Why Are Returns To Private Business Wealth So Dispersed?

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Motivation

- Large, persistent differences in returns to private business wealth
  - Fagereng et al. (2020), Bach et al. (2020), Smith et al. (2021)
  - e.g. in Norway annual returns range from -0.05 to 0.36
- Often interpreted as evidence of financial frictions
  - borrowing constraints or uninsurable idiosyncratic risk
- But evidence is on *accounting* (average), not *financial* (marginal) returns
  - so may also reflect returns to fixed factor
  - e.g. managerial talent or market power

Question

- How large and dispersed are financial returns?
  - do they reflect risk premia or borrowing constraints?
  - implications for misallocation?

- Implications for tax policy:
  - increasing wealth share of high return entrepreneurs
  - increases output and productivity, but leads to higher inequality
  - Itskhoki-Moll (2019), Guvenen et al. (2019), Boar-Midrigan (2021)

Our Paper

- Use micro data on balance sheets and income statements to document
  - private businesses experience large fluctuations in income
  - because large changes in output not acompanied by changes in wage bill
- Interpret using model of entrepreneurship with
  - risky capital and labor choices, collateral constraints
- Financial returns half as dispersed as accounting returns
  - mostly reflect risk, even with moderate risk aversion
  - in contrast to model where labor flexibly chosen
- Even though model matches dispersion in returns, modest misallocation
  - suggestive evidence that most k/y differences not due to financial frictions

Outline

1. Accounting vs. Financial Returns
2. Data and Motivating Facts
3. Model
4. Importance of Financial Frictions
5. Dispersion in Capital-Output Ratio

Accounting vs. Financial Returns

Simple Example

- Technology $y = f(k,l)$, user cost $R = r + \delta$, wage W
  - let $\alpha_k = f_k k/y$ and $\alpha_l = f_l l/y$

- Income $\pi = ra + y - Rk - Wl$

- Accounting return if unconstrained:
  - $f_k = R \quad \Longrightarrow \quad Rk = \alpha_k y$
  - $f_l = W \quad \Longrightarrow \quad Wl = \alpha_l y$

$$\Longrightarrow \; \pi/a = r + (1 - \alpha_k - \alpha_l)\, y/a$$

- Financial return: $\partial\pi/\partial a = r + [f_k - R]\,\partial k/\partial a + [f_l - W]\,\partial l/\partial a$
  - $\partial\pi/\partial a = r$ unless firm financially constrained $(\partial k/\partial a, \partial l/\partial a > 0)$

Accounting vs. Financial Returns

- Suppose collateral constraint $k \leq \lambda a$

- Even if constraint binds, accounting return overstates financial return
  - financial return:
$$\frac{\partial \pi}{\partial a} = r + \left[\alpha_k \frac{y}{k} - R\right] \lambda$$
  - accounting return:
$$\frac{\pi}{a} = r + \left[(1 - \alpha_l) \frac{y}{k} - R\right] \lambda$$

- Unless constant returns: $\alpha_k + \alpha_l = 1$
  - Angeletos (2007), Moll (2014)

Data and Motivating Facts

Data

- Orbis Global Database
  - firm-level data from national registers and other sources
  - annual balance sheet and income statements, 1995 – 2018

- Focus on Spain, similar results for other countries

- Restrict to partnerships and private limited companies
  - all sectors except FIRE, Public Administration, Defense
  - keep firms with data for at least 10 years
  - 228,000 firms with 15 years of data on average
  - $1/4^{th}$ of firms, $3/4^{th}$ of output, equity

Definitions

- output y_{it} value added = production – all non-labor costs – taxes
- labor l_{it} wages and benefits
- capital k_{it} book value of property, plant, equipment, intangibles
- equity a_{it} total assets – total liabilities
- income π_{it} output – labor – depreciation – interest expenses

retained earnings = income − dividends

$$a_{it+1} - a_{it} = \pi_{it} - c_{it}$$

summary stats

Large And Persistent Differences in Returns

- Exclude firms with negative equity, all statistics equity-weighted

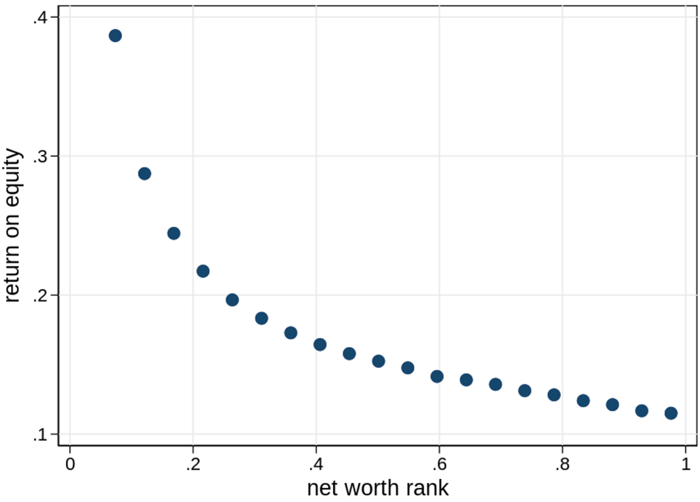
	mean	std	p10	p25	p50	p75	p90	p95
π/a	0.08	0.22	-0.03	0.01	0.06	0.13	0.24	0.34

Large And Persistent Differences in Returns

- Exclude firms with negative equity, all statistics equity-weighted
- Mean return for each firm over time, $\overline{\pi/a}$

	mean	std	p10	p25	p50	p75	p90	p95
π/a	0.08	0.22	-0.03	0.01	0.06	0.13	0.24	0.34
$\overline{\pi/a}$	0.08	0.12	0.00	0.03	0.07	0.11	0.17	0.22

Net Worth and Returns Negatively Correlated



Output Growth Rates Dispersed, Fat-Tailed

- Distribution $\log(y_{it}) - \log(y_{it-1})$
  - compare to Gaussian with the same variance

	std	p0.1	p0.5	p25	p75	p99.5	p99.9
Data	0.41	-2.78	-1.78	-0.12	0.15	1.68	2.68
Gaussian	0.41	-1.27	-1.06	-0.28	0.28	1.06	1.27

- Considerable kurtosis: 16.4
  - even though we truncated top/bottom 0.1%

Δ Output Not Accompanied by Δ Inputs

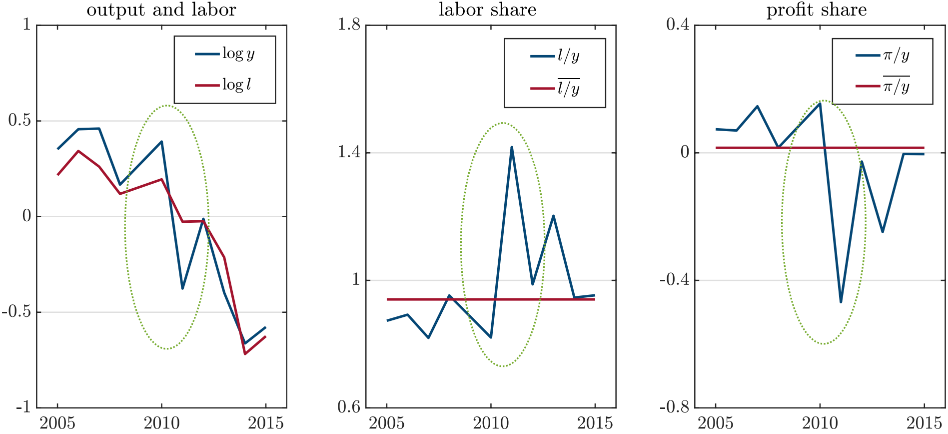
- When output falls, labor and capital respond little
  - so labor share and capital-output ratio increase
  - reducing profit share

- In contrast to model where labor and capital flexibly chosen

- Illustrate with example of firm, then document systematically

Example of a Firm

Example of a Firm



Δ Output Not Accompanied by Δ Inputs

- Regress $\Delta \log l_{it}$ and $\Delta \log k_{it}$ on $\Delta \log y_{it}$
  - all observations

	$\Delta \log l$	$\Delta \log k$
$\Delta \log y$	0.372	0.152
	(0.001)	(0.001)

  - observations with $|\Delta \log y| < 0.5$ ($\approx 90\%$ observations)

	$\Delta \log l$	$\Delta \log k$
$\Delta \log y$	0.561	0.303
	(0.001)	(0.002)

profits low frequency

Summary

- Differences in accounting returns are large and persistent
- Net worth and returns negatively correlated
- Output growth rates dispersed, fat-tailed
- Changes in output unaccompanied by changes in capital and labor

Model

Overview

- Fixed labor supply, equilibrium wage W, constant interest rate r
- Entrepreneurs heterogeneous in productivity
- Decreasing returns technology
  - persistent and transitory productivity shocks
  - capital and labor chosen one period in advance
- Profits from business only source of income
- Collateral constraint limits ability to borrow

Problem of Entrepreneur

- Entrepreneur maximizes

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\frac{c_t^{1-\theta}}{1-\theta}$$

- Budget constraint

$$c_t + k_{t+1} - b_{t+1} = y_t - Wl_t + (1-\delta)\,k_t - (1+r)\,b_t$$

- Using net worth $a_t = k_t - b_t$

$$a_{t+1} - a_t = \pi_t - c_t \qquad \text{where} \qquad \pi_t = y_t - Wl_t - Rk_t + ra_t$$

- Borrowing constraint

$$b_{t+1} \leq \xi k_{t+1} \quad \text{or} \quad k_{t+1} \leq \frac{1}{1-\xi}a_{t+1}$$

Technology

- Production function

$$y_t = z_t \varepsilon_t \left(k_t^{\alpha} l_t^{1-\alpha}\right)^{\eta} \quad \text{with} \quad \eta < 1$$

- Productivity has two components evidence
  - persistent z_t
  - transitory ε_t

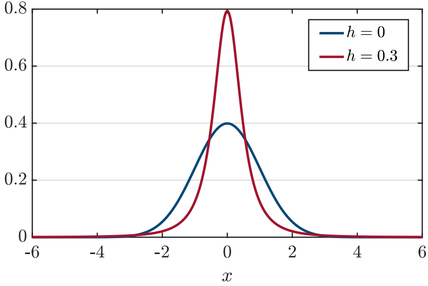
- Choose k_{t+1}, l_{t+1} before observing z_{t+1} and ε_{t+1}

Productivity Process

- Persistent component

$$\log z_{t+1} = \rho \log z_t + u_{t+1}$$

- Innovations u_t and ε_t drawn from fat-tailed distribution
  - transform standard normal x using $x \exp\left(\frac{h}{2}x^2\right)(1-2h)^{3/4}$



Optimal Choices

- Labor:

$$\mathbb{E}_t c_{t+1}^{-\theta} \left[(1-\alpha)\, \eta \frac{y_{t+1}}{l_{t+1}} - W \right] = 0$$

- Capital:

$$\mathbb{E}_t c_{t+1}^{-\theta} \left[\alpha\eta \frac{y_{t+1}}{k_{t+1}} - R \right] \geq 0 \qquad \left(= 0 \quad \text{if} \quad k_{t+1} < \frac{1}{1-\xi} a_{t+1} \right)$$

- Net worth:

$$c_t^{-\theta} = \beta \left(1 + r + \mu_t\right) \mathbb{E}_t c_{t+1}^{-\theta}$$

- Multiplier on collateral constraint:

$$\mu_t = \frac{1}{1-\xi} \mathbb{E}_t \frac{c_{t+1}^{-\theta}}{\mathbb{E}_t c_{t+1}^{-\theta}} \left[\alpha\eta \frac{y_{t+1}}{k_{t+1}} - R \right] \equiv \frac{1}{1-\xi} \hat{\mathbb{E}}_t \left[\alpha\eta \frac{y_{t+1}}{k_{t+1}} - R \right]$$

Optimal Choices

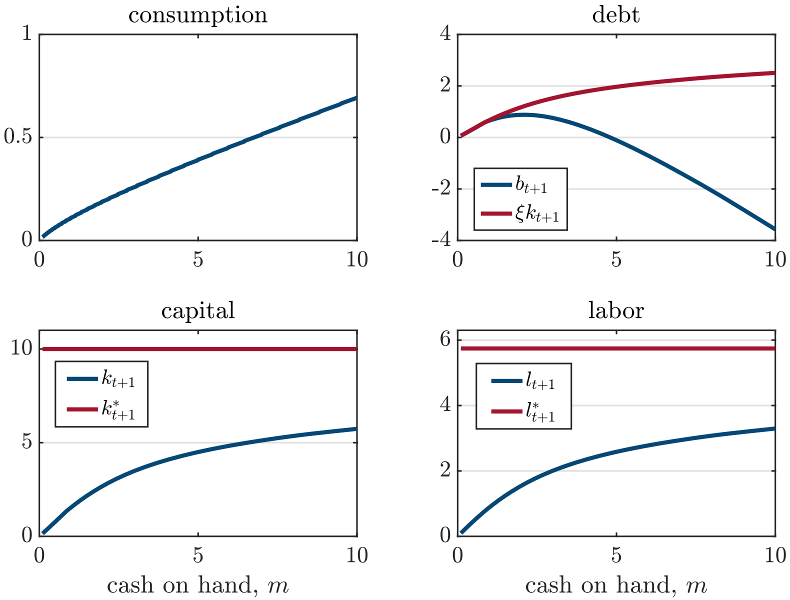
- Even if constraint does not bind, k_{it+1} and l_{it+1} reduced
  - relative to environment with full insurance
  - due to covariance between consumption c_{t+1} and productivity $z_{t+1}\varepsilon_{t+1}$

- Contrast to frictionless choices: objective

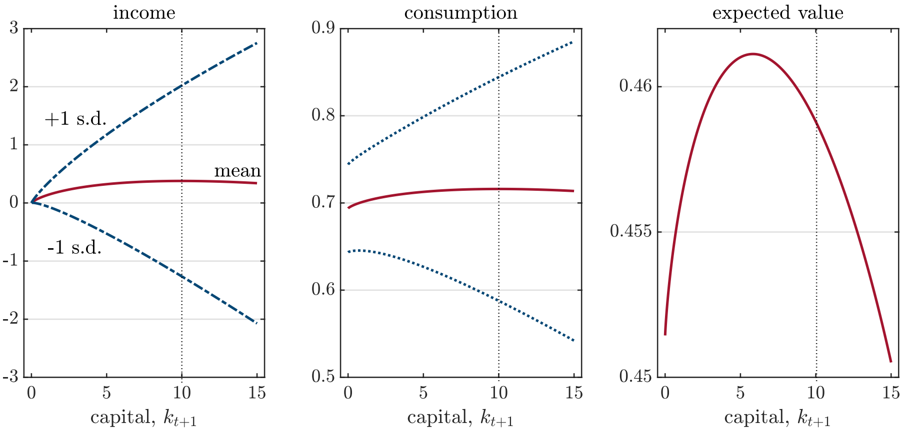
$$\frac{k_{t+1}}{k^*_{t+1}} = \frac{l_{t+1}}{l^*_{t+1}} = \left(\frac{\hat{\mathbb{E}}_t z_{t+1}\varepsilon_{t+1}}{\mathbb{E}_t z_{t+1}\varepsilon_{t+1}}\right)^{\frac{1}{1-\eta}} = \left(1 + \frac{\mathbb{COV}_t\left(c_{t+1}^{-\theta},\ z_{t+1}\varepsilon_{t+1}\right)}{\mathbb{E}_t c_{t+1}^{-\theta}\mathbb{E}_t z_{t+1}\varepsilon_{t+1}}\right)^{\frac{1}{1-\eta}}$$

- Wealthier entrepreneurs:
  - consumption less sensitive to changes in income: $c_t + a_{t+1} = \pi_t + a_t$
  - so hire more capital and labor

Decision Rules



Mean-Variance Tradeoff



Implications for Accounting Returns

- Expected income:

$$\mathbb{E}_{t-1}\pi_t = ra_t + \mathbb{E}_{t-1}\left[y_t - Wl_t - Rk_t\right]$$

- Financial frictions reduce payments to capital and labor

$$Wl_t + Rk_t = \underbrace{\eta\mathbb{E}_{t-1}y_t + \eta\frac{\mathbb{COV}_{t-1}\left(c_t^{-\theta}, z_t\varepsilon_t\right)}{\mathbb{E}_{t-1}c_t^{-\theta}}}_{\eta\hat{\mathbb{E}}_{t-1}y_t} - \mu_{t-1}a_t$$

- Increasing profit share

$$\mathbb{E}_{t-1}\pi_t = ra_t + (1-\eta)\,\mathbb{E}_{t-1}y_t - \eta\frac{\mathbb{COV}_{t-1}\left(c_t^{-\theta}, z_t\varepsilon_t\right)}{\mathbb{E}_{t-1}c_t^{-\theta}} + \mu_{t-1}a_t$$

- Aggregating across firms and scaling by aggregate wealth

$$\frac{\Pi_t}{A_t} = \underbrace{r}_{\text{risk-free rate}} + \underbrace{(1-\eta)\frac{Y_t}{A_t}}_{\text{fixed factor}} + \underbrace{\Omega_t}_{\text{finance frictions}}$$

Implications for Financial Returns

- Expected income:

$$\mathbb{E}_{t-1}\pi_t = ra_t + \mathbb{E}_{t-1}\left[y_t - Wl_t - Rk_t\right]$$

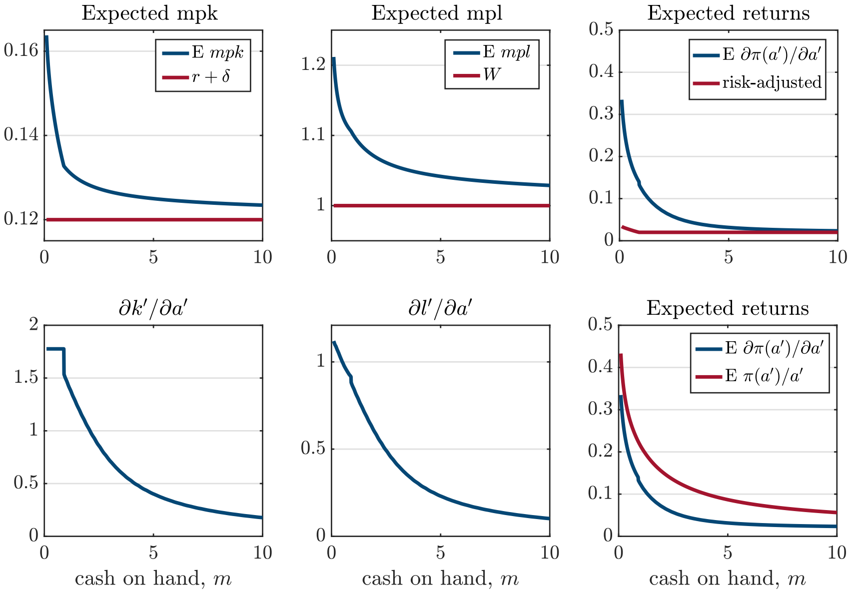
- So expected financial returns:

$$\frac{\partial \mathbb{E}_{t-1}\pi_t}{\partial a_t} = r + \mathbb{E}_{t-1}\left[\alpha\eta\frac{y_t}{k_t} - R\right]\frac{\partial k_t}{\partial a_t} + \mathbb{E}_{t-1}\left[(1-\alpha)\,\eta\frac{y_t}{l_t} - W\right]\frac{\partial l_t}{\partial a_t}$$

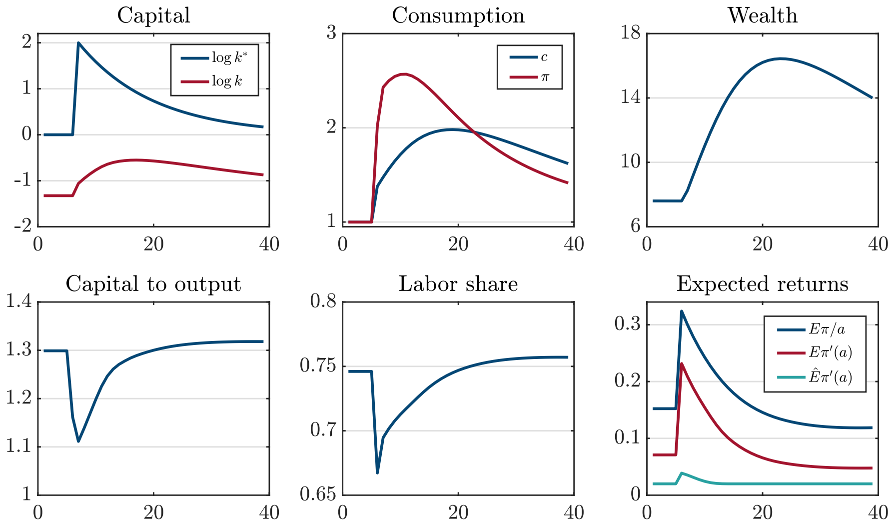
- Larger than risk-adjusted expected returns

$$\frac{\partial \hat{\mathbb{E}}_{t-1}\pi_t}{\partial a_t} = r + \hat{\mathbb{E}}_{t-1}\left[\alpha\eta\frac{y_t}{k_t} - R\right]\frac{\partial k_t}{\partial a_t} = r + \mu_{t-1}$$

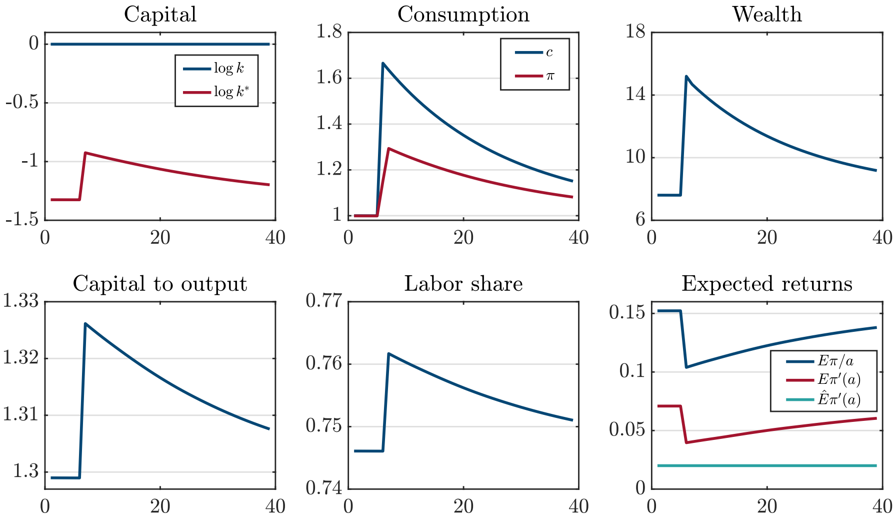
Expected Returns



Response to Persistent Productivity Shock



Response to Wealth Transfer



Parameterization

Strategy

- Period 1 year. Assign values to

θ	relative risk aversion	2
r	interest rate	0.02
δ	depreciation rate	0.10

  - then illustrate sensitivity to $\theta = 1/2$

- Calibrate remaining parameters to match moments in Spain
  - discount factor β
  - technology parameters α and η
  - maximum loan to value ξ
  - process for shocks ρ_z, σ_z, σ_e, h

Targeted Moments

	Data	Model		Data	Model
std $\log y_{it}$	1.26	1.31	aggregate a/y	1.57	1.55
std $\log y_{it}/y_{it-1}$	0.41	0.37	aggregate k/y	1.24	1.27
std $\log y_{it}/y_{it-2}$	0.52	0.51	aggregate l/y	0.71	0.74
std $\log y_{it}/y_{it-3}$	0.60	0.62	aggregate π/y	0.12	0.14
iqr $\log y_{it}/y_{it-1}$	0.28	0.27	corr $\log y_{it}, \log y_{it-1}$	0.95	0.96
iqr $\log y_{it}/y_{it-2}$	0.41	0.42	corr $\log y_{it}, \log y_{it-2}$	0.91	0.92
iqr $\log y_{it}/y_{it-3}$	0.52	0.54	corr $\log y_{it}, \log y_{it-3}$	0.88	0.89
iqr $l_{it}/y_{it} - \overline{l_{it}/y_{it}}$	0.12	0.11	p90 k/a	1.73	1.72

Parameter Values

β	0.916	discount factor	ρ_z	0.926	AR(1) z
α	0.173	capital elasticity	σ_z	0.041	std. dev. z shocks
η	0.948	span of control	σ_e	0.219	std. dev. e shocks
ξ	0.437	max loan to value	h	0.374	Tukey h parameter

- Intuition for identification:
  - $\beta \longleftarrow a/y$
  - $\alpha, \eta \longleftarrow k/y, l/y, \pi/y$
  - $\xi \longleftarrow$ p90 k/a
  - $\rho_z, \sigma_z, \sigma_e \longleftarrow \rho(y), \sigma(\Delta y), \text{iqr}(l/y)$
  - $h \longleftarrow \text{iqr}(\Delta y) \,/\, \sigma(\Delta y)$

Untargeted Moments

- Model also reproduces volatility and persistence of labor and capital

	Data	Model		Data	Model
std $\Delta \log l_{it}$	0.30	0.36	std $\Delta \log k_{it}$	0.54	0.36
iqr $\Delta \log l_{it}$	0.19	0.23	iqr $\Delta \log k_{it}$	0.25	0.23
autocorr $\log l_{it}$	0.97	0.96	autocorr $\log k_{it}$	0.96	0.96

Untargeted Moments

- Model reproduces low elasticity $\Delta \log l_{it}$ and $\Delta \log k_{it}$ to $\Delta \log y_{it}$
  - observations with $|\Delta \log y_{it}| < 1/2$

	$\Delta \log l$	$\Delta \log k$
Data	0.56	0.30
Model	0.54	0.54

- As well as correlation net worth a_{it} and productivity $z_{it}\varepsilon_{it}$
  - 0.24 rank correlation in both model and data

Distribution of Accounting Returns

- Model reproduces well large and persistent differences returns
- Distribution of π/a, equity-weighted

	mean	std	p10	p25	p50	p75	p90	p95
Data	0.08	0.22	-0.03	0.01	0.06	0.13	0.24	0.34
Model	0.09	0.17	0.00	0.02	0.05	0.12	0.24	0.35

Distribution of Accounting Returns

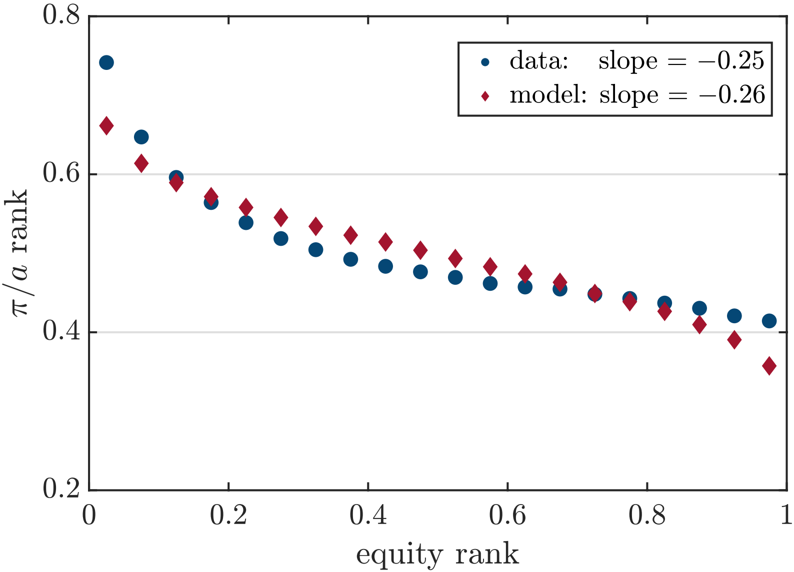
- Model reproduces well large and persistent differences returns
- Distribution of π/a, equity-weighted

	mean	std	p10	p25	p50	p75	p90	p95
Data	0.08	0.22	-0.03	0.01	0.06	0.13	0.24	0.34
Model	0.09	0.17	0.00	0.02	0.05	0.12	0.24	0.35

- Distribution of $\overline{\pi/a}$ (15-year averages), equity-weighted

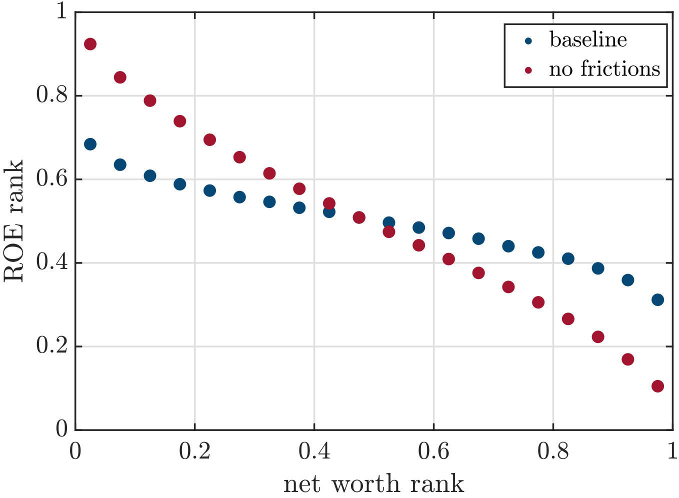
	mean	std	p10	p25	p50	p75	p90	p95
Data	0.08	0.12	0.00	0.03	0.07	0.11	0.17	0.22
Model	0.09	0.07	0.02	0.04	0.07	0.13	0.18	0.23

Net Worth and Returns Negatively Correlated



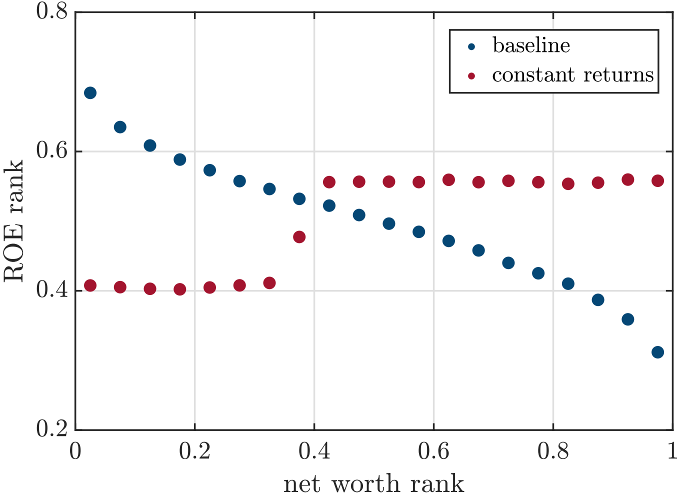
Net Worth and Returns Negatively Correlated

- Absent financial frictions: slope = -0.78



Net Worth and Returns Negatively Correlated

- Absent fixed factor: slope coefficient = 0.21



Importance of Financial Frictions

Decompose Accounting Returns

- Recall aggregate accounting returns

$$\frac{\Pi_t}{A_t} = \underbrace{r}_{\text{risk-free rate}} + \underbrace{(1-\eta)\frac{Y_t}{A_t}}_{\text{fixed factor}} + \underbrace{\Omega_t}_{\text{finance frictions}}$$

- Aggregate returns: 0.092
  - risk-free rate: 0.02
  - fixed factor: 0.033
  - finance frictions: 0.038

Sources of Dispersion in Returns

- Use model to calculate dispersion in accounting and financial returns

	mean	std	p10	p50	p90
π_t/a_t	0.092	0.166	-0.001	0.047	0.244
$\mathbb{E}_{t-1}\pi_t/a_t$	0.092	0.084	0.025	0.065	0.205
$\mathbb{E}_{t-1}\partial\pi_t/\partial a_t$	0.051	0.060	0.020	0.025	0.115
$\hat{\mathbb{E}}_{t-1}\partial\pi_t/\partial a_t$	0.021	0.003	0.020	0.020	0.020

- Large dispersion in financial returns
  - mostly due to risk, not collateral constraints

Valuation of Private Businesses

- Calculate firm values using three approaches:
  - p_1: price at which entrepreneur willing to sell business

- Scale by book value of equity, report equity-weighted statistics

	mean	p10	p25	p50	p75	p90
p_1/a	2.1	1.2	1.4	1.8	2.4	3.3

Valuation of Private Businesses

- Calculate firm values using three approaches:
  - p_1: price at which entrepreneur willing to sell business
  - p_2: pdv of income flows π_{it}, discounted at r

- Scale by book value of equity, report equity-weighted statistics

	mean	p10	p25	p50	p75	p90
p_1/a	2.1	1.2	1.4	1.8	2.4	3.3
p_2/a	4.7	1.4	1.9	3.0	5.4	9.4

Valuation of Private Businesses

- Calculate firm values using three approaches:
  - p_1: price at which entrepreneur willing to sell business
  - p_2: pdv of income flows π_{it}, discounted at r
  - p_3: pdv of income flows π^*_{it} absent risk, discounted at r
- Scale by book value of equity, report equity-weighted statistics

	mean	p10	p25	p50	p75	p90
p_1/a	2.1	1.2	1.4	1.8	2.4	3.3
p_2/a	4.7	1.4	1.9	3.0	5.4	9.4
p_3/a	17.4	2.0	3.3	7.4	17.7	39.0

Macroeconomic Implications

- Recall $\partial\pi/\partial a = r + [f_k - R]\,\partial k/\partial a + [f_l - W]\,\partial l/\partial a$

- Large average financial returns reflect
  - lower aggregate capital-output and labor share
  - absent frictions $K_t/Y_t = \alpha\eta/R$ and $WL_t/Y_t = (1-\alpha)\,\eta$

- Dispersed financial returns reflect
  - dispersion in marginal products across firms: misallocation
  - absent frictions $Z_t = Y_t/\left(K_t^{\alpha}L_t^{1-\alpha}\right)^{\eta} = \left(\int\left(\mathbb{E}_{t-1}z_{it}\varepsilon_{it}\right)^{\frac{1}{1-\eta}}\,\mathrm{d}i\right)^{1-\eta}$

Macroeconomic Implications

- Financial frictions have important consequences
  - in their absence labor share would increase from 0.73 to 0.78
  - capital to output ratio would increase from 1.27 to 1.37

- Large TFP, output and wage losses

	Z	Y	W
% deviation from baseline	5.7	8.4	15.5

- Mostly due to uninsurable risk

role of collateral constraint

Role of Risk and Collateral Constraints

- Isolate role of each by studying alternative economies
  - flexibly chosen labor, risky capital
  - flexibly chosen labor and capital
  - no financial frictions

- Each recalibrated to match same targets

  details

  - discount factor β increases from 0.916 to 0.927 to 0.936 to 0.937
  - span of control η falls from 0.948 to 0.931 to 0.917 to 0.904

Distribution of Accounting Returns

- $\mathbb{E}_{t-1}\pi_t/a_t$ dispersed in all models

	mean	std	p10	p50	p90
Baseline	0.092	0.084	0.025	0.065	0.205
Labor flexible	0.088	0.104	0.022	0.049	0.196
Both flexible	0.088	0.082	0.025	0.059	0.187
No frictions	0.085	0.134	0.022	0.039	0.192

Distribution of Accounting Returns

- Returns persistent in all models. Distribution of $\overline{\pi/a}$

	mean	std	p10	p50	p90
Baseline	0.091	0.070	0.023	0.073	0.185
Labor flexible	0.086	0.079	0.022	0.055	0.189
Both flexible	0.087	0.069	0.026	0.064	0.182
No frictions	0.081	0.125	0.022	0.039	0.195

- Dispersed, persistent accounting returns $\not\Rightarrow$ financial frictions

Distribution of Financial Returns

- If labor choice flexible, financial returns dispersed
  - but mostly due to collateral constraint

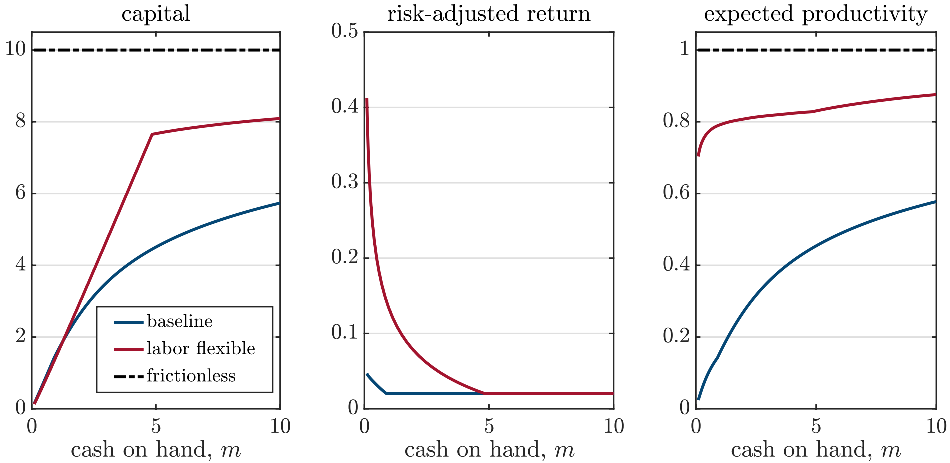
- Distribution $\mathbb{E}_{t-1}\partial\pi_t/\partial a_t$ and $\hat{\mathbb{E}}_{t-1}\partial\pi_t/\partial a_t$

	mean	std	p10	p50	p90
Baseline	0.051	0.060	0.020	0.025	0.115
	0.021	0.003	0.020	0.020	0.020
Labor flexible	0.042	0.063	0.020	0.020	0.092
	0.038	0.054	0.020	0.020	0.076
Both flexible	0.035	0.039	0.020	0.021	0.067
	0.033	0.036	0.020	0.020	0.061

Role of Labor Risk

- If labor choice flexible, expected returns lower
  - mostly reflect collateral constraint
- Because productivity shocks generate smaller fluctuations in profits
  - so covariance c_{t+1} and z_{t+1} lower, increasing desired k_{t+1}
- Optimal capital choice if collateral constraint does not bind:
  - baseline: $\frac{k_{t+1}}{k^*_{t+1}} = \left(\frac{\hat{\mathbb{E}}_t z_{t+1}\varepsilon_{t+1}}{\mathbb{E}_t z_{t+1}\varepsilon_{t+1}}\right)^{\frac{1}{1-\eta}}$
  - flexible labor: $\frac{k_{t+1}}{k^*_{t+1}} = \left(\frac{\hat{\mathbb{E}}_t \left(z_{t+1}\varepsilon_{t+1}\right)^{\frac{1}{1-(1-\alpha)\eta}}}{\mathbb{E}_t \left(z_{t+1}\varepsilon_{t+1}\right)^{\frac{1}{1-(1-\alpha)\eta}}}\right)^{\frac{1-(1-\alpha)\eta}{1-\eta}}$

Comparison of Decision Rules



Dispersion in Capital-Output Ratio

Dispersion in Capital-Output Ratio

- Data: large, persistent differences in k/y for firms in a given industry
  - often interpreted as evidence of misallocation
  - perhaps due to financial frictions

- Our model: reproduces accounting returns well, but low dispersion in k/y
  - even though model reproduces low correlation wealth and productivity
  - 0.24 rank correlation in both model and data

- Two questions:
  - how can model reproduce high π/a dispersion despite low k/y dispersion?
  - are financial frictions responsible for k/y differences in the data?

Cross-Sectional Dispersion k/y and l/y

- Output-weighted moments of k_i/y_i and l_i/y_i
  - capital-output ratio

	mean	p10	p25	p50	p75	p90
Data	1.24	0.07	0.22	0.65	1.45	2.76
Model	1.27	1.04	1.17	1.28	1.37	1.48

  - labor share

	mean	p10	p25	p50	p75	p90
Data	0.71	0.37	0.57	0.74	0.88	0.96
Model	0.74	0.61	0.68	0.74	0.79	0.85

- Most dispersion due to *persistent, within-industry* differences

evidence

Labor vs. Capital Intensity

- Sort firms in the data into 5 equally-sized bins by time-series mean k/y
  - report aggregate k/y, l/y, π/y in each bin

bin	k/y	l/y	π/y
1	0.11	0.85	0.11
2	0.35	0.78	0.14
3	0.70	0.74	0.12
4	1.29	0.67	0.13
5	3.50	0.55	0.13

- Low k/y firms have large labor share
  - so profit shares π/y similar across bins
  - so large differences in k/y do not generate large differences in π/a

What Explains Differences in k/y?

- Open question: technology differences across producers vs. distortions?

- Are financial frictions responsible for most dispersion k/y?
  - model: financially constrained firms have high k relative to a
  - so finance frictions imply negative correlation k/y and k/a
  - data: exactly the opposite pattern

Distribution of k/a

- Divide firms into 5 equally sized bins by time-series mean k/y
- Contrast k/a for firms in lowest and highest k/y quintile
  - all statistics are output-weighted

	Model		Data	
k/a	low k/y	high k/y	low k/y	high k/y
p10	0.74	0.26	0.03	0.62
p25	1.10	0.46	0.08	0.95
p50	1.46	0.74	0.20	1.49
p75	1.67	1.05	0.43	2.32
p90	1.75	1.30	0.85	4.26
fraction borrow	0.80	0.29	0.08	0.71

Conclusions

- Using Orbis data document
  - large, persistent differences in accounting returns
  - negatively correlated with net worth
  - low correlation growth rate of output and inputs
- Model of entrepreneurship consistent with evidence
  - accounts for dispersion in accounting returns well
  - large dispersion in financial returns, mostly uninsurable risk
- Financial frictions generate modest differences in k/y
  - both in the model and in the data

Robustness

Robustness

- Role of preferences
  - reduce CRRA/IIES to 1/2
- Role of fat-tailed shocks
  - assume Gaussian shocks
- Role of transitory shocks
  - eliminate transitory shocks
- Concern that book value lower than replacement value
  - K/Y in Orbis lower than EU-KLEMS (1.43 vs. 1.86)
  - increase each firm's capital by 30% and adjust equity accordingly
- Evidence on ROE for other countries

Role of Preferences

	Data	Baseline	$\theta = 1/2$
aggregate a/y	1.57	1.55	1.57
aggregate k/y	1.24	1.27	1.24
aggregate l/y	0.71	0.74	0.75
aggregate π/y	0.12	0.14	0.14
std $\Delta \log y_{it}$	0.41	0.37	0.38
iqr $\Delta \log y_{it}$	0.28	0.27	0.27
autocorr y_{it}	0.95	0.96	0.95
iqr $l_{it}/y_{it} - \overline{l_{it}/y_{it}}$	0.12	0.11	0.12
p90 k/a	1.73	1.72	1.72
β		0.916	0.959
η		0.948	0.934

Distribution of Expected Returns

	Baseline		Lower θ	
	mean	std	mean	std
$\mathbb{E}_{t-1}\pi_t/a_t$	0.092	0.084	0.086	0.068
$\mathbb{E}_{t-1}\partial\pi_t/\partial a_t$	0.051	0.060	0.038	0.041
$\hat{\mathbb{E}}_{t-1}\partial\pi_t/\partial a_t$	0.021	0.003	0.020	0.001

- Even with less risk aversion, financial returns dispersed
  - mostly due to risk, not collateral constraints

Role of Fat-Tailed Shocks

	Data	Baseline	Gaussian
aggregate a/y	1.57	1.55	1.57
aggregate k/y	1.24	1.27	1.24
aggregate l/y	0.71	0.74	0.75
aggregate π/y	0.12	0.14	0.13
std $\Delta \log y_{it}$	0.41	0.37	0.37
iqr $\Delta \log y_{it}$	0.28	0.27	0.43
autocorr y_{it}	0.95	0.96	0.96
iqr $l_{it}/y_{it} - \overline{l_{it}/y_{it}}$	0.12	0.11	0.12
p90 k/a	1.73	1.72	1.72
β		0.916	0.931
η		0.948	0.934

Distribution of Expected Returns

	Baseline		Gaussian	
	mean	std	mean	std
$\mathbb{E}_{t-1}\pi_t/a_t$	0.092	0.084	0.085	0.074
$\mathbb{E}_{t-1}\partial\pi_t/\partial a_t$	0.051	0.060	0.036	0.037
$\hat{\mathbb{E}}_{t-1}\partial\pi_t/\partial a_t$	0.021	0.003	0.028	0.023

- Financial returns 1/3 less dispersed
  - collateral constraints much more important

Role of Transitory Shocks

	Data	Baseline	No trans.
aggregate a/y	1.57	1.55	1.57
aggregate k/y	1.24	1.27	1.24
aggregate l/y	0.71	0.74	0.75
aggregate π/y	0.12	0.14	0.14
std $\log y_{it} - \log y_{it-1}$	0.41	0.37	0.41
std $\log y_{it} - \log y_{it-3}$	0.60	0.62	0.74
corr y_{it}, y_{it-1}	0.95	0.96	0.95
corr y_{it}, y_{it-3}	0.88	0.89	0.83
iqr $l_{it}/y_{it} - \overline{l_{it}/y_{it}}$	0.12	0.11	0.03
p90 k/a	1.73	1.72	1.72
β		0.916	0.931
η		0.948	0.928

Distribution of Expected Returns

	Baseline		No transitory	
	mean	std	mean	std
$\mathbb{E}_{t-1}\pi_t/a_t$	0.092	0.084	0.087	0.089
$\mathbb{E}_{t-1}\partial\pi_t/\partial a_t$	0.051	0.060	0.038	0.051
$\hat{\mathbb{E}}_{t-1}\partial\pi_t/\partial a_t$	0.021	0.003	0.032	0.031

- Financial returns 1/6 less dispersed
  - collateral constraints much more important

Scale Capital Stock by 30%

	Data	Model
aggregate a/y	1.95	1.94
aggregate k/y	1.61	1.62
aggregate l/y	0.71	0.71
aggregate π/y	0.12	0.13
std $\Delta \log y_{it}$	0.41	0.39
iqr $\Delta \log y_{it}$	0.28	0.28
autocorr y_{it}	0.95	0.95
iqr $l_{it}/y_{it} - \overline{l_{it}/y_{it}}$	0.12	0.11
p90 k/a	1.75	1.73

- With higher capital target
  - η increases from 0.948 to 0.964
  - α increases from 0.173 to 0.217
  - β increases from 0.916 to 0.932

Accounting Returns

- Distribution of π/a less dispersed, both in data and model

	mean	std	p10	p25	p50	p75	p90
Data	0.06	0.17	-0.03	0.01	0.05	0.11	0.20
Model	0.07	0.15	-0.01	0.02	0.04	0.09	0.18

Accounting Returns

- Distribution of π/a less dispersed, both in data and model

	mean	std	p10	p25	p50	p75	p90
Data	0.06	0.17	-0.03	0.01	0.05	0.11	0.20
Model	0.07	0.15	-0.01	0.02	0.04	0.09	0.18

- Distribution of $\overline{\pi/a}$ also less dispersed

	mean	std	p10	p25	p50	p75	p90
Data	0.06	0.09	0.00	0.02	0.06	0.09	0.14
Model	0.07	0.05	0.02	0.03	0.05	0.09	0.14

- Model reproduces well large and persistent differences across firms

Distribution of Expected Returns

	Baseline		Scaled k	
	mean	std	mean	std
$\mathbb{E}_{t-1}\pi_t/a_t$	0.095	0.084	0.071	0.058
$\mathbb{E}_{t-1}\partial\pi_t/\partial a_t$	0.051	0.060	0.044	0.043
$\hat{\mathbb{E}}_{t-1}\partial\pi_t/\partial a_t$	0.021	0.003	0.022	0.007

- Financial returns 1/3 less dispersed
  - mostly due to risk

Distribution of Accounting Returns

- All observations, contrast public vs. private firms. After taxes

	Spain		Italy		France	
	private	public	private	public	private	public
p10	-0.04	-0.01	-0.06	-0.05	-0.03	-0.01
p25	0.01	0.02	0.00	0.00	0.03	0.02
p50	0.05	0.08	0.04	0.05	0.10	0.07
p75	0.13	0.15	0.12	0.12	0.21	0.14
p90	0.25	0.26	0.26	0.19	0.37	0.23
p95	0.37	0.39	0.38	0.26	0.51	0.32
obs.	5.9M	1.0M	6.0M	0.4M	7.9M	0.8M

Distribution of Accounting Returns

- Privately-held firms in Spain. Compare pre- and post-tax returns

	after tax	before tax
p10	-0.04	-0.04
p25	0.01	0.01
p50	0.05	0.07
p75	0.13	0.17
p90	0.25	0.33
p95	0.37	0.48
obs.	5.9M	5.9M

Correlation Labor and Profit Share

- Compute deviation of labor and profit share from firm's mean
  - $l_{it}/y_{it} - \frac{1}{T}\sum l_{it}/y_{it}$ and $\pi_{it}/y_{it} - \frac{1}{T}\sum \pi_{it}/y_{it}$
- These deviations are very dispersed

	p01	p10	p25	p50	p75	p90	p99
labor share	-0.41	-0.15	-0.07	-0.01	0.05	0.13	1.81
profit share	-2.43	-0.16	-0.05	0.01	0.08	0.18	0.48

- And strongly negatively correlated: -0.91

back

Transitory Changes in Output

- Calculate
  - std changes $\log(y_{it}) - \log(y_{it-k})$
  - correlation $\log(y_{it})$ and $\log(y_{it-k})$

	std changes	autocorrelation
$k = 1$	0.41	0.95
$k = 2$	0.52	0.91
$k = 3$	0.60	0.88

- Changes in output have important transitory component
  - volatility of growth rates increases slowly with horizon
  - autocorrelation decays slowly with horizon

back

Frictionless Choices

- With full insurance and no collateral constraint, problem reduces to

$$\max_{k_{t+1},l_{t+1}} \quad -k_{t+1}+\frac{1}{1+r}\left(\mathbb{E}_t z_{t+1}\varepsilon_{t+1}\left(k_{t+1}^{\alpha}l_{t+1}^{1-\alpha}\right)^{\eta}-Wl_{t+1}+(1-\delta)\,k_{t+1}\right)$$

- So optimal choices are

$$k_{t+1}^{*}=\left(\frac{\alpha\eta}{R}\right)^{\frac{1-(1-\alpha)\eta}{1-\eta}}\left(\frac{(1-\alpha)\eta}{W}\right)^{\frac{(1-\alpha)\eta}{1-\eta}}\left(\mathbb{E}_t z_{t+1}\varepsilon_{t+1}\right)^{\frac{1}{1-\eta}}$$

$$l_{t+1}^{*}=\left(\frac{\alpha\eta}{R}\right)^{\frac{\alpha\eta}{1-\eta}}\left(\frac{(1-\alpha)\,\eta}{W}\right)^{\frac{1-\alpha\eta}{1-\eta}}\left(\mathbb{E}_t z_{t+1}\varepsilon_{t+1}\right)^{\frac{1}{1-\eta}}$$

back

Implications for Return on Wealth

- Expected income:

$$\mathbb{E}_t \pi_{t+1} = r a_{t+1} + \mathbb{E}_t \left[z_{t+1} \varepsilon_{t+1} \left(k_{t+1}^{\alpha} l_{t+1}^{1-\alpha} \right)^{\eta} - W l_{t+1} - R k_{t+1} \right]$$

- So expected financial returns:

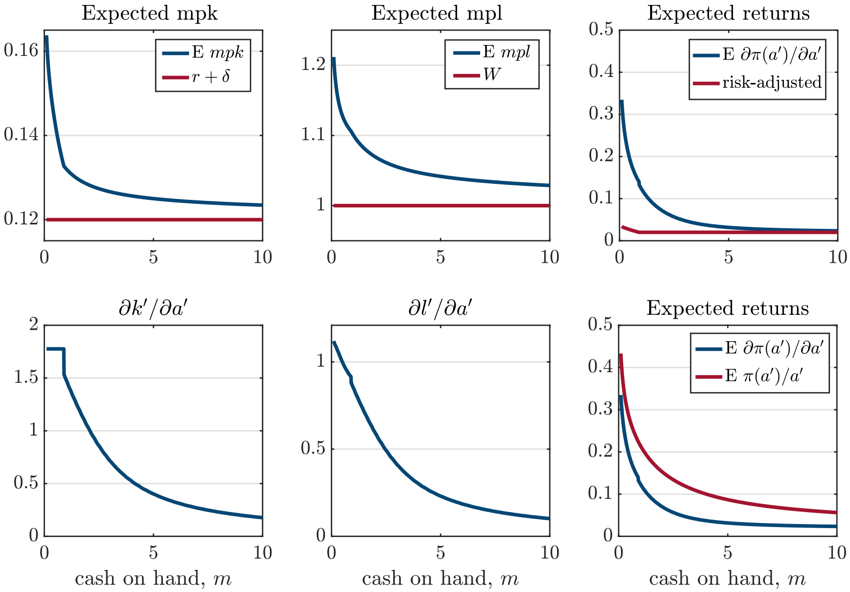
$$\frac{\partial \mathbb{E}_t \pi_{t+1}}{\partial a_{t+1}} = r + \mathbb{E}_t \left[\alpha \eta \frac{y_{t+1}}{k_{t+1}} - R \right] \frac{\partial k_{t+1}}{\partial a_{t+1}} + \mathbb{E}_t \left[(1-\alpha) \eta \frac{y_{t+1}}{l_{t+1}} - W \right] \frac{\partial l_{t+1}}{\partial a_{t+1}}$$

- Larger than risk-adjusted expected returns

$$\frac{\partial \hat{\mathbb{E}}_t \pi_{t+1}}{\partial a_{t+1}} = r + \hat{\mathbb{E}}_t \left[\alpha \eta \frac{y_{t+1}}{k_{t+1}} - R \right] \frac{\partial k_{t+1}}{\partial a_{t+1}} = r + \mu_t$$

Expected Returns

back



Cross-Sectional Dispersion k/y and l/y

- Differences very persistent
  - average capital-output ratio, $\overline{k_{it}/y_{it}}$

	mean	p10	p25	p50	p75	p90
Data	1.24	0.10	0.29	0.74	1.49	2.67
Model	1.27	1.17	1.22	1.28	1.33	1.37

  - average labor share, $\overline{l_{it}/y_{it}}$

	mean	p10	p25	p50	p75	p90
Data	0.71	0.43	0.60	0.75	0.87	0.94
Model	0.74	0.68	0.71	0.74	0.76	0.79

Cross-Sectional Dispersion k/y and l/y

- Large dispersion even when adjust for industry differences

- E.g. $\frac{k_{it}(s)/y_{it}(s)}{K_t(s)/Y_t(s)} \frac{K_t}{Y_t}$. Sector: 4-digit NACE

  - capital-output ratio

	mean	p10	p25	p50	p75	p90
actual	1.24	0.07	0.22	0.65	1.45	2.76
industry-adjusted	1.22	0.12	0.35	0.83	1.46	2.46

  - labor share

	mean	p10	p25	p50	p75	p90
raw	0.71	0.37	0.57	0.74	0.88	0.96
industry-adjusted	0.72	0.46	0.60	0.72	0.82	0.92

back

Role of Collateral Constraint

- Isolate role of collateral constraint by varying ξ
  - report % deviation from baseline ($\xi = 0.44$)

	Z	Y	W
$\xi = 0$	-0.93	-3.73	-2.03
$\xi = 1$	0.02	0.15	0.02

- Collateral constraints play minor role in baseline model
  - so most distortion due to uninsurable risk

back

Calibration

	Data	Baseline	Labor Flex	Both Flex	No FF
aggregate a/y	1.57	1.55	1.57	1.56	1.58
aggregate k/y	1.24	1.27	1.24	1.24	1.24
aggregate l/y	0.71	0.74	0.75	0.75	0.75
aggregate π/y	0.12	0.14	0.14	0.14	0.13
std $\Delta \log y_{it}$	0.41	0.37	0.38	0.40	0.38
iqr $\Delta \log y_{it}$	0.28	0.27	0.31	0.28	0.28
autocorr y_{it}	0.95	0.96	0.95	0.95	0.95
iqr $l_{it}/y_{it} - \overline{l_{it}/y_{it}}$	0.12	0.11	0	0	0
p90 k/a	1.73	1.72	1.73	1.73	1.96
β		0.916	0.927	0.936	0.937
η		0.948	0.931	0.917	0.904

back

Labor vs. Capital Intensity

- Is negative correlation k/y and l/y at odds with our mechanism?
  - model: constrained firms reduce both k/y and l/y

- Not necessarily: may reflect persistent differences in technologies

- Illustrate by sorting firms into 5 bins by time-series mean of π/a

Data			Model		
π/a	k/y	l/y	π/a	k/y	l/y
-0.06	1.72	0.82	0.03	1.37	0.79
0.03	1.65	0.74	0.07	1.32	0.76
0.07	1.25	0.70	0.11	1.28	0.74
0.11	1.31	0.67	0.15	1.24	0.72
0.22	0.68	0.69	0.24	1.17	0.69

Summary Statistics

- Baseline sample: 3.6M obs., '000 2015 USD

	mean	p10	p25	p50	p75	p90
output	604	44	90	195	438	963
labor	431	34	70	150	330	710
capital	748	10	34	126	395	1,070
equity	953	6	41	148	475	1,362
income	74	-25	0	8	37	126
employment	15	2	3	6	12	25

- Full sample: 5.9M obs., '000 2015 USD

	mean	p10	p25	p50	p75	p90
output	455	29	62	142	331	749
labor	328	22	48	111	253	561
capital	556	5	19	75	273	789
equity	693	-2	18	84	302	948
income	51	-26	-1	5	27	94
employment	12	1	2	4	10	21

back

Low Frequency

- Regress $\Delta \log l_{it}$ and $\Delta \log k_{it}$ on $\Delta \log y_{it}$

	$\Delta \log l$	$\Delta \log k$
$\Delta \log y$	0.372 (0.001)	0.152 (0.001)

- Regress $\log l_{it}$ and $\log k_{it}$ on $\log y_{it}$

	$\log l$	$\log k$
$\log y$	0.925 (0.001)	0.862 (0.002)

back