Individual Strategy Choice in Electoral Decisions

Chase R. Abram

Department of Economics
Indiana University

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The Problem

- Suppose there is a population that makes decisions electorally - a simple majority wins
- Individuals know their own preferences over the possible selections
  - Simply voting for their favorite option may not be optimal
- The 2016 U.S. presidential election raised a question for many - Should I vote for my second or third favorite choice?

More Motivated Problems

- University student government elections
- Small group dynamics - choosing where to dine?
- Notation and jargon in academic fields

The Model's Assumptions

- Simple majority wins
- Citizens know their own preferences at the time of the election, and they are independent of how others vote
- Under campaigning allowance, there is probability zero of a tie
- Risk neutrality (optional)

Starting Simple - 2 Citizens, 2 Options

- Even number of citizens - ties are possible
- Allow a draw to simply be counted as a half win for each option
- Thus we have outcome matrix:

		Other Citizen	
		X	Y
Citizen	X	X	$.5X, .5Y$
	Y	$.5X, .5Y$	Y

Starting Simple - 2 Citizens, 2 Options

- Consider Citizen's possible perspectives
  1. Indifferent $\Rightarrow$ zero matrix
  2. Prefer X $\Rightarrow$ matrix below
  3. Prefer Y $\Rightarrow$ analogous to X preference

		Other Citizen X	Other Citizen Y
Citizen	X	1	$.5$
Citizen	Y	$.5$	0

- Clearly, picking X is a dominant strategy

Enough with the Obvious!

- With only two options, choosing the preferred option is always a dominant strategy
- The analysis with a population of n is simple, and assuming our Citizen prefers one option to the other, the utility of choosing that preferred option is always greater than or equal to choosing the alternative option

Three Options

- Our Citizen's preferences are now more complicated, but can be categorized into 13 bins:
  - (1) - (6): ordered preferences ($A > B > C$)
  - (7) - (9): aversion preferences ($A = B > C$
  - (10) - (12): affinity preferences ($A > B = C$)
  - (13): indifference preference ($A = B = C$)
- Assume no fair weather fans (vote for popular choice) or counterculture (vote against popular choice)
- No need to analyze all 13 possibilities; instead consider one from each (nontrivial) bin

Three Options, Two Citizens

- Using the same rules as before, we construct the new outcome matrix:

Other Citizen

		A	B	C
	A	A	$.5A, .5B$	$.5A, .5C$
Citizen	B	$.5A, .5B$	B	$.5B, .5C$
	C	$.5A, .5C$	$.5B, .5C$	C

Preferences and Payoffs

Other Citizen

Citizen		A	B	C
	A	1	.5	0
	B	.5	0	$-.5$
	C	0	$-.5$	-1

- $A > B > C \Rightarrow$ Strategy A is dominant

Other Citizen

Citizen		A	B	C
	A	0	0	$-.5$
	B	0	0	$-.5$
	C	$-.5$	$-.5$	-1

- $A = B > C \Rightarrow$ Strategy C is dominated

Other Citizen

Citizen		A	B	C
	A	1	.5	.5
	B	.5	0	0
	C	.5	0	0

- $A > B = C \Rightarrow$ Strategy A is dominant

Three Citizens

- The potential scenarios our Citizen will face grow quickly with the size of population - we will look at a population of 3 before jumping to n
- There are 6 possible scenarios
  1. Both pick A
  2. One picks A, one picks B
  3. One picks A, one picks C
  4. Both pick B
  5. One picks B, one picks C
  6. Both pick C

Outcome Matrix

- The outcome matrix also becomes more involved
- Again, split the outcome for ties, as if the tie will be decided by a uniform random distribution (invoke risk neutrality)

		Two Other Citizens					
		1	2	3	4	5	6
	A	A	A	A	B	$.33A, .33B, .33C$	C
Citizen	B	A	B	$.33A, .33B, .33C$	B	B	C
	C	A	$.33A, .33B, .33C$	C	B	C	C

Payoffs and Progress

- Same results as before:
  (a) Ordered preferences: pick the favorite
  (b) Aversion preferences: do not pick the averse option
  (c) Affinity preferences: pick the favorite
- Thus far: Citizen should always pick the favorite
- This machinery is consistent with intuition, and easy to extend

n Citizens

- Sort the possible scenarios presented into 18 bins, based on how our Citizen's vote will affect the outcome
- The 18 bins can be summarized:
  (a) Citizen's vote is meaningless
  (b) Two or three options are tied, Citizen can potentially break the tie
  (c) Citizen can potentially create a tie
- The outcome and payoff matrices are at an unwieldy 3×18 size

n Citizens

- When considering a population of n, the results become more interesting:
  (a) Ordered preferences: do not pick the least favorite
  (b) Aversion preferences: do not pick the least favorite
  (c) Affinity preferences: pick the favorite
- Contrary to the previous results, picking the favorite is not necessarily Pareto dominant anymore
- This feature arises because it may be best in some cases to vote for the second favorite option, in order to block the least favorite choice

Incorporating Campaigning

- Thus far, our population is entirely unrealistic: no one interacts, everyone just guesses what everyone else will do and votes accordingly
- Fix: introduce campaigning, so citizens can have a wider impact and share their ideas, and have unequal representation and costs
- Campaigning: any action through which an individual incurs a cost in an effort to promote their preferences in any way other than merely voting (pecuniary, non-pecuniary, or any combination)

Campaigning Scenarios and Outcomes

- Now only 10 scenarios exist, because this more realistic model eliminates ties, which are assumed to occur with probability zero:
  (a) Campaigning is futile
  (b) Campaigning *may* affect outcome
  (c) Campaigning *will* affect outcome

$n-1$ Other Citizens

		1	2	3	4	5	6	7	8	9	10
	A	A	A	B	C	A	A	A	B	A	C
Citizen	B	B	A	B	C	B	A	B	B	C	B
	C	C	A	B	C	A	C	B	C	C	C

Ordered Preferences: General Result

- Suppose $A > B > C$
- Choosing C is weakly dominated, but that is all that can be said without imposing further assumptions

$n-1$ Other Citizens

		1	2	3	4	5	6	7	8	9	10
Citizen	A	1	1	0	-1	1	1	1	0	1	-1
	B	0	1	0	-1	0	1	0	0	-1	0

Aversion Preferences: General Result

- Suppose $A = B > C$
- Choosing C is weakly dominated, but that is all that can be said without imposing further assumptions

$n-1$ Other Citizens

		1	2	3	4	5	6	7	8	9	10
Citizen	A	0	0	0	-1	0	0	0	0	0	-1
	B	0	0	0	-1	0	0	0	0	-1	0

Affinity Preferences: General Result

- Suppose $A > B = C$
- Choosing A is weakly dominant, so we are finished analyzing this possibility!

$n - 1$ Other Citizens

		1	2	3	4	5	6	7	8	9	10
	A	1	1	0	0	1	1	1	0	1	0
Citizen	B	0	1	0	0	0	1	0	0	0	0
	C	0	1	0	0	1	0	0	0	0	0

Extending Assumptions

- Generality is the goal, so we minimize further assumptions that allow complete descriptions
- Recall that for ordered and aversion preferences optimal strategic behavior has not yet been fully prescribed
- Method: eliminate scenarios yielding indifferent outcomes, summarize results assuming:
    (a) Some outcomes will not occur
    (b) A mixed strategy may be employed
    (c) The disparity between option utilities is constant

Ordered Preferences: $A > B > C$

		$n-1$ Other Citizens				
		1	5	7	9	10
Citizen	A	1	1	1	1	-1
	B	0	0	0	-1	0

(1) Do not choose C

(2) If 10 will not happen, choose A

(3) If none of 1, 5, 7, or 9 will occur, choose B

(4) If we assign relative probabilities p, q, r, and t, to scenarios 1, 5, 7, and 9, respectively, assign relative probability $1-p-q-r-t$ to scenario 10, and Citizen estimates $p+q+r+t < \frac{1}{2}$, then Citizen should choose A with probability $\frac{-t}{2p+2q+2r+t-1}$

Aversion Preferences: $A = B > C$

		$n-1$ Other Citizens	
		9	10
Citizen	A	0	-1
	B	-1	0

(1) Do not choose C

(2) If 10 will not occur, choose A

(3) If 9 will not occur, choose B

(4) If the relative probabilities of scenarios 9 and 10 occurring are known, with probability of 9 occurring p, it is a weakly dominant strategy to choose A with probability p

Practical Application - Where to Eat?

- Steven and 4 friends are going out to dinner
- Steven is indifferent between Burgertown and Spaghettitopia, but has a strong distaste for Veggieville
- His friends are quite predictable: Cassidy and James usually choose Veggieville, and Oliver and Felicia usually choose Spaghettitopia
- Steven chooses Spaghettitopia

		4 Friends	
		9	10
Steven	Spaghettitopia	0	-1
	Burgertown	-1	0

Practical Applications - Deciding a Verdict

- A jury is deciding a case with multiple related charges (i.e. breaking and entering, theft, property damage, etc.)
- They realize they are almost a hung jury, so decide to take a vote: all will agree to the outcome for ruling
- The options are: guilty on all charges, guilty on some charges, or innocent
- One juror believes the accused is innocent, and the rest of the jury is split between some charges and all charges
- The sympathetic juror should vote guilty on some charges

		11 Other Jurors
		9
Juror	Innocent	-1
	Guilty on Some Charges	0

Conclusions

- Largely achieved goal of constructing rigorous strategic voting optimization, though at the cost of stringent assumptions
- Many times, one would be able to apply real-world assumptions to this model and reach a simple strategy, but at other times, a lack of ability to estimate probabilities of scenarios would hinder the model's scope
- Plethora of other assumptions or concepts could be introduced, which might alter results

Thank You

- Abram, Chase. *Individual Strategy Choice in Electoral Decisions With Up to Three Options*. 2017.
- If you are interested in learning more or reading my paper, feel free to contact me at chabram@indiana.edu
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