Product Entry in the Global Automobile Industry

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- Product entry: important for understanding changes in market outcomes
 - 1980-2018: Num. of firms stable, num. products doubled (Grieco et al. 2023)
- Main question: what is the role of global product entry in determining how national policies affect global consumer and firm-level outcomes?

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 - Product portfolio fixed costs \rightarrow complementary offerings across markets
 - Key: new methods to study entry in settings with heterogeneous firms/products

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- Challenges in static entry games with asymmetric firms:
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- Challenges in static entry games with asymmetric firms:
 - 1. Equilibrium multiplicity
 - 2. Computational infeasibility
- **Contribution:** method to estimate and solve entry games with multiple asymmetric firms with multiple discrete choices
 - 1. Estimation: new moment inequalities \rightarrow bound fixed cost parameters
 - 2. Solution method: algorithm based on inequalities \rightarrow bound counterfactual outcomes

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 - Extensive margin response amplifies the increase in US-brand share bounds by 25%
 - Lower bound of consumer surplus gains twice as large in developing countries

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 - Lower bound of consumer surplus gains twice as large in developing countries
- 3. Effect of a US 50% consumption subsidy favoring domestic brands:
 - Induces entry of additional US products worldwide
 - New US products not popular absent a reduction in cost
 - \implies profit-shifting attributed to extensive margin is small

Overview

- 1. Literature
- 2. Data & Key Model-Relevant Facts
- 3. Structural Model
- 4. Bounds on Choice Probabilities
- 5. Estimation Method
- 6. Solution Method
- 7. Estimation Results
- 8. Counterfactual Exercises

Literature

- Multi-Product Entry / IO of Autos: Goldberg (1995), Berry et al (1995), Petrin (2002), Eizenberg (2014), Wollmann (2018), Fan & Yang (2020), Grieco et al (2021), Montag (2023), Allcott et al. (2024)
 - This paper: focus on global product portfolio decisions \rightarrow cross-market interdependence
- **Global Firms and Trade:** Krugman (1980), Atkeson and Burstein (2008), Bernard et al. (2011), Mayer et al. (2014), Tintelnot (2016), Antras et al. (2017), Bernard et al. (2018), Head and Mayer (2019), Alfaro-Ureña et al. (2023), Castro-Vincenzi et al. (2024), Head et al. (2024)
 - This paper (similarity): interdependent choices, mechanisms
 - This paper (difference): global product portfolio rather than export platform / production location / dynamic market entry decision, strategic behavior + heterogeneity
- Solving/Estimating Discrete Choice Models: Seim (2006), Jia (2008), Ciliberto & Tamer (2009), Pakes (2010), Pakes et al (2015), Dickstein & Morales (2018), Fan and Yang (2020), Arkolakis et al. (2023), Magnolfi and Roncoroni (2023), Fan and Yang (2024), Porcher et al. (2024), Dickstein et al. (2024)
 - This paper: overcome jointly key challenges in the entry game literature using inequalities to bound parameters *and* counterfactual outcomes

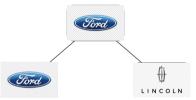
Data and Empirical Facts



- 1. IHS Markit Prices and Characteristics Data 2019: observe universe of sales, prices and characteristics for US, China, Japan, Germany, UK, France, Italy, Spain, Australia, Brazil, Mexico, India at the trim level
 - 12 countries account for 77% of 2019 global sales
 - 49 firms, 130 brands, 371 products with positive sales
- 2. World Bank Gini and PPP Income Data
- 3. Micromoments: MRI-Simmons 2019 Crosstab Report (US)
- 4. **CEPII**: gravity variables

Empirical Definitions

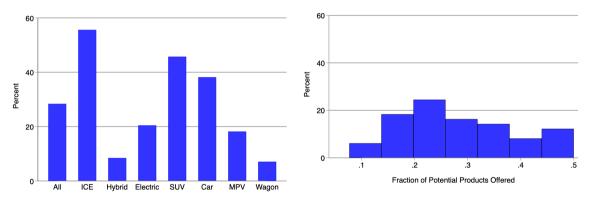
- Firms: parent company e.g., Ford Motor Company
- Brands of each firm: e.g., Ford, Lincoln



• Potential products of a firm: all possible brand - body type- fuel type combinations:

- Body type: "Car", SUV, Wagon, Multi-Purpose Vehicle (MPV), Convertible
- Fuel type: (Plug-in) Electric, Hybrid, ICE
- Aggregate across trims and impute other product characteristics at this level Imputation

Fraction of Potential Products Offered

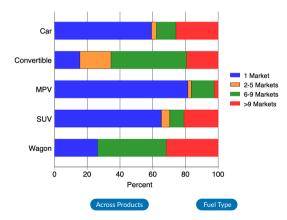


Panel A: Across Product Categories

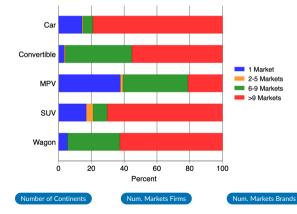
Panel B: Across Firms

Number of Markets Entered by Body Type Conditional on Portfolio

Panel A: Not Quantity Weighted



Panel B: Quantity Weighted



Model

Structural Model: Ingredients

- Firms: indexed by *f*, with potential products A^f
 - Information set $\mathcal{I} \to \mathsf{firms}$ know DGP and policies in the counterfactual
 - Global portfolio fixed cost:

$$F_j^g = \exp(Z_j' heta_g + \sigma_g
u_j^g), \quad
u_j^g | \mathcal{I} \sim_{iid} Normal(0, 1)$$

Market entry fixed cost:

$$F_{jm}^{e} = \exp(Z'_{jm} heta_{e} + \sigma_{e}
u^{e}_{jm}), \quad
u^{e}_{jm}|\mathcal{I} \sim_{iid} Normal(0,1)$$

• Strategic product introduction and product-market entry $ightarrow \mathsf{BNE}^{ extstence \,\& \, \mathsf{Purification}}$



Structural Model: Timing

- 1. Firms observe own $\{\nu_i^g\}_{j\in\mathcal{A}^f}$ and choose their global product portfolio \mathcal{G}^f

2. Firms observe own $\{\nu_{jm}^{e}\}_{j\in\mathcal{G}^{f},m}$ and choose which products to offer in each market Ω_{m}^{f}

Offerings Ω_m and demand/marginal cost shocks (ξ, ω) realized and observed by all firms

3. Prices determined in a Nash-Bertrand game in each market

Consumers choose which vehicle to purchase

Demand and Marginal Costs

Demand: Consumer *i* in country *m* derives the following indirect utility from product *j*:

$$\boldsymbol{u}_{ijm} = \gamma_m + \gamma_{\boldsymbol{b}(j)} - \alpha_i \boldsymbol{p}_{jm} + \beta_i \boldsymbol{X}_{jm} + \xi_{jm} + \varepsilon_{ijm}$$

$$lpha_{i} = \exp\left(lpha_{0} - lpha_{ extsf{CHN}} imes extsf{china}_{i} + lpha_{1} \log(extsf{income}_{i}) + \sigma^{y} extsf{v}_{i}^{y}
ight)$$

Outside option: $u_{i0m} = \varepsilon_{i0m}$ Characteristics:

• Fuel type-, size-, body-type-market dummies; horsepower, hp/weight, home bias (Constant) Marginal costs:

$$\log(\textit{mc}_{\textit{jm}}) = \kappa_{\textit{m}} + \kappa_{\textit{b}(\textit{j})} + \kappa \textit{X}_{\textit{jm}} + \omega_{\textit{jm}}$$

Characteristics:

• Fuel-type, size, horsepower, horsepower/weight, distance to HQ country

Stage 3: Nash-Bertand Pricing Game

Given product offerings ($\Omega_m^f, \Omega_m^{-f}$), equilibrium prices in each country solve:

$$\max_{\boldsymbol{p}_{im}} \pi_m^f = \sum_{j \in \Omega_m^f} (p_{jm} - mc_{jm}) M_m s_{jm}(\boldsymbol{p}_{fm}, \boldsymbol{p^*}_{-fm})$$

- Yields product-market specific markups: $p_{jm} = \mu_{jm}(\Omega_m, \xi_m, \omega_m)mc_{jm}$
- Given prices and markups \rightarrow obtain marginal costs for estimation

Stage 2: Market Entry

Given global portfolio, \mathcal{G}^{f} , firms choose subset of products Ω_{m}^{f} to offer in market m

$$\Pi_{m}^{f}\left(\mathcal{G}^{f}, \{\nu_{jm}^{e}\}_{j\in\mathcal{G}^{f},m}, \mathcal{I}\right) = \max_{\Omega_{m}^{f}\subseteq\mathcal{G}^{f}}\sum_{j\in\mathcal{G}^{f}}O_{jm}\left[\mathbb{E}\left[\pi_{jm}(\Omega_{m}^{f}, \Omega_{m}^{-f})\middle|\mathcal{I}\right] - F_{jm}^{e}(\nu_{jm}^{e})\right]$$

- Expectation \mathbb{E} over rival firms' offerings
- Key interdependence: cannibalization within market

Stage 1: Portfolio Choice

Firms choose their product portfolio \mathcal{G}^{f} by maximizing expected profits:

$$\max_{\mathcal{G}' \subseteq \mathcal{A}'} \mathbb{E} \left[\sum_m \Pi_m^f \left(\mathcal{G}^f, \{ \nu_{jm}^{\mathbf{e}} \}_{j \in \mathcal{G}', m} \right) \, \bigg| \mathcal{I} \right] -$$



Complementary Offerings Across Markets

- Expectation $\mathbb E$ over own and rival firms' offerings
- Key interdependence: complementarity across markets

Bounds on Choice Probabilities

Deriving Bounds on Choice Probabilities

Key Technical Properties

- 1. Unobserved rival fixed costs shocks $\{\nu_{jm}^{e}\}_{j \in \mathcal{G}^{t}, m \in \mathcal{M}}$ and $\{\nu_{j}^{g}\}_{j \in \mathcal{A}^{t}}$
- 2. Submodular variable profit function: For each product *j* and market *m*,

$$MV_{jm}\left(\Omega_{m}^{f},\Omega_{m}^{-f}\right) := \pi_{jm}\left(\Omega_{m}^{f},\Omega_{m}^{-f}\right) + \underbrace{\sum_{j'\neq j,j'\in\Omega_{m}^{f}}\left[\pi_{j'm}\left(\Omega_{m}^{f},\Omega_{m}^{-f}\right) - \pi_{j'm}\left(\Omega_{m}^{f}\setminus\{j\},\Omega_{m}^{-f}\right)\right]}_{\text{cannibalization}}$$

is (weakly) decreasing in Ω_m^f and Ω_m^{-f}

- Constant marginal costs \rightarrow firms view products as substitutes like consumers
- Can be relaxed, especially for estimation

To overcome computational and multiplicity issues, use necessary conditions Formal Derivation

$$O_{jm}^{*} = 1, \Omega_{m}^{f,*} \implies \mathbb{E} \left[MV_{jm} \left(\Omega_{m}^{f,*}, \Omega_{m}^{-f,*} \right) \middle| \underbrace{\mathcal{I}, \{\nu_{jm}^{e}\}_{j \in \mathcal{G}^{f},m}, \mathcal{G}^{f}}_{I \in \mathcal{G}^{f},m} \right] - F_{jm}^{e}(\nu_{jm}^{e}) \ge 0$$

To overcome computational and multiplicity issues, use necessary conditions Formal Derivation

• Example:

$$O_{jm}^{*} = 1, \Omega_{m}^{f,*} \implies \mathbb{E}\left[MV_{jm}\left(\Omega_{m}^{f,*}, \Omega_{m}^{-f,*}\right) \middle| \underbrace{\mathcal{I}, \{\nu_{jm}^{e}\}_{j \in \mathcal{G}^{f},m}, \mathcal{G}^{f}}_{j \in \mathcal{G}^{f},m} \right] - F_{jm}^{e}(\nu_{jm}^{e}) \ge 0$$

• Compare to probit/logit (single agent, binary): Enter iff $\pi - \sigma \nu \ge 0$, probability $\Phi(\pi/\sigma)$

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- Compare to probit/logit (single agent, binary): Enter iff $\pi \sigma \nu \ge 0$, probability $\Phi(\pi/\sigma)$
- Issue: we don't know $\nu_{im}^{e} \rightarrow$ want to integrate it using knowledge of distribution
 - Challenge 1: strategic interactions mean that ν_{im}^{e} potentially correlated with $\Omega_{m}^{-f,*}$
 - Challenge 2: multiple choices by the firm mean that ν_{jm}^{e} potentially correlated with $\Omega_{m}^{f,*}$

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- Under model assumptions:
 - Unobserved rival fixed cost shocks: ν_{jm}^{e} (conditionally on \mathcal{I}) independent of $\Omega_{m}^{-f,*}$
 - •

To overcome computational and multiplicity issues, use necessary conditions Formal Derivation

$$O_{jm}^* = 1 \implies \mathbb{E} \big[MV_{jm} \big(\{j\}, \Omega_m^{-f,*} \big) \big| \mathcal{I}, \mathcal{G}^f \big] - F_{jm}^e(\nu_{jm}^e) \ge 0$$

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- Under model assumptions:
 - Unobserved rival fixed cost shocks: ν_{im}^{e} (conditionally on \mathcal{I}) independent of $\Omega_{m}^{-f,*}$
 - Submodularity: ν_{jm}^{e} (conditionally on \mathcal{I}) independent of upper and lower bound bundles

Deriving Bounds on Probabilities of Product Entry

Under two properties, obtain bounds on firms' choice probabilities

Stage 2:
$$\underbrace{\Gamma\left(\mathbb{E}\left[MV_{jm}\left(\mathcal{A}^{f},\Omega_{m}^{-f}\right)|\mathcal{I}\right]\right)}_{\text{prob. smallest }\Delta \text{ in profits is positive}} \leq \underbrace{\mathbb{P}\left[O_{jm}=1|\mathcal{I}\right]}_{\text{prob. }j \text{ is offered in }m} \leq \underbrace{\Gamma\left(\mathbb{E}\left[MV_{jm}\left(\{j\},\Omega_{m}^{-f}\right)|\mathcal{I}\right]\right)}_{\text{prob. largest }\Delta \text{ in profits is positive}}$$

- Γ : CDF of market entry fixed cost F^e_{jm}(ν^e_{jm})
- Intuition: necessary conditions \rightarrow min and max cannibalization
- Similar intuition to derive Stage 1 inequalities; + deal with subgame perfection Proof Sketch

Key Takeaways

- Submodularity: weaker necessary conditions to deal with the multi-product problem
- Incomplete info: firms' entry decisions conditionally independent
- Bounds depend on fixed cost parameters, and expectations over rivals' actions given ${\cal I}$
- Two challenges:
 - **1**. Given \mathcal{I} , estimate parameters (moment inequalities)
 - 2. Given parameters, solve for new equilibrium given new \mathcal{I} (solution method)

Moment Inequalities Based on Bounds on Choice Probabilities

Fixed Cost Estimation: Moment Inequality Theorem (Informative) (Inference) (Misspec.

- Goal: Estimate $(\theta_e, \sigma_e, \theta_g, \sigma_g)$ without having to solve the model
- Recall:

$$\Gamma\left(\mathbb{E}_{\boldsymbol{\mu}_{\boldsymbol{m}}^{*}}\left[\boldsymbol{M}\boldsymbol{V}_{jm}\left(\{j\},\boldsymbol{\Omega}_{\boldsymbol{m}}^{-\boldsymbol{f}}\right)\big|\mathcal{I}\right];\boldsymbol{\theta}_{\boldsymbol{e}},\boldsymbol{\sigma}_{\boldsymbol{e}}\right)\geq\mathbb{E}\left[\boldsymbol{O}_{jm}|\mathcal{I}\right]$$

• New approach: Use a convex upper bound CDF Bounds of CDF Γ and apply Jensen's inequality to obtain moment inequality:

$$\mathbb{E}\bigg[\overline{\Gamma}\bigg(MV_{jm}(\{j\}, \Omega_m^{-f}); \theta_e, \sigma_e\bigg) - O_{jm}\bigg|\mathcal{I}\bigg] \geq 0.$$

Theorem

The set of parameter vectors consistent with the moment inequalities in Moment Inequality Theorem contains the true parameter vector ($\theta_e, \sigma_e, \theta_a, \sigma_a$).

Key Takeaways

- Approach does not require solving the model \rightarrow computationally feasible
- Private info \rightarrow can use ex-post realization \rightarrow informative
- Submodularity stronger than needed for estimation
- Convex/concave bounds \rightarrow average out firms' expectational errors in a strategic setting
- Bounds on $(\theta_e, \sigma_e) \rightarrow$ can compute counterfactuals

Solution Method Based on Bounds on Choice Probabilities

• Goal: Given $(\theta_e, \sigma_e, \theta_g, \sigma_g)$, bound firms' equilibrium offerings given any policy

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 $\mathbb{E}_{\boldsymbol{\mu}_{\boldsymbol{m}}^{*}}\big[\boldsymbol{M}\boldsymbol{V}_{j\boldsymbol{m}}\big(\{j\},\boldsymbol{\Omega}_{\boldsymbol{m}}^{-f}\big)\big|\mathcal{I}\big] \qquad \mathbb{E}_{\boldsymbol{\mu}_{\boldsymbol{m}}^{*}}\big[\boldsymbol{M}\boldsymbol{V}_{j\boldsymbol{m}}\big(\mathcal{A}^{f},\boldsymbol{\Omega}_{\boldsymbol{m}}^{-f}\big)\big|\mathcal{I}\big]$

• Goal: Learn about μ_m^*

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- Goal: Learn about μ_m^*
- Step 2 (initialization): compute weaker probability bounds μ_m^1 and $\overline{\mu}_m^1$ depending on

$$\mathbb{E}[MV_{jm}(\{j\}, \emptyset) | \mathcal{I}] \qquad \mathbb{E}[MV_{jm}(\mathcal{A}^{f}, \mathcal{A}^{-f}) | \mathcal{I}]$$

- Goal: Given $(\theta_e, \sigma_e, \theta_g, \sigma_g)$, bound firms' equilibrium offerings given any policy
- Step 1: given policies \mathcal{I} probability bounds depend on:

 $\mathbb{E}_{\boldsymbol{\mu}_{\boldsymbol{m}}^{*}}\left[\boldsymbol{M}\boldsymbol{V}_{j\boldsymbol{m}}\left(\{j\},\boldsymbol{\Omega}_{\boldsymbol{m}}^{-f}\right)\big|\mathcal{I}\right] \qquad \mathbb{E}_{\boldsymbol{\mu}_{\boldsymbol{m}}^{*}}\left[\boldsymbol{M}\boldsymbol{V}_{j\boldsymbol{m}}\left(\mathcal{A}^{f},\boldsymbol{\Omega}_{\boldsymbol{m}}^{-f}\right)\big|\mathcal{I}\right]$

- Goal: Learn about μ^{*}_m
- Step 3 (iteration): simulate tighter probability bounds μ_m^2 and $\overline{\mu}_m^2$ depending on

$$\mathbb{E}_{\underline{\boldsymbol{\mu}}_{m}^{1}}[\boldsymbol{M}\boldsymbol{V}_{jm}(\{j\},\Omega_{m}^{-f})|\mathcal{I}] \qquad \mathbb{E}_{\overline{\boldsymbol{\mu}}_{m}^{1}}[\boldsymbol{M}\boldsymbol{V}_{jm}(\mathcal{A}^{f},\Omega_{m}^{-f})|\mathcal{I}]$$

•
$$\underline{\mu}_m^1, \overline{\mu}_m^1 \rightarrow \text{obtained in first iteration}$$

Solution Algorithm: Theorem

Theorem

Under modelling assumptions, the algorithm converges monotonically to bounds of any equilibrium distribution of product offerings decisions in each market m given any information set \mathcal{I} . That is, for any iteration k > 0 and any $m \in \mathcal{M}$,

$$\underline{\boldsymbol{\mu}}_m^k \leq_{FOSD} \boldsymbol{\mu}_m^* \leq_{FOSD} \overline{\boldsymbol{\mu}}_m^k.$$

Useful:

$$\texttt{e.g.}, \mathbb{E}_{\underline{\boldsymbol{\mu}}_m}\left[\boldsymbol{CS}_m | \mathcal{I}\right] \leq \mathbb{E}_{\boldsymbol{\mu}_m^*}\left[\boldsymbol{CS}_m | \mathcal{I}\right] \leq \mathbb{E}_{\overline{\boldsymbol{\mu}}_m}\left[\boldsymbol{CS}_m | \mathcal{I}\right]$$

Key Takeaways

- Method bounds any entry equilibrium Simple Example
- Key: Works even when there are more than two asymmetric firms
- No heuristics nor equilibrium selection assumptions
- Global submodularity (or supermodularity) within markets required
- Intuition: iterated elimination of dominated strategies in incomplete-info setting
- Could also use for estimation Estimation w. Sol. Method

Estimation Results

Demand and Marginal Cost Estimates

	Parameter estimate	Standard error	
emand			
Mean parameters			
price (α_0)	2.88	0.690	
home market	1.01	0.153	
horsepower (log)	5.00	2.46	
horsepower/weight (log)	-2.19	1.44	
Non-linear parameters (price coefficient)			
Income (α_1)	-0.790	0.117	
China	-1.51	0.297	
Shock Std (σ^{y})	0.809	0.131	
rginal Costs (log)			
lectric	0.340	0.051	
ybrid	0.272	0.030	
norsepower/weight (log)	-0.426	0.111	
horsepower (log)	1.00	0.112	
size (log)	0.251	0.209	
distance to brand HQ (log)	0.062	0.007	
ervations	1,414	1,414	
an Share-Weighted Implied Own Price Elasticity	-8.41	-8.41	
rcent Implied Negative Marginal Costs	0	0	

Notes: The demand specification includes body-type-market, electric-hybrid-market, brand, and market fixed effects. It also includes size-market interactions. Both specifications include brand and market fixed effects. Standard errors are clustered at the brand level.

stribution

on/Moments

Fixed Cost Estimates

Compute 95% Andrews and Soares (2010) confidence sets Plots (Vs/Implementation Inference

[-4.8, -4.2]
[2.9, 4.4]
[-1.8, -0.6]
[1.6, 3.3]
3240
739

95% Confidence Set Limits

- Product portfolio fixed cost significantly larger than market entry fixed cost
 - Median of market entry fixed cost $-\exp(\theta_e)$ is USD \$8-15 million
 - Median of portfolio fixed cost-exp (θ_g) is USD \$138-549 million
- pprox \$1-6 billion provided by IHS consultants after converting into "dynamic" estimate

Impact of US Policies on Global Market Outcomes

Counterfactual Exercises

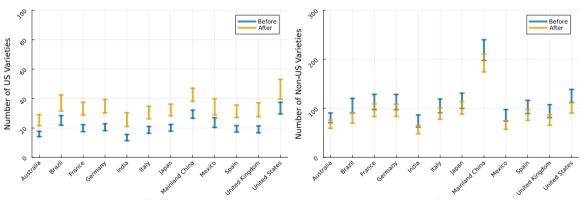
- 1. 20% Marginal Cost Subsidy on US Brands
 - Motivation: IRA 10% production subsidy + significant additional state incentives
 - Caveat: HQ vs production location
- 2. 50% Consumer Subsidy on US Brands
 - Motivation: Large consumer subsidies on clean vehicles in many jurisdictions (peaked at 40-60% in China according to EESI)

Report mean outcomes (e.g., consumer surplus) integrating over:

- 1. Bounds on the distribution of offerings (before and after policies)
- 2. Demand and marginal cost shocks

Exercise 1: 20% Marginal Cost Subsidy on US Brands

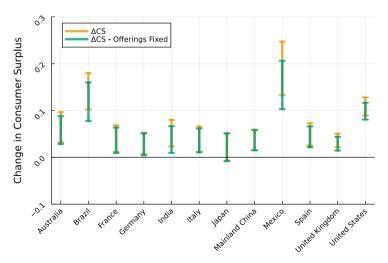
Change in Number of Varieties



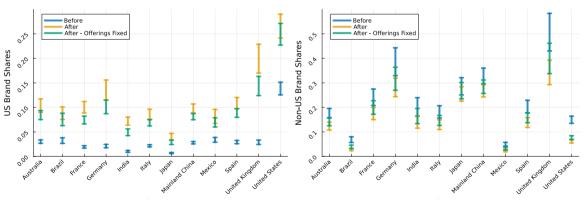
Panel B: Non-US Brand Varieties

Exercise 1: 20% Marginal Cost Subsidy on US Brands

Change in Consumer Surplus



Exercise 1: 20% Marginal Cost Subsidy on US Brands Change in Brand Shares

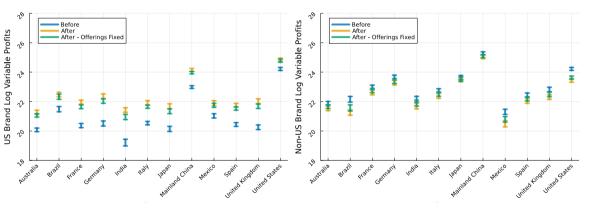


Panel A: US Brand Share

Panel B: Non-US Brand Share

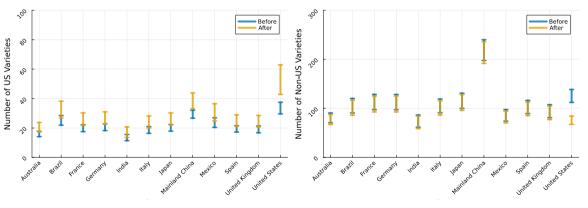
Exercise 1: 20% Marginal Cost Subsidy on US Brands

Change in Brand (Variable) Profits



Panel B: Non-US Brand Variable Profits

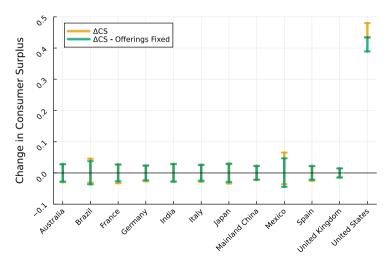
Change in Number of Varieties



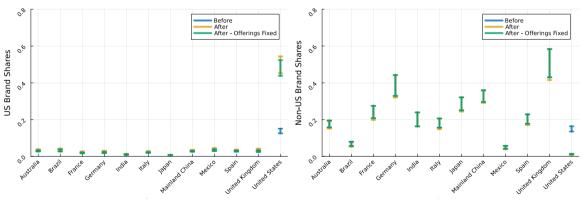
Panel A: US Brand Varieties

Panel B: Non-US Brand Varieties

Change in Consumer Surplus



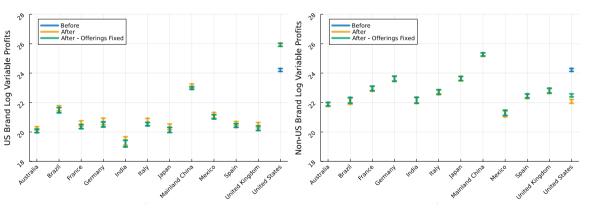
Change in Brand Shares



Panel A: US Brand Share

Panel B: Non-US Brand Share

Change in Brand (Variable) Profits



Panel B: Non-US Brand Variable Profits

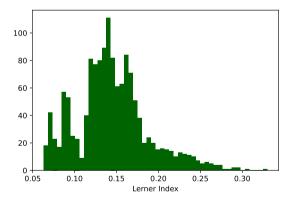
Key Takeaways

- 1. Consumers: Beneficial US entry dominates harmful non-US exit
 - Gains are heterogeneous across countries
 - Offsetting effects on the extensive margin for consumers
- 2. Subsidies increase offerings of US products worldwide
 - Greater domestic market \rightarrow "home market effect"
- 3. Producer subsidy improves the appeal of US products
 - Extensive margin response amplifies profit-shifting towards US brands worldwide \rightarrow strategic interactions matter
- 4. Consumer subsidy does not improve the appeal of US products abroad (unsubsidized)
 - Extensive margin response has small effects on foreign outcomes

Conclusion

- Quantified effects of national policies across markets through product portfolio choices
- Showed how to use inequalities to estimate and solve the model
- Moment inequalities: product-level fixed costs >> market entry fixed costs
- Product entry amplifies the global profit-shifting effects of national policies
- Induced entry does not lead to profit-shifting when new products are unattractive

Markup Distribution



1

Back

Micromoments Matched

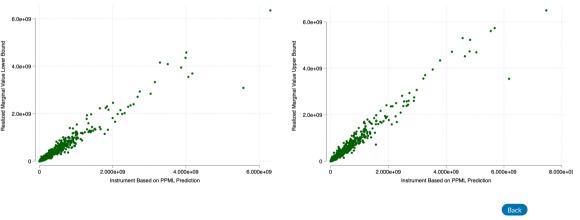
Match: 1. $P(income_i > \$100k | price_j > \$50k, US)$ 2. $P(\$60k \le income_i \le \$100k | price_i > \$50k, US)$

Micromoments Fit

Moment	Observed	Estimated	Difference
1	0.631	0.612	0.0188
2	0.212	0.245	-0.0329

Instrument Relevance - Second Stage

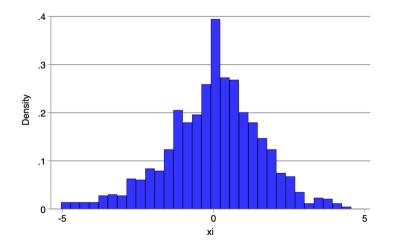
Pseudo- $R^2 \approx 95\%$ in both cases



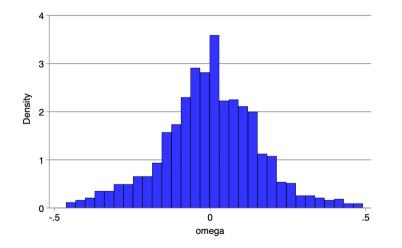
Lower Bound

Upper Bound

ξ Distribution



ω Distribution



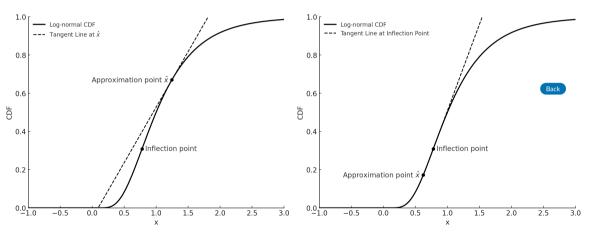
Existence and Purification

Theorem

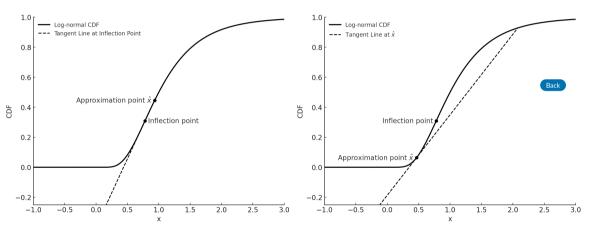
If a Nash-Betrand pricing equilibrium exists in the final stage, a Bayesian Nash Equilibrium in global entry decisions exists. Moreover, for any mixed strategy equilibrium there exists a pure strategy equilibrium that generates the same distribution of entry decisions.

Proof uses Milgrom and Weber (1985) and Balder (1988).

CDF Bounds



Concave Lower Bounds



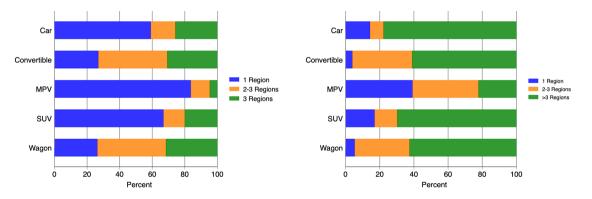
Stage 2 Inequalities

$$\mathbb{E}\left[\overline{\mathsf{\Gamma}}_{jm}\left(\pi_{jm}\left(\{j\},\Omega_{m}^{-f}\right),\hat{x}_{jm};\theta_{o},\sigma_{o}\right)-O_{jm}\middle|\mathcal{I},\mathcal{G}^{f}\right]\geq0$$



$$\mathbb{E}\left[\underline{\Gamma}_{jm}\left(\pi_{jm}\left(\mathcal{G}^{f},\Omega_{m}^{-f}\right)+\sum_{\substack{j'\neq j,\\j'\in\mathcal{G}^{f}}}\left[\pi_{j'm}\left(\mathcal{G}^{f},\Omega_{m}^{-f}\right)-\pi_{j'm}\left(\mathcal{G}^{f}\setminus\{j\},\Omega_{m}^{-f}\right)\right],\hat{x}_{jm};\theta_{o},\sigma_{o}\right)-O_{jm}\middle|\mathcal{I},\mathcal{G}^{f}\right]\leq0.$$

Number of Continents Conditional on Portfolio

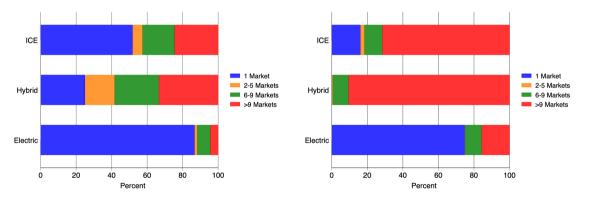


Panel A: Not Quantity Weighted

Panel B: Quantity Weighted

Figure: Number of Continents Offered Conditional on Portfolio, by Body Type

Number of Markets Conditional on Portfolio

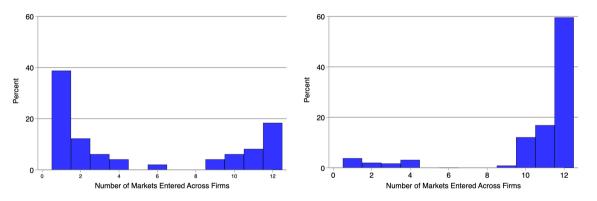


Panel A: Not Quantity Weighted

Panel B: Quantity Weighted

Figure: Number of Markets Offered Conditional on Portfolio, by Fuel Type

Number of Markets Entered Across Firms

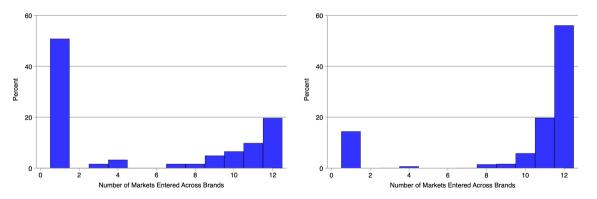


Panel A: Not Quantity Weighted

Panel B: Quantity Weighted

Number of Markets Entered Across Firms

Number of Markets Entered Across Brands

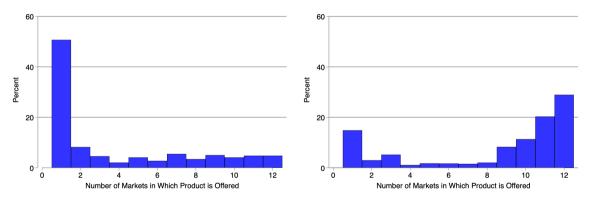


Panel A: Not Quantity Weighted

Panel B: Quantity Weighted

Number of Markets Entered Across Brands

Number of Markets Conditional on Portfolio

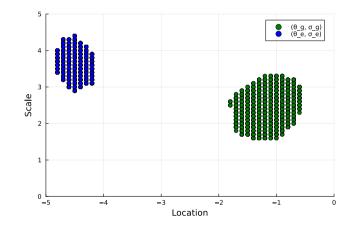


Panel A: Not Quantity Weighted

Panel B: Quantity Weighted

Number of Markets Offered Conditional on Portfolio

95% Confidence Set Plot



16

Moment Inequality Implementation

- 1. Use the demand model to fit a bivariate normal distribution for (ξ, ω) (ξ, ω)
- 2. Use the demand model to obtain $\overline{\pi}_{jm}$ and $\underline{\pi}_{jm}$ at realized rival entry decisions
- 3. Construct instruments using PPML by projecting bounds on objects in the information set *I*:

$$\hat{\underline{\pi}}_{jm}, \hat{\overline{\pi}}_{jm} = \exp\left(\hat{arphi}_{0} X_{m} + \hat{arphi}_{1,m} \widetilde{\delta}_{jm} + \hat{arphi}_{2,m} \widetilde{mc}_{jm} + \hat{arphi}_{3,m} \widetilde{\delta}_{jm} imes \widetilde{mc}_{jm}
ight)$$

where $\tilde{\delta}_{jm}$ is mean non-price utility and $\tilde{m}c_{jm}$ is mean marginal cost, net of unobserved heterogeneity.

4. Instruments are then **IV** Relevance :

$$\mathbb{1}\left[\hat{\pi}_{\textit{jm}} \in [\pmb{q}_{\ell-1}, \pmb{q}_{\ell}]
ight]$$

 q_ℓ denotes the $\ell^{ ext{th}}$ percentile of the predicted profit bounds.

Moment Inequality Theorem

Theorem

The following conditional moment inequalities partially identify the true fixed cost parameters ($\theta_{\theta}, \sigma_{\theta}$) and (θ_{g}, σ_{g}):

$$\mathbb{E}\left[\overline{\Gamma}_{jm}(MV_{jm}(\{j\},\Omega_m^{-f});\theta_{\theta},\sigma_{\theta})-O_{jm}|\mathcal{I}\right]\geq 0,\tag{1}$$

$$\mathbb{E}\left[\underline{\Gamma}_{jm}(MV_{jm}(\mathcal{G}^{t},\Omega_{m}^{-t});\theta_{\theta},\sigma_{\theta})-O_{jm}|\mathcal{I}\right]\leq0,$$
(2)

$$\mathbb{E}\left[\overline{\Lambda}_{j}\left(\sum_{m\in\mathcal{M}}\Gamma_{jm}(\pi_{jm}(\{j\},\Omega_{m}^{-t}))\times\left[\pi_{jm}(\{j\},\Omega_{m}^{-t})-\mathbb{E}[F_{jm}^{e}(\nu_{jm}^{e})|\mathcal{I},F_{jm}^{e}(\nu_{jm}^{e})\leq\pi_{jm}(\{j\},\Omega_{m}^{-t})]\right];\theta_{g},\sigma_{g}\right)-G_{j}\left|\mathcal{I}\right]\geq0,$$
(3)

$$\mathbb{E}\Big[\underline{\Lambda}_{j}\Big(\sum_{m\in\mathcal{M}}\mathsf{\Gamma}_{jm}(\hat{x}_{jm})\big[\mathsf{MV}_{jm}\left(\mathcal{A}^{f},\Omega_{m}^{-f}\right)-\mathbb{E}\big[\mathsf{F}_{jm}^{e}(\nu_{jm}^{e})+\mathsf{F}_{jm}^{e}(\nu_{jm}^{e})\leq\hat{x}_{jm},\mathcal{I}\big]\big];\theta_{g},\sigma_{g}\Big)-G_{j}\Big|\mathcal{I}\Big]\leq0,\tag{4}$$

where \hat{x}_{jm} is an \mathcal{I} – measurable approximation of $MV_{jm}\left(\mathcal{A}^{f}, \Omega_{m}^{-f}\right)$ and $\overline{\Lambda}_{j}/\overline{\Gamma}_{jm}$ and $\underline{\Lambda}_{j}/\underline{\Gamma}_{jm}$ are convex/concave upper/lower bounds of the CDFs of F_{i}^{g} and F_{im}^{e} , respectively.

Demand and Marginal Cost Estimation

- Match micro-moments to pin down non-homotheticity in demand Matched Micro-Moments
- Instruments:
 - 1. Gandhi-Houde (2019) differentiation IVs \rightarrow characteristics of similar products
 - 2. Hausman et al. (1994) IVs \rightarrow prices of the same/similar products in other markets
- Identifying assumption: Conditional on observed characteristics, and brand and country fixed effects, instruments uncorrelated with unobserved heterogeneity ξ_{jm} and ω_{jm}

Simulations: Informativeness of Moment Inequalities

Simulate N symmetric 3-product firms competing in 12 markets

$$\Pi_m^f(N_m^f, N_m^{-f}) = A_m \frac{N_m^f}{1 + \left(N_m^f\right)^{\kappa_o} \left(N_m^{-f}\right)^{\kappa_r}},$$

- $\kappa_o \rightarrow$ substitution within the firm
- $\kappa_r \rightarrow$ substitution across firms

Baseline: N = 10, $\kappa_o = 0.1$, $\kappa_r = 0.1$

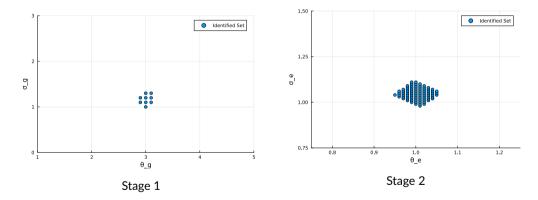


Figure: Identified Sets

High Substitutability Within Firms: $N = 10, \kappa_o = 0.25, \kappa_r = 0.1$

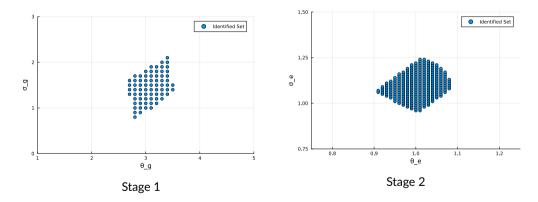


Figure: Identified Sets

Low Substitutability Within Firms: N = 10, $\kappa_o = 0.01$, $\kappa_r = 0.1$

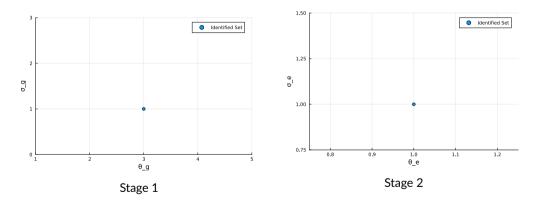


Figure: Identified Sets

Inference Under a Single Realization of Entry Game

Coverage of True $(\theta_{\theta}, \sigma_{\theta})$ Parameters (%)

SE Type	N = 5	N = 25	N = 50	N = 75
Robust	92.9	93.8	94.1	93.7
Clustered (country)	92.4	94.1	94.1	95.0



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Notes: This table reports the average coverage across simulations.

Median Length of Confidence Set Along (θ_e, σ_e)

SE Type	N = 5	N = 25	<i>N</i> = 50	N = 75
Robust	(0.8, 2.8)	(0.4, 0.9)	(0.2, 0.6)	(0.2, 0.5)
Clustered (country)	(0.9, 4.4)	(0.4, 1.6)	(0.3, 1.1)	(0.3, 0.9)

Notes: This table reports the median length across simulations of the confidence set along each of the dimensions of parameters (θ_e, σ_e) . The first coordinate reports the median length of the θ_e dimension of the confidence set, conditional on σ_e being at the true value. The second coordinate reports the median length of the σ_e dimension, conditional on θ_e being at the true value.

Upper Bound, Market Entry

Notation: $\mathbb{1}_{\Omega_m^f}$ is an indicator function denoting that firm *f* chooses bundle Ω_m^f **Best response**:

$$\left(\mathbb{1}_{\Omega_m^{f}} + \mathbb{1}_{\Omega_m^{f} \setminus \{j\}}\right) \times \left[\mathbb{1}\underbrace{\left\{\underbrace{\mathbb{E}[MV_{jm}(\Omega_m^{f}, \Omega_m^{-f}) | \mathcal{I}, \mathcal{G}^{f}, \{\nu_{jm}^{e}\}_{j \in \mathcal{G}^{f}, m}]}_{\Omega_m^{f} \text{ is preferred to } \Omega_m^{f} \setminus \{j\}} - \underbrace{\mathbb{E}[MV_{jm}(\Omega_m^{f}, \Omega_m^{-f}) | \mathcal{I}, \mathcal{G}^{f}, \{\nu_{jm}^{e}\}_{j \in \mathcal{G}^{f}, m}]}_{\Omega_m^{f} \text{ is preferred to } \Omega_m^{f} \setminus \{j\}} - \underbrace{\mathbb{E}[MV_{jm}(\Omega_m^{f}, \Omega_m^{-f}) | \mathcal{I}, \mathcal{G}^{f}, \{\nu_{jm}^{e}\}_{j \in \mathcal{G}^{f}, m}]}_{\Omega_m^{f} \text{ is preferred to } \Omega_m^{f} \setminus \{j\}}$$

Upper Bound, Market Entry

Submodularity:

$$\mathbb{E}[MV_{jm}(\{j\}, \Omega_m^{-f})|\mathcal{I}] \geq \mathbb{E}[MV_{jm}\left(\Omega_m^f, \Omega_m^{-f}\right)|\mathcal{I}] \geq \mathbb{E}[MV_{jm}(\mathcal{A}^f, \Omega_m^{-f})|\mathcal{I}]$$

Submodularity + best response:

$$(\mathbb{1}_{\Omega'_m} + \mathbb{1}_{\Omega'_m \setminus \{j\}}) \times \left[\mathbb{1}\left\{\mathbb{E}[MV_{jm}(\{j\}, \Omega_m^{-f}) | \mathcal{I}] - \exp(Z'_{jm}\theta_e + \sigma_e \nu_{jm}^e) \ge 0\right\} - \mathbb{1}_{\Omega'_m}\right] \ge 0$$

Note that:

$$\sum_{\Omega_m^t:j\in\Omega_m^t}\mathbb{1}_{\Omega_m^t}=O_{jm};\qquad \sum_{\Omega_m^t:j\in\Omega_m^t}\mathbb{1}_{\Omega_m^t\setminus\{j\}}=1-O_{jm}$$

Upper Bound, Market Entry

Summing, we obtain a bound on any entry opportunity (no selection),

$$\mathbb{1}\big\{\mathbb{E}[\textit{MV}_{\textit{jm}}(\{j\},\Omega_m^{-f})|\mathcal{I}] - \overbrace{\exp(Z'_{\textit{jm}}\theta_e + \sigma_e\nu_{\textit{jm}}^e)}^{F^e_{\textit{jm}}} \ge 0\big\} \ge O_{\textit{jm}}$$

Taking expectations conditional on \mathcal{I} we obtain,

$$\mathbb{P}\left(\mathcal{O}_{jm}=1|\mathcal{I}\right) \leq \Gamma\left(\mathbb{E}[MV_{jm}(\{j\},\Omega_m^{-f})|\mathcal{I}]\right)$$

Upper Bound, Market Entry

Summing, we obtain a bound on any entry opportunity (no selection),

$$\mathbb{1}\big\{\mathbb{E}[\textit{MV}_{jm}(\{j\},\Omega_m^{-f})|\mathcal{I}] - \overbrace{\exp(Z'_{jm}\theta_e + \sigma_e\nu_{jm}^e)}^{F^e_{jm}} \ge 0\big\} \ge O_{jm}$$

Taking expectations conditional on \mathcal{I} we obtain,

$$\mathsf{\Gamma}_{jm} \Big(\mathbb{E}[\mathsf{MV}_{jm}(\mathcal{A}^{f}, \Omega_{m}^{-f}) | \mathcal{I}] \Big) \leq \mathbb{P} \left(\mathcal{O}_{jm} = \mathsf{1} | \mathcal{I} \right) \leq \mathsf{\Gamma} \Big(\mathbb{E}[\mathsf{MV}_{jm}(\{j\}, \Omega_{m}^{-f}) | \mathcal{I}] \Big)$$

Deriving Bounds on Portfolio Probabilities

Upper Bound, Portfolio Choice (Sketch)

Notation: $\mathbb{1}_{G^{f}}$ is an indicator function denoting that firm *f* chooses bundle \mathcal{G}^{f}

Best response:

$$\left(\mathbb{1}_{\mathcal{G}^{f}} + \mathbb{1}_{\mathcal{G}^{f} \setminus \{j\}}\right) \times \left[\mathbb{1}\underbrace{\left\{\underbrace{\mathcal{V}_{f}(\mathcal{G}^{f}) - \mathcal{V}_{f}(\mathcal{G}^{f} \setminus \{j\})}_{\mathcal{G}^{f} \text{ is preferred to } \mathcal{G}^{f} \setminus \{j\}} - \underbrace{\mathsf{exp}(Z_{j}^{\prime} \theta_{g} + \sigma_{g} \nu_{j}^{g}) \geq 0}_{\mathcal{G}^{f} \text{ is preferred to } \mathcal{G}^{f} \setminus \{j\}} - \mathbb{1}_{\mathcal{G}^{f}}\right] = 0$$

Key: bound by above by bounding $\mathcal{V}_f(\mathcal{G}^f \setminus \{j\})$ by below using the market entry decisions optimal under \mathcal{G}^f for for products $\mathcal{G}^f \setminus \{j\}$

Deriving Bounds on Portfolio Probabilities

Upper Bound, Portfolio Choice (Sketch)

Obtain,

$$(\mathbb{1}_{\mathcal{G}^{f}} + \mathbb{1}_{\mathcal{G}^{f} \setminus \{j\}}) \times \\ \mathbb{1} \{ \sum_{m \in \mathcal{M}} (\mathbb{E} [\mathcal{O}_{jm}^{\mathcal{G}^{f}} [\mathbb{E} [\mathcal{M}_{jm}(\Omega_{m}^{\mathcal{G}^{f}} \setminus \{j\}, \Omega_{m}^{-f}) | \mathcal{I}, \{\nu_{jm}^{e}\}_{j \in \mathcal{G}^{f}, m}, \mathcal{G}^{f}] - \mathcal{F}_{jm}^{e}(\nu_{jm}^{e}) | \mathcal{I}]) - \mathcal{F}_{j}^{g}(\nu_{j}^{g}) \geq \mathbf{0} \}$$

 $\geq (\mathbb{1}_{\mathcal{G}^{f}} + \mathbb{1}_{\mathcal{G}^{f} \setminus \{j\}})\mathbb{1}_{\mathcal{G}^{f}}$

Applying submodularity,

$$\begin{aligned} (\mathbb{1}_{\mathcal{G}'} + \mathbb{1}_{\mathcal{G}' \setminus \{j\}}) & \times \mathbb{1} \Big\{ \sum_{m \in \mathcal{M}} \left(\mathbb{E} \big[\mathcal{O}_{jm}^{\{j\}} \big[\mathbb{E} \big[\mathcal{M} \mathcal{V}_{jm}(\{j\}, \Omega_m^{-f}) | \mathcal{I} \big] - \mathcal{F}_{jm}^{e}(\nu_{jm}^{e}) | \mathcal{I} \big] \right) - \mathcal{F}_{j}^{g}(\nu_{j}^{g}) \geq \mathbf{0} \Big\} \\ & \geq (\mathbb{1}_{\mathcal{G}'} + \mathbb{1}_{\mathcal{G}' \setminus \{j\}}) \mathbb{1}_{\mathcal{G}'} \end{aligned}$$

where

$$O_{jm}^{\{j\}} = \mathbb{1}\{\mathbb{E}ig[MV_{jm}(\{j\},\Omega_m^{-f})|\mathcal{I}ig] - F_{jm}^e(
u_{jm}^e) \ge 0\}$$

Deriving Bounds on Portfolio Probabilities

Upper Bound, Portfolio Choice (Sketch)

Inequality holds for all \mathcal{G}^{f} with $j \in \mathcal{G}^{f}$; summing across all such inequalities yields,

$$\mathbb{1}\big\{\sum_{m\in\mathcal{M}}\big(\mathbb{E}\big[\mathcal{O}_{jm}^{\{j\}}\big[\mathbb{E}\big[\mathcal{MV}_{jm}(\{j\},\Omega_m^{-f})|\mathcal{I}\big]-\mathcal{F}_{jm}^{e}(\nu_{jm}^{e})\big|\mathcal{I}\big]\big)-\mathcal{F}_{j}^{g}(\nu_{j}^{g})\geq 0\big\}\geq G_{j}$$

Taking expectations conditional on \mathcal{I} obtains,

$$\mathbb{P}(G_{j} = 1 | \mathcal{I}) \leq \Lambda \left(\sum_{m \in \mathcal{M}} \left(\mathbb{E} \left[\mathcal{O}_{jm}^{\{j\}} \left[\mathbb{E} \left[\mathcal{M} \mathcal{V}_{jm}(\{j\}, \Omega_{m}^{-f}) | \mathcal{I} \right] - \mathcal{F}_{jm}^{e}(\nu_{jm}^{e}) | \mathcal{I} \right] \right) \right)$$

where Λ is the CDF of $F_j^g(\nu_j^g)$

Similar arguments apply for the lower bound

Imputing Product Characteristics



Size:

- 1. Use the mean size of observed products of the same body type sold by the same brand;
- 2. if there are no such products, use the mean size of observed products of the same body type sold by the same parent company;
- 3. if there are no such products, use the mean size across observed products of said body type

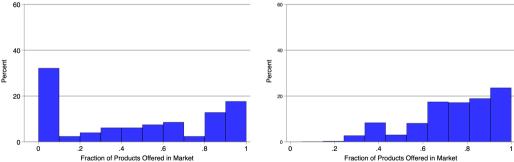
Horsepower and horsepower/weight:

- 1. Use the mean horsepower (horsepower/weight) of observed products sold of the same fuel type and body type sold by the same parent company;
- 2. if there are no such products, use the mean horsepower (horsepower/weight) of observed products with the same body type and fuel type offered in that country;
- 3. if there are no such products, use the mean horsepower (horsepower/weight) of all products with the same fuel type and body type.

Fraction of Products Offered (Firm-Markets)

Panel A: Not Quantity Weighted

Panel B: Quantity Weighted



No PSNE in Complete-Info Game with Strategic Subs & 3 Players Counterexample

Consider the following game:

Р3	pl	lays	1
----	----	------	---

P3 plays 0

P1/ P2	1	0	P1 / P2	1	0
1	(-5, -5, -2)	(-4,0,1)	1	(1,-1,0)	(2, 0, 0)
0	(0,1,-1)	(0,0,2)	0	(0,1,0)	(0,0,0)

This is a static binary choice complete information entry game. Each player's payoff from entering is weakly decreasing in the set of entry decisions chosen by other players. No pure strategy Nash equilibrium exists. (Similar to the counterexample in Appendix B.2 of Jia 2008.)



Identified Sets Under Misspecification

Simulate two 2-product firms competing in 12 markets

$$\Pi_m^f(N_m^f, N_m^{-f}) = A_m \frac{N_m^f}{1 + \left(N_m^f\right)^{\kappa_o} \left(N_m^{-f}\right)^{\kappa_r}},$$

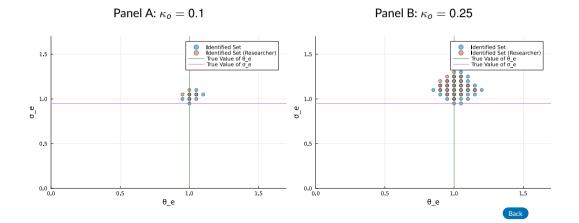
True model:

$$F_{jm}^{e} = \exp(\theta_{e} + \underbrace{\sigma_{e}\nu_{jm}^{e}}_{\text{private info}} + \underbrace{\sigma_{b}\nu_{fm}^{b}}_{\text{public info}})$$

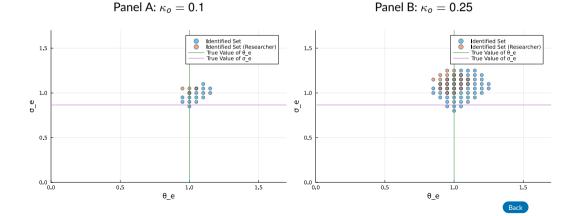
Question: how do moment inequalities assuming only private info perform when the true model also contains a public info component?

• Normalize
$$\sigma_{e}^{2} + \sigma_{b}^{2} = 1$$

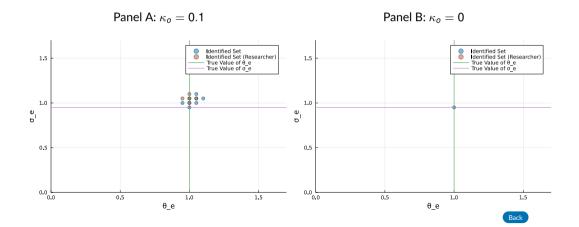
Smaller Degrees of Unobserved Complete Info $\sigma_b^2 = 0.1$



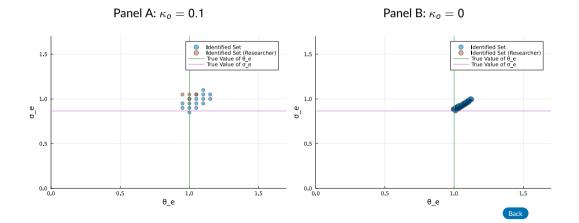
Larger Degrees of Unobserved Complete Info $\sigma_b^2 = 0.25$



Smaller Degrees of Unobserved Complete Info $\sigma_b^2 = 0.1$



Larger Degrees of Unobserved Complete Info $\sigma_b^2 = 0.25$



Key Takeaways: Misspecification Simulations

• Inequality used for estimation based on e.g., $\mathbb{E}[\overline{\Gamma}_{jm}(\overline{MV}_{jm}; \theta_e, \sigma_e)|\mathcal{I}]$ misses a complete-information component unobserved to the econometrician:

$$\mathbb{E}[\overline{\Gamma}_{jm}(\exp(-\sigma_b\nu_{jm}^b)\overline{MV}_{jm};\theta_e,\sigma_e)|\mathcal{I},\boldsymbol{\nu}^b]$$

- Misspecification causes confidence sets to be smaller than they should be
- When non-empty, typically do not contain true $\sigma_e \rightarrow$ biased
- **Possible way out:** use solution method to form inequalities that permit simulating the complete-info component (Estimation W/ Sol. Method):

$$\begin{split} \hat{\mathbb{E}}_{\boldsymbol{\nu}^{b}} \bigg[\mathsf{\Gamma}_{jm} \bigg(\exp(-\sigma_{b} \nu_{jm}^{b}) \mathbb{E} \big[\boldsymbol{M} \boldsymbol{V}_{jm} \big(\boldsymbol{\mathcal{A}}^{f}, \Omega_{m}^{-f} \big) \big| \boldsymbol{\mathcal{I}}, \boldsymbol{\nu}_{s}^{b} \big]; \boldsymbol{\theta}_{e}, \sigma_{e}, \sigma_{b} \bigg) \bigg] &\leq \mathbb{P} \left[\boldsymbol{O}_{jm} = 1 | \boldsymbol{\mathcal{I}} \right] \\ &\leq \hat{\mathbb{E}}_{\boldsymbol{\nu}^{b}} \bigg[\mathsf{\Gamma}_{jm} \bigg(\exp(-\sigma_{b} \nu_{jm}^{b}) \mathbb{E} \big[\boldsymbol{M} \boldsymbol{V}_{jm} \big(\{ \boldsymbol{j} \}, \Omega_{m}^{-f} \big) \big| \boldsymbol{\mathcal{I}}, \boldsymbol{\nu}_{s}^{b}; \boldsymbol{\theta}_{e}, \sigma_{e}, \sigma_{b} \big] \bigg) \bigg] \end{split}$$

Solution Method Under Equilibrium Multiplicity

Consider the model with two firms i = 1, 2 each making a binary entry decision. Payoffs are:

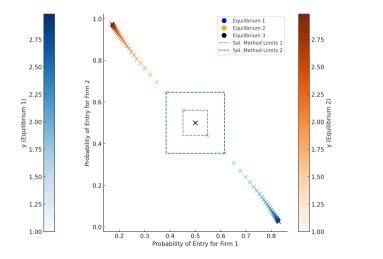
P1/ P2	1	0
1	$(-x - \nu_1, -y - \nu_2)$	$(x - \nu_1, 0)$
0	$(0, \mathbf{y} - \nu_2)$	(0,0)

- $\nu_i \sim Normal(0, 1)$ and private information
- Results:
 - 1. Multiple equilibria if and only if $xy > \pi/2$
 - 2. Equilibrium enter if and only $\nu_i < 0$ (*i* enters w.p. 1/2) always exists

Normalize x = 1

• Intuition: as y increases, probability of Firm 2 entry rises in one equilibrium but declines in another equilibrium

Solution Method Under Equilibrium Multiplicity



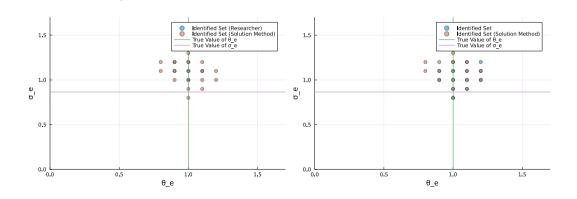
Estimation with Solution Method $\kappa_o = 0.25, \sigma_b^2 = 0.25$

Panel A: Misspecified vs Solution Method

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Back to Takeaways - Main

Panel B: "No Selection" vs Solution Method



Economies of Scope

In the Appendix, I show how to deal with can deal with cases of the form:

$$m{F}^{e,f}_m= heta_0\mathbbm{1}\{|\Omega^f_m|\geq 1\}+\sum_{j\in\Omega^f_m}m{F}^e_{jm}.$$

When $\theta_0 \ge 0$ can always obtain bounds on additional fixed costs from offering a product in country *m*:

$$F_{jm}^{e} \leq \Delta F_{jm}^{e} \leq \theta_{0} + F_{jm}^{e}$$

