Optimal Monetary Policy with Redistribution

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Some facts about labor earnings

- Fact 1. There are large, systematic, forecastable differences in labor earnings
- Fact 2. Households face heterogeneous exposure of their labor earnings to the business cycle
  - earnings of low-income households exhibit a greater covariance with aggregate fluctuations than that of high-income households
  - countercyclical earnings inequality

> "We find that the fortunes during recessions are predictable by observable characteristics before the recession."

- Guvenen and Smith (2014); Guvenen, Ozkan, Song (2014); Schulhofer-Wohl (2011); Guvenen(2011); Guvenen, Schulhofer-Wohl, Song, Yogo (2017); Parker Vissing-Jorgenson (2009)

A third "fact"

Fact 3. Markups are Counter-cyclical

- Bils (1987); Chari, Kehoe, and McGrattan (2007); Bils, Klenow, Malin (2018)

This Paper: Optimal Monetary Policy with Redistribution

- We write down a general equil business cycle model with heterogeneity and nominal rigidities

- We take the Ramsey approach to redistribution (not insurance)

- Given a restricted set of available tax instruments:

  We solve for optimal fiscal & monetary policy following the primal approach

  Lucas Stokey (1983); Chari Kehoe (1999); Correia, Nicolini, Teles (2008)

- We find that, given Facts 1 & 2, Fact 3 is consistent with optimal monetary policy

Our Framework

- heterogeneous agent economy à la Werning (2007)
  - workers differ in type-specific labor productivities, "skills"
  - skills are state-contingent, but markets are complete
  - no missing insurance markets
- firms face nominal rigidities = informational friction
  - must set nominal prices before observing demand
- shocks to aggregate productivity and the labor skill distribution
- Ramsey taxation: full set of linear tax instruments, with measurability restrictions
  - tax rates are non-state-contingent [or set one period in advance]
  - state-contingent lump sum transfers: uniform across household types

What We Do and What We Show

- We consider a utilitarian planner with arbitrary Pareto weights and solve for optimal policy
- When shocks to the skill distribution are proportional (no movement in *relative* productivities):
  - all redistribution is done via the tax system
  - optimal for monetary policy to implement flexible-price allocations $\rightarrow$ target price stability
- When shocks affect relative productivities:
  - tax instruments are insufficient to implement the constrained efficient allocation
  - optimal for monetary policy to deviate from implementing flexible-prices
- Optimal markup co-varies positively with a sufficient statistic for labor income inequality

The Environment

The Environment

- $t = 0, 1, \ldots$
- finite states $s_t \in S$
- history $s^t = (s_0, ..., s_t) \in S^t$
  - conditional probabilities $\mu(s^t | s^{t-1})$
  - unconditional probabilities $\mu(s^t)$

Household preferences

- unit mass continuum of households
- identical preferences over consumption and effort

$$U(c,h) = \frac{c^{1-\gamma}}{1-\gamma} - \frac{h^{1+\eta}}{1+\eta}$$

Household types

- finite types $i \in I$ of relative size π^i
- types correspond to workers' state-contingent skill

$$\theta^i(s_t) > 0$$

- efficiency units of labor

$$\ell^i(s^t) = \theta^i(s_t) h^i(s^t)$$

- expected lifetime utility

$$\sum_t \sum_{s^t} \beta^t \mu(s^t) U\left(c^i(s^t), \frac{\ell^i(s^t)}{\theta^i(s_t)}\right)$$

Household budget set

$$(1+\tau_c)P(s^t)c^i(s^t)+b^i(s^t)+\sum_{s^{t+1}|s^t}Q(s^{t+1}|s^t)z^i(s^{t+1}|s^t)$$

$$\leq (1-\tau_\ell)W(s^t)\ell^i(s^t)+P(s^t)T(s^t)+(1-\tau_\Pi)\Pi(s^t)+z^i(s^t|s^{t-1})+(1+i(s^{t-1}))b^i(s^{t-1})$$

Firms

- intermediate good firms. monopolistically-competitive, indexed by $j \in \mathcal{J} = [0,1]$

$$y^j(s^t) = A(s_t) n^j(s^t)$$

$$\text{profits}^j(s^t) = (1-\tau_r) p_t^j(\cdot) y^j(s^t) - W(s^t) n^j(s^t)$$

- final good firm. perfectly competitive:

$$Y(s^t) = \left[\int_{j \in J} y^j(s^t)^{\frac{\rho-1}{\rho}} \mathrm{d}j \right]^{\frac{\rho}{\rho-1}} \quad \rightarrow \quad y^j(s^t) = \left(\frac{p_t^j(\cdot)}{P(s^t)} \right)^{-\rho} Y(s^t)$$

The Government: Consolidated Fiscal and Monetary Authority

- tax revenue

$$\mathcal{T}(s^t) \equiv \tau_c P(s^t)C(s^t) + \tau_\ell W(s^t)L(s^t) + \tau_r P(s^t)Y(s^t) + \tau_\Pi \Pi(s^t)$$

- government budget

$$(1+i(s^{t-1}))B(s^{t-1}) + Z(s^t) + P(s^t)T(s^t) = B(s^t) + \sum_{s^{t+1}|s^t} Q(s^{t+1}|s^t)Z(s^{t+1}) + \mathcal{T}(s^t)$$

Market Clearing

- aggregates

$$C(s^t) = \sum_{i \in I} \pi^i c^i(s^t), \qquad L(s^t) = \sum_{i \in I} \pi^i \ell^i(s^t), \qquad \Pi(s^t) \equiv \int_0^1 \text{profits}^j(s^t) \mathrm{d}j$$

- market clearing

$$C(s^t) = Y(s^t), \qquad L(s^t) = \int_{j \in J} n^j(s^t) \mathrm{d}j$$

$$B(s^t) \equiv \sum_{i \in I} \pi^i b^i(s^t), \qquad Z(s^t) \equiv \sum_{i \in I} \pi^i z^i(s^t | s^{t-1}),$$

Nominal Rigidities

Nominal Rigidity = Informational Friction

- Nature draws the aggregate state

$$s_t \in S$$

- the state determines

$$A(s_t), (\theta^i(s_t))_{i \in I}$$

- $\kappa \in [0,1)$ of intermediate-good firms, $j \in \mathcal{J}^s \subset \mathcal{J}$, are "inattentive" to the current state
- $1-\kappa$ of intermediate-good firms, $j \in \mathcal{J}^f \subset \mathcal{J}$, are "attentive" to the current state

Nominal Rigidity = Informational Friction

- inattentive, "sticky-price" firms do not observe s_t
  - make their pricing decisions based only on knowledge of past states

$$p_t^s(s^{t-1}), \qquad \forall j \in \mathcal{J}^s$$

- attentive, "flexible-price" firms observe s_t perfectly
  - make their pricing decisions under complete information

$$p_t^f(s^t), \qquad \forall j \in \mathcal{J}^f$$

Implicit Timing Assumption

1. Nature draws the aggregate state $s_t \in S$
2. intermediate-good firms make their nominal pricing decisions

$$p_t^s(s^{t-1}) \qquad \text{and} \qquad p_t^f(s^t)$$

3. once prices are set, the state s_t is revealed/becomes common knowledge
4. all other market outcomes, allocations adjust to the aggregate state
   - given prices, the final good firm chooses its inputs
   - households make consumption, effort, and savings decisions
   - inputs adjust so that supply = demand

Feasible Allocations: satisfy technology and resource constraints

- allocation

$$x \equiv \{(c^i(s^t), \ell^i(s^t))_{i \in I}, (y^j(s^t), n^j(s^t))_{j \in \mathcal{J}}, C(s^t), Y(s^t), L(s^t)\}_{s^t \in S^t}$$

Definition

An allocation x is feasible if it satisfies, for all $s^t \in S^t$:

$$y^j(s^t) = A(s_t) n^j(s^t), \qquad \forall j \in \mathcal{J};$$

$$Y(s^t) = \left[\int_{j \in J} y^j(s^t)^{\frac{\rho-1}{\rho}} \mathrm{d}j\right]^{\frac{\rho}{\rho-1}}; \qquad L(s^t) = \int_{j \in J} n^j(s^t) \mathrm{d}j;$$

$$C(s^t) = \sum_{i \in I} \pi^i c^i(s^t); \qquad L(s^t) = \sum_{i \in I} \pi^i \ell^i(s^t); \qquad C(s^t) = Y(s^t).$$

Let $\mathcal{X}$ denote the set of all feasible allocations.

We are interested in allocations $x \in \mathcal{X}$ that can be supported in equilibrium

- policy

$$\phi \equiv \{\tau_c, \tau_\ell, \tau_r, \tau_\Pi, (T(s^t), i(s^t))_{s^t \in S^t}\}$$

- price system

$$\rho \equiv \{p_t^f(s^t), p_t^s(s^{t-1}), P(s^t), W(s^t), (Q(s^{t+1}|s^t))_{s^{t+1} \in S^{t+1}}\}_{s^t \in S^t}$$

- financial positions

$$\zeta \equiv \{(b^i(s^t))_{i \in I}, B(s^t), (z^i(s^{t+1}|s^t), Z(s^{t+1}))_{s^{t+1} \in S^{t+1}|S^t}\}_{s^t \in S^t}$$

Equilibrium Definitions

Definition

A sticky-price equilibrium is an allocation x, a price system ρ, a policy ϕ, and financial positions ζ such that:

(i) $p_t^s(s^{t-1})$ is optimal for firms $j \in \mathcal{J}^s$; $p_t^f(s^t)$ is optimal for firms $j \in \mathcal{J}^f$;

(ii) prices and allocations jointly satisfy the CES demand function;

(iii) the allocation and financial asset holdings solve household i's problem, for each $i \in I$;

(iv) the government budget constraint is satisfied;

(v) markets clear.

Definition

A flexible-price equilibrium is an allocation x, a price system ρ, a policy ϕ, and financial positions ζ such that:

$p_t^f(s^t)$ is optimal for firms $j \in \mathcal{J}$, and parts (ii)-(vi) of the previous definition hold.

Equilibrium Characterization

There exists a "Fictitious" Representative Household

Lemma

(Negishi 1960; Werning 2007)

For any equilibrium there exist "market" or "Negishi" weights $\varphi \equiv (\varphi^i)_{i \in I}$ *with* $\varphi^i \geq 0$ *such that*

$$\{c^i(s^t), \ell^i(s^t)\}_{i \in I}$$

solve the following static sub-problem

$$U^m(C(s^t), L(s^t); \varphi) \equiv \max \sum_{i \in I} \varphi^i \pi^i U(c^i(s^t), \ell^i(s^t)/\theta^i(s_t))$$

subject to

$$C(s^t) = \sum_{i \in I} \pi^i c^i(s^t), \qquad \text{and} \qquad L(s^t) = \sum_{i \in I} \pi^i \ell^i(s^t)$$

- the superscript "m" stands for "market"

Equilibrium prices thereby satisfy

$$-\frac{U_L^m(s^t)}{U_C^m(s^t)} = \left(\frac{1-\tau_\ell}{1+\tau_c}\right)\frac{W(s^t)}{P(s^t)}$$

$$\frac{U_C^m(s^t)}{P(s^t)} = \beta(1+i(s^t))\sum_{s^{t+1}|s^t}\mu(s^{t+1}|s^t)\frac{U_C^m(s^{t+1})}{P(s^{t+1})}$$

$$Q(s^{t+1}|s^t) = \beta\mu(s^{t+1}|s^t)\frac{U_C^m(s^{t+1})}{U_C^m(s^t)}\frac{P(s^t)}{P(s^{t+1})}$$

- solution to sub-problem with iso-elastic utility:

$$c^i(s^t) = \omega_C^i(\varphi)C(s^t) \qquad \text{and} \qquad \ell^i(s^t) = \omega_L^i(\varphi, s_t)L(s^t),$$

$$\omega_C^i(\varphi) \equiv \frac{(\varphi^i)^{1/\gamma}}{\sum_{j\in I}\pi^j(\varphi^j)^{1/\gamma}}, \qquad \omega_L^i(\varphi, s_t) \equiv \frac{(\varphi^i)^{-1/\eta}\theta^i(s_t)^{\frac{1+\eta}{\eta}}}{\sum_{k\in I}\pi^k(\varphi^k)^{-1/\eta}\theta^i(s_t)^{\frac{1+\eta}{\eta}}}$$

Equilibrium prices thereby satisfy

$$-\frac{U_L^m(s^t)}{U_C^m(s^t)} = \left(\frac{1-\tau_\ell}{1+\tau_c}\right)\frac{W(s^t)}{P(s^t)}$$

$$\frac{U_C^m(s^t)}{P(s^t)} = \beta(1+i(s^t))\sum_{s^{t+1}|s^t}\mu(s^{t+1}|s^t)\frac{U_C^m(s^{t+1})}{P(s^{t+1})}$$

$$Q(s^{t+1}|s^t) = \beta\mu(s^{t+1}|s^t)\frac{U_C^m(s^{t+1})}{U_C^m(s^t)}\frac{P(s^t)}{P(s^{t+1})}$$

- solution to sub-problem with iso-elastic utility:

$$c^i(s^t) = \omega_C^i(\varphi)C(s^t) \qquad \text{and} \qquad \ell^i(s^t) = \omega_L^i(\varphi, s_t)L(s^t),$$

$$\omega_C^i(\varphi) \equiv \frac{(\varphi^i)^{1/\gamma}}{\sum_{j\in I}\pi^j(\varphi^j)^{1/\gamma}}, \qquad \omega_L^i(\varphi, s_t) \equiv \frac{(\varphi^i)^{-1/\eta}\theta^i(s_t)^{\frac{1+\eta}{\eta}}}{\sum_{k\in I}\pi^k(\varphi^k)^{-1/\eta}\theta^i(s_t)^{\frac{1+\eta}{\eta}}}$$

Primal approach: budget implementability conditions

$$\sum_t \sum_{s^t} \beta^t \mu(s^t) \left[U_C^m(s^t) \omega_C^i(\varphi) C(s^t) + U_L^m(s^t) \omega_L^i(\varphi, s_t) L(s^t) \right] = U_C^m(s_0) \bar{T}, \quad \forall i \in I$$

$$\bar{T} \equiv \frac{1}{U_C^m(s_0)(1+\tau_c)} \sum_t \sum_{s^t} \beta^t \mu(s^t) U_C^m(s^t) \left[T(s^t) + (1-\tau_\Pi) \frac{\Pi(s^t)}{P(s^t)} \right]$$

- implementability conditions: one for each type $i \in I$ (Werning, 2007)
  - similar to Lucas Stokey (1983) implementability condition for rep household's budget constraint
  - however, unlike Lucas Stokey: existence of lump-sum taxes + multiple household types
- profits are isomorphic to lump-sum transfers
  - we relax this in an extension with heterogeneous equity shares

Flexible Price Firm's Problem

Firm $j \in \mathcal{J}^f$ *solves*

$$\max_{p'} \left\{ (1-\tau_r) p' y^j(s^t) - W(s^t) \frac{y^j(s^t)}{A(s_t)} \right\}$$

subject to

$$y^j(s^t) = \left(\frac{p'}{P(s^t)} \right)^{-\rho} Y(s^t), \qquad \forall s^t \in S^t.$$

Sticky Price Firm's Problem

Firm $j \in \mathcal{J}^s$ *solves*

$$\max_{p'} \sum_{s^t | s^{t-1}} \mu(s^t | s^{t-1}) \frac{U_C^m(s^t)}{P(s^t)} \left\{ (1-\tau_r) p' y^j(s^t) - W(s^t) \frac{y^j(s^t)}{A(s_t)} \right\}$$

subject to

$$y^j(s^t) = \left(\frac{p'}{P(s^t)} \right)^{-\rho} Y(s^t), \qquad \forall s^t \in S^t.$$

Firm Optimality

- flex-price firm: price = mark-up over marginal cost

$$p_t^f(s^t) = \left[(1-\tau_r)\left(\frac{\rho-1}{\rho}\right)\right]^{-1} \frac{W(s^t)}{A(s_t)}$$

- sticky-price firm: price = mark-up over expected marginal cost

$$p_t^s(s^{t-1}) = \left[(1-\tau_r)\left(\frac{\rho-1}{\rho}\right)\right]^{-1} \sum_{s^t|s^{t-1}} \left[\frac{W(s^t)}{A(s_t)}\right] q(s^t|s^{t-1})$$

where

$$q(s^t|s^{t-1}) \equiv \frac{\mu(s^t|s^{t-1})U_C^m(s^t)Y(s^t)P(s^t)^{\rho-1}}{\sum_{s^t|s^{t-1}} \mu(s^t|s^{t-1})U_C^m(s^t)Y(s^t)P(s^t)^{\rho-1}}$$

Proposition

A feasible allocation $x \in \mathcal{X}$ is implementable as a flexible-price equilibrium iff

$\exists$ market weights $\varphi \equiv (\varphi^i)$ and constants $\bar{T} \in \mathbb{R}$ and $\chi \in \mathbb{R}_+$, such that:

(i) for all $s^t \in S^t$:

$$y^j(s^t) = y^{j'}(s^t) = Y(s^t) \qquad \forall j, j' \in \mathcal{J};$$

(ii) for all $s^t \in S^t$:

$$-\frac{U_L^m(s^t)}{U_C^m(s^t)} = \chi A(s_t);$$

(iii) for all $i \in I$:

$$\sum_t \sum_{s^t} \beta^t \mu(s^t) \left[U_C^m(s^t) \omega_C^i(\varphi) C(s^t) + U_L^m(s^t) \omega_L^i(\varphi, s_t) L(s^t) \right] = U_C^m(s_0) \bar{T}.$$

What can fiscal policy do?

- the fiscal authority has the power to move around allocations through χ and $\bar{T}$
- the labor wedge results from linear taxes and markups

$$\chi \equiv \left(\frac{\rho-1}{\rho}\right)\frac{(1-\tau_\ell)(1-\tau_r)}{1+\tau_c}$$

- lump sum taxes/transfers + profits affect budgets via $\bar{T}$

Proposition

A feasible allocation $x \in \mathcal{X}$ is implementable as a sticky-price equilibrium iff $\exists$ market weights $\varphi \equiv (\varphi^i)$ and constants $\bar{T} \in \mathbb{R}$ and $\chi \in \mathbb{R}_+$, such that:

(i) for all $s^t \in S^t$:

$$\begin{aligned} y^j(s^t) &= y^f(s^t), \qquad \forall j \in \mathcal{J}^f \\ y^j(s^t) &= y^s(s^t), \qquad \forall j \in \mathcal{J}^s \end{aligned}$$

(ii) for all $s^t \in S^t$:

$$\chi U_C^m(s^t)\left(\frac{y^f(s^t)}{Y(s^t)}\right)^{-1/\rho} + U_L^m(s^t)\frac{1}{A(s_t)} = 0,$$

for all $s^{t-1} \in S^{t-1}$:

$$\sum_{s^t|s^{t-1}} y^s(s^t)\left\{\chi U_C^m(s^t)\left[\frac{y^s(s^t)}{Y(s^t)}\right]^{-1/\rho} + U_L^m(s^t)\frac{1}{A(s_t)}\right\}\mu(s^t|s^{t-1}) = 0,$$

(iii) for all $i \in I$:

$$\sum_t \sum_{s^t} \beta^t \mu(s^t)\left[U_C^m(s^t)\omega_C^i(\varphi)C(s^t) + U_L^m(s^t)\omega_L^i(\varphi, s_t)L(s^t)\right] = U_C^m(s_0)\bar{T}.$$

What can monetary policy do vis-a-vis fiscal policy?

- sticky-price firm: price = mark-up over realized marginal cost, modulo a forecast error

$$p_t^s(s^{t-1}) = \left[(1-\tau_r)\left(\frac{\rho-1}{\rho}\right)\right]^{-1} \varepsilon(s^t)\frac{W(s^t)}{A(s_t)}$$

- then:

$$-\frac{U_L^m(s^t)}{U_C^m(s^t)} = \underbrace{\chi\left[\kappa\varepsilon(s^t)^{1-\rho} + (1-\kappa)\right]^{-\frac{1}{1-\rho}}}_{\text{labor wedge}} \times A(s_t)$$

- monetary policy: state-contingent wedge $\varepsilon(s^t)$
  - cost of using monetary wedge is loss in production efficiency: $y^s(s^t) \neq y^f(s^t)$
  - constraint on $\varepsilon(s^t)$: forecast error $\rightarrow$ on average, equal to 1

Lemma

Let $\mathcal{X}^f$ denote the set of flexible-price allocations. Let $\mathcal{X}^s$ denote the set of sticky-price allocations.

$$\mathcal{X}^f \subset \mathcal{X}^s \subset \mathcal{X}.$$

Proof.

Take any $x \in \mathcal{X}^f$. x can be implemented under sticky prices with:

$$\frac{y^s(s^t)}{Y(s^t)} = \frac{y^f(s^t)}{Y(s^t)} = 1, \qquad \forall s^t \in S^t.$$

[i.e. $\varepsilon(s^t) = 1$ for all $s^t \in S^t$.] □

The Ramsey Problem

Utilitarian Welfare Function

- social welfare function with Pareto weights $\lambda^i > 0$

$$\mathcal{U} \equiv \sum_{i \in I} \lambda^i \pi^i \sum_t \sum_{s^t} \beta^t \mu(s^t) U(c^i(s^t), \ell^i(s^t)/\theta^i(s_t))$$

- our goal: characterize the welfare-maximizing allocation $x \in \mathcal{X}^s$

Definition

A Ramsey optimum x^* is an allocation that maximizes welfare subject to $x^* \in \mathcal{X}^s$.

- $\mathcal{X}^s$ is a complicated set

- We first solve an *easier* problem, the "relaxed Ramsey planning problem"

  Correia, Nicolini, Teles (2008)

The Relaxed Ramsey Planner

Definition

The relaxed set of allocations $\mathcal{X}^R$ is the set of feasible allocations $x \in \mathcal{X}$ that satisfy, for all $i \in I$:

$$\sum_t \sum_{s^t} \beta^t \mu(s^t) \left[U_C^m(s^t) \omega_C^i(\varphi) C(s^t) + U_L^m(s^t) \omega_L^i(\varphi, s_t) L(s^t) \right] \leq U_C^m(s_0) \bar{T}.$$

A Relaxed Ramsey optimum x^{R*} is an allocation that maximizes welfare subject to

$$x^{R*} \in \mathcal{X}^R.$$

- our Relaxed Ramsey planner = "Lucas-Stokey-Werning" planner

Corollary

The relaxed set is a strict superset of $\mathcal{X}^s$

$$\mathcal{X}^f \subset \mathcal{X}^s \subset \mathcal{X}^R \subset \mathcal{X}.$$

Why look at the Relaxed Ramsey planner's problem?

- the relaxed set is a strict superset

$$\mathcal{X}^f \subset \mathcal{X}^s \subset \mathcal{X}^R$$

- we will derive conditions under which

$$x^{R*} \in \mathcal{X}^f \subset \mathcal{X}^s$$

- under these conditions, x^{R*} solves the (unrelaxed) Ramsey problem!

The Relaxed Ramsey Planner's Problem

- let $\pi^i \nu^i$ be the Lagrange multiplier on the implementability condition of type i
- define the pseudo-welfare function by:

$$\mathcal{W}(C,L;\varphi,\nu,\lambda) \equiv \sum_{i\in I} \pi^i \left\{ \lambda^i U^i(\omega_C^i(\varphi)C(s^t), \omega_L^i(\varphi,s_t)L(s^t)) + \nu^i \left[U_C^m(s^t)\omega_C^i(\varphi)C(s^t) + U_L^m(s^t)\omega_L^i(\varphi,s_t)L(s^t) \right] \right\}$$

Relaxed Ramsey Planner's Problem

$$\max_{x,\varphi,\bar{T}} \quad \sum_t \sum_{s^t} \beta^t \mu(s^t) \mathcal{W}(C(s^t), L(s^t); \varphi, \nu, \lambda) - U_C^m(s_0) \sum_{i\in I} \pi^i \nu^i \bar{T}$$

subject to feasibility.

Proposition

The Relaxed Ramsey optimum $x^{R*} \in \mathcal{X}^R$ *satisfies*

$$-\frac{\mathcal{W}_L(s^t)}{\mathcal{W}_C(s^t)} = A(s_t), \qquad \forall s^t \in S^t$$

and

$$y^j(s^t) = y^{j'}(s^t) = Y(s^t) \qquad \forall j, j' \in \mathcal{J};$$

- Lucas-Stokey-Werning optimum features zero output dispersion across firms
- preserves Diamond and Mirrlees (1971) production efficiency

When can you implement x^{R*} under flexible prices?

Theorem

If $\exists$ *positive scalars* $(\vartheta^1, \vartheta^2, \dots \vartheta^I) \in \mathbb{R}_+^I$ *and a function* $\Theta : S \to \mathbb{R}_+$ *such that*

$$\theta^i(s_t) = \vartheta^i \Theta(s_t), \qquad \forall s_t \in S,$$

then

$$x^{R*} \in \mathcal{X}^f.$$

It follows that

$$x^{R*} \in \mathcal{X}^s.$$

It is therefore optimal for monetary policy to replicate flexible price allocations.

Proof

- relaxed Ramsey optimality condition

$$-\frac{\mathcal{W}_L(s^t)}{\mathcal{W}_C(s^t)} = A(s_t)$$

- with homothetic preferences this can be written as:

$$-\frac{U_L^m(s^t)}{U_C^m(s^t)}\left[\frac{\sum_{i\in I}\pi^i\omega_L^i(\varphi,s_t)\left(\frac{\lambda^i}{\varphi^i}+\nu^i(1+\eta)\right)}{\sum_{i\in I}\pi^i\omega_C^i(\varphi)\left(\frac{\lambda^i}{\varphi^i}+\nu^i(1-\gamma)\right)}\right] = A(s_t)$$

- with this condition on the skill distribution:

$$\omega_L^i(\varphi,s_t) = \omega_L^i(\varphi) \equiv \frac{(\varphi^i)^{-1/\eta}(\vartheta^i)^{\frac{1+\eta}{\eta}}}{\sum_{k\in I}\pi^k(\varphi^k)^{-1/\eta}(\vartheta^k)^{\frac{1+\eta}{\eta}}}$$

- in which case the relaxed optimum can be implemented under flexible prices:

$$-\frac{U_L^m(s^t)}{U_C^m(s^t)} = \chi^* A(s_t)$$

Why should monetary policy implement flexible price allocations?

- relaxed Ramsey planner uses distortionary taxes to redistribute: $\chi^* \neq 1$
  - high-skilled, high-income households pay more taxes than low-skilled, poor households
  - higher tax rate implies more redistribution (Werning 2007, Correia 2010)
- planner trades-off the benefit of distortionary taxation (redistribution) with cost (efficiency)
- when there are no shocks to the *relative* skill distribution and preferences are homothetic:
  - both the marginal cost & marginal benefit of taxation are invariant to the state
  - it follows that the optimal tax rate is constant, as in Lucas Stokey (1983)
- optimal level of redistribution is accomplished through the tax system
  - monetary policy implements flexible-price allocations, preserves production efficiency

What if fiscal policy is set suboptimally?

Proposition

If $\exists$ *positive scalars* $(\vartheta^1, \vartheta^2, \ldots \vartheta^I) \in \mathbb{R}^I_+$ *and a function* $\Theta : S \rightarrow \mathbb{R}_+$ *such that*

$$\theta^i(s_t) = \vartheta^i \Theta(s_t), \qquad \forall s_t \in S,$$

and

$$\chi \neq \chi^*,$$

then it remains optimal for monetary policy to replicate flexible prices.

- tax rate is suboptimal, but monetary policy is unable to substitute for the missing tax
- why? missing tax rate is constant, but $\varepsilon(s^t)$ is a forecast error!

The (Unrelaxed) Ramsey Problem

Ramsey Planner's Problem

$$\max_{\{y^s(s^t),y^f(s^t),C(s^t),Y(s^t),L(s^t)\}_{s^t\in S^t},\varphi,\chi,\bar{T}} \quad \sum_t \sum_{s^t} \beta^t \mu(s^t) \mathcal{W}(C(s^t),L(s^t);\varphi,\nu,\lambda) - U_C^m(s_0) \sum_{i\in I} \pi^i \nu^i \bar{T}$$

subject to feasibility and implementability conditions:

$$C(s^t) = Y(s^t) = \left[\kappa y^s(s^t)^{\frac{\rho-1}{\rho}} + (1-\kappa) y^f(s^t)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}, \qquad L(s^t) = \kappa \frac{y^s(s^t)}{A(s_t)} + (1-\kappa)\frac{y^f(s^t)}{A(s_t)},$$

$$\chi U_C^m(s^t)\left(\frac{y^f(s^t)}{Y(s^t)}\right)^{-1/\rho} + U_L^m(s^t)\frac{1}{A(s_t)} = 0,$$

$$\sum_{s^t|s^{t-1}} y^s(s^t)\left\{\chi U_C^m(s^t)\left[\frac{y^s(s^t)}{Y(s^t)}\right]^{-1/\rho} + U_L^m(s^t)\frac{1}{A(s_t)}\right\}\mu(s^t|s^{t-1}) = 0,$$

Proposition

The Ramsey optimum $x^* \in \mathcal{X}^s$ *satisfies*

$$-\frac{\mathcal{W}_L(s^t)+U_L^m(s^t)\left[\kappa\varsigma(s^{t-1})\frac{y^s(s^t)}{Y(s^t)}+(1-\kappa)\xi(s^t)\frac{y^f(s^t)}{Y(s^t)}\right]\left\{\frac{U_{LL}^m(s^t)L(s^t)}{U_L^m(s^t)}+1\right\}\frac{Y(s^t)}{A(s_t)L(s^t)}}{\mathcal{W}_C(s^t)+\chi U_C^m(s^t)\left[\kappa\varsigma(s^{t-1})\left[\frac{y^s(s^t)}{Y(s^t)}\right]^{\frac{\rho-1}{\rho}}+(1-\kappa)\xi(s^t)\left(\frac{y^f(s^t)}{Y(s^t)}\right)^{\frac{\rho-1}{\rho}}\right]\left\{\frac{U_{CC}^m(s^t)C(s^t)}{U_C^m(s^t)}+1\right\}}=\frac{Y(s^t)}{L(s^t)}, \qquad \forall s^t\in S^t$$

Implicit Monetary Wedge

- we define an implicit monetary wedge $1-\tau_M^*(s^t)$ by

$$-\frac{U_L^m(s^t)}{U_C^m(s^t)} = \chi^*(1-\tau_M^*(s^t))\frac{Y(s^t)}{L(s^t)}$$

- portion of the labor wedge implemented by monetary policy at the Ramsey optimum

The Optimal Monetary Wedge and Income Inequality

Theorem

Let $\mathcal{I} : S \to \mathbb{R}_+$ *be the function defined by:*

$$\mathcal{I}(s_t) \equiv \frac{\sum_{i \in I} \tilde{\pi}^i (\varphi^i)^{-1/\eta} (\theta^i(s_t))^{\frac{1+\eta}{\eta}}}{\sum_{i \in I} \pi^i (\varphi^i)^{-1/\eta} (\theta^i(s_t))^{\frac{1+\eta}{\eta}}} > 0, \qquad \text{where} \qquad \tilde{\pi}^i \equiv \pi^i \left[\frac{\lambda^i}{\varphi^i} + \nu^i (1+\eta) \right]$$

There exists a threshold $\bar{\mathcal{I}}(s^{t-1}) > 0$ *such that:*

$$\begin{aligned}
\tau_M^*(s^t) > 0 \quad & \text{if and only if} \quad \mathcal{I}(s_t) > \bar{\mathcal{I}}(s^{t-1}), \\
\tau_M^*(s^t) = 0 \quad & \text{if and only if} \quad \mathcal{I}(s_t) = \bar{\mathcal{I}}(s^{t-1}), \\
\tau_M^*(s^t) < 0 \quad & \text{if and only if} \quad \mathcal{I}(s_t) < \bar{\mathcal{I}}(s^{t-1}).
\end{aligned}$$

- $\mathcal{I}(s_t)$ is a sufficient statistic for labor income inequality in our model

Strict Monotonicity of the Optimal Monetary Wedge

Theorem

Suppose tax rates can be set one period in advance $\rightarrow \chi(s^{t-1})$. *Then:*

(i) $\tau_M^*(s^t)$ *is strictly increasing in* $\mathcal{I}(s_t)$.

(ii) $\exists$ *a threshold* $\bar{\mathcal{I}}(s^{t-1}) > 0$ *such that:*

$$\tau_M^*(s^t) = 0 \quad \text{iff} \quad \mathcal{I}(s_t) = \bar{\mathcal{I}}(s^{t-1})$$

(iii) the derivative of $\tau_M^*(s^t)$ *at zero satisfies:*

$$\delta_0 \equiv \left.\frac{d\tau_M^*(s^t)}{d\mathcal{I}(s_t)}\right|_{\mathcal{I}(s_t)=\bar{\mathcal{I}}(s^{t-1})} = (1-\kappa)(\eta+\gamma) + \kappa\frac{1}{\rho} > 0 \qquad \text{and} \qquad \frac{d\delta_0}{d\rho} < 0.$$

The monetary wedge is increasing in inequality

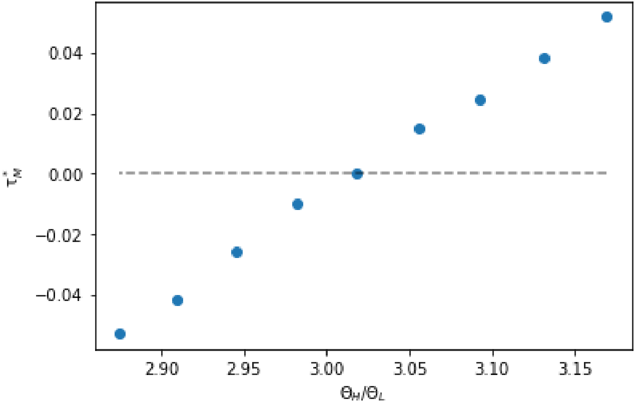


Figure: The optimal monetary tax $\tau_M^*(s^t)$ as a function of $\theta^H(s_t)/\theta^L(s_t)$

Why should monetary policy deviate from implementing flexible prices?

- $\mathcal{I}(s_t)$ is a sufficient statistic for labor income inequality
- when $\mathcal{I}(s_t)$ increases above the threshold:
  - marginal benefit of taxation (greater redistribution) increases
  - marginal cost of taxation (efficiency) remains the same
  - it follows that the optimal tax rate, were it state-contingent, would increase
- it is thus optimal for monetary policy to mimic a higher tax rate in this state
- the monetary authority can do so by targeting a higher markup

Optimal Monetary Policy

Theorem

Optimal monetary policy targets a state-contigent mark-up

$$\log \mathcal{M}(s^t) \equiv \log P(s^t) - \log(W(s^t)/A(s^t))$$

that satisfies:

$$\begin{array}{lll} \log \mathcal{M}(s^t) > 0 & \textit{if and only if} & \mathcal{I}(s_t) > \bar{\mathcal{I}}(s^{t-1}), \\ \log \mathcal{M}(s^t) = 0 & \textit{if and only if} & \mathcal{I}(s_t) = \bar{\mathcal{I}}(s^{t-1}), \\ \log \mathcal{M}(s^t) < 0 & \textit{if and only if} & \mathcal{I}(s_t) < \bar{\mathcal{I}}(s^{t-1}). \end{array}$$

If tax rates can be set one period in advance, then $\log \mathcal{M}(s^t)$ *is strictly increasing in* $\mathcal{I}(s_t)$.

- optimal markup co-varies positively with a sufficient statistic for labor income inequality
- higher markup $\rightarrow$ high-skilled, high-income households pay more than low-skilled, poor households

Heterogeneous Equity Shares

What if profit shares are heterogeneous?

- we relax our assumption of uniform equity shares
- let $1+\sigma^i$ denote the fraction of equity held by household $i \in I$

$$\sum_{i \in I} \pi^i \sigma^i = 0$$

- then household i's nominal income from dividends:

$$(1-\tau_\Pi)(1+\sigma^i)\Pi(s^t) \qquad \text{with} \qquad \tau_\Pi \in [0,1]$$

Implementability Conditions

- implementability condition for budget of household i:

$$\sum_t \sum_{s^t} \beta^t \mu(s^t) \left[U_C^m(s^t)\omega_C^i(\varphi)C(s^t) + U_L^m(s^t)\omega_L^i(\varphi, s_t)L(s^t) - \sigma^i U_C^m(s^t) \frac{1-\tau_\Pi}{1+\tau_c} \frac{\Pi(s^t)}{P(s^t)} \right] = U_C^m(s_0)\bar{T}$$

- we assume

$$\frac{1-\tau_\Pi}{1+\tau_c} > 0$$

The optimal monetary wedge is still increasing in inequality!

Theorem

Suppose tax rates can be set one period in advance $\rightarrow \chi(s^{t-1})$. *Then:*

(i) $\exists$ *a threshold* $\bar{\mathcal{I}}(s^{t-1}) > 0$ *such that*

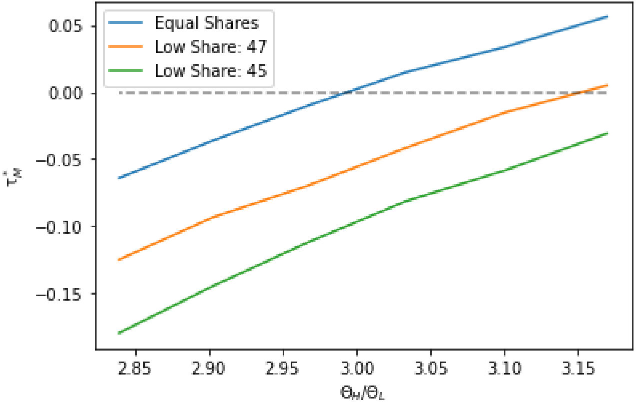
$$\tau_M^*(s^t) = 0 \quad \textit{iff} \quad \mathcal{I}(s_t) = \bar{\mathcal{I}}(s^{t-1})$$

(ii) the derivative of $\tau_M^*(s^t)$ *at zero satisfies:*

$$\delta_0 \equiv \left.\frac{d\tau_M^*(s^t)}{d\mathcal{I}(s_t)}\right|_{\mathcal{I}(s_t)=\bar{\mathcal{I}}(s^{t-1})} > 0.$$

(iii) the threshold $\bar{\mathcal{I}}(s^{t-1})$ *is increasing in* $\sum_I \pi^i \nu^i \sigma^i$.

Heterogeneity in equity shares do not disrupt the qualitative result



Conclusion

- When shocks to the skill distribution are proportional (no movement in *relative* productivities):
  - all redistribution is done via the tax system
  - optimal monetary policy implements flexible-price allocations $\rightarrow$ targets price stability
  - optimal to implement flex-price allocations *even if* fiscal policy is set suboptimally
- When shocks affect relative productivities:
  - tax instruments are insufficient to implement constrained efficient allocation
  - optimal for monetary policy to deviate from implementing flexible-prices
  - monetary policy targets a state-contingent markup
  - optimal markup co-varies positively with a sufficient statistic for labor income inequality
- Results are robust to heterogeneity in profit shares

Thank You!