Optimal Monetary Policy with Redistribution

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Some facts about labor earnings

- **Fact 1.** There are *large, systematic, forecastable* differences in labor earnings

- **Fact 2.** Households face heterogeneous exposure of their labor earnings to the business cycle
  - earnings of low-income households exhibit a greater covariance with aggregate fluctuations than that of high-income households
  - countercyclical earnings inequality

  “We find that the fortunes during recessions are predictable by observable characteristics before the recession.”

Guvenen and Smith (2014); Guvenen, Ozkan, Song (2014); Schulhofer-Wohl (2011); Guvenen(2011); Guvenen, Schulhofer-Wohl, Song, Yogo (2017); Parker Vissing-Jorgenson (2009)
A third “fact”

Fact 3. Markups are Counter-cyclical

Bils (1987); Chari, Kehoe, and McGrattan (2007); Bils, Klenow, Malin (2018)
This Paper: Optimal Monetary Policy with Redistribution

- We write down a general equil business cycle model with heterogeneity and nominal rigidities

- We take the Ramsey approach to redistribution (not insurance)

- Given a restricted set of available tax instruments:

  We solve for optimal fiscal & monetary policy following the primal approach

  Lucas Stokey (1983); Chari Kehoe (1999); Correia, Nicolini, Teles (2008)

- We find that, given Facts 1 & 2, Fact 3 is consistent with optimal monetary policy
Our Framework

- heterogeneous agent economy à la Werning (2007)
  - workers differ in type-specific labor productivities, “skills”
  - skills are state-contingent, but markets are complete
  - no missing insurance markets

- firms face nominal rigidities = informational friction
  - must set nominal prices before observing demand

- shocks to aggregate productivity and the labor skill distribution

- Ramsey taxation: full set of linear tax instruments, with measurability restrictions
  - tax rates are non-state-contingent [or set one period in advance]
  - state-contingent lump sum transfers: uniform across household types
What We Do and What We Show

- We consider a utilitarian planner with arbitrary Pareto weights and solve for optimal policy

- When shocks to the skill distribution are proportional (no movement in relative productivities):
  - all redistribution is done via the tax system
  - optimal for monetary policy to implement flexible-price allocations \(\rightarrow\) target price stability

- When shocks affect relative productivities:
  - tax instruments are insufficient to implement the constrained efficient allocation
  - optimal for monetary policy to deviate from implementing flexible-prices

- Optimal markup co-varies positively with a sufficient statistic for labor income inequality
The Environment
The Environment

- $t = 0, 1, \ldots$
- finite states $s_t \in S$
- history $s^t = (s_0, \ldots, s_t) \in S^t$
  - conditional probabilities $\mu(s^t | s^{t-1})$
  - unconditional probabilities $\mu(s^t)$
Household preferences

- unit mass continuum of households
- identical preferences over consumption and effort

\[ U(c,h) = \frac{c^{1-\gamma}}{1-\gamma} - \frac{h^{1+\eta}}{1+\eta} \]
Household types

- finite types \( i \in I \) of relative size \( \pi^i \)

- types correspond to workers' state-contingent skill

\[
\theta^i(s_t) > 0
\]

- efficiency units of labor

\[
\ell^i(s^t) = \theta^i(s_t) h^i(s^t)
\]

- expected lifetime utility

\[
\sum_t \sum_{s^t} \beta^t \mu(s^t) U \left( c^i(s^t), \frac{\ell^i(s^t)}{\theta^i(s_t)} \right)
\]
Household budget set

\[(1 + \tau_c)P(s^t)c^i(s^t) + b^i(s^t) + \sum_{s^{t+1}|s^t} Q(s^{t+1}|s^t)z^i(s^{t+1}|s^t)\]

\[\leq (1 - \tau_{\ell})W(s^t)\ell^i(s^t) + P(s^t)T(s^t) + (1 - \tau_{\Pi})\Pi(s^t) + z^i(s^t|s^{t-1}) + (1 + i(s^{t-1}))b^i(s^{t-1})\]
Firms

- **intermediate good firms.** monopolistically-competitive, indexed by $j \in J = [0, 1]$

  \[ y^j(s^t) = A(s_t)n^j(s^t) \]

  \[ \text{profits}^j(s^t) = (1 - \tau_r)p^j(\cdot)y^j(s^t) - W(s^t)n^j(s^t) \]

- **final good firm.** perfectly competitive:

  \[ Y(s^t) = \left[ \int_{j \in J} y^j(s^t) \frac{\rho - 1}{\rho} \, dj \right]^{\frac{\rho}{\rho - 1}} \rightarrow y^j(s^t) = \left( \frac{p^j(\cdot)}{P(s^t)} \right)^{-\rho} Y(s^t) \]
The Government: Consolidated Fiscal and Monetary Authority

- tax revenue

\[ T(s^t) \equiv \tau_c P(s^t) C(s^t) + \tau_l W(s^t) L(s^t) + \tau_r P(s^t) Y(s^t) + \tau_\Pi \Pi(s^t) \]

- government budget

\[
(1 + i(s^{t-1})) B(s^{t-1}) + Z(s^t) + P(s^t) T(s^t) = B(s^t) + \sum_{s^{t+1}|s^t} Q(s^{t+1}|s^t) Z(s^{t+1}) + T(s^t)
\]
Market Clearing

- aggregates

\[
C(s^t) = \sum_{i \in I} \pi^i c^i(s^t), \quad L(s^t) = \sum_{i \in I} \pi^i \ell^i(s^t), \quad \Pi(s^t) \equiv \int_0^1 \text{profits}^j(s^t) \, dj
\]

- market clearing

\[
C(s^t) = Y(s^t), \quad L(s^t) = \int_{j \in J} n^j(s^t) \, dj
\]

\[
B(s^t) \equiv \sum_{i \in I} \pi^i b^i(s^t), \quad Z(s^t) \equiv \sum_{i \in I} \pi^i z^i(s^t | s^{t-1})
\]
Nominal Rigidities
Nominal Rigidity = Informational Friction

- Nature draws the aggregate state
  \[ s_t \in S \]

- the state determines
  \[ A(s_t), (\theta^i(s_t))_{i \in I} \]

- \( \kappa \in [0, 1) \) of intermediate-good firms, \( j \in \mathcal{J}^s \subset \mathcal{J} \), are “inattentive” to the current state

- \( 1 - \kappa \) of intermediate-good firms, \( j \in \mathcal{J}^f \subset \mathcal{J} \), are “attentive” to the current state
inattentive, “sticky-price” firms do not observe $s_t$

- make their pricing decisions based only on knowledge of past states

$$p_t^s(s_{t-1}), \quad \forall j \in J^s$$

attentive, “flexible-price” firms observe $s_t$ perfectly

- make their pricing decisions under complete information

$$p_t^f(s_t), \quad \forall j \in J^f$$
Implicit Timing Assumption

1. Nature draws the aggregate state $s_t \in S$

2. intermediate-good firms make their nominal pricing decisions

\[
p_t^s(s^{t-1}) \quad \text{and} \quad p_t^f(s^t)
\]

3. once prices are set, the state $s_t$ is revealed/becomes common knowledge

4. all other market outcomes, allocations adjust to the aggregate state
   - given prices, the final good firm chooses its inputs
   - households make consumption, effort, and savings decisions
   - inputs adjust so that supply = demand
Feasible Allocations: satisfy technology and resource constraints

- Allocation

\[ x \equiv \{ (c^i(s^t), \ell^i(s^t))_{i \in I}, (y^j(s^t), n^j(s^t))_{j \in J}, C(s^t), Y(s^t), L(s^t) \}_{s^t \in S^t} \]

**Definition**

An allocation \( x \) is **feasible** if it satisfies, for all \( s^t \in S^t \):

\[ y^j(s^t) = A(s_t)n^j(s^t), \quad \forall j \in J; \]

\[ Y(s^t) = \left[ \int_{j \in J} y^j(s^t) \frac{\rho-1}{\rho} \, dj \right] \frac{\rho}{\rho-1} ; \quad L(s^t) = \int_{j \in J} n^j(s^t) \, dj; \]

\[ C(s^t) = \sum_{i \in I} \pi^i c^i(s^t) ; \quad L(s^t) = \sum_{i \in I} \pi^i \ell^i(s^t) ; \quad C(s^t) = Y(s^t). \]

Let \( \mathcal{X} \) denote the set of all feasible allocations.
We are interested in allocations $x \in \mathcal{X}$ that can be supported in equilibrium

- policy
  \[ \phi \equiv \{ \tau_c, \tau_\ell, \tau_r, \tau_\Pi, (T(s^t), i(s^t))_{s^t \in S^t} \} \]

- price system
  \[ \rho \equiv \{ p^f_t(s^t), p^s_t(s^{t-1}), P(s^t), W(s^t), (Q(s^{t+1}|s^t))_{s^t+1 \in S^{t+1}} \}_{s^t \in S^t} \]

- financial positions
  \[ \zeta \equiv \{ (b^i(s^t))_{i \in I}, B(s^t), (z^i(s^{t+1}|s^t), Z(s^{t+1}))_{s^t+1 \in S^{t+1}|S^t} \}_{s^t \in S^t} \]
Equilibrium Definitions

Definition
A **sticky-price equilibrium** is an allocation \(\mathbf{x}\), a price system \(\mathbf{\rho}\), a policy \(\phi\), and financial positions \(\zeta\) such that:

(i) \(p^s_t(s^{t-1})\) is optimal for firms \(j \in \mathcal{J}^s\); \(p^f_t(s^t)\) is optimal for firms \(j \in \mathcal{J}^f\);
(ii) prices and allocations jointly satisfy the CES demand function;
(iii) the allocation and financial asset holdings solve household \(i\)'s problem, for each \(i \in I\);
(iv) the government budget constraint is satisfied;
(v) markets clear.

Definition
A **flexible-price equilibrium** is an allocation \(\mathbf{x}\), a price system \(\mathbf{\rho}\), a policy \(\phi\), and financial positions \(\zeta\) such that:

\(p^f_t(s^t)\) is optimal for firms \(j \in \mathcal{J}\), and parts (ii)-(vi) of the previous definition hold.
Equilibrium Characterization
There exists a “Fictitious” Representative Household

Lemma

(Negishi 1960; Werning 2007)

For any equilibrium there exist “market” or “Negishi” weights $\varphi \equiv (\varphi^i)_{i \in I}$ with $\varphi^i \geq 0$ such that

$$\{c^i(s^f), \ell^i(s^f)\}_{i \in I}$$

solve the following static sub-problem

$$U^m(C(s^f), L(s^f); \varphi) \equiv \max \sum_{i \in I} \varphi^i \pi^i U(c^i(s^f), \ell^i(s^f)/\theta^i(s_t))$$

subject to

$$C(s^f) = \sum_{i \in I} \pi^i c^i(s^f), \quad \text{and} \quad L(s^f) = \sum_{i \in I} \pi^i \ell^i(s^f)$$

the superscript “m” stands for “market”
Equilibrium prices thereby satisfy

\[- \frac{U_m^L(s^t)}{U_m^C(s^t)} = \left( 1 - \frac{\tau_c}{1 + \tau_c} \right) \frac{W(s^t)}{P(s^t)} \]

\[
\frac{U_m^m(s^t)}{P(s^t)} = \beta (1 + i(s^t)) \sum_{s^{t+1} | s^t} \mu(s^{t+1} | s^t) \frac{U_m^m(s^{t+1})}{P(s^{t+1})} 
\]

\[
Q(s^{t+1} | s^t) = \beta \mu(s^{t+1} | s^t) \frac{U_m^m(s^{t+1})}{U_m^m(s^t)} \frac{P(s^t)}{P(s^{t+1})} 
\]

- solution to sub-problem with iso-elastic utility:

\[
c^i(s^t) = \omega^i_C(\varphi) C(s^t) \quad \text{and} \quad \ell^i(s^t) = \omega^i_L(\varphi, s_t) L(s^t),
\]

\[
\omega^i_C(\varphi) \equiv \frac{(\varphi^i)^{1/\gamma}}{\sum_{j \in I} \pi^j(\varphi^j)^{1/\gamma}}, \quad \omega^i_L(\varphi, s_t) \equiv \frac{(\varphi^i)^{-1/\eta} \theta^i(s_t)^{1+\eta}}{\sum_{k \in I} \pi^k(\varphi^k)^{-1/\eta} \theta^i(s_t)^{1+\eta}}
\]
Equilibrium prices thereby satisfy

\[- \frac{U^m_L(s^t)}{U^m_C(s^t)} = \left( \frac{1 - \tau_\ell}{1 + \tau_c} \right) \frac{W(s^t)}{P(s^t)} \]

\[\frac{U^m_C(s^t)}{P(s^t)} = \beta (1 + i(s^t)) \sum_{s^t+1|s^t} \mu(s^t+1|s^t) \frac{U^m_C(s^t+1)}{P(s^t+1)} \]

\[Q(s^t+1|s^t) = \beta \mu(s^t+1|s^t) \frac{U^m_C(s^t+1)}{U^m_C(s^t)} \frac{P(s^t)}{P(s^t+1)} \]

- solution to sub-problem with iso-elastic utility:

\[c^i(s^t) = \omega^i_C(\varphi)C(s^t) \quad \text{and} \quad \ell^i(s^t) = \omega^i_L(\varphi, s_t)L(s^t), \]

\[\omega^i_C(\varphi) \equiv \frac{(\varphi^i)^{1/\gamma}}{\sum_{j \in I} \pi^j(\varphi^j)^{1/\gamma}}, \quad \omega^i_L(\varphi, s_t) \equiv \frac{(\varphi^i)^{-1/\eta} \theta^i(s_t)^{1+\eta}}{\sum_{k \in I} \pi^k(\varphi^k)^{-1/\eta} \theta^i(s_t)^{1+\eta}} \]
Primal approach: budget implementability conditions

\[
\sum_{t} \sum_{s} \beta^t \mu(s^t) \left[ U^{m}_{C}(s^t) \omega^{i}_{C}(\varphi) C(s^t) + U^{m}_{L}(s^t) \omega^{i}_{L}(\varphi, s_t) L(s^t) \right] = U^{m}_{C}(s_0) \bar{T}, \quad \forall i \in I
\]

\[
\bar{T} \equiv \frac{1}{U^{m}_{C}(s_0)(1 + \tau_c)} \sum_{t} \sum_{s} \beta^t \mu(s^t) U^{m}_{C}(s^t) \left[ T(s^t) + (1 - \tau) \Pi(s^t) \right] \frac{\Pi(s^t)}{P(s^t)}
\]

- implementability conditions: one for each type \( i \in I \) (Werning, 2007)
  - similar to Lucas Stokey (1983) implementability condition for rep household’s budget constraint
  - however, unlike Lucas Stokey: existence of lump-sum taxes + multiple household types

- profits are isomorphic to lump-sum transfers
  - we relax this in an extension with heterogeneous equity shares
Flexible Price Firm’s Problem

Firm $j \in J^f$ solves

$$\max_{p'} \left\{ (1 - \tau_r) p' y^j(s^t) - W(s^t) \frac{y^j(s^t)}{A(s^t)} \right\}$$

subject to

$$y^j(s^t) = \left( \frac{p'}{P(s^t)} \right)^{-\rho} Y(s^t), \quad \forall s^t \in S^t.$$ 

Sticky Price Firm’s Problem

Firm $j \in J^s$ solves

$$\max_{p'} \sum_{s' | s^t - 1} \mu(s' | s^t - 1) \frac{U^m(s^t)}{P(s^t)} \left\{ (1 - \tau_r) p' y^j(s^t) - W(s^t) \frac{y^j(s^t)}{A(s^t)} \right\}$$

subject to

$$y^j(s^t) = \left( \frac{p'}{P(s^t)} \right)^{-\rho} Y(s^t), \quad \forall s^t \in S^t.$$
Firm Optimality

- **flex-price firm**: price = mark-up over marginal cost

\[ p^f_t(s^t) = \left[ (1 - \tau_r) \left( \frac{\rho - 1}{\rho} \right) \right]^{-1} \frac{W(s^t)}{A(s_t)} \]

- **sticky-price firm**: price = mark-up over expected marginal cost

\[ p^s_t(s^{t-1}) = \left[ (1 - \tau_r) \left( \frac{\rho - 1}{\rho} \right) \right]^{-1} \sum_{s^t|s^{t-1}} \left[ \frac{W(s^t)}{A(s_t)} \right] q(s^t|s^{t-1}) \]

where

\[ q(s^t|s^{t-1}) \equiv \frac{\mu(s^t|s^{t-1}) U^m_C(s^t) Y(s^t) P(s^t) \rho^{-1}}{\sum_{s^t|s^{t-1}} \mu(s^t|s^{t-1}) U^m_C(s^t) Y(s^t) P(s^t) \rho^{-1}} \]
Proposition

A feasible allocation $x \in \mathcal{X}$ is implementable as a flexible-price equilibrium iff

$\exists$ market weights $\varphi \equiv (\varphi^i)$ and constants $\bar{T} \in \mathbb{R}$ and $\chi \in \mathbb{R}_+$, such that:

(i) for all $s^t \in S^t$:

$$y^j(s^t) = y^{j'}(s^t) = Y(s^t) \quad \forall j, j' \in \mathcal{J};$$

(ii) for all $s^t \in S^t$:

$$- \frac{U^m_L(s^t)}{U^m_C(s^t)} = \chi A(s_t);$$

(iii) for all $i \in I$:

$$\sum_t \sum_{s^t} \beta^t \mu(s^t) \left[ U^m_C(s^t) \omega^i_C(\varphi) C(s^t) + U^m_L(s^t) \omega^i_L(\varphi, s_t) L(s^t) \right] = U^m_C(s_0) \bar{T}.$$
What can fiscal policy do?

- the fiscal authority has the power to move around allocations through $\chi$ and $\bar{T}$
- the labor wedge results from linear taxes and markups

$$\chi \equiv \left( \rho - 1 \right) \frac{(1 - \tau_\ell)(1 - \tau_r)}{1 + \tau_c}$$

- lump sum taxes/transfers + profits affect budgets via $\bar{T}$
Proposition

A feasible allocation $x \in \mathcal{X}$ is implementable as a **sticky-price equilibrium** iff

$\exists$ market weights $\varphi \equiv (\varphi^i)$ and constants $\bar{T} \in \mathbb{R}$ and $\chi \in \mathbb{R}^+$, such that:

(i) for all $s^t \in S^t$:

$$
\begin{align*}
  y^j(s^t) &= y^f(s^t), & \forall j \in \mathcal{J}^f \\
  y^j(s^t) &= y^s(s^t), & \forall j \in \mathcal{J}^s
\end{align*}
$$

(ii) for all $s^t \in S^t$:

$$
\chi U^m_C(s^t) \left( \frac{y^f(s^t)}{Y(s^t)} \right)^{-1/\rho} + U^m_L(s^t) \frac{1}{A(s^t)} = 0,
$$

for all $s^{t-1} \in S^{t-1}$:

$$
\sum_{s^t | s^{t-1}} y^s(s^t) \left\{ \chi U^m_C(s^t) \left[ \frac{y^s(s^t)}{Y(s^t)} \right]^{-1/\rho} + U^m_L(s^t) \frac{1}{A(s^t)} \right\} \mu(s^t | s^{t-1}) = 0,
$$

(iii) for all $i \in I$:

$$
\sum_t \sum_{s^t} \beta^t \mu(s^t) \left[ U^m_C(s^t) \omega^i_C(\varphi) C(s^t) + U^m_L(s^t) \omega^i_L(\varphi, s^t) L(s^t) \right] = U^m_C(s_0) \bar{T}.
$$
What can monetary policy do vis-a-vis fiscal policy?

- sticky-price firm: price = mark-up over realized marginal cost, modulo a forecast error

\[ p_t^s(s_t^{-1}) = \left[ (1 - \tau_r) \left( \frac{\rho - 1}{\rho} \right) \right]^{1-\rho} \frac{W(s_t)}{A(s_t)} \]

- then:

\[ -\frac{U_m^L(s_t)}{U_m^C(s_t)} = \chi \left[ \kappa \epsilon(s_t) \right]^{1-\rho} + (1 - \kappa) \left[ \frac{1}{1-\rho} \right] \times A(s_t) \]

- monetary policy: state-contingent wedge \( \epsilon(s_t) \)
  - cost of using monetary wedge is loss in production efficiency: \( y^s(s_t) \neq y^f(s_t) \)
  - constraint on \( \epsilon(s_t) \): forecast error → on average, equal to 1
Lemma

Let $\mathcal{X}^f$ denote the set of flexible-price allocations. Let $\mathcal{X}^s$ denote the set of sticky-price allocations.

$$\mathcal{X}^f \subset \mathcal{X}^s \subset \mathcal{X}.$$ 

Proof.

Take any $x \in \mathcal{X}^f$. $x$ can be implemented under sticky prices with:

$$\frac{y^s(s^t)}{Y(s^t)} = \frac{y^f(s^t)}{Y(s^t)} = 1, \quad \forall s^t \in S^t.$$ 

[i.e. $\varepsilon(s^t) = 1$ for all $s^t \in S^t$.]
The Ramsey Problem
Utilitarian Welfare Function

- social welfare function with Pareto weights $\lambda^i > 0$

$$U ≡ \sum_{i \in I} \lambda^i \pi^i \sum_t \sum_{s^t} \beta^t \mu(s^t)U(c^i(s^t), \ell^i(s^t)/\theta^i(s_t))$$

- our goal: characterize the welfare-maximizing allocation $x \in X^s$

**Definition**

A **Ramsey optimum** $x^*$ is an allocation that maximizes welfare subject to $x^* \in X^s$. 
\( \mathcal{X}^s \) is a complicated set

We first solve an \textit{easier} problem, the “relaxed Ramsey planning problem”

Correia, Nicolini, Teles (2008)
The Relaxed Ramsey Planner

**Definition**

The relaxed set of allocations \( \mathcal{X}^R \) is the set of feasible allocations \( x \in \mathcal{X} \) that satisfy, for all \( i \in I \):

\[
\sum_t \sum_{s_t} \beta^t \mu(s_t) \left[ U_C^m(s_t) \omega_C^i(\phi) C(s_t) + U_L^m(s_t) \omega_L^i(\phi, s_t) L(s_t) \right] \leq U_C^m(s_0) \bar{T}.
\]

A **Relaxed Ramsey optimum** \( x^R \) is an allocation that maximizes welfare subject to

\[
x^R \in \mathcal{X}^R.
\]

- our Relaxed Ramsey planner = “Lucas-Stokey-Werning” planner
Corollary

The relaxed set is a strict superset of $\mathcal{X}^s$

$$\mathcal{X}^f \subset \mathcal{X}^s \subset \mathcal{X}^R \subset \mathcal{X}.$$
Why look at the Relaxed Ramsey planner’s problem?

- the relaxed set is a strict superset
  \[ \mathcal{X}^f \subset \mathcal{X}^s \subset \mathcal{X}^R \]

- we will derive conditions under which
  \[ x^R \in \mathcal{X}^f \subset \mathcal{X}^s \]

- under these conditions, \( x^R \) solves the (unrelaxed) Ramsey problem!
The Relaxed Ramsey Planner’s Problem

- let $\pi^i \nu^i$ be the Lagrange multiplier on the implementability condition of type $i$
- define the pseudo-welfare function by:

$$
\mathcal{W}(C,L;\phi,\nu,\lambda) \equiv \sum_{i \in I} \pi^i \left\{ \lambda^i U^i(\omega^i_C(\phi)C(s^t), \omega^i_L(\phi,s_t)L(s^t)) + \nu^i \left[ U^m_C(s^t)\omega^i_C(\phi)C(s^t) + U^m_L(s^t)\omega^i_L(\phi,s_t)L(s^t) \right] \right\}
$$

Relaxed Ramsey Planner’s Problem

$$
\max_{x,\phi,\bar{T}} \sum_t \sum_{s^t} \beta^t \mu(s^t) \mathcal{W}(C(s^t),L(s^t);\phi,\nu,\lambda) - U^m_C(s_0) \sum_{i \in I} \pi^i \nu^i \bar{T}
$$

subject to feasibility.
Proposition

The Relaxed Ramsey optimum $x^{R*} \in X^R$ satisfies

$$-\frac{W_L(s^t)}{W_C(s^t)} = A(s_t), \quad \forall s^t \in S^t$$

and

$$y^j(s^t) = y^{j'}(s^t) = Y(s^t), \quad \forall j, j' \in J;$$

- Lucas-Stokey-Werning optimum features zero output dispersion across firms
- preserves Diamond and Mirrlees (1971) production efficiency
When can you implement $x^{R^*}$ under flexible prices?

**Theorem**

If $\exists$ positive scalars $(\vartheta^1, \vartheta^2, \ldots, \vartheta^I) \in \mathbb{R}^I_+$ and a function $\Theta : S \to \mathbb{R}_+$ such that

$$\theta^i(s_t) = \vartheta^i \Theta(s_t), \quad \forall s_t \in S,$$

then

$$x^{R^*} \in X^f.$$  

It follows that

$$x^{R^*} \in X^s.$$  

It is therefore optimal for monetary policy to replicate flexible price allocations.
Proof

- relaxed Ramsey optimality condition

\[- \frac{W_L(s^t)}{W_C(s^t)} = A(s_t)\]

- with homothetic preferences this can be written as:

\[- \frac{U^m_L(s^t)}{U^m_C(s^t)} \left[ \frac{\sum_{i \in I} \pi^i \omega^i_L(\phi, s_t) \left( \frac{\lambda^i}{\varphi} + v^i(1 + \eta) \right)}{\sum_{i \in I} \pi^i \omega^i_C(\phi) \left( \frac{\lambda^i}{\varphi} + v^i(1 - \gamma) \right)} \right] = A(s_t)\]

- with this condition on the skill distribution:

\[\omega^i_L(\phi, s_t) = \omega^i_L(\phi) = \frac{(\phi^i)^{-\frac{1}{\eta}}(v^i)^{\frac{1+\eta}{\eta}}}{\sum_{k \in I} \pi^k(\phi^k)^{-\frac{1}{\eta}}(v^k)^{\frac{1+\eta}{\eta}}}\]

- in which case the relaxed optimum can be implemented under flexible prices:

\[- \frac{U^m_L(s^t)}{U^m_C(s^t)} = \chi^* A(s_t)\]
Why should monetary policy implement flexible price allocations?

- relaxed Ramsey planner uses distortionary taxes to redistribute: \( \chi^* \neq 1 \)
  - high-skilled, high-income households pay more taxes than low-skilled, poor households
  - higher tax rate implies more redistribution (Werning 2007, Correia 2010)

- planner trades-off the benefit of distortionary taxation (redistribution) with cost (efficiency)

- when there are no shocks to the relative skill distribution and preferences are homothetic:
  - both the marginal cost & marginal benefit of taxation are invariant to the state
  - it follows that the optimal tax rate is constant, as in Lucas Stokey (1983)

- optimal level of redistribution is accomplished through the tax system
  - monetary policy implements flexible-price allocations, preserves production efficiency
What if fiscal policy is set suboptimally?

Proposition

If \( \exists \) positive scalars \( (\vartheta^1, \vartheta^2, \ldots, \vartheta^I) \in \mathbb{R}_+^I \) and a function \( \Theta : S \to \mathbb{R}_+^I \) such that

\[
\theta^i(s_t) = \vartheta^i \Theta(s_t), \quad \forall s_t \in S,
\]

and

\[
\chi \neq \chi^*,
\]

then it remains optimal for monetary policy to replicate flexible prices.

- tax rate is suboptimal, but monetary policy is unable to substitute for the missing tax
- why? missing tax rate is constant, but \( \varepsilon(s^t) \) is a forecast error!
The (Unrelaxed) Ramsey Problem
Ramsey Planner's Problem

\[
\max \left\{ y(s'), y^f(s'), C(s'), Y(s'), L(s') \right\}_{s' \in S}, \varphi, \chi, \bar{T} \quad \sum_i \sum_{s'} \beta_i \mu(s') \mathcal{W}(C(s'), L(s'); \varphi, v, \lambda) - \sum_{i \in I} \pi^i v^i \bar{T}
\]

subject to feasibility and implementability conditions:

\[
C(s^t) = Y(s^t) = \left[ \kappa y^s(s^t) \frac{\rho - 1}{\rho} + (1 - \kappa) y^f(s^t) \frac{\rho - 1}{\rho} \right]^{\frac{\rho}{\rho - 1}}, \quad L(s^t) = \kappa \frac{y^s(s^t)}{A(s^t)} + (1 - \kappa) \frac{y^f(s^t)}{A(s^t)},
\]

\[
\chi U^m_C(s^t) \left( \frac{y^f(s^t)}{y(s^t)} \right)^{-1/\rho} + U^m_L(s^t) \frac{1}{A(s^t)} = 0,
\]

\[
\sum_{s' | s^t - 1} y^s(s^t) \left\{ \chi U^m_C(s^t) \left[ \frac{y^s(s^t)}{Y(s^t)} \right]^{-1/\rho} + U^m_L(s^t) \frac{1}{A(s^t)} \right\} \mu(s' | s^t - 1) = 0,
\]
The Ramsey optimum $x^* \in \mathcal{X}^s$ satisfies

$$
\frac{\mathcal{W}_L(s^t) + U_L^m(s^t)}{\mathcal{W}_C(s^t) + \chi U_C^m(s^t)} \left[ \kappa \zeta(s^t - 1) \frac{y^v(s^t)}{Y(s^t)} + (1 - \kappa) \xi(s^t) \frac{y^f(s^t)}{Y(s^t)} \right] \left\{ \frac{U_L^m(s^t)L(s^t)}{U_L^m(s^t)} + 1 \right\} \frac{Y(s^t)}{A(s)L(s^t)} = \frac{Y(s^t)}{L(s^t)}, \quad \forall s^t \in S^t
$$
we define an implicit monetary wedge \( 1 - \tau^*_M(s^t) \) by

\[
- \frac{U_L^m(s^t)}{U_C^m(s^t)} = \chi^*(1 - \tau^*_M(s^t)) \frac{Y(s^t)}{L(s^t)}
\]

portion of the labor wedge implemented by monetary policy at the Ramsey optimum
The Optimal Monetary Wedge and Income Inequality

**Theorem**

Let $\mathcal{I}: S \rightarrow \mathbb{R}_+$ be the function defined by:

$$
\mathcal{I}(s_t) \equiv \frac{\sum_{i \in I} \tilde{\pi}^i(\phi^i)^{-1/\eta}(\theta^i(s_t))^{1+\eta}}{\sum_{i \in I} \pi^i(\phi^i)^{-1/\eta}(\theta^i(s_t))^{1+\eta}} > 0, \quad \text{where} \quad \tilde{\pi}^i \equiv \pi^i \left[ \frac{\lambda^i}{\phi^i} + v^i(1+\eta) \right]
$$

There exists a threshold $\bar{\mathcal{I}}(s^{t-1}) > 0$ such that:

- $\tau^*_M(s^t) > 0$ if and only if $\mathcal{I}(s_t) > \bar{\mathcal{I}}(s^{t-1})$,
- $\tau^*_M(s^t) = 0$ if and only if $\mathcal{I}(s_t) = \bar{\mathcal{I}}(s^{t-1})$,
- $\tau^*_M(s^t) < 0$ if and only if $\mathcal{I}(s_t) < \bar{\mathcal{I}}(s^{t-1})$.

$\mathcal{I}(s_t)$ is a sufficient statistic for labor income inequality in our model.
Strict Monotonicity of the Optimal Monetary Wedge

**Theorem**

Suppose tax rates can be set one period in advance \( \rightarrow \chi(s^{t-1}) \). Then:

(i) \( \tau^*_M(s^t) \) is strictly increasing in \( \mathcal{I}(s_t) \).

(ii) \( \exists \) a threshold \( \bar{\mathcal{I}}(s_t) > 0 \) such that:

\[
\tau^*_M(s^t) = 0 \quad \text{iff} \quad \mathcal{I}(s_t) = \bar{\mathcal{I}}(s^{t-1})
\]

(iii) the derivative of \( \tau^*_M(s^t) \) at zero satisfies:

\[
\delta_0 \equiv \left. \frac{d\tau^*_M(s^t)}{d\mathcal{I}(s_t)} \right|_{\mathcal{I}(s_t)=\bar{\mathcal{I}}(s^{t-1})} = (1 - \kappa)(\eta + \gamma) + \kappa \frac{1}{\rho} > 0 \quad \text{and} \quad \frac{d\delta_0}{d\rho} < 0.
\]
The monetary wedge is increasing in inequality

Figure: The optimal monetary tax $\tau^*_M(s^t)$ as a function of $\theta^H(s_t)/\theta^L(s_t)$
Why should monetary policy deviate from implementing flexible prices?

- $I(s_t)$ is a sufficient statistic for labor income inequality

- when $I(s_t)$ increases above the threshold:
  - marginal benefit of taxation (greater redistribution) increases
  - marginal cost of taxation (efficiency) remains the same
  - it follows that the optimal tax rate, were it state-contingent, would increase

- it is thus optimal for monetary policy to **mimic a higher tax rate** in this state

- the monetary authority can do so by targeting a **higher markup**
Optimal Monetary Policy

**Theorem**

*Optimal monetary policy targets a state-contingent mark-up*

\[
\log M(s^t) \equiv \log P(s^t) - \log (W(s^t)/A(s^t))
\]

*that satisfies:*

\[
\log M(s^t) > 0 \quad \text{if and only if} \quad I(s_t) > \bar{I}(s^{t-1}),
\]

\[
\log M(s^t) = 0 \quad \text{if and only if} \quad I(s_t) = \bar{I}(s^{t-1}),
\]

\[
\log M(s^t) < 0 \quad \text{if and only if} \quad I(s_t) < \bar{I}(s^{t-1}).
\]

*If tax rates can be set one period in advance, then \( \log M(s^t) \) is strictly increasing in \( I(s_t) \).*

- optimal markup co-varies positively with a sufficient statistic for labor income inequality
- higher markup \( \rightarrow \) high-skilled, high-income households pay more than low-skilled, poor households
Heterogeneous Equity Shares
What if profit shares are heterogeneous?

- we relax our assumption of uniform equity shares
- let $1 + \sigma^i$ denote the fraction of equity held by household $i \in I$

\[
\sum_{i \in I} \pi^i \sigma^i = 0
\]

then household $i$’s nominal income from dividends:

\[
(1 - \tau_\Pi)(1 + \sigma^i)\Pi(s^i) \quad \text{with} \quad \tau_\Pi \in [0, 1]
\]
Implementability Conditions

- Implementability condition for budget of household $i$:

\[
\sum_t \sum_{s^t} \beta^t \mu(s^t) \left[ U_C^m(s^t) \omega_C^i(\phi) C(s^t) + U_L^m(s^t) \omega_L^i(\phi, s_t) L(s^t) - \sigma^i U_C^m(s^t) \frac{1 - \tau_\Pi \Pi(s^t)}{1 + \tau_c P(s^t)} \right] = U_C^m(s_0) \bar{T}
\]

- We assume

\[
\frac{1 - \tau_\Pi}{1 + \tau_c} > 0
\]
The optimal monetary wedge is still increasing in inequality!

**Theorem**

*Suppose tax rates can be set one period in advance → χ(s⁻¹). Then:*

(i) ∃ a threshold $\bar{I}(s⁻¹) > 0$ such that

$$\tau^*_M(s^t) = 0 \quad \text{iff} \quad I(s_t) = \bar{I}(s⁻¹)$$

(ii) the derivative of $\tau^*_M(s^t)$ at zero satisfies:

$$\delta_0 \equiv \frac{d\tau^*_M(s^t)}{dI(s_t)} \bigg|_{I(s_t)=\bar{I}(s⁻¹)} > 0.$$  

(iii) the threshold $\bar{I}(s⁻¹)$ is increasing in $\sum_I \pi^i v^i \sigma^i$. 

Heterogeneity in equity shares do not disrupt the qualitative result
Conclusion

- When shocks to the skill distribution are proportional (no movement in relative productivities):
  - all redistribution is done via the tax system
  - optimal monetary policy implements flexible-price allocations → targets price stability
  - optimal to implement flex-price allocations even if fiscal policy is set suboptimally

- When shocks affect relative productivities:
  - tax instruments are insufficient to implement constrained efficient allocation
  - optimal for monetary policy to deviate from implementing flexible-prices
  - monetary policy targets a state-contingent markup
  - optimal markup co-varies positively with a sufficient statistic for labor income inequality

- Results are robust to heterogeneity in profit shares
Thank You!