Unconventional Monetary Policy According to HANK

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Fed Asset Purchases

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fred.stlouisfed.org
Research Question

How does quantitative easing (QE) transmit to the aggregate economy when there are heterogeneous households with uninsurable income risk?

▷ Is QE more or less effective?
▷ Is much lost by using a representative agent framework?
▷ What are the distributional consequences of QE shocks?
▷ Do micro- and macro-level wealth distributions matter or not?
Literature

Marrying two literatures:

1. DSGE models to study QE (RANK)
   - Gertler and Karadi (2011, 2013); Carlstrom, Fuerst, and Paustian (2017); Sims and Wu (2020, 2021); Sims, Wu, and Zhang (forthcoming)

2. DSGE models to study monetary policy with heterogeneous agents (HANK)
   - McKay, Nakamura, and Steinsson (2016); Kaplan, Moll, and Violante (2018); Auclert (2019); Acharya and Dogra (2020); Alves, Kaplan, Moll, and Violanta (2020); Ravn and Sterk (2020)
Approach

DSGE model with constrained financial intermediaries (Sims and Wu 2020) and heterogeneous households

- Intermediaries borrow short and lend long, subject to a leverage constraint
- Production firms float long-term debt to finance investment
- QE shocks ease these constraints, stimulate investment
- Households similar to Krusell and Smith (1998) with endogenous labor supply, subject to borrowing constraint and uninsurable income risk

Solve model using perturbation methods
Findings

Aggregate effects of a QE shock are very similar in a HANK version of the model compared to a RANK version

- QE slightly more stimulative; driven by poorest households
- Micro wealth distribution (Gini coefficient, Lorenz curve) not important for aggregate transmission
- Macro parameters (unemployment rate, unemployment benefit) have small implications for aggregate transmission, in direction one might expect

Conclusion: RANK model good approximation to HANK model for understanding QE
Plan

1. Model
2. Solution method
3. HANK vs. RANK
4. Micro wealth distribution
5. Macro wealth distribution
6. Conclusion
Model
Overview

1. Households: uninsurable, idiosyncratic employment risk, borrowing constraint, save via short-term deposits

2. Production firms: float long-term bonds to finance investment

3. Financial intermediaries: stand between households and production firms

4. Price and wage stickiness

5. Central bank

6. Fiscal authority
Households

\[
\max_{c_{j,t}, l_{j,t}, d_{j,t}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \log c_{j,t} - \chi \frac{l_{j,t}^{1+\eta}}{1+\eta} \right)
\]

s.t.

\[
d_{j,t} = \frac{R_{t-1}^d}{\Pi_t} \left( d_{j,t-1} + m_{rs_t} \left[ (1 - \tau) l_{j,t} \varepsilon_{j,t} + \mu (1 - \varepsilon_{j,t}) \right] \right)
- c_{j,t} - \tau_{j,t} + \text{div}_{j,t} - X_j
\]

\[
d_{j,t} \geq d
\]

\[
l_{j,t} \leq \bar{l}
\]
\( \varepsilon_{jt} \in \{0, 1\} \)

\[
\begin{bmatrix}
    p(\varepsilon_{j,t+1} = 0|\varepsilon_{jt} = 0) & p(\varepsilon_{j,t+1} = 1|\varepsilon_{jt} = 0) \\
    p(\varepsilon_{j,t+1} = 0|\varepsilon_{jt} = 1) & p(\varepsilon_{j,t+1} = 1|\varepsilon_{jt} = 1)
\end{bmatrix}
\]

\[
= \begin{bmatrix}
    p & 1 - p \\
    \frac{U}{1-U}(1 - p) & 1 - \frac{U}{1-U}(1 - p)
\end{bmatrix}
\]

\( U = p(\varepsilon_{jt} = 0) \)

\( L_t = \int_0^1 l_{j,t} \, dj \)
\[ c_{j,t}^{-1} \geq \beta R_t^d E_t^{c_{j,t+1}} \frac{c_{j,t+1}^{-1}}{\Pi_{t+1}} \]

\[ l_{j,t}^\eta \leq \frac{(1 - \tau) mrs_t}{\Lambda c_j, t} \]
Wholesale Producer

Production:

\[ Y_{m,t} = Z_t (u_t K_t)^\alpha L_{d,t}^{1-\alpha} \]

Accumulation:

\[ K_{t+1} = \hat{I}_t + (1 - \delta(u_t)) K_t \]

Long-term bonds: coupons decay at rate \( \kappa \in [0, 1] \), trade at \( Q_t \)

Investment constraint:

\[ \psi P_t^k \hat{I}_t \leq Q_t (F_{m,t} - \kappa F_{m,t-1}) \]
Financial Intermediaries

Probability of exit: $1 - \sigma$

New intermediaries get $X$ in startup net worth

Balance sheet:

$$Q_t F_t + Q_{B,t} B_t + RE_t = D_t + N_t$$

Law of motion

$$N_t = \left( R_t^F - R_{t-1}^d \right) Q_{t-1} F_{t-1} + \left( R_t^B - R_{t-1}^d \right) Q_{B,t-1} B_{t-1}$$

$$+ \left( R_{t-1}^{re} - R_{t-1}^d \right) RE_{t-1} + R_{t-1}^d N_{t-1}$$

$$R_t^F = \frac{1 + \kappa Q_t}{Q_{t-1}}, \quad R_t^B = \frac{1 + \kappa Q_{B,t}}{Q_{B,t-1}}$$
Costly Enforcement Constraint

As in Gertler and Karadi (2011, 2013) and Sims and Wu (2021):

\[ V_t \geq \theta (Q_t f_t + \Delta Q_{B,t} b_t) \]

Optimality:

\[
\mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1} \left( R_{t+1}^F - R_t^d \right) = \frac{\lambda_t}{1 + \lambda_t} \theta \\
\mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1} \left( R_{t+1}^B - R_t^d \right) = \frac{\lambda_t}{1 + \lambda_t} \theta \Delta \\
\mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1} \left( R_{t+1}^{re} - R_t^d \right) = 0
\]

Where:

\[
\Omega_t = 1 - \sigma + \sigma \theta \phi_t \\
\phi_t = \frac{1 + \lambda_t}{\theta} \mathbb{E}_t [\Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1}] R_t^d.
\]
Endogenous Leverage Constraint

Value of firm:

\[ V_t = \theta \phi_t n_t. \]

When the constraint binds

\[ \phi_t = \frac{Q_t f_t + \Delta Q_{B,t} b_t}{n_t} \]

So:

\[ \theta \phi_t \geq 1 + \lambda_t \]
Fiscal and Monetary Policy

Fiscal budget:

\[ P_t G_t + P_{t-1} \bar{b}_G + MRS_t \mu U = \]
\[ P_t T_t + P_t T_{cb,t} + Q_{B,t} P_t \bar{b}_G (1 - \kappa \Pi_t^{-1}) + \tau MRS_t L_t \]

Taylor rule:

\[ \ln R^{re}_t = (1 - \rho_r) \ln R^{re}_{t-1} + \rho_r \ln R^{re}_t + \]
\[ (1 - \rho_r) \left[ \phi_\pi (\ln \Pi_t - \ln \Pi) + \phi_y (\ln Y_t - \ln Y_{t-1}) \right] + \sigma_r \epsilon_{r,t} \]

Balance sheet

\[ Q_t F_{cb,t} + Q_{B,t} B_{cb,t} = RE_t \]
Solution Method
Solving heterogeneous agent models is hard

Similar to Winberry (2018), develop a way to solve the model via perturbation methods in Dynare

- Quick
- People understand it
- Easy to have lots of state variables and many sources of aggregate uncertainty
- Different than Winberry (2018), use non-parametric approximation for cross-sectional wealth distribution
Equilibrium

Equilibrium conditions

- Individual decisions: $c_t(d_{t-1}, \varepsilon_t)$, $l_t(d_{t-1}, \varepsilon_t)$, $d_t(d_{t-1}, \varepsilon_t)$
- Cross-sectional distribution: $p(d_{t-1}, \varepsilon_t)$
- Aggregation

\[
d_{t-1} = \sum_{\varepsilon_t} \int d_{t-1} p(d_{t-1}, \varepsilon_t) \, dd_{t-1}
\]

\[
L_t = \sum_{\varepsilon_t} \int l_t(d_{t-1}, \varepsilon_t) p(d_{t-1}, \varepsilon_t) \, dd_{t-1}
\]

- Aggregate variables
Individual Decisions

- Approximate conditional expectation on \( \{d_m\}_{m=1}^M \) grid

\[
\beta R^d_t \mathbb{E}_t \left[ \frac{c_{t+1}^{-1}}{\Pi_{t+1}} \right] = T(\varepsilon_t, d_{t-1} = d_m) \approx \exp \left\{ \sum_{n=0}^{M-1} \theta_{n,t}(\varepsilon_t) T_n(d_{t-1} = d_m) \right\}
\]

- Solve policy function with a system of equations

\[
d_t = \max \left\{ \frac{R^d_{t-1}}{\Pi_t} d_{t-1} + mrs_t[(1 - \tau)l_t \varepsilon_t + \mu(1 - \varepsilon_t)] - T(d_{t-1}, \varepsilon_t)^{-1} - T_t + \text{div}_t - X, d \right\}
\]

\[
l_t = \min \left\{ \left[ \frac{(1 - \tau)mrs_t}{\chi c_t} \right]^{\frac{1}{\eta}}, l \right\}
\]

\[
c_t = \frac{R^d_{t-1}}{\Pi_t} d_{t-1} + mrs_t[(1 - \tau)l_t \varepsilon_t + \mu(1 - \varepsilon_t)] - d_t - T_t + \text{div}_t - X,
\]

\[\Rightarrow (M + 1) \times 2 \times 3 \text{ equations for } d_t(d_m, \varepsilon_t), l_t(d_m, \varepsilon_t), c_t(d_m, \varepsilon_t)\]

- \(c_{t+1}\) can be expressed as a function of \(T(d_t, \varepsilon_{t+1})\)

\[
\beta R^d_t \mathbb{E}_t \left[ \frac{1}{\Pi_{t+1}} c_{t+1} \left( \exp \left\{ \sum_{n=0}^{M-1} \theta_{n,t+1}(\varepsilon_{t+1}) T_n(d_t) \right\} \right)^{-1} \right] \approx \exp \left\{ \sum_{n=0}^{M-1} \theta_{n,t}(\varepsilon_t) T_n(d_{t-1} = d_m) \right\}
\]

\[\Rightarrow M \times 2 \text{ equations for } M \times 2 \text{ variables } \theta_{n,t}(\varepsilon_t)\]
Cross-Sectional Distribution: Young (2010)

Transition dynamics for $p(d_t, \varepsilon_{t+1})$

$$p(d_t, \varepsilon_{t+1}) = \sum_{\varepsilon_t} \sum_{d_m} p(d_t|d_{t-1} = d_m, \varepsilon_t) p(\varepsilon_{t+1}|\varepsilon_t) p(d_{t-1} = d_m, \varepsilon_t)$$

Young (2010): approximate $p(d_t|d_{t-1} = d_m, \varepsilon_t)$ with the $d_m$ grid:

▶ find the two neighboring grids $d_{m'}$, $d_{m'+1}$ that are closest to $d_t$

▶ Assign weights to the two grids based on distance

$$p(d_t = d_{m'}|d_{t-1} = d_m, \varepsilon_t) = 1 - \frac{d_t - d_{m'}}{d_{m'+1} - d_{m'}}$$

$$p(d_t = d_{m'+1}|d_{t-1} = d_m, \varepsilon_t) = \frac{d_t - d_{m'}}{d_{m'+1} - d_{m'}}$$

$(M + 1) \times 2$ equations for $(M + 1) \times 2$ probabilities
Stationary Equilibrium

We solve a fixed point problem over $D$ and $L$

1. Given $D$ and $L$, solve for aggregate variables
2. Solve for Chebyshev coefficients: a fixed point problem
3. Solve for stationary distribution $p(d, \varepsilon)$: a fixed point problem
4. With the policy function from step 2 and distribution from step 3, update $\tilde{D}$ and $\tilde{L}$

Repeat until convergence
Calibration of non-HANK Parameters

We follow Sims and Wu (2020)

- Financial intermediary parameters (e.g. leverage, bond coupon decay, steady-state bond holdings and issuance)
- Standard medium-scale NK parameters (e.g. price and wage stickiness, investment adjustment cost, utilization costs)
Calibration of HANK Parameters

<table>
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<tr>
<th>Parameters</th>
<th>Value</th>
<th>Target</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L^U$</td>
<td>0.05</td>
<td></td>
<td>Fraction unemployed</td>
</tr>
<tr>
<td>$p$</td>
<td>0.5</td>
<td>2 quarter duration $L^{RANK} = 0.95$</td>
<td>Unemployment duration</td>
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<tr>
<td>$\chi$</td>
<td>0.4</td>
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<td>Labor disutility scaling parameter</td>
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<tr>
<td>$\mu$</td>
<td>0.3</td>
<td></td>
<td>Unemployment benefit</td>
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<tr>
<td>$\tau$</td>
<td>0</td>
<td></td>
<td>Labor income tax rate</td>
</tr>
<tr>
<td>$d$</td>
<td>0</td>
<td></td>
<td>Borrowing constraint</td>
</tr>
<tr>
<td>$\bar{l}$</td>
<td>1.5</td>
<td></td>
<td>Time endowment</td>
</tr>
</tbody>
</table>

Other parameters as in Sims and Wu (2021)
Lump Sum Distribution

We assume throughout that all households receive the same fiscal transfer.

As a baseline, assume that all households receive the same dividends each period.
Main Results
Stationary Wealth Distribution

![Graph showing stationary wealth distribution with deposited/labor income on the x-axis and some distribution on the y-axis. The graph includes two lines, one for unemployed and one for employed individuals.](image)
Impulse Responses: QE Shock

- **Output**
- **Consumption**
- **Investment**
- **Inflation**
- **Policy Rate**
- **Corporate Bond Spread**
- **Labor**
- **Deposit**
First-Period Individual Responses

- Consumption
- Deposit
- Labor
Take-Aways

HANK responses **nearly identical** to RANK responses

Aggregate consumption falls less, driven by the behavior of the poorest households

Suggests RANK is a good approximation to HANK
Micro Distribution of Wealth
Inequality

What do micro-level measures of inequality (e.g. Gini coefficient, Lorenz curve) look like in our model?

Do they matter for aggregate transmission of QE shocks?

How does distribution of lump sum transfers (dividends) matter for both inequality metrics and aggregate transmission?
Dividend Distribution

\[
\frac{\text{div}_{j,t}}{\text{div}_t} = \left( a_t + b_t d^{\vartheta}_{j,t-1} \right).
\]

\(a_t\) and \(b_t\) chosen so that:

\[
\int_{0}^{1} \left( a_t + b_t d^{\vartheta}_{j,t-1} \right) dj = 1,
\]

\[
\frac{a_t + b_t d^{\vartheta}}{a_t + b_t d^{\vartheta}} = n.
\]

Vary \(n\) as well as \(\vartheta\); influences how dividends are distributed across the wealth distribution.
Relative Weights

![Graph of Relative Weights](image)
Stationary Distribution

![Graph of Stationary Distribution]

The graphs show the distribution of deposit/labor income for different scenarios, labeled as HANK, L10, L100, S10, and C10. The x-axis represents the deposit/labor income, while the y-axis shows the probability density. The graphs illustrate how the distribution changes across different income levels for each scenario.
Lorenz Curves
## Gini Coefficients

<table>
<thead>
<tr>
<th></th>
<th>HANK</th>
<th>L10</th>
<th>L100</th>
<th>S10</th>
<th>C10</th>
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</thead>
<tbody>
<tr>
<td>Gini coeff.</td>
<td>0.44</td>
<td>0.39</td>
<td>0.34</td>
<td>0.48</td>
<td>0.51</td>
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</tbody>
</table>
Impulse Responses

- Output
- Consumption
- Investment
- Inflation
Take-Aways

Seems to be little relationship between micro-level wealth distribution and macro aggregates

Consumption responses slightly different with more curvature in dividend distribution rule, but hardly noticeable for macro aggregates
Macro Distribution of Wealth
Experiments

Not many people located near borrowing constraint in our baseline model

Consider varying three macro parameters related to employment, with affect stationary distributions

1. Unemployment rate, $U$
2. Unemployment benefit, $\mu$
3. Unemployment duration, $p$
Stationary Distribution: HANK vs. Higher $U$

The diagram illustrates the stationary distribution for HANK and different values of $U$ (10%, 20%, 40%). The x-axis represents deposit/labor income, while the y-axis shows the probability density. The graph shows how the distribution changes with varying $U$ values.
Impulse Responses: HANK vs. RANK with Higher $U$
Stationary Distribution: HANK with Different $\mu$
Impulse Responses: HANK with Different $\mu$

- **Output**: $HANK = 30\% = 50\%$
- **Consumption**
- **Investment**
- **Inflation**
- **Policy Rate**
- **Corporate Bond Spread**
- **Labor**
- **Deposit**
Impulse Responses: HANK with Different $\rho$
Take-Aways

Only small differences in aggregate effects of QE shock for different:

1. Unemployment rate
2. Unemployment benefit
3. Unemployment duration
Conclusion

We combine financial intermediaries, long-term bonds, and scope for central bank bond purchases (QE) to matter with heterogeneous households with uninsurable income risk.

Does household heterogeneity matter for aggregate QE transmission?

- No

- Micro-level wealth heterogeneity does not matter

- Macro-level parameters (unemployment, unemployment benefit, unemployment duration) might matter a little more, but not much for macro aggregates

Conclusion: at least with our modeling of the frictions allowing QE to matter, **RANK is a good approximation to HANK**
Chebyshev Polynomials

\[ T(\varepsilon_t, d_{t-1}) \approx \exp \left\{ \sum_{n=0}^{N_T} \theta_{n,t}(\varepsilon_t) T_n(\zeta(d_{t-1})) \right\} \]

- Chebyshev polynomials are defined as following:

\[ T_n(x) = \cos(n \arccos x) \]

- can also be defined recursively

- The algorithm aims to fit a set of nodes \( \{d_m\}_{m=1}^{M_T} \)

- \( \zeta(d) = 2 \frac{d-d_\bar{d}}{d-d_\bar{d}} - 1 \) transforms the interval \( a \in (d, \bar{d}) \) to \((-1, 1)\)

- The Chebyshev nodes defined on \((-1, 1)\) are

\[
x_m = -\cos \left( \frac{2m - 1}{2M} \pi \right)
\]

- \( d_m = \zeta^{-1}(x_m) \):

\[
d_m = (x_m + 1) \left( \frac{\bar{d} - d}{2} \right) + d.
\]
## Additional Parameters

<table>
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<tr>
<th>Parameters</th>
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<th>Target</th>
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<td><strong>SW parameters</strong></td>
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