# Uniform Priors for Impulse Responses

Jonas E. Arias,[1] Juan F. Rubio-Ramírez,[2] Daniel F. Waggoner[2]

[1]Federal Reserve Bank of Philadelphia

[2]Emory University and Federal Reserve Bank of Atlanta

February 7, 2023

* The views expressed here are the authors' and not necessarily those of the Federal Reserve Bank of Atlanta, the Federal Reserve Bank of Philadelphia, or the Federal Reserve System.

# Motivation

- SVARs identified with sign restrictions are popular:
$$\mathbf{y}_t' \mathbf{A}_0 = \mathbf{x}_t' \mathbf{A}_+ + \varepsilon_t'.$$
- Many researchers use the conventional methods by Faust (1998), Canova and De Nicoló (2002), Uhlig (2005), and Rubio-Ramírez et al. (2010).
- They can be used to independently draw from any posterior distribution over the parameterization of interest subject to the identifying restrictions.
- Typically, the parameterization of interest is either the structural parameters or the impulse responses and the posterior is conjugate.
- Independently draw from a conjugate posterior over the orthogonal reduced-form parameterization and transform into the parameterization of interest:
$$\mathbf{y}_t' = \mathbf{x}_t' \mathbf{B} + \varepsilon_t' \mathbf{Q}' h(\mathbf{\Sigma}).$$
- A central ingredient is the uniform prior distribution over the set of orthogonal matrices with respect to the Haar measure.
- The normal-inverse-Wishart part of the prior is uncontroversial.

# Motivation

- Some researchers strongly caution against using it in applied work:

  *Figure 1 establishes that although the prior is uninformative about the angle of rotation, it can be highly informative for the objects about which the researcher intends to form an inference, namely the impulse response functions.(Page 1975 Baumeister and Hamilton, 2015)*

  *the distributions in Figures 1 and 2 are simply an unintended side effect of the Haar prior. (Pages 1978 Baumeister and Hamilton, 2015)*

  *Good priors lead to good inference and conversely for bad priors. Sorting out the good from the bad requires careful presentation and justification for the prior actually used, a point forcefully and convincingly made in theory and practice in Baumeister and Hamilton (2015, 2019). In this regard, the kinds of flat (Haar) priors made on the rotation matrix ... seem counterproductive (page 192). (Pages 192 Watson, 2020)*

# Motivation

## Summary of the line of reasoning underlying the critique

- Abstract from uncertainty about the reduced-form parameters by fixing them
- Draw orthogonal matrices from the uniform distribution and argue that the prior distributions over the identified sets of individual impulse responses may be nonuniform. Because the prior and the posterior coincide over the identified sets, posterior distributions over the identified sets of individual impulse responses may also be nonuniform.
- Consequently, they conclude that posterior inference could be governed by the prior over the set of orthogonal matrices.

# This Paper has Three Objectives

## Objective 1

- It is true that the conditional prior distributions of individual impulse responses are nonuniform but the problem is not as severe as Baumeister and Hamilton (2015) and Watson (2020) imply. Using Watson (2020) application we show:

  1. The conditional prior distributions of individual impulse responses are different from unconditional prior distributions of individual impulse responses.
  2. The unconditional prior distributions of individual impulse responses do not drive unconditional posterior distributions of individual impulse responses.

# This Paper has Three Objectives

## Objective 2

- Baumeister and Hamilton (2015) and Watson (2020) have a reasonable concern: it is not desirable that priors over identified sets are nonuniform.
- This could be an issue in uncommon large-sample settings.
- Inoue and Kilian (2022) argue this less problematic in tightly identified models.
- We show that the conventional method implies a uniform *joint* prior and posterior distributions over the identified set for the *vector* of impulse responses.
- This is an "if and only if" result: If we want a uniform *joint* prior and posterior distributions over the identified set for the *vector* of impulse responses we have to have a uniform prior over the set of orthogonal matrices.
- Implications for Giacomini and Kitagawa (2021).
- Holds for other typical parameterizations of interest: the structural parameters.

# This Paper has Three Objectives

## Objective 3

- We prove that the following conjecture by Baumeister and Hamilton (2015) is not true:
  *Because the objects of interest in structural VARs are highly nonlinear functions of the underlying parameters, the quest for 'noninformative' priors for structural VARs is destined to fail (Pages 1979 Baumeister and Hamilton, 2015)*
- We can have a uniform joint prior distribution for the vector of impulse responses.
- We need a particular prior distribution over the orthogonal reduced-form parameterization.
- We can use the conventional methods to draw from the joint posterior distribution for the vector of impulse responses implied by this prior.
- The key is to set a particular prior over the reduced-form parameters.
- We generalize it for a large class of parameterizations of interest.

# The Algorithm

The following algorithm independently draws from the $NGN(\tilde{\nu}, \tilde{\mathbf{\Phi}}, \tilde{\mathbf{\Psi}}, \tilde{\mathbf{\Omega}})$ posterior distribution over the structural parameterization conditional on the sign restrictions.

1. Draw $(\mathbf{B}, \mathbf{\Sigma})$ independently from the $NIW(\tilde{\nu}, \tilde{\mathbf{\Phi}}, \tilde{\mathbf{\Psi}}, \tilde{\mathbf{\Omega}})$ posterior distribution.
2. Draw $\mathbf{Q}$ independently from the uniform distribution over $\mathcal{O}(n)$.
3. Keep $(\mathbf{A}_0, \mathbf{A}_+) = f_h^{-1}(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q})$ if the sign restrictions are satisfied.
4. Return to Step 1 until the required number of draws has been obtained.

We could easily modify the algorithm (Step 3) to consider the impulse response parameterization.

# The IR Parameterization

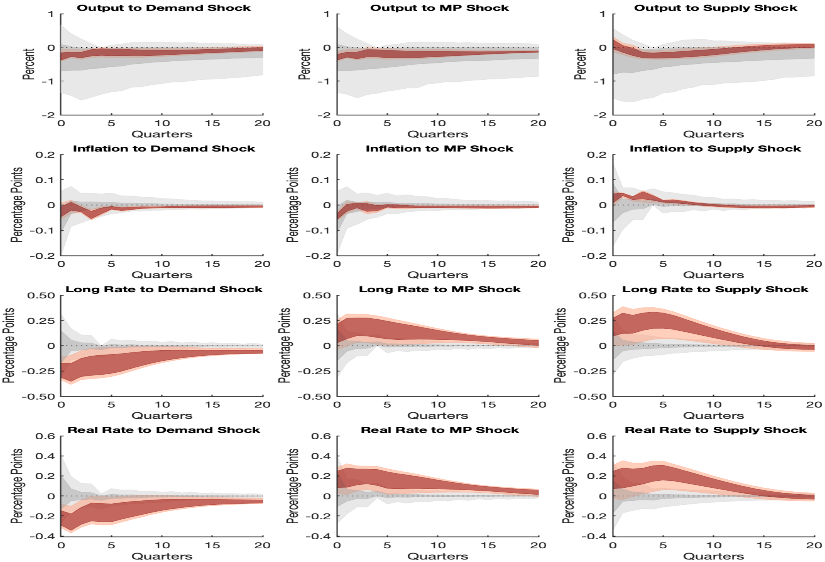
- The natural parameterization for our analysis is the IR parameterization.
- The IR parameterization is $(\mathbf{L}_0, \cdots, \mathbf{L}_p, \mathbf{c})$.
- The matrices $\mathbf{L}_k$ are functions of the structural parameterization, $\mathbf{L}_0 = \left(\mathbf{A}_0^{-1}\right)'$ and $\mathbf{L}_k = \sum_{\ell=1}^{k} \left(\mathbf{A}_\ell \, \mathbf{A}_0^{-1}\right)' \mathbf{L}_{k-\ell}$, for $1 \leq k \leq p$.
- The matrices $\mathbf{A}_k$ are also functions of the IR parameterization, $\mathbf{A}_0 = \left(\mathbf{L}_0^{-1}\right)'$ and $\mathbf{A}_k = \left(\mathbf{L}_k \, \mathbf{L}_0^{-1}\right)' \mathbf{A}_0 - \sum_{\ell=1}^{k-1} \left(\mathbf{L}_{k-\ell} \, \mathbf{L}_0^{-1}\right)' \mathbf{A}_\ell$, for $1 \leq k \leq p$.
- Let $\mathbf{L}_+' = \begin{bmatrix} \mathbf{L}_1' & \cdots & \mathbf{L}_p' & \mathbf{c}' \end{bmatrix}$, the IR parameterization is $(\mathbf{L}_0, \mathbf{L}_+)$.
- We will denote the mapping from the IR parameterization to the structural parameterization by $f_{ir}$.
- We will denote the mapping from the IR parameterization to the orthogonal reduced-form parameterization by $\phi_h = f_h \circ f_{ir}$.
- Marginal distributions of individual IRs are marginal distributions of individual parameters in the IR parameterization.
- A joint distribution of IRs is a joint distribution over the IR parameterization.

# Revisiting

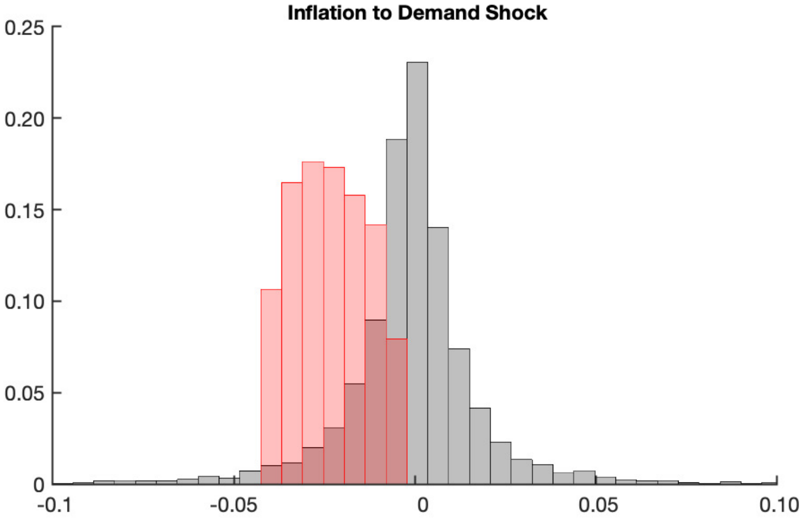
- The model used in the empirical section follows Watson (2020), quarterly U.S. data over the period 1984Q1:2007Q4 on $\mathbf{y}_t' = (\Delta(y_t - n_t), n_t, \Delta p_t, i_t^L)$ with 4 lags and an intercept. We use a Normal-Inverse Wishart prior over the reduced-form parameters.
- Shocks identified with sign and zero restrictions.

| Variable \Shock | Technology | Demand | Monetary Policy | Supply |
|---|---|---|---|---|
| Restrictions on 4-quarter ahead IRs | | | | |
| Output | | $-$ | $-$ | $-$ |
| Price Level | | $-$ | $-$ | |
| Inflation | | | | $+$ |
| 10-Year Treasury Bond Rate | | $-$ | $+$ | |
| Restrictions on long-run IRs | | | | |
| Labor Productivity Growth | | 0 | 0 | 0 |

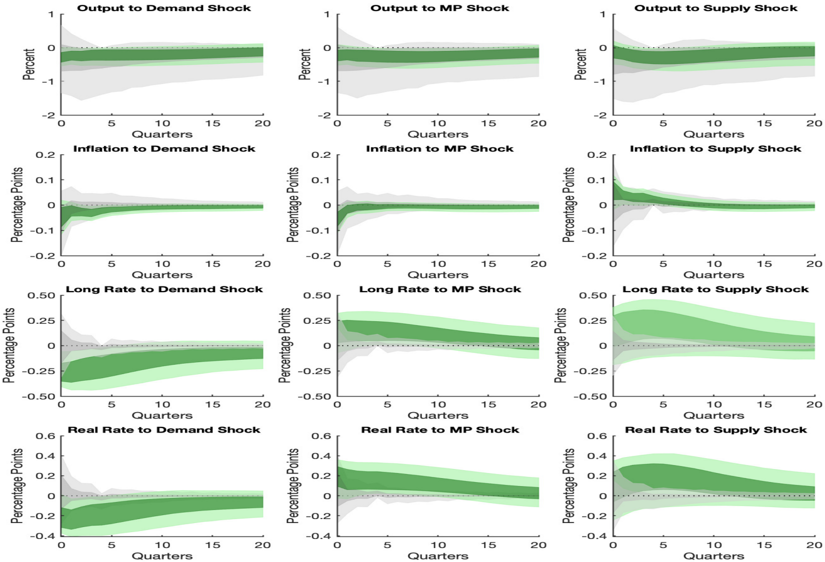
# Effects of Prior



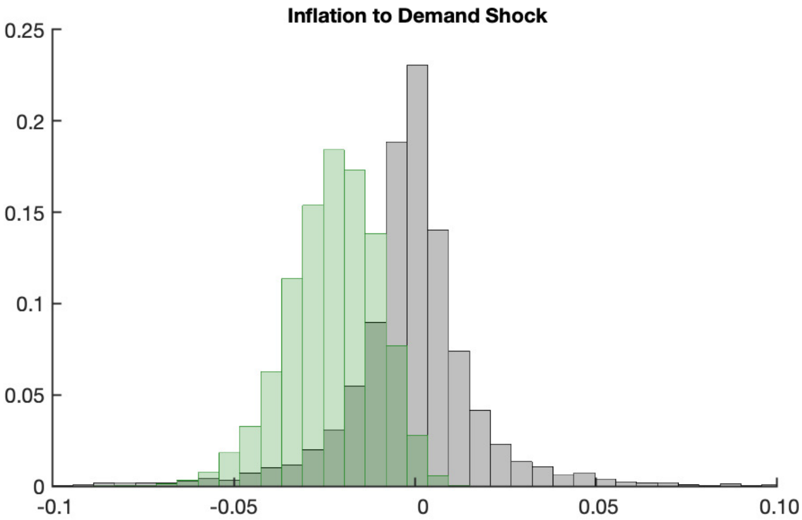
# Effects of Prior



# Effects of Prior



# Effects of Prior



# Conditional Joint Prior for Impulse Responses

- Let $f_p$ denote the projection from the orthogonal reduced-form parameterization onto the reduced-form parameters
- Then, $\phi = f_p \circ \phi_h$ is the mapping from the IR parameterization to the reduced-form parameters, and $\phi$ does not depend on $h$.
- The set $\phi^{-1}(\mathbf{B}, \mathbf{\Sigma})$ will be the submanifold that is the support of the joint distribution of the IR parameterization conditional on $(\mathbf{B}, \mathbf{\Sigma})$.
- The submanifold structure induces the volume measure over $\phi^{-1}(\mathbf{B}, \mathbf{\Sigma})$.
- If $\pi(\mathbf{L}_0, \mathbf{L}_+)$ is a density over the IR parameterization with respect to the Lebesgue measure ...
- ... then the density conditional on $(\mathbf{B}, \mathbf{\Sigma})$ with respect to the volume measure over $\phi^{-1}(\mathbf{B}, \mathbf{\Sigma})$ will be proportional to $\pi(\mathbf{L}_0, \mathbf{L}_+)$.
- The density with respect to the volume measure over $\phi^{-1}(\mathbf{B}, \mathbf{\Sigma})$ will be uniform if and only if $\pi(\mathbf{L}_0, \mathbf{L}_+)$ is constant over $\phi^{-1}(\mathbf{B}, \mathbf{\Sigma})$.

# Conditional Joint Prior for Impulse Responses

## Proposition

*For every density over the IR parameterization with respect to Lebesgue measure, the density conditional on $(\mathbf{B}, \boldsymbol{\Sigma})$ with respect to the volume measure over $\phi^{-1}(\mathbf{B}, \boldsymbol{\Sigma})$ is uniform for every $(\mathbf{B}, \boldsymbol{\Sigma})$ if and only if the induced distributions over the orthogonal reduced-form parameters $(\mathbf{B}, \boldsymbol{\Sigma})$ and $\mathbf{Q}$ are independent and the distribution of $\mathbf{Q}$ is uniform with respect to the Haar measure.*

## Proposition

*For every density over the IR parameterization with respect to Lebesgue measure, the density conditional on $(\mathbf{B}, \boldsymbol{\Sigma})$ with respect to the volume measure over $\phi^{-1}(\mathbf{B}, \boldsymbol{\Sigma})$ is constant over observationally equivalent vectors of impulse responses if and only if the induced distributions over the orthogonal reduced-form parameters $(\mathbf{B}, \boldsymbol{\Sigma})$ and $\mathbf{Q}$ are independent and the distribution of $\mathbf{Q}$ is uniform with respect to the Haar measure.*

# Conditional Joint Prior for Impulse Responses

- This is an "if and only if" result and holds for typical parameterizations of interest: the structural parameters or the impulse responses.
- A uniform joint prior distribution over the identified set for the vector of impulse responses one *must* use a uniform prior distribution over the orthogonal matrices.
- Any other choice of prior over the orthogonal matrices *will* imply a nonuniform joint prior distribution over the identified set for the vector of impulse responses.
- This is true for any prior distribution over the reduced-form parameters.
- This result is important for the robust methodology in Giacomini and Kitagawa (2021), they consider all possible priors over the orthogonal matrices.
- These are *joint* claims.

# Uniform Joint Prior for Impulse Responses

- We show that it is possible to conduct Bayesian inference based on a uniform joint prior distribution for the vector of impulse responses and that such type of prior can be implemented by slightly modifying the methods in Rubio-Ramírez, Waggoner, and Zha (2010).
- If $\pi(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q})$ is any density over the orthogonal reduced-form parameterization, the mapping $\phi_h = f_h \circ f_{ir}$ induces a distribution over the IR parameterization.
- The induced density over the IR parameterization will be $\pi(\phi_h(\mathbf{L}_0, \mathbf{L}_+))v_{\phi_h}(\mathbf{L}_0, \mathbf{L}_+)$.
- The volume element is $v_{\phi_h}(\mathbf{L}_0, \mathbf{L}_+) = 2^{\frac{n(n+1)}{2}} |\det(\mathbf{L}_0)|^{-(m-3)}$.
- We also have $v_{\phi_h}(\phi_h^{-1}(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q})) = 2^{\frac{n(n+1)}{2}} |\det(\mathbf{\Sigma})|^{-\frac{(m-3)}{2}}$.

# Uniform Joint Prior for Impulse Responses

- If $\pi(\mathbf{L}_0, \mathbf{L}_+)$ is any density over the IR parameterization, then the induced density over the orthogonal reduced-form parameterization will be

$$\pi(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q}) = \frac{\pi(\phi_h^{-1}(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q}))}{v_{\phi_h}(\phi_h^{-1}(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q}))} = \frac{\pi(\phi_h^{-1}(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q}))}{2^{\frac{n(n+1)}{2}} |\det(\mathbf{\Sigma})|^{-\frac{(m-3)}{2}}}$$

.

## Proposition

*A density over the IR parameterization with respect to the Lebesgue measure is uniform if and only if the induced prior distribution over the orthogonal reduced-form parameterization has density proportional to* $|\det(\mathbf{\Sigma})|^{\frac{m-3}{2}}$ *with respect to volume measure.*

# Uniform Joint Prior for Impulse Responses

### Proposition (by DeJong (1992))

*Let $a > 2n + 2 + m - T$. If the reduced-form prior density is proportional to $|\det(\mathbf{\Sigma})|^{-\frac{a}{2}}$, then the normal-inverse-Wishart posterior density over the reduced-form is defined by*

$$NIW_{\left(\hat{\nu}(a), \hat{\mathbf{S}}, \hat{\mathbf{B}}, (\mathbf{X}'\mathbf{X})^{-1}\right)}(\mathbf{B}, \mathbf{\Sigma})$$

*where $\hat{\nu}(a) = T + a - m - n - 1$.*

### Proposition

*If the prior density over the orthogonal reduced-form parameterization is proportional to $|\det(\mathbf{\Sigma})|^{\frac{m-3}{2}}$, the posterior density over the orthogonal reduced-form parameterization is*

$$UNIW_{\left(\hat{\nu}(-(m-3)), \hat{\mathbf{S}}, \hat{\mathbf{B}}, (\mathbf{X}'\mathbf{X})^{-1}\right)}(\mathbf{B}, \mathbf{\Sigma}).$$

# The Algorithm

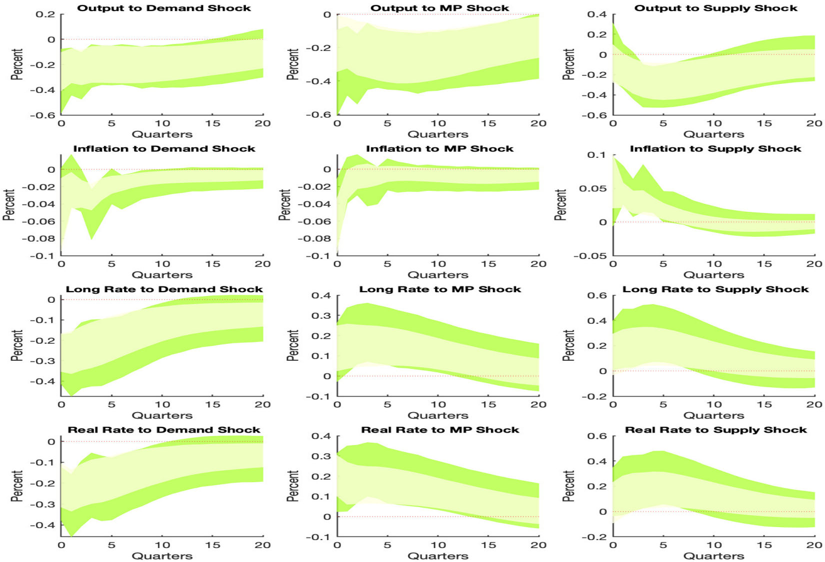
The following algorithm independently draws from the posterior distribution over the IR parameterization conditional on the sign restrictions implied by a uniform prior distribution over the IR parameterization.

1. Draw $(\mathbf{B}, \mathbf{\Sigma})$ independently from the $NIW\left(\hat{\nu}(-(m-3)), \hat{\mathbf{S}}, \hat{\mathbf{B}}, (\mathbf{X}'\mathbf{X})^{-1}\right)$ distribution.
2. Draw $\mathbf{Q}$ independently from the uniform distribution over $\mathcal{O}(n)$.
3. Keep $(\mathbf{L}_0, \mathbf{L}_+) = \phi_h^{-1}(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q})$ if the sign restrictions are satisfied.
4. Return to Step 1 until the required number of draws has been obtained.

# The Application

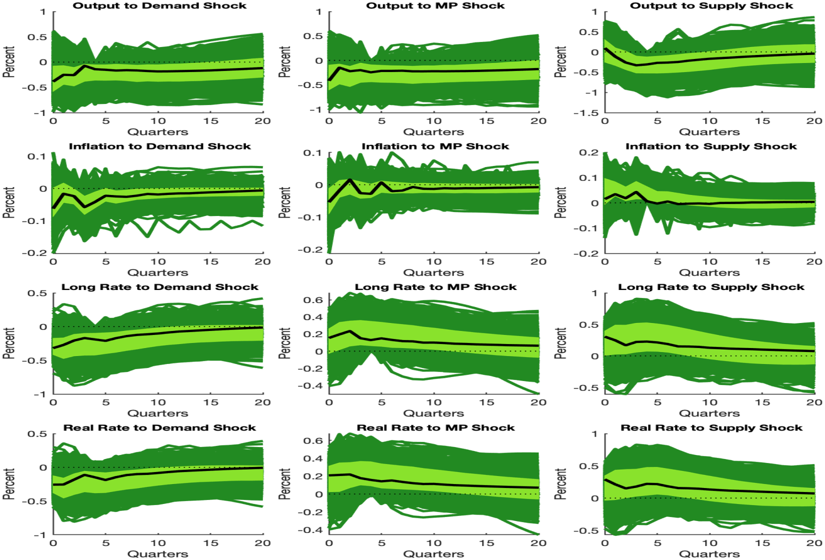
- Illustration uniform joint prior distribution for the vector of impulse responses.
- We begin by comparing the unconditional posterior distributions of individual impulse responses implied by uniform joint prior distribution for the vector of impulse responses and the commonly used Minnesota prior.
- Next we show the difference between marginal and joint inference.
- We finalize by comparing the joint posterior distribution for the vector of impulse responses implied by uniform joint prior distribution for the vector of impulse responses and the commonly used Minnesota prior.

# Marginals

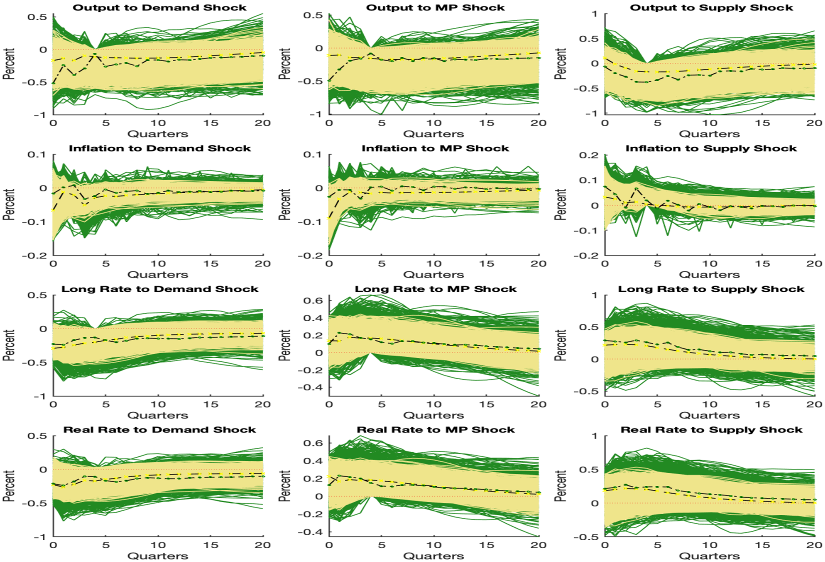


Output to Demand Shock
Output to MP Shock
Output to Supply Shock
Inflation to Demand Shock
Inflation to MP Shock
Inflation to Supply Shock
Long Rate to Demand Shock
Long Rate to MP Shock
Long Rate to Supply Shock
Real Rate to Demand Shock
Real Rate to MP Shock
Real Rate to Supply Shock
Percent
Quarters

# Marginal versus Joint



# Joints


Output to Demand Shock
Output to MP Shock
Output to Supply Shock
Inflation to Demand Shock
Inflation to MP Shock
Inflation to Supply Shock
Long Rate to Demand Shock
Long Rate to MP Shock
Long Rate to Supply Shock
Real Rate to Demand Shock
Real Rate to MP Shock
Real Rate to Supply Shock
Percent
Quarters

## Objects of Interest

- The insights of the previous sections generalize to a general class of vectors of objects of interest, $\boldsymbol{\Upsilon}$, that can be represented by a one-to-one and onto continuously differentiable transformation of the orthogonal reduced-form parameterization.
- Let us assume a simple model with two variables, no constant, and no lags:

$$\mathbf{y}_t' \, \mathbf{A}_0 = \boldsymbol{\varepsilon}_t'.$$

- Let us assume that we want to define joint priors about individual elements of:
  - $\mathbf{A}_0$
  - $\mathbf{L}_0$
- In particular, we want to write a joint prior over:
  - $A_{0,1,1}$
  - $A_{0,2,2}$
  - $A_{0,2,1}$
  - $L_{0,1,1}$
- The object of interest is $\boldsymbol{\Upsilon} = (A_{0,1,1}, A_{0,2,2}, A_{0,2,1}, L_{0,1,1})'$.
- And we want to have a prior over the object of interest $\pi(\boldsymbol{\Upsilon})$.

# Uniform Joint Priors for Objects of Interest

- If $\pi(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q})$ is any density over the orthogonal reduced-form parameterization, the invertible mapping $\phi_o$ induces a distribution over $\mathbf{\Upsilon}$:

$$\pi(\phi_o(\mathbf{\Upsilon}))v_{\phi_o}(\mathbf{\Upsilon}).$$

- A prior over the vector of objects of interest implies:

$$\pi(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q}) = \pi(\phi_o^{-1}(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q}))v_{\phi_o^{-1}}(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q}).$$

- A uniform prior over the vector of objects of interest implies:

$$\pi(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q}) \propto v_{\phi_o^{-1}}(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q}).$$

- The induced density may depend on $\mathbf{Q}$, hence the induced prior over the set of orthogonal matrices is not uniform.

# Conditional Joint Prior for Objects of Interest

- Let $f_p$ denote the projection from the orthogonal reduced-form parameterization onto the reduced-form parameters
- Then $\hat{\phi}_o = f_p \circ \phi_o$ is the mapping from the vector of objects of interest to the reduced-form parameters.
- The set $\hat{\phi}_o^{-1}(\mathbf{B}, \mathbf{\Sigma})$ will be the submanifold that is the support of the joint distribution of the vector of object of interest conditional on $(\mathbf{B}, \mathbf{\Sigma})$.
- The submanifold structure induces the volume measure over $\hat{\phi}_o^{-1}(\mathbf{B}, \mathbf{\Sigma})$.
- If $\pi(\mathbf{\Upsilon})$ is a density over the vector of object of interest with respect to the Lebesgue measure ...
- ... then the density conditional on $(\mathbf{B}, \mathbf{\Sigma})$ with respect to the volume measure over $\hat{\phi}_o^{-1}(\mathbf{B}, \mathbf{\Sigma})$ will be proportional to $\pi(\mathbf{\Upsilon})$.
- The density with respect to the volume measure over $\hat{\phi}_o^{-1}(\mathbf{B}, \mathbf{\Sigma})$ will be uniform if and only if $\pi(\mathbf{\Upsilon})$ is constant over $\hat{\phi}_o^{-1}(\mathbf{B}, \mathbf{\Sigma})$.

# Conditional Joint Prior for Objects of Interest

### Proposition

*For every density over the vector of objects of interest with respect to Lebesgue measure, the density conditional on* $(\mathbf{B}, \mathbf{\Sigma})$ *with respect to the volume measure over* $\hat{\phi}_o^{-1}(\mathbf{B}, \mathbf{\Sigma})$ *is uniform for every* $(\mathbf{B}, \mathbf{\Sigma})$ *if and only if the induced distribution over the orthogonal reduced-form parameterization is such that* $\pi(\mathbf{Q} \,|\, \mathbf{B}, \mathbf{\Sigma})$ *is proportional to* $v_{\phi_o^{-1}}(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q})$.

### Proposition

*For every density over the vector of objects of interest with respect to Lebesgue measure, the density conditional on* $(\mathbf{B}, \mathbf{\Sigma})$ *with respect to the volume measure over* $\phi^{-1}(\mathbf{B}, \mathbf{\Sigma})$ *is constant over observationally equivalent vectors of interest if and only if the induced distributions over the orthogonal reduced-form parameters* $(\mathbf{B}, \mathbf{\Sigma})$ *and* $\mathbf{Q}$ *are independent and the distribution of* $\mathbf{Q}$ *is such that* $\pi(\mathbf{Q} \,|\, \mathbf{B}, \mathbf{\Sigma})$ *is proportional to* $v_{\phi_o^{-1}}(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q})$.

# An Algorithm

The following algorithm independently draws from the posterior distribution over the vector of objects of interest conditional on the sign restrictions implied by a uniform prior distribution over the vector of objects of interest.

1. Draw $(\mathbf{B}, \mathbf{\Sigma})$ independently from the $NIW_{(\hat{\nu}, \hat{\mathbf{\Phi}}, \hat{\mathbf{\Psi}}, \hat{\mathbf{\Omega}})}(\mathbf{B}, \mathbf{\Sigma})$ distribution.
2. Draw $\mathbf{Q}$ independently from the uniform distribution over $\mathcal{O}(n)$.
3. If $\mathbf{\Upsilon} = \phi_o^{-1}(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q})$ satisfies the sign restrictions, then set its importance weight to:
$$\frac{1}{v_{\phi_o^{-1}}(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q})}.$$
   Otherwise, set its importance weight to zero.
4. Return to Step 1 until the required number of draws has been obtained.

# An Example

Consider a version of the two-variable SVAR described in Baumeister and Hamilton (2015) in order to illustrate the algorithm. Accordingly, let

$$\Delta n_t = k^d + \beta^d \Delta w_t + b_w^d \Delta w_{t-1} + b_n^d \Delta n_{t-1} + \sigma^d u_t^d, \tag{1}$$
$$\Delta n_t = k^s + \alpha^s \Delta w_t + b_w^s \Delta w_{t-1} + b_n^s \Delta n_{t-1} + \sigma^s u_t^s, \tag{2}$$

where the vector $\left(u_t^d, u_t^s\right)'$, conditional on past information and the initial conditions, is Gaussian with mean zero and covariance matrix $\boldsymbol{I}_2$. Letting $\mathbf{y}_t$ denote the endogenous variables (i.e., $\mathbf{y}_t = (\Delta w_t, \Delta n_t)'$), it should be clear that Baumeister and Hamilton's (2015) $(\mathbf{A}, \mathbf{D}, \mathbf{\Pi})$ parameterization of Equations (1) and (2) is

$$\mathbf{A}\mathbf{y}_t = \mathbf{\Pi}\mathbf{x}_{t-1} + \mathbf{u}_t \tag{3}$$

where $\mathbf{u}_t = \left(u_t^d, u_t^s\right)'$, $\mathbf{x}_{t-1} = (\mathbf{y}_{t-1}, 1)'$, and

$$\mathbf{A} = \begin{bmatrix} -\beta^d & 1 \\ -\alpha^s & 1 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} \sigma^d & 0 \\ 0 & \sigma^s \end{bmatrix}, \text{and } \mathbf{\Pi}' = \begin{bmatrix} b_w^d & b_n^d & k^d \\ b_w^s & b_n^s & k^s \end{bmatrix}.$$

# An Example

- For compactness, we let $\boldsymbol{\Upsilon} = \left(\beta^d, \alpha^s, \sigma^d, \sigma^s, b_w^d, b_n^d, k^d, b_w^s, b_n^s, k^s\right)'$ denote the vector of objects of interest. Given $\boldsymbol{\Upsilon}$, we will construct a mapping $f_o$ from $\boldsymbol{\Upsilon}$ to the structural parameters and we will obtain the joint prior over the orthogonal reduced-form parameterization implied by a uniform joint prior distribution for the vector of objects of interest.
- The mapping $f_o$ is defined as follows:

$$f_o(\boldsymbol{\Upsilon}) = \left( \underbrace{\mathbf{A}'\boldsymbol{D}^{-1}}_{\mathbf{A}_0}, \underbrace{\boldsymbol{\Pi}\boldsymbol{D}^{-1}}_{\mathbf{A}_+} \right).$$

The inverse of the $f_o$ mapping is given by:

$$f_o^{-1}(\mathbf{A}_0, \mathbf{A}_+) = (-\mathbf{A}(1,1), -\mathbf{A}(2,1), \text{diag}(\mathbf{D}), \text{vec}(\boldsymbol{\Pi}))$$

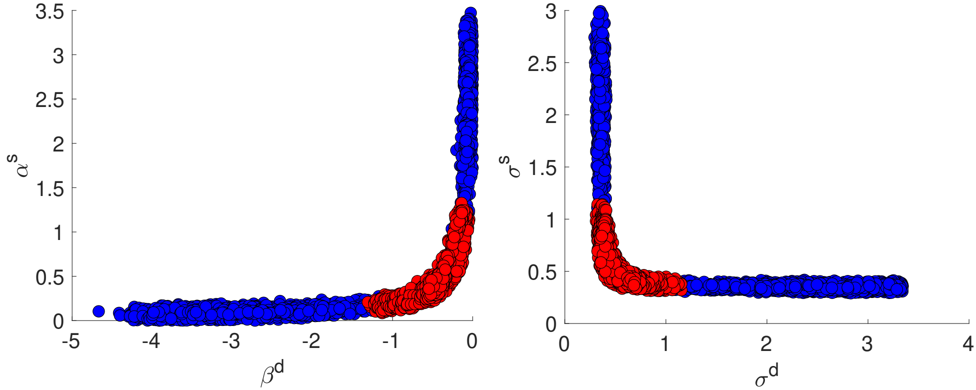
where

$$\mathbf{D} = \begin{bmatrix} \mathbf{A}_0^{-1}(2,1) & 0 \\ 0 & \mathbf{A}_0^{-1}(2,2) \end{bmatrix}, \mathbf{A} = \mathbf{D}\mathbf{A}_0', \text{and } \boldsymbol{\Pi} = \mathbf{A}_+ \mathbf{D}.$$

# An Example

- The volume element is $v_{f_o}(\boldsymbol{\Upsilon}) = |\sigma^d \sigma^s|^{-6}$. This implies that the importance weights are:
$$|\sigma^d \sigma^s|^6.$$
- Finally, following Baumeister and Hamilton (2015), we impose the following sign restrictions $\beta^d < 0$ and $\alpha^s > 0$.
- We will compare the results to ones obtained with a conjugate uniform-normal-inverse-Wishart prior distribution over the orthogonal reduced-form parameterization where the normal-inverse-Wishart part of the prior is a standard Minnesota.

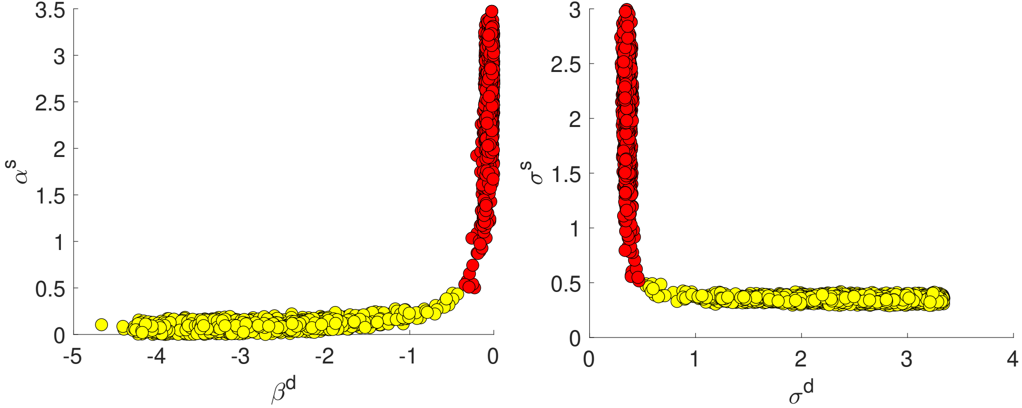
# An Example




(a) Elasticities (b) Shock Sizes

Figure: Uniform joint prior distribution for the vector of objects of interest (Blue) versus Minnesota prior. The 68 percent credible sets under the absolute value loss function using a uniform joint prior distribution for the vector of objects of interest and a Minnesota prior over the reduced-form parameters.

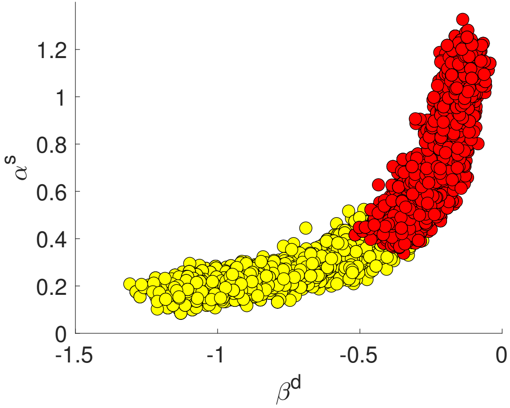
# An Example



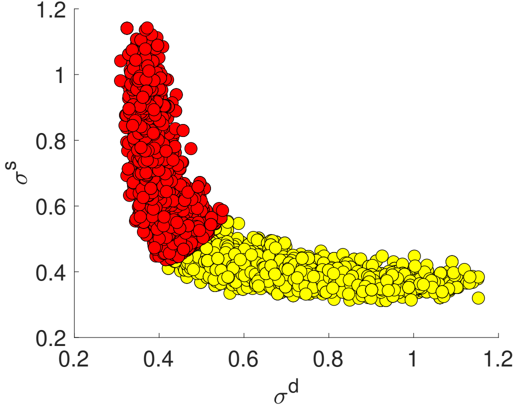

(a) Elasticities (b) Shock Sizes

FIGURE: The 68 percent credible sets under the absolute value loss function using a uniform joint prior distribution for the vector of objects of interest.

# An Example




(a) Elasticities




(b) Shock Sizes

Figure: The 68 percent credible sets under the absolute value loss function using a Minnesota prior.

# Conclusions

- The conventional method implies prior and posterior distributions over the identified sets of individual impulse responses that may be nonuniform.
- But, we should not discard the conventional method.
- It implies joint prior and posterior distributions over the identified set for the vector of impulse responses that are uniform.
- The uniform distribution over the set of orthogonal matrices with respect to the Haar measure is not only sufficient, it is also necessary.
- We can implement uniform joint prior distribution for the vector of impulse responses using the conventional methods.
- Offers a practical complement to Giacomini and Kitagawa (2021).
- Their robust prior numerical method only marginal.
- We offer a compromise for researchers whose goal is to perform joint posterior inference without favoring some vector of impulse responses over others a priori.

# REFERENCES


Baumeister, C. and J. D. Hamilton (2015): "Sign Restrictions, Structural Vector Autoregressions, and Useful Prior Information," *Econometrica*, 83, 1963–1999.

——— (2019): "Structural Interpretation of Vector Autoregressions with Incomplete Identification: Revisiting the Role of Oil Supply and Demand Shocks," *American Economic Review*, 109, 1873–1910.

Canova, F. and G. De Nicoló (2002): "Monetary Disturbances Matter for Business Fluctuations in the G-7," *Journal of Monetary Economics*, 49, 1131–1159.

DeJong, D. N. (1992): "Co-integration and Trend-stationarity in Macroeconomic Time Series: Evidence from the Likelihood Function," *Journal of Econometrics*, 52, 347–370.

Faust, J. (1998): "The Robustness of Identified VAR Conclusions about Money," *Carnegie-Rochester Conference Series on Public Policy*, 49, 207–244.

Giacomini, R. and T. Kitagawa (2021): "Robust Bayesian Inference for Set-identified Models," *Econometrica*, 89, 1519–1556.

Inoue, A. and L. Kilian (2022): "The Role of the Prior in Estimating VAR Models with Sign Restrictions," *FRB of Dallas Working Paper No. 2030*.

Rubio-Ramírez, J., D. Waggoner, and T. Zha (2010): "Structural Vector Autoregressions: Theory of Identification and Algorithms for Inference," *Review of Economic Studies*, 77, 665–696.

Uhlig, H. (2005): "What are the Effects of Monetary Policy on Output? Results from an Agnostic Identification Procedure," *Journal of Monetary Economics*, 52, 381–419.

Watson, M. W. (2020): "Comment," *NBER Macroeconomics Annual*, 34, 182–193.