Trade, Leakage, and the Design of a Carbon Tax*

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Abstract

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1 Introduction

Concerns about leakage have been central to the design of carbon policies in the United States. Leakage arises when different nations adopt different prices on greenhouse gases. If industries relocate where carbon prices are low, the result is an increase in emissions in low-tax countries, undermining the efficacy of climate change policies while at the same time distorting the location of production.

The most common response to leakage is to impose carbon border adjustments or more simply border adjustments. Border adjustments combine taxes on the emissions associated with imports and rebates of prior taxes paid for exports. They effectively shift the tax downstream, for example, from emissions from domestic production to emissions associated with domestic consumption. They are thought to help insulate the tax from leakage because, with border adjustments, the tax would be the same regardless of the location of production. Every carbon tax bill introduced in the current Congress includes border adjustments. The European Union has proposed border adjustments for its cap and trade system. They have also been subject to significant study. (For a recent review of the literature, see Böhringer et al (2022).)

Notwithstanding their prominence, it is still not clear whether, or the extent to which, they are desirable, or whether alternative approaches may be preferable. To answer this question, we consider the design of a carbon tax in a simple setting where one region of the world imposes a carbon policy and the rest of the world does not. The taxing region sets policies to address climate change while taking into account the possibility of leakage. We solve the model to find the optimal choices for the taxing region, constraining those choices to commonly proposed policies to allow us to compare those policies.[1]

We get the following results.

(1) *Impose the tax on both the supply and the demand for fossil fuels.* The usual result in taxation is that in the absence of avoidance, evasion, or administrative costs, the legal incidence of a tax does not matter. As a result, in the absence of trade (or if the tax were global), a carbon tax could be imposed entirely upstream

[1]The approach builds on but simplifies the model in Kortum and Weisbach (2021). The two key differences are (1) this paper restricts the set of policies that the taxing region can impose to those that are similar to existing or proposed policies while Kortum and Weisbach find the unrestricted optimal policy, and (2) because we restrict choices to simpler policies, we use a somewhat more general model here, while Kortum and Weisbach (2021) use more specific functional forms.

on extractors to minimize administrative costs, as suggested by Metcalf and Weisbach (2009). With trade and the possibility of leakage, this is no longer true.

In particular, carbon taxes are commonly imposed on the use of fossil fuels in production or on the implicit consumption of fossil fuels embodied in goods, but in both cases, on the demand for fossil fuels. Taxes on the demand for fossil fuels lower their global price, inducing an increase in their use or consumption abroad. Taxes on the extraction of fossil fuels, that is, on their supply, by contrast, raise their global price, inducing an increase in extraction abroad. The optimal policy combines taxes on supply and demand, so that these effects offset, allowing the taxing region to control responses in the rest of the world.[2]

Incorporating this principle into the design of carbon taxes involves an almost trivial adjustment to proposed carbon taxes yet offers potentially enormous gains in terms of the effectiveness of the tax. In particular, many current carbon tax bills impose the tax nominally on extraction. They then impose border adjustments on energy (that is taxes on the imports of fossil fuels and rebate taxes paid on exports of fossil fuels), to shift the tax downstream to domestic production. If the border adjustments on energy are imposed at a lower rate than the nominal extraction tax, a portion of the tax would remain on extraction. Our simulations here show that this minor change has the potential to dramatically improve the effectiveness of the tax in reducing global emissions.

While this hybrid policy—combining a tax on extraction and a demand-side tax—is always desirable, there remains the question of how to impose the demand-side tax. Should it be on production, consumption, or some combination?

(2) *Impose demand side taxes on both production and consumption.* Our second result is that in the absence of administrative costs, the taxing region should impose the tax both on emissions from domestic production and on emissions associated with domestic consumption. The tax rate on production, however, should be lower than the tax rate on consumption to address concerns about

[2]This result was previously seen in Markusen (1974) and Hoel (1994) although it does not appear to have been incorporated into the design of carbon taxes. In fact, we are not aware of any carbon taxes (or cap and trade systems) that incorporate this principle. One possible explanation is that the models in those papers were restrictive. In particular, those papers assumed an economy with extraction and direct consumption of fossil fuels, such as for transportation or residential heating. They did not include a manufacturing or production sector of the economy. Leakage concerns, however, are largely focused on the production of goods. We show that the result applies in a more general economy with production and the possibility of leakage due to shifts in the location of production.

leakage. If leakage is zero, the tax rate on production should equal the tax rate on consumption. If leakage is 100%, the tax rate on production should be zero.

This result answers the widely posed question of whether border adjustments should include export rebates in addition to import tariffs. In particular. To implement this set of taxes, the taxing region starts with a nominal extraction tax and shifts part of it downstream to production by imposing border adjustments (at a lower rate than the nominal extraction tax) on imports and exports of energy. To shift the tax further downstream to consumption, the taxing region imposes border taxes on imports of goods at the same rate as the border adjustments on energy. On rebate on exports, which removes taxes on domestic production for goods sold abroad, however, is lower to account for leakage, leaving part of the tax on domestic production. That is, the rebate on exports should be partial and, we show, proportional to leakage.

(3) *Administrative costs may make border adjustments on goods undesirable.* Once we include administrative costs, however, it may no longer be desirable to impose taxes on both production and consumption. The key reason is that to impose taxes on domestic consumption, the taxing region must impose border adjustments on imports of goods. As discussed in Kortum and Weisbach (2017), doing so will be complex and expensive. Imposing a tax on domestic production only requires border adjustments on imports and exports of energy. These border adjustments are simple to impose.

Our simulations show that a combination of a tax on domestic extraction and domestic production often performs nearly as well as a tax that also falls on domestic consumption. The key reason our simulations differ from those in the prior literature is that we always simulate taxes on production and consumption as hybrid taxes that also include a tax on domestic extraction (point (1) above). A comparison of these hybrid taxes shows that the benefit of border adjustments on goods may be modest and, therefore, not worth the administrative costs. Instead, just combining a tax on extraction and a tax on domestic production may be the best policy.

The key variable in this comparison is the foreign elasticity of energy supply. If this parameter is low, the combination of an extraction and production tax performs almost as well as taxes that also fall on consumption. If, however, the foreign elasticity of energy supply is high, the simpler combination of an extraction and production tax no longer performs well, and shifting the tax downstream to consumption via border adjustments may be desirable notwithstanding the administrative costs.

(4) *Ensure that countries with a high elasticity of energy supply are in the taxing coalition.* Building on point (3), one way to improve the effectiveness of the tax without having to impose border adjustments on goods is to ensure that the foreign elasticity of energy supply is low. To do this, the taxing coalition can work to include countries with a high elasticity of energy supply. In effect, this strategy—including countries with a high elasticity of energy supply in the taxing coalition—acts as a substitute for border adjustments.

We develop these results in four parts. Section 2 presents a model, similar to Hoel (1994) where individuals directly consume fossil fuels (for example, for transportation and residential heating), to illustrate the logic of combining taxes on supply and demand. Section 3 introduces trade in goods to allow us to study leakage. It shows that the results from section 2 carry over to this more realistic setting as well as showing how the various demand side policies compare to one another. Section 4 provides our simulations. Section 5 discusses the results and concludes.

2 Trade in Energy but Not Goods

We start by reviewing and extending the theory of optimal carbon policy in a two-region world where energy is used directly in consumption, such as for transportation or residential heating, but is not embodied in traded goods. This case provides intuition for why carbon policy should act on both the supply and demand side of the energy market. The same intuition carries over to the more general case. The setting is similar to the setting in Hoel (1994).[3] In Section 3 we introduce traded goods that are produced in either country with energy as an input, which allows us to consider leakage.

2.1 Graphical Intuitions

To develop intuitions, we use a graphical illustration of how domestic taxes affect trade. We assume that there are two regions of the world, Home and Foreign,

[3] The solution was implicit in Markusen's seminal paper from 1975. Keen and Kotsogiannis (2014) generalize the result and consider Pareto-optimal policies as we do here.

that extract fossil fuel energy, Q_e and Q_e^*, respectively, and directly consume it, C_e and C_e^*. Home imposes a carbon policy while Foreign is passive.

The left hand panel of Figure 1 shows the conventional diagram with supply and demand of a good, here fossil fuel energy, and a tax, t_c, imposed on consumers. The usual assumption is, equivalently, that the taxing region is the entire world or that there is no trade between the taxing region and the rest of the world (autarky). The tax creates a wedge between the amount consumers pay, $p_e + t_c$, and the amount sellers (here extractors of energy) receive, p_e. The equilibrium sets $Q_e = C_e$ given the wedge between extractors and consumers. As is conventional, in autarky it does not matter if the tax is imposed on extractors or consumers because the wedge between the two would be the same regardless.

If there is trade in energy, illustrated by the right hand panel of Figure 1, we can see that this cannot be an equilibrium. If the price of energy goes down from p_0 to p_e, Foreign extractors would extract less energy while Foreign consumers would demand more, generating a net demand for Home exports, a demand which cannot be met if $Q_e = C_e$.

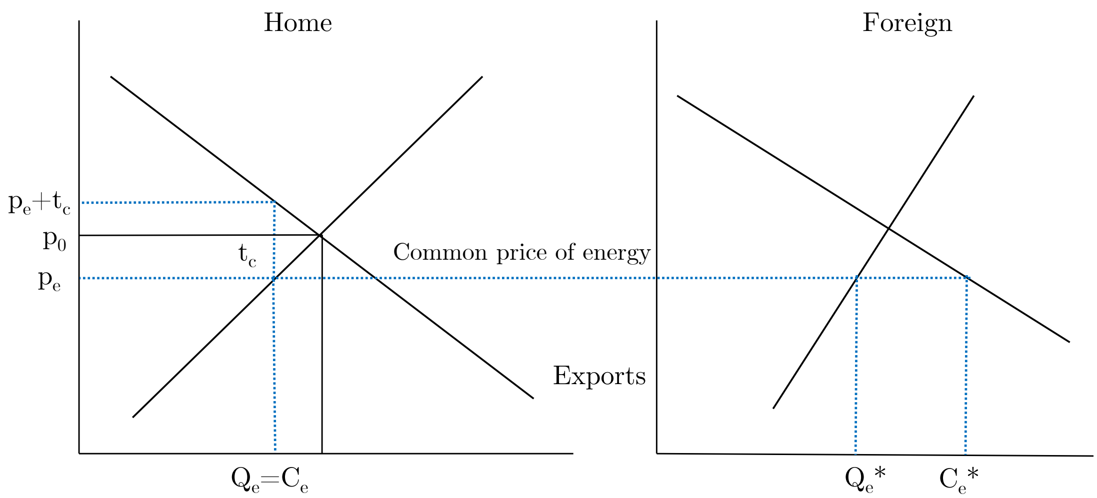


Figure 1: Autarky

Figure 2 shows the equilibrium that would arise if Home taxes the consumption of energy and trades with Foreign. The price of energy, p_e would still go down relative to the price without a tax, but it would go down less that it would with autarky. The lower price of energy would still induce excess demand, X_e, in Foreign (though less than illustrated in Figure 1), but Home would now have excess supply because $C_e < Q_e$ at the equilibrium price. The price of energy would go down just enough that Home's excess supply matches Foreign's excess demand. At that price, global supply, $Q_e + Q_e^*$ would equal global demand $C_e + C_e^*$.

The quantity extracted in Home, Q_e, goes down less with trade than it would without trade. Offsetting this somewhat, Foreign extraction, Q_e^*, goes down. On net, however, the global supply of fossil fuel goes down less with trade than without, which means that trade makes the tax less effective.

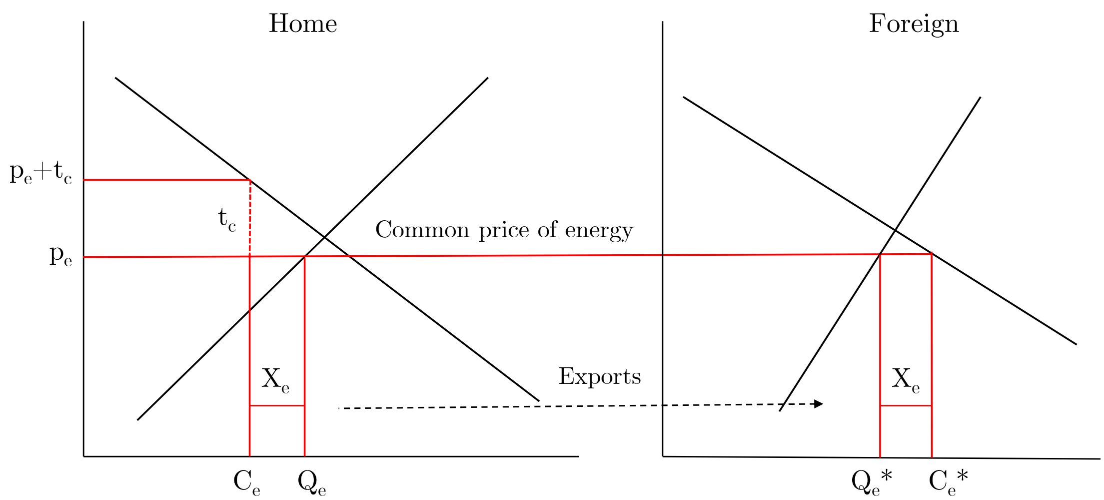


Figure 2: Trade: consumption tax

Figure 3 shows the equilibrium if Home instead chooses to tax extractors, imposing a tax of t_e instead of t_c at the same rate. The logic is the same as with the consumption tax except now the price of energy seen by Foreign actors goes up. Foreign consumers demand less energy while Foreign extractors produce more, resulting in excess supply in Foreign. To be in equilibrium, the price of energy goes up less that it would in autarky, inducing excess demand in Home ($C_e > Q_e$). In equilibrium, the price of energy adjusts so that Home's excess demand equals Foreign's excess supply. As with a consumption tax, a pure extraction tax is less effective with trade than in autarky because of how Foreign actors respond to the tax.

The question, which we address immediately below, is how Home optimizes in this situation. As we will show, rather than choosing either a pure consumption tax or a pure extraction tax, Home mixes the two, which allows it to better control responses in Foreign.

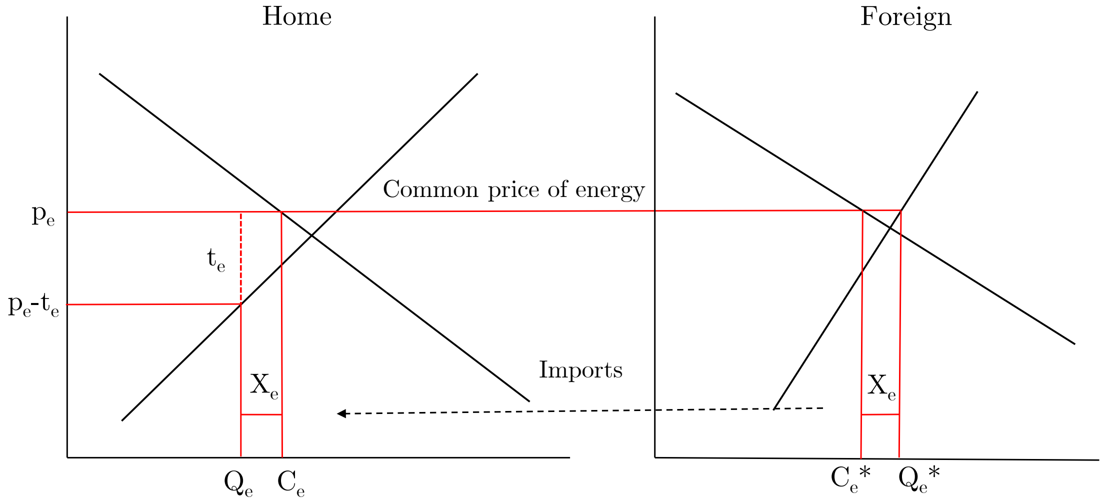


Figure 3: Trade: extraction tax

2.2 Basic Model

To formalize the problem illustrated in Part 2.1, continue to assume that there are two regions, Home, which implements a carbon policy, and Foreign, which is passive. Home and Foreign are endowed with labor, L and L^* (* means Foreign). They both extract carbon-based energy and trade it at price p_e. The labor required to extract a quantity of energy Q_e in Home is $c(Q_e)$ while to extract Q_e^* in Foreign requires $c^*(Q_e^*)$. Both c and c^* are strictly increasing and convex. A numeraire good, which we call services, is produced one-for-one with labor and is traded at price 1. Consumption of services in the two regions is constrained by the labor available to produce them, $C_s + C_s^* = L + L^* - c(Q_e) - c^*(Q_e^*)$. Consumption of energy is constrained by global extraction of energy $C_e + C_e^* = Q_e + Q_e^* = Q_e^W$, where we choose units so that global carbon emissions equal global extraction, $E = Q_e^W$.

The welfare functions in the two regions, U and U^*, depend on consumption of goods and services as well as global emissions. To keep the analysis transparent we assume they are additively separable:

$$U = C_s + u(C_e) - \varphi E$$
$$U^* = C_s^* + u^*(C_e^*) - \varphi^* E,$$

where u and u^* are strictly increasing and concave. (In Appendix A.2 we consider the consequences of removing the restriction of additive separability.) The global externality that motivates carbon policy is $\varphi^W = \varphi + \varphi^*$. We can think of φ^W as the global social cost of carbon.

Foreign's energy supply curve, $Q_e^*(p_e)$, satisfies $c^{*\prime}(Q_e^*(p_e)) = p_e$, with slope $Q_e^{*\prime} > 0$. Foreign's energy demand curve, $C_e^*(p_e)$, satisfies $u^{*\prime}(C_e^*(p_e)) = p_e$, with slope $C_e^{*\prime} < 0$. Thus if p_e increases, Foreign extraction rises and Foreign consumption falls. Home indirectly influences Foreign extraction and consumption by manipulating the global price of energy through its carbon policy. If Home reduces C_e the energy price declines while if it reduces Q_e the energy price rises. This means that we can think of Home as choosing p_e rather than choosing Q_e and C_e.

Following Keen and Kotsogiannis (2014), we assume that Home cannot adopt policies that make Foreign worse off. All policies are Pareto improvements. This approach eliminates terms of trade considerations and, in addition, helps motivate the assumption that Foreign remains passive. (In Appendix A.3 we consider the consequences of instead imposing trade balance.) Within the model, this means that Home transfers services to keep Foreign welfare at a threshold $\bar{U}^*$. With that transfer Foreign can consume services:

$$C_s^*(p_e, E) = \bar{U}^* + \varphi^* E - u^*(C_e^*(p_e)).$$

The particular value of $\bar{U}^*$ doesn't matter for the analysis that follows.

Finally, we assume that Home chooses its carbon policy to meet a global emissions goal of $\bar{E}$. We assume that Home focuses on global emissions rather than domestic emissions because its harm is the same regardless of the source of emissions.[4] Moreover, focusing on global emissions forces Home's policy to take leakage into account.

Home's optimal policy is the solution to:

$$\max_{p_e} C_s + u(C_e) - \varphi \bar{E},$$

[4]In the Paris Agreement, nations set domestic emissions goals rather than global goals, but the joint aim was to produce a global goal.

subject to labor market clearing and energy market clearing:

$$C_s = L + L^* - c(\bar{E} - Q_e^*(p_e)) - c^*(Q_e^*(p_e)) - C_s^*(p_e, \bar{E})$$
$$C_e = \bar{E} - C_e^*(p_e).$$

The first-order condition implies:[5]

$$(p_e - c')Q_e^{*\prime} = (u' - p_e)|C_e^{*\prime}|. \quad (1)$$

(The absolute value on the slope of Foreign demand makes all terms positive.)

To interpret this equation, define the extraction wedge as the difference between the marginal cost of extracting energy in Foreign and Home, $p_e - c'$, and the consumption wedge as the difference between the marginal value of consuming energy in Home and Foreign, $u' - p_e$. A higher extraction wedge, corresponding to lower Q_e, raises the energy price while a higher consumption wedge, corresponding to lower C_e, reduces the energy price. Either wedge represents a global inefficiency. The optimal balance is for Home to equate the product of the price response of Foreign extraction and the extraction wedge to the product of the price response of Foreign consumption (in absolute value) and the consumption wedge. In this way Home minimizes the global inefficiency due to its being unable to separately set Foreign extraction and consumption. Crucially, both wedges must be positive since Foreign supply is increasing and Foreign demand is decreasing in the energy price.

This condition will be satisfied with a combination of taxes in Home: an extraction tax equal to the extraction wedge and a consumption tax equal to the consumption wedge. Since both wedges are positive so are both taxes: it is optimal for Home to tax both the demand side and the supply side of the energy market. Rearranging equation (1), the relative tax rates satisfy:

$$\frac{t_e}{t_c} = \frac{|C_e^{*\prime}|}{Q_e^{*\prime}}. \quad (2)$$

[5] Substituting the two constraints into Home's objective function, along with the expression for $C_s^*(p_e, \bar{E})$, and then differentiating with respect to p_e, the first-order condition is:

$$c'Q_e^{*\prime} - c^{*\prime}Q_e^{*\prime} + u^{*\prime}C_e^{*\prime} - u'C_e^{*\prime} = 0.$$

Applying the competitive-market conditions in Foreign, $c^{*\prime} = u^{*\prime} = p_e$, the first-order condition reduces to (1).

Equation (2) has a standard elasticity-type explanation, which is that Home wants to avoid taxes on highly responsive items. A tax on the the demand for energy, t_c, lowers the energy price seen in Foreign, causing Foreign demand to go up. The more responsive Foreign demand is to the price of energy, the lower the tax on domestic consumption. Similarly, a tax on domestic extraction increases the price of energy in Foreign, causing an increase in extraction there. The more responsive Foreign supply is to the price of energy, the lower the tax on domestic extraction. The optimal ratio of the taxes balances these concerns.

If we allow Home to choose an emissions goal rather than than meeting an exogenously imposed goal $\bar{E}$, Home sets the sum of the taxes equal to the global social cost of carbon: $t_e + t_c = \varphi^W$.[6] The individual taxes are:

$$\begin{aligned} t_e &= \varphi^W \frac{|C_e^{*\prime}|}{Q_e^{*\prime} + |C_e^{*\prime}|} \\ t_c &= \varphi^W \frac{Q_e^{*\prime}}{Q_e^{*\prime} + |C_e^{*\prime}|}, \end{aligned} \tag{3}$$

with sum equal to the global social cost of carbon φ^W and ratio satisfying (2). The intuitions for these values are the same as for Equation (2). Looking at the expression for t_e, the higher $Q_e^{*\prime}$, the lower the value of t_e. Similarly, the higher the value of $|C_e^{*\prime}|$, the lower the value of t_c.

Figure 4 illustrates. Equation (2) requires that the ratio of the tax rates equals the ratio of the slopes of Foreign's supply and demand curves. The height of each rectangle is the tax. The ratio of the widths is equal to the ratio of the slopes of the supply and demand curves, $Q_e^{*\prime}$ and $|C_e^{\prime}|$ (with p_e on the y axis, slopes are read off the x axis).[7] At the optimum, the mix of t_e and t_c is set so that the size of the two rectangles are the same, as shown in Figure 4. Because we have assumed that supply is steeper than demand, i.e. $Q_e^{*\prime} < |C_e^{\prime}|$, the optimal extraction tax in this illustration exceeds the optimal consumption tax.

[6]Substituting the two constraints into Home's objective function and then differentiating with respect to $\bar{E}$, the first-order condition is:

$$-c' - \varphi^* + u' - \phi = (p_e - c') + (u' - p_e) - \varphi^W = 0.$$

[7]This is because the widths are the base of the triangles underneath the supply and demand curves, above $p_e - t_e$, and centered on the intersection of supply and demand.

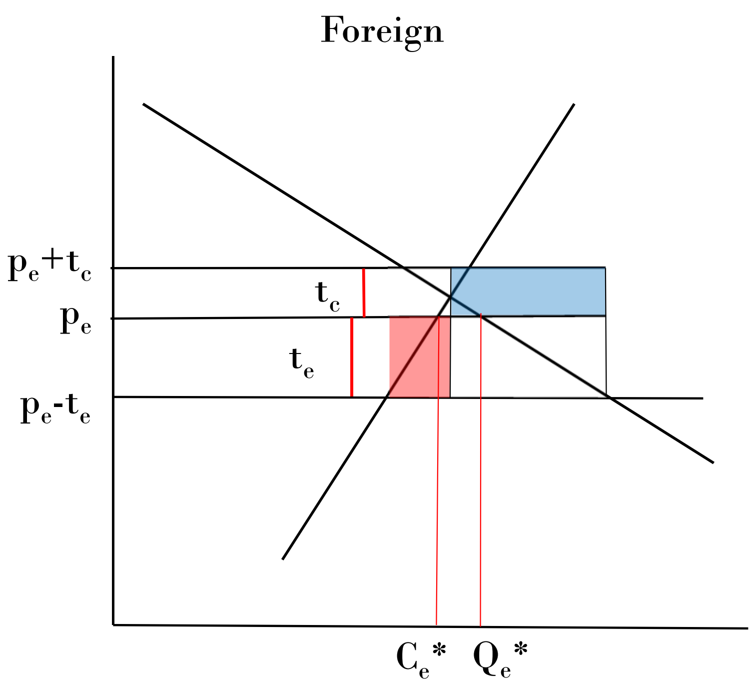


Figure 4: Optimal Mix of Extraction and Consumption Taxes

2.3 Implementation

The taxes described in (2) and (3) are effective taxes. As noted, current carbon tax bills in the United States begin with a nominal tax τ on domestic extraction.[8] They then impose taxes on US energy imports and rebate prior taxes paid on US energy exports, which we call border adjustments on energy and denote by β_e. Border adjustments on energy shift the nominal tax τ on extraction downstream. In the present model, with no manufacturing sector, border adjustments on energy shift the tax all the way downstream to consumption.[9]

Current carbon tax bills set $\beta_e = \tau$ so that they shift the entire tax downstream, setting the effective tax on extraction to zero. Our basic model says that's not

[8] list some current bills.

[9] Adding goods production, as we do in Section 3, border adjustments on energy only shift the tax to producers. As we discuss in Section 3.5, in that case border adjustments on goods are needed to shift the tax to consumption.

optimal. To get to the optimal policy Home could follow the same general strategy but impose the border adjustments on energy at a lower rate than the underlying extraction tax, i.e. $\beta_e < \tau$. A partial border adjustment shifts only a portion of the tax downstream to consumption. To implement the optimal effective taxes t_e and t_c in (3) Home would impose a nominal extraction tax at rate $\tau = t_e + t_c = \varphi^W$, and the border adjustments at rate $\beta_e = t_c$ on energy imports and exports. This strategy of a nominal tax and border adjustments leaves the optimal effective tax on extraction, $t_e = \tau - \beta_e$.

3 Trade in Energy and Goods

The key concern for unilateral carbon taxes is how those taxes affect the location of production. In particular, a unilateral carbon tax on production might cause production, and the resulting emissions, to shift offshore, an effect known as leakage. The basic model in Part 2, however, had only extraction and consumption of energy. It did not include the use of energy in production of traded goods.

We now extend the model to include production in both regions. The production sector in each region manufactures an array of tradable final goods using carbon-based energy. Goods are produced with varying levels of efficiency in different locations using a combination of labor and energy. They are traded based on Ricardian comparative advantage as in Dornbusch, Fisher, and Samuelson (1977). Taxes on production alter the regions' comparative advantage, generating leakage.

In Kortum and Weisbach (2021) we derive the optimal carbon policy for Home in this setting, without restricting the choices available to Home. Here, in order to connect directly with current policy, we restrict Home to particular combinations of taxes: (i) the optimal combination of an extraction and consumption tax, (ii) the optimal combination of an extraction and production tax, and (iii) the optimal combination of all three. The details of our earlier analysis aren't needed for these simpler policies, so we leave them out. (More of the details are relevant for the numerical illustrations in the next section, hence we include them in Appendix C.)

3.1 Model Structure

The welfare functions from the basic model still hold but with the utility from consuming carbon-based energy replaced by the utility from consuming goods, both domestically produced and imported.[10] These goods embody the energy used in their production in either Home or Foreign.

To trace and possibly tax emissions from production and the implicit emissions associated with consumption, we denote the implicit consumption of energy embodied in goods as C_e with a superscript denoting the source of the good and the location of consumption: C_e^d is energy in goods produced domestically and consumed domestically, C_e^m is energy in goods Home imports, C_e^x is energy in goods Home exports, and C_e^f is energy in goods Foreign both produces and consumes. The total quantity of energy consumed in Home is $C_e = C_e^d + C_e^m$. Similarly $C_e^* = C_e^f + C_e^x$. We can also account for all energy used in producing goods in Home, $G_e = C_e^d + C_e^x$ and in Foreign $G_e^* = C_e^f + C_e^m$. Note that these values are functions of p_e and tax policy. They are the demand curves of producers and consumers. For notational compactness, however, we omit these arguments.

Table 1 shows how these values relate to one another, with rows showing emissions associated with consumption and columns showing emissions from production. As we will discuss in Part 4, we use these values to calibrate our model for simulation. Table 1 shows the calibration of these values for the year 2015 under the assumption that Home is the OECD. Global emissions in 2015 were 32.3 $GtCO_2$ and of that, the OECD emitted 12.2 $GtCO_2$. Most of that, 11.3 $GtCO_2$ was consumed domestically. The OECD imported 2.5 $GtCO_2$ so that it consumed 13.8 $GtCO_2$.

In the basic model of Section 2.2 we started with a planner in Home setting quantities (implicitly via its choice of p_e and explicitly via its choice of $\bar{E}$). Here we directly model a competitive market economy with a policy maker choosing tax rates. In addition to an extraction tax, t_e, we will need to consider three demand-side taxes corresponding to the three sources of demand that Home can influence through its taxes: (i) a tax t_d on the energy C_e^d used to produce goods in Home for the domestic market, (ii) a tax t_m on the energy C_e^m used to produce the goods Home imports, and (iii) a tax t_x on the energy C_e^x used to produce Home exports. The consumption tax considered in Section 3.2 restricts $t_d = t_m = t_c$

[10]In this extended model we drop the direct consumption of energy as in the basic model. Including both would be relevant for a more detailed quantification than we provide in Part 4.

Table 1: Carbon matrix, OECD, 2015

	Home	Foreign	Total
Home	$C_e^d = 11.3$	$C_e^m = 2.5$	$C_e = 13.8$
Foreign	$C_e^x = 0.9$	$C_e^f = 17.6$	$C_e^* = 18.5$
Total	$G_e = 12.2$	$G_e^* = 20.1$	$C_e^W = 32.3$
Extraction	$Q_e = 8.6$	$Q_e^* =23.7$	$Q_e^W = 32.3$

Units: gigatons of CO_2.

and $t_x = 0$. The production tax considered in Section 3.3 restricts $t_d = t_x = t_p$ and $t_m = 0$. The combination of all three taxes considered in Section 3.4 removes these restrictions, allowing arbitrary combinations of production and consumption taxes.

Note that these taxes are effective taxes. While effective taxes are unique, there are a number of different ways to implement them. In particular, instead of directly imposing the effective taxes, Home could start with a nominal extraction tax and impose border adjustments on imports and exports of energy and of goods. Various combinations of border adjustments produce each of the policies we consider. We defer the discussion of implementation to Section 3.5, and here work with effective taxes.

Because we are working with prices and taxes, it is convenient to use indirect utility functions, which give the maximum welfare that a region can attain given spending and prices. We interpret those prices as being the effective prices of the energy embodied in the goods that are consumed. Under a consumption tax that price is $p_e + t_c$ for goods consumed in Home and p_e for goods consumed in Foreign, no matter where they are produced. Under a production tax it is $p_e + t_p$ for goods produced in Home and p_e for goods produced in Foreign, no matter where they are consumed. Production and trade in services means wages (and the price of services) are 1 in both regions.

Exploiting the separability assumptions of the basic model, welfare becomes:

$$U = Y + \tilde{u} - \varphi E$$
$$U^* = Y^* + \tilde{u}^* - \varphi^* E.$$

The tilde on $\tilde{u}$ and $\tilde{u}^*$ distinguishes indirect utility (with price arguments implicit to avoid clutter) from direct utility u and u^* in the basic model. Here Y and Y^* represent the levels of spending in Home and Foreign.

Spending in each region comes from labor income, rents to the energy sector, tax revenue, and transfers (from Home to Foreign): $Y = L + R_e + R_t - T$ and $Y^* = L^* + R_e^* + T$. Because Foreign has no carbon policy it gets no tax revenue. Rents to the energy sector in Home (with an extraction tax t_e) are $R_e = (p_e - t_e)Q_e - c(Q_e)$. As in the basic model, we assume that the level of transfers keep Foreign welfare at $\bar{U}^*$, so $T = \bar{U}^* + \varphi^* E - \tilde{u}^* - L^* - R_e^*$.

Substituting these sources of spending into Home welfare, dropping constants, and imposing the global emissions constraint $\bar{E}$, Home's objective is to choose taxes that maximize the Lagrangian:

$$\mathcal{L} = R_e + R_e^* + R_t + \tilde{u} + \tilde{u}^* - \varphi^W \bar{E} - \mu(E - \bar{E}), \tag{4}$$

with Lagrangian multiplier μ on the global emissions constraint. Recall that global emission are equal to global extraction, $E = Q_e^W$. In solving this maximization problem, the policy maker accounts for how its choice of taxes affects the energy price and quantities of energy supplied and demanded in the global energy market. When there is no ambiguity, we denote the response of any variable x to the energy price by $x' = \partial x / \partial p_e$.

3.2 Taxing Extraction and Consumption

Our first application of this model is to solve for the optimal combination of an extraction tax t_e and a consumption tax t_c, by requiring $t_d = t_m = t_c$ and $t_x = 0$. With a consumption tax the effective cost of energy is $p_e + t_c$ to produce goods for consumption in Home and p_e to produce goods for consumption in Foreign. Home's tax revenue is $R_t = t_e Q_e + t_c C_e$. Home's optimal policy is to maximize the Lagrangian $\mathcal{L}$ in (4) by choosing t_e and t_c (the full derivation is in Appendix B.1).

Taking first order conditions yields equation (2) from the basic model, $t_e/t_c = |C_e^{*\prime}|/Q_e^{*\prime}$. Adding trade in goods that embody carbon-based energy doesn't matter when we limit the policy to consist of an extraction tax and a consumption tax. Home uses both taxes, with the optimal ratio of the two taxes being the relative price sensitivity of implicit energy demand to energy supply in Foreign.

The taxes sum to the Lagrangian multiplier:

$$t_e + t_c = \mu. \tag{5}$$

A more ambitious emissions goal leads to a higher shadow value on the constraint and hence higher tax rates. The expressions for the tax rates are also the same as in the simple case:

$$\begin{aligned} t_e &= \mu \frac{|C_e^{*\prime}|}{Q_e^{*\prime} + |C_e^{*\prime}|} \\ t_c &= \mu \frac{Q_e^{*\prime}}{Q_e^{*\prime} + |C_e^{*\prime}|}. \end{aligned} \tag{6}$$

If Home optimizes its emissions goal, $\mu = \varphi^W$, as with the optimal policy for the basic model. That is, even with no carbon taxes in Foreign, the optimal wedge between the after-tax price paid (implicitly) by Home's consumers and the after-tax price received by its extractors—is equal to the global social cost of carbon.[11]

While the bottom line here looks like the solution to the basic model, there is a key distinction. In Section 2 we found that the combination of an extraction tax and a consumption tax was optimal. Here that's not necessarily true. We simply required that the carbon policy consist of only those two taxes and then found a condition for their optimal magnitudes.

3.3 Taxing Extraction and Production

Suppose instead that Home is restricted to imposing only an extraction tax t_e and a production tax t_p. This implies that $t_d = t_x = t_p$ and $t_m = 0$. With a production tax the effective cost of energy is $p_e + t_p$ for producing goods in Home for consumption in either region and p_e for producing goods in Foreign for consumption in either region. Home's tax revenue is $R_t = t_e Q_e + t_p G_e$.

While we didn't need to consider leakage in the combination of an extraction tax and a consumption tax, with a production tax we do. Unlike with a tax on consumption, a tax on Home's production reduces its comparative advantage, causing a shift in the location of production and hence leakage. Leakage is

[11] That feature replicates the key optimality condition for a globally harmonized tax (see Appendix A.1, for example). With the unilateral extraction-consumption tax here, that condition holds even with no carbon taxes in Foreign.

conventionally defined as the increase in Foreign emissions relative to the decrease in domestic emissions, for a given change in t_p:

$$\Lambda = -\frac{\partial G_e^*/\partial t_p}{\partial G_e/\partial t_p} > 0. \tag{7}$$

It follows that $\partial G_e^W/\partial t_p = (1-\Lambda)\partial G_e/\partial t_p$.

Note that there are two sources of leakage captured by Λ. Foreign can increase its use of energy to serve its own consumers: C_e^f might go up in response to Home's policies. In addition, Home can increase its imports from Foreign: C_e^m might go up. With only a production tax, Home is subject to both sources of leakage. As we will see, if Home is also able to tax consumption (i.e. taxing imports), it can eliminate the latter source, leaving only changes in C_e^f or what we will call "Foreign leakage."[12]

Home's optimal policy is to maximize the Lagrangian $\mathcal{L}$ in (4) by choosing t_e and t_p (the full derivation is in Appendix B.2). Taking first order conditions yields the analogue of equation (5):

$$t_e + \frac{t_p}{1-\Lambda} = \mu. \tag{8}$$

and the analogue of (6), but now for the optimal ratio of an extraction tax to a production tax:

$$\frac{t_e}{t_p} = \frac{|G_e^{*\prime}| + \Lambda|G_e'|}{(1-\Lambda)Q_e^{*\prime}}. \tag{9}$$

Higher leakage, as measured by Λ, makes it optimal to tax extraction at a higher rate relative to production. The reason is that with higher leakage, the production tax becomes less effective in lowering global emissions. Moreover, as leakage goes up the (unweighted) sum of the two taxes goes down. The policy becomes less effective as leakage goes up, and Home responds by taxing less.

[12]The literature sometimes refers to what we call Foreign leakage as the "fuel price effect." This term confusing because both sources of leakage, changes to C_e^m and C_e^f, are due to changes in p_e. Therefore, we use the term Foreign leakage instead.

Combining (8) and (9), we get:

$$\begin{aligned} t_e &= \mu \frac{|G_e^{*\prime}| + \Lambda |G_e^{\prime}|}{Q_e^{*\prime} + |G_e^{*\prime}| + \Lambda |G_e^{\prime}|} \\ t_p &= \mu \frac{(1-\Lambda) Q_e^{*\prime}}{Q_e^{*\prime} + |G_e^{*\prime}| + \Lambda |G_e^{\prime}|} \end{aligned} \tag{10}$$

As with an extraction-consumption hybrid, a more ambitious emissions goal leads to a higher shadow value on the constraint and hence higher tax rates.

If Home optimizes over the emissions goal, then it sets $\mu = \varphi^W$. The optimal extraction-production tax loses the feature that the sum of the taxes imposed by Home—here the wedge it creates between the after-tax price paid by its producers and the after-tax price received by its extractors—is equal to the global social cost of carbon, as in the extraction-consumption policy. Leakage limits Home's ability to tax carbon. The sum of the taxes is now:

$$t_e + t_p = \varphi^W - \frac{\Lambda \varphi^W Q_e^{*\prime}}{Q_e^{*\prime} + |G_e^{*\prime}| + \Lambda |G_e^{\prime}|}.$$

Unless leakage is zero, this is less than φ^W. Moreover, as leakage goes up, the sum of the taxes goes down.

3.4 Taxing Extraction, Consumption, and Production

Finally, suppose Home is free to choose t_d, t_x, and t_m independently (together with an extraction tax, t_e).[13] The effective cost of energy is $p_e + t_d$ for Home producers supplying the domestic market, $p_e + t_m$ for Foreign producers supplying imports to Home, and $p_e + t_x$ for Home exporters. Home's tax revenue is $R_t = t_e Q_e + t_d C_e^d + t_m C_e^m + t_x C_e^x$. Home's optimal policy is to maximize the Lagrangian $\mathcal{L}$ in (4) by choosing t_e, t_d, t_m, and t_x (the full derivation is in Appendix B.3).

Although Home has the flexibility to tax imports differently than domestically produced goods, the first order conditions for the relevant tax rates, t_m and t_d, give us $t_d = t_m$: Home taxes the energy embodied in Home's consumption at the same rate regardless of source. This allows us to simplify notation by setting $t_d = t_m = t_c$.

[13] This freedom still does not allow Home to reach the optimal policy found in Kortum and Weisbach (2021). That policy also includes per-unit export subsidies for exported goods. We ignore that feature of an optimal policy in this paper.

Because this policy involves elements of a production tax, in the form of t_x, we need to introduce leakage again. Due to the consumption tax element, however, there is no leakage in serving Home consumers—producers in both Home and Foreign face the same price of energy when selling in Home. If $t_x > 0$, however, Foreign producers still have an advantage relative to Home producers when serving Foreign consumers. This results in what we call Foreign leakage (denoted with a *). Foreign leakage is the increase in Foreign production to serve Foreign consumers relative to the decrease in Home production to serve Foreign consumers, both for a given change in t_x:

$$\Lambda^* = -\frac{\partial C_e^f/\partial t_x}{\partial C_e^x/\partial t_x} > 0. \tag{11}$$

It follows that $\partial C_e^*/\partial t_x = (1 - \Lambda^*)\partial C_e^x/\partial t_x$.

The first order conditions for t_e and t_c give us $t_e + t_c = \mu$, as with the extraction-consumption hybrid. When Home chooses its emissions goal, these taxes sum to the social cost of carbon, φ^W. The production tax (i.e. the tax on energy embodied in Home's exports), however, is lower because of Foreign leakage. The first order condition for t_x, in combination with the others, yields an expression analogous to the extraction-production hybrid:

$$t_x = (1 - \Lambda^*)t_c.$$

The analogue of expression (2) is now:

$$\frac{t_e}{t_c} = \frac{|C_e^{f\prime}| + \Lambda^*|C_e^{x\prime}|}{Q_e^{*\prime}}.$$

Note that the numerator is less than $|C_e^{*\prime}|$ as long as $\Lambda^* < 1$. With Foreign leakage below 100% it is optimal to raise the consumption tax relative to the extraction tax, compared to the case for an extraction-consumption tax (2). The reason is that keeping a tax on Home's exports, via the production tax, eliminates a reason for relying on the extraction tax to keep the consumption tax low.

Combining the results above, we can express the optimal policy as:

$$
\begin{aligned}
t_e &= \mu \frac{|C_e^{f\prime}| + \Lambda^* |C_e^{x\prime}|}{Q_e^{*\prime} + |C_e^{f\prime}| + \Lambda^* |C_e^{x\prime}|} \\
t_c &= \mu \frac{Q_e^{*\prime}}{Q_e^{*\prime} + |C_e^{f\prime}| + \Lambda^* |C_e^{x\prime}|} \\
t_x &= (1 - \Lambda^*) t_c,
\end{aligned}
\tag{12}
$$

with $\mu = \varphi^W$ if Home optimizes its emissions goal. While we refer to it as an extraction-production-consumption tax, the production component is only present in the tax on exports, t_x.

Two restricted versions of this policy are insightful. The first is to simply set $t_x = 0$, ignoring the corresponding first-order condition. The resulting problem is equivalent to taxing only extraction and consumption, as in Section 3.2. It emerges as the optimal here if $\Lambda^* = 1$. If leakage is 100%, taxing exports doesn't reduce global emissions so it is best to set $t_x = 0$.

Suppose instead Home sets $t_x = t_c = t_{cp}$. This condition would be optimal if $\Lambda^* = 0$. If there were no leakage, there would be no reason to lower the tax on exports relative to the tax on domestic consumption.

The policy might also arise because of legal or policy constraints. For example, trade law might require exports to be taxed at the same rate as domestic consumption. If it is a constraint, Home optimizes over t_{cp} (a combined consumption-production tax) and t_e. To solve the resulting first-order conditions requires introducing yet a third measure of leakage:

$$
\tilde{\Lambda}^* = -\frac{\partial C_e^f / \partial t_{cp}}{\partial C_e / \partial t_{cp} + \partial C_e^x / \partial t_{cp}},
$$

so that $\tilde{\Lambda}^* < \Lambda^*$. The solution for optimal tax rates is:

$$
\begin{aligned}
t_e &= \mu \frac{|C_e^{f\prime}| + \tilde{\Lambda}^* |C_e' + C_e^{x\prime}|}{Q_e^{*\prime} + |C_e^{f\prime}| + \tilde{\Lambda}^* |C_e' + C_e^{x\prime}|} \\
t_{cp} &= \mu \frac{(1 - \tilde{\Lambda}^*) Q_e^{*\prime}}{Q_e^{*\prime} + |C_e^{f\prime}| + \tilde{\Lambda}^* |C_e' + C_e^{x\prime}|}.
\end{aligned}
$$

The form of these expressions is familiar from the solution for taxing extraction and production, in Section 3.3.

3.5 Implementation

In Section 2.3 we noted that if we start with a nominal extraction tax of τ, adding partial border adjustments $0 < \beta_e < \tau$ on the imports and exports of energy shifts a portion of the tax downstream. In the basic model (i.e., without manufacturing) these border adjustments shift β_e of the tax all the way downstream to consumers of energy leaving an effective tax $t_e = \tau - \beta_e$ on extraction.

When we add manufacturing and trade in goods, border adjustments on energy only shift the tax to producers who use energy to manufacture goods. Home needs additional border adjustments on the imports and exports of goods to shift the tax to the implicit consumption of carbon. Because the extraction-production-consumption policy treats imports and exports of goods differently (that is, $t_p \neq t_c$), Home needs separate border adjustments to implement this policy: a border adjustment on the energy content of imports of goods (β_m), and a border adjustment on the energy content of exports of goods (β_x). With these three border adjustments (β_e, β_m, and β_x) and a nominal tax on the extraction of energy of (τ), Home can implement any of the three hybrids considered in this paper. Table 2 shows the mapping, specific to each policy, from effective tax rates to the nominal tax on extraction together with border adjustments that achieves the same outcome.

Table 2: Implementation with Border Adjustments

Policy	τ	β_e	β_m	β_x
Extraction-Production	$t_e + t_p < \mu$	t_p	0	0
Extraction-Consumption	$t_e + t_c = \mu$	t_c	t_c	t_c
Extraction-Production-Consumption	$t_e + t_c = \mu$	t_c	t_c	$t_c - t_x$

where τ is the nominal extraction tax, β_e is the border adjustment on energy,and β_m (imports) and β_x (exports) are border adjustments on goods, and μ is the Lagrange multiplier.

To implement the extraction-production hybrid (10), the first row of Table 2 shows that Home would impose a nominal extraction tax of $\tau = t_e + t_p$ and border adjustments on imports and exports of energy at a lower rate of $\beta_e = t_p$. This shifts t_p downstream to production, leaving $\tau - t_p$ on extraction.

Because this border adjustment is only on energy, it would be simple to implement—energy imports and exports are already highly regulated and monitored. It would, moreover, only require a slight rewording of existing legislative proposals, namely reducing the magnitude of the border adjustment on energy from τ to β_e (as well as eliminating any border adjustments on goods found in the legislation).

To implement the extraction-consumption hybrid (6), Home would impose a nominal extraction tax of $\tau = t_e + t_c$ and border adjustments on imports of energy at a lower rate of $\beta_e = t_c$, much like for the extraction-production case. To shift the tax downstream to consumption, however, it would also have to impose border adjustments on imports and exports of goods (β_m and β_x respectively) also at rate t_c. This leaves a tax of $\tau - t_c$ on extraction, and no tax on production.

As we discuss in Kortum and Weisbach (2017), computing accurate border adjustments on goods is expensive and complex because there is no straightforward way to determine the implicit energy content of imports (or even exports). Any resulting border adjustments are likely to be inaccurate. Whether it is desirable to incur these costs to impose border adjustments on goods depends on whether, and if so by how much, the extraction-consumption hybrid outperforms the extraction-production hybrid, an issue we explore in our quantitative illustrations, found in Part 4.

Finally, to implement the combination of all three taxes (12), Home again imposes a nominal extraction tax of $\tau = t_e + t_c$. The border adjustment on energy and on imports of goods is $\beta_e = \beta_m = t_c$. Unlike with the extraction-consumption tax, however, there is an even lower border adjustment on the export of goods, $\beta_x = t_c - t_x$. That is, to tax exports at an effective rate of t_x under this implementation, producers of goods would receive an export rebate of $\beta_x = \Lambda^* t_c$.

The tax on production is proportional to Foreign leakage. If Λ^* is zero, there should be no rebate on exports. As Foreign leakage goes up, so does the export rebate. With $\Lambda^* = 1$, the rebate on exports of goods would equal the tax on imports of goods. There would be no tax on production, and the combined policy would be an extraction-consumption tax. That is, the value of Λ^* gives us the answer to the commonly-posed policy question of whether border adjustments should include rebates of prior taxes paid for exports of goods. While implementing this three-way hybrid involves all the difficulties associated with computing the carbon content of goods, it would be no more difficult to administer than the extraction-consumption tax.

4 Quantitative Illustrations

To get a sense of the size of the benefits from the various types of hybrid taxes, we calibrate and simulate the model described in Part 3. Our sufficient-statistic formulas for optimal taxes in Sections 2 and 3 give intuitions, but they do not allow us to compute numerical values or to compare welfare across all policies. To do this, we need to add considerable additional detail, including functional forms for extraction and production, and for the efficiency of production of goods in each region. We follow the approach taken in Kortum and Weisbach (2021). Details can be found there.

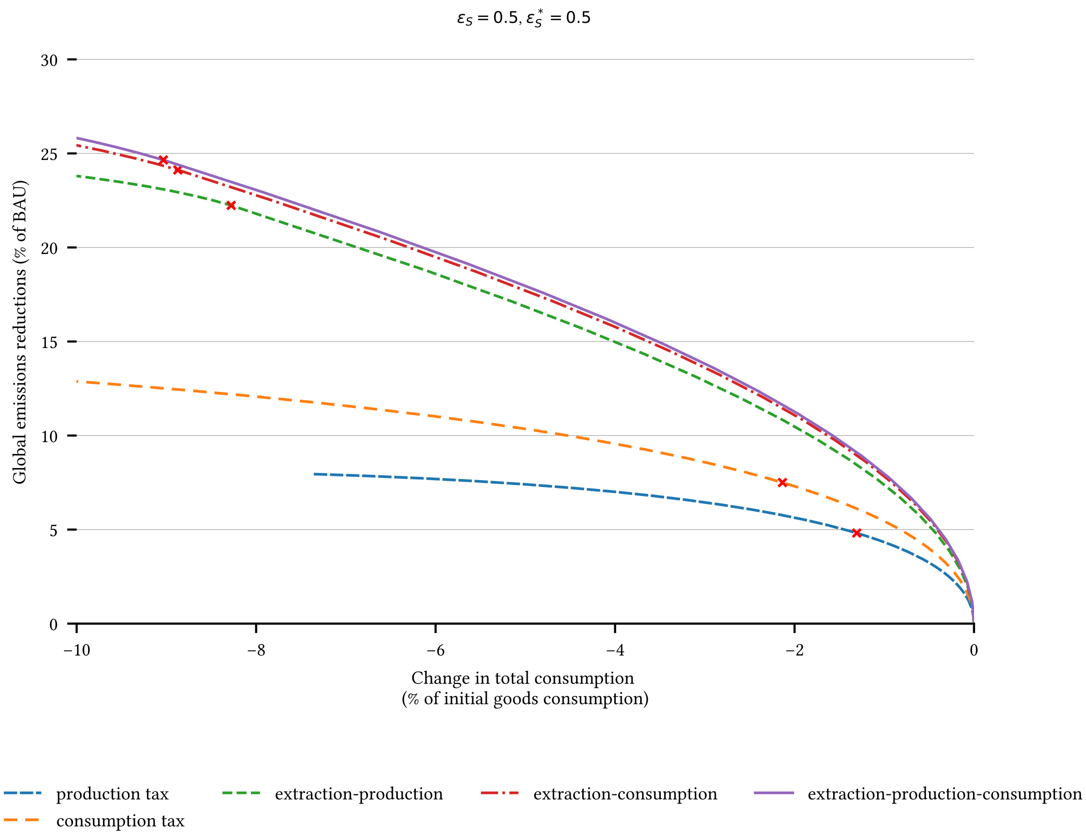
Following Dekle, Eaton, and Kortum (2007), we calibrate the business as usual competitive equilibrium to data on global carbon flows and compute the effects of various policies relative to this baseline. (Calibrating the model this way subsumes transport costs for goods, which Kortum and Weisbach (2021) model as iceberg costs.) In particular, we match the energy variables in Table **??** to values from the Trade Embodied in CO_2 database made available by the OECD (2019). Table 1 (which corresponds to Table 5 in Kortum and Weisbach (2021)) in Part 3 shows the baseline calibration, where the taxing region is the OECD and the calibration year is 2015.

In addition to our calibration to the CO_2 matrix, we also need values for several elasticities. As we will discuss, a key parameter is Foreign's elasticity of energy supply, $\epsilon_s^* = p_e Q_e^{*\prime}/Q_e^*$. Our baseline value is $\epsilon_s^* = 0.5$ but because of uncertainty in this value, we also show simulations for $\epsilon_s^* = 2$.[14]

Our figures show what we call "production possibility frontiers" for various policies. Along the x-axis of the frontier is the cost of the policy, measured as the decline in services consumption as a percent of the business-as-usual level of spending on goods consumption (ignoring the benefits of emissions reductions). The y-axis shows the resulting emissions reductions as a percent of business-as-usual emissions (which with no policy is 32.3 gigatons of CO_2). The frontier for a given policy (when optimized) is traced out by ranging over values of φ^W, so that each point on the line shows the emissions reductions that Home's policy would achieve for a given social cost of carbon. The red dots show the policy that Home would choose in each case when $\varphi^W = 2$, which means that the global social cost

[14] We also set the share of energy in production equal to 0.15, the Foreign demand elasticity ($\epsilon_d^* = p_e C_e^{*\prime}/C_e^*$) equal to 1, and the trade elasticity equal to 4. More details on the calibration are in Kortum and Weisbach (2021).

Figure 5: Effects of different taxes on emissions of OECD



of a unit of carbon is twice the value of energy containing a unit of carbon).

Figure 5 compares the three hybrid policies and the two standard approaches to carbon taxes, a tax on domestic production and that same tax with border adjustments on goods (which shifts it to domestic consumption). As can be seen, with this calibration, all three hybrid policies perform similarly and substantially outperform the standard approaches. For example, adding an extraction tax component to a production tax nearly doubles the global emissions reductions the policy would achieve at any given cost.

In this calibration, there is almost no advantage to adding border adjustments on goods. The emissions reductions that are achievable with a simpler tax – the combination of an extraction and production tax – are about the same. Given the complexities of imposing border adjustments on goods, the modest additional emissions reductions are unlikely to be worth the costs.

Figure 6: Effects of different taxes on emissions of OECD

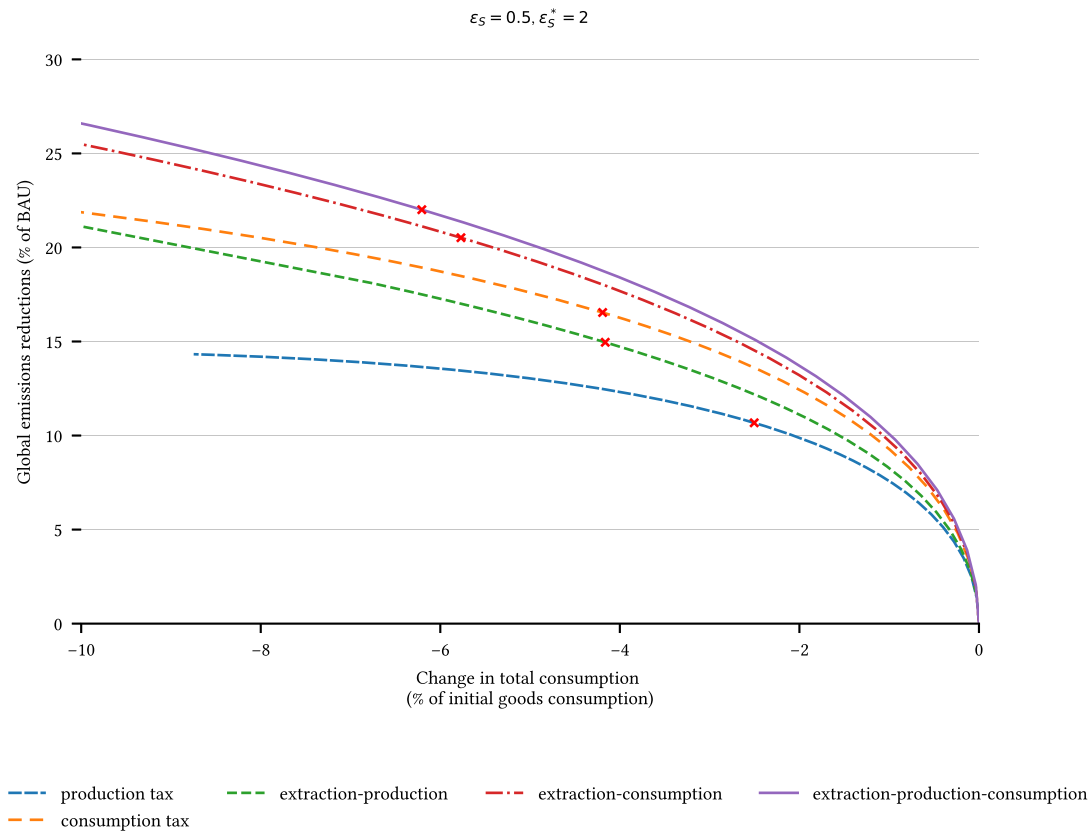


Figure 6 explores the robustness of these results to ϵ_s^* by setting $\epsilon_s^* = 2$ instead of 0.5. The extraction-production tax now performs less well. The reason is that with a high value of ϵ_s^*, the extraction component of the various hybrid policies, which raise the global energy price, induce a significant positive response by Foreign extractors. The policies must, as a result, rely more on demand-side taxes, and the leakage costs of the production tax therefore play a larger role. In this case, all of the policies that use a consumption tax as the demand-side tax (including a pure consumption tax) out perform the policies that rely on a production tax. Because a demand-side tax on consumption does not cause leakage, policies that impose the demand side tax on consumption are more robust to the value of ϵ_s^*.

Whether the gains from imposing border adjustments on goods (to shift the tax downstream to consumption) are worth the costs depends primarily on (1)

Table 3: Calibration for the OECD plus China

	Home	Foreign	Total
Home	$C_e^d = 20.1$	$C_e^m = 1.7$	$C_e = 21.8$
Foreign	$C_e^x = 1.4$	$C_e^f = 9.1$	$C_e^* = 10.5$
Total	$G_e = 21.5$	$G_e^* = 10.8$	$C_e^W = 32.3$
Extraction	$Q_e = 16.24$	$Q_e^* =$16.1	$Q_e^W = 32.3$

the risk of a high value of ϵ_s^* and (2) the costs of imposing border adjustments on goods.

To explore the role of coalition size, particularly as measured by production, we include China in the taxing region. To do so we recalibrate the model to the values of embodied CO_2 shown in Table 3 (which corresponds to Table 9 in Kortum and Weisbach (2021)). The table shows that including China nearly doubles the baseline amount of CO_2 emitted in production by the coalition (Home), with a somewhat smaller increase in implicit consumption of CO_2.

Figures 7 and 8 illustrate the effects of adding China to the taxing coalition. As expected, under all policies, adding China to the taxing coalition dramatically increases the possible emissions reductions. Once again, the hybrid policies substantially outperform the traditional approaches, indicating that the benefits of the hybrid policies continue even with the larger taxing coalition.[15]

Another effect of adding China to the taxing coalition is that now the extraction-production hybrid is more robust to the value of ϵ_s^*. Since the coalition now represents two-thirds of the CO_2 emitted in production, there are fewer opportunities for leakage with China in the taxing coalition (Λ declines). As a consequence, the production tax performs relatively better than with the smaller taxing coalition.

We suspect that this result is general, in the sense that the choice of the taxing coalition affects the relative performance of the various taxes. Because the extraction-production tax is so much simpler to implement, a promising strategy is to have a taxing coalition where this tax performs well. In particular,

[15] At the limit, however, where the taxing coalition is the entire world, all the taxes would perform the same. Therefore, at some point, the simple taxes should perform about as well as the hybrid policies.

Figure 7: Effects of different taxes on emissions of OECD and China

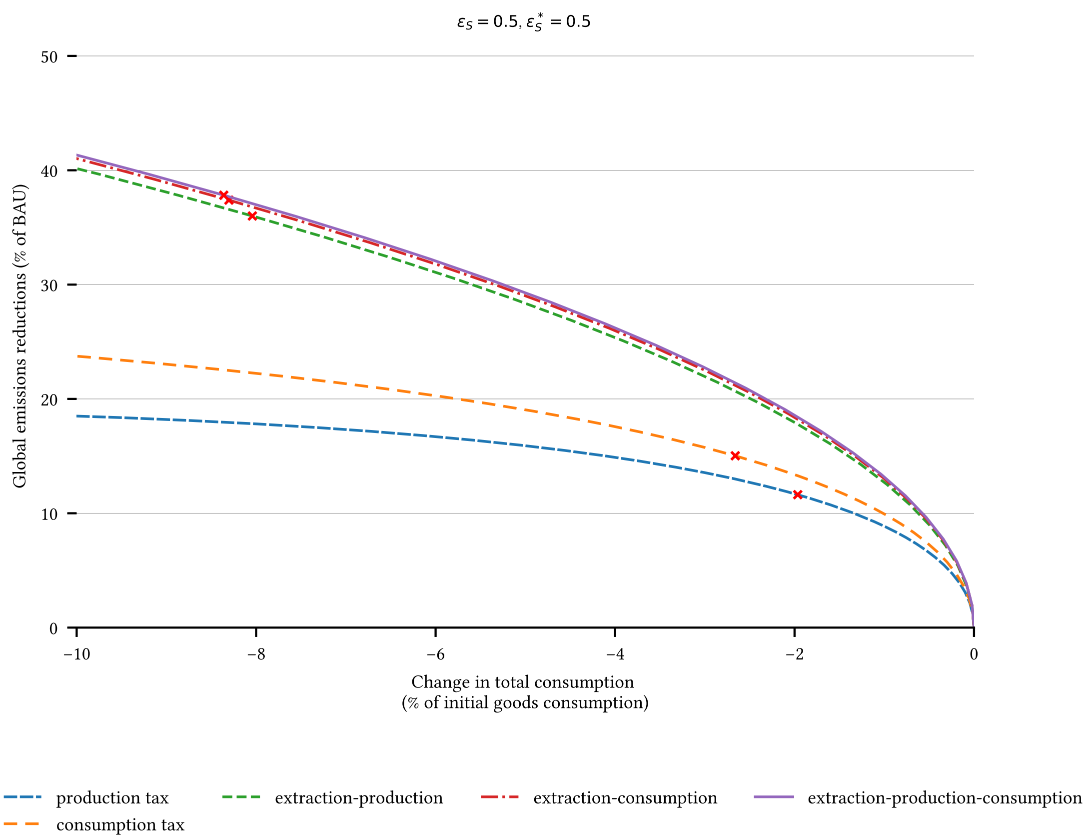
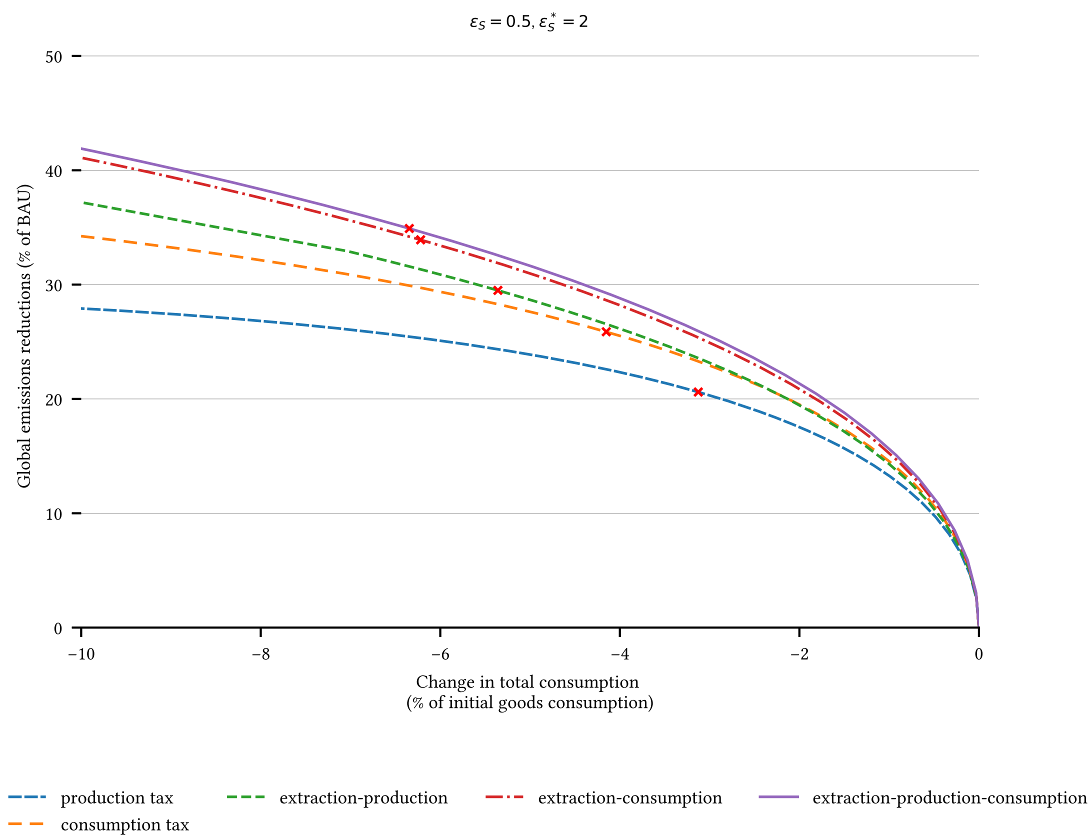


Figure 8: Effects of different taxes on emissions of OECD and China



including countries with a substantial base of production and a high elasticity of energy supply in the taxing coalition might be a promising strategy because doing so lowers both Λ and ϵ_s^*, allowing the taxing region to use the simpler extraction-production hybrid, thereby avoiding border adjustments on goods.[16]

5 Discussion and Conclusion

Most of the key points from this analysis were highlighted in the introduction. We add further discussion here. Key points:

- As noted, there are potentially large gains from combining supply and demand side taxes. This point, which is known as early as Markusen (1975) appears not to be widely appreciated. It involves a simple change to current proposals, and there seems to be no reason not to pursue this approach to improving the functioning of carbon taxes.

- The relative portion of the tax that should stay on extraction depends on the foreign reaction to the different taxes, as measured by the slope of its supply and demand curves. Steep supply . . . Steep demand . . .

- If we do not take administrative costs into account, the taxing region should tax the use of fossil fuels at all stages of its use as it flows through the economy: extraction, production, and consumption. The production component of the tax, however, is muted by leakage. If leakage were zero, the production tax would be at the same rate as the consumption tax. If leakage is 100%, the production tax would be zero, with the tax falling only on extraction and consumption.

- To implement this, impose a nominal tax on extraction. Shift a portion of it downstream to production via border adjustments on energy at a lower rate. In addition, further shift the tax to consumption via border adjustments on imports of goods. Finally, to lower the production tax to account for leakage, rebate a portion of the tax on exports of goods.

- Note that carbon tax bills and analyses of border adjustments vary in how the treat exports of goods. This analysis explains and rationalizes the arguments over border adjustments on exports.

[16]This strategy, as it relates to the extraction elasticity, has similarities to Harstad (2012).

- The administrative costs, however, of imposing a tax on consumption would be high because there is no straightforward way to observe the emissions associated with imports of goods. Moreover, in our baseline simulation, the gains from taxing consumption relative to the extraction-production hybrid are small. As a result, the extraction-production tax may be a superior instrument. It could be implemented simply and accurately by imposing a nominal tax on extraction and border adjustments on the imports and exports of energy at a lower rate.

- This latter conclusion, however, depends on the foreign elasticity of energy supply. Our baseline calibration assumed it was 0.5. As it goes up, the effectiveness of the extraction-production tax goes down relative to combinations that include a consumption tax. The reason is that the extraction tax component becomes less effective when foreign extraction is more sensitive to the price of energy. As a result, the policies have to rely more on demand-side taxes. Therefore, as the foreign elasticity of energy supply goes up, border adjustments become more desirable.

- Finally, include high elasticity of supply countries in the taxing coalition. This makes the extraction production tax more effective and, therefore, reduces the need to rely on border adjustments.

Caveats and extensions.

- Considered here only constrained policies. In KW (2021) we considered the full optimum. There . . .

- Key limitation is the assumption that the non-taxing regions are passive. Extension of model to consider setting carbon policy in a game theoretic setting would be valuable.

References

Böhringer, Christoph, Carolyn Fischer, Knut Einar Rosendahl, and Thomas Fox Rutherfod (2022), "Potential impacts and challenges of border carbon adjustments," *Nature Climate Change,* 12, 22-29.

Dekle, Robert, Jonathan Eaton, and Samuel Kortum (2007), "Unbalanced Trade," *American Economic Review: Papers and Proceedings,* 97, 351–355.

Dornbusch, Rudiger, Stanley Fischer, and Paul A. Samuelson (1977), "Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods," *American Economic Review,* 67, 823–839.

Harstad, Bård (2012), "Buy Coal! A Case for Supply-Side Environmental Policy,"*Journal of Political Economy*, 120, 77–115.

Hoel, Michael (1994), "Efficient Climate Policy in the Presence of Free Riders," *Journal of Environmental Economics and Management*, 27, 259-274.

Keen, Michael and Christos Kotsogiannis (2014), "Coordinating climate and trade policies: Pareto efficiency and the role of border tax adjustments," *Journal of International Economics*, 94, 119-128.

Kortum, Samuel and David Weisbach (2017), "The Design of Border Adjustments for Carbon Prices," *National Tax Journal*, 70, 421–446.

Kortum, Samuel, and David Weisbach (2021), "Optimal Unilateral Carbon Policy," Cowles Foundation Discussion Paper No. 2311.

Markusen, James R. (1975), "International Externalities and Optimal Tax Structures," *Journal of International Economics*, 5, 15–29.

OECD, Trade in Embodied CO2 Database (TECO2), April 2019.

A Appendix: Basic Model

Here we drop the assumption in the basic model that welfare in either country is linearly separable. We therefore express welfare with general functions satisfying the typical regularity conditions:

$$U = u(C_s, C_e, E)$$
$$U^* = u^*(C_s^*, C_e^*, E).$$

In this setting the marginal social costs of carbon for Home and Foreign, in terms of the numeraire, are:

$$\varphi = -(\partial u/\partial E)/(\partial u/\partial C_s) = -u_E/u_s$$
$$\varphi^* = -(\partial u^*/\partial E)/(\partial u^*/\partial C_s^*) = -u_E^*/u_s^*.$$

The global externality remains $\varphi^W = \varphi + \varphi^*$.

We will solve for optimal policies by first fixing a global emissions goal of $\bar{E}$ and later optimizing over the goal. We start with the global optimum before turning to the unilateral optimum, solved in the paper, in which Home can't directly control outcomes in Foreign.

A.1 Global Optimum

Suppose that Home can dictate a policy for Foreign too, as long as it transfers services T to keep Foreign welfare at some threshold, $\bar{U}^*$. The optimal policy is then the solution to the Lagrangian:

$$\max_{\{T, C_e^*, Q_e^*\}} u(C_s, C_e, \bar{E}) + \mu \left[u^*(C_s^*, C_e^*, \bar{E}) - \bar{U}^* \right],$$

subject to:

$$C_s = L - c(\bar{E} - Q_e^*) - T$$
$$C_e = \bar{E} - C_e^*$$
$$C_s^* = L^* - c^*(Q_e^*) + T.$$

The first order conditions are:

$$u_s = \mu u_s^*$$
$$u_e = \mu u_e^*$$
$$u_s c' = \mu u_s^* c'^*.$$

They can be distilled down to two:

$$u_e/u_s = u_e^*/u_s^*$$
$$c' = c^{*\prime}$$

These two conditions rule out any wedge between Home and Foreign in the marginal value of energy consumption (in terms of the numeraire) or in the marginal cost of extracting energy.

But, they do admit the possibility of a wedge between the marginal cost of extracting energy and it's marginal value in either country, $u_e/u_s - c' = u_e^*/u_s^* - c^{*\prime}$. The level of this wedge is determined by the emissions goal, with a more ambitious goal requiring a larger wedge. The optimal emissions goal satisfies the first-order condition is:

$$u_e/u_s - c' = -(u_E/u_s + u_E^*/u_s^*).$$

Note that there is no need to distinguish between the consumption wedge, $u_e/u_s - p_e$, and the extraction wedge, $p_e - c'$.

These conditions will hold in a competitive equilibrium with taxes. With a consumption tax of t_c consumers equate their marginal rate of substitution between energy and services to $p_e + t_c$ while with an extraction tax of t_e extractors equate their marginal extraction costs to $p_e - t_e$. The first optimality condition says that a consumption tax must be harmonized between Home and Foreign, $t_c = t_c^*$, while the second says that an extraction tax must be harmonized, $t_e = t_e^*$. The third condition says that taxes on extraction and consumption must sum to the global externality:

$$t_c + t_e = \phi^W.$$

Conditional on their sum, the allocation of the tax across consumption and extraction is arbitrary. Any combination adding to the marginal global social cost of carbon attains the global optimum.

A.2 Unilateral Optimum

We now consider the optimal policy when Home can only indirectly influence Foreign extraction and consumption, by manipulating the price of energy. We will continue to assume that Home uses transfers T to keep Foreign welfare above a threshold of $\bar{U}^*$.

To solve this problem we follow Keen and Kotsogiannis (2014) and employ Foreign's expenditure function, defined as:

$$e^*(p_e, \bar{U}^*, \bar{E}) = \min \left\{ C_s^* + p_e C_e^* \;\middle|\; u^*(C_s^*, C_e^*, \bar{E}) = \bar{U}^* \right\}$$

Two key properties of the expenditure function are:

$$\begin{aligned} e^{*\prime} &= \partial e^*/\partial p_e = C_e^*(p_e, \bar{U}^*, \bar{E}) \\ e_E^* &= \partial e^*/\partial \bar{E} = -u_E^*/u_s^*. \end{aligned}$$

Here $C_e^*(p_e, \bar{U}^*, \bar{E})$ is simply Foreign's compensated demand for energy. Its partial derivative with respect to the global emissions goal is denoted by $C_{e,E}^*$. We treat the slope of this energy demand curve (the partial derivative with respect to the energy price) as strictly negative, $C_e^{*\prime} < 0$.

Foreign obtains income from labor, L^*, and rents from the energy sector, $p_e Q_e^* - c(Q_e^*)$. It also gets transfers, T, and net energy exports, $p_e X_e$, from Home. Foreign absorption, A^*, is the sum of income, transfers, and net imports of energy:

$$A^* = L^* + p_e Q_e^* - c(Q_e^*) + T + p_e X_e = L^* + p_e C_e^* - c(Q_e^*) + T.$$

If Foreign absorption is $A^* = e^*(p_e, \bar{U}^*, \bar{E})$ it can achieve welfare of $\bar{U}^*$ when the energy price is p_e and global emissions goal is $\bar{E}$. Foreign's energy supply curve, $Q_e^*(p_e)$, satisfies $c^{*\prime}(Q_e^*(p_e)) = p_e$. The slope of this energy supply curve is $Q_e^{*\prime} > 0$.

Home's optimal policy is then the solution to the Lagrangian:

$$\max_{\{T, p_e\}} u(C_s, C_e, \bar{E}) + \mu \left[L^* + p_e C_e^* - c^*(Q_e^*) + T - e^*(p_e, \bar{U}^*, \bar{E}) \right],$$

subject to:

$$\begin{aligned} Q_e^* &= Q_e^*(p_e) \\ C_e^* &= C_e^*(p_e, \bar{U}^*, \bar{E}) \\ C_s &= L - c(\bar{E} - Q_e^*(p_e)) - T \\ C_e &= \bar{E} - C_e^*(p_e, \bar{U}^*, \bar{E}). \end{aligned}$$

Here, to control outcomes in Foreign, we maximize over p_e whereas in the first-best problem we maximized separately over C_e^* and Q_e^*.

The first order conditions are:

$$\begin{aligned} u_s &= \mu \\ u_s c' Q_e^{*\prime} - u_e C_e^{*\prime} &= \mu \left(-C_e^* - p_e C_e^{*\prime} + c^{*\prime} Q_e^{*\prime} + e^{*\prime}\right). \end{aligned}$$

Applying $e^{*\prime} = C_e^*$ and $c^{*\prime} = p_e$ they can be distilled down to:

$$(p_e - c')Q_e^{*\prime} = (u_e/u_s - p_e)|C_e^{*\prime}|.$$

This result is identical to the one in the paper, showing that it is robust to welfare being non-separable in it's arguments. The key point is that, unlike the global optimum, it now matters how the overall wedge is split between an extraction wedge (on the left-hand side) and a consumption wedge (on the right-hand side). This condition will hold in a competitive equilibrium with t_e equal to the extraction wedge and t_c equal to the consumption wedge.

The first-order condition for the emissions goal is:

$$-u_s c' + u_e - u_e C_{e,E}^* + u_E = \mu \left(-p_e C_{e,E}^* + e_E^*\right)$$

and hence the overall wedge is:

$$u_e/u_s - c' = (u_e/u_s - p_e)\, C_{e,E}^* - (u_E/u_s + u_E^*/u_s^*).$$

This conditions looks like that for the global optimum, but with an additional term. Suppose $C_{e,E}^* > 0$ so that Foreign's compensated demand for energy is increasing in $\bar{E}$.[17] This term gives an added reason for Home to lower emissions, as doing so

[17] For example, if Foreign welfare is $u^*(C_s^*, C_e^*, \bar{E}) = (C_s^*)^\gamma (C_e^*)^{1-\gamma} (\bar{E})^{-\phi}$ then its compensated

shifts energy consumption away from Foreign. Such a shift is beneficial because the value of energy consumption is higher in Home then in Foreign, $u_e/u_s > p_e$, as dictated by the second condition. For linearly separable Foreign welfare $C^*_{e,W} = 0$ and this first condition collapses to the corresponding condition for the global optimum.

Both conditions will be satisfied in a competitive equilibrium with taxes satisfying:

$$t_c + t_e = \varphi^W \left(1 + \frac{Q^{*\prime}_e C^*_{e,E}}{Q^{*\prime}_e - C^*_{e,E} Q^{*\prime}_e - C^{*\prime}_e}\right)$$
$$t_e Q^{*\prime}_e + t_c C^{*\prime}_e = 0.$$

Solving for the individual taxes:

$$t_c = \varphi^W \frac{Q^{*\prime}_e}{Q^{*\prime}_e - C^*_{e,E} Q^{*\prime}_e - C^{*\prime}_e}$$
$$t_e = \varphi^W \frac{-C^{*\prime}_e}{Q^{*\prime}_e - C^*_{e,E} Q^{*\prime}_e - C^{*\prime}_e}.$$

It remains optimal for Home to tax both the demand side and the supply side of the energy market.

A.3 Trade Balance

The labor constraint in Home is $L = Q_s + c(Q_e)$ while the trade balance constraint is $p_e(C_e - Q_e) = Q_s - C_s$. Substituting these two constraints into the utility function, letting X_e denote Home net exports of energy, and dropping the constant L, Home welfare is:

$$U = u(C_e) - c(Q_e) + p_e X_e - \varphi Q^W_e.$$

demand for energy is:

$$C^*_e(p_e, \bar{U}^*, \bar{E}) = ((1-\gamma)/\gamma)^\gamma p_e^{-\gamma} (\bar{E})^\phi \bar{U}^*,$$

and $C^*_{e,E} = \phi C^*_e/\bar{E} > 0$. If instead Foreign welfare is linearly separable, $u^*(C^*_s, C^*_e, \bar{E}) = C_s + u^*(C^*_e) - \varphi^* \bar{E}$ (with $u^{*\prime} > 0$ and $u^{*\prime\prime} < 0$), then its compensated demand for energy depends only on the energy price and $C^*_{e,E} = 0$.

Home chooses C_e and Q_e to maximize welfare taking account of Foreign energy demand and supply functions, which are implicitly defined by $u^{*\prime}(D^*(p_e)) = c^{*\prime}(S^*(p_e)) = p_e$. The equilibrium price of energy p_e clears the global energy market, $X_e = D^*(p_e) - S^*(p_e)$, so p_e is a function of X_e, with $p_e' = -1/(S^{*\prime} - D^{*\prime}) < 0$.

The first order conditions for Home's problem are:

$$\begin{aligned} u' &= p_e + X_e p_e' - \varphi S^{*\prime} p_e' \\ c' &= p_e + X_e p_e' - \varphi D^{*\prime} p_e', \end{aligned}$$

where we've used the result that:

$$Q_e^W = Q_e + S^*(p_e) = C_e + D^*(p_e),$$

with p_e a function of $X_e = Q_e - C_e$.

These conditions can be replicated in a competitive equilibrium with a consumption tax t_c and an extraction tax t_e satisfying:

$$\begin{aligned} t_c &= u' - p_e = X_e p_e' - \varphi S^{*\prime} p_e' \\ t_e &= p_e - c' = -X_e p_e' + \varphi D^{*\prime} p_e'. \end{aligned}$$

Using the expression for p_e' it follows that $t_e + t_c = \varphi$ and

$$t_c = \frac{-X_e p_e + \varphi \epsilon_S^* Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^*},$$

where ϵ_S^* and ϵ_D^* are the price elasticities of Foreign supply and demand.

To implement this policy Home can impose a nominal tax on extraction, $t_e^N = \varphi$, together with a partial border adjustment on energy trade (a tariff on imports or a partial removal of the nominal tax on exports) at rate t_c. The partial nature of the border adjustment is a key feature of the optimal policy. In the case of balanced trade in energy, the border adjustment is only a fraction $\epsilon_S^* Q_e^* / (\epsilon_S^* Q_e^* + \epsilon_D^* C_e^*)$ of the nominal extraction tax. This fraction is small if the elasticity of Foreign supply is small or the elasticity of Foreign demand is large. It is then better to have more of the tax land on extraction, raising the global energy price. If Home is an energy exporter, there is an additional terms-of-trade rationale for leaving more of the tax on supply rather than demand.

B Appendix: Trade in Energy and Goods

Here we provide derivations of the optimality conditions in Section 3.

B.1 Taxing Extraction and Consumption

Home's optimal policy is to maximize the Lagrangian $\mathcal{L}$ in (4) by choosing t_e and t_c. To simplify the first order conditions we exploit various envelope conditions. Roy's identity gives $\partial\tilde{u}/\partial p_e = \partial\tilde{u}/\partial t_c = -C_e$ and $\partial\tilde{u}^*/\partial p_e = -C_e^*$. Hotelling's lemma implies $\partial R_e/\partial p_e = -\partial R_e/\partial t_e = Q_e$ and $R_e^*/\partial p_e = Q_e^*$. The derivatives of Home's tax revenue are $\partial R_t/\partial p_e = t_c\partial C_e/\partial p_e$, $\partial R_t/\partial t_c = C_e + t_c\partial C_e/\partial t_c$, and $\partial R_t/\partial t_e = Q_e + t_e\partial Q_e/\partial t_e$.

Applying these results, the first order conditions collapse to:

$$t_e\frac{\partial Q_e}{\partial t_e} + t_eQ_e'\frac{dp_e}{dt_e} + t_cC_e'\frac{dp_e}{dt_e} = \mu\left(\frac{\partial Q_e}{\partial t_e} + Q_e^{W\prime}\frac{dp_e}{dt_e}\right)$$
$$t_c\frac{\partial C_e}{\partial t_c} + t_cC_e'\frac{dp_e}{dt_c} + t_eQ_e'\frac{dp_e}{dt_c} = \mu Q_e^{W\prime}\frac{dp_e}{dt_c}.$$

The market-clearing conditions for energy imply:

$$\frac{dp_e}{dt_e} = \left(\frac{-1}{Q_e^{W\prime} - C_e^{W\prime}}\right)\frac{\partial Q_e}{\partial t_e}$$
$$\frac{dp_e}{dt_c} = \left(\frac{1}{Q_e^{W\prime} - C_e^{W\prime}}\right)\frac{\partial C_e}{\partial t_c}.$$

Substituting these price derivatives into the first-order conditions, canceling $\partial Q_e/\partial t_e$ from the first, and canceling $\partial C_e/\partial t_c$ from the second, we end up with:

$$t_e\left(C_e^{W\prime} - Q_e^{*\prime}\right) + t_cC_e' = \mu C_e^{W\prime}$$
$$t_c\left(Q_e^{W\prime} - C_e^{*\prime}\right) + t_eQ_e' = \mu Q_e^{W\prime}.$$

The results shown in the paper follow immediately.

B.2 Taxing Extraction and Production

Home's optimal policy is to maximize the Lagrangian $\mathcal{L}$ in (4) by choosing t_e and t_p.

The first order conditions can be reduced to:

$$t_e \frac{\partial Q_e}{\partial t_e} + t_e Q_e' \frac{dp_e}{dt_e} + t_p G_e' \frac{dp_e}{dt_e} = \mu \left(\frac{\partial Q_e}{\partial t_e} + Q_e^{W\prime} \frac{dp_e}{dt_e} \right)$$
$$t_p \frac{\partial G_e}{\partial t_p} + t_p G_e' \frac{dp_e}{dt_p} + t_e Q_e' \frac{dp_e}{dt_p} = \mu Q_e^{W\prime} \frac{dp_e}{dt_p},$$

while the market-clearing conditions for energy imply:

$$\frac{dp_e}{dt_e} = \left(\frac{-1}{Q_e^{W\prime} - G_e^{W\prime}} \right) \frac{\partial Q_e}{\partial t_e}$$
$$\frac{dp_e}{dt_p} = \left(\frac{1}{Q_e^{W\prime} - G_e^{W\prime}} \right) \frac{\partial G_e^W}{\partial t_p}.$$

Substituting the market-clearing conditions into the first order conditions, canceling $\partial Q_e / \partial t_e$ from the first, and using the leakage expression (7) to cancel $\partial G_e^W / \partial t_p$ from the second, we get:

$$t_e \left(G_e^{W\prime} - Q_e^{W\prime} \right) + t_e Q_e' + t_p G_e' = \mu G_e^{W\prime}$$
$$\frac{t_p}{1 - \Lambda} \left(Q_e^{W\prime} - G_e^{W\prime} \right) + t_p G_e' + t_e Q_e' = \mu Q_e^{W\prime}.$$

Subtracting the second equation above from the first:

$$t_e + \frac{t_p}{1 - \Lambda} = \mu. \tag{13}$$

Substituting this value of μ back into the first equation we get the analog of (2).

B.3 Taxing Extraction, Consumption, and Production

Home's optimal policy is to maximize the Lagrangian $\mathcal{L}$ in (4) by choosing t_e, t_d, t_m, and t_x.

The first order conditions for t_d and t_m are:

$$t_d \frac{\partial C_e^d}{\partial t_d} + \left(t_d C_e^{d\prime} + t_m C_e^{m\prime} + t_x C_e^{x\prime} + t_e Q_e' \right) \frac{dp_e}{dt_d} = \varphi^W Q_e^{W\prime} \frac{dp_e}{dt_d}$$
$$t_m \frac{\partial C_e^m}{\partial t_m} + \left(t_d C_e^{d\prime} + t_m C_e^{m\prime} + t_x C_e^{x\prime} + t_e Q_e' \right) \frac{dp_e}{dt_m} = \varphi^W Q_e^{W\prime} \frac{dp_e}{dt_m}.$$

Substituting in the corresponding market clearing conditions:

$$\frac{dp_e}{dt_d} = \frac{\partial C_e^d/\partial t_d}{Q_e^{W\prime} - C_e^{W\prime}}$$
$$\frac{dp_e}{dt_m} = \frac{\partial C_e^m/\partial t_m}{Q_e^{W\prime} - C_e^{W\prime}},$$

the first-order conditions reduce to $t_d = t_m$. Hence, it is optimal to tax energy embodied in Home's consumption at the same rate, $t_c = t_d = t_m$, whether the goods are produced domestically or imported. Applying this condition, we can add C_e^d and C_e^m to form a single first-order condition for t_c.

To handle the remaining three first-order conditions we need to introduce leakage again, but here with a twist. Due to the consumption tax, there is no leakage in serving Home consumers – producers in both Home and Foreign face the same price of energy when selling in Home. Foreign producers, however, still have an advantage relative to Home producers when serving Foreign consumers. This results in what we call Foreign leakage (denoted with a *). We define Foreign leakage as the increase in Foreign production to serve Foreign consumers relative to the decrease in Home production to serve Foreign consumers, for a given change in t_x:

$$\Lambda^* = -\frac{\partial C_e^f/\partial t_x}{\partial C_e^x/\partial t_x} > 0. \tag{14}$$

It follows that $\partial C_e^x/\partial t_x = (\partial C_e^*/\partial t_x)/(1 - \Lambda^*)$.

The first order conditions for t_e, t_c, and t_x can now be reduced to:

$$t_e \frac{\partial Q_e}{\partial t_e} + (t_e Q_e' + t_c C_e' + t_x C_e^{x\prime}) \frac{dp_e}{dt_e} = \mu \left(\frac{\partial Q_e}{\partial t_e} + Q_e^{W\prime} \frac{dp_e}{dt_e} \right)$$
$$t_c \frac{\partial C_e}{\partial t_c} + (t_c C_e' + t_x C_e^{x\prime} + t_e Q_e') \frac{dp_e}{dt_c} = \mu Q_e^{W\prime} \frac{dp_e}{dt_c}$$
$$t_x \frac{\partial C_e^x}{\partial t_x} + (t_x C_e^{x\prime} + t_c C_e' + t_e Q_e') \frac{dp_e}{dt_x} = \mu Q_e^{W\prime} \frac{dp_e}{dt_x},$$

with associated market-clearing conditions:

$$\frac{dp_e}{dt_e} = \left(\frac{-1}{Q_e^{W\prime} - C_e^{W\prime}}\right)\frac{\partial Q_e}{\partial t_e}$$
$$\frac{dp_e}{dt_c} = \left(\frac{1}{Q_e^{W\prime} - C_e^{W\prime}}\right)\frac{\partial C_e}{\partial t_c}$$
$$\frac{dp_e}{dt_x} = \left(\frac{1}{Q_e^{W\prime} - C_e^{W\prime}}\right)\frac{\partial C_e^*}{\partial t_x}.$$

Substituting each market-clearing condition into its corresponding first order condition, canceling $\partial Q_e/\partial t_e$ from the first, canceling $\partial C_e/\partial t_c$ from the second, and using (14) to eliminate $\partial C_e^*/\partial t_x$ from the third, we get:

$$t_e\left(C_e^{W\prime} - Q_e^{W\prime}\right) + (t_e Q_e' + t_c C_e' + t_x C_e^{x\prime}) = \mu C_e^{W\prime}$$
$$t_c\left(Q_e^{W\prime} - C_e^{W\prime}\right) + (t_e Q_e' + t_c C_e' + t_x C_e^{x\prime}) = \mu Q_e^{W\prime}$$
$$\frac{t_x}{1-\Lambda^*}\left(Q_e^{W\prime} - C_e^{W\prime}\right) + (t_e Q_e' + t_c C_e' + t_x C_e^{x\prime}) = \mu Q_e^{W\prime}.$$

Subtracting the third from the first, much like for the extraction-production tax, gives:

$$t_e + \frac{t_x}{1-\Lambda^*} = \mu.$$

Substituting in this value of μ and subtracting the second from the first, we get:

$$t_x = (1-\Lambda^*)t_c,$$

so that we have $t_e + t_c = \mu$ as in the extraction-consumption case. Substituting both of these results back into the first we get the analog of (2), which is now:

$$\frac{t_e}{t_c} = \frac{|C_e^{f\prime}| + \Lambda^*|C_e^{x\prime}|}{Q_e^{*\prime}}.$$

C Appendix: Online

Here we demonstrate how our results hold in the setting of Kortum and Weisbach (2021), which explicitly follows the Dornbusch, Fisher, and Samuelson (1977) Ricardian model of trade with a unit continuum of goods. The relative efficiency of producing good $j \in [0,1]$ in Home is $a_j^*/a_j = F(j)$, where a_j is Home's total input requirement and a_j^* is Foreign's. The function F is assumed to be continuous and strictly decreasing in j.

Producers combine inputs of labor and energy in a constant-returns-to-scale production function to produce any good in any region. The wage is 1 and the relevant after-tax energy price is some value p. The associated unit cost function for Home producers to supply good j is

$$f_j(p) = a_j f(p).$$

By Shepard's lemma, the unit energy requirement is:

$$e_j = a_j f'(p).$$

The same holds for Foreign producers, with a_j replaced by a_j^*, taking account of the energy price they face.

We will consider goods produced for the Home market, but the argument will carry over in an obvious way to the Foreign market. Suppose we allow for a tax t_d on energy embodied in domestic production and a tax t_m on energy embodied in imports. The after-tax prices of energy are therefore $p_e^d = p_e + t_d$ and $p_e^m = p_e + t_m$. At these after-tax prices, Home producers can supply good j at cost $a_j f(p_e^d)$ while the cost to Foreign producers is $\tau^* a_j^* f(p_e^m)$. Here τ^* is an iceberg trade cost.

Home consumers will purchase j from the cheapest supplier. Thus they buy goods $j \in [0, \bar{j}_m)$ from domestic producers and goods $j \in (\bar{j}_m, 1]$ from Foreign producers. The threshold satisfies:

$$F(\bar{j}_m) = \frac{1}{\tau^*} \frac{f(p_e^d)}{f(p_e^m)},$$

making Home consumers indifferent about where they buy good $\bar{j}_m$ (since it costs the same from either source). Thus consumers in Home, buying from the low-cost supplier, face prices:

$$\begin{aligned} p_j &= a_j f(p_e^d) \quad j \leq \bar{j}_m \\ p_j &= \tau^* a_j^* f(p_e^m) \quad j \geq \bar{j}_m. \end{aligned}$$

Welfare of a representative consumer in Home is:

$$U = C_s + \frac{\sigma}{\sigma - 1}\eta^{1/\sigma}\left(C_g^{(\sigma-1)/\sigma} - 1\right) - \varphi E,$$

where E is global carbon emissions and C_g is a CES index of the consumption of individual goods:

$$C_g = \left(\int_0^1 c_j^{(\sigma-1)/\sigma} dj\right)^{\sigma/(\sigma-1)}.$$

Facing prices p_j, the utility maximizing consumption of good j is:

$$c_j = \eta p_j^{-\sigma}.$$

In Section 3 we express Home welfare in terms of indirect utility:

$$U = Y + \tilde{u} - \varphi Q_e^W.$$

We therefore have:

$$\tilde{u} = \frac{\eta}{\sigma - 1} p_g^{-(\sigma-1)} - \frac{\eta^{1/\sigma}\sigma}{\sigma - 1},$$

where p_g is the price index associated with C_g:

$$p_g = \left(\int_0^1 p_j^{1-\sigma} dj\right)^{\frac{1}{1-\sigma}}.$$

We can express the price index as a function of the after-tax prices of energy in Home and Foreign:

$$p_g(p_e^d, p_e^m) = \left(\int_0^{\bar{j}_m} \left(a_j f(p_e^d)\right)^{1-\sigma} dj + \int_{\bar{j}_m}^1 \left(\tau^* a_j^* f(p_e^m)\right)^{1-\sigma} dj\right)^{\frac{1}{1-\sigma}}.$$

While the threshold $\bar{j}_m$ is also a function of these after-tax prices of energy, by the envelope theorem that doesn't matter for the derivatives (the cost of sourcing the threshold good from Home or Foreign producers is the same). For example:

$$
\begin{aligned}
\partial p_g / \partial p_e^d = & \frac{1}{1-\sigma} p_g^\sigma \left\{ \int_0^{\bar{j}_m} (1-\sigma) p_j^{-\sigma} a_j f'(p_e^d) dj \right. \\
& \left. + \left(\left(a_{\bar{j}_m} f(p_e^d) \right)^{1-\sigma} - \left(\tau^* a^*_{\bar{j}_m} f(p_e^m) \right)^{1-\sigma} \right) \partial \bar{j}_m / \partial p_e^d \right\} \\
& = p_g^\sigma \int_0^{\bar{j}_m} p_j^{-\sigma} a_j f'(p_e^d) dj.
\end{aligned}
$$

We can also treat $\tilde{u}$ as a function of the after-tax prices of energy in Home and Foreign:

$$
\tilde{u} = \tilde{u}(p_e^d, p_e^m) = \frac{\eta}{\sigma - 1} p_g (p_e^d, p_e^m)^{-(\sigma-1)} - \frac{\eta^{1/\sigma} \sigma}{\sigma - 1}. \tag{15}
$$

By expressing Home welfare this way, Roy's identity implies:

$$
\begin{aligned}
\partial U / \partial p_e^d &= -C_e^d \\
\partial U / \partial p_e^m &= -C_e^m.
\end{aligned}
$$

It follows that:

$$
\begin{aligned}
\partial U / \partial t_d &= -C_e^d \\
\partial U / \partial p_m &= -C_e^m \\
\partial U / \partial p_e = -C_e^d - C_e^m &= -C_e.
\end{aligned}
$$

We employ these results in the paper.

To confirm that such results are applicable despite the extensive margin of trade, we can differentiate (15) to get:

$$
\begin{aligned}
\partial \tilde{u} / \partial p_e^d &= -\eta p_g^{-\sigma} \partial p_g / \partial p_e^d \\
&= - \int_0^{\bar{j}_m} a_j f'(p_e^d) \eta p_j^{-\sigma} dj \\
&= - \int_0^{\bar{j}_m} e_j c_j dj = -C_e^d.
\end{aligned}
$$

Roy's identity clearly applies to this setting.