Concerns for Long-Run Risks and Natural Resource Policy[1]

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Abstract

The legislature in many countries requires that short-run risk and long-run risk be considered in making natural resource policy. In this paper, we explore this issue by analyzing how natural resource conservation policy should optimally respond to long-run risks in a resource management framework where the social evaluator has recursive preferences. The response of resource conservation policy to long-run risks is reflected into a matrix whose coefficients measure precaution toward short-run risk, long-run risk and covariance risk. Attitudes toward the temporal resolution of risk underlies both precaution and the response of resource conservation policy toward long-run risks. We formally compare the responses of natural resource policy to long-run risks under recursive utility and under time-additive expected utility. A stronger preference for earlier resolution of uncertainty prompts a more conservative resource policy as a response to long-run risks. In the very particular case where the social evaluator preferences are represented by a standard expected utility, long-run risks are not factored in resource conservation policy decisions.

Key words: Natural Resource Policy, Long-Run Risk, Short-Run Risk, Recursive Utility, Nonindifference Toward Temporal Resolution of Uncertainty.

JEL Classification: Q2, D81, O44

1 Introduction

How does natural resource policy optimally respond to long-run risk considerations? How much resource conservation is the society willing to exert today in order to mitigate long-run risks? Future uncertainty is widely recognized as a key challenge for natural resource policy. The National Environmental Policy Act (NEPA,1969), for instance, requires that federal agencies consider both short-run and long-run effects and risks in implementing natural resource policy programs. The matter of precaution toward long-run risks is pertinent, with relevance to both a national and a global perspective (NAPA, 1997; English, 2000; Slimak and Dietz, 2006; Fischhoff, 1990; Viscusi, 1990; Pindyck, 2007, 2010; Hartzell-Nichols, 2012).

Concerns for long-run risks deserve attention in natural resource policy because decisions involve both uncertainty and long time horizons. In this paper we formally analyze and compare the responses of natural resource policy to long-run risks under recursive utility and under time-additive expected utility. We show that a stronger preference for earlier resolution of uncertainty prompts a more conservative resource policy as a response to long-run risks. In the very particular case where the social evaluator preferences are represented by a standard expected utility, long-run risks are not factored in resource conservation policy decisions.

In recent years, there has been a growing interest in the finance literature to account for concerns for long-run risks in exploring policy analysis in equity markets. For the most part, the main insight from this literature is that allowing for nonindifference toward the temporal resolution of long-run uncertainty can help illuminate some puzzling facts observed in financial market.[1] Concerns for long-run risks hinge crucially on nonindifference toward the temporal resolution of uncertainty.[2] However, to the best of our knowledge, the issue of concerns for long-run risks has not yet been analyzed from a natural resource policy approach. While there has been a substantial body of papers focussed on the issue of uncertainty in natural resource management,[3] so far a formal analysis of how natural resource policy should respond to long-run risks has not been explored. Therefore, this paper contributes to the literature by formally analyzing concerns for

[1]For instance, accounting for concerns for long-run risks has provided a reconciliation of the so-called equity premium puzzle with financial theory. [See for instance Epstein et al. (2014); Bansal (2007); Bansal and Yaron (2004); Sargent (2007); Brown and Kim (2014); Bansal et al. (2010); Bansal and Ochoa (2011); Strzalecki (2013) and CóRdoba and J. Ripoll (2016).] In the economics of longevity literature, CóRdoba and J. Ripoll (2016) analyze how attitudes towards the temporal resolution of uncertainty affect the value of statistical life (VSL) by calibrating a version of a discrete-time recursive utility framework.

[2]To illustrate the concept of nonindifference to the temporal resolution of uncertainty, let us consider the following three options: In the first option, a coin is flipped in each future date. If heads you get a high consumption payoff and if tails a low one. In the second option, a coin is flipped once. If heads you get a high consumption payoff in all future dates and if tails you get a low one in all future dates. In a third option all the coins are tossed at once in the first period, but the timing of the payoffs being the same as in the other two options. A decision maker may not be indifferent about the three options. A decision maker may prefer a late resolution of uncertainty or an earlier resolution of uncertainty as a result of his/her attitudes toward correlation of payoffs across periods, long-run uncertainty (Duffie and Epstein, 1992).

[3]See for instance Pindyck (1980); Epaulard and Pommeret (2003); Dasgupta and Heal (1974); Howitt et al. (2005); Knapp and Olson (1996); Young and Ryan (1996); Lewis (1977); Sundaresan (1984); Ackerman et al. (2013); Bansal and Ochoa (2011); Peltola and Knapp (2001); Kakeu and Bouaddi (2017); Pindyck (2007); Bansal et al. (2008)

long-run risks from a natural resource policy framework.

It is worth investigating alternatives resource policy analysis under frameworks that are broader than the time-additive traditional expected utility (Heal and Millner, 2014; Pindyck, 2007; Dasgupta, 2008). While insightful, the time-additive expected utility is overly restrictive in expressing sensitivity to future uncertainty (Skiadas, 2007; Pindyck, 2010; Dasgupta, 2008). The recursive utility framework provides greater flexibility for understanding plausible channels by which long-term uncertainty matters in current decision making (Kreps and Porteus, 1978; Skiadas, 2007; Hansen, 2010, 2012; Sargent, 2007; Duffie and Epstein, 1992). Recursive utility allows for nonindifference toward the temporal resolution of uncertainty (Epstein et al., 2014; Kreps and Porteus, 1978), an aspect of risk preferences that plays an important role in understanding attitudes toward long-run risks.

Our work is related to three strands in the economic literature. First, our model fits in with the literature that adress environmental and natural resource management problems under uncertainty using recursive utility approach (Ackerman et al., 2013; Kakeu and Bouaddi, 2017; Young and Ryan, 1996; Pindyck, 2010; Hambel et al., 2015; Pindyck, 2007; Bansal et al., 2010, 2008; Cai et al., 2017). For instance, empirical studies by Howitt et al. (2005); Kakeu and Bouaddi (2017) suggest that natural resource management under uncertainty is consistent with recursive utility. Numerical simulations done by Ackerman et al. (2013) using a discrete-time recursive utility into the DICE model show that optimal climate policy calls for rapid abatement of carbon emissions. Hambel et al. (2015) uses a model a climate model with recursive utility to show that postponing abatement action could reduce the probability that the climate can be stabilized. Using an empirical model with recursive utility, Bansal et al. (2008) find that preference for early resolution of uncertainty matter for investigating policies designed to mitigate climate change. While Lontzek and Narita (2011) uses numerical techniques to show that risk aversion plays a central role in environmental decisions made under uncertainty, they use the time-additive expected utility and do not adress how concerns for long-run risks affects policy making. The paper by Cai et al. (2017) provides powerful stochastic simulation approaches to compute stylized climate-economy model with recursive preferences.[4] Second, our framework adds to the stochastic growth-theoretic literature [see Olson and Roy (2006); Nyarko and Olson (1994); Rankin (1998); Femminis (2001) for an extensive review) with a Duffie and Epstein (1992) recursive utility and treating natural resource as an economic asset.[5] Third our paper can be related to the growing financial literature that use recursive utility in analyzing aversion to how long-run risk factors play out in asset pricing and business cycle models (Epstein et al., 2014; Bansal, 2007; Bansal and Yaron, 2004; Sargent, 2007; Brown and Kim, 2014; Bansal et al., 2010; Bansal and Ochoa, 2011; Strzalecki, 2013).

In this paper, we compare the implications of assuming a recursive utility versus time-additive expected utility in analyzing the optimal response of natural resource policy to short-run and long-

[4] Alternative numerical modeling approaches dealing with climate risk include Daniel et al. (2016) and Traeger (2014).

[5] Natural resources are increasing analyzed as natural assets that provide a flow of beneficial goods and services over time (Arrow et al., 2012; Fenichel and Abbott, 2014; Arrow et al., 2004; Daily et al., 2000; Dasgupta, 1990).

run risks.[6] We show that the optimal response of natural resource policy under uncertainty can be associated with a matrix of weights on short-run and long-run risks. The weighting matrix reflects prudence toward short-run and long-run risks. Resource policy under time-additive utility is insensitive toward long-run risks whereas a resource policy under recursive preferences incorporates long-run risks. A stronger preference for earlier resolution of uncertainty prompts a more stringent conservative resource policy as a response to long-run risks. In the very particular case where preferences are represented by a standard expected utility, long-run risks are not factored in resource conservation policy decisions.

The paper is organized as follows: In section 2, we lay out the basic theoretical framework and show that the optimal response of the resource policy to uncertainty depends upon a matrix that awards weights to short-run and long-run risks . In section 3, the weighting matrices corresponding to the Schroder and Skiadas (1999) recursive utility and the time-additive utility are presented. In section 4, we show that the optimal response of natural resource policy to short-run and long-run risks are associated with the sign of a weighting matrix. In section 5, we derive general conditions for comparing the responses of two resource policies to short-run and long-run risks . As example, we compare policies corresponding to the Schroder and Skiadas (1999) recursive utility and the time-additive utility. In the final section, we offer concluding comments. Some proofs are consigned to the Appendix.

2 Stochastic natural resource economy

2.1 Stochastic resource dynamics

Consider an economy built on natural resources, whose stock at time t is $S(t)$. The initial stock of natural resources is $S(0) = S_0 > 0$. For clarity of exposition, we assume that disposing of natural resources is costless. The natural resource stock at any time t follows the following dynamic:

$$dS(t) = [N(S(t)) - x(t)]dt + \sigma_S(t)dB(t), \tag{1}$$

where $N(S(t))$ is the stock-growth function of the natural resource at time t,[7] $x(t)$ is the depletion rate or the harvesting rate of natural resource at time t, and $\sigma_S(t)dB(t)$ represents exogenous stochastic shocks.[8] The term $\sigma_S(t)$ represents the conditional variance of these exogenous shocks at time t. The term $dB(t)$ is the increment of a wiener process, i.e $dB = \varepsilon(t)\sqrt{dt}$, with $\varepsilon(t) \sim N(0,1)$.

These stochastic shocks can arise from uncertainties surrounding the evolution of the natural natural geological or biological process, and might be positive, then contributing to increase the stock of natural resource, or might be negative, then aggravating the natural resource loss.

[6]To keep matters as simple as possible, we have not admit a role for technology. However allowing a role for technology will not change the main insights brought out by this paper.

[7]The function $N(.)$ is assumed to be differentiable, linear or concave.

[8]The volatility does not depend of the size natural resource stock and is assumed to be additive. See Pindyck (1980) for a similar assumption. This specification of the dynamics of uncertainty does not prevent other forms of uncertainty dynamics to be analyzed. The point here is to allow unexpected shocks on the evolution of the natural resource over time.

If the stock-growth function $N(S) = 0$, this corresponds to a depletable natural resource (See for instance Hotelling (1931)) while the case where $N(S) = AS$, with $A > 0$ corresponds to an exponential growth renewable capital (See Dawid and Kopel (1999); Merks et al. (2003); Gauteplass and Skonhoft (2014); Levhari and Mirman (1980) for a similar assumption). The function $N(S)$ can also be assumed to be a logistic growth function as in the economics of fisheries and marine ecosystems (Clark, 1976; Smith, 2014; Zhang and Smith, 2011; Stavins, 2011).

2.2 The Duffie and Epstein (1992) continuous-time recursive utility framework

Let us first present the continuous-time Duffie and Epstein (1992) recursive utility framework, namely stochastic differential utility, used to define dynamic preferences. With recursive preferences, the instantaneous utility, $f(x,V)$, depends not only on the current extraction but also on expectations about future extraction through the single variable index $V(t)$. The future utility index $V(t)$ is also called prospective utility by Koopmans (1960). The future utility index $V(t)$ is attributable to the distribution of the future extraction stream$\{x(\tau) : \tau > t\}$ given the current information available at time t. In other words, $V(t)$ depends upon expectations about future extraction prospects. The flexible forward looking structure of the recursive utility allows a wide range of attitude toward the entire future. Indeed, recursive utility pushes beyond time-additive utility by allowing a rich structure to implement asymmetric attitudes over time and states through an endogenous marginal rate of substitution of current for future utility. With recursive utility, there is a flexible tradeoff between current-period utility and the utility to be derived from all future periods. The current utility $f(x,V)$ is assumed to be continuous, increasing and concave in the extraction, and to satisfy some growth and Liptschitz condition.[9] Preferences over future extraction process profiles $\{x(\tau) : \tau > t\}$ are defined recursively by future utility as follows:

$$V(t) = E_t \left[\int_t^\infty f(x(\tau),V(\tau))d\tau \right], \tag{2}$$

where E_t denotes the conditional expectation given the information available at date t.[10] V must also satisfy a transversality condition of the form

$$\lim_{t\to\infty} e^{-\nu t} E\left(|V(t)|\right) = 0. \tag{3}$$

The functional specification and the properties of the aggregator $f(x(t),V(t))$ can be thought of as reflecting some dimensions of arbitrariness that involve but are not limited to ethical questions, principles of equity between current and future generations, political systems, ideologies, and institutions. The point to keep in mind is that different possible specifications of the aggregator can

[9] Allowing the aggregator f to satisfy certain continuity-Lipschitz-growth types conditions ensure the existence of the recursive utility (Duffie and Epstein, 1992, p.366).

[10] An alternative way to express the integral equation (2) is to use the following differential notation $dV(t) = -f(x(t),V(t))dt + \sigma_V(t)dB(t)$, with initial value V_0, and where $\sigma_V(t)$ is the volatility of the short-run and long-run well-being given the information available at time t.

be used to compare competing offer value judgments; that is a pluralist approach to intertemporal issues (Dasgupta, 2005, 12).

The time-additive expected utility can be derived as a very special case of the recursive utility framework. Note that if the aggregator is linear upon the future utility, $f(x,V) = u(c) - \beta V$, where β represents the pure rate of time preference, the solution to the recursive integral equation (2) is given by

$$V(t) = E_t \int_t^{\infty} e^{-\beta(s-t)} u(c(\tau)) d\tau,$$

which is the standard time-additive expected utility over the time interval (t, ∞).

2.3 Stochastic resource management under a Duffie and Epstein (1992) recursive utility

The problem of the social evaluator is to choose an extraction profile $\{x(t) : t \geq 0\}$, so as to maximize the future utility subject to the stochastic natural resource dynamics constraint.

Let us denote by $V(S(t))$ the maximized future utility at t achievable from a stock $S(t)$ of natural resource.

$$V(S_0) = \max_{\{x(t):t\geq 0\}} E\Big[\int_0^{\infty} (f(x(t),V(t))dt\Big], \tag{4}$$

subject to:

$$\begin{aligned} dS(t) \quad &= \quad \Big[N(S(t)) - x(t)\Big]dt + \sigma_S(t)dB(t), \\ &\quad\quad x(t) \geq 0, \\ &\quad\quad S(t) \geq 0,\ S(0) = S_0 > 0, \end{aligned} \tag{5}$$

where over a period $(t, t+dt)$, the evolution of the natural resource size, as shown in equation (5), is the resultant of a deterministic component $[N(S(t)) - x(t)]$ and a stochastic component $\sigma_S(t)dB(t)$ capturing random exogenous shocks on natural resource over time. The corresponding Hamilton-Jacobi-Bellman equation of the resource management problem described above is then given by:[11]

$$0 = \sup_x \Big[f(x(t),V((t)) + \frac{1}{dt}E_t dV(S(t))\Big], \tag{6}$$

where

$$\frac{1}{dt}E_t dV(S(t)) \quad = \quad V_S(S(t))\Big[N(S(t)) - x(t)\Big] + \frac{1}{2}V_{SS}(S(t))\sigma_S(t)^2, \tag{7}$$

with V_S denoting the derivative of V with respect to S and V_{SS} denoting the second derivative

[11] The Bellman's characterization of optimality with a continuous-time recursive utility is shown by Duffie and Epstein (1992, proposition 9). Some general theorems on the existence and the unicity of the solution to the Hamilton-Jacobi-Bellman equation require that the aggregator or both the drift coefficient and the diffusion coefficient of the state variable satisfy certain continuity-Lipschitz-growth types conditions. See for instance Duffie and Lions (1992); Schroder and Skiadas (1999).

with respect to S.[12]

The Bellman equation (6) tells us that the sum of the current utility, $f(x(t),V(S(t)))$, and the expected change in the future utility $\frac{1}{dt}E_t dV(S(t))$ is zero. This represents a no-arbitrage condition between the current utility and the expected change in the future utility, which captures the future welfare consequences in extracting the natural resource today. In other words, the future utility itself can be viewed as a stock variable that provides a flow of utility returns over time when the natural resource stock is being managed optimally across generations by the social evaluator.

From the Bellman equation (6), the first order condition with respect to the extraction rate is:[13]

$$f_x(x(t),V(t)) = V_S(t). \tag{9}$$

Equation (9) tells us that, in general, the shadow price of the natural resource $V_S(t)$ depends on both the current extraction rate and the future utility $V(t)$. Differentiating the maximized Hamilton-Jacobi-Bellman equation and using the first order equation along with envelope theorem, it can be shown [see Appendix] that the expected rate of change of the resource extraction is given by

$$\underbrace{\frac{1}{x(t)}\frac{1}{dt}E_t dx(t)}_{Expected\, pace\, of\, depletion\, of\, natural\, capital} = \underbrace{\mu_f(t)}_{Certainty-equivalent\, pace\, at\, time\, t} + \underbrace{\mathcal{R}_f(t)}_{Risk-driven\, pace\, premium\, at\, time\, t}, \tag{10}$$

where the second term on the right-hand side of Equation (10) decomposes into a short-run risk component, a long-run risk component, and a covariance risk component as follows:

$$\mathcal{R}_f(t) = \frac{1}{2}\left[\underbrace{\frac{-x(t)^2(t)f_{xxx}(t)}{x(t)f_{xx}(t)}}_{weight\, on\, short-run\, risk}\underbrace{\sigma_x^2(t)}_{Short-run\, risk} + \underbrace{-\frac{V^2(t)f_{xVV}(t)}{x(t)f_{xx}(t)}}_{weight\, on\, long-run\, risk}\underbrace{\sigma_V^2(t)}_{Long-run\, risk}\underbrace{-\frac{2x(t)V(t)f_{xxV}(t)}{x(t)f_{xx}(t)}}_{weight\, on\, covariance\, risk}\underbrace{\sigma_{xV}(t)}_{Covariance\, risk}\right]. \tag{11}$$

The weight $-\frac{x^2 f_{xxx}}{x()f_{xx}}$ is a measure of prudence associated with the short-run risk, σ_S. The weight $-\frac{V^2(t)f_{xVV}}{xf_{xx}}$ is a measure of prudence associated with long-run risk over future utility, σ_V. The weight $\frac{xVf_{xxV}}{xf_{xx}}$ is a measure of cross-prudence associated with the covariance risk, σ_{SV}, which captures stochastic links between short-run and long-run uncertainty. Therefore, with a recursive utility, it appears that the specification of the aggregator, $f(x,V)$, will have implications on how short-run and long-run risks are incorporated into natural resource conservation policy decisions

[12] For ease of notation, throughout we shortly use $V(t)$ to refer to $V(S(t))$, and $f(t)$ to refer to $f(x(t),V(t))$, unless otherwise stated.

[13] In addition, V must satisfy another a transversality, condition of the form

$$\lim_{t\to\infty} V_S(t)S(t) = 0. \tag{8}$$

under uncertainty.

The risk-driven component $\mathcal{R}_f$, can be rewritten in a compact form as the trace function of the product of the weighting symmetric $\mathcal{W}_f$ and the risk symmetric matrix Σ as follows:[14]

$$\mathcal{R}(t) = \frac{1}{2} trace\Big(\mathcal{W}(t)\Sigma(t)\Big). \tag{12}$$

The symmetric matrix

$$\mathcal{W}_f(t) = \begin{pmatrix} \underbrace{\frac{-x(t)^2(t) f_{xxx}(t)}{x(t) f_{xx}(t)}}_{Weight\, on\, short-run\, risk} & \underbrace{-\frac{x(t)V(t) f_{xxV}(t)}{x(t) f_{xx}(t)}}_{Weight\, on\, covariance\, risk} \\ \underbrace{-\frac{x(t)V(t) f_{xxV}(t)}{x(t) f_{xx}(t)}}_{Weight\, on\, covariance\, risk} & \underbrace{-\frac{V^2(t) f_{xVV}(t)}{x(t) f_{xx}(t)}}_{Weight\, on\, long-run\, risk} \end{pmatrix} \tag{13}$$

is a weighting matrix whose coefficients characterize the weights used by the social evaluator to factor short-run risk, long-run risks, and covariance risks in natural resource conservations decisions under uncertainty.

The positive symmetric semi-definite matrix

$$\Sigma(t) = \begin{pmatrix} \underbrace{\sigma_x^2(t)}_{Short-run\, risk} & \underbrace{\sigma_{xV}(t)}_{Covariance\, risk} \\ \underbrace{\sigma_{xV}(t)}_{Covariance\, risk} & \underbrace{\sigma_V^2(t)}_{Long-run\, risk} \end{pmatrix}$$

is composed of risks elements involved in natural resource conservations decisions.

Using the derivative of the *trace function* of the product of two matrices and noting that the short-run and long-run risk matrix $\sigma_S(t)$ is symmetric, it is easy to show that:[15]

$$\underbrace{\frac{\partial \left[\frac{1}{x(t)} \frac{1}{dt} E_t dx(t) - \mu_f(t)\right]}{\partial \Sigma(t)}}_{Sensitivity\, of\, optimal\, policy\, to\, all\, risks} = \frac{1}{2} \underbrace{\mathcal{W}_f(t)}_{Weighting\, matrix}. \tag{15}$$

[14]The function $trace(A)$ of a square matrix A is defined to be the sum of its diagonal elements.

[15]Indeed, the derivative of the *trace function* for the product of two matrices is given by:

$$\frac{\partial trace\Big(\mathcal{W}_f(t)\Sigma(t)\Big)}{\partial \Sigma(t)} = transpose(\mathcal{W}_f(t)) = \mathcal{W}_f(t). \tag{14}$$

The *transpose* of a matrix is a new matrix whose rows are the columns of the original (which makes its columns the rows of the original).

Indeed, equation (15) suggests that the sensitivity of the optimal choice to short-run and long-run risks at time t is characterized by the structure of the weighting matrix at time t.

The weighting matrix $\mathcal{W}_f$ is a generalization of the concept of relative prudence, first discussed by Kimball (1990) in the presence of a unidimensional risk, to a multidimensional setting involving a short-run risk, a long-run risk and a covariance risk.[16] The symmetric matrix $\mathcal{W}_f$ can be viewed as a local matrix-measure of multivariate prudence toward short-run risk, long-run risk, and covariance risk. The diagonal coefficient $-\frac{x^2 f_{xxx}}{x f_{xx}}$ measures the social evaluator's precautionary attitude per unit of short-run risk taken in isolation. This element is reminiscent of the concept of relative prudence index first developed by Kimball (1990) in the presence of a unidimensional risk. The other diagonal coefficient $-\frac{V^2 f_{xVV}}{x(t) f_{xx}}$ measures the social evaluator's precautionary attitude per unit of long-run risk taken in isolation. The off-diagonal element $-\frac{xV f_{xxV}}{x f_{xx}}$ is a measure of cross-prudence, i.e., the social evaluator's precautionary attitude toward covariance risk taken in isolation.

When there is no uncertainty, the risk term $\mathcal{R}_f(t)$ in equation (10) reduces to zero, and therefore the first term

$$\mu_f(t) = \left[\frac{-x f_{xx}(t)}{f_x(t)}\right]^{-1} \left[f_V(x(t),V(t)) - N'(S(t)) + \left[\frac{V(t) f_{xV}(t)}{f_x(t)}\right]\left(\frac{1}{V(t)}\frac{1}{dt}E_t dV(t)\right)\right] \quad (16)$$

can be thought of as the certainty-equivalent component of the expected pace of depletion.

3 Weighting of short-run and long-run risks in natural resource policy: Schroder and Skiadas (1999) recursive utility versus time-additive utility

3.1 Weighting matrix with a Schroder and Skiadas (1999) recursive utility

Let us consider the following Schroder and Skiadas (1999) parametric homothetic recursive utility:

$$f(x,V) = (1+\alpha)\left[\frac{x^\gamma}{\gamma} V^{\frac{\alpha}{1+\alpha}} - \beta V\right], \quad (17)$$

with parameters satisfying α, β, γ such that $\beta \geq 0, \alpha > -1, 0 < \gamma < \min(1, 1/(1+\alpha))$. The parameter β represents the pure rate of time preference and is assumed to be positive. The ratio $\frac{1}{1-\gamma}$ is the elasticity of intertemporal substitution, and α captures the dependency of current utility to future utility $J(t)$.[17] A negative α penalizes uncertainty about future utility, whereas a positive α rewards uncertainty about future utility. The parameter α can be viewed as a measure risk attitude toward uncertainty shocks to changes in future utility (long-run uncertainty).

[16] This notion of prudence was first defined by Kimball (1990) as the sensitivity of the optimal choice to risk. The coefficient of absolute prudence of Kimball (1990) is defined as the ratio between the third derivative and the second derivative of the current utility function, while the coefficient of relative prudence is defined as absolute prudence, multiplied by the extraction rate.

[17] When $\gamma = 0$, this aggregator becomes $f(x,V) = (1+\alpha V)\left[log(x) - \frac{\beta}{\alpha} log(1+\alpha V)\right]$.

Preference toward the temporal resolution of uncertainty is related to the idea that in situations where uncertainty does not resolve in one shot, agents may distinguish between future prospects based on their attitudes toward the temporal resolution of uncertainty (Kreps and Porteus, 1978; Skiadas, 1998; Schroder and Skiadas, 1999).[18] Preferences for early or late resolution of uncertainty can be related to the curvature of the aggregator (Kreps and Porteus, 1978). With the Schroder and Skiadas (1999) parametric recursive utility, the sign of the parameter α expresses the curvature of the aggregator with respect to the second argument, and therefore it captures attitudes toward the temporal resolution of uncertainty.[19] A value of α different from zero expresses nonindifference toward the temporal resolution of uncertainty. A negative-sign of the parameter α expresses a preference for early resolution of uncertainty whereas a positive sign of the parameter α expresses a preference for late resolution of uncertainty. There is a link between preference for earlier resolution of uncertainty and aversion to long-run risks.[20] A value of $\alpha = 0$ characterizes indifference to long-run risks. In the very particular case where $\alpha = 0$, which corresponds to the indifference toward the timing of resolution, the aggregator of the standard time-additive expected utility is obtained as $f(x,V) = \frac{x^\gamma}{\gamma} - \beta V$.

With the Schroder and Skiadas (1999) parametric recursive utility, the associated weighting matrix is computed as follows:

$$\mathcal{W}_f(t) = \begin{pmatrix} \underbrace{2-\gamma}_{weight\,on\,short-run\,risk} & \underbrace{-\frac{\alpha}{1+\alpha}}_{weight\,on\,covariance\,risk} \\ \underbrace{-\frac{\alpha}{1+\alpha}}_{weight\,on\,covariance\,risk} & \underbrace{\frac{-\alpha}{(1+\alpha)^2(1-\gamma)}}_{weight\,on\,long-run\,risk} \end{pmatrix}. \quad (18)$$

Expressed another way, the composition of short-run and long-run risks in shaping resource

[18] To illustrate the concept of temporal resolution of uncertainty, let us consider the following three options: In the first option, a coin is flipped in each future date. If heads you get a high consumption payoff and if tails a low one. In the second option, a coin is flipped once. If heads you get a high consumption payoff in all future dates and if tails you get a low one in all future dates . In a third option all the coins are tossed at once in the first period, but the timing of the payoffs being the same as in the other two options. A decision maker may not be indifferent about the three options. A decision maker may prefer a late resolution of uncertainty or an earlier resolution of uncertainty as a result of his/her attitudes toward correlation of payoffs across periods, long-run uncertainty (Duffie and Epstein, 1992).

[19] There is a connection between preferences for the timing of resolution of uncertainty and preferences for information (Skiadas, 1998).

[20] The concept of aversion to long-run risk is similar to the concept of correlation aversion of payoffs across time periods (Strzalecki, 2013; Duffie and Epstein, 1992). The idea of correlation aversion was first discussed by Richard (1975). Along the same lines, see Crainich et al. (2013).

conservation is given by:

$$\mathcal{R}_f(t) = \frac{1}{2}\left[\underbrace{2-\gamma}_{weight\,on\,short-run\,risk}\sigma_x^2(t) + \underbrace{\frac{-\alpha}{(1+\alpha)^2(1-\gamma)}}_{weight\,on\,long-run\,risk}\sigma_V^2(t) \quad \underbrace{-\frac{\alpha}{1+\alpha}}_{weight\,on\,covariance\,risk}\sigma_{xV}(t)\right]. \tag{19}$$

As shown in (18), the weighting matrix depends on the preference parameters. The parameter α plays an important role for understanding the optimal response of resource policy to long-run risks. A negative sign of the parameter α expresses preference for the early resolution of uncertainty and plays an important role for understanding the social evaluator's conservation attitudes in the face of long-run risk and covariance risk. A more negative α; that is decreasing α toward -1, implies that a larger weight in absolute value is given to the long-run risk and the covariance risk. This suggests that the stronger the social evaluator's preference for early resolution of uncertainty, the larger the weights attached to long-run and covariance risks. Increasing α to infinity $(+\infty)$ implies that a smaller weight in absolute value is given to the long-run risk. Decreasing γ toward 1 implies a smaller magnitude of the weight being put on the short-run risk while t it implies a larger magnitude of the weight being put on the long-run risk. With the Schroder and Skiadas (1999) parametric recursive utility, the pace of extraction (50) becomes

$$\mu_f(t) \quad = \quad [1-\gamma]^{-1}\left[(1+\alpha)\left[\frac{x^\gamma}{\gamma}V^{\frac{-1}{1+\alpha}} - \beta\right] - N'(S(t)) + \left[\frac{\alpha}{1+\alpha}\right]\left(\frac{1}{V}\frac{1}{dt}E_t dV\right)\right]. \tag{20}$$

This component does not depend on long-run nor short-run risks.

3.2 Weighting matrix with a time-additive utility

Let us consider the following linear aggregator, which is related to the time-additive utility:

$$g(\tilde{x}, \widetilde{V}) = \frac{\tilde{x}^\gamma}{\gamma} - \beta\widetilde{V},\ with\ \gamma < 1. \tag{21}$$

It is worth mentioning that this aggregator of the time-additive utility can be obtained as a very special case of the Schroder and Skiadas (1999) parametric when $\alpha = 0$.

With a time-additive utility, the time t corresponding weighting matrix is:

$$\mathcal{W}_g(t) = \begin{pmatrix} \underbrace{2-\tilde{\gamma}}_{Weight\,on\,short-run\,risk} & \underbrace{0}_{weight\,on\,covariance\,risk} \\ \underbrace{0}_{Weight\,on\,covariance\,risk} & \underbrace{0}_{Weight\,on\,long-run\,risk} \end{pmatrix}. \tag{22}$$

In other words,

$$\mathcal{R}_g(t) = \frac{1}{2} \underbrace{2-\tilde{\gamma}}_{Weight\,on\,short-run\,risk} \underbrace{\sigma_x^2(t)}_{Short-run\,risk} . \tag{23}$$

The weighting matrix corresponding to the linear aggregator $g(\tilde{x},\widetilde{V})$ is such that the weights $-\frac{\widetilde{V}^2 g_{\tilde{x}\widetilde{V}\widetilde{V}}}{\tilde{x} g_{\tilde{x}\tilde{x}}}$ and $-\frac{\widetilde{V}^2 g_{\tilde{x}\widetilde{V}\widetilde{V}}}{\tilde{x} g_{\tilde{x}\tilde{x}}}$ related to long-run risk and covariance risk reduce to zero. This shows that the social evaluator endowed with a time-additive utility does not care about long-run risks in natural resource policy decisions. Put another way, the social evaluator endowed with a time-additive utility completely discriminates against long-run risks and covariance risks.[21] The social evaluator cares only about the short-run risk and its intertemporal choices are not affected by the long-run and covariance risks. As shown in (30), the weight which characterizes the social evaluator's sensitity to short-run risk faced by current generations is $-\frac{\tilde{x}^2 g_{\tilde{x}\tilde{x}\tilde{x}}}{\tilde{x}(t) g_{\tilde{x}\tilde{x}}} = 2-\tilde{\gamma} > 0$. This coefficient is reminiscent of the concept of prudence that was defined by Kimball (1990) as the sensitivity of the optimal choice to risk in a unidimensional risk setting.

With a time-additive utility, the pace of extraction (50) at time t becomes

$$\mu_g(t) \;\; = \;\; [1-\gamma]^{-1} \left[-\beta - N'(S(t))\right]. \tag{24}$$

This component does not depend on short-run nor long-run risks.

4 The sign of the weighting matrix and the response of resource policy to short-run and long-run risks

Let us define the extraction pace premium as the difference in extraction pace between the stochastic natural resource economy and its equivalent deterministic counterpart. It is computed as follows:

$$\underbrace{\frac{1}{x(t)}\frac{1}{dt}E_t dx(t) - \mu_f(t)}_{Pace\,premium\,at\,time\,t} = \frac{1}{2} trace \left(\underbrace{\mathcal{W}_f(t)}_{Weights} . \underbrace{\Sigma(t)}_{Matrice\,of\,risks} \right). \tag{25}$$

The pace premium provides insights about links between the sign of the weighting matrix and local conservation attitudes at time t. Note that in a world without risk ($\Sigma = 0$) or for a risk-neutral social evaluator ($\mathcal{W}_f = 0$), the pace premium reduces to zero.

Definition 1 *At time t, a social evaluator endowed with a recursive utility aggregator $f(x,V)$ is said to be more conservative in the face of short-run and long-run risks than under certainty if and only if*

$$at\,time\,t,\; \frac{1}{x(t)}\frac{1}{dt}E_t dx(t) - \mu_f(t) \geq 0\, for\; any\; \Sigma(t). \tag{26}$$

[21] With a time-additive expected utility, there is a sense that long-run risks are irrelevant to the social evaluator. An intuitive connection can be made with the concept of risk independence defined on multiattributed utility functions by Fishburn (1965); Keeney (1973); Pollak (1973). Broadly this research agenda shows that risk independence implies that the utility function is additive.

The following proposition relates a social evaluator's conservations attitudes to the characteristics of the weighting matrix.

Proposition 4.1 *Let $f(x(t),V(t))$ and $\mathcal{W}_f(t)$ be the aggregator of a recursive utility and its corresponding weighting matrix at time t. The following conditions are equivalent:*

1. *The pace premium $\frac{1}{x(t)}\frac{1}{dt}E_t dx(t) - \mu_f(t) \geq (\leq)\, 0$ for any risk matrix $\Sigma(t)$.*
2. *The weighting matrix $\mathcal{W}_f(t)$ is respectively positive semi-definite (negative semi-definite).*

Proof. See Appendix B.
This proposition relates conservation attitudes of a social evaluator to the sign of the weighting matrix, which is a multivariate matrix-measure of prudence toward short-run and long-run risks. It tells us that when the weighting matrix is positive semi-definite, there is a negative precautionary pace premium required in order to respond to short-run and long-run risks. In such a case, the response of resource policy is more conservative in the face of short-run and long-run risks than in the absence of short-run and long-run risks. Put another way, the social evaluator is willing to accept a decrease in the natural resource size in order to conserve the same amount as in the absence of short-run and long-run risks. The reverse interpretation holds true if the weighting matrix is negative semi-definite.

In what follows we use some parametric aggregators to illustrate how a social evaluator's conservative attitudes relate to the elements of the weighting matrix, which captures the sensitivity of the social evaluator's optimal decisions to short-run and long-run risks (short-run risk, long-run risk, covariance risk).

4.1 Response of resource policy to short-run and long-run risks with a Schroder and Skiadas (1999)'s parametric recursive utility

With the Schroder and Skiadas (1999) parametric recursive utility, the determinant of the weighting matrix

$$\mathcal{W}_f(t) = \begin{pmatrix} 2-\gamma & -\frac{\alpha}{1+\alpha} \\ -\frac{\alpha}{1+\alpha} & \frac{-\alpha}{(1+\alpha)^2(1-\gamma)} \end{pmatrix}. \tag{27}$$

is given by

$$det(\mathcal{W}_f(t)) = \frac{\alpha(2-\gamma)}{(1+\alpha)^2(\gamma-1)} - \frac{\alpha^2}{(1+\alpha)^2} \tag{28}$$

The time-t matrix $\mathcal{W}_f(t)$ is positive semi-definite if and only if it has nonnegative principal minors, which leads to

$$\left.\begin{aligned} 2-\gamma \geq 0. \\ \alpha \leq 0. \end{aligned}\right\} \tag{29}$$

The restrictions on the parameters of the aggregator shown in equation (17) ensure that the condition $2-\gamma \geq$ is satisfied. Therefore, the conditions (29) are equivalent to saying that α is negative. So for a social evaluator, endowed with aggregator shown in equation (17), to be more conservative in the face of short-run and long-run risks than under certainty, the parameter α needs to be negative. It is worth mentioning that an alternative interpretation of a negative sign of the parameter α is that it expresses preference for early resolution of uncertainty, a concept discussed by Kreps and Porteus (1978); Skiadas (1998). This suggests that a social evaluator endowed with the aggregator shown in equation (17) and who exhibits a stronger preference for early resolution of uncertainty (a more negative $\alpha > -1$) will be more conservative in the face of short-run and long-run risks than under certainty. With a stronger preference for early resolution of uncertainty, the social evaluator is willing to pay a higher pace premium to resolve long-run risk and covariance risk, and therefore adopts a more stringent natural resource conservation policy.

4.2 Response of resource policy to short-run and long-run risks with a parametric time-additive expected utility

With a time-additive utility, the weighting matrix

$$\mathcal{W}_g(t) = \begin{pmatrix} 2-\tilde{\gamma} & 0 \\ & \\ 0 & 0 \end{pmatrix}. \tag{30}$$

is positive semi-definite, as the determinant is zero and the matrix $\mathcal{W}_g(t)$ has nonnegative principal minors. this implies that the pace premium is negative, meaning that the social evalutor is more conservative in the face of short-run and long-run risks than under the absence of short-run and long-run risks. Futhermore, as the structure of the weighting matrix $\mathcal{W}_f(t)$ shows, the social evaluator's conservation decisions are not affected by the long-run risk or the covariance risk, each being taken in isolation. This corresponds to a case where the social evaluator who is indifferent to the timing of the resolution of uncertainty of the future extraction path.

5 Comparison of two resource policy responses to short-run and long-run risks

In this section we will formally analyze and compare the responses of two social evaluators to short-run and long-run risks. Let us start with the following definition that relates the difference in two social evaluators conservations attitudes to the characteristics of the difference of their weighting matrices.

Definition 2 *A social evaluator endowed with a recursive utility aggregator $f(x,V)$ is said to be more conservative than a social evaluator with a recursive utility aggregator $g(\tilde{x},\tilde{V})$ when facing the same short-run and long-run risks if and only if*

$$at\,time\,t \quad \left[\frac{1}{x(t)}\frac{1}{dt}E_t dx(t) - \mu_f(t)\right] \geq \left[\frac{1}{\tilde{x}(t)}\frac{1}{dt}E_t d\tilde{x}(t) - \mu_g(t)\right] \geq 0 \tag{31}$$

The following result compares the pace premia of two social evaluators endowed with different aggregators.

Proposition 5.1 *Let $f(x,V)$ and $g(\tilde{x},\tilde{V})$ be the welfare metrics of two social evaluators when facing the short-run risk and long-run risk represented by the matrix Σ.*[22] *Let us denote by $\mathcal{W}_f(t)$ and $\mathcal{W}_g(t)$ the time-t associated weighting matrices with $f(x,V)$ and $g(\tilde{x},\tilde{V})$. Let us denote by $\mu_f(t)$ and $\mu_g(t)$ the certainty-equivalent paces corresponding associated with $f(x,V)$ and $g(\tilde{x},\tilde{V})$.*

The following conditions are equivalent:

1. $\left[\frac{1}{x(t)}\frac{1}{dt}E_t dx(t)-\mu_f(t)\right] \geq \left[\frac{1}{\tilde{x}(t)}\frac{1}{dt}E_t d\tilde{x}(t)-\mu_g(t)\right] \geq 0$ *for any risk matrix $\Sigma(t)$.*
2. *The matrices $\mathcal{W}_f(t)$, $\mathcal{W}_g(t)$, and $\mathcal{W}_f(t)-\mathcal{W}_g(t)$ are positive semi-definite.*

Proof. See Appendix C

This proposition tells us that the resource risk policy under f is more conservative than the resource risk policy under g if the sign of their weighting matrices as well as the differential of the two weighting matrices are positive semi-definite.

5.1 Comparison of two resource policy responses to short-run and long-run risks: Schroder and Skiadas (1999)'s recursive utility versus time-additive utility

Let us consider two social evaluators endowed respectively with the following aggregators

$$f(x,V)=(1+\alpha)\left[\frac{x^{\gamma}}{\gamma}V^{\frac{\alpha}{1+\alpha}}-\beta V\right],\ with\,\beta\geq 0,\ -1<\alpha<0,\ 0<\gamma<\min\left(1,\frac{1}{1+\alpha}\right). \tag{32}$$

and

$$g(\tilde{x},\tilde{V})=\left[\frac{x^{\tilde{\gamma}}}{\tilde{\gamma}}-\tilde{\beta}\tilde{V}\right],\ with\,\tilde{\beta}\geq 0,\ 0<\gamma<1 \tag{33}$$

The time-t weighting matrices associated to the Schroder and Skiadas (1999) parametric ho-

[22]More formally, it is assumed that at time t the positive semi-definite matrices $\Sigma(t)$ and $\tilde{\Sigma}(t)$ are equal. In other words, at time t the following equality is satisfied:

$$\begin{pmatrix} \sigma_S^2(t) & \sigma_{SV}(t) \\ \sigma_{SV}(t) & \sigma_V^2(t) \end{pmatrix} = \begin{pmatrix} \sigma_{\tilde{S}}^2(t) & \sigma_{\tilde{S}\tilde{V}}(t) \\ \sigma_{\tilde{S}\tilde{V}}(t) & \sigma_{\tilde{V}}^2(t) \end{pmatrix}.$$

mothetic recursive utility and the time-additive utility are respectively given by:

$$\mathcal{W}_f(t) = \begin{pmatrix} \underbrace{2-\gamma}_{weight\,on\,short-run\,risk} & \underbrace{-\frac{\alpha}{1+\alpha}}_{weight\,on\,covariance\,risk} \\ \underbrace{-\frac{\alpha}{1+\alpha}}_{weight\,on\,covariance\,risk} & \underbrace{\frac{-\alpha}{(1+\alpha)^2(1-\gamma)}}_{weight\,on\,long-run\,risk} \end{pmatrix}, \tag{34}$$

(35)

(36)

$$\mathcal{W}_g(t) = \begin{pmatrix} \underbrace{2-\gamma}_{weight\,on\,short-run\,risk} & \underbrace{0}_{weight\,on\,covariance\,risk} \\ \underbrace{0}_{weight\,on\,covariance\,risk} & \underbrace{0}_{weight\,on\,long-run\,risk} \end{pmatrix}.$$

The time-t weighting matrix $\mathcal{W}_g(t)$ is positive semi-definite. It is already shown in (29) that the time-t matrix $\mathcal{W}_f(t)$ is positive semi-definite if and only if $2-\gamma \geq 0$ and $\alpha \leq 0$, which are satisfied.

The difference of the two weighting matrices $\mathcal{W}_f(t) - \mathcal{W}_g(t)$ is positive semi definite if and only if it has nonnegative principal minors, or more formally,

$$\begin{cases} 2-\gamma \geq 2-\tilde{\gamma}, & \text{(37a)} \\ \dfrac{-\alpha}{(1+\alpha)^2(1-\gamma)}\left[(\tilde{\gamma}-\gamma)-\alpha(\gamma-1)\right] \geq 0 & \text{(37b)} \end{cases}$$

which implies

$$\begin{cases} \dfrac{1}{1-\gamma} \leq \dfrac{1}{1-\tilde{\gamma}}, & \text{(38a)} \\ \alpha = 0. & \text{(38b)} \end{cases}$$

or

$$\begin{cases} \dfrac{1}{1-\gamma} = \dfrac{1}{1-\tilde{\gamma}}, & \text{(39a)} \\ -1 < \alpha < 0. & \text{(39b)} \end{cases}$$

or

$$\begin{cases} \dfrac{1}{1-\gamma} < \dfrac{1}{1-\tilde{\gamma}}, & \text{(40a)} \\ \max\left(\dfrac{-(\tilde{\gamma}-\gamma)}{1-\gamma}, -1\right) < \alpha < 0. & \text{(40b)} \end{cases}$$

Both attitudes toward the resolution of uncertainty and attitudes toward consumption smoothing are important factors to take into account in comparing the response to two resource policies to short-run and long-run risks. When conditions (37a)-(37b) are satisfied, the social evaluator endowed with the aggregator f is more conservative than the social evaluator endowed with the aggregator g, in the face of short-run and long-run risks. Condition (37a) compares the elasticities of intertemporal subsitution while the second condition (37b) imposes a lower bound and a negative sign to the coefficient α, which expresses preference for early resolution of uncertainty.

Conditions (38a)-(38b) correspond to the limiting case where $\alpha = 0$; that is, where the two social evaluators are indifferent toward long-run risks. In this case, only the elasticities of intertemporal substitution should be used in comparing the response to the two resource policies to short-run risks. In a standard time-additive expected utility economy, long-run risks are not considered a pressing issue as they are awarded a zero weight.

Conditions (39a)-(39b) correspond to the case where the elasticities of intertemporal substitution are equal. In such a case only the parameter $\alpha \in [-1\,,0]$, which expresses preference for early resolution of uncertainty, should be used to compare the responses of the two resource policies to long-run risks. In such as case, the more negative is the parameter α, holding other parameters unchanged, the greater is the weight awarded to long-run risks, and the more conservative is the social evaluator endowed with the aggregator f relative to the social evaluator endowed with the aggregator g.

Conditions (40a)-(40b) correspond to the case where the elasticities of intertemporal substitution are different and the value of α is negative. In such a case the more conservative social evaluator is the one that has the lowest elasticity of intertemporal substitution and who cares about long-run risk or prefers an earlier resolution of uncertainty.

6 Concluding remarks

In this paper, we have examined the responses of the optimal resource policy to long-run risk, covariance risk, and short-run risk, under time-additive preferences and recursive preferences. A weighting matrix used for balancing short-run and long-run risks is derived. The weights used for balancing short-run and long-run risks in optimal conservation policy are heavily dependent on the intertemporal structure of short-run and long-run preferences. The weighting matrix can also be thought of as a measure of multivariate prudence. The sign of the weighting matrix, which represents a matrix measure of prudence, is a major determinant of conservation policy under the short-run and long-run multivariate risk profile faced by the social evaluator.

The sign of the difference between two weighting matrices plays a critical role in comparing two alternative conservation policies under uncertainty. Examples are given to illustrate how the weights given to short-run risk, long-run risk, and correlation risk relate to the parameters of the aggregator of the recursive utility. We have also found a formal link between the response of natural resource policy conservations to long-run risks and the preference for early resolution of uncertainty, a concept developed by Kreps and Porteus (1978); Skiadas (1998). The social evaluator with a preference for early resolution of uncertainty is more conservative in the face of uncertainty than a social evaluator who is indifferent toward the timing of resolution of uncertainty. The social evaluator with a preference for early resolution accounts for long-run risks and covariance risks by adopting a more stringent natural resource conservation policy.[23]

Finally, let us mention that while most of the debate over natural resource policy is framed in terms of discount rate, this paper suggests that differences in policy recommendations may also result from differences in attitudes toward short-run and long-run risk. The point is that attitudes toward long-run risks matter in formulating natural resource policy. Differences in attitudes toward long-run risks may to some degree be related to differences in social factors including culture (Slimak and Dietz, 2006; Fischhoff, 1990; Viscusi, 1990; English, 2000). For instance, there is suggestive evidence that the United States (US) and the European Union (EU) exhibit some differences when it comes to risk attitudes involved in the design of environmental and resource policies (Wiener and Rogers, 2002; Vogel, 2012). This paper clearly illustrates that even small differences in attitude toward long-run risk can lead to differences in policy recommendations.

[23]There are empirical studies that suggest that concerns for long run risks, represented by preferences for early resolution of uncertainty, call for a more stringent climate policy (Bansal and Ochoa, 2011; Bansal et al., 2008) .

Appendix

A The derivation of equation (10)

To derive equation (10), differentiate the maximized Hamilton-Jacobi-Bellman equation (6) with respect to S to obtain

$$f_x \frac{\partial x}{\partial S} + f_V V_S = -\frac{\partial}{\partial S}\Big(\frac{1}{dt} E_t dV(S(t))\Big). \tag{41}$$

Assuming that the optimal extraction policy can represented by a smooth function $x(t) = h(S(t))$ of the natural resource stock, the right-hand side of equation (41) can be computed from (7) as follows:

$$\begin{aligned} -\frac{\partial}{\partial S}\Big(\frac{1}{dt} E_t dV(S(t))\Big) &= -\frac{\partial}{\partial S}\Big(V_S\Big[N(S(t) - x(t)\Big] + \frac{1}{2} V_{SS}\sigma_S^2(t)\Big) \\ &= -\underbrace{\Big(V_{SS}\Big[N(S(t) - x(t)\Big] + \frac{1}{2} V_{SSS}\sigma_S^2(t)\Big)}_{\frac{1}{dt} E_t dV_S(S(t))} - V_S N'(S(t) + \frac{\partial h}{\partial S} V_S \\ &= -\frac{1}{dt} E_t dV_S(S(t)) - V_S N'(S(t) + \frac{\partial h}{\partial S} V_S \end{aligned} \tag{42}$$

Plugging (42) back into equation (41) leads to

$$f_V V_S = -\frac{1}{dt} E_t dV_S(S(t)) - V_S N'(S(t)) + \underbrace{[V_S - f_x]}_{0} \frac{\partial h}{\partial S}, \tag{43}$$

where the last term vanishes as it contains the first order condition (9).

From the equation (43), it follows that :

$$\frac{1}{dt} E_t dV_S(S(t)) = [-f_V - N'(S(t))] V_S(t) \tag{44}$$

or

$$\frac{1}{V_S(t)} \frac{1}{dt} E_t dV_S(t) = -f_V(x(t), V(t)) - N'(S(t)). \tag{45}$$

The term $-f_V(x(t), V(t))$ generalizes the notion of discount rate to the stochastic continuous-time recursive utility. [24]

[24] The shadow price of the marginal unit of natural resource stock $V_S(t)$ is the increase in well-being which would be enjoyed if a unit more of natural resource stock were made available costlessly (Drèze and Stern, 1990; Arrow et al., 2012; Dasgupta, 2010a,b). Equation (45) represents the shadow pricing equation of a natural resource under a general class of stochastic continuous-time recursive utility. In the very particular case where the aggregator takes the form $f(x,V) = U(x) - \beta V$ so that $-f_V(x,V) = \beta$, and there is no growth in natural resource, $N(S) = 0$, the basic Hotelling (1931) rule of exhaustible resource extraction is obtained. That is $\frac{1}{V_S(t)} \frac{1}{dt} E_t dV_S(t) = \beta$.

Using the first-order equation (9), we may replace V_S by $f_x(x,V)$, to obtain:

$$\frac{1}{f_x(x(t),V(t))}\frac{1}{dt}E_t df_x(x(t),V(t)) = -f_V - N'(S(t)). \tag{46}$$

Using the Multivariate Itô Lemma, the left-hand side of equation (46) can be computed as:

$$\begin{aligned}\frac{1}{f_x(x(t),V(t))}\frac{1}{dt}E_t df_x(x(t),V(t)) &= \left(\frac{x(t)f_{xx}(t)}{f_x(t)}\right)\left[\frac{1}{x(t)}\frac{1}{dt}E_t dx(t)\right] + V(t)\frac{f_{xV}(t)}{f_x(t)}\left[\frac{1}{V(t)}\frac{1}{dt}E_t dV(t)\right] \\ &+ \frac{1}{2}x^2(t)\frac{f_{xxx}(t)}{f_x(t)}\sigma_x^2(t) + \frac{1}{2}V^2(t)\frac{f_{xVV}(t)}{f_x(t)}\sigma_V^2(t) + x(t)V(t)\frac{f_{xxV}(t)}{f_x(t)}\sigma_x(t)\sigma_V(t).\end{aligned} \tag{47}$$

Plugging (47) back into equation (46) leads to

$$\begin{aligned}&\left(\frac{x(t)f_{xx}(t)}{f_x(t)}\right)\left[\frac{1}{x(t)}\frac{1}{dt}E_t dx(t)\right] + V(t)\frac{f_{xV}(t)}{f_x(t)}\left[\frac{1}{V(t)}\frac{1}{dt}E_t dV(t)\right] \\ +\ &\frac{1}{2}x^2(t)\frac{f_{xxx}(t)}{f_x(t)}\sigma_x^2(t) + \frac{1}{2}V^2(t)\frac{f_{xVV}(t)}{f_x(t)}\sigma_V^2(t) + x(t)V(t)\frac{f_{xxV}(t)}{f_x(t)}\sigma_x(t)\sigma_V(t) \\ =\ &f_V - N'(S(t)).\end{aligned} \tag{48}$$

It follows that

$$\underbrace{\frac{1}{x(t)}\frac{1}{dt}E_t dx(t)}_{Expected\ pace\ of\ depletion\ of\ natural\ capital} = \underbrace{\mu_f(t)}_{Certainty-equivalent\ pace\ at\ time\ t} + \underbrace{\mathcal{R}_f(t)}_{Risk-driven\ pace\ premium\ at\ time\ t}, \tag{49}$$

where

$$\mu_f(t) = \left[\frac{-x(t)f_{xx}(t)}{f_x(t)}\right]^{-1}\left[f_V(x(t),V(t)) - N'(S(t)) + \left[\frac{V(t)f_{xV}(t)}{f_x(t)}\right]\left(\frac{1}{V(t)}\frac{1}{dt}E_t dV(t)\right)\right], \tag{50}$$

and

$$\mathcal{R}_f(t) = \frac{1}{2}\left[\underbrace{-\frac{x(t)^2(t)f_{xxx}(t)}{x(t)f_{xx}(t)}}_{weight\ on\ short-run\ risk}\underbrace{\sigma_x^2(t)}_{Short-run\ risk}\underbrace{-\frac{V^2(t)f_{xVV}(t)}{x(t)f_{xx}(t)}}_{weight\ on\ long-run\ risk}\underbrace{\sigma_V^2(t)}_{Long-run\ risk}\underbrace{-\frac{2x(t)V(t)f_{xxV}(t)}{x(t)f_{xx}(t)}}_{weight\ on\ covariance\ risk}\underbrace{\sigma_{xV}(t)}_{Covariance\ risk}\right]. \tag{51}$$

B Derivation of Proposition 4.1

Proof. $(1) \rightrightarrows (2)$: Assume that at time t, $\frac{1}{x(t)}\frac{1}{dt}E_t dx(t) - \mu(t) \leq (\geq) 0$ for any positive semi-definite matrix $\sigma_S(t)$. Since $\frac{1}{x(t)}\frac{1}{dt}E_t dx(t) - \mu(t) \geq 0 = -\frac{1}{2} trace\Big(\mathcal{W}_f(t)\Sigma(t)\Big)$, it follows that

$$-\frac{1}{2} trace\Big(\mathcal{W}_f(t)\Sigma_f(t)\Big) \leq (\geq)\, 0\, for\, any\, \Sigma(t). \tag{52}$$

For any given column vector $\Psi(t)$ of size 2×1,[25] assume that the positive semi-definite matrix is of the form $\Sigma(t) = \Psi(t)\Psi'(t)$. Since $\mathcal{W}_f(t)\Psi(t)\Psi'(t) = \Psi'(t)\mathcal{W}_f(t)\Psi(t)$, then it follows from (52) that

$$-\frac{1}{2}\Psi'(t)\mathcal{W}_f(t)\Psi(t) \leq (\geq)\, 0 \;\; for\; any\, \Psi(t), \tag{53}$$

where the row vector $\Psi'(t)$, of size 1×2, is the transpose of the vector $\Psi(t)$. The inequalities shown in (53) are satisfied for any $\Psi(t)$ if and only if the time-t matrix $\mathcal{W}_f(t)$ is positive semi-definite (negative semi-definite).

$(2) \rightrightarrows (1)$: The converse is immediate.

C Derivation of Proposition 4.2

Proof. $(1) \rightrightarrows (2)$: Assume that $\Big[\frac{1}{x(t)}\frac{1}{dt}E_t dx(t) - \mu_f(t)\Big] - \Big[\frac{1}{\tilde{x}(t)}\frac{1}{dt}E_t d\tilde{x}(t) - \mu_g(t)\Big] \leq 0$ for any positive semi-definite matrix $\Sigma(t)$.

Since $\Big[\frac{1}{x(t)}\frac{1}{dt}E_t dx(t) - \mu_f(t)\Big] - \Big[\frac{1}{\tilde{x}(t)}\frac{1}{dt}E_t d\tilde{x}(t) - \mu_g(t)\Big] = -\frac{1}{2} trace\Big\{\big[\mathcal{W}_f(t) - \mathcal{W}_g(t)\big]\Sigma(t)\Big\}$, it follows that at time t,

$$-\frac{1}{2} trace\Big\{\big[\mathcal{W}_f(t) - \mathcal{W}_g(t)\big]\Sigma(t)\Big\} \leq 0\; for\, any\, \Sigma(t). \tag{54}$$

For any column vector $\Psi(t)$ of size 2×1, choosing $\Sigma(t)$ of the form $\Psi(t)\Psi'(t)$ and pursuing to (54) leads to

$$-\frac{1}{2}\Psi'(t)\big[\mathcal{W}_f(t) - \mathcal{W}_g(t)\big]\Psi(t) \leq 0\; for\; any\, \Psi(t), \tag{55}$$

where the row vector $\Psi'(t)$, of size 1×2, is the transpose of the vector $\Psi(t)$.

The inequality (55) holds for any vector $\Psi(t)$ if and only if the time-t symmetric matrix $\mathcal{W}_f(t) - \mathcal{W}_g(t)$ is positive semi-definite. From proposition 4.1, it is readily seen that $\mathcal{W}_f(t)$ and $\mathcal{W}_g(t)$ are positive semi-definite.

$(2) \rightrightarrows (1)$: The converse is immediate.

[25] In other words, $\Psi(t)$ is a vector with 2 rows and 1 column.

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