Diversity and Offshoring*

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Abstract

How do skill dispersion and the average skill level affect offshoring? To study this question, we extend Grossman and Maggi (2000) to allow workers in different countries to collaborate in teams, which is referred to as offshoring. We first demonstrate that if the skill distributions in the two countries have the same form, the pattern of trade is determined by a comparison of the coefficients of variation of skills in the two countries. Moreover, we demonstrate that if two countries have the same average skill level but different skill dispersions, as in Grossman and Maggi (2000), the wages of workers with the same skill level are equalized under free trade, and thus, offshoring is impossible. However, if two countries have the same skill dispersion but different average skill levels, the wages of workers with the same skill level are not equalized across countries, and thus, offshoring is possible. Furthermore, exporting and offshoring have different effects on the welfare of the workers with the highest and lowest skill levels and the results for a country with a higher average skill level are consistent with the findings in Hummel, et al. (2014).

1 Introduction

In recent years, production processes have increasingly involved multiple countries, with each country specializing in certain tasks.[1] This phenomenon of offshoring has attracted considerable attention from policy makers and economists. Our goal in this paper is to investigate how the average skill level and skill dispersion affect offshoring. To do so, we introduce offshoring into the seminal study by Grossman and Maggi (2000), which highlights how skill

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[1] This phenomenon is referred to as fragmentation in Jones and Kierzkowski (1990), trade in tasks in Grossman and Rossi-Hansberg (2008), and vertical specialization in Hummels, Ishii, and Yi (2001).

dispersion affects the pattern of trade. This question is important because countries differ in the distribution of skills, as found by Bombardini, Gallipoli, and Pupato (2012).[2]

To investigate how the average skill level affects the possibility of offshoring, we first analyze how the average skill level affects trade patterns between two countries with a common skill dispersion when there is no offshoring. Following Grossman and Maggi (2000), there are two sectors: The technology of one sector exhibits supermodularity, i.e, a technology that requires complementarity between tasks; the technology of the other sector exhibits submodularity, i.e, tasks are substitutable. As Grossman and Maggi (2000) note, given a symmetric density function, if two countries have the same average skill levels, the country with a higher skill dispersion exports the good in the submodular sector. Since the submodular sector matches the highest and lowest skilled workers, an increase in skill dispersion raises the relative supply of the factors used in the submodular sector. Therefore, a more diverse country has a comparative advantage in the submodular sector. We demonstrate that if two countries have the same skill dispersion, the country with the higher average skill exports the good produced in the supermodular sector, because an increase in the average skill raises the relative supply of factors used in the supermodular sector. Moreover, given a symmetric density function, we demonstrate that if the skill distributions in the two countries have the same form, the pattern of trade is determined by a comparison of the coefficients of variation of the skills in the two countries.[3] Specifically, a country that has a higher coefficient of variation of skills has a comparative advantage in the supermodualr sector.

As in Antras, Garricano and Rossi-Hansberg (2006), offshoring occurs when workers in different countries collaborate in teams. If two countries have the same average skill level but different skill dispersions, as in Grossman and Maggi (2000), offshoring will not occur because wages are equalized under free trade. However, if two countries have the same skill dispersion but different average skills, the wages of workers with the same skill level are not equalized and offshoring can occur.

We also separately examine the effects of trade and offshoring on the income distribution. We demonstrate that exporting and offshoring have different effects on workers' welfare in the submodular sector, in which workers with the highest and lowest skills collaborate. Trade affects the welfare of workers in the same direction: All gain from trade, or all lose from trade. However, offshoring affects the welfare of workers with the highest skills and that of workers with the lowest skills in opposite directions. Intuitively, workers with the highest skills in the country with the higher average skill level have the opportunity to form teams with better partners in the foreign country, and thus, they gain from offshoring. It follows that workers who are replaced by foreign workers with lower skills lose. These results are consistent with the findings of a recent study by Hummels, Jorgensen, Munch, and Xiang (2014), who estimate how offshoring and exporting affect wages by skill type using matched worker-firm data from

[2] See Figure 1 in Bombardini, Gallipoli, and Pupato (2012). They use scores on the International Adult Literacy Survey (IALS), an internationally comparable measure of work-related skills, to document the differences in the mean and standard deviation of skills among 19 countries during the period 1994–1998. Their results support that skill dispersion affects the pattern of trade, as predicted by Grosman and Maggi (2000).

[3] The definition of diversity in our paper is strict than that in Grossman and Maggi (2000).

Denmark.[4] Offshoring affects workers in the foreign country in opposite directions, as workers with the lowest skills find better partners, and workers with the highest skills lose their best partners.

Our paper is related to two strands of the trade literature. One is the literature on offshoring, which is large and diverse.[5] Several papers, mostly notably Feenstra and Hanson (1996), Yi (2003), and Grossman and Rossi-Hansberg (2008), have investigated the effects of offshoring on trade volumes, trade patterns and the income distribution. Costinot, Vogel, and Wang (2013) highlight how global or local technological changes affect countries participating in the same global supply chain. Antras and Chor (2013) emphasize the optimal allocation of ownership rights along the global value chain. Baldwin and Venables (2013) reveal the implications of production processes for offshoring. Our paper is closely related to, among others, Antras, Garricano and Rossi-Hansberg (2006), who study the impact of the formation of cross-country teams with one manager and several workers on the organization of production and wages. They assume a uniform distribution of skills, and thus, how the distribution of skills affects offshoring is not their focus. In contrast, our paper investigates how the average skill level and skill dispersion affect offshoring with a symmetric distribution function.

Moreover, our paper is related to a growing literature that uses matching and assignment models in an international context, for example, Grossman and Maggi (2000), Nocke and Yeaple (2008), Costinot (2009), Ngienthi, Ma, and Dei (2011), and Ma (2017). Our article is closely connected with, among others, studies on talent (human capital) and trade pioneered by Grossman and Maggi (2000), who analyze how the skill distribution affects countries' comparative advantages and trade patterns. Research in this area includes Grossman (2004), Bougheas and Riezman (2007), Ohnsorge and Trefler (2007), Bombardini, Giovanni, and Germán (2012, 2014), and Chang and Huang (2014). None of these articles, however, consider offshoring. Ngienthi, Ma, and Dei (2011) and Ma (2017) focus on offshoring among countries at different stages of development. In contrast, our paper studies how the distribution of human capital affects offshoring.

Our paper is organized as follows. In section 2, we investigate the trade pattern between two countries that share the same skill dispersion but have different average skill levels. We consider how free trade affects the income distribution in section 3. In section 4, we examine the effects of offshoring. Section 5 provides concluding remarks.

2 The Setup

In this section, we first introduce the setup and some results in Grossman and Maggi (2000) (hereafter, GM (2000)). Next, we extend GM (2000) to analyze how the average skill level affects trade patterns between two countries with a common skill dispersion.

[4] Denmark has a relatively higher average skill level than most of the countries in Figure 1 of Bombardini, Gallipoli, and Pupato (2012). Thus, our results are consistent with the following findings in Hummels et al. (2014, p. 1). That is, within job spells, (1) offshoring tends to increase the high-skilled wage and decrease the low-skilled wage; (2) exporting tends to increase the wages of all skill types.

[5] See Antras and Rossi-Hansberg (2009) for a review.

2.1 The Skill Dispersion and Comparative Advantage

There are two countries: Home and Foreign. In each country, there is a continuum of workers who differ in their skill levels. The skill distribution in Home (Foreign) is represented by cumulative distribution functions $\Phi(t)$, $t \in [t_{\min}, t_{\max}]$, ($\Phi^*(t)$, $t \in [t^*_{\min}, t^*_{\max}]$). Let $L(L^*)$, $\bar{t}$ ($\bar{t}^*$) and $I \equiv t_{\max} - t_{\min}$ ($I^* \equiv t^*_{\max} - t^*_{\min}$) represent the measures of workers, the mean level of skills, and the skill gap between the most and least able workers in Home (Foreign).

There are two sectors in each country: One is sector C, where C represents "complementarity", for example, the car industry. The other is sector S, where S represents "substitutability", for example, the software industry. We assume that two countries share the same technologies in both sectors. Production in each sector requires a team of two workers. Output by a pair of workers in sector i ($i = C, S$) is $F^i(t_1, t_2)$, where t_j ($j = 1, 2$) represents the skill level of the worker performing task j. F^i is assumed to be monotonically increasing in t_j, symmetric, and to exhibit constant returns to skills. In sector C, a pair of workers perform complementary tasks, and the production function exhibits supermodularity such that $F^C_{12} > 0$; i.e, the marginal product of an individual's skill is greater the more able his co-worker is. In sector S, the workers exert effort on substitutable tasks, and the production function exhibits submodularity such that $F^S_{12} < 0$; i.e., the marginal product of skills is higher the less capable his partner is. Given constant returns to skills, the production functions in sectors C and S imply decreasing returns to "individual" skill ($F^C_{jj} < 0$) in sector C, but increasing returns to individual skill ($F^S_{jj} > 0$) in sector S. Since the two sectors differ in the nature of the production technology, the industry output of sector C and that of sector S are maximized through self-matching and cross-matching of workers' skill levels, respectively.

Lemma 1: (GM (2000), Lemmas 1 and 3) For any given output Y_S of good S, the output Y_C of good C is maximized by 1) allocating all workers with skill levels $t \leq \hat{t}$ and all workers with skill levels $m(\hat{t}) \leq t$ to sector S, where $m(\hat{t})$ is defined implicitly by $\Phi[m(t)] = 1 - \Phi(t)$ and $\hat{t}$ solves $Y_S = L \int_{t_{min}}^{\hat{t}} F^S(t, m(t))\, d\Phi(t)$; (2) allocating the remaining workers with $t \in [\hat{t}, m(\hat{t})]$ to sector C such that $t_1 = t_2$ in all teams, i.e., $Y_C = \frac{\lambda_C L}{2} \int_{\hat{t}}^{m(\hat{t})} t d\Phi(t)$, where $\lambda_C = F^C(1, 1)$.[6]

As in GM (2000), we assume that the density function in Home (Foreign) $\phi \equiv d\Phi/dt$ ($\phi^* \equiv d\Phi^*/dt$) is symmetric about its mean. This implies that $m(t) = 2\bar{t} - t$ ($m^*(t) = 2\bar{t}^* - t$). It follows that

$$Y_C = \frac{L}{2} \lambda_C \bar{t} [\Phi(2\bar{t} - \hat{t}) - \Phi(\hat{t})], \tag{1}$$

$$Y_S = L \int_{t_{min}}^{\hat{t}} F^S(t, 2\bar{t} - t)\, \phi(t) dt. \tag{2}$$

Let $MRT \equiv -\frac{dY_S/d\hat{t}}{dY_C/d\hat{t}}$ denote the marginal rate of transformation (MRT) of the production possibility frontier (PPF), and we have

$$MRT = \frac{F^S(\hat{t}, 2\bar{t} - \hat{t})}{\lambda_C \bar{t}}. \tag{3}$$

[6]We borrow the approach of summarizing Lemmas 1 and 3 in GM (2000) from Chang and Huang (2014).

It is clear that the MRT depends on the average skill level $\bar{t}$ and the least skilled worker in sector C, i.e., $\hat{t}$. Similarly, we can obtain Foreign's outputs Y_C^*, Y_S^*, and MRT^*.

Let $p = p_C/p_S$ denote the relative price of good C. In equilibrium, $p = MRT$ pins down $\hat{t}$. GM (2000, Proposition 4) demonstrate that if two countries have a common average skill level, i.e., $\bar{t} = \bar{t}^*$, the country with a more diverse distribution of skills has a comparative advantage in good S.[7] Since the submodular sector matches the highest and lowest skilled workers, an increase in skill dispersion raises the relative supply of the factors used in sector S. Therefore, a more diverse country has a comparative advantage in good S.

2.2 The Average Skill and Comparative Advantage

We extend GM (2000) to examine the trade pattern between two countries with a common skill dispersion but different average skill levels. We assume that the average skill level in Home $\bar{t}$ is $\beta(>1)$ times that in Foreign $\bar{t}^*$, i.e., $\bar{t} = \beta\bar{t}^*$. In other words, the two countries share the same skill dispersion in the sense that if we shift the foreign country's density function horizontally $\delta \equiv \bar{t} - \bar{t}^*(= (\beta-1)\bar{t}^*)$, we obtain the home country's density function, i.e, $\phi(t) = \phi^*(t-\delta)$ for $t \in [t_{\min}, t_{\max}]$ and $I = I^*$.

To investigate the effect of the difference in the average skill level on the trade pattern between two countries with a common skill dispersion, we examine how an increase in β affects Y_S/Y_C, with MRT held constant. Substituting $\bar{t} = \beta\bar{t}^*$ into (3) to obtain

$$MRT = \frac{F^S\left(\hat{t}, 2\beta\bar{t}^* - \hat{t}\right)}{\lambda_C \beta\bar{t}^*}. \tag{4}$$

Since $F^S\left(\hat{t}, 2\beta\bar{t}^* - \hat{t}\right)$ is homogeneous of degree one, holding MRT constant implies that $\hat{t} = \beta\hat{t}^*$, where $\hat{t}^*$ satisfies $MRT^* = \frac{F^S\left(\hat{t}^*, 2\bar{t}^* - \hat{t}^*\right)}{\lambda_C \bar{t}^*}(= MRT)$. That is, with MRT held constant, an increase in the average skill level leads to an increase in $\hat{t}$. It follows that [8]

$$\frac{d(Y_S/Y_C)}{d\beta}\ |_{MRT=const} < 0. \tag{5}$$

Hence, an increase in β leads to a decline in Y_S/Y_C with MRT held constant, as shown in Figure 1. An increase in the average skill level has two effects on output in sector C: One is that sector C expands from $t \in [\hat{t}^*, 2\bar{t}^* - \hat{t}^*]$ to $t \in [\beta\hat{t}^*, 2\beta\bar{t}^* - \beta\hat{t}^*]$, which increases the output of good C, and the other is that workers in sector C are more productive due to an increase in their skills, which also increases the output of good C. However, an increase in the average skill level has two conflicting effects on the output in sector S: One is that sector S shrinks, which leads to a decline in the output of S, and the other is that workers in sector S become more

[7] The definition of diversity in GM is as follows: The distribution of skill Φ is more diverse than Φ^* if $\Phi(t) > \Phi^*(t)$ for $t < t'$ and $\Phi(t) < \Phi^*(t)$ for $t > t'$, for some $t' > t_{\min}$. PROPOSITION 4 in GM: Suppose that $\bar{t} = \bar{t}^*$ and Φ is more diverse than Φ^*. Then, the home country exports good S and imports good C in a free trade equilibrium.

[8] See Appendix B for the calculation.

productive due to an increase in their skills, which increases the sector's output. As a result, an increase in the average skill level leads to a decline in Y_S/Y_C with MRT held constant. It follows that Home, which has a higher average skill level, has a comparative advantage in good C.

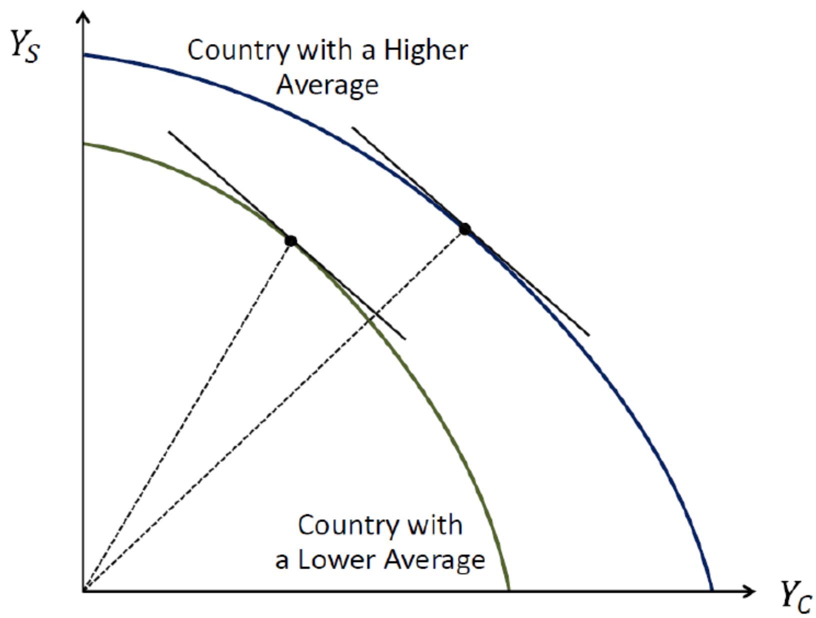

Figure 1: The Average Skill Level and Trade Pattern

Proposition 1 *If two countries have identical skill dispersions, the country with a higher average skill level exports the good produced in the supermodular sector.*

When two countries have the same skill dispersion but different average skill levels, the pattern of trade between them is implied by Proposition $4'$ in GM (2000).[9] When two countries share the same dispersion, the country with the higher average skill level exports the good produced in the supermodular sector because this country is relatively less diverse than the other country. However, Proposition $4'$ does not indicate whether a higher average skill level and a more diverse skill dispersion affect the pattern of trade in the same or different directions.

Let σ (σ^*) represent the standard deviation of skills in Home (Foreign). By comparing the coefficient of variations of skills in the two countries, we obtain Proposition 2.

[9] Proposition $4'$ in GM (2000) is as follows: Let $\Phi(t)$ and $\Phi^*(t)$ be symmetric distributions. Define $\Phi^{ma}(\cdot)$ such that $\Phi^{ma}(rt) = \Phi(t)$ for all t, where $r \equiv \bar{t}^*/\bar{t}$. If Φ^{ma} is more diverse than Φ^*, the home country exports good S and imports good C in a free-trade equilibrium.

Proposition 2 *Suppose that the distributions of skills in the two countries have the same form. If* $\frac{\bar{t}}{\sigma} > \frac{\bar{t}^*}{\sigma^*}$*, the country with a higher coefficient of variation exports the good in the supermodular sector; if* $\frac{\bar{t}}{\sigma} < \frac{\bar{t}^*}{\sigma^*}$*, the country with a lower coefficient of variation exports the good in the submodular sector; and if* $\frac{\bar{t}}{\sigma} = \frac{\bar{t}^*}{\sigma^*}$*, the free-trade equilibrium has no trade in final goods.*

See Appendix C.

Proposition 2 illustrates that the pattern of trade is determined by a comparison of the coefficients of variation of skills in the two countries, which is shown in Figure 2. Note that along the horizontal line of $\frac{\bar{t}}{\bar{t}^*} = 1$, we obtain the same result as in GM (2000); along the vertical line of $\frac{\sigma}{\sigma^*} = 1$, we obtain Proposition 1; and along the 45 degree line, we demonstrate that there is no trade in final goods, which is confirmed by Proposition 3 in GM (2000).[10]

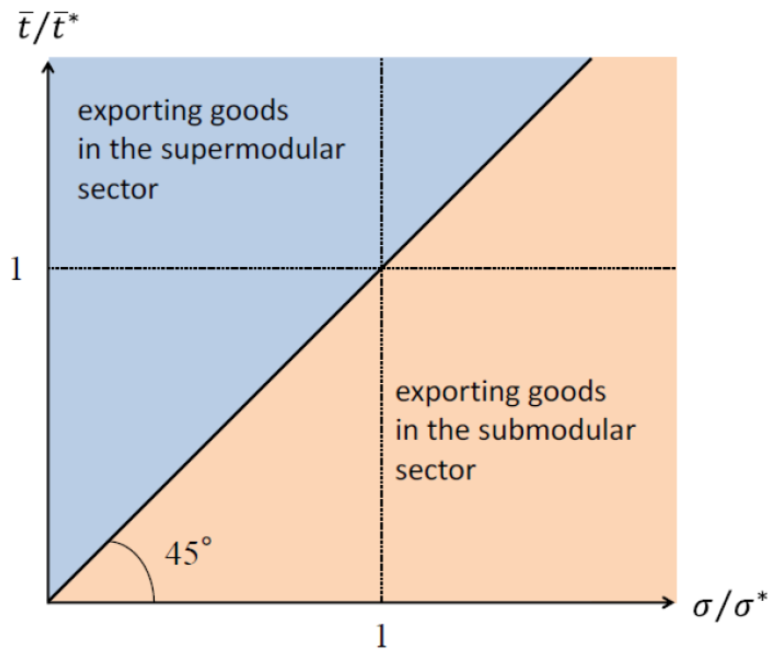

Figure 2:Trade Patterns and the Comparison of Coefficients of Variation

3 Effects of Trade

In this section, we focus on the effects of free trade on wages. We first introduce how opening to trade affects nominal wages in GM (2000). Next, we investigate how opening to trade affects

[10] Theorem 3 in GM (2000) notes that if $\Phi(t) = \Phi^*(\beta t)$ for all $t \in [t_{\min}, t_{\max}]$, then the free-trade equilibrium has no trade. Clearly, when $\Phi(t) = \Phi^*(\beta t)$ for all $t \in [t_{\min}, t_{\max}]$, we have $\frac{\bar{t}}{\sigma} = \frac{\bar{t}^*}{\sigma^*}$.

wages and the income distribution if two countries have the same skill dispersion but different average skills.

3.1 Skill Dispersion and Wage Equalization

We introduce how opening to trade affects the nominal wages in GM (2000), in which two countries share the same average skill level and Home is assumed to be more diverse. To do this, we obtain the equations that determine the equilibrium wages of workers with different skill levels. As stated in Lemma 1, workers with skills between $\hat{t}$ and $m(\hat{t})$ are allocated to sector C. Since each member of a matched pair contributes equally to output, the members of a team are paid the same. Since wages paid to a team are equal to the revenue it earn, we have

$$w(t) = \frac{p\lambda_C t}{2}, \text{ for } t \in [\hat{t}, m(\hat{t})]. \tag{6}$$

Firms in sector S also earn zero profits in the competitive equilibrium. Thus, we have

$$w(t) + w[(m(t)] = F^S[t, m(t)], \text{ for } t \in [t_{\min}, \hat{t}]. \tag{7}$$

Profit maximization implies that $F_1^S[s, m(s)] = w'(s)$ for all $s \leq \hat{t}$.[11] Moreover, equation (6) yields the wage for the marginal worker in sector S with skill $\hat{t}$. For workers with less skill than $\hat{t}$, we have

$$w(t) = \frac{p\lambda_C \hat{t}}{2} - \int_t^{\hat{t}} F_1^S\left(\tau, 2\bar{t} - \tau\right) d\tau, \text{ for } t \in [t_{min}, \, \hat{t}). \tag{8}$$

The wages for workers with skills at the upper end of the distribution can be obtained as follows by combining (7) and (8):

$$w(t) = F^S\left(t, 2\bar{t} - t\right) - \frac{p\lambda_C \hat{t}}{2} + \int_{2\bar{t}-t}^{\hat{t}} F_1^S\left(\tau, 2\bar{t} - \tau\right) d\tau, \text{ for } t \in (2\bar{t} - \hat{t}, t_{max}]. \tag{9}$$

The wage distributions associated with a given relative price under autarky are shown in Figure 3. As noted by GM (2000), Home's (Foreign's) schedule is linear in the range $[\hat{t}', m(\hat{t}')]$ ($[\hat{t}^{*\prime}, m(\hat{t}^{*\prime})]$) and convex outside this range, where $\hat{t}'(\hat{t}^{*\prime})$ represents the least skilled worker in Home (Foreign), and $m(\hat{t}')(m(\hat{t}^{*\prime}))$ represents the highest skilled worker in sector C in Home (Foreign). Since the wage schedule in each country is obtained in the same way, in the following, we only explain how to obtain the Home wage schedule. The linearity of $t \in [\hat{t}', m(\hat{t}')]$ follows directly from (6). For $t < \hat{t}'$, (8) implies that $w''(t) = F_{11}^S\left(t, 2\bar{t} - t\right) - F_{12}^S\left(t, 2\bar{t} - t\right)$. We have $F_{12}^S\left(t, 2\bar{t} - t\right) < 0$ due to submodularity and $F_{11}^S\left(t, 2\bar{t} - t\right) > 0$ because of the assumption of constant returns to skill. Therefore, we have $w''(t) > 0$. Convexity in the range of $[m(\hat{t}'), t_{\max}]$ can be established similarly.

[11] See details in GM (2000, page 1266).

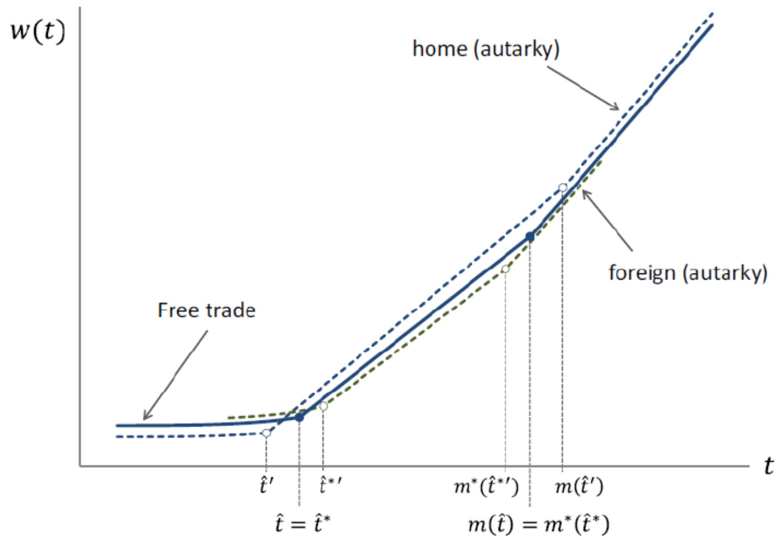

Figure 3: Skill Dispersion and Wages

Since Home has a more diverse distribution of skills, Home exports good S in the free-trade equilibrium. Thus, opening to trade leads to a decline in the relative price of good C, p. Since we have that $p = MRT$ in any equilibrium, (3) implies that $d\hat{t}'/dp < 0$. Therefore, a decline in p increases $\hat{t}'$ and decreases $m(\hat{t}')$, i.e., workers exit sector C, as shown in Figure 3. In Foreign, which has a less diverse distribution of skills, workers exit sector S. Note that (3) implies that $\hat{t}$, and thus, $m(\hat{t})$, i.e., the least skilled and the highest skilled workers in sector C under free trade, are equalized between Home and Foreign, because $p = MRT$ holds in equilibrium. It follows that the wages of workers with the same skill level are equalized between two countries under free trade.

3.2 The Average Skill and Wage Schedule

We turn to examining how free trade affects nominal wages and the income distribution if two countries share the same skill dispersion but the average skill level in Home $\bar{t}$ is β (> 1) times of that in Foreign $\bar{t}^*$. To do so, we first derive the wage schedules of both countries under autarky. Note that (6), (8), and (9) hold for each country. Substituting the average skills of both countries into the above equations, we obtain the nominal wage of workers in the home country, $w(t)$, for $t \in [t_{\min}, t_{\max}]$, and that of Foreign, $w^*(t)$, for $t \in [t^*_{\min}, t^*_{\max}]$ as follows:

$$
w(t) = \begin{cases} p\lambda_C \hat{t}'/2 - \int_t^{\hat{t}'} F_1^S\left(\tau, 2\beta\bar{t}^* - \tau\right) d\tau, & t \in [t_{min}, \hat{t}') \\ p\lambda_C t/2, & t \in [\hat{t}', 2\beta\bar{t}^* - \hat{t}'] \\ F^S\left(2\beta\bar{t}^* - t, t\right) - p\lambda_C \hat{t}'/2 + \int_{2\beta\bar{t}^* - t}^{\hat{t}'} F_1^S\left(\tau, 2\beta\bar{t}^* - \tau\right) d\tau, & t \in (2\beta\bar{t}^* - \hat{t}', t_{max}] \end{cases},
$$

$$w^*(t) = \begin{cases} p\lambda_C \hat{t}^{*\prime}/2 - \int_t^{\hat{t}^{*\prime}} F_1^S(\tau, 2\bar{t}^* - \tau)\, d\tau, & t \in [t^*_{min}, \hat{t}^{*\prime}) \\ p\lambda_C t/2, & t \in [\hat{t}^{*\prime}, 2\bar{t}^* - \hat{t}^{*\prime}] \\ F^S(2\bar{t}^* - t, t) - p\lambda_C \hat{t}^{*\prime}/2 + \int_{2\bar{t}^*-t}^{\hat{t}^{*\prime}} F_1^S(\tau, 2\bar{t}^* - \tau)\, d\tau, & t \in (2\bar{t}^* - \hat{t}^{*\prime}, t^*_{max}] \end{cases},$$

where $m(\hat{t}') = 2\beta\bar{t}^* - \hat{t}'$ and $m(\hat{t}^{*\prime}) = 2\bar{t}^* - \hat{t}^{*\prime}$. The wage schedule is linear in the range $[\hat{t}', m(\hat{t}')]$ for the home country ($[\hat{t}^{*\prime}, m(\hat{t}^{*\prime})]$ for the foreign country) and convex outside this range.

3.2.1 The Wage Schedule Under Free Trade

We investigate how opening to trade affects the nominal wages in each country. Since the home country has a higher average skill, it exports good C in the free-trade equilibrium. Thus, the relative price of p rises in the home country, and (3) implies that $d\hat{t}/dp < 0$. Therefore, an increase in p decreases $\hat{t}$ and increases $m(\hat{t})$, i.e., workers move from sector S into sector C. In the foreign country, a decline in p increases $\hat{t}^*$ and decreases $m(\hat{t}^*)$, i.e., workers exit industry C.

Note that $\hat{t}$ is not equal to $\hat{t}^*$ in the free-trade equilibrium, where $\hat{t}$ and $\hat{t}^*$ represent the least skilled worker in sector C in Home and Foreign under free trade, respectively. In any equilibrium, $p = MRT$, and thus, (3) together with $\bar{t} = \beta\bar{t}^*$ ($\beta > 1$) implies that $\hat{t} = \beta\hat{t}^*$ in the free-trade equilibrium. The nominal wages in Home and Foreign are illustrated in Figure 4. Note that the wages of workers with the same skill levels are not equalized across countries under free trade. Since $m(t) = 2\beta\bar{t}^* - \hat{t}$ and $m(\hat{t}^*) = 2\bar{t}^* - \hat{t}^*$ under free trade, we demonstrate in Appendix D that

$$\begin{cases} w^*(t) \leq w(t), & t \in [t_{min},\, \hat{t}) \\ w^*(t) = w(t), & t \in [\hat{t},\, 2\bar{t}^* - \hat{t}^*] \\ w^*(t) \geq w(t), & t \in (2\bar{t}^* - \hat{t}^*,\, t^*_{max}] \end{cases}.$$

Thus, we have the following Proposition.

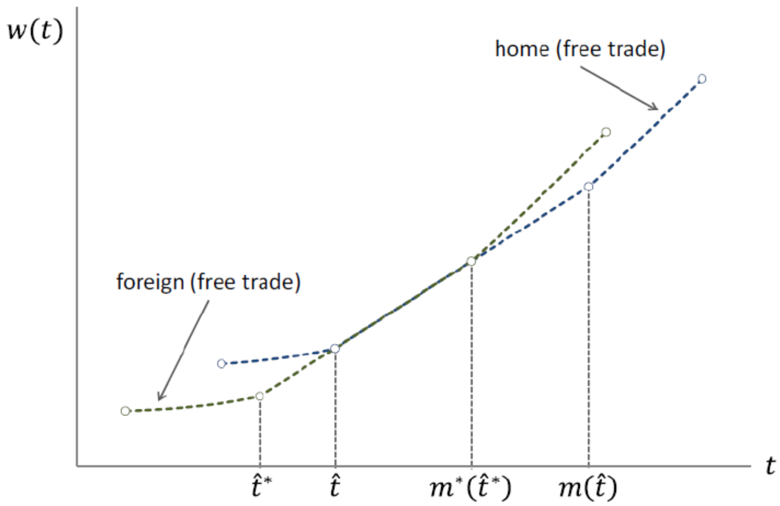

Figure 4: The Average Skill and Wages Under Free Trade

Proposition 3 *If two countries have the same average skill level but different skill dispersions, the wages of workers with the same level of skills are equalized across countries under free trade. If two countries have the same skill dispersion but different average skills, the wages of workers with the same level of skills are not equalized under free trade.*

Interestingly, in the case in which there is no trade in the free-trade equilibrium, the wages of workers with the same level of skills are not equalized in the free-trade equilibrium if two countries have different average skills.

Corollary 1 *Suppose that the distributions of skills in the two countries have the same form. If $\frac{\bar{t}}{\sigma} = \frac{\bar{t}^*}{\sigma^*}$ but $\bar{t} \neq \bar{t}^*$, then the free-trade equilibrium has no trade in final goods. However, the wages of workers with same level of skills are not equalized in the free-trade equilibrium.*

Whether wages are equalized depends solely on the average skill levels under free trade. If two countries have the same average skills, all workers have paired with their best partners under free trade, and thus, there is no incentive for offshoring. However, if two countries have different average skills, then there exist incentives for workers in the submodular sector to pair with Foreign workers under free trade. Hence, offshoring can occur between two such countries.

3.2.2 The Income Distribution under Free Trade

We examine the income distribution under free trade using a numerical example, as shown in Figure 5.[12] In Home, which has a higher average skill level, trade benefits the middle-skilled

[12]We use a beta distribution function with both shape parameters equal to 1.6. Moreover, $L = L^* = 1$, $t_{\min} = 7$, $t_{\max} = 17$, $t^*_{\min} = 5$, and $t^*_{\max} = 15$. Thus, we have $\bar{t} = 12$, $\bar{t}^* = 10$ and standard deviation $\sigma = 2.44$ in

workers ($t \in [\hat{t}, m(\hat{t})]$) who work in the exportables sector both before and after opening to trade because their real wages increase in terms of either good. We also know that workers who work in the importables sector before and after opening to trade are worse off. The lowest skilled workers lose because their real wage decreases in terms of either good. The highest skilled workers also lose because the increases in their nominal wages are less than the increase in the relative price. Whether the remaining workers who switch from the importables to the exportables sector are better or worse off depends on the consumption shares.

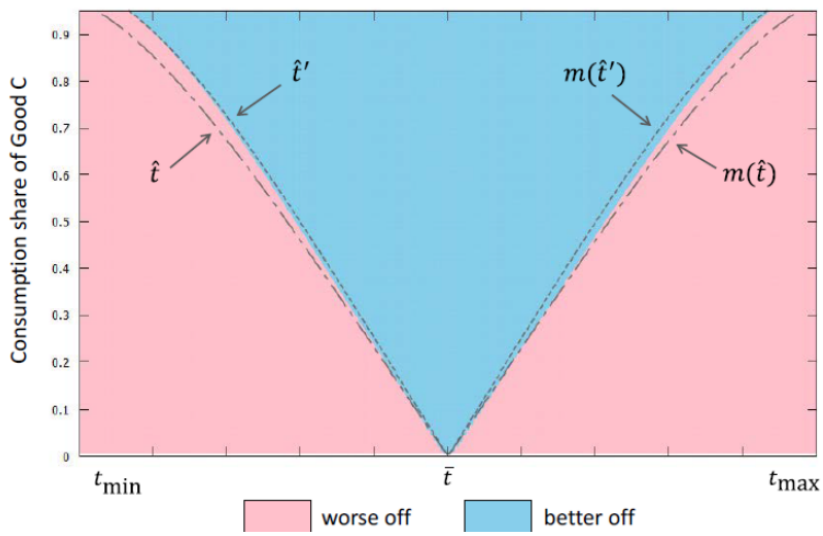

Figure 5: The Income Distribution under Free Trade

4 The Effects of Offshoring

In this section, we examine the effects of offshoring between two countries that have different average skills. Following Antras, Garricano and Rossi-Hansberg (2006), offshoring means that workers in different countries can collaborate in teams. For simplicity, we assume that there are no offshoring costs and both countries have the same labor size, i.e., $L = L^*$. We first investigate the effect of offshoring on the relative price and then explore the effects of offshoring on the income distribution.

Since workers can collaborate in international teams, we need to consider the global distribution of skills. Let $\phi^W(t)$ represent the density function of skills under offshoring, which

both countries. In addition, we use the CES production function $F(t_1, t_2) = (t_1^\theta + t_2^\theta)^{\frac{1}{\theta}}$ with $\theta = 4$ in sector S and $\theta = 0.5$ in sector C. We use a Cobb-Douglas utility function with the consumption share of good C $\mu = 0.6$ to plot the wage schedule and explore the effects of offshoring on the income distribution for $\mu \in [0.005, 0.965]$. We obtain similar results by using different parameters of the beta distribution function, which includes a uniform distribution function.

is given by

$$\phi^W(t) = \begin{cases} \dfrac{\phi^*(t)}{2} & t \in [t^*_{min}, t_{min}) \\ \dfrac{\phi(t)}{2} + \dfrac{\phi^*(t)}{2} & t \in [t_{min}, t^*_{max}) \\ \dfrac{\phi(t)}{2} & t \in [t^*_{max}, t_{max}] \end{cases}. \tag{10}$$

Since $\phi(t)$ and $\phi^*(t)$ are symmetric about the mean levels of skill in Home ($\bar{t}$) and Foreign ($\bar{t}^*$), respectively, $\phi^W(t)$ is symmetric about the mean level of skills worldwide, $\bar{t}^W \equiv (\bar{t} + \bar{t}^*)/2$. It follows that Lemma 1 holds under offshoring with $\hat{t}^W$ and $m^W(\hat{t}^W)$ being the least skilled and the highest skilled workers in Sector C.[13] Thus, we obtain

$$Y^W_C = L\lambda_C \bar{t}^W \int_{\hat{t}}^{2\bar{t}^W - \hat{t}^W} \phi^W(t)dt,$$
$$Y^W_S = 2L \int_{t^*_{min}}^{\hat{t}^W} F^S\left(t, 2\bar{t}^W - t\right) \phi^W(t)dt.$$

It is clear that offshoring occurs in the submodular sector, which leads to labor reallocation from the free-trade equilibrium.

We first investigate the effect of offshoring on the world relative price. To do this, we need to consider the demand side. We assume that preferences are represented by a Cobb-Douglas utility function as follows:

$$U = \frac{X^\mu_C X^{1-\mu}_S}{\mu^\mu (1-\mu)^{1-\mu}}, \tag{11}$$

where X_C and X_S represent the consumption of good C and good S, respectively. The world relative price p in equilibrium is determined by the following two equations.

$$\frac{F^S\left(\hat{t}^W, 2\bar{t}^W - \hat{t}^W\right)}{\lambda_C \bar{t}^W} = p,$$
$$\frac{2}{\lambda_C \bar{t}^W} \frac{\int_{t^*_{min}}^{\hat{t}^W} F^S\left(t, 2\bar{t}^W - t\right)\phi^W(t)dt}{\int_{\hat{t}^W}^{2\bar{t}^W - \hat{t}^W} \phi^W(t)dt} = \frac{1-\mu}{\mu} p.$$

The first equation is $MRT = p$, and the second equation reflects that the relative supply is equal to the relative demand. As proved in Appendix E, there exists a unique equilibrium under offshoring. By using a numerical example, we show that the world price under offshoring is higher than that under free trade.[14] Intuitively, offshoring leads to workers with the highest

[13] The density function $\phi^W(t)$ may not be continuous at $t_{\min}$ and $t^*_{\max}$. However, the cumulative distribution function is everywhere continuous, and thus, Lemma 1 holds.

[14] It is difficult to compare the world relative price under offshoring with that under free trade, because $\int_{t^*_{min}}^{\hat{t}^W} F^S\left(t, 2\bar{t}^W - t\right)\phi^W(t)dt$ cannot be calculated even when we assume that $F^S\left(t, 2\bar{t}^W - t\right)$ is a CES production function.

skills worldwide forming teams with workers with the lowest skills, which improves productivity in the submodular sector. As a result, the relative price of good C increases.

Next, we turn to the effects of offshoring on nominal wages. Clearly, the wages of workers with the same skill level are equalized, as shown in Figure 6. In the country with the higher average skill level, offshoring increases the wages of workers who work in the supermodular sector before and after offshoring. Their nominal wages increase in proportion to the increase in p. The convexity of $w(t)$ outside $(\hat{t}^W, m(\hat{t}^W))$ ensures a more-than-proportional nominal wage increase (reduction) for workers with the highest skills (workers with the lowest skills) whether they work in the submodular sector before and after offshoring or switch from the supermodular (submodular) sector to the submodular (supermodular) sector. In other words, offshoring increases wage inequality in the country with a higher average skill level. This result is consistent with the fact that developed countries have experienced a rise in wage inequality since the 1980s (see Feenstra and Hanson (1996, 1997, 1999)).

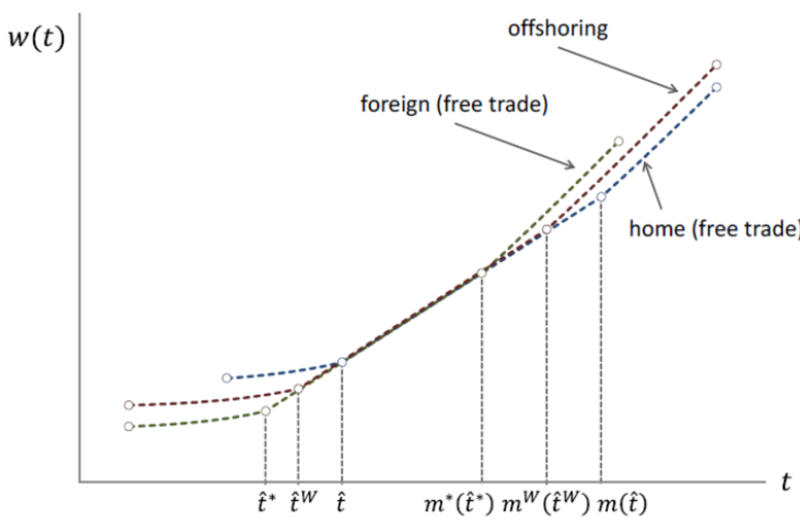

Figure 6: Wage Schedules under Outsourcing

The effects of offshoring on the income distribution are shown in Figure 7. From Figure 6, we can see that the increase in the relative price from free trade is small relative to the changes in nominal wages. Hence, the effects of offshoring on welfare are similar to those on the nominal wages. Interestingly, offshoring affects the workers in the submodular sector in opposite directions: workers with the highest skill levels benefit from offshoring in the country with a higher average skill level, while workers with the lowest skills lose. Note that offshoring and free trade have different effects on the welfare of workers in the submodular sector.

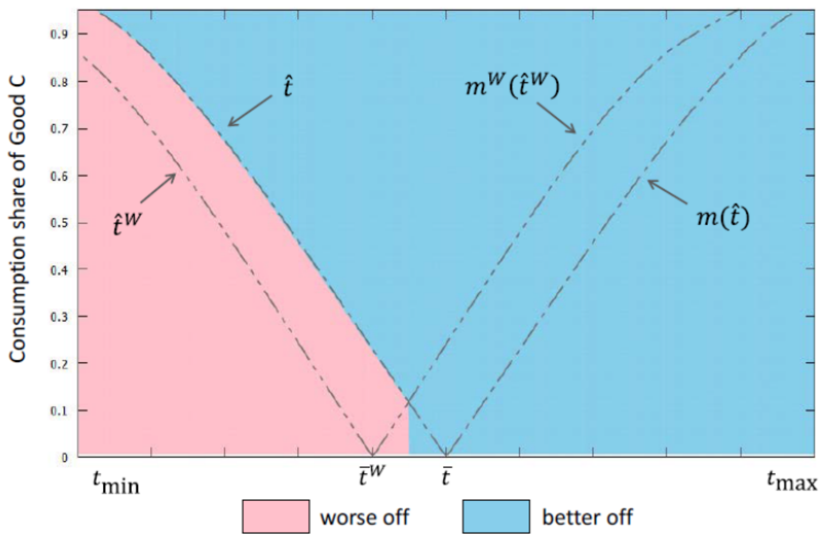

Figure 7: The Income Distribution under Offshoring

Proposition 4 *Offshoring and free trade have different effects on the welfare of workers in the submodular sector. Trade affects the welfare of workers in the same direction: All gain from trade, or all lose from trade. However, offshoring affects the welfare of workers with the highest skills and that of workers with the lowest skills in opposite directions.*

Offshoring offers an incentive for workers in the submodular sector to form international teams, which leads to labor reallocation. Workers with the highest skills in the country with the higher average skill have the opportunity to form teams with better partners in the foreign country. Therefore, those workers gain from offshoring, and workers with lower skills who are replaced by foreign workers lose. These results are consistent with the findings in Hummels et al. (2014), who estimate how offshoring and exporting affect wages by skill type using matched worker-firm data from Denmark. They find that within job spells, (1) offshoring tends to increase the high-skilled wage and decrease the low-skilled wage; (2) exporting tends to increase the wages of all skill types. Offshoring affects workers in the foreign country in opposite directions, as workers with the lowest skills find better partners and workers with the highest skills lose their best partners.

5 Concluding Remarks

In this paper, we investigate how skill dispersion and the average skill level affect the possibility of offshoring. We demonstrate that if two countries have the same skill dispersion but different average skills, the wages of workers with the same level of skills are not equalized, and thus, offshoring is possible. When two countries have the same average skill level but different skill dispersions, as in Grossman and Maggi (2000), the wages of workers with the same level of skills are equalized under free trade, and thus, offshoring is impossible. Moreover, we demonstrate that exporting and offshoring have different effects on the wages of workers with highest and lowest skill levels. Specifically, the results in the country with a higher skill level are consistent with the findings in Hummels et al. (2014).

We show that offshoring leads to an increase in wage inequality in the country with a higher average skill. This result is consistent with the fact that developed countries have experienced a rise in wage inequality since the 1980s (see Feenstra and Hanson (1996, 1997, 1999)). In addition, our paper suggests that the difference in the average skill levels between two high-income countries may be another possible reason for the finding in Hummels et al. (2014, p 1619). That is, although most Danish trade is with other high-income countries, offshoring tends to reduce the wage of low-skilled workers. They also restrict their sample to only include Danish trade with high-income partners and find a similar sign pattern for offshoring.

Our paper considers two important factors related to workers' skills, the average skill and the skill dispersion, and is useful to interpret several observations on offshoring in the real world. However, we abstract from offshoring costs and imperfect observation of workers' skills in the current setup. These issues are left for future work.

Appendix A: The proof for $F_1^S(t_1,t_2) - F_2^S(t_1,t_2) \leq 0$ if $t_1 \leq t_2$.

Since the tasks are symmetric, i.e., $F^S(t_1,t_2) = F^S(t_2,t_1)$, we have $F_2^S(t_1,t_2) = F_1^S(t_2,t_1)$. It follows that

$$F_1^S(t_1,t_2) - F_2^S(t_1,t_2) = F_1^S(t_1,t_2) - F_1^S(t_2,t_1) = F_1^S\left(\frac{t_1}{t_2},1\right) - F_1^S\left(\frac{t_2}{t_1},1\right).$$

The last equality is obtained because $F^S(t_1,t_2)$ is homogeneous of degree one, and thus $F_1^S(t_1,t_2)$ is homogeneous of degree zero. From $F^S(t_1,t_2)$ exhibiting submodularity, we have $F_{11}^S \geq 0$. Thus, if $t_1 \leq t_2$, we obtain $F_1^S(t_1,t_2) - F_2^S(t_1,t_2) \leq 0$.

Appendix B: The sign of $\frac{d(Y_S^*/Y_C^*)}{d\beta}$

From (1), (2), $\phi(t) = \phi^*[t-(\beta-1)\bar{t}^*]$ for $t \in [t_{\min}, t_{\max}]$, and $t_{\min} = t_{\min}^* + (\beta-1)\bar{t}^*$, we have

$$\begin{aligned}
\frac{Y_S}{Y_C} &= \frac{2}{\lambda_C \beta \bar{t}^*} \frac{\int_{t_{\min}}^{\hat{t}} F^S\left(t, 2\beta\bar{t}^* - t\right)\phi(t)dt}{\int_{\hat{t}}^{2\beta\bar{t}^*-\hat{t}} \phi(t)dt} \\
&= \frac{2}{\lambda_C \bar{t}^*} \frac{\int_{t_{\min}^*+(\beta-1)\bar{t}^*}^{\beta\hat{t}^*} F^S\left(t, 2\beta\bar{t}^* - t\right)\cdot\phi^*[t-(\beta-1)\bar{t}^*]dt}{\beta\int_{\beta\hat{t}^*}^{2\beta\bar{t}^*-\beta\hat{t}^*} \phi^*[t-(\beta-1)\bar{t}^*]dt}.
\end{aligned}$$

Let $A \equiv \int_{\beta\hat{t}^*}^{2\beta\bar{t}^*-\beta\hat{t}^*} \phi^*[t-(\beta-1)\bar{t}^*]dt$ and $sign[\frac{d(Y_S/Y_C)}{d\beta}] = sign[D]$. Holding MRT constant, we have

$$D = A[\int_{t_{\min}}^{\beta\hat{t}^*} \{[2\beta\bar{t}^* F_2^S(t, 2\beta\bar{t}^*-t) - F^S(t, 2\beta\bar{t}^*-t)]\phi(t) - F^S(t, 2\beta\bar{t}^*-t)\,\beta\bar{t}^*\phi'(t)\}dt \tag{B1}$$
$$+ A[\beta\hat{t}^* F^S\left(\beta\hat{t}^*, 2\beta\bar{t}^*-\beta\hat{t}^*\right)\phi(\beta\hat{t}^*) - \beta\bar{t}^* F^S(t_{\min}, t_{\max})\,\phi(t_{\min})]$$
$$- [\int_{t_{\min}}^{\beta\hat{t}^*} F^S(t, 2\beta\bar{t}^*-t)\cdot\phi(t)dt][\phi(2\beta\bar{t}^*-\beta\hat{t}^*)(2\beta\bar{t}^*-\beta\hat{t}^*) - \beta\hat{t}^*\phi(\beta\hat{t}^*)]$$

Due to $\phi(t) = \phi(2\beta\bar{t}^*-t)$ and $\phi(t) = \phi^*(t-\delta)$, we have $\int_{\beta\hat{t}^*}^{2\beta\bar{t}^*-\beta\hat{t}^*} \phi'^*(\cdot)dt = 0$, $\phi^*(t^*_{\min}) = \phi(t_{\min})$, $\phi^*(\beta\hat{t}^*-(\beta-1)\bar{t}^*) = \phi(\beta\hat{t}^*)$ and $\phi^*(t-(\beta-1)\bar{t}^*) = \phi(t)$, and $\phi'^*(t-(\beta-1)\bar{t}^*) = \phi'(t)$.

First, we obtain

$$\int_{t^*_{\min}+(\beta-1)\bar{t}^*}^{\beta\hat{t}^*} [2\beta\bar{t}^* F_2^S(t, 2\beta\bar{t}^*-t) - F^S(t, 2\beta\bar{t}^*-t)]\phi(t)dt \tag{B2}$$
$$= \int_{t^*_{\min}+(\beta-1)\bar{t}^*}^{\beta\hat{t}^*} t[F_2^S(t, 2\beta\bar{t}^*-t) - F_1^S(t, 2\beta\bar{t}^*-t)]\phi(t)dt > 0.$$

Next, let us consider

$$\beta\hat{t}^* F^S\left(\beta\hat{t}^*, 2\beta\bar{t}^*-\beta\hat{t}^*\right)\phi(\beta\hat{t}^*) - \beta\bar{t}^* F^S(t_{\min}, t_{\max})\,\phi(t_{\min})$$
$$= \beta\hat{t}^* F^S\left(\beta\hat{t}^*, 2\beta\bar{t}^*-\beta\hat{t}^*\right)\phi(\beta\hat{t}^*) - F^S(t_{\min}, t_{\max})\,\phi(t_{\min})(t_{\min}+\frac{I}{2})$$
$$= \int_{t_{\min}}^{\beta\hat{t}^*} \frac{d[F^S(t, 2\beta\bar{t}^*-t)\,\phi(t)t]}{dt}dt - F^S(t_{\min}, t_{\max})\,\phi(t_{\min})\frac{I}{2}$$
$$= \int_{t_{\min}}^{\beta\hat{t}^*} [F^S(t, 2\beta\bar{t}^*-t)\,\phi(t) + F^S(t, 2\beta\bar{t}^*-t)\,\phi'(t)t]dt - F^S(t_{\min}, t_{\max})\,\phi(t_{\min})\frac{I}{2} \tag{B3}$$
$$+ \int_{t_{\min}}^{\beta\hat{t}^*} \phi(t)t[F_1^S(t, 2\beta\bar{t}^*-t) - F_2^S(t, 2\beta\bar{t}^*-t)]dt.$$

Substituting (B2) and (B3) into (B1), we obtain

$$D = A[\int_{t_{\min}}^{\beta\hat{t}^*} (t-\beta\bar{t}^*)F^S(t, 2\beta\bar{t}^*-t)]\phi'(t)dt] \tag{B4}$$
$$+ \int_{t_{\min}}^{\beta\hat{t}^*} [F^S(t, 2\beta\bar{t}^*-t)\,\phi(t)dt - F^S(t_{\min}, t_{\max})\,\phi(t_{\min})\frac{I}{2}]$$
$$- [\int_{t^*_{\min}+(\beta-1)\bar{t}^*}^{\beta\hat{t}^*} F^S(t, 2\beta\bar{t}^*-t)\cdot\phi(t)dt][\phi(\beta\hat{t}^*)(2\beta\bar{t}^*-2\beta\hat{t}^*)].$$

It follows that

$$\int_{t_{\min}}^{\beta\hat{t}^*}(t-\beta\bar{t}^*)F^S\left(t,2\beta\bar{t}^*-t\right)\phi'(t)dt]$$
$$=\beta(\hat{t}^*-\bar{t}^*)F^S\left(\beta\hat{t}^*,2\beta\bar{t}^*-\beta\hat{t}^*\right)\phi(\beta\hat{t}^*)+\frac{I}{2}F^S\left(t_{\min},t_{\max}\right)\phi(t_{\min})$$
$$-\int_{t_{\min}}^{\beta\hat{t}^*}\phi(t)F^S\left(t,2\beta\bar{t}^*-t\right)dt+\int_{t_{\min}}^{\beta\hat{t}^*}\phi(t)(t-\beta\bar{t}^*)(F_1^S\left(t,2\beta\bar{t}^*-t\right)-F_2^S\left(t,2\beta\bar{t}^*-t\right))dt, \tag{B5}$$

where $t_{\min}-\beta\bar{t}^*=-\frac{I}{2}$. Substituting (B5) into (B4) to obtain

$$D=-A[\beta(\bar{t}^*-\hat{t}^*)F^S\left(\beta\hat{t}^*,2\beta\bar{t}^*-\beta\hat{t}^*\right)\phi(\beta\hat{t}^*)+\int_{t_{\min}}^{\beta\hat{t}^*}\phi(t)(t-\beta\bar{t}^*)(F_2^S-F_1^S)dt]$$
$$-[\int_{t_{\min}}^{\beta\hat{t}^*}F^S\left(t,2\beta\bar{t}^*-t\right)\cdot\phi(t)dt][\phi(\beta\hat{t}^*)(2\beta\bar{t}^*-2\beta\hat{t}^*)].$$

Since $t\le 2\beta\bar{t}^*-t$, following Appendix A1, we obtain $F_1^S\left(t,2\beta\bar{t}^*-t\right)\le F_2^S\left(t,2\beta\bar{t}^*-t\right)$ for $t\in[t_{min},\beta\hat{t}^*)$. Clearly, we have $sign[\frac{d(Y_S/Y_C)}{d\beta}]<0$.

Appendix C: A Comparison of Coefficients of Variation and Trade Patterns

We assume that the skill levels of both countries follow the same type of distribution functions, by which we mean that one country's skill distribution function can be derived by linearly transforming the other country's skill distribution function. Under this assumption, the standardized skill distribution of Home is coincident with that of Foreign and the following properties hold: $\phi(t)=\frac{1}{\sigma}\phi_N(\frac{t-\bar{t}}{\sigma})$, $\phi^*(t)=\frac{1}{\sigma^*}\phi_N(\frac{t-\bar{t}^*}{\sigma^*})$ and $\frac{t_{\min}-\bar{t}}{\sigma}=\frac{t^*_{\min}-\bar{t}^*}{\sigma^*}$, where $\phi_N(\cdot)$ represents the density function with mean 0 and standard deviation 1. In addition, we assume that $\bar{t}=\beta\bar{t}^*$, $\beta>0$ and $\sigma=\gamma\sigma^*$, with $\gamma>0$. In the following, we prove that (a) $\frac{Y_S}{Y_C}=\frac{Y_S^*}{Y_C^*}$, if $\frac{\beta}{\gamma}=1$; (b) $\frac{\partial(\frac{Y_S}{Y_C})}{\partial(\frac{\beta}{\gamma})}<0$, with MRT held constant.

(a) (3) implies that $\hat{t}=\beta\hat{t}^*$, if two countries have the same MRT. From (1), (2), we have

$$\frac{Y_S}{Y_C}=\frac{2}{\lambda_C\bar{t}}\frac{\int_{t_{\min}}^{\hat{t}}F^S\left(t,2\bar{t}-t\right)\frac{1}{\sigma}\phi_N(\frac{t-\bar{t}}{\sigma})dt}{\int_{\hat{t}}^{2\bar{t}-\hat{t}}\frac{1}{\sigma}\phi_N(\frac{t-\bar{t}}{\sigma})dt}.$$

Let $\tau = \frac{t-\bar{t}}{\sigma}$ and the above equation becomes

$$\begin{aligned}
\frac{Y_S}{Y_C} &= \frac{2}{\lambda_C \bar{t}} \frac{\int_{\frac{t_{\min}-\bar{t}}{\sigma}}^{\frac{\hat{t}-\bar{t}}{\sigma}} F^S\left(\bar{t}+\tau\sigma, \bar{t}-\tau\sigma\right)\phi_N(\tau)d\tau}{\int_{\frac{\hat{t}-\bar{t}}{\sigma}}^{\frac{\bar{t}-\hat{t}}{\sigma}} \phi_N(\tau)d\tau} \\
&= \frac{2}{\lambda_C \frac{\bar{t}}{\sigma}} \frac{\int_{\frac{t_{\min}-\bar{t}}{\sigma}}^{\frac{\hat{t}-\bar{t}}{\sigma}} F^S\left(\frac{\bar{t}}{\sigma}+\tau, \frac{\bar{t}}{\sigma}-\tau\right)\phi_N(\tau)d\tau}{\int_{\frac{\hat{t}-\bar{t}}{\sigma}}^{\frac{\bar{t}-\hat{t}}{\sigma}} \phi_N(\tau)d\tau} \\
&= \frac{2}{\lambda_C} \frac{\int_{\frac{t^*_{\min}-\bar{t}^*}{\sigma^*}}^{\frac{\beta(\hat{t}^*-\bar{t}^*)}{\gamma\sigma^*}} F^S\left(\frac{\beta}{\gamma}\frac{\bar{t}^*}{\sigma^*}+\tau, \frac{\beta}{\gamma}\frac{\bar{t}^*}{\sigma^*}-\tau\right)\phi_N(\tau)d\tau}{\left(\frac{\beta}{\gamma}\frac{\bar{t}^*}{\sigma^*}\right)\int_{\frac{\beta(\hat{t}^*-\bar{t}^*)}{\gamma\sigma^*}}^{\frac{\beta(\bar{t}^*-\hat{t}^*)}{\gamma\sigma^*}} \phi_N(\tau)d\tau}. \qquad \text{(C1)}
\end{aligned}$$

Similarly, we have

$$\frac{Y^*_S}{Y^*_C} = \frac{2}{\lambda_C \frac{\bar{t}^*}{\sigma^*}} \frac{\int_{\frac{t^*_{\min}-\bar{t}^*}{\sigma^*}}^{\frac{\hat{t}-\bar{t}^*}{\sigma^*}} F^S\left(\frac{\bar{t}^*}{\sigma^*}+\tau, \frac{\bar{t}^*}{\sigma^*}-\tau\right)\frac{1}{\sigma^*}\phi_N(\tau)d\tau}{\int_{\frac{\hat{t}-\bar{t}^*}{\sigma^*}}^{\frac{\bar{t}^*-\hat{t}}{\sigma^*}} \frac{1}{\sigma^*}\phi_N(\tau)d\tau}. \qquad \text{(C2)}$$

Clearly, if $\frac{\beta}{\gamma} = 1$, we have $\frac{\bar{t}}{\sigma} = \frac{\bar{t}^*}{\sigma^*}$ and (C1) is the same as (C2).

(b) (C1) is rewritten as

$$\frac{Y_S}{Y_C} = \frac{2}{\lambda_C} \frac{\int_{\frac{t^*_{\min}-\bar{t}^*}{\sigma^*}}^{\frac{\beta(\hat{t}^*-\bar{t}^*)}{\gamma\sigma^*}} F^S\left(\frac{\bar{t}^*}{\sigma^*}+\frac{\tau}{\frac{\beta}{\gamma}}, \frac{\bar{t}^*}{\sigma^*}-\frac{\tau}{\frac{\beta}{\gamma}}\right)\phi_N(\tau)d\tau}{\frac{\bar{t}^*}{\sigma^*}\int_{\frac{\beta(\hat{t}^*-\bar{t}^*)}{\gamma\sigma^*}}^{\frac{\beta(\bar{t}^*-\hat{t}^*)}{\gamma\sigma^*}} \phi_N(\tau)d\tau}$$

It is clear that the dominator increases in $\frac{\beta}{\gamma}$. In the following, we show that the numerator decreases with $\frac{\beta}{\gamma}$. Let $M \equiv \int_{\frac{t^*_{\min}-\bar{t}^*}{\sigma^*}}^{\frac{\beta(\hat{t}^*-\bar{t}^*)}{\gamma\sigma^*}} F^S\left(\frac{\bar{t}^*}{\sigma^*}+\frac{\tau}{\frac{\beta}{\gamma}}, \frac{\bar{t}^*}{\sigma^*}-\frac{\tau}{\frac{\beta}{\gamma}}\right)\phi_N(\tau)d\tau$. Thus, we obtain

$$\begin{aligned}
sign[\frac{\partial M}{\partial(\frac{\beta}{\gamma})}] &= -F^S\left(\frac{\hat{t}^*}{\sigma^*}, 2\frac{\bar{t}^*}{\sigma^*}-\frac{\hat{t}^*}{\sigma^*}\right)\phi_N(\frac{\beta(\hat{t}^*-\bar{t}^*)}{\gamma\sigma^*})\frac{(\bar{t}^*-\hat{t}^*)}{\sigma^*} \\
&\quad - \int_{-\frac{\bar{t}^*-t^*_{\min}}{\sigma^*}}^{-\frac{\beta(\bar{t}^*-\hat{t}^*)}{\gamma\sigma^*}} [F_1^S\left(\frac{\bar{t}^*}{\sigma^*}+\frac{\tau}{\frac{\beta}{\gamma}}, \frac{\bar{t}^*}{\sigma^*}-\frac{\tau}{\frac{\beta}{\gamma}}\right) - F_2^S\left(\frac{\bar{t}^*}{\sigma^*}+\frac{\tau}{\frac{\beta}{\gamma}}, \frac{\bar{t}^*}{\sigma^*}-\frac{\tau}{\frac{\beta}{\gamma}}\right)]\frac{\tau}{(\frac{\beta}{\gamma})^2}\phi_N(\tau)d\tau \\
&\quad < 0.
\end{aligned}$$

where from Appendix 1, we have $F_1^S\left(\frac{\bar{t}^*}{\sigma^*}+\frac{\tau}{\frac{\beta}{\gamma}}, \frac{\bar{t}^*}{\sigma^*}-\frac{\tau}{\frac{\beta}{\gamma}}\right) - F_2^S\left(\frac{\bar{t}^*}{\sigma^*}+\frac{\tau}{\frac{\beta}{\gamma}}, \frac{\bar{t}^*}{\sigma^*}-\frac{\tau}{\frac{\beta}{\gamma}}\right) \leq 0.$

Appendix D: The Comparison of Wage Schedules

The comparison of the wages in the home country with those in the foreign country is shown as follows:

$$
\begin{aligned}
&w^*(t) \leq w(t), \text{ if } t \in [t_{min}, \hat{t}^*) \text{ and } t \in [\hat{t}^*, \hat{t}), \\
&w^*(t) = w(t), \text{ if } t \in [\hat{t}, 2\bar{t}^* - \hat{t}^*], \\
&w^*(t) \geq w(t), \text{ if } t \in (2\bar{t}^* - \hat{t}^*, 2\beta\bar{t}^* - \hat{t}) \text{ and } t \in [2\beta\bar{t}^* - \hat{t}, t^*_{max}].
\end{aligned}
$$

Proof:

(1) If $t \in [t_{min}, \hat{t}^*)$, we have

$$
\begin{aligned}
&w^*(t) - w(t) \\
&= \frac{p\lambda_C \hat{t}^*}{2} - \int_t^{\hat{t}^*} F_1^S(\tau, 2\bar{t}^* - \tau)\, d\tau - \left[\frac{p\lambda_C \hat{t}}{2} - \int_t^{\hat{t}} F_1^S(\tau, 2\bar{t} - \tau)\, d\tau\right] \\
&= \frac{p\lambda_C \hat{t}^*(1-\beta)}{2} + \int_{\hat{t}^*}^{\beta\hat{t}^*} F_1^S(\tau, 2\beta\bar{t}^* - \tau)\, d\tau + \int_t^{\hat{t}^*} \left[F_1^S(\tau, 2\beta\bar{t}^* - \tau) - F_1^S(\tau, 2\bar{t}^* - \tau)\right] d\tau
\end{aligned}
$$

Because of the submodularity of F^S, we have $F_{12}^S \leqq 0$. It follows that

$$F_1^S(\tau, 2\beta\bar{t}^* - \tau) - F_1^S(\tau, 2\bar{t}^* - \tau) \leqq 0$$

for all $\tau \in [t, \hat{t}^*)$. Next, we examine the sign of $\frac{p\lambda_C \hat{t}^*(1-\beta)}{2} + \int_{\hat{t}^*}^{\beta\hat{t}^*} F_1^S(\tau, 2\beta\bar{t}^* - \tau)\, d\tau$ as follows:

$$
\begin{aligned}
&\frac{p\lambda_C \hat{t}^*(1-\beta)}{2} + \int_{\hat{t}^*}^{\beta\hat{t}^*} F_1^S(\tau, 2\beta\bar{t}^* - \tau)\, d\tau \\
&\leq \frac{p\lambda_C \hat{t}^*(1-\beta)}{2} + \left(\beta\hat{t}^* - \hat{t}^*\right) F_1^S\left(\beta\hat{t}^*, 2\beta\bar{t}^* - \beta\hat{t}^*\right) \\
&= \frac{\hat{t}^*(1-\beta)}{2}\left[p\lambda_C - 2F_1^S\left(\hat{t}^*, 2\bar{t}^* - \hat{t}^*\right)\right] \\
&= \frac{\hat{t}^*(1-\beta)}{2\bar{t}^*}\left[F^S\left(\hat{t}^*, 2\bar{t}^* - \hat{t}^*\right) - 2\bar{t}^* F_1^S\left(\hat{t}^*, 2\bar{t}^* - \hat{t}^*\right)\right] \\
&= \frac{\hat{t}^*(1-\beta)\left(2\bar{t}^* - \hat{t}^*\right)}{2\bar{t}^*}\left[F_2^S\left(\hat{t}^*, 2\bar{t}^* - \hat{t}^*\right) - F_1^S\left(\hat{t}^*, 2\bar{t}^* - \hat{t}^*\right)\right] \leqq 0.
\end{aligned}
$$

We substitute (4) into the third equality to obtain the fourth equality. The fifth equality is obtained because of the assumption of constant returns to skill of F^S.

(2) If $t \in [\hat{t}^*,\, \hat{t})$, we have

$$\begin{aligned}
w^*(t) - w(t) &= \frac{p\lambda_C t}{2} - \left[\frac{p\lambda_C \hat{t}}{2} - \int_t^{\hat{t}} F_1^S\left(\tau, 2\bar{t} - \tau\right) d\tau \right] \\
&= \frac{p\lambda_C\left(t - \hat{t}\right)}{2} + \int_t^{\hat{t}} F_1^S\left(\tau, 2\bar{t} - \tau\right) d\tau \\
&\leqq \frac{p\lambda_C\left(t - \hat{t}\right)}{2} + \left(\hat{t} - t\right) F_1^S\left(\hat{t}, 2\bar{t} - \hat{t}\right) \\
&= \left(t - \hat{t}\right)\left[\frac{p\lambda_C}{2} - F_1^S\left(\hat{t}, 2\bar{t} - \hat{t}\right)\right] \\
&= \frac{t - \hat{t}}{2\bar{t}}\left[F^S\left(\hat{t}, 2\bar{t} - \hat{t}\right) - 2\bar{t} F_1^S\left(\hat{t}, 2\bar{t} - \hat{t}\right)\right] \\
&= \frac{\left(t - \hat{t}\right)\left(2\bar{t} - \hat{t}\right)}{2\bar{t}}\left[F_2^S\left(\hat{t}, 2\bar{t} - \hat{t}\right) - F_1^S\left(\hat{t}, 2\bar{t} - \hat{t}\right)\right] \leqq 0
\end{aligned}$$

(3) If $t \in (2\bar{t}^* - \hat{t}^*,\, 2\beta\bar{t}^* - \hat{t})$, we have

$$\begin{aligned}
w^*(t) - w(t) &= F^S\left(2\bar{t}^* - t, t\right) - \frac{p\lambda_C \hat{t}^*}{2} + \int_{2\bar{t}^* - t}^{\hat{t}^*} F_1^S\left(\tau, 2\bar{t}^* - \tau\right) d\tau - \frac{p\lambda_C t}{2} \\
&= F^S\left(2\bar{t}^* - t, t\right) + \int_{2\bar{t}^* - t}^{\hat{t}^*} F_1^S\left(\tau, 2\bar{t}^* - \tau\right) d\tau - \frac{p\lambda_C\left(t + \hat{t}^*\right)}{2}.
\end{aligned}$$

Converging t to $2\bar{t}^* - \hat{t}^*$ yields $\lim_{t \to 2\bar{t}^* - \hat{t}^*}\left(w^*(t) - w(t)\right) = 0$. In addition, we have

$$\begin{aligned}
&\frac{\partial\left(w^*(t) - w(t)\right)}{\partial t} \\
&= F_2^S\left(2\bar{t}^* - t, t\right) - F_1^S\left(2\bar{t}^* - t, t\right) + F_1^S\left(2\bar{t}^* - t, t\right) - \frac{p\lambda_C}{2} \\
&= F_2^S\left(2\bar{t}^* - t, t\right) - \frac{1}{2\bar{t}^*} F^S\left(\hat{t}^*, 2\bar{t}^* - \hat{t}^*\right) \\
&= \left[F_2^S\left(2\bar{t}^* - t, t\right) - F_2^S\left(\hat{t}^*, 2\bar{t}^* - \hat{t}^*\right)\right] + \frac{\hat{t}^*}{2\bar{t}^*}\left[F_2^S\left(\hat{t}^*, 2\bar{t}^* - \hat{t}^*\right) - F_1^S\left(\hat{t}^*, 2\bar{t}^* - \hat{t}^*\right)\right]
\end{aligned}$$

We have $F_2^S\left(\hat{t}^*, 2\bar{t}^* - \hat{t}^*\right) \geq F_1^S\left(\hat{t}^*, 2\bar{t}^* - \hat{t}^*\right)$ due to Appendix A. Due to the submodularity of F^S and the assumption of constant returns to skill, we obtain $F_{22}^S \geq 0 \geq F_{21}^S$. It follows that $F_2^S\left(2\bar{t}^* - t, t\right) \geq F_2^S\left(\hat{t}^*, 2\bar{t}^* - \hat{t}^*\right)$ for $t > 2\bar{t}^* - \hat{t}^*$. Thus, we have $\partial\left(w^*(t) - w(t)\right)/\partial t \geqq 0$. Therefore, we have $w^*(t) \geq w(t)$ for $t \in \left(2\bar{t}^* - \hat{t}^*,\, 2\beta\bar{t}^* - \hat{t}\right)$.

(4) If $t \in [2\beta\bar{t}^* - \hat{t},\, t^*_{max}]$, we have

$$\begin{aligned}
& w^*(t) - w(t) \\
&= F^S\left(2\bar{t}^* - t, t\right) + \frac{p\lambda_C(\hat{t} - \hat{t}^*)}{2} + \int_{2\bar{t}^*-t}^{\hat{t}^*} F_1^S\left(\tau, 2\bar{t}^* - \tau\right) d\tau - F^S\left(2\beta\bar{t}^* - t, t\right) \\
&- \int_{2\beta\bar{t}^*-t}^{\hat{t}} F_1^S\left(\tau, 2\beta\bar{t}^* - \tau\right) d\tau \\
&= \frac{p\lambda_C\hat{t}^*(\beta - 1)}{2} + \left[F^S\left(2\bar{t}^* - t, t\right) - F^S\left(2\beta\bar{t}^* - t, t\right)\right] + \int_{2\bar{t}^*-t}^{\hat{t}^*} F_1^S\left(\tau, 2\bar{t}^* - \tau\right) d\tau \\
&- \int_{2\beta\bar{t}^*-t}^{\beta\hat{t}^*} F_1^S\left(\tau, 2\beta\bar{t}^* - \tau\right) d\tau
\end{aligned}$$

Clearly, $\lim_{\beta\to 1}\left(w^*(t) - w(t)\right) = 0$. In addition, we have

$$\begin{aligned}
& \frac{\partial\left(w^*(t) - w(t)\right)}{\partial\beta} \\
&= \frac{p\lambda_C\hat{t}^*}{2} - \hat{t}^* F_1^S\left(\beta\hat{t}^*, 2\beta\bar{t}^* - \beta\hat{t}^*\right) - 2\bar{t}^* \int_{2\beta\bar{t}^*-t}^{\beta\hat{t}^*} F_{12}^S\left(\tau, 2\beta\bar{t}^* - \tau\right) d\tau \\
&= \frac{\hat{t}^*}{2\bar{t}^*}\left[\frac{F^S\left(\beta\hat{t}^*, 2\beta\bar{t}^* - \beta\hat{t}^*\right)}{\beta} - 2\bar{t}^* F_1^S\left(\beta\hat{t}^*, 2\beta\bar{t}^* - \beta\hat{t}^*\right)\right] - 2\bar{t}^* \int_{2\beta\bar{t}^*-t}^{\beta\hat{t}^*} F_{12}^S\left(\tau, 2\beta\bar{t}^* - \tau\right) d\tau \\
&= \frac{\hat{t}^*}{2\bar{t}^*}\left[F^S\left(\hat{t}^*, 2\bar{t}^* - \hat{t}^*\right) - 2\bar{t}^* F_1^S\left(\hat{t}^*, 2\bar{t}^* - \hat{t}^*\right)\right] - 2\bar{t}^* \int_{2\beta\bar{t}^*-t}^{\beta\hat{t}^*} F_{12}^S\left(\tau, 2\beta\bar{t}^* - \tau\right) d\tau \\
&= \frac{\hat{t}^*\left(2\bar{t}^* - \hat{t}^*\right)}{2\bar{t}^*}\left[F_2^S\left(\hat{t}^*, 2\bar{t}^* - \hat{t}^*\right) - F_1^S\left(\hat{t}^*, 2\bar{t}^* - \hat{t}^*\right)\right] - 2\bar{t}^* \int_{2\beta\bar{t}^*-t}^{\beta\hat{t}^*} F_{12}^S\left(\tau, 2\beta\bar{t}^* - \tau\right) d\tau.
\end{aligned}$$

We have $F_1^S\left(\hat{t}^*, 2\bar{t}^* - \hat{t}^*\right) \leq F_2^S\left(\hat{t}^*, 2\bar{t}^* - \hat{t}^*\right)$ due to Appendix A. Because of the supermodularity of F^S, we have $F_{12} \leqq 0$. It follows that $\frac{\partial(w^*(t) - w(t))}{\partial\beta} \geq 0$. Therefore, we have $w^*(t) \geqq w(t)$ for $t \in [2\beta\bar{t}^* - \hat{t},\, t^*_{max}]$ when $\beta > 1$.

Appendix E: Existence and Uniqueness of Equilibrium

We begin with the equilibrium of the country with a higher average skill under autarky. The equilibrium is determined by the following two equations:

$$\frac{F^S\left(\hat{t}', 2\beta\bar{t}^* - \hat{t}'\right)}{\lambda_C\beta\bar{t}^*} = p_a,$$

$$\frac{2}{\lambda_C\beta\bar{t}^*}\frac{\int_{t^*_{min}+(\beta-1)\bar{t}^*}^{\hat{t}'} F^S\left(t, 2\beta\bar{t}^* - t\right)\phi^*(t - (\beta - 1)\bar{t}^*)dt}{\int_{\hat{t}'}^{2\beta\bar{t}^*-\hat{t}'} \phi^*(t - (\beta - 1)\bar{t}^*)dt} = \frac{1 - \mu}{\mu}p_a,$$

where p_a and $\hat{t}'$ represent the relative price of good C and the marginal skill level between the two sectors under autarky, respectively. Let $\theta_a \equiv \hat{t}'/\beta\bar{t}^*$. Thus, from the above two equations, we obtain

$$\mathcal{A}(\theta_a, \beta) \equiv \frac{\mu}{1-\mu}\frac{2}{\beta\bar{t}^*}\frac{\int_{t^*_{min}+(\beta-1)\bar{t}^*}^{\theta_a\beta\bar{t}^*} F^S\left(t, 2\beta\bar{t}^* - t\right)\phi^*(t-(\beta-1)\bar{t}^*)dt}{\int_{\theta_a\beta\bar{t}^*}^{\beta\bar{t}^*(2-\theta_a)} \phi^*(t-(\beta-1)\bar{t}^*)dt} = F^S\left(\theta_a, 2-\theta_a\right), \tag{E1}$$

where $\theta_a \in [t_{min}/\beta\bar{t}^*, 1]$. The left-hand side of (E1) converges to 0 when $\theta_a \to t_{min}/\beta\bar{t}^*$ and diverges to $+\infty$ when $\theta_a \to 1$. In addition, $\mathcal{A}(\theta_a, \beta)$ is continuous and monotonically increasing in θ_a. The right-hand side of (E1) is continuous and monotonically decreases in θ_a. Moreover, $F^S\left(\theta_a, 2-\theta_a\right)$ takes finite values not only when $\theta_a = t_{min}/\beta\bar{t}^*$ but also when $\theta_a = 1$. Therefore, there exists a unique θ_a satisfying (E1). It follows that there exists a unique autarky equilibrium $(\hat{t}', p_a)$ in the home country. By letting $\beta = 1$, we can demonstrate the existence and uniqueness of the autarky equilibrium $(\hat{t}^{*\prime}, p_a^*)$ in the foreign country.

Similarly, we can show there exists a unique equilibrium under free trade. We begin with the case of imperfect specialization. The equilibrium is determined by the following equations:

$$\frac{F^S\left(\hat{t}, 2\beta\bar{t}^* - \hat{t}\right)}{\lambda_C\beta\bar{t}^*} = \frac{F^S\left(\hat{t}^*, 2\bar{t}^* - \hat{t}^*\right)}{\lambda_C\bar{t}^*} = p, \tag{E2}$$

$$\left(\frac{Y_S+Y_S^*}{Y_C+Y_C^*} =\right)\frac{2}{\lambda_C\bar{t}^*}\frac{\int_{t_{min}}^{\hat{t}} F^S\left(t, 2\beta\bar{t}^* - t\right)\phi(t)dt + \int_{t^*_{min}}^{\hat{t}^*} F^S\left(t, 2\bar{t}^* - t\right)\phi^*(t)dt}{\beta\int_{\hat{t}}^{2\beta\bar{t}^*-\hat{t}}\phi(t)dt + \int_{\hat{t}^*}^{2\bar{t}^*-\hat{t}^*}\phi^*(t)dt} = \frac{1-\mu}{\mu}p.$$

From (E2), we obtain $\hat{t} = \beta\hat{t}^*$. Let us define $\theta \equiv \hat{t}/\beta\bar{t}^* = \hat{t}^*/\beta\bar{t}^*$, where $\theta \in [t_{min}/\beta\bar{t}^*,\ 1)$. Thus, we summarize the equilibrium condition as

$$\frac{\mu}{1-\mu}\frac{2}{\bar{t}^*}\frac{\int_{t_{min}}^{\theta\beta\bar{t}^*} F^S\left(t, 2\beta\bar{t}^* - t\right)\phi(t)dt + \int_{t^*_{min}}^{\theta\bar{t}^*} F^S\left(t, 2\bar{t}^* - t\right)\phi^*(t)dt}{\beta\int_{\theta\beta\bar{t}^*}^{\beta\bar{t}^*(2-\theta)}\phi(t)dt + \int_{\theta\bar{t}^*}^{\bar{t}^*(2-\theta)}\phi^*(t)dt} = F^S\left(\theta, 2-\theta\right).$$

Next, we consider the case in which complete specialization occurs in one or both countries. Since $\hat{t} = \beta\hat{t}^*$, the case in which one country completely specializes in producing good S cannot occur as long as good C is consumed. It follows that only the home country can completely specialize in producing good C. Thus, the free trade equilibrium condition under complete specialization is

$$\frac{\mu}{1-\mu}\frac{2}{\bar{t}^*}\frac{\int_{t^*_{min}}^{\theta\bar{t}^*} F^S\left(t, 2\bar{t}^* - t\right)\phi^*(t)dt}{\beta + \int_{\theta\bar{t}^*}^{\bar{t}^*(2-\theta)}\phi^*(t)dt} = F^S\left(\theta, 2-\theta\right),$$

where $\theta \in [t^*_{min}/\bar{t}^*, t_{min}/\beta\bar{t}^*]$, $\hat{t}^* = \theta\bar{t}^*$, and $\hat{t} = t_{min}$.

By combining the incomplete and complete specialization cases, we summarize the free trade equilibrium as follows:

$$\mathcal{F}\left(\theta, \beta\right) = F^S\left(\theta, 2-\theta\right), \tag{E3}$$

where

$$\mathcal{F}(\theta,\beta)=\begin{cases}\frac{\mu}{1-\mu}\frac{2}{\bar{t}^*}\frac{\int_{t^*_{min}}^{\theta\bar{t}^*}F^S(t,2\bar{t}^*-t)\phi^*(t)dt}{\beta+\int_{\theta\bar{t}^*}^{\bar{t}^*(2-\theta)}\phi^*(t)dt}, & \theta\in[t^*_{min}/\bar{t}^*,\, t_{min}/\beta\bar{t}^*)\\ \frac{\mu}{1-\mu}\frac{2}{\bar{t}^*}\frac{\int_{t_{min}}^{\theta\beta\bar{t}^*}F^S(t,2\beta\bar{t}^*-t)\phi(t)dt+\int_{t^*_{min}}^{\theta\bar{t}^*}F^S(t,2\bar{t}^*-t)\phi^*(t)dt}{\beta\int_{\theta\beta\bar{t}^*}^{\beta\bar{t}^*(2-\theta)}\phi(t)dt+\int_{\theta\bar{t}^*}^{\bar{t}^*(2-\theta)}\phi^*(t)dt}, & \theta\in[t_{min}/\beta\bar{t}^*,\, 1)\end{cases} \tag{E4}$$

The left-hand side of (E3) is continuous and monotonically increases in θ over $\theta\in[t^*_{min}/\bar{t}^*,1)$. In addition, $\mathcal{F}(\theta,\beta)$ equals 0 at $\theta=t^*_{min}/\bar{t}^*$ and diverges to $+\infty$ when $\theta\to 1$. The right-hand side of (E3) is continuous and monotonically decreases in θ. $F^S(\theta,2-\theta)$ takes finite values when both $\theta_a=t_{min}/\beta\bar{t}^*$ and $\theta_a=1$. Therefore, the equilibrium under free trade uniquely exists.

Finally, the equilibrium under offshoring is determined by the following equations:

$$\frac{F^S\left(\hat{t}^W,2\bar{t}^W-\hat{t}^W\right)}{\lambda_C\bar{t}^W}=p,$$

$$\frac{2}{\lambda_C\bar{t}^W}\frac{\int_{t^*_{min}}^{\hat{t}^W}F^S\left(t,2\bar{t}^W-t\right)\phi^W(t)dt}{\int_{\hat{t}^W}^{2\bar{t}^W-\hat{t}^W}\phi^W(t)dt}=\frac{1-\mu}{\mu}p.$$

These conditions are essentially same as the autarky case except for the difference in the skill distribution function. Defining $\theta^W\equiv\hat{t}^W/\bar{t}^W$, we derive

$$\mathcal{I}\left(\theta^W,\beta\right)\equiv\frac{\mu}{1-\mu}\frac{2}{\bar{t}^W}\frac{\int_{t^*_{min}}^{\theta^W\bar{t}^W}F^S\left(t,2\bar{t}^W-t\right)\phi^W(t)dt}{\int_{\theta^W\bar{t}^W}^{\bar{t}^W(2-\theta^W)}\phi^W(t)dt}=F^S\left(\theta^W,2-\theta^W\right). \tag{E5}$$

Since (E5) is similar to (E1), we can demonstrate that a unique equilibrium exists by following the process of proving the existence and uniqueness of equilibrium under autarky.

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