Why Are Returns to Private Business Wealth So Dispersed?*

Corina Boar† Denis Gorea‡ Virgiliu Midrigan§

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Abstract

We use micro data from Orbis on firm level balance sheets and income statements to document that accounting returns for privately held businesses are dispersed, persistent, and negatively correlated with firm equity. We also show that firms experience large, fat-tailed, and partly transitory changes in output that are not fully accompanied by changes in their capital stock and wage bill. This implies that capital and labor choices are risky, as fluctuations in output are accompanied by large changes in firm profits. We interpret this evidence using a model of entrepreneurial dynamics in which return heterogeneity can arise from both limited span of control, as well as from financial frictions which generate differences in financial returns to saving. The model matches the evidence on accounting returns and predicts that financial returns to saving are half as large and dispersed as accounting returns. Financial returns mostly reflect risk, as opposed to collateral constraints which play a negligible role due to firms' unwillingness to expand and take on more risk.

Keywords: inequality, entrepreneurship, rate of return.

JEL classifications: E2, E44, G32

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†New York University and NBER, corina.boar@nyu.edu.

‡Danmarks Nationalbank, dego@nationalbanken.dk.

§New York University and NBER, virgiliu.midrigan@nyu.edu.

1 Introduction

Our paper is motivated by the large wealth inequality observed in the United States and other developed economies, as well as the large concentration of wealth with owners of private businesses. According to the 2013 Survey of Consumer Finances, the 12% of households who own a private pass-through business account for 46% of wealth in the United States and are especially prevalent at the top of the wealth distribution. Recent theoretical work has convincingly argued that heterogeneity in *rates of return on savings* is an important determinant of wealth inequality (Benhabib et al., 2011, Benhabib et al., 2017). Indeed, several empirical studies document large and persistent differences in rates of return, especially returns to private business wealth (Fagereng et al., 2020, Bach et al., 2020, Smith et al., 2021), concluding that entrepreneurship is key to understanding wealth inequality.

Our paper asks: What accounts for the heterogeneity in returns to private business wealth? The empirical work mentioned above uses accounting measures of returns, computed as business income divided by the net worth of a business. Such measures may indeed reflect differences in rates of return on savings, that is, in *financial returns*.[1] These differences in financial returns would arise in the presence of financial frictions that preclude higher-return business owners from borrowing from lower-return ones and arbitraging away such differences. However, accounting measures of returns may also reflect *returns to a fixed factor*, such as managerial talent or market power.

Understanding how dispersed financial returns are has important implications for tax policy, and in particular the relative merits of wealth and capital income taxation. An important recent contribution by Guvenen et al. (2019) studies an economy in which financial frictions generate large differences in financial returns across private business owners and finds that replacing a capital income tax with a tax on wealth improves welfare. Intuitively, a capital income tax disproportionately falls on financially constrained owners of private businesses. Replacing it with a wealth tax thus allows the government to redistribute towards these agents, increasing efficiency and welfare. In contrast, Boar and Midrigan (2020) study an economy in which financial rates of return are much less dispersed and find the opposite result. In their economy, the government's concern for redistribution swamps efficiency considerations and leads it to prefer capital income as opposed to wealth taxes.

[1] As we explain in Section 2, we define accounting returns as average returns, that is the ratio of income to equity. In contrast, financial returns are marginal returns, that is the return to investing an additional dollar in firm equity.

Our paper uses micro data on firm balance sheets and income statements for a number of European countries from Orbis, for the period 1995-2018. We document that dispersion in accounting returns is large and persistent, and that accounting returns are negatively correlated with firm equity. We also show that firms in the data experience large and fat-tailed changes in output that are not fully accompanied by changes in their capital or wage bill. In this sense, capital and labor choices are risky because an unexpected drop in output leads to an increase in the capital-output ratio and the labor share, thus reducing the profit share. We interpret these facts through the lens of a model of entrepreneurship in which heterogeneity in returns can arise from both a fixed factor, which we model as limited span of control, as well as from financial frictions arising due to borrowing constraints and uninsurable investment and labor risk. The model predicts that financial returns are half as large and dispersed as accounting returns, suggesting an important role for financial frictions. Importantly, differences in financial rates of return mostly reflect uninsurable risk, as opposed to collateral constraints which play a negligible role due to firms' unwillingness to expand and take on more risk.

The model we study is a relatively standard model of entrepreneurial dynamics, in which firms face two sources of financial frictions. First, motivated by the evidence in Moskowitz and Vissing-Jørgensen (2002) that private business ownership is poorly diversified, we assume that each firm is entirely owned by a single entrepreneur who consumes the income generated by the business and can only insure the business risk by saving in a risk-free asset. Second, we assume a collateral constraint that limits firms' ability to borrow. Firms produce a homogeneous good with a decreasing returns to scale technology and differ in their productivity. Importantly, we assume that capital and labor are chosen prior to the firm observing its current productivity and that productivity shocks are drawn from a fat-tailed distribution.

These timing assumptions imply that firms choose labor and capital to equate their expected marginal products with the respective factor prices, using their owner's stochastic discount factor to weigh future states. The negative covariance between firm productivity and the stochastic discount factor implies that firms underproduce relative to an environment with perfect risk sharing.[2] Because firms choose to restrict their capital and labor choices, the expected marginal products of capital and labor are higher than their user costs, depressing the labor share and the capital-output ratio. We note that in our environment with imperfect risk sharing there is an important distinction between expected marginal products under the

[2]See Arellano et al. (2019), who explore a similar mechanism to study the role of increased firm volatility in explaining the large decline in output and labor during the Great Recession.

physical and the risk-neutral probability measures. Though the former are elevated, owing to the presence of risk, the latter are not, unless the collateral constraint binds. As the firm grows wealthier, the owner's consumption becomes insulated from the fortunes of its business, leading to more risk-taking. Risk therefore generates a positive financial return on equity.

We calibrate the model to match salient features of the micro data from Spain, a country for which the Orbis dataset has particularly good coverage. Matching the evidence requires that firms face large, both transitory and persistent, and fat-tailed shocks to their productivity. Such shocks lead to large changes in the firms' profit share, consistent with the evidence, and imply an important role for risk, even though business owners have only a moderate degree of risk aversion. The model matches well the dispersion and persistence in accounting rates of return in the data, which range from 0.05 at the median to 0.35 at the 95^{th} percentile. As in the data, accounting rates of return are negatively correlated with firm equity: the slope of a rank-rank regression of accounting returns against equity is -0.26.

We use the model to calculate how dispersed financial returns are and to understand the sources of their dispersion and their macroeconomic consequences. We find that expected financial returns are dispersed and range from 0.02 at the 10^{th} percentile to 0.17 at the 95^{th} percentile. To illustrate the implications of these financial returns, we note that the average business would be worth more than twice as much if one were to discount its cash flows at the risk-free rate of 2% as opposed to the owner's stochastic discount factor, and 17 times as much if the owner could fully insure risk. Crucially, this dispersion in expected financial returns is largely driven by risk, as the dispersion in expected risk-neutral financial returns is substantially smaller and is equal to the risk-free interest rate even at the 90^{th} percentile. Eliminating financial frictions would increase aggregate productivity by 5.7% and, assuming that labor is in fixed supply, aggregate output by 8.4% and the equilibrium wage by 15.5%. The reason the wage is substantially depressed compared to output is that financial frictions in our environment greatly depress the labor share owing to the risk inherent in the labor choice.

We conduct a number of experiments to clarify what drives our findings. We show that even though economies in which the labor choice is flexible can match the distribution of accounting returns in the data, they imply that financial returns are much less dispersed and, in contrast to our baseline model, are mostly driven by collateral constraints. Similarly, in the absence of fat-tailed or transitory shocks, the model can match the distribution of accounting returns, but attributes a much smaller role to risk. Interestingly, we find that even a model that abstracts from financial frictions altogether can reproduce the distribution of accounting returns. Intuitively, absent financial frictions the model attributes a much larger role to the limited span of control and heterogeneity in productivity in explaining accounting rates of return. We show, however, that such a model is at odds with the data in that it predicts a much stronger negative relationship between accounting returns and equity: the slope of a rank-rank regression falls from -0.26 in our baseline model and in the data to -0.78. The broader lesson that emerges is that the evidence that accounting rates of return are dispersed, on its own, is not necessarily evidence that firms are financially constrained and face dispersed rates of return on savings.

Related Work. Our paper is related to the work of Smith et al. (2018), who also ask whether pass-through business profits accrue to human capital in addition to financial capital. Though we use a different methodology, we corroborate these authors' findings that business income partly reflects non-financial factors. Our paper is also related to Karabarbounis and Neiman (2019) who attempt to decompose business income into profits (Π), unmeasured investments in intangible or organizational capital (K), or financial frictions (R). Using their language, we seek to understand the importance of R relative to the two other factors, which we refer to collectively as the fixed factor. Though we focus on heterogeneity in returns across private businesses, and they focus on the aggregate time series, these researchers also find an important role for R in explaining recent macroeconomic trends.

Our findings have implications for the design of optimal tax policy. In addition to the work of Guvenen et al. (2019) and Boar and Midrigan (2020) mentioned above, our paper is therefore also related to several other studies by Brüggemann (2019), Itskhoki and Moll (2019), Boar and Knowles (2020), Gaillard and Wangner (2021), who analyze optimal tax policy in an environment with financially constrained businesses.

Finally, our paper studies the macroeconomic consequences of financial frictions and is therefore related to the work of Buera et al. (2011), Midrigan and Xu (2014), Moll (2014), and Gopinath et al. (2017) who study the role of collateral constraints. In our framework, risk as opposed to collateral constraints play a more important role, thus relating our paper to Tan (2018) and Robinson (2021), who study the role of risk in distorting firm investment. In contrast to these papers, we emphasize the role of risky labor choices, as in the work of Arellano et al. (2019).

The remainder of the paper proceeds as follows. Section 2 presents a simple example to clarify the distinction between accounting and financial returns. Section 3 discusses the data and the facts that motivate our quantitative analysis. Section 4 presents the model and the parameterization. Section 5 discusses the results. Section 6 studies a number of extensions and robustness checks of the model. Finally, Section 7 concludes.

2 Accounting vs. Financial Returns

We begin with a simple example to clarify the distinction between accounting and financial rates of return, and to motivate our empirical and quantitative analysis. Assume a production technology that uses labor l and capital k to produce output y according to

$$y = f\left(z, k, l\right),$$

where z denotes the firm's productivity. Suppose that the firm can freely save and borrow at an interest rate r, faces a constant depreciation rate δ and hires labor at a competitive wage rate W. Letting b denote the amount the firm borrows and $a = k - b$ denote the firm's equity,[3] accounting income is given by output net of labor costs, capital depreciation and interest expenses

$$\pi = y - Wl - \delta k - rb, \tag{1}$$

or, equivalently,

$$\pi = ra + y - Rk - Wl,$$

where $R = r + \delta$ denotes the user cost of capital. Let us first calculate the *accounting* returns if the firm is unconstrained in its choice of capital and labor. The first order conditions for capital and labor are

$$f_k = R \qquad \text{and} \qquad f_l = W.$$

Equivalently, letting $\alpha_k = f_k k/y$ and $\alpha_l = f_l l/y$ denote the output elasticites with respect to capital and labor, it follows that capital and labor are paid

$$Rk = \alpha_k y \qquad \text{and} \qquad Wl = \alpha_l y.$$

The firm's accounting returns are therefore

$$\frac{\pi}{a} = r + \left(1 - \alpha_k - \alpha_l\right) \frac{y(z)}{a}.$$

[3]Throughout the paper we use the terms equity, wealth and net worth interchangeably.

Since output is increasing in productivity, more productive firms earn higher returns, holding wealth constant.

Consider next the firm's *financial* return, which we define as the change in firm income resulting from an incremental change in wealth. Differentiating (1) with respect to wealth gives

$$\frac{\partial \pi}{\partial a} = r + [f_k - R]\frac{\partial k}{\partial a} + [f_l - W]\frac{\partial l}{\partial a}.$$

If the firm is financially unconstrained, that is $\partial k/\partial a$, $\partial l/\partial a = 0$ and additional wealth does not change the firm's production choices, the financial return is equal to r and is less than the accounting return.

Consider next the case of a firm that is constrained. Let us assume for simplicity that the firm faces a collateral constraint $k \leq \lambda a$ or, equivalently, $b \leq \left(1 - \frac{1}{\lambda}\right) k$, where $\lambda \geq 1$ is the maximum leverage ratio. If the constraint binds, the marginal product of capital f_k or, equivalently, $\alpha_k y/k$ is above the user cost R, so the financial return exceeds the interest rate r and is equal to

$$\frac{\partial \pi}{\partial a} = r + \left[\alpha_k \frac{y}{k} - R\right]\lambda.$$

In contrast, the accounting return is

$$\frac{\pi}{a} = r + \left[(1 - \alpha_l)\frac{y}{k} - R\right]\lambda,$$

where we use that $k = \lambda a$ for a constrained firm. We conclude that unless the firm operates a constant returns to scale technology as in Angeletos (2007) and Moll (2014), that is, unless $\alpha_k + \alpha_l = 1$, accounting returns overstate financial returns. Intuitively, if $\alpha_k + \alpha_l < 1$ part of the firm's income accrues to a fixed factor (e.g. managerial talent as in Lucas, 1978 or market power as in Melitz, 2003), so accounting or average returns capture both financial returns as well as the returns to that fixed factor.

Since financial returns for private businesses are not readily observable in the data, in the remaining sections we use micro data and a richer quantitative model to measure how dispersed they are.

3 Data and Motivating Facts

In this section we describe the dataset we use and document several facts that motivate our modeling choices and inform our quantitative analysis. Specifically, we document that differences in accounting returns for private businesses are large and persistent, and that returns are negatively correlated with wealth. We also document that firms experience large and fat-tailed changes in output that are not fully accompanied by changes in their capital or wage bill. In this sense, capital and labor choices are risky because an unexpected drop in output leads to a large increase in the capital-output ratio and labor share, reducing the profit share.

3.1 Data

The dataset we use is the historical product of Orbis, compiled by the Moody's Bureau van Dijk. The data covers the period 1995-2018 and is compiled from national registers and other sources, and has harmonized information on annual balance sheets and income statements of privately and publicly traded firms (see Gopinath et al., 2017 and Kalemli-Ozcan et al., 2015 for a more detailed description of the data). We focus our empirical analysis on Spain, a country with excellent coverage of firms across the entire size distribution, as shown by Gopinath et al. (2017). However, all of our results hold for other countries, as we show in the Appendix.

Given our interest in private business firms, we restrict attention to partnerships and private limited companies. We also exclude firms that operate in Finance, Insurance and Real Estate, Public Administration and Defense. To minimize the concern that variables are measured with error, we exclude observations in the top and bottom 0.1% of the distribution of growth rates of value added, capital and wage bill, as well as the distribution of returns to wealth, capital-to-wealth ratio, capital-output ratio, the labor share, the profit share and wealth-to-value added ratio. Since our goal is to document the persistence of returns to wealth, we restrict the sample to firms for which we observe at least 10 years of data. Our final sample consists of 228,394 firms for which we observe, on average, 15.5 years of data. These firms represent 25% of all firms and account for 72% of all value added and 74% of all wealth. As we show in the Appendix, none of our substantive findings change if we calculate statistics based on the entire sample of firms.

We next define the variables we use in the analysis. Our measure of output y_{it} is value added, which we compute as the difference between revenues and all non-labor costs, including taxes. Our measure of labor l_{it} is the firm's wage bill, including benefits. The capital stock k_{it} is the book value of property, plant, equipment and intangibles. We calculate equity (wealth) a_{it} as the difference between the firm's total assets and total liabilities. Finally, we define income π_{it} as output net of labor, depreciation and interest expenses. All variables are inflation adjusted. Given these definitions, we have the following identity which relates the change in the firm's equity (retained earnings) to its income net of dividends c_{it}

$$a_{it+1} - a_{it} = \pi_{it} - c_{it}.$$

Alternatively, dividing this budget constraint by a_{it} allows us to express the growth rate of a firm's equity to the difference between its accounting return and the dividend to equity ratio

$$\frac{a_{it+1} - a_{it}}{a_{it}} = \frac{\pi_{it}}{a_{it}} - \frac{c_{it}}{a_{it}}.$$

3.2 Facts

Dispersion and Persistence in Accounting Returns. We start by corroborating the findings of Fagereng et al. (2020) and Bach et al. (2020) about the dispersion and persistence of accounting returns to private business wealth, which we refer to for brevity as *returns*. Differently from these papers, our measure of returns is net of taxes. The first row of Table 1 reports moments of the cross-sectional distribution of returns. To compute these, we restrict the sample to firms with positive equity, which represent 92.5% of the firms in the baseline sample and weight all observations by the firm's equity. The mean rate of return is 0.08 and the distribution features substantial dispersion, with the 10^{th} percentile equal to -0.03 and the 90^{th} equal to 0.24.[4]

To document persistence in these returns, we calculate for each firm the time series (equity weighted) average of its return π_{it}/a_{it} and denote it by $\overline{\pi_i/a_i}$. The second row of Table 1 reports the distribution of these time series averages across firms. They range from 0 at the 10^{th} percentile to 0.17 at the 90^{th} percentile. Since we observe firms for an average of 15 years, these numbers suggest that some firms earn extremely high returns over a long period of time.

Returns and Equity Are Negatively Correlated. We next document that rates of return are negatively correlated with equity, suggesting that firms face decreasing returns to scale.[5] Figure 1 presents a binscatter plot of rates of return against the firm's rank in the equity distribution. That is, we calculate for firms in each bin of the equity distribution the average rate of return. The figure shows a clear negative relationship between the two.

[4] The distribution of pre-tax returns is even more dispersed, with the 10^{th} percentile equal to -0.04 and the 90^{th} equal to 0.33.

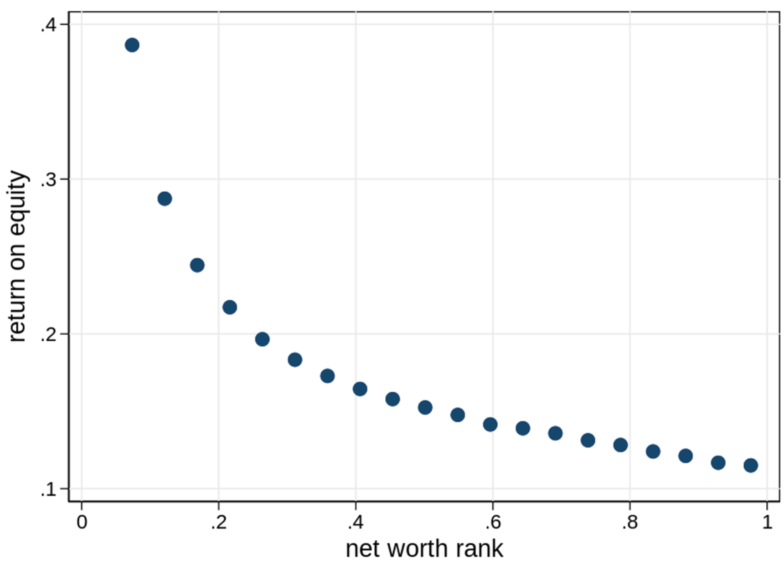
[5] Absent decreasing returns, this correlation would be positive.

Table 1: Accounting Rates of Return

	mean	p10	p25	p50	p75	p90	p95
π/a	0.08	-0.03	0.01	0.06	0.13	0.24	0.34
$\overline{\pi/a}$	0.08	0.00	0.03	0.07	0.11	0.17	0.22

Notes: All statistics are equity weighted.

Figure 1: Rates of Return and Equity



Notes: For visual clarity we exclude the bottom 5% of the net worth rank, which have an average return on equity of 1.2.

Firms at the bottom of the equity distribution have rates of return that average 0.4, while those at the top have average returns of 0.1. This fact may appear to contradict the findings of Fagereng et al. (2020) and Bach et al. (2020) who show that wealthier households earn higher returns. There is, in fact, no contradiction. The positive correlation between rates of return and wealth documented by these authors reflects heterogeneity in portfolios, whereby the wealthy hold larger shares of high return assets, such as private equity or risky financial assets. In contrast, our finding pertains to a specific asset class: private business wealth.

Table 2: Distribution of Output Growth Rates

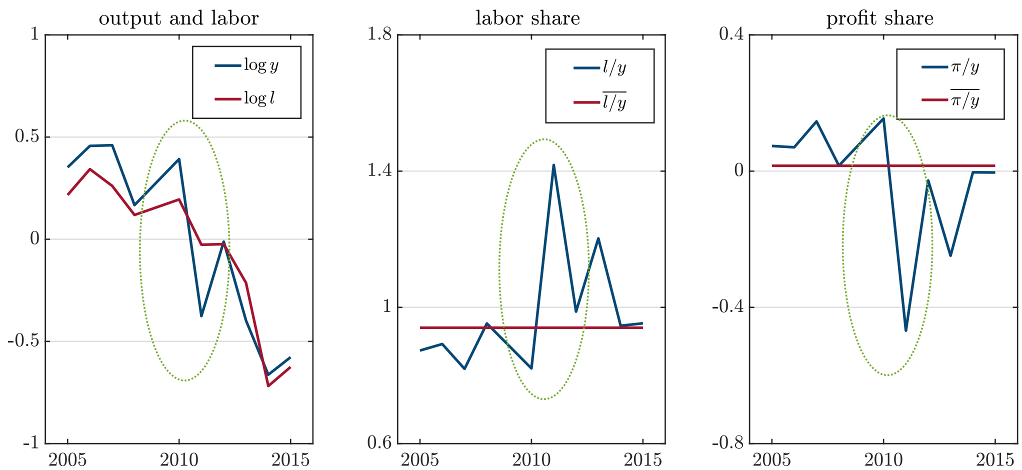
	s.d.	p0.1	p0.5	p25	p75	p99.5	p99.9
Data	0.41	-2.78	-1.78	-0.12	0.15	1.68	2.68
Gaussian	0.41	-1.27	-1.06	-0.28	0.28	1.06	1.27

Output Growth Rates Are Dispersed and Fat-Tailed. We next document that firms experience large and fat-tailed changes in output. To see this, Table 2 reports percentiles of the distribution of the growth rate of output $\log y_{it}/y_{it-1}$. For comparison, we also report in the second row of the table percentiles of a Gaussian distribution with the same variance. Relative to the Gaussian distribution, the distribution of output growth rates in the data features much heavier tails. For example, at the $1/10^{th}$ percentile the output growth rate is -2.78 in the data and -1.27 under a Gaussian distribution. Similarly, at the 99.9^{th} percentile the output growth rate is 2.68 in the data and 1.27 under a Gaussian distribution. Thus, even though most firms experience relatively small changes in output, as evidenced by the small interquartile range, a few firms experience extremely large changes in output. Even though we truncated the top and bottom 0.1% of output growth rates, the distribution has considerable excess kurtosis of 13.4.

Capital and Labor Choices Are Risky. We next document that capital and labor choices are risky in that they appear to be made in advance of the realization of the output growth rate. That is, when output falls, labor and capital change little, leading to a large increase in the firm's labor share and capital-ouput ratio, thus reducing the firm's profit share. We first illustrate this point by means of an example of a firm and then document the pattern more systematically.

The left panel of Figure 2 plots the logarithm of output and labor for a firm in our data. We normalize units so that the logarithm of output centers around zero. Notice that this firm experiences a large decline in output from 2005 to 2015 and that labor tracks output fairly closely at low frequencies. Zooming in on higher frequencies, the firm experiences a sharp decline in output between 2010 and 2011 that is accompanied by a more modest decline in the wage bill. In fact, in 2011 the firm's wage bill substantially exceeds its output, resulting

Figure 2: Example of a Firm



in a labor share that exceeds unity (middle panel) and a profit share that falls below zero (right panel).

In Table 3 we document this pattern more systematically. We regress the growth rate of labor and capital against the growth rate of output and report the slope coefficients from these regressions. The first row of the table shows that the elasticity of the wage bill with respect to output is 0.37 and that of capital is 0.15. That is, a 10% decline in output is associated with only a 3.7% in the firm's wage bill and a 1.5% decline in its capital stock. Though the pattern for capital is not surprising, given the evidence that investment is subject to large capital adjustment costs (Cooper and Haltiwanger, 2006), the pattern for labor is less well documented. To address the concern that measurement error in output reduces these elasticities, the second row of the table reports the results from a regression where we only include observations for which the growth rate of output is less than 50% in absolute value. Though the elasticities are somewhat larger, once again, we find an incomplete pass-through of output changes to labor and capital.

The observation that capital and labor react imperfectly to changes in output implies that a decline in output leads to a sharp increase in the labor share and capital-output ratio, thus decreasing the profit share. To see how large are the fluctuations in the labor and profit shares, we calculate for each firm the deviation of its labor and profit share from their time series means. Table 4 shows that these deviations are very dispersed. At the top the labor share substantially exceeds its time series mean, by as much as 1.81 at the 99^{th} percentile. Similarly, the profit share can fall by as much as 2.43 below its time series average for firms

Table 3: Comovement Between Capital, Labor and Output

	$\Delta \log l$	$\Delta \log k$
$\Delta \log y$	0.372	0.152
	(0.001)	(0.001)
$\Delta \log y$ (restricted)	0.561	0.303
	(0.001)	(0.002)

Notes: The second row restricts the sample to observations for which $|\Delta \log y| < 0.5$. Standard errors are clustered at the firm level.

Table 4: Deviation of Labor and Profit Shares from Time Series Mean

	p01	p10	p25	p50	p75	p90	p99
labor share	-0.41	-0.15	-0.07	-0.01	0.05	0.13	1.81
profit share	-2.43	-0.16	-0.05	0.01	0.08	0.18	0.48

Notes: All statistics are output weighted.

at the 1^{st} percentile of the distribution. The correlation between the deviations of the labor share and of the profit share from their respective means is equal to -0.91, suggesting that fluctuations in the profit shares are largely driven by changes in the labor share arising from the imperfect pass-through of output changes.

To summarize, we documented that there are large and persistent differences in accounting returns to business wealth, and that these returns are negatively correlated with firm equity. Additionally, firms experience large and fat-tailed changes in output that are only partially accompanied by changes in the wage bill and capital. In this sense, capital and labor choices are risky because a the decline in output increases the firm's labor share and capital-output ratio and reduces the profit share.

4 Model

Motivated by these observations, we next study a relatively standard model of entrepreneurial dynamics which we use to quantify the sources of dispersion in accounting rates of return. We assume that firms face two sources of financial frictions. First, motivated by the evidence in Moskowitz and Vissing-Jørgensen (2002) that entrepreneurial investment is poorly diversified, we follow Quadrini (2000), Cagetti and De Nardi (2006) and Buera et al. (2011) in assuming that each firm is entirely owned by a single entrepreneur who consumes the income generated by the business and can only insure the business risk by saving in a risk-free asset. Second, we assume a collateral constraint that limits the firm's ability to borrow. Additionally, following Lucas (1978), firms produce a homogeneous good with a decreasing returns to scale technology and differ in their productivity.[6] Thus, as in our earlier example, entrepreneurial income, and thus accounting returns, are determined both by a fixed factor (managerial ability) as well as financial constraints. We make two additional assumptions that are motivated by our empirical analysis. First, motivated by the evidence that labor and capital choices are risky, we assume that capital and labor are chosen one period in advance, prior to the firm observing its current productivity. Second, motivated by the evidence that changes in output are large and fat-tailed, we assume that productivity shocks are drawn from a fat-tailed distribution.

4.1 Environment

We assume a small open economy populated by a unit mass of entrepreneurs who can save and borrow at a constant interest rate r. There is no aggregate uncertainty. We assume a fixed labor supply that is remunerated at an equilibrium wage rate W. An entrepreneur's lifetime utility is

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\frac{c_t^{1-\theta}}{1-\theta},$$

where c_t denotes consumption in period t, β is the discount factor and θ is the relative risk aversion. The budget constraint is

$$c_t + k_{t+1} - b_{t+1} = y_t - Wl_t + (1-\delta)\,k_t - (1+r)\,b_t,$$

where k_t and l_t are the amounts of capital and labor used to produce output y_t and b_t is the amount the firm borrows. We assume that capital depreciates at rate δ. As in the earlier

[6]The firm's problem in this environment is equivalent to the problem faced by a monopolistically competitive firm that sells a differentiated variety and faces a constant demand elasticity.

example, letting $a_t = k_t - b_t$ denote the entrepreneur's wealth, the budget constraint can be rewritten as

$$a_{t+1} - a_t = \pi_t - c_t,$$

where

$$\pi_t = y_t - Wl_t - Rk_t + ra_t$$

is the entrepreneur's income, defined as in the data.

We assume that the firm is subject to a borrowing constraint

$$b_{t+1} \leq \xi k_{t+1}$$

that restricts the amount the firm can borrow to a fraction ξ of its capital stock. Equivalently, this constraint can be written as

$$k_{t+1} \leq \frac{1}{1-\xi} a_{t+1},$$

so that the firm's capital can be at most a multiple $1/(1-\xi)$ of its equity.

We assume a Cobb-Douglas production function

$$y_t = z_t \varepsilon_t \left(k_t^{\alpha} l_t^{1-\alpha}\right)^{\eta},$$

where $\eta < 1$ is the span of control parameter, α determines the elasticity of output to capital, z_t is the persistent productivity component and ε_t is an iid shock to firm productivity. The persistent component evolves according to

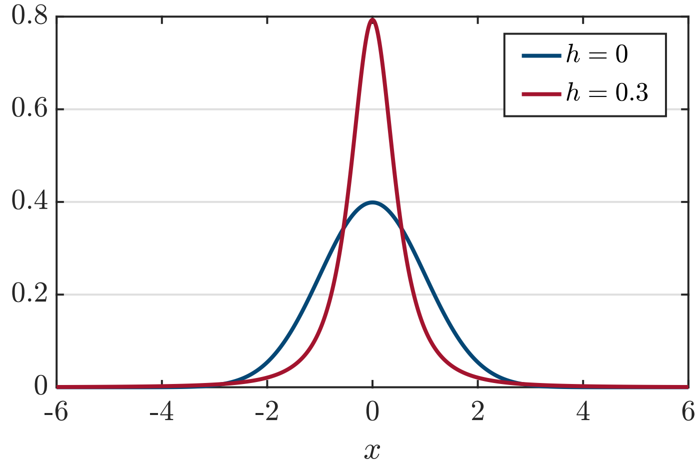
$$\log z_{t+1} = \rho \log z_t + u_{t+1}.$$

The innovations to firm productivity u_t and ε_t have standard deviations σ_u and σ_ε and are drawn from a Tukey g-h distribution, a flexible family of distributions that transforms a standard normal variable x according to

$$x \exp\left(\frac{h}{2} x^2\right) (1 - 2h)^{3/4},$$

where h is a parameter that governs the thickness of the tails. A higher value of h elongates the tails of the distribution relative to a standard normal ($h = 0$), as illustrated in Figure 3. As we show below, allowing for both persistent and transitory shocks is necessary to match the rate at which the autocorrelation of output and the standard deviation of its growth rate change with the horizon.

Figure 3: Shape of Tukey g-h Distribution



4.2 Decision Rules

Since labor and capital are chosen in advance, the optimal choices satisfy

$$\mathbb{E}_t c_{t+1}^{-\theta} \left[(1-\alpha)\,\eta \frac{y_{t+1}}{l_{t+1}} - W \right] = 0$$

and

$$\mathbb{E}_t c_{t+1}^{-\theta} \left[\alpha\eta \frac{y_{t+1}}{k_{t+1}} - R \right] \geq 0.$$

Intuitively, the firm chooses labor and capital to equate their expected marginal products with the respective factor prices. Since business income risk is not diversified, the owner uses its own stochastic discount factor to weigh future states. The first order condition for capital holds with equality if the collateral constraint does not bind.

The optimality condition for equity can be written as

$$c_t^{-\theta} = \beta \left(1 + r + \mu_t\right) \mathbb{E}_t c_{t+1}^{-\theta},$$

where μ_t is the multiplier on the collateral constraint and satisfies

$$\mu_t = \frac{1}{1-\xi} \mathbb{E}_t \frac{c_{t+1}^{-\theta}}{\mathbb{E}_t c_{t+1}^{-\theta}} \left[\alpha\eta \frac{y_{t+1}}{k_{t+1}} - R \right].$$

Intuitively, since an additional unit of wealth allows the firm to acquire an additional $1/(1-\xi)$ units of capital, the excess return on saving is equal to $1/(1-\xi)$ times the risk-adjusted expected difference between the marginal product of capital and its user cost. Henceforth, we use $\hat{\mathbb{E}}_t \equiv \mathbb{E}_t \frac{c_{t+1}^{-\theta}}{\mathbb{E}_t c_{t+1}^{-\theta}}$ to denote the expectation under the risk-neutral measure.

Absent the collateral constraint, the optimal choices of labor and capital are

$$l_{t+1} = \left(\frac{\alpha\eta}{R}\right)^{\frac{\alpha\eta}{1-\eta}} \left(\frac{(1-\alpha)\,\eta}{W}\right)^{\frac{1-\alpha\eta}{1-\eta}} \left(\hat{\mathbb{E}}_t z_{t+1}\varepsilon_{t+1}\right)^{\frac{1}{1-\eta}}$$

and

$$k_{t+1} = \left(\frac{\alpha\eta}{R}\right)^{\frac{1-(1-\alpha)\eta}{1-\eta}} \left(\frac{(1-\alpha)\eta}{W}\right)^{\frac{(1-\alpha)\eta}{1-\eta}} \left(\hat{\mathbb{E}}_t z_{t+1}\varepsilon_{t+1}\right)^{\frac{1}{1-\eta}}.$$

Notice that it is the risk-neutral expected productivity that determines how much labor and capital the firm hires.

To see the impact risk has on the firm's input choices, we next contrast them with the optimal choices under full insurance, assuming no collateral constraints. In this case, the problem of the entrepreneur reduces to

$$\max_{k_{t+1},l_{t+1}} \quad -k_{t+1} + \frac{1}{1+r}\left(\mathbb{E}_t z_{t+1}\varepsilon_{t+1}\left(k_{t+1}^{\alpha} l_{t+1}^{1-\alpha}\right)^{\eta} - W l_{t+1} + (1-\delta)\,k_{t+1}\right),$$

so the optimal choices are

$$l^*_{t+1} = \left(\frac{\alpha\eta}{R}\right)^{\frac{\alpha\eta}{1-\eta}} \left(\frac{(1-\alpha)\,\eta}{W}\right)^{\frac{1-\alpha\eta}{1-\eta}} \left(\mathbb{E}_t z_{t+1}\varepsilon_{t+1}\right)^{\frac{1}{1-\eta}}$$

and

$$k^*_{t+1} = \left(\frac{\alpha\eta}{R}\right)^{\frac{1-(1-\alpha)\eta}{1-\eta}} \left(\frac{(1-\alpha)\eta}{W}\right)^{\frac{(1-\alpha)\eta}{1-\eta}} \left(\mathbb{E}_t z_{t+1}\varepsilon_{t+1}\right)^{\frac{1}{1-\eta}}.$$

In this case it is the expectation under the physical measure that determines input choices.

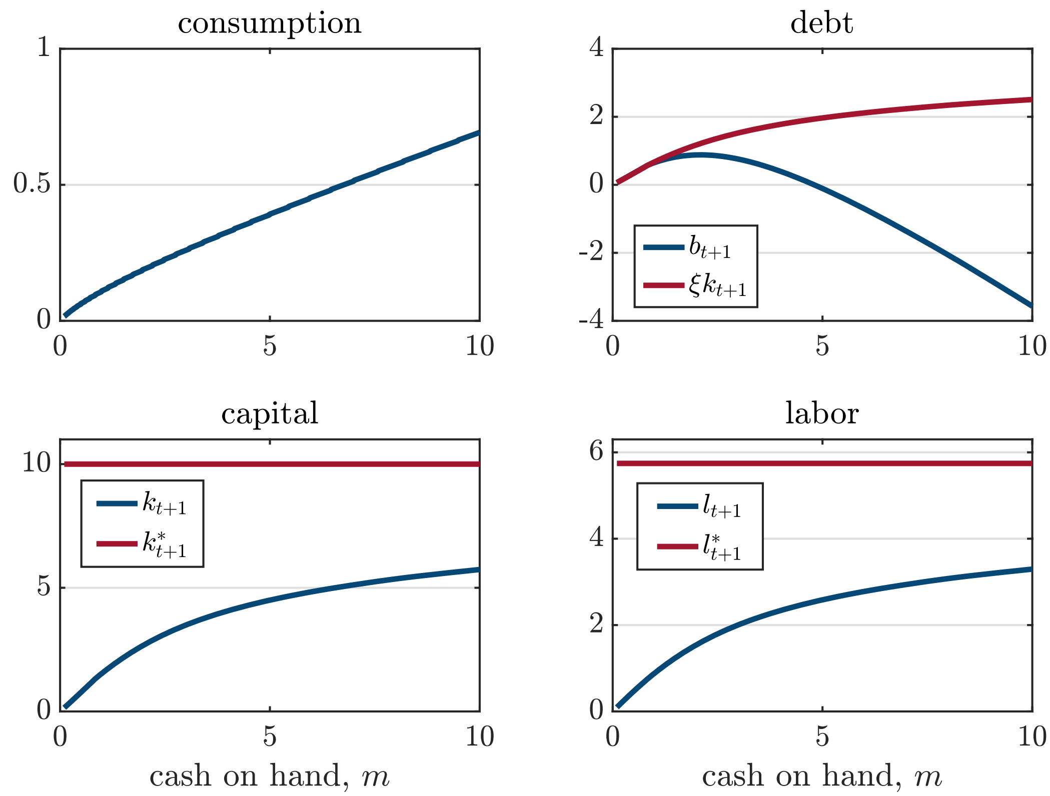
The ratio between the firm's labor and capital relative to the frictionless counterpart is therefore

$$\frac{l_{t+1}}{l^*_{t+1}} = \frac{k_{t+1}}{k^*_{t+1}} = \left(\frac{\hat{\mathbb{E}}_t z_{t+1}\varepsilon_{t+1}}{\mathbb{E}_t z_{t+1}\varepsilon_{t+1}}\right)^{\frac{1}{1-\eta}} = \left(1 + \frac{\mathbb{COV}_t\left(c_{t+1}^{-\theta},\ z_{t+1}\varepsilon_{t+1}\right)}{\mathbb{E}_t c_{t+1}^{-\theta}\mathbb{E}_t z_{t+1}\varepsilon_{t+1}}\right)^{\frac{1}{1-\eta}}$$

and is less than unity because of the negative covariance between the owner's marginal utility of consumption and firm productivity. In other words, even if the collateral constraint does not bind, labor and capital choices are reduced relative to the environment with full insurance.

Figure 4 illustrates the impact of the two financial frictions on the the firm's optimal savings and production choices against the firm's cash on hand $m = a + \pi$. The consumption function is concave. As the entrepreneur becomes wealthier, it acquires more capital, which increases its debt limit ξk_{t+1}. Notice in the upper right panel that this debt limit binds for low levels of cash on hand. Even for high levels of cash on hand, when the debt limit does not bind and, in fact, the producer has net savings, capital and labor choices are still below

Figure 4: Decision Rules



Notes: Cash on hand m is defined as $a + \pi$.

the frictionless levels k^* and l^* as a consequence of risk. As the bottom panels show, the gap between capital and labor choices and their frictionless counterparts falls with cash on hand as wealthier entrepreneurs are willing to take on more risk. This is because the consumption of wealthier entrepreneurs is less sensitive to changes in the firm's income, so the covariance between the owner's marginal utility of consumption and firm productivity decreases with wealth.

We consider next the implications for expected rates of return. Note that the entrepreneur's expected income is

$$\mathbb{E}_t \pi_{t+1} = r a_{t+1} + \mathbb{E}_t \left[z_{t+1} \varepsilon_{t+1} \left(k_{t+1}^{\alpha} l_{t+1}^{1-\alpha} \right)^{\eta} - W l_{t+1} - R k_{t+1} \right].$$

Differentiating with respect to a_{t+1} gives the expected financial return

$$\frac{\partial \mathbb{E}_t \pi_{t+1}}{\partial a_{t+1}} = r + \mathbb{E}_t \left[\alpha \eta \frac{y_{t+1}}{k_{t+1}} - R \right] \frac{\partial k_{t+1}}{\partial a_{t+1}} + \mathbb{E}_t \left[(1-\alpha) \eta \frac{y_{t+1}}{l_{t+1}} - W \right] \frac{\partial l_{t+1}}{\partial a_{t+1}},$$

which sums up the interest rate r, the expected difference between the marginal product of capital and its user cost multiplied by the marginal impact of wealth on capital, and the expected difference between the marginal product of labor and the wage rate multiplied by the marginal impact of wealth on labor.

Figure 5 shows how each of these components of expected return change with the firm's cash on hand. The expected financial return decreases with cash on hand, reflecting not only declining marginal products of capital and labor, but also the declining marginal effect of wealth on capital and labor choices.

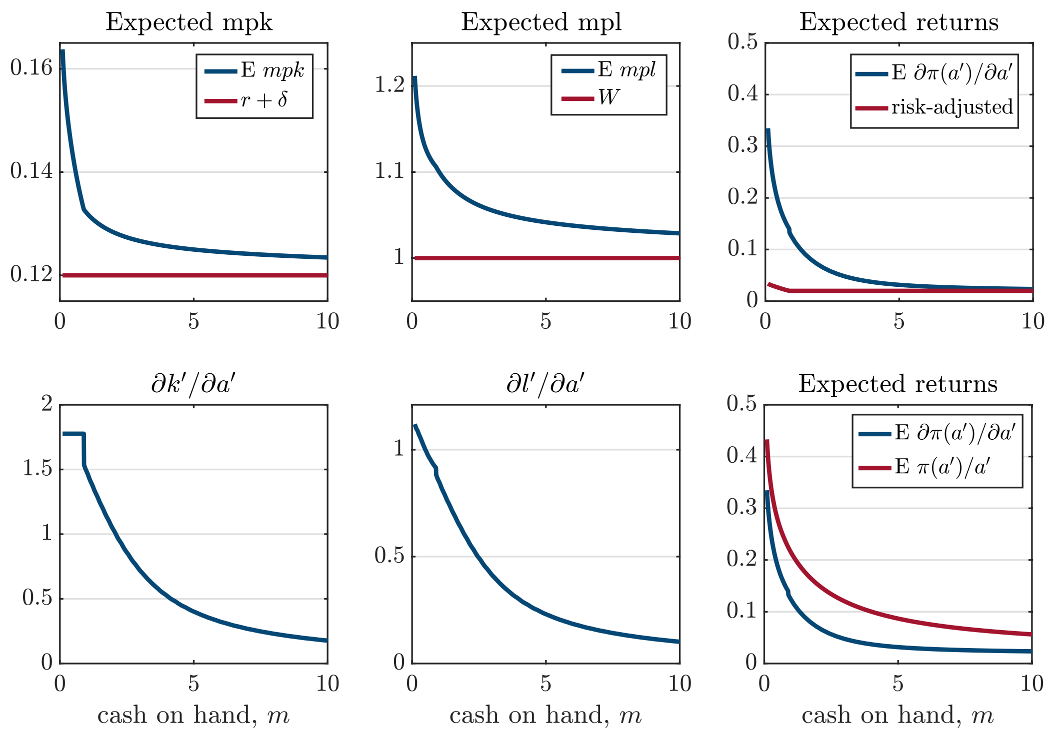
We next compute the risk-adjusted expected return, that is the expected financial return calculated under the risk-neutral measure. Because the optimal choices of capital and labor equate their risk-adjusted marginal products to their user costs, unless the collateral constraint binds, we have

$$\frac{\partial \hat{\mathbb{E}}_t \pi_{t+1}}{\partial a_{t+1}} = r + \hat{\mathbb{E}}_t \left[\alpha \eta \frac{y_{t+1}}{k_{t+1}} - R \right] \frac{\partial k_{t+1}}{\partial a_{t+1}} = r + \mu_t.$$

Thus, due to the negative covariance between marginal utility and firm productivity, risk-adjusted returns are smaller than the expected return under the physical probability and, in fact, are equal to r unless the borrowing constraint binds. This is illustrated in the top right panel of Figure 5, which contrasts the expected financial return under the two measures. Notice also in the bottom right panel that expected financial returns are smaller than expected accounting returns, owing to the decreasing returns to scale.

To provide a better sense for the mechanism of the model, Figure 6 shows impulse responses to a persistent idiosyncratic productivity shock. To compute this, we first set the shocks to ε_t and u_t to zero and calculate the firm's optimal choices that satisfy the optimality conditions above, that is, under uncertainty. Under this particular sequence of zero innovations to the firm's productivity, its wealth converges to a particular level. At this wealth level, the firm's capital stock is below its frictionless level. We then shock the persistent component of productivity once and trace out the firm's choices. Notice that in response to this shock the gap between k and k^* widens. Over time, the higher productivity increases the firm's profits and leads it to accumulate more wealth, which allows it to increase both capital and labor. The capital-output ratio and the labor share fall persistently, reflecting the gradual adjustment of the firm's wealth. Because the marginal products of capital and labor are high, expected returns, both accounting and financial, increase. Notice, however, that the risk-adjusted expected return hardly changes, suggesting that risk, rather than collateral constraints, account for the bulk of the deviation of k from k^*. Thus, persistent productivity shocks lead to persistent changes in the firm's returns, owing to the gradual adjustment of the entrepreneur's wealth.

Figure 5: Expected Rates of Return



Notes: Cash on hand m is defined as $a + \pi$.

4.3 Parameterization

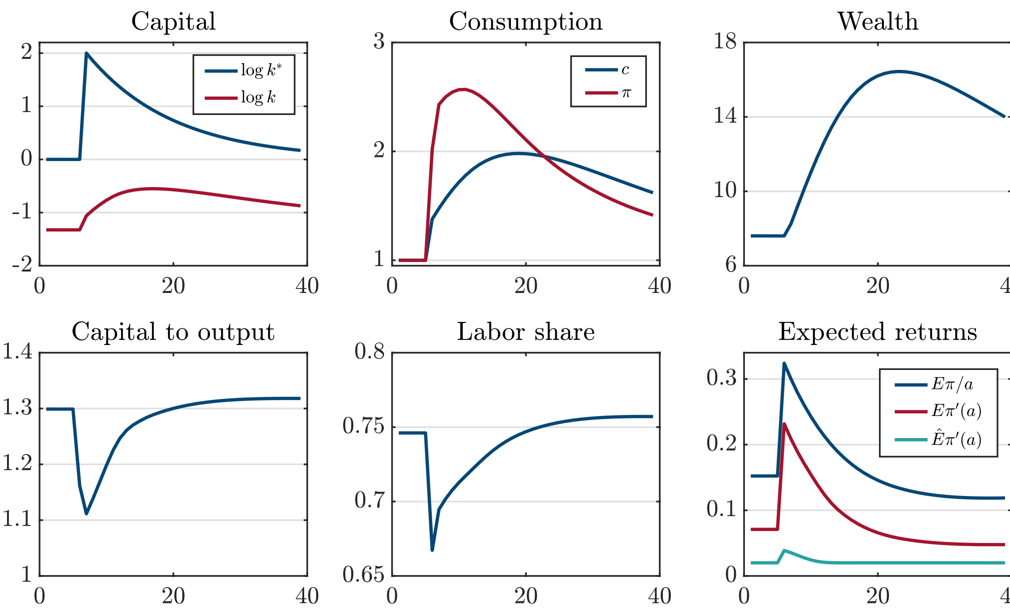
We next describe how we choose parameters for our quantitative analysis.

Assigned Parameters. We assume that a period in the model is one year and set the depreciation rate of capital $\delta = 0.10$ and the interest rate $r = 0.02$. We set the relative risk aversion $\theta = 2$, a commonly used value in the literature. Because this parameter matters for how important risk is, in the robustness section below we experiment with a lower value of $\theta = 0.5$.

Calibrated Parameters. We calibrate the remaining parameters that describe preferences (β), technology (α and η), the maximum loan to value (ξ) and the process for productivity (ρ_z, σ_z, σ_e and h) to match moments in the firm level data from Spain. We report these moments in Table 5 and the calibrated parameter values in Table 6.

The discount factor β is pinned down by the aggregate wealth to output ratio, which is equal to 1.57 in the data and 1.55 in the model. The technology parameters are jointly

Figure 6: Response to a Persistent Productivity Shock



Notes: The firm experiences a persistent productivity shock in period 5.

pinned down by the aggregate capital-output ratio (1.24 vs 1.27), the aggregate labor share (0.71 vs 0.74) and the aggregate income to output ratio (0.12 vs 0.14). Because we only have two technology parameters α and η for three moments, the model does not reproduce them perfectly but nevertheless does a reasonable job. The maximum loan to value ξ is pinned down by the 90^{th} percentile of the capital to wealth ratio (1.73 vs 1.72). Intuitively, since $k/a \leq 1/(1-\xi)$, a higher value of ξ increases the leverage ratio at the top of the distribution. As we show below, however, the value of ξ is not critical for our results because risk rather than collateral constraints are the main source of financial frictions.

The persistence ρ_z and volatility σ_z and σ_e of the two productivity shocks are jointly pinned down by the autocorrelation of output at horizons one to three, the cross-sectional standard deviation of output, as well as the standard deviation of output growth rates at horizons one to three. Our model matches all these targets well. In addition to the speed at which the autocorrelation and volatility of growth rates changes with the horizon, the relative importance of transitory versus persistent shocks is pinned down by the extent to which the labor share for any given firm fluctuates over time relative to its time series mean. We thus also target the inter-quartile range of these deviations (0.12 vs 0.11). Finally, the parameter h that governs the thickness of the tails of the distribution of productivity shocks is pinned

Table 5: Targeted Moments

	Data	Model		Data	Model
s.d. $\log y_{it}$	1.26	1.31	aggregate a/y	1.57	1.55
s.d. $\log y_{it}/y_{it-1}$	0.41	0.37	aggregate k/y	1.24	1.27
s.d. $\log y_{it}/y_{it-2}$	0.52	0.51	aggregate l/y	0.71	0.74
s.d. $\log y_{it}/y_{it-3}$	0.60	0.62	aggregate π/y	0.12	0.14
iqr $\log y_{it}/y_{it-1}$	0.28	0.27	corr $\log y_{it}$, $\log y_{it-1}$	0.95	0.96
iqr $\log y_{it}/y_{it-2}$	0.41	0.42	corr $\log y_{it}$, $\log y_{it-2}$	0.91	0.92
iqr $\log y_{it}/y_{it-3}$	0.52	0.54	corr $\log y_{it}$, $\log y_{it-3}$	0.88	0.89
iqr $l_{it}/y_{it} - \overline{l_{it}/y_{it}}$	0.12	0.11	p90 k/a	1.73	1.72

down by the inter-quartile range of the distribution of output growth rates. Intuitively, a fat-tailed distribution is characterized by a low inter-quartile range relative to the standard deviation, a feature that our model matches well.

The calibrated discount factor is $\beta = 0.916$. Because the capital-output ratio in the data is relatively low, the implied capital elasticity is low as well, $\alpha = 0.173$. One concern is that the capital-output ratio in the data, which reflects the book value of capital, is below the replacement cost. To address this, our robustness section targets a larger value of k/y from the EU-KLEMS data. The span of control parameter is $\eta = 0.948$, which is high compared to values of 0.85 or 0.9 typically used in the literature. As we discuss below, this is because risk plays an important role in our model and a substantial fraction of profits accrue to capital income as opposed to managerial span of control. Absent risk, the value of η required to match the profit share in the data would be much lower. The maximum loan to value is $\xi = 0.437$, implying that firms can at most lever up to 1.78. The persistence and standard deviation of the persistent productivity component are 0.926 and 0.041, while the standard deviation of the transitory shock is 0.219. Notice that productivity is much less persistent than output, suggesting an important role for financial frictions in determining the dynamics of output. Intuitively, since wealth accumulates gradually and it affects production choices,

Table 6: Parameter Values

β	0.916	discount factor	ρ_z	0.926	AR(1) z
α	0.173	capital elasticity	σ_z	0.041	std. dev. z shocks
η	0.948	span of control	σ_e	0.219	std. dev. e shocks
ξ	0.437	max loan to value	h	0.374	Tukey h parameter

output can be very persistent even if productivity is not. Lastly, the Tukey h parameter is high and equal to 0.374, implying substantial excess kurtosis.

Untargeted Moments. We next evaluate the model's ability to reproduce several untargeted moments. As Table 7 shows, the model reproduces well the volatility and persistence of labor and capital in the data. For example, focusing on employment, the standard deviation of changes is 0.30 in the data and 0.36 in the model, the inter-quartile range is 0.19 vs 0.23, and the autocorrelation is 0.97 vs 0.96. In addition, the model also reproduces well the low comovement of employment and output. Recall that in the data a regression of $\Delta \log l_{it}$ on $\Delta \log y_{it}$ gives a coefficient of 0.56 when we restrict the sample to observations with $|\Delta \log y_{it}| \leq 0.5$. The corresponding regression coefficient in the model is 0.54.

As is well understood, the correlation between wealth and productivity is a crucial determinant of the severity of financial frictions (Moll, 2014). Using the parameters α and η reported above, we calculate productivity in the model as the Solow residual in the production function and find that the rank correlation between wealth and productivity in the data is quite low, 0.24. Reassuringly, the model matches this correlation perfectly, owing to the fat tails of the distribution of productivity shocks and the presence of transitory shocks.

4.4 Distribution of Accounting Returns

We conclude this section by gauging the model's ability to reproduce the distribution of accounting rates of return in the data and their correlation with equity. Panel A of Table 8 contrasts the cross-sectional, equity weighted, distribution of π/a in the data and in the model. The average accounting return is 0.08 in the data and 0.09 in the model. Importantly, the distribution is very dispersed. Though the model fails to generate the negative returns at

Table 7: Untargeted Moments

	Data	Model		Data	Model
s.d. $\Delta \log l_{it}$	0.30	0.36	s.d. $\Delta \log k_{it}$	0.54	0.36
iqr $\Delta \log l_{it}$	0.19	0.23	iqr $\Delta \log k_{it}$	0.25	0.23
autocorr $\log l_{it}$	0.97	0.96	autocorr $\log k_{it}$	0.96	0.96
elast l_{it} to y_{it}	0.56	0.54	elast k_{it} to y_{it}	0.30	0.54

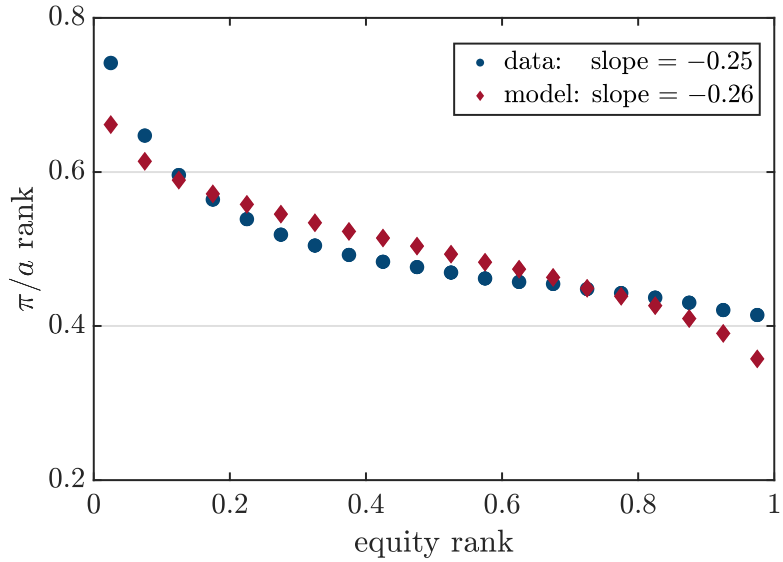
Table 8: Distribution of Accounting Returns

	mean	p10	p25	p50	p75	p90	p95
			A. π/a, equity weighted				
Data	0.08	-0.03	0.01	0.06	0.13	0.24	0.34
Model	0.09	0.00	0.02	0.05	0.12	0.24	0.35
			B. $\overline{\pi/a}$, equity weighted				
Data	0.08	0.00	0.03	0.07	0.11	0.17	0.22
Model	0.09	0.02	0.04	0.07	0.13	0.18	0.23

the bottom of the distribution, it reproduces all other percentiles closely, especially the top ones. For example, the 90^{th} percentile of the distribution of returns is 0.24 in both the data and the model. Consider next the persistence of these returns. Panel B of Table 8 reports the distribution of the time series (equity weighted) average return $\overline{\pi/a}$ for each firm. Since we observe firms for an average of 15 years in the data, we calculate these averages for 15 years in the model. Once again, the model reproduces well the distribution of these averages. For example, the 90^{th} percentile of the distribution of $\overline{\pi/a}$ is 0.17 in the data and 0.18 in the model.

Figure 7 shows a binscatter of accounting returns against equity. We first calculate, for

Figure 7: Accounting Returns and Equity



Notes: The figure plots the rank-rank correlation between accounting rates of return and equity for 20 bins.

each firm, its rank in the equity distribution as well as its rank in the π/a distribution. We then bin firms according to their equity rank and report, for each equity bin, the average π/a rank. As in the data, firms at the bottom of the equity distribution have higher returns than firms at the top. Specifically, the slope of a rank-rank regression is equal to -0.25 in the data and -0.26 in the model.

5 Sources of Dispersion in Returns

We now use the model to study how dispersed financial returns are, understand the sources of their dispersion, their implications for the valuation of firms and their macroeconomic consequences. We also disentangle the role of the various frictions in the model.

5.1 Distribution of Financial Returns

We start by calculating the distribution of expected accounting returns, $\mathbb{E}_{t-1}\pi_t/a_t$, which allows us to isolate the role of transitory shocks in driving the cross-sectional distribution of realized accounting returns. The first row of Table 9 shows that expected accounting returns are dispersed, ranging from 0.03 at the 10^{th} percentile to 0.20 at the 90^{th} percentile. Recall from Table 8 that realized accounting returns range from 0 at the 10^{th} percentile to 0.24 at the 90^{th} percentile, suggesting that transitory shocks account for approximately 30% of the

Table 9: Dispersion in Financial Returns

	mean	p10	p25	p50	p75	p90	p95
$\mathbb{E}_{t-1}\pi_t/a_t$	0.09	0.03	0.04	0.06	0.12	0.20	0.25
$\mathbb{E}_{t-1}\partial\pi_t/\partial a_t$	0.05	0.02	0.02	0.03	0.05	0.11	0.17
$\hat{\mathbb{E}}_{t-1}\partial\pi_t/\partial a_t$	0.02	0.02	0.02	0.02	0.02	0.02	0.03

Notes: All statistics are equity weighted.

observed dispersion in realized accounting returns $(1 - 0.17/0.24)$.

The second row shows the distribution of expected financial returns, $\mathbb{E}_{t-1}\partial\pi_t/\partial a_t$. As discussed in Section 4.2, dispersion in expected financial returns arises due to financial frictions. Notice that the average financial return is 0.05, approximately half the average accounting return. This suggests that approximately half of the accounting returns accrue to the fixed factor, the managerial span of control. Expected financial returns are approximately half as dispersed as expected accounting returns: they range from 0.02 at the 10^{th} percentile to 0.11 at the 90^{th} percentile. Importantly, most of this dispersion reflects risk, as opposed to collateral constraints. To see this, the last row of Table 9 reports the distribution of expected financial returns calculated under the risk-neutral measure, $\hat{\mathbb{E}}_{t-1}\partial\pi_t/\partial a_t$. As discussed in Section 4.2, these risk-adjusted returns exceed the interest rate r only if the collateral constraint binds. Notice that the majority of firms in our economy are unconstrained as risk-adjusted returns are equal to the interest rate even at the 90^{th} percentile.

In summary, we find that financial returns are half as dispersed as accounting returns and mostly reflect compensation for risk rather than collateral constraints.

5.2 Valuation of Private Businesses

We next provide an alternative metric for the importance of financial constraints by studying their implications for the valuation of firms.

Entrepreneur's Valuation of the Business. We first calculate the price p_{1i} at which the owner of firm i is willing to sell their business. Specifically, let V_i denote the lifetime value of the business owner under the status quo. If the entrepreneur were to sell the business and forfeit all claims to its future profits as well as the current equity, it would simply consume out of its existing wealth and consequently enjoy lifetime utility

$$\hat{V}\left(p_{1i}\right) = \frac{\left(1 - \beta^{\frac{1}{\theta}}\left(1+r\right)^{\frac{1}{\theta}-1}\right)^{-\theta} p_{1i}^{1-\theta}}{1-\theta},$$

where we used the Euler equation and the budget constraint that equates the present value of consumption, discounted at r, to the price at which the business is sold. The indifference price p_{1i} is defined as the implicit solution to $V_i = \hat{V}\left(p_{1i}\right)$ and reflects the initial equity as well as the present value of the income the entrepreneur expects to receive, evaluated using discount rates that reflect the entrepreneur's current and future marginal utility of consumption. The first row of Table 10 reports the distribution of p_1. We scale each p_{1i} by the book value of equity a_i and report equity weighted statistics. The ratio of the entrepreneur's valuation of its business to the book value of its equity ranges from 1.2 at the 10^{th} percentile to 3.3 at the 90^{th} percentile, with an average of 2.1. That is, the average entrepreneur values their business by approximately twice as much as the book value of equity.

Present Value of Income Flows, Discounted at r. We let p_{2i} denote the present value of the expected income flows π_{it} that firm i generates under the status quo allocations, but discounted at rate r

$$p_{2i} = a_i + \mathbb{E}\sum_{s=0}^{\infty}\frac{1}{\left(1+r\right)^s}\pi_{is}.$$

Relative to p_1, this statistic keeps the stochastic process for income flows unchanged, but discounts them at the lower interest rate r and thus allows us to gauge the role of discount rates in depressing the value of firms. The second row of Table 10 shows that the average ratio of p_2 to equity is 4.7 and ranges from 1.4 at the 10^{th} percentile to 9.4 at the 90^{th} percentile. That this valuation is approximately twice as large as the entrepreneur's valuation of the business shows that the implicit discount rates faced by entrepreneurs are large.

Present Value of Riskless Income Flows, Discounted at r. The calculation above simply allows us to contrast the owners' discount rate with the interest rate r, given the cash flows in the status quo allocations. If the entrepreneur were to face the lower discount rate r, that is, if it had access to full insurance and faced no collateral constraints, it would

Table 10: Valuation of Private Businesses

	mean	p10	p25	p50	p75	p90
p_1/a	2.1	1.2	1.4	1.8	2.4	3.3
p_2/a	4.7	1.4	1.9	3.0	5.4	9.4
p_3/a	17.4	2.0	3.3	7.4	17.7	39.0

Notes: All statistics are equity weighted.

earn higher expected profits π_{it}^* corresponding to the capital and labor choices k_{it}^* and l_{it}^* in Section 4.2. We let p_{3i} denote the valuation of these income flows

$$p_{3i} = a_i + \mathbb{E} \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} \pi_{is}^*.$$

A comparison of p_{2i} and p_{3i} allows us to gauge the role of frictions in depressing the firm's capital and labor choices and, therefore, expected profits. The last row of Table 10 shows that the average ratio of p_3 to equity is 17.4 and ranges from 2 at the 10^{th} percentile to 39 at the 90^{th} percentile. That the ratio of p_3 to p_2 is approximately 4 on average suggests that financial frictions indeed play a sizable role.

We conclude that financial frictions play an important role in determining firm valuations, both by increasing discount rates and especially by depressing capital and labor. Because p_3 represents the value at which a firm would sell if the owner were able to access frictionless financial markets, for example by incorporating the business as in Boar and Midrigan (2019) and Peter (2019), while p_1 represents the actual value of the business to its owners, the sizable gap between these two valuations suggests that financial frictions, and specifically imperfect risk sharing, have large consequences.

5.3 Macroeconomic Implications

We next study the macroeconomic consequences of financial frictions in our model. To do so, we note that dispersion in financial returns in our model reflects differences in the marginal product of capital and labor across producers, that is, *misallocation.* Moreover, a high average level of financial returns reflects a low aggregate capital-output ratio and a low labor share.

To see the first effect, notice that in the absence of frictions aggregate TFP in the economy would be equal to

$$Z_t^* = \frac{Y_t^*}{\left(K_t^\alpha L_t^{1-\alpha}\right)^\eta} = \left(\int \left(\mathbb{E}_{t-1} z_{it}\varepsilon_{it}\right)^{\frac{1}{1-\eta}} \mathrm{d}i \right)^{1-\eta},$$

reflecting that the efficient capital and labor choices are also made a period in advance of the realization of the productivity shocks. In contrast, in our economy, aggregate TFP is

$$Z_t = \frac{Y_t}{\left(K_t^\alpha L_t^{1-\alpha}\right)^\eta} = \left(\int \left(\mathbb{E}_{t-1}\left(z_{it}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}} \tau_{it}^{-\frac{1}{1-\eta}} \right)^{1-\eta},$$

where $\tau_{it} = \left(\left(\tau_{it}^k\right)^\alpha \left(\tau_{it}^l\right)^{1-\alpha}\right)^\eta$ reflects the wedge in the first order conditions for capital and labor induced by the two financial frictions, namely lack of risk-sharing and collateral constraints. The wedges τ_{it}^k and τ_{it}^l are equal to

$$\tau_{it}^k = \frac{\mathbb{E}_{t-1} y_{it}/k_{it}}{Y_t/K_t} \qquad \text{and} \qquad \tau_{it}^l = \frac{\mathbb{E}_{t-1} y_{it}/l_{it}}{Y_t/L_t}.$$

The first row of Table 11 shows that absent financial frictions aggregate productivity would be 5.7% higher compared to the baseline. Notice also that since we normalize the aggregate labor supply to unity, aggregate output in our economy is equal to

$$Y_t = Z_t^{\frac{1}{1-\alpha\eta}} \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha\eta}{1-\alpha\eta}}.$$

Because the capital-output ratio in our economy is equal to 1.27, whereas absent financial frictions it would be equal to $\alpha\eta = 1.37$, output in the absence of financial frictions would be 8.4% higher, a gap that reflects both higher TFP as well as capital deepening. Finally, financial frictions also depress the labor share, which in our model is equal to 0.736 and absent financial frictions would be equal to 0.784. This decline in the labor share further reduces the equilibrium wage, which would be 15.5% higher absent financial frictions. These results assume that labor is supplied inelastically. The consequences for output would be even larger with an elastic labor supply.

The second and third rows of Table 11 explore the consequences of removing the firms' ability to borrow ($\xi = 0$) and of eliminating the collateral constraint ($\xi = 1$). Notice that productivity would fall by 0.93% if firms were unable to borrow, output would fall by 3.7% and wages by 2%. Eliminating the collateral constraint would, in contrast, have a negligible effect. We thus once again conclude that risk, as opposed to collateral constraints, accounts for the bulk of distortions due to financial frictions.

Table 11: Macroeconomic Implications

	Z	Y	W
No financial frictions	5.73	8.39	15.51
No borrowing, $\xi = 0$	-0.93	-3.73	-2.03
No collateral constraint, $\xi = 1$	0.02	0.15	0.02

Notes: All numbers are percent deviations from the baseline model.

5.4 Role of Risk and Collateral Constraints

In our baseline economy we assume that both labor and capital have to be chosen in advance and that firms face collateral constraints. We next study the importance of each of these ingredients in generating dispersion in returns. To isolate the role of each ingredient, we study three alternative economies: *(i)* an economy where labor is flexibly chosen, that is after productivity is realized, but capital is still chosen in advance as in Midrigan and Xu (2009) and Gopinath et al. (2017); *(ii)* an economy where both labor and capital are flexibly chosen, as in Buera et al. (2011); and *(iii)* an economy with flexible inputs and no collateral constraints.

We calibrate each of these models to match the same set of targets as in Table 5 as closely as possible. As we show in the Appendix, we match these targets well, with two exceptions. First, the labor share is constant when labor is flexibly chosen, so the model cannot reproduce the time series variation in the labor share we see in the data. Second, the model without a collateral constraint overstates the 90^{th} percentile of the capital to equity ratio, which is equal to 1.96 in the model and 1.73 in the data. We note that the discount factor β required to match the wealth to output ratio increases from 0.916 in the baseline model to 0.927 in the model with flexible labor, 0.936 in the model with flexible labor and capital, and 0.937 when we also eliminate the collateral constraint. Intuitively, eliminating these financial frictions reduces financial rates of return and a higher discount factor is needed to reproduce the wealth accumulation in the data. The span of control parameter η required to match the profit share in the data falls from 0.948 in the baseline model to 0.931 in the model with flexible labor, 0.917 in the model with flexible labor and capital, and 0.904 when

Table 12: Distribution of Expected Accounting Returns

	p50	p75	p90	p95
Baseline	0.06	0.12	0.20	0.25
Labor flexible	0.05	0.11	0.18	0.24
Both flexible	0.06	0.11	0.18	0.23
No frictions	0.04	0.08	0.18	0.27

Notes: All statistics are equity weighted.

we also eliminate the collateral constraint. Intuitively, profits in our baseline model accrue both to the fixed factor, $1-\eta$, as well as to wealth due to the presence of financial frictions. Naturally, eliminating the financial frictions increases the importance of the fixed factor.

Table 12 reports moments of the distribution of expected accounting returns $\mathbb{E}_{t-1}\pi_t/a_t$ implied by these models. For comparison, the top row of the table reproduces the results from the baseline model. As the table shows, all models imply similar dispersion in accounting rates of return, suggesting that dispersion in accounting returns is not indicative of firms facing financial frictions and thus high rates of return on saving. Intuitively, as we move towards the model without financial frictions, the importance of the fixed factor and heterogeneity in productivity increases, so the model can reproduce the dispersion in accounting returns without relying on financial frictions. We note, however, that the model without financial constraints is greatly at odds with the data in that it predicts a much stronger negative relationship between accounting returns and equity: the slope of a rank-rank regression falls from -0.26 in our baseline model and in the data to -0.78.

Table 13 reports moments of the distribution of the expected financial returns across alternative models, computed both under the physical and risk-neutral probabilities. When labor is flexibly chosen, financial returns are less dispersed: the difference between the 90^{th} and the 50^{th} percentile falls from 0.09 in the baseline model to 0.06. Importantly, most of the dispersion in these financial returns now reflects the collateral constraints. As Panel B of Table 13 shows, the risk-neutral expected financial returns are almost as dispersed as those computed under the physical probabilities. When both factors are flexible, this dispersion in expected financial returns is further reduced and entirely reflects the collateral constraint.

Table 13: Distribution of Financial Returns

	p50	p75	p90	p95
	A. Baseline			
$\mathbb{E}_{t-1}\partial\pi_t/\partial a_t$	0.03	0.05	0.11	0.17
$\hat{\mathbb{E}}_{t-1}\partial\pi_t/\partial a_t$	0.02	0.02	0.02	0.03
	B. Labor flexible			
$\mathbb{E}_{t-1}\partial\pi_t/\partial a_t$	0.02	0.03	0.08	0.12
$\hat{\mathbb{E}}_{t-1}\partial\pi_t/\partial a_t$	0.02	0.02	0.07	0.11
	C. Both flexible			
$\mathbb{E}_{t-1}\partial\pi_t/\partial a_t$	0.02	0.03	0.06	0.09
$\hat{\mathbb{E}}_{t-1}\partial\pi_t/\partial a_t$	0.02	0.03	0.06	0.09

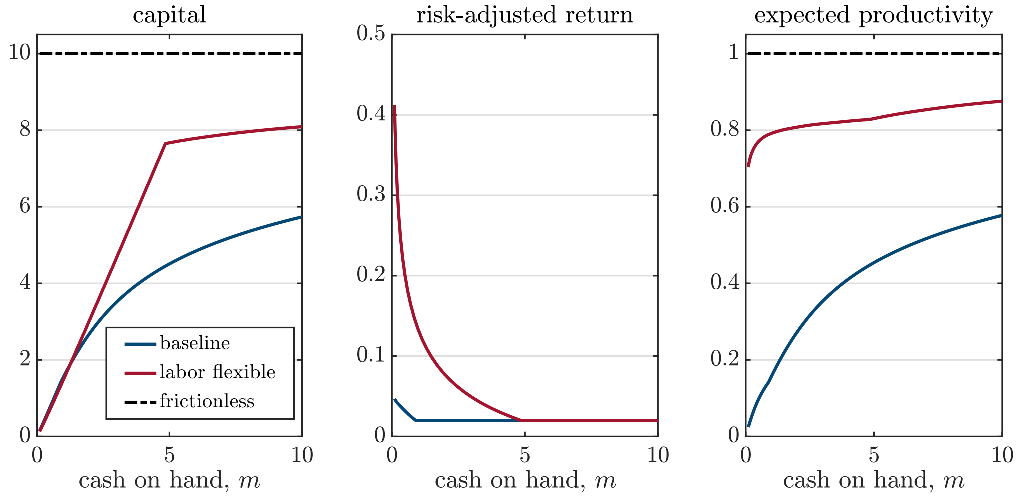
Notes: All statistics are equity weighted.

The results above reveal that assuming that labor is chosen in advance, a feature needed to match the evidence on the comovement of output and labor, greatly amplifies the importance of risk in generating dispersion in financial returns. To see why this is the case, we next contrast the optimal capital choice of a firm to its frictionless counterpart in an environment in which labor is flexible as opposed to chosen in advance. We first note that when labor is flexibly chosen the ratio of the firm's capital to its frictionless counterpart is

$$\frac{k_{t+1}}{k^*_{t+1}} = \left(\frac{\hat{\mathbb{E}}_t\left(z_{t+1}\varepsilon_{t+1}\right)^{\frac{1}{1-(1-\alpha)\eta}}}{\mathbb{E}_t\left(z_{t+1}\varepsilon_{t+1}\right)^{\frac{1}{1-(1-\alpha)\eta}}}\right)^{\frac{1-(1-\alpha)\eta}{1-\eta}},$$

so once again depends on the ratio between the expected productivity under the risk-neutral and the physical measure. Figure 8 contrasts the optimal choice of capital in the baseline model to that in the economy with flexible labor. We choose units for productivity so that in both of these economies the frictionless optimal level of capital is equal to 10. Notice that a firm chooses a much higher level of capital in the flexible labor model, owing to a higher risk-adjusted expected productivity, as shown in the right panel of the figure. Intuitively, because the firm chooses labor after it learns its productivity and because labor accounts for a substantial fraction of the firm's costs, being able to chose labor flexibly insulates the

Figure 8: Decision Rules When Labor Is Flexible



Notes: The expected productivity in the right panel is computed under the risk-neutral measure.

firm's profits and therefore consumption from productivity shocks. As the middle panel of the figure shows, because the firm desires a higher level of capital when labor is flexible, the collateral constraint binds in a larger region of the state space and, consequently, the risk-adjusted return is much higher.

6 Extensions

We next investigate the role of the assumptions we made on the process for productivity shocks, namely the presence of fat-tailed and transitory shocks, that of entrepreneurs' attitudes towards risk and address the concern that the high accounting returns in the data may reflect that the book value of capital is lower than the replacement value. We also show that our empirical findings for Spain are robust across countries.

6.1 Role of Fat-Tailed Shocks

In our baseline model we assumed that the distribution of both persistent and transitory shocks is fat tailed, to match the kurtosis of the distribution of output growth rates in the data. We next show that with normally distributed shocks the model predicts much less dispersion in financial returns, even though it reproduces the distribution of accounting returns. Moreover, most of the dispersion in financial returns is due to a binding collateral constraint. Thus our conclusion that risk plays an important role is, to a large extent, driven by the fat-tailed nature of the shocks, an essential feature needed to match the distribution of output growth rates in the data.

We now assume that both productivity shocks are drawn from a Gaussian distribution and calibrate the model to match the same targets as in Table 5. As we show in the Appendix, we match these targets well, with the exception of the interquartile range of output growth rates, which the model overstates considerably. We also note that the model requires a higher discount factor, $\beta = 0.931$, as opposed to 0.916 in the baseline, as well as a lower span of control parameter, $\eta = 0.934$, as opposed to 0.948 in the baseline.

As Panel A of Table 14 shows, the model with Gaussian shocks generates a similar distribution of expected accounting returns as the baseline model. In contrast, expected financial returns are smaller and less dispersed, as shown in Panel B. For example, at the 95^{th} percentile the expected financial return falls from 0.17 in the baseline model to 0.11 in the model with Gaussian shocks. Moreover, these financial returns now reflect binding collateral constraints. As shown in Panel C, the risk-adjusted expected financial returns at the 95^{th} percentile increase from 0.03 in our baseline to 0.07 in the model without fat-tailed shocks.

These results suggest that in our baseline model capital and labor choices are too low relative to their frictionless values because firms are concerned about the possibility of rare disasters: large declines in output which require a large decline in wealth to pay for labor expenses.

6.2 Role of Transitory Shocks

In the baseline model we assume that, in addition to persistent shocks to productivity, firms also face transitory shocks, a feature required to match the rate at which the autocorrelation and the standard deviation of output growth rates change with the horizon. We show that without these transitory shocks the model can also match the distribution of accounting returns, but predicts lower and less dispersed financial returns which now reflect binding collateral constraints.

As earlier, we recalibrate the model without transitory shocks to match the original targets in the data and report the results in the Appendix. Not surprisingly, the model without transitory shocks overstates the rate at which the standard deviation of output growth rates increases with the horizon and the rate at which the autocorrelation of output decays. Additionally, the model is also no longer able to match the large volatility of a firm's

Table 14: Distribution of Expected Returns

	p50	p75	p90	p95
	A. Accounting			
Baseline	0.06	0.12	0.20	0.25
Gaussian	0.06	0.11	0.18	0.23
No transitory shocks	0.05	0.10	0.18	0.24
Lower risk aversion	0.06	0.11	0.19	0.24
	B. Financial			
Baseline	0.03	0.05	0.11	0.17
Gaussian	0.02	0.03	0.08	0.11
No transitory shocks	0.02	0.03	0.07	0.11
Lower risk aversion	0.02	0.03	0.08	0.13
	C. Risk-adjusted			
Baseline	0.02	0.02	0.02	0.03
Gaussian	0.02	0.02	0.04	0.07
No transitory shocks	0.02	0.02	0.06	0.09
Lower risk aversion	0.02	0.02	0.03	0.05

Notes: All statistics are equity weighted.

labor share around its time series mean. Since firms now face considerably less risk, the model requires a higher discount factor, $\beta = 0.931$, and a lower span of control, $\eta = 0.928$, to match the aggregate wealth and income to output ratios in the data.

Panels A and B of Table 14 show that the model without transitory shocks generates the same distribution of accounting returns as in the baseline, but predicts smaller and less dispersed expected financial returns. For example, the 95^{th} percentile is 0.11 in this model and 0.17 in the baseline. As shown in Panel C, most of the dispersion in financial returns is now due to the collateral constraint: the 95^{th} percentile of the risk-adjusted returns is 0.09, much larger than in the baseline.

We therefore conclude that the presence of transitory shocks plays an important role in elevating the amount of risk that firms face in our baseline model.

6.3 Role of Preferences for Risk

In the baseline model we assume that entrepreneurs have a coefficient of relative risk aversion $\theta = 2$, a common choice in the literature. Motivated by the important role we found for risk, we next investigate the extent to which our results are driven by this parameter by assuming a lower relative risk aversion $\theta = 0.5$. We find that even though the dispersion in financial returns falls, it is nevertheless large and, once again, mostly accounted for by risk rather than collateral constraints.

We recalibrate the model with $\theta = 0.5$ to match the same targets as in the baseline and report the results in the Appendix. With a lower coefficient of risk aversion the model requires a much higher discount factor $\beta = 0.959$, and a lower span of control $\eta = 0.934$.

As Panel A of Table 14 shows, the model generates the same distribution of expected accounting returns as the baseline. Expected financial returns are, however, one-fourth less dispersed than in the baseline. For example, the 95^{th} percentile falls from 0.17 in the baseline to 0.13 with a lower risk aversion. As in our baseline model, risk-adjusted expected financial returns are much smaller than the returns computed under the physical measure. For example, the 95^{th} percentile is only equal to 0.05. Thus, once again, risk as opposed to collateral constraints account for the bulk of the dispersion in financial returns.

6.4 Book Value of Capital

One concern is that the book value of capital in Orbis data is too low compared to its replacement value, leading us to understate the overall amount of wealth that firms have and thus overstate accounting returns. Indeed, in the Orbis data for Spain, the aggregate capital-output ratio is only 1.43.[7] In contrast, the corresponding capital-output ratio in the EU-KLEMS data for Spain is 1.86, 30% larger.

To assess the quantitative importance of this potential bias, we scale up each firm's capital stock in the Orbis data by 30% and increase its equity to reflect the higher value of the firm's capital. We recalculate the targets in Table 5 and recalibrate the model to match the updated values. We report the results of the calibration in the Appendix and here we note that with

[7]This value is larger than the target we use in Table 5 because it includes all firms, including publicly traded ones.

Table 15: Distribution of Accounting Returns, Scaled Capital

	mean	p10	p25	p50	p75	p90	p95
			A. π/a, equity weighted				
Data	0.06	-0.03	0.01	0.05	0.11	0.20	0.28
Model	0.07	-0.01	0.02	0.04	0.09	0.18	0.27
			B. $\overline{\pi/a}$, equity weighted				
Data	0.06	-0.01	0.02	0.06	0.09	0.14	0.18
Model	0.07	0.02	0.03	0.05	0.09	0.14	0.17

a higher capital stock and equity, the model requires a higher discount factor, $\beta = 0.932$, a higher span of control, $\eta = 0.964$, and a higher capital elasticity, $\alpha = 0.217$.

Table 15 reports the distribution of accounting returns resulting from scaling up the capital stock, in the data and in the recalibrated model. Compared to Table 8, accounting returns in the data are smaller on average and less dispersed. For example, the average return is now equal to 0.06 and is 0.08 in the unscaled data. The 95^{th} percentile is 0.28, smaller than 0.34 originally. Long-run accounting returns are also slightly less dispersed: the 95^{th} percentile is 0.18 in the scaled data and 0.22 in the original data. The recalibrated model matches these numbers well, especially at the top of the distribution.

Since agents now are more patient and save more compared to the baseline, the recalibrated model predicts less dispersion in both accounting and financial returns. For example, as shown in Panels A and B of Table 16, at the 95^{th} percentile the expected accounting return is 0.18, whereas the 95^{th} percentile of the distribution of financial returns is 0.13. As in our baseline model, most of the dispersion in financial returns is due to risk, not collateral constraints: at the 95^{th} percentile the risk-adjusted expected financial return is only 0.03.

Our conclusion that firms' capital and labor choices are considerably distorted by risk is therefore not an artifact of the low book value of capital in the Orbis data.

Table 16: Distribution of Expected Returns, Scaled Capital

	p50	p75	p90	p95
		A. Accounting		
Baseline	0.06	0.12	0.20	0.25
Scaled capital	0.05	0.09	0.14	0.18
		B. Financial		
Baseline	0.03	0.05	0.11	0.17
Scaled capital	0.03	0.05	0.09	0.13
		C. Risk-adjusted		
Baseline	0.02	0.02	0.02	0.03
Scaled capital	0.02	0.02	0.02	0.03

Notes: All statistics are equity weighted.

6.5 Evidence From Other Countries

In Section 3 we used data from Spain to show that accounting rates of return are dispersed, persistent, and negatively correlated with equity, that the distribution of output growth rates displays fat tails and that labor and capital choices are both risky. In this section, we document that these findings hold more generally and are remarkably similar in five other countries: Italy, France, Norway, Portugal and Slovakia.

Table A.1 shows that accounting rates of return are dispersed and persistent in all these countries. Table A.2 shows that the slope coefficient in a rank-rank regression of accounting returns on equity ranges from -0.31 to -0.25, numbers similar to that we find for Spain. Table A.3 shows that the standard deviation of output growth rates is higher than the interquartile range, and that the kurtosis of the output growth rate distribution ranges from 16 to 20 across countries. Lastly, Table A.4 shows that the coefficient of a regression of changes in the logarithm of the wage bill agains log output ranges from 0.42 to 0.58, and that of changes in log capital against log output ranges from 0.22 to 0.34.

7 Conclusion

In this paper we ask: What accounts for the heterogeneity in returns to private business wealth? To answer this question, we first use micro data from Orbis on firm level balance sheets and income statements to document that differences in accounting returns for privately held businesses are large, persistent and negatively correlated with equity. We also document that firms experience large, fat-tailed, and partly transitory changes in output that are not accompanied by equally sized changes in their capital stock and wage bill. This implies that capital and labor choices are risky, as fluctuations in output are accompanied by large changes in firm profits.

We then study a model of entrepreneurial dynamics that is quantitatively consistent with this evidence. The model accounts well for the dispersion in accounting returns measured in the data, and allows us to back out the distribution of financial returns, that is the marginal returns to saving in the business. We find that financial returns are large and dispersed, considerably depressing the valuation of firms. Differences in financial returns are half as large as those in accounting returns, suggesting an important role for heterogeneity in managerial ability or other fixed factors, in addition to financial frictions. Financial returns mostly reflect risk, as opposed to collateral constraints which play a negligible role owing to the firms' unwillingness to expand and take on more risk.

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Appendix

A Facts on Accounting Returns in Other Countries

In this section we revisit the main facts reported in Section 3 for five other countries in the Orbis data: Italy, France, Norway, Portugal and Slovakia. We also show that our results are robust to not restricting the Orbis sample to firms with at least ten years of data.

Dispersion and Persistence in Accounting Returns. Panel A of Table A.1 reports moments of the cross-sections distribution of returns, π/a. Panel B of the table reports the distribution of long-term returns, $\overline{\pi/a}$.

Table A.1: Accounting Rates of Return in Other Countries

	mean	p10	p25	p50	p75	p90	p95
			A. π/a				
Italy	0.06	-0.06	0.00	0.04	0.12	0.24	0.35
France	0.14	-0.02	0.04	0.11	0.21	0.36	0.50
Norway	0.18	-0.01	0.04	0.12	0.25	0.48	0.68
Portugal	0.08	-0.02	0.01	0.06	0.14	0.25	0.34
Slovakia	0.08	-0.03	0.00	0.02	0.11	0.27	0.43
Spain*	0.08	-0.04	0.01	0.05	0.13	0.25	0.37
			B. $\overline{\pi/a}$				
Italy	0.06	-0.04	0.00	0.05	0.11	0.18	0.25
France	0.13	0.00	0.05	0.11	0.18	0.30	0.41
Norway	0.18	0.02	0.06	0.12	0.25	0.42	0.64
Portugal	0.08	-0.01	0.03	0.06	0.12	0.19	0.25
Slovakia	0.07	-0.02	0.00	0.03	0.12	0.24	0.35
Spain*	0.08	-0.01	0.02	0.07	0.11	0.19	0.25

Notes: All statistics are equity weighted. Spain* refers to the unrestricted data for Spain.

Returns and Equity and Negatively Correlated. Table A.2 reports the rank-rank slope of a regression of returns π/a on equity a.

Table A.2: Rates of Return and Equity in Other Countries

	rank-rank slope
Italy	-0.25
France	-0.27
Norway	-0.31
Portugal	-0.28
Slovakia	-0.27
Spain*	-0.32

Notes: Spain* refers to the unrestricted data for Spain.

Output Growth Rates and Dispersed and Fat-Tailed. Table A.3 reports the standard deviation, inter-quartile range and the kurtosis of the distribution of output growth rates.

Table A.3: Distribution of Output Growth Rates in Other Countries

	s.d.	iqr	kurtosis
Italy	0.41	0.27	17.5
France	0.29	0.21	19.7
Norway	0.34	0.24	19.3
Portugal	0.47	0.31	16.3
Slovakia	0.49	0.34	14.9
Spain*	0.50	0.33	13.0

Notes: Spain* refers to the unrestricted data for Spain.

Capital and labor Choices are Risky. Table A.4 reports the slope coefficients of regressions of the growth rate of labor and capital againts the growth rate of output.

Table A.4: Comovement Between Capital, Labor and Output in Other Countries

	$\Delta \log l$	$\Delta \log k$
Italy	0.576	0.253
	(0.001)	(0.002)
France	0.549	0.258
	(0.001)	(0.002)
Norway	0.510	0.222
	(0.003)	(0.006)
Portugal	0.424	0.293
	(0.002)	(0.004)
Slovakia	0.429	0.341
	(0.006)	(0.011)
Spain*	0.588	0.318
	(0.001)	(0.001)

Notes: The sample is restricted to observations for which $|\Delta \log y| < 0.5$. Standard errors are clustered at the firm level. Spain* refers to the unrestricted data for Spain.

B Parameterization of Alternative Models

Tables B.1 and B.2 report the targeted moments and calibrated parameter values for the alternative models discussed in Section 5.4.

Tables B.3 and B.4 report the targeted moments and calibrated parameter values for the alternative models discussed in Section 6.

Tables B.5 and B.6 report the targeted moments and calibrated parameter values for the economy with a higher level of capital-output ratio discussed in Section 6.4.

Table B.1: Targeted Moments, Remove Frictions

	Data	Baseline	Labor Flexible	Both Flexible	No Frictions
s.d. $\log y_{it}$	1.26	1.31	1.26	1.26	1.26
s.d. $\log y_{it}/y_{it-1}$	0.41	0.37	0.38	0.40	0.38
s.d. $\log y_{it}/y_{it-2}$	0.52	0.51	0.52	0.52	0.51
s.d. $\log y_{it}/y_{it-3}$	0.60	0.62	0.62	0.61	0.61
iqr $\log y_{it}/y_{it-1}$	0.28	0.27	0.31	0.28	0.28
iqr $\log y_{it}/y_{it-2}$	0.41	0.42	0.41	0.40	0.42
iqr $\log y_{it}/y_{it-3}$	0.52	0.54	0.50	0.51	0.53
iqr $l_{it}/y_{it} - \overline{l_{it}/y_{it}}$	0.12	0.11	0	0	0
corr $\log y_{it}$, $\log y_{it-1}$	0.95	0.96	0.95	0.95	0.95
corr $\log y_{it}$, $\log y_{it-2}$	0.91	0.92	0.92	0.92	0.92
corr $\log y_{it}$, $\log y_{it-3}$	0.88	0.89	0.88	0.88	0.88
aggregate a/y	1.57	1.55	1.57	1.56	1.58
aggregate k/y	1.24	1.27	1.24	1.24	1.24
aggregate l/y	0.71	0.74	0.75	0.75	0.75
aggregate π/y	0.12	0.14	0.14	0.14	0.13
p90 k/a	1.73	1.72	1.73	1.73	1.96

Table B.2: Parameter Values, Remove Frictions

		Baseline	Labor Flexible	Both Flexible	No Frictions
β	discount factor	0.916	0.927	0.936	0.937
α	capital elasticity	0.173	0.198	0.187	0.165
η	span of control	0.948	0.931	0.917	0.904
ξ	max loan to value	0.437	0.421	0.420	–
ρ_z	AR(1) z	0.926	0.935	0.949	0.962
σ_z	std. dev. z shocks	0.041	0.053	0.040	0.043
σ_e	std. dev. e shocks	0.219	0.087	0.021	0.016
h	Tukey h parameter	0.374	0.417	0.333	0.401

Table B.3: Targeted Moments, Extensions

	Data	Baseline	No Fat Tails	No Transitory Shocks	Lower $\theta = 0.5$
s.d. $\log y_{it}$	1.26	1.31	1.26	1.26	1.26
s.d. $\log y_{it}/y_{it-1}$	0.41	0.37	0.37	0.41	0.38
s.d. $\log y_{it}/y_{it-2}$	0.52	0.51	0.52	0.61	0.52
s.d. $\log y_{it}/y_{it-3}$	0.60	0.62	0.63	0.74	0.62
iqr $\log y_{it}/y_{it-1}$	0.28	0.27	0.43	0.28	0.27
iqr $\log y_{it}/y_{it-2}$	0.41	0.42	0.63	0.48	0.41
iqr $\log y_{it}/y_{it-3}$	0.52	0.54	0.78	0.66	0.53
iqr $l_{it}/y_{it} - \overline{l_{it}/y_{it}}$	0.12	0.11	0.12	0.03	0.12
corr $\log y_{it}$, $\log y_{it-1}$	0.95	0.96	0.96	0.95	0.95
corr $\log y_{it}$, $\log y_{it-2}$	0.91	0.92	0.91	0.88	0.92
corr $\log y_{it}$, $\log y_{it-3}$	0.88	0.89	0.87	0.83	0.88
aggregate a/y	1.57	1.55	1.57	1.57	1.57
aggregate k/y	1.24	1.27	1.24	1.24	1.24
aggregate l/y	0.71	0.74	0.75	0.75	0.75
aggregate π/y	0.12	0.14	0.13	0.14	0.14
p90 k/a	1.73	1.72	1.72	1.72	1.72

Table B.4: Parameter Values, Extensions

		Baseline	No Fat Tails	No Transitory Shocks	Lower $\theta = 0.5$
β	discount factor	0.916	0.931	0.931	0.959
α	capital elasticity	0.173	0.177	0.184	0.173
η	span of control	0.948	0.934	0.928	0.934
ξ	max loan to value	0.437	0.420	0.420	0.420
ρ_z	AR(1) z	0.926	0.935	0.907	0.944
σ_z	std. dev. z shocks	0.041	0.036	0.054	0.053
σ_e	std. dev. e shocks	0.219	0.118	–	0.362
h	Tukey h parameter	0.374	–	0.322	0.436

Table B.5: Targeted Moments, Scaled Capital

	Data	Model		Data	Model
s.d. $\log y_{it}$	1.26	1.27	aggregate a/y	1.95	1.94
s.d. $\log y_{it}/y_{it-1}$	0.41	0.39	aggregate k/y	1.61	1.62
s.d. $\log y_{it}/y_{it-2}$	0.52	0.52	aggregate l/y	0.71	0.71
s.d. $\log y_{it}/y_{it-3}$	0.60	0.61	aggregate π/y	0.12	0.13
iqr $\log y_{it}/y_{it-1}$	0.28	0.28	corr $\log y_{it}$, $\log y_{it-1}$	0.95	0.95
iqr $\log y_{it}/y_{it-2}$	0.41	0.42	corr $\log y_{it}$, $\log y_{it-2}$	0.91	0.92
iqr $\log y_{it}/y_{it-3}$	0.52	0.53	corr $\log y_{it}$, $\log y_{it-3}$	0.88	0.88
iqr $l_{it}/y_{it} - \overline{l_{it}/y_{it}}$	0.12	0.11	p90 k/a	1.75	1.73

Table B.6: Parameter Values, Scaled Capital

β	0.932	discount factor	ρ_z	0.930	AR(1) z
α	0.217	capital elasticity	σ_z	0.031	std. dev. z shocks
η	0.964	span of control	σ_e	0.255	std. dev. e shocks
ξ	0.454	max loan to value	h	0.376	Tukey h parameter