Reputation and Sovereign Default

Manuel Amador and Christopher Phelan

University of Minnesota
Federal Reserve Bank of Minneapolis
NBER

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Indiana University

Introduction:

- Consider five (or six) characteristics of actual Sovereign Debt.
  1. Some countries are "Serial Defaulters".
  2. "Debt Intolerance".
  3. Some countries do eventually "graduate".
  4. Countries with positive trade deficits do sometimes default.
  5. Defaults are not perfectly predictable.
  6. Bigger defaults have bigger effects. (extension with partial default).

- Difficult to reconcile these with standard models of Sovereign Debt.

Introduction:

- This paper considers a continuous time model of debt, "reputation", and sovereign default which captures each of these characteristics of actual economies.
- Reputation: Outsiders belief that a country's government is a *Commitment type* which can't default.
- *Optimizing* type can not only default on debt (not make required coupon payments), but completely repudiate it (rip it up, so the debt holders guaranteed to never receive another payment on current debt).

Results.

- Serial Default: Countries which have recently defaulted more likely to default.
- Debt Intolerance: Countries which have recently defaulted pay higher interest rates and have lowest debt levels.
- Graduation: Finite amount of time since last default T where if a country makes it that long, pays lowest interest rate.
- Countries default even if they could borrow more.
- Defaults, in our model like in the data, not predictable.
- (extension) Partial defaults have smaller effect on interest rates than full default.

Results. (More technical)

- In **any** Markov equilibrium, there exists time since last default T such that optimizing government randomly defaults for $\tau < T$ and this probability goes to one as T approaches. Immediately defaults if born into world where $\tau > T$.
- Debt smoothly increases in τ.
- Reputation smoothly increases with τ and reaches 1 when $\tau = T$.
- Bond price smoothly increases in τ.
- Constant net imports (current account deficit) if $\tau < T$ for some amount of time T. Eventually, current account surplus ($\tau > T$).

Environment:

- Time continuous and infinite.
- Small open economy with constant endowment flow y.
- Countable list of potential *governments* with alternating types.
  - With probability ρ_0, first government is *commitment type*.
  - With probability $1 - \rho_0$, first government is *opportunistic type*.
  - With Poisson arrival rate ϵ, commitment type replaced by next government on list, an opportunistic type.
  - With Poisson arrival rate δ, opportunistic type replaced by next government on list, a commitment type.
  - Government type not directly observable.
  - Initially set $\rho_0 = 0$.
- Continuum of risk-neutral foreign *lenders* who discount at rate i.

Environment:

- A *bond* sold at date t is a promise to pay an exponentially shrinking coupon flow $(i+\lambda)e^{-\lambda(s-t)}$ for all $s \geq t$.
- Value of a single bond to a lender if coupon payments certain to be made is

$$\int_t^{\infty} \underbrace{(i+\lambda)e^{-\lambda(s-t)}}_{\text{coupon}} e^{-i(s-t)} ds = \frac{i+\lambda}{i+\lambda} = 1.$$

- We initially assume debt level $b_0 = 0$. (along with setting $\rho_0 = 0$.)
- Over time, debt and reputation will evolve.

Strategies:

- Commitment types cannot default.
- Optimizing types can default — we define *default* as
  - No coupon payments made to existing debt forevermore.
  - Thus debt b set to zero.
  - Reputation ρ becomes zero as a consequence (since commitment type can't default).
  - Since $b_0 = 0$ and $\rho_0 = 0$, default restarts game.
  - Later will allow $(b_0, \rho_0) \neq (0, 0)$.
  - Later will have extension allowing for partial default.

Markov Strategies:

- We assume strategies are *Markov* — depend only on b and ρ.
- Implies b and ρ and everything else depend only on time since last default, τ.
- Thus we make state variable τ, not b and ρ directly.
- Will look for equilibrium objects
  - $b(\tau)$ (the level of debt),
  - $\rho(\tau)$ (the country's reputation),
  - $q(\tau)$ (the price of the country's bonds).
  - $\{F_\tau\}_{\tau \geq 0}$ (default behavior of optimizing government).

Markov Strategies:

Commitment Government:

- Commitment type assumed non-strategic.
- Can't default.
- Follows a set net borrowing rule $b'(\tau) = H(b, q)$.
  - Buys tractability.
  - Avoids informed type having rich choice space.
- Check later if optimal for it to follow H if by deviating, foreign lenders assume it is the optimizing type.

Markov Strategies:

Commitment Government Borrowing:

- $H(b,q)$ assumed continuous, differentiable, decreasing in b, and increasing in q.
- $H(b,q) : [0, \frac{y}{i+\lambda}] \times [0,1] \to \mathbb{R}$
- (bounded debt):$H(0,0) \geq 0$ and $H(\frac{y}{i+\lambda}, 1) \leq 0$.
- (impatience relative to outside lenders): $H(0, \frac{i+\lambda}{i+\lambda+\delta+\epsilon}) > 0$.
- Implies consumption rule

$$C(b,q) \equiv y - \underbrace{(i+\lambda)b}_{\text{coupon payments}} + q\underbrace{\left(H(b,q) + \lambda b\right)}_{\text{bond sales}}.$$

Markov Strategies:

Optimizing Government:

- Can default. (Set $b = 0$.)
- If it doesn't default, follows borrowing rule of commitment type. (No reason to reveal type without defaulting).
- Only strategic decision is default decision.

Markov Strategies:

Optimizing Government:

- Formally, strategy is collection of functions $\{F_\tau\}_{\tau=0}^{\infty}$ subject to a Markov restriction.
- F_τ is the **cumulative distribution function** (weakly increasing, right-continuous) of default for a government alive at date τ.
- That is, $1 - F_\tau(s)$ is probability of *not* defaulting between τ and s, conditional on being continuously in power.

Markov Strategies:

Optimizing Government:

- Markov restriction implies F_0 determines F_τ for all τ such that $F_0(\tau) < 1$.

$$1 - F_\tau(s) = (1 - F_\tau(m^-))(1 - F_m(s)) \text{ for all } 0 < \tau \leq m \leq s$$

- Need to specify F_τ for τ such that $F_0(\tau) = 1$ since an opportunistic government can be "born", due to a type switch, after an earlier opportunistic government would have defaulted.
- If $F_\tau(\tau) > 0$, then a positive probability of defaulting at *exactly* at date τ.
- If $F_\tau(\tau) = 0$ but $F'_\tau(\tau) > 0$, then zero probability of default at exactly date τ, but positive hazard rate. (Positive probability of defaulting in the next few seconds.)

Payoffs:

Optimizing Government:

- $u(c)$, discounted by r.
- preview: $u(c)$ and r won't matter at all. Same outcomes as long as more is preferred to less and now is preferred to later.

Commitment Government:

- No need to have preferences (for now) since not strategic.

Beliefs:

Non-recursive

$$\rho(\tau) = \frac{\text{Prob of no default in } [0,\tau] \textbf{ and } \text{commitment type at } \tau}{\text{Prob of no default in } [0,\tau]}$$

Beliefs:

Recursive

- If $F_\tau(\tau) > 0$, reputation jumps if no default,

$$\rho(\tau) = \frac{\rho(\tau^-)}{\rho(\tau^-) + (1 - \rho(\tau^-))(1 - F_\tau(\tau))}.$$

- If $F_\tau(\tau) = 0$, reputation smoothly moves if no default (but it moves). Drift toward unconditional and up higher unconditional default rate. Note this is a differential equation.

$$\rho'(\tau) = (1 - \rho(\tau))\epsilon + \rho(\tau)((1 - \rho(\tau))F'_\tau(\tau) - \delta).$$

- For both cases, unconditional default behavior determines evolution of reputation if no default.

Prices:

- $q(\tau) = \rho(\tau)q^c(\tau) + (1-\rho(\tau))q^o(\tau)$.
- $q^c(\tau) = \mathbb{E}_t\left[\int_\tau^t (i+\lambda)e^{-(i+\lambda)(s-\tau)}ds + e^{-(i+\lambda)(t-\tau)}q^o(t)\right]$.
- $q^o(\tau)$ more complicated expectation. Have to keep track of whether default or type switch comes first.
- However, if $F_\tau(\tau) = 0$, then $q(\tau)$ obeys the following differential equation:

$$\underbrace{\left[i+\lambda+(1-\rho(\tau))F'_\tau(\tau)\right]}_{\text{effective discount rate}} q(\tau) = \underbrace{(i+\lambda)}_{\text{coupon}} + \underbrace{q'(\tau)}_{\text{capital gain}} .$$

Markov Perfect Equilibrium Definition

$$(b(\tau), \rho(\tau), q(\tau), \{F_\tau\}_{\tau \geq 0}) \text{ such that}$$

1. Foreign investors break even in equilibrium. (pricing equations hold).
2. Market beliefs are rational.
   - $\rho(0) = 0$ (or after a default, beliefs revert to certainty of the optimizing type) and
   - *conditional on no default*, beliefs follow Bayes' rule.
3. Debt evolution follows rule. $b(0) = 0$, $b'(\tau) = H(b(\tau), q(\tau))$.
4. Optimizing government optimizes. Taking $(b(\tau), \rho(\tau), q(\tau))$ as given, no other default strategy improves its payoff.
5. $1 - F_\tau(s) = (1 - F_\tau(m^-))(1 - F_m(s))$ for all $0 < \tau \leq m \leq s$

Constructing An Equilibrium

Conjecture constant indifference by optimizing type

- **constant** $c(\tau) = c^\star > y$ for $\tau < T$.
- for $\tau < T$, $F_\tau(\tau) = 0$, $F'_\tau(\tau) > 0$, $F_0(\tau) < 1$. (continuously increasing cdf, no jumps).
- $F_0(T) = 1$.
- for $\tau \geq T$, $F_\tau(\tau) = 1$. (certain immediate default).
- guarantees optimizing type always eats $c^\star > y$, and thus indifferent.

Later prove this is essentially only equilibrium

Constructing An Equilibrium

$c^\star$ and T enough to pin down everything else.

- Two ordinary differential equations in b and q.
  1. $b'(\tau) = H(b(\tau), q(\tau))$.
  2. $q'(\tau) = \frac{-C_b(b(\tau),q(\tau))}{C_q(b(\tau),q(\tau))} H(b(\tau), q(\tau))$.
  3. Second equation is derivative of consumption with respect to τ set to equal to zero (since consumption is constant). Given debt increasing (since $c^\star > y$), prices must increase at rate necessary to keep consumption constant.
- $b(0) = 0$.
- guess $c^\star$, then solve for $c^\star = C(0, q(0))$ to get $q(0)$.
- Roll out $b(\tau)$, $q(\tau)$. This is *candidate* $b(\tau)$, and $q(\tau)$ for $\tau \leq T$.

Constructing An Equilibrium

- Again, all we are doing is finding path for b and q which causes commitment government to have constant consumption equal to $c^\star$.
- Not yet using that prices have to make sense given default probabilities.
- Fill out rest of candidate equilibrium (belief evolution and default behavior) by finding what justifies these prices.

Constructing An Equilibrium, continued.

$$\underbrace{\left[i + \lambda + (1 - \rho(\tau))F'_\tau(\tau)\right]}_{\text{effective discount rate}} q(\tau) = \underbrace{(i + \lambda)}_{\text{coupon}} + \underbrace{q'(\tau)}_{\text{capital gain}} .$$

- use this pricing equation to back out what unconditional default rates must be to justify this price path.

$$\rho'(\tau) = (1 - \rho(\tau))\epsilon + \rho(\tau)((1 - \rho(\tau))F'_\tau(\tau) - \delta).$$

- Use this Bayes' rule differential equation, with starting condition $\rho(0) = 0$ to solved for unconditional default hazard rates to roll out path for $\rho(\tau)$.
- Having solved for $(1 - \rho(\tau))F'_\tau(\tau)$ and $\rho(\tau)$ then implies $F'_\tau(\tau)$.

Constructing An Equilibrium, continued.

- define T to be date where $q(T) = \frac{i+\lambda}{i+\lambda+\delta}$. This is price if $\rho = 1$, assuming immediate default after a type switch. Ensures $q(\tau)$ path is continuous at T.
- Set $F_\tau(\tau) = 1$ for $\tau \geq T$. This pins down everything for $\tau \geq T$ and gets price right after T.
- Lastly check if $\rho(T) = 1$ which is necessary **and sufficient** for equilibrium. If $\rho(T) < 1$, guessed $c^\star$ too low. If $\rho(T) > 1$, guessed $c^\star$ too high. (Not quite a uniqueness proof, but close.)

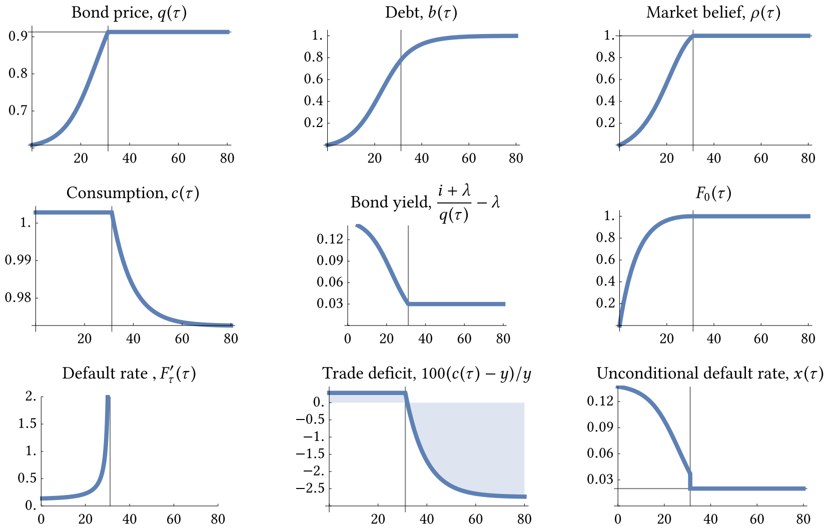
Characterization

- Will skip Proposition/proof here, but basic result is that this is the **only** Markov equilibrium.
- We prove necessary that $V(\tau)$ is constant.
- We prove necessary that c is constant if defaulting with less than full probability.
- We prove necessary that country eventually defaults with full probability at some date T and that any opportunistic government born after T defaults immediately with full probability.
- Constructed equilibrium only used constant c and existence of T. Everything else implied by equilibrium conditions.

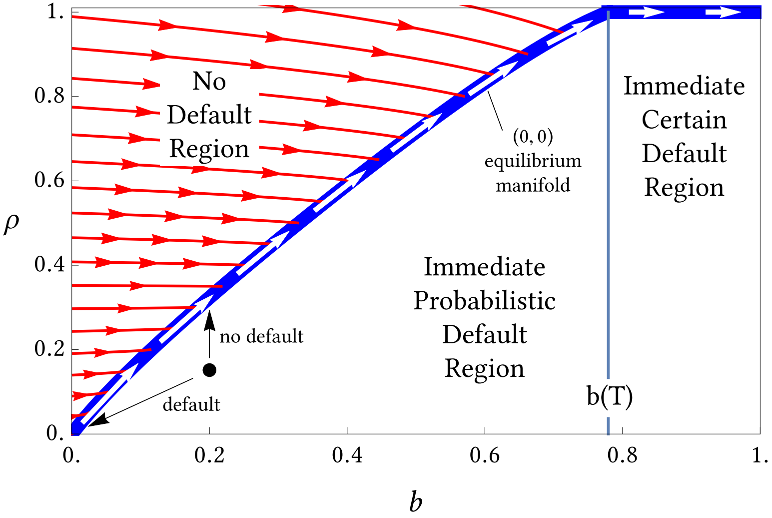
An Example:

- $y = 1$.
- $\epsilon = .01$, $\delta = .02$. (long run chance of being commitment type, $\frac{\epsilon}{\epsilon+\delta} = \frac{1}{3}$.)
- $i = .01$, $\lambda = .2$ (five year debt).
- Functional form of $H(b,q)$ falls out of log utility maximizer who discounts at .15 and can borrow at the yield associated with q, except debt limit changed to y from $\frac{y}{i}$.
- $$H(b,q) = \left(.15 - \left(\frac{i+\lambda}{q} - \lambda\right)\right)(y-b).$$

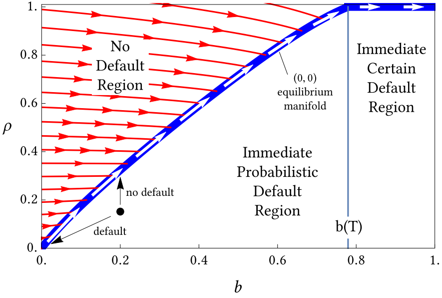
An Example:


Bond price, $q(\tau)$
Debt, $b(\tau)$
Market belief, $\rho(\tau)$
Consumption, $c(\tau)$
Bond yield, $\frac{i+\lambda}{q(\tau)} - \lambda$
$F_0(\tau)$
Default rate , $F'_\tau(\tau)$
Trade deficit, $100(c(\tau) - y)/y$
Unconditional default rate, $x(\tau)$

General (b_0, ρ_0): Key Figure.

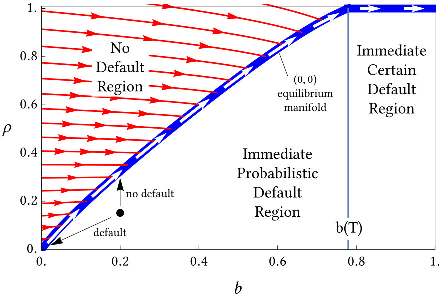


General (b_0, ρ_0): Key Figure.



- Start on blue line: Debt is "appropriate" relative to reputation. Act as if you got there from $(0, 0)$.
- Start above blue line: Debt is **lower** than appropriate level given reputation. No default (and thus higher prices and higher consumption). Follow red path until reach blue line.

General (b_0, ρ_0): Key Figure.



- Start below blue line: Debt is **higher** than appropriate level given reputation.
- If $b \geq b(T)$ immediately default with probability one.
- If $b < b(T)$, probabilistically default to either raise reputation by not defaulting, or start again at $(0, 0)$.
- No other behaviors compatible with equilibrium.

General (b_0, ρ_0): Key Figure.

- Solving for off manifold (b_0, ρ_0) allow for
  - Checking if following $H(b, q)$ is optimal for both types.
  - Probability zero permanent shocks to endowment flow.
  - Extension to Partial Default.

Optimality of following $H(b, q)$

- Optimizing. $\rho \to 0$ if it deviates from H. Either doesn't change value (ρ below line) or lower it (ρ above line).
- Commitment.
  - Again, $\rho \to 0$ if it deviates from H. Since strategy calls for immediate certain default, same as not being able to borrow.
  - Best deviation is to not borrow and pay down b for a time. We show not profitable if commitment type discounts enough.

Aggregate Shocks

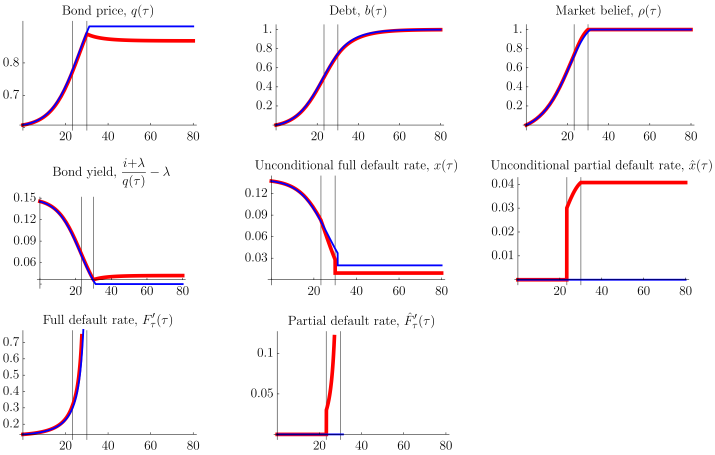
- Suppose country receives one-time probability zero **good** shock. (y goes permanently up.)
  - b and ρ unaffected, but manifold moves **down**.
  - Equilibrium then calls for zero probability of default for a while.
- Suppose country receives one-time probability zero **bad** shock. (y goes permanently down.)
  - b and ρ again unaffected, but manifold moves **up**.
  - Equilibrium then calls immediate probabilistic default. Bigger the shock, higher the probability.
  - Lack of predictability of reaction to bad shock.
  - Asymmetry between reaction to good and bad shocks.

Partial Default

- Suppose $b^* > 0$ such that if $b > b^*$, **commitment** type is forced to partially default so that b is reset to b^* (and each bondholder gets coupon payments proportionally reduced) with constant exogenous arrival rate $\theta > 0$.
- Opportunistic type can mimic this as well.
- Opportunistic type now chooses $F_\tau(s)$ (full default behavior) and $\hat{F}_\tau(s)$ (partial default behavior).

Partial Default

- Equilibrium looks almost the same - consumption constant.
- Full default resets τ to zero. Partial default resets τ to the amount of time it takes to get from zero to b^*.
- Probability of partial default chosen in equilibrium so that if it happens, reputation drops exactly as much as it needs to in order to stay on manifold.
  - Can't have too little probability of opportunistic type partially defaulting, or else reputation too high after a partial default (and thus incentive to immediately partially default.)
  - Can't have too high probability of opportunistic type partially defaulting, relative to fully, or else reputation too low after a partial default — off manifold equilibrium calls for immediate probabilistic full default



- 3% yearly risk of exogenous partial default.
- Bond price falls more for full default than partial default.
- Could have had multiple b_n^*. Just set clock back further for smaller b_n^*. Bigger default means more of a bond price drop.

Conclusion

- Presented tractable sovereign debt model where borrower's reputation and its interaction with default events generate dynamics of debt and asset prices that are consistent with several facts.
- In model, a government that defaults loses its reputation, and it takes periods of borrowing and not defaulting to eventually restore it.
- During these periods, bond prices are low and default frequencies are high, as in the data.
- Relative to countries that have not recently defaulted, debt levels are low.
  - In model, as in the data, countries with low debt levels face relatively high interest rates, a phenomenon referred to as "debt intolerance."

Conclusion

- In model, a country can “graduate” into the set of “debt-tolerant” countries by not defaulting for a sufficiently long period of time, as perhaps Mexico has done by not defaulting since the 1980s.
- In the data, default is less than fully predictable and somewhat untied to fundamentals.
- Recent work has emphasized this fact as an argument for introducing features that lead to multiple equilibria in the standard sovereign debt model. In our environment, such an outcome arises naturally.
- Equilibrium default in our model is necessarily random, both in our baseline model and in our consideration of when a country is hit by a bad shock. Such randomness is a fundamental ingredient for the dynamics of learning and reputation.