discussion of the algorithm.

4.2 Numerical Results

Figures 4-7 show the percentage deviations of nondurables consumption, durables spending, and aggregate consumption \((C + S)\) from their initial values. There are four runs with a separable utility function and two each for the cases of Edgeworth complementarity and Edgeworth substitutability. In the figure for aggregate consumption, the dashed line is the path when all consumption is nondurable. The vertical distance between this line and the actual path reflects the impact of durables expenditure at each point in the cycle.

What stands out in a quick pass through the figures are the big numbers for aggregate consumption and durables spending. The peak increase ranges from 12% to 27% for aggregate consumption and from 46% to 117% for durables. Furthermore, the “fit” between the numerical results and the stylized facts pertaining to the composition of consumption growth and the volatility of durables expenditure is remarkably good. In the panel regressions reported by DeGregorio, Guidotti and Vegh (1998), annual growth of durables spending averages 21% for the first three years, with the 95% confidence interval bracketing 8.5%-33.7%.\(^{15}\) The point estimate of 21% is 3.5 times larger than the estimate for total consumption growth. By comparison, in the eight cases covered by Figures 4-7, the peak increase in durables spending is 3-5 times larger than the peak increase in aggregate consumption and the average growth rate for durables ranges from 13% to 29%.\(^{16}\)
5. Habit Formation

While the model delivers good results for the magnitude and the composition of the consumption boom, it does less well in accounting for the slope of the consumption path. The time profiles of aggregate consumption and durables spending are essentially the same in each case: a sharp jump at \( t = 0 \), a succeeding flat stretch for the next 1.5 years, and then rapid, accelerating growth until the end of year three. This is at variance with the facts. Consumption rose continuously in some ERBS episodes (e.g., Argentina, 1967-1970; Brazil, 1964-1968; Mexico, 1988-1994; Peru, 1985-1987; Paraguay, 1991-1997); in others, a flat stretch or a downturn materialized, but only in the last year of the program.

The results in Uribe (2002) suggest that a model with habit formation might generate a more realistic path for consumption. Following Carroll et al. (2000) and Fuhrer (2000), let the stock of habit \( H \) grow at the rate

\[
\dot{H} = v(N - H),
\]

where \( v > 0 \) and

\[
N \equiv [a_1C^{(\sigma - 1)/\sigma} + a_2D^{(\sigma - 1)/\sigma}]^{\sigma/(\sigma - 1)}.
\]

Habit enters the utility function with the persistence parameter \( \alpha \). Two formulations dominate the literature:

\[
U(N, H) = \frac{(N - \alpha H)^{1-1/\tau}}{1 - 1/\tau}, \quad 0 < \alpha < 1
\]

(46a)

and

\[
U(N, H) = \frac{(N/H^\alpha)^{1-1/\tau}}{1 - 1/\tau}, \quad 0 < \alpha < 1.
\]

(46b)

In the additive specification, utility is a function of the difference between current consumption and the stock of habit (also called the “subsistence level of consumption”). The multiplicative specification assumes that felicity depends instead on consumption relative to the stock of habit. The difference in emphasis is slight but important. Both specifications capture the basic idea of habit formation — that the private agent desires to smooth the change as well as the level of consumption. The choice of functional form also affects, however, the
ease of intertemporal substitution and the degree of complementarity/substitutability be-
tween durables and nondurables during periods of adjustment where \( H \neq N \). We elaborate
on this later in sub-sections 5.1 and 5.2.

There is general agreement in the literature that the persistence parameter \( \alpha \) is on the
order of .70. Unfortunately, no similar consensus about the likely value of \( v \). Numerous
papers postulate very rapid habit formation, citing Fuhrer (2000) for support. It is common
practice, for example, to set the stock of habit equal to last quarter’s consumption. At the
opposite extreme, the evidence marshalled in Constantinides (1991), Heaton (1995), and
Boldrin et al. (1997) suggests that it takes years for habits to fully adjust. Carroll et al.
(2000) and Mansoorian and Michelis (2005) subscribe to this view, calibrating their models
with \( v = .20 \).

In the next two sections we present results for the case where \( \tau = \sigma = .25 \), \( \Omega = 10 \) and
\( \alpha = .70 \). Since our approach is exploratory, we do not take sides in the debate about the
right value for \( v \) or the right way to put habit in the utility function. Habit may enter the
utility function additively or multiplicatively and \( v \) takes either the low value .5 or the high
value 6.5. In the runs with \( v = 6.5 \), the stock of habit covers 80% of the distance to its new
long-run level within one quarter.\(^{18}\) For \( v = .5 \), the 80% point is not reached until 3.25 years
elapse.

5.1 The Additive Specification

Habit formation creates an incentive to adjust consumption gradually in a series of steps.
This moderates consumption growth most in the early and late stages of ERBS as the private
agent tries to smooth the large jumps in \( C \) and \( S \) at \( t = 0 \) and \( t = T \). Under the additive
specification, the consumption boom is also weakened by temporarily lower values for the
intertemporal elasticity of substitution.\(^{19}\) At \( t = 0 \), the stock of habit is fixed and

\[
- \frac{U_N}{U_{NN}N} \bigg|_{H \text{ constant}} \equiv \bar{\tau} = \tau \left( 1 - \alpha \frac{H}{N} \right).
\]

But after full adjustment

\[
- \frac{U_N}{U_{NN}N} \bigg|_{dH = dN} = \tau.
\]

In the case where \( v = 6.5 \), rapid habit formation brings \( \bar{\tau} \) close to \( \tau \) in the space of six
months. For \( v = .5 \), however, the difference between \( \bar{\tau} \) and \( \tau \) is large in the first half of \( \text{ERBS} \) and nontrivial at the end of the program. Consequently, the downward pull to the consumption path is stronger and more persistent.

The relationship between habit formation and the trajectory of nondurables consumption is more complex. As usual, higher consumption today raises the marginal utility of consumption tomorrow by increasing the stock of habit. If the story ended here, we could be sure that habit formation would impart an upward tilt to the path of nondurables consumption. But it doesn’t. There are two complicating factors in our model. First, the desire to smooth the fall in consumption at \( T \) tends to ratchet the consumption path southward immediately after the jump in at \( t = 0 \). Second and more important, habit formation changes the impact of variations in \( D \) on \( U_c \) while habit adjustment is incomplete. With full adjustment and \( \tau = \sigma \),

\[
\left. \frac{U_{CD}D}{U_C} \right|_{dH=0} \equiv \eta = 0,
\]

whereas

\[
\left. \frac{U_{CD}D}{U_C} \right|_{H \text{ constant}} \equiv \bar{\eta} = \frac{\theta_d}{\tau} \left( 1 - \frac{N}{N - \alpha H} \right)
\]

at \( t = 0 \). In the model without habit formation, \( \eta = \bar{\eta} = 0 \) and we get the flat paths for nondurables consumption seen in Figures 4 and 5. Introducing habit formation causes the elasticity to decrease temporarily from zero to a negative number. Moreover, the number may be large. Evaluated at a steady state (where \( H = N \)), \( U_{CD}D/U_C = -\theta_d\alpha/\tau(1 - \alpha) \).

Proponents of habit formation believe that \( \alpha \) is a number on the order of .70. When this value of \( \alpha \) combines with \( \theta_d = \tau = .25 \), the elasticity is a hefty -2.33. The overall impact on the trajectory of nondurables consumption is thus highly uncertain.

Panel A in Figure 8 is an example where the complicating factors and slow habit formation lead to mixed results. In some respects, the transition path looks better. Habit formation cuts the initial jump in \( S \) from 40% to 22% and shortens the flat stretch of the consumption path by half a year. But the price for these gains is a three percentage point reduction in peak expenditure growth and a much less realistic path for nondurables consumption, which drops below its pre-ERBS level at the end of year two. This underlying source of trouble is that \( H \) lags \( N \) far into the \( \text{ERBS} \) period: hence \( \bar{\tau} < \tau \) has a significant effect on the peak
level of consumption (although an 18.7% increase is still pretty good), while the post-jump path of $C$ angles sharply to the southeast because $D \uparrow \rightarrow U_C \downarrow$ (the indirect effect) trounces $H \uparrow \rightarrow U_C \uparrow$ (the direct effect).

The obverse conclusion is that increasing $v$ to 6.5 should improve the fit with the stylized facts. Although it is not clear a priori that the improvement will be significant, the results in Panel B are encouraging. Rapid habit formation gives rise to a nice hump-shaped path for nondurables consumption just as in Fuhrer (2000) and Uribe (2002); it also pushes the peak increase in aggregate consumption back above 20% and eliminates the embarrassing U-shaped portion of the path. Problems remain, especially with regard to the distribution of expenditure growth across periods, but the adjustments are in the right direction. The time profiles for aggregate consumption and its principal components are uniformly superior to the time profiles in the model without habit formation.

5.2 The Multiplicative Specification

In the additive specification with $\tau = \sigma$, durables and nondurables are Edgeworth substitutes ($U_{CD} < 0$) and the intertemporal elasticity of substitution is less than $\tau$ until habit adjustment is complete. The multiplicative specification has very different implications. In the extreme short run (where $H$ is fixed), the intertemporal elasticity equals $\tau$ and the utility function is separable in durables and nondurables consumption ($U_{CD} = 0$). Over time, however, intertemporal substitution becomes easier (Carroll et al., 2000) and accumulation of durables exerts a positive effect on the marginal utility of nondurables consumption. After habits have fully adjusted,

$$- \left. \frac{U_N}{U_{NN} N} \right|_{dH=dN} \equiv \tilde{\tau} = \frac{\tau}{\tau \alpha + 1 - \alpha} > \tau \quad \text{for} \quad \tau < 1$$

and

$$\left. \frac{U_{CD} D}{U_C} \right|_{dH=dN} \equiv \tilde{\eta} = \frac{\theta_d \alpha (1 - \tau)}{\tau} > 0 \quad \text{for} \quad \tau < 1.$$  

Since $\tilde{\tau} > \tau$, the multiplicative specification is sure to deliver a bigger consumption boom than the additive specification; with rapid habit formation, the boom may even be stronger than in the model of sections 2-4. Note also from the positive sign for $\tilde{\eta}$ that the path of nondurables consumption may slope upward during part or all of the ERBS phase.
Figure 9 confirms these analytic-based conjectures. Switching to the multiplicative specification affects mainly the height of the consumption paths. The difference is substantial in the run for \( v = 6.5 \). Absent habit formation, nondurables consumption jumps 3.4% at \( t = 0 \) and then stays flat for the rest of the ERBS period. In Panel B, \( C \) surges 6% in the first six months, rising to a peak of 8% at \( t = 2.25 \). Since durables spending also rises more at the outset, total consumption growth at \( t = 1 \) and \( t = 2 \) is 5-6 percentage points higher than in the model without habit formation. The gap narrows in the third year but does not disappear. Thus, when the multiplicative specification combines with rapid habit formation, there is no tradeoff of better slope for lower height — the path of aggregate consumption is continuously above the no habit formation path.

Two additional points merit comment. First, regardless of whether one favors the multiplicative or the additive specification, the case for habit formation requires very fast adjustment in the stock of habit. There is not much to choose from between a model with slow or moderately fast habit formation and a model with no habit formation. Second, while the introduction of rapid habit formation produces better results, it is not a complete fix. The paths for durables spending and aggregate consumption still have the wrong shape — growth should be greater in the third year than in the second, not the other way around. This motivates us to investigate one more scenario.

5.3 Habit Formation in Durables Spending

So far we have followed Bernanke (1985) in assuming that deliberation costs depend on how fast the stock of durables changes. This is not the only sensible specification. It is equally plausible that the private agent experiences psychological unease when durables spending \( S \) varies from its customary level. Suppose therefore

\[
R(S, H) = x \frac{(S/H - 1)^2}{2} H, \tag{47}
\]

where

\[
\dot{H} = v(S - H), \quad v > 0. \tag{48}
\]

Naturally, we hope that habit formation will now generate a smooth hump-shaped path for \( S \) and, by extension, for aggregate consumption. The idea is not new. Burnside et al. (2004)
have employed a similar strategy to dampen the volatility of investment in a real business cycle model.

Figure 10 shows the outcome when \( \tau = \sigma = .25, \Omega = 5, v = 3, \) and habit formation appears only in the deliberation cost function. We would like to include habit formation in the utility function as well, but, at present, our computer program cannot solve for the nonlinear saddle path in systems that have more than three jump variables.\(^{20}\) The new specification is *perforce* less than ideal.

Even so, the run in Figure 10 gives us almost everything we want. The paths for durables spending and aggregate consumption are hump-shaped, with expenditure contracting rapidly in the last six months of the program. Equally important, the pace of consumption growth does not slow until the start of the third year. Durables spending increases 46% in year one, 42% in year two, and 11% in the first half of year three. The corresponding numbers for aggregate consumption are 12%, 11% and 4%. Perhaps the most surprising result is that the peak increases in durables spending and aggregate consumption are 2-2.5 times as large as in the model without habit formation. This is a natural consequence, however, of substituting \( H \) for \( D \) in the deliberation cost function. Increases in \( H \) reduce marginal deliberation costs in (47) in the same way that increases in \( D \) do when \( R = x(S/D - \delta)^2D. \) But habit formation is extremely fast relative to durables formation (\( v \) is large and \( H \) is only a tenth the size of \( D \)).\(^{21}\) Hence, after about six months, the higher level of \( S \) becomes routine and marginal deliberation costs decrease sharply. This paves the way for a boom that is both smoother and much more powerful than in the model without habit formation.

6. **Concluding Remarks**

Many supporters of the weak credibility (WC) hypothesis probably sympathize with the theorist who complained that “facts are inconvenient things.” In empirical tests, models reliant on the hypothesis have not come close to explaining the dramatic consumption boom seen in ERBS programs. But maybe the problem is with the models as opposed to the hypothesis. The tests were conducted on models that assume all consumption is nondurable. Since the response of nondurables consumption is limited by the low value of the intertemporal elasticity of substitution, it was almost a foregone conclusion that the models would
fare poorly when confronted with the data.

In this paper we have used a mix of theory and numerical methods to investigate the explanatory power of the WC hypothesis in a more general model that allows for both durable and nondurable consumer goods. Inclusion of durables is essential for a fair test. Because refrigerators, TVs, etc. provide a service flow for many years, large-scale purchases of durables do not unbalance the consumption path in the same way as a spike in nondurables expenditure. Consequently, low values of the intertemporal elasticity of substitution do not preclude a durables-driven consumption boom. Our results go further and argue that the WC hypothesis is a very promising hypothesis. In numerical simulations based on conservative assumptions about the expenditure share of durables (20%) and wealth effects (none), aggregate consumption increases 12-27% during the low-crawl phase. And in every case, the boom is powered by spectacular, eye-catching growth in durables spending; when durables are removed from the model, the peak increase in consumption is only 4-8%.

The main variant of the model suffers from the shortcoming that the path to the peak of the consumption boom has the wrong shape. Consumption grows too fast at the beginning and the end of ERBS and too slow in the middle. Seeking better results, we experimented with different ways of adding habit formation to the model. This refinement generates smoother, more realistic paths provided the stock of habit adjusts quickly. For conventional specifications of habit formation, the trajectory of nondurables consumption is hump-shaped but the improvement in the paths of durables spending and aggregate consumption is comparatively modest. Shifting habit formation from the utility function to the deliberation cost function solves the latter problem by strengthening the incentive to smooth the path of durables spending. In the run with this specification, the paths of durables spending and aggregate consumption are steeply sloped and hump-shaped: spending rises apace for two years, then slows and contracts sharply in the last year. The curse of dimensionality restricted us to runs where habit formation appears in either the utility function or the deliberation cost function. It should be clear, however, that a model with habit formation in both places has the potential to explain not only the magnitude and the composition of the consumption boom but also the time profiles of durables and nondurables expenditure over the entire ERBS cycle.
NOTES

1. Tradable consumption drops below its pre-stabilization level in a single jump at the time of the policy reversal. Nontradable consumption may fall below its pre-stabilization level before ERBS is abandoned.

2. The peak increase in consumption occurs at $t = 0$. The solution stated in the text is obtained by solving the Calvo-Vegh model for small changes. It is approximately correct for large changes.

3. The Reinhart-Vegh model delivers much larger increases in consumption when the nominal interest rate falls several hundred percentage points (according to their calculations, 1,270 points in the case of Argentina’s Austral Plan). But then doubts arise about how the change in the nominal interest rate is computed and about the story told by the cash-in-advance (liquidity costs) constraint: Are Reinhart and Vegh correct in assuming that the difference between the initial interest rate and the lowest rate observed during ERBS is close to the average change in the rate? And is it believable that temporary huge variations in the nominal interest rate generate equally huge variations in the price of current vs. future consumption?

4. Seeking stronger quantitative effects, theoretical research moved on to investigate the properties of models with assorted wealth and supply-side effects. This class of models also failed to produce a satisfactory fit with the stylized facts. The consumption surge is bigger but still far from a boom. See Rebelo and Vegh (1995) and Uribe (2002).

5. The data for the U.S. and other developed countries indicate that durables spending is far smoother than predicted by a frictionless Permanent Income/Life Cycle model.

6. This description of fiscal policy is included only to motivate the failure of ERBS. None of our results depend on the assumption that fiscal policy is completely passive. The solutions for the paths of consumption and net foreign debt are independent of the path postulated $g$. (The only restriction on $g$ is that eventually it must adjust to align the fiscal deficit with seigniorage.)

7. Although $\pi$ returns to its original level, $k$ and $m$ do not. The central bank suffers a loss in foreign exchange reserves and the increase in net foreign debt reduces aggregate consumption and holdings of real money balances.

8. Write the term multiplying $(D - \bar{D})$ as

$$\frac{U_{CD}^2}{U_{CC}} - U_{DD} = \frac{U_D}{D} \left[ \left( \frac{U_{CD}D}{U_C} \right) \left( \frac{U_{DC}C}{U_D} \right) \frac{U_C}{U_{CC}C} - \frac{U_{DD}D}{U_D} \right]$$

$$= -\frac{U_D}{D} \left[ \frac{\tau \theta_c + \sigma \theta_d}{\sigma \tau} - \frac{(\tau - \sigma)^2 \theta_d \theta_c}{(\tau \theta_d + \sigma \theta_c) \sigma \tau} \right]$$

$$= -\frac{U_D}{(\tau \theta_d + \sigma \theta_c)D} \left[ \frac{(\tau \theta_d + \sigma \theta_c)(\tau \theta_c + \sigma \theta_d)}{\sigma \tau} - \frac{(\tau - \sigma)^2 \theta_d \theta_c}{\sigma \tau} \right].$$
Mercifully, the term in square brackets simplifies to unity. Moreover, $U_D = U_C(\rho + \delta)$ at a steady state. Hence

$$\frac{U_{CD}^2}{U_{CC}} - U_{DD} = -\frac{(\rho + \delta)U_C}{(\tau_\delta + \sigma C)D}.$$

9. $\omega_2$ is the shadow price of the durable. A foreseen jump in $\omega_2$ implies therefore that the private agent could achieve a welfare gain by altering the paths of $S$ and $D$.

10. The expression for $f$ is derived from the elasticity formulas for $U_{CD}D/U_C$ and $-U_C/U_{CC}C$.

11. The assumption that net foreign debt equals zero at the initial equilibrium does not affect the generality of the results.

12. The solution for $D_1 - D_o$ was derived from (32) and (37a). In writing the solution we exploited the fact that $D_o \approx D_1$ for small changes.

13. $S$ and $C$ decrease proportionately in the long run. This and overshooting imply that the contraction in durables spending is greater than the contraction in nondurables expenditure throughout the post-ERBS adjustment process $[(S(t) - S_o)/S_o < (C_2 - C_o)/C_o, \ t > T]$.

14. We ignore Arrau et al.’s estimate for Brazil (3.26 is implausibly high) and use the average of Arrau et al. and Reinhart and Veghs’ estimates for Argentina and Chile (.27 and .30, respectively). The simple average of the estimated interest elasticities for Chile, Argentina, Mexico, Brazil and Uruguay then works out to .51.

15. The numbers cited here are for failed, noncredible ERBS programs.

16. Our results for the post-ERBS collapse in durables spending are also fairly close to those in DeGregorio, Guidotti and Vegh (1998). Their 95% confidence interval has durables expenditure plunging 53-90% (relative to the pre-ERBS trend line) in the year after ERBS collapses. The range in our simulations is -31% to -75%.

17. In the Brazilian episode, the path has an early flat stretch much as in Figures 4-7. Consumption rose sharply in year 1, decreased slightly in year 2, and accelerated rapidly in years 3 and 4. See DeGregorio, Guidotti and Vegh (1998, Table 1) and Calvo and Vegh (1994b) for data on the episodes in Mexico, Argentina and Brazil. For data on consumption growth in the Paraguayan and Peruvian programs, see Economic Survey of Latin America and the Caribbean.

18. Following a permanent increase in $N$ from $N_o$ to $N_1$, the solution to (45) reads $[H(t) - H_1]/(H_1 - H_o) = e^{-vt}$ (where $H_1 = N_1$). For $v = 6.5$ and $t = .25$, this gives $[H(t) - H_1]/(H_1 - H_o) = e^{-1.625t} = .197$.


20. In the model without habit formation, the dynamic system has two jump variables, $\omega_1$ and $\omega_2$ (or $C$ and $S$), the multipliers associated with the constraints in equations (4) and (5). (Even though $\dot{\omega}_1 = 0$, the computer has to solve for the initial jump in $\omega_1$.)
habit formation enters both the utility function and the deliberation cost function, the multipliers associated with the equations governing habit accumulation become part of the dynamic system. This adds two more jump variables to the system.

21. The magnitude of the consumption boom is sensitive to $v$. Lowering $v$ from three to two reduces the peak increase in aggregate consumption from 28.6% to 21.6% (still nine percentage points above the peak in the model without habit formation).
Appendix

In this appendix we supply the algebra that lies behind assertions made in section 3 of the main text.

The Post-Jump Level of $S$

To see that $S$ jumps to a point above $A_1A_1$, note from (19) that

$$S(0) = S_1 - (\lambda_2 + \delta)(D_1 - D_o) + h_1(\lambda_1 - \lambda_2). \quad (A1)$$

The sufficient conditions in Propositions 1 and 2 ensure that $h_1$ is positive [the condition in (41a) holds]. Since $S_1 - (\lambda_2 + \delta)(D_1 - D_o)$ is the value of $S$ on the saddle path running through $(S_1, D_1)$, this implies that $S(0)$ is north of the $A_1A_1$ schedule.

In Proposition 3 the sufficient condition does not pin down the sign of $h_1$. It is still the case, however, that $h_1$ is positive as long as $\Omega$ and $\sigma$ are not unusually small.

$\dot{S}$ at $t = T$

Under the conditions stated in Propositions 1 and 2, the path of $S$ must cross the $\dot{S} = 0$ schedule because $\dot{S}(T) > 0$. From (19) and (23),

$$\dot{S}(T) = h_1 e^{\lambda_1 T} [\lambda_1 (\lambda_1 + \delta) - \lambda_2 (\lambda_2 + \delta) e^{\lambda_2 T} / e^{\lambda_1 T} - \lambda_2 (\lambda_2 + \delta) e^{\lambda_2 T} (D_1 - D_o)].$$

Since $\lambda_1 > 0 > \lambda_2$, the term $e^{\lambda_2 T} / e^{\lambda_1 T}$ is less than unity. Hence

$$h_1 e^{\lambda_1 T} [(\lambda_1 - \lambda_2)\delta + (\lambda_1 + \lambda_2)(\lambda_1 - \lambda_2)] - \lambda_2 (\lambda_2 + \delta) e^{\lambda_2 T} (D_1 - D_o) \geq 0,$$

$$\implies h_1 e^{\lambda_1 T} (\rho + \delta)(\lambda_1 - \lambda_2) - \lambda_2 (\lambda_2 + \delta) e^{\lambda_2 T} (D_1 - D_o) \geq 0 \quad (A2)$$

is sufficient for $\dot{S}(T) > 0$. From (37b),

$$D_1 - D_o = \frac{\tau D \mu}{JK} (\pi_o - \pi_1)[e^{-rT} + e^{-\lambda_1 T} r(1 - \gamma)x_2/\delta],$$

where

$$J \equiv 1 + L - L'm/X > 1,$$

$$K \equiv 1 + r(1 - \gamma)x_2/\delta > 0. \quad \text{(See next section.)}$$
Substitute this solution for $D_1 - D_o$ and

$$h_1 = \frac{e^{-\lambda_1 T}}{\lambda_1 - \lambda_2} \frac{\delta \Omega D \mu}{J} x_2 (\pi_o - \pi_1)$$

into (A2). After canceling a few terms, there emerges

$$\frac{\delta \Omega}{\tau} x_2 (\rho + \delta) \geq \lambda_2 (\lambda_2 + \delta) e^{-\lambda_1 T} \left[ 1 + e^{\lambda_2 T} r (1 - \gamma) x_2 / \delta \right] / K.$$ 

For the cases covered by Propositions 1 and 2, $x_2$ is non-negative and the term in square brackets is positive and less than unity. Thus it is sufficient that

$$\frac{\delta \Omega}{\tau} \left( 1 + \frac{\lambda_2 \tau}{\delta \Omega} \right) (\rho + \delta) \geq \lambda_2 (\lambda_2 + \delta)$$

$$\implies \frac{\delta \Omega}{\tau} (\rho + \delta) \geq \lambda_2 \left( \lambda_2 - \rho \right)$$

$$\implies \frac{\delta \Omega}{\tau} x_2 (\rho + \delta) \geq -\lambda_2 \lambda_1.$$ 

Recall that $-\lambda_1 \lambda_2 = -c = \delta \Omega (\rho + \delta) / (\tau \theta_d + \sigma \theta_c)$ [see equation (18)]. The sufficient condition simplifies therefore to

$$\frac{\tau}{\tau \theta_d + \sigma \theta_c} \leq 1.$$ 

This holds as $\sigma \geq \tau$ in Propositions 1 and 2. The path of $S$ must cross the $\dot{S} = 0$ schedule before ERBS collapses.

**The Sign of K**

$K = 1 + r (1 - \gamma) x_2 / \delta$ is positive even when $x_2$ is negative. For $x_2 < 0$, a positive sign is assured for $K$ if

$$x_2 + \frac{\delta}{\lambda_1 (1 - \gamma)} > 0$$

(A3)

because $\lambda_1 > r$ ($\lambda_1 + \lambda_2 = \rho = r$). Furthermore, brute force algebra shows that $x_2 = 1 - V / \lambda_1$, where $V \equiv (\rho + \delta) \tau / (\tau \theta_d + \sigma \theta_c)$. Hence the condition in (A3) can be restated as

$$\lambda_1 + \frac{\delta}{1 - \gamma} > V.$$
Substituting for $\lambda_1$ produces

$$\bar{\delta} + \sqrt{\rho^2 - 4c} > 2V - \rho - \bar{\delta},$$

where $\bar{\delta} \equiv \delta/(1 - \gamma)$. Squaring both sides leads to

$$\bar{\delta}(\sqrt{\rho^2 - 4c} - \rho - c > V(V - \rho - \bar{\delta}).$$

The left side is positive ($c < 0$). On the right side, $V > 0$ and $V - \rho - \bar{\delta} = -\sigma(\rho + \delta)\theta_c/\theta_d(\tau\theta_d + \sigma\theta_c) < 0$. Thus the condition holds.
References


Lluch, C. et al., 1977, Patterns in Household Demand and Saving (London, Oxford University Press).


Figure 1: The transition path in the benchmark case of a separable utility function.
Figure 2: The transition path when durables and nondurables are Edgeworth substitutes.
Figure 3: The transition path when durables and nondurables are Edgeworth complements.
Panel A: $\Omega = 5$.

Panel B: $\Omega = 10$.

Figure 4: Transition Path when $\sigma = \tau = .25$. 
Panel A: $\Omega = 5$.

Panel B: $\Omega = 10$.

Figure 5: Transition Path when $\sigma = \tau = .50$. 
Figure 6: Transition Path when Durables and Nondurables are Edgeworth Complements ($\tau = .50$ and $\sigma = .25$).
Figure 7: Transition Path when Durables and Nondurables are Edgeworth Substitutes ($\sigma = .75$, $\tau = .25$).
Panel A: $v = .5$.

Panel B: $v = 6.5$.

Figure 8: Transition Path with the Additive Specification of Habit Formation (--- is the path without habit formation).
Panel A: $v = .5$.

Panel B: $v = 6.5$.

Figure 9: Transition Path with the Multiplicative Specification of Habit Formation (--- is the path without habit formation).
Figure 10: Transition Path when Habit Formation Affects Deliberation Costs (---is the path without habit formation).