# Individual Strategy Choice in Electoral Decisions With Up to Three Options 

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## 1 Introduction

In any electoral decision, citizens are presented with a variety of options to which they may lend their support. While at first it may seem a simple decision to give support to one's favorite option, this is not always a Pareto optimal strategy. The winning choice is not decided solely by one citizen, so each citizen must take into account their own preferences over all outcomes as well as how the conglomerate of other citizens will vote, in order to maximize their personal utility.

The 2016 presidential election in the United States has caused this dilemma to become especially interesting, as many citizens claim to have aversion preferences, where all candidates give equal utility except for one, which gives less utility. Traditionally, ordered preferences are assumed to exist, where all candidates can be ordered by their utility, none of which is equal. Furthermore, many previous elections have centered on the two major party candidates, the Democrats and Republicans, but due to the lack of popularity with both parties, it now seems prudent to introduce the Libertarian candidate as a potential choice.

This paper first sets out to construct a theoretical analysis of strategic voting from the individual's perspective. We will begin with the simplest case, two citizens voting on two options, and build upon our findings until we have a model for $n$ citizens choosing from three options. Next we will introduce the concept of campaigning, or citizens incurring costs for the sake of promoting a certain option. Campaigning significantly complicates the models, and forces us to consider mixed strategies in certain contexts. Throughout, dominant strategies are found when possible, and when not possible, conditions are set for the minimal additional assumptions needed to unearth a dominant strategy. After satisfactorily constructing models for voting and campaigning behavior, we will apply these models to hypothetical real-world examples concerning the 2016 election, to demonstrate the strength of our results.

## 2 Two Options

In order to properly model how a citizen within a population should vote, it is logical to begin with the simplest nontrivial example of a two citizen population with two options. A one citizen population is too trivial, as she will always pick
her first choice and win. After analyzing the two citizen, two option model, the cases of three and $n$ citizens populations will be explored, each with two options.

### 2.1 Two Citizens

Two options are given, $X$ and $Y$. Since there are an even number of citizens, there is the possibility of a tie, which is problematic. For the models presented in this paper, it will be assumed that the two tied options each get half of a win. This can best be understood by saying that a fair coin toss decides the winner of the tie, and the election is repeated an infinite number of times, thus each tied option will be declared the winner half of the time. Now, the outcome matrix is:

> |  |  | Other Citizen |  |
| :---: | :---: | :---: | :---: |
|  |  | $X$ | $Y$ |
|  |  | $X$ | $.5 X, .5 Y$ |
|  |  | $X$ |  |
|  |  | $.5 X, .5 Y$ | $Y$ |
|  |  |  |  |

In order to assign payoffs for Citizen, preferences must be assigned. Citizen either prefers $X$ to $Y$, prefers $Y$ to $X$, or is indifferent between $X$ and $Y$. The indifference case will clearly give the same utility regardless of outcome, so we turn to $X$ preferred to $Y$. Again, we note that in the event of a tie, Citizen only gets half of the utility she would achieve if $X$ has won outright. Furthermore, only relative preferences matter, so we assign utilities of 1 and 0 to $X$ and $Y$ respectively:


It is clearly a dominant strategy for Citizen to always pick $X$, and the symmetry of this model allows us to say that if Citizen preferred $Y$ to $X$, it would be a dominant strategy to always pick $Y$.

### 2.2 Three Citizens

We now turn to the three citizen model, where two options are given: $X$ and $Y$. Thus each citizen is faced with a choice of $X$ or $Y$, and his fellow citizens will also create one of three situations for him to be faced with, though he will not know which he is receiving at the time of the vote:
(1) Both of his fellow citizens picked $X$.
(2) One picked $X$, one picked $Y$.
(3) Both of his fellow citizens picked $Y$.

Furthermore, it is assumed that a simple majority is needed for a choice to prevail, thus the outcome matrix for each citizen is as follows:

|  |  | Two Citizens |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 1 |  | 2 | 3 |
| Citizen | $X$ | $X$ | $X$ | $Y$ |
|  | $Y$ | $X$ | $Y$ | $Y$ |
|  |  |  |  |  |

Now preferences can determine payoffs assigned to the outcomes. Since there are only two options under consideration, the preference assigned to the single citizen under analysis will be $X$ is preferred to $Y$, though $Y$ is preferred to $X$ would have an analogously reversed payoff matrix. Again, the indifference preference is not worth examining in detail because it will yield the same payoffs regardless of outcome. Assuming Citizen prefers $X$ over $Y$, relative payoffs are:

> Two Citizens

Citizen

|  |  |  | 1 |  | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $X$ | 1 | 1 | 0 |  |  |  |
| $Y$ | 1 | 0 | 0 |  |  |  |
|  |  |  |  |  |  |  |

It becomes quite clear that Citizen should always choose X , as it is a weakly dominant choice.

## $2.3 n$ Citizens

The two option game can be extended to a population of $n$, and the outcome and payoff matrices will depend on whether $n$ is even or odd. We begin with the case where $n$ is odd.

The first striking observation we make is that we need not analyze every distinct outcome of $n-1$ citizens' choices, which will be the possible scenarios presented to Citizen. Instead, the outcomes can be grouped into three bins:
(1) Enough fellow citizens pick $X$ that $X$ will prevail, regardless of Citizen's choice.
(2) Half pick $X$, half pick $Y$.
(3) Enough fellow citizens pick $Y$ that $Y$ will prevail, regardless of Citizen's choice.

The realization that there are really only three outcomes dealt to Citizen allows for drastic simplification of the outcome and payoff matrices:

\[

\]

Citizen

| $n-1$ Citizens  <br>   <br> 1  |  |  |  |
| :---: | :---: | :---: | :---: |
|  | 2 | 3 |  |
|  | 1 | 1 | 0 |
|  | 1 | 0 | 0 |
|  |  |  |  |

We now turn to the case where $n$ is even. In this case, ties must be incorporated into the model, so there are four scenarios that may be presented to Citizen:
(1) Enough fellow citizens pick $X$ that $X$ will prevail, regardless of Citizen's choice.
(2) $\frac{n}{2}$ citizens pick $X$.
(3) $\frac{n}{2}$ citizens pick $Y$.
(4) Enough fellow citizens pick $Y$ that $Y$ will prevail, regardless of Citizen's choice.

The outcome matrix is then:

\[

\]

The corresponding payoff matrix:

$$
n-1 \text { Citizens }
$$

Citizen

|  | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| $X$ | 1 | 1 | . 5 | 0 |
| $Y$ | 1 | . 5 | 0 | 0 |

Perhaps unsurprisingly, choosing $X$ is a weakly dominant choice for a citizen with preference $X>Y$, regardless of size of the population.

## 3 Three Options

The above analysis proves that it is always a weakly dominant strategy for a citizen to pick their favorite if there are only two options. This bit of insight is not particularly interesting or surprising, so our focus now turns to the possibility of three options under consideration. Again, we build our analysis of a three option ballot by starting with a two citizen population, moving to three citizens, and finishing with $n$ citizens.

### 3.1 Citizen Support Preferences for Three Options

Citizens in this model are assumed to have preferences independent of the choice of the remainder of the population. In other words, this approach does not account for fair weather fans (voting for the popular choice) or counterculture (voting against the popular choice). Rather, it is assumed that for any given two of the three options, a citizen always either prefers one of them or is indifferent between them. For example, $A=B>C$ means Citizen is indifferent between $A$ and $B$, but prefers either to $C . B>A=C$ means Citizen is indifferent between $A$ and $C$, but would prefer $B$ over either. Therefore, with three options, there are 13 possible preferences:
(1) $A>B>C$ ( $A$ affinity, $C$ aversion)
(2) $A>C>B(A$ affinity, $B$ aversion $)$
(3) $B>A>C$ ( $B$ affinity, $C$ aversion)
(4) $B>C>A(B$ affinity, $A$ aversion $)$
(5) $C>A>B$ ( $C$ affinity, $B$ aversion)
(6) $C>B>A(C$ affinity, $A$ aversion $)$
(7) $A=B>C(C$ aversion $)$
(8) $A=C>B$ ( $B$ aversion $)$
(9) $B=C>A(A$ aversion $)$
(10) $A>B=C$ ( $A$ affinity)
(11) $B>A=C(B$ affinity $)$
(12) $C>A=B$ ( $C$ affinity $)$
(13) $A=B=C$ (Indifference)

Each unique preference will have a unique payout matrix.

### 3.2 Two Citizens

With three options, the outcome matrix becomes more complicated:

|  |  | Other Citizen |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $A$ |  |  |  |
| Citizen | $B$ | $C$ |  |  |
|  | $A$ | $A$ | $.5 A, .5 B$ | $.5 A, .5 C$ |
|  | $B$ | $.5 A, .5 B$ | $B$ | $.5 B, .5 C$ |
|  | $C$ | $.5 A, .5 C$ | $.5 B, .5 C$ | $C$ |

Now the preferences of Citizen have also become more complicated, but we can categorize the preference types into sets. Ordered preferences is the set containing all preferences that distinctly order the three options. Aversion preferences is the set containing all preferences where Citizen is indifferent between two options, but prefers either to the third option. Affinity preferences is the set of all preferences where Citizen is indifferent between options, but prefers the third option to either. The indifference preference is its own set. We again care about relative payoffs, so simply assign utilities of $-1,0$, and 1 to the three options in an ordered preference. For an aversion preference, we assign utilities 0 and -1 . Finally, for affinity preferences we assign utilities 1 and 0 .

Let us first look at the payoff matrix for ordered preference $A>B>C$ :
Other Citizen

Citizen

|  | $A$ |  | $B$ |
| :---: | :---: | :---: | :---: |
| $C$ |  |  |  |
| $A$ | 1 | .5 | 0 |
| $B$ | .5 | 0 | -.5 |
| $C$ | 0 | -.5 | -1 |
|  |  |  |  |

It remains a dominant strategy for Citizen to choose their favorite, $A$. Now we look at the aversion preference $A=B>C$ :


In this case, the dominant strategy is to choose either option that is not the averse option (the averse option is $C$, in this case). Finally, we look at affinity preference $A>B=C$ :

## Other Citizen

Citizen

| $A$ |  | $B$ |  |
| :---: | :---: | :---: | :---: |
| $C$ |  |  |  |
| $A$ | 1 | .5 | .5 |
| $B$ | .5 | 0 | 0 |
| $C$ | .5 | 0 | 0 |
|  |  |  |  |

We see that choosing the preferred option is always a dominant strategy.

### 3.3 Three Citizens

For the two citizen population, there were only three scenarios that could have been presented to Citizen, namely the other citizen picking $A, B$, or $C$. Now we must explicitly list all of the outcomes that could be presented to Citizen when two other citizens are voting as well:
(1) Both pick $A$.
(2) One picks $A$, one picks $B$.
(3) One picks $A$, one picks $C$.
(4) Both pick $B$.
(5) One picks $B$, one picks $C$.
(6) Both pick $C$.

Let us now examine the outcome and payoff matrices for three citizens with three choices:

Two Other Citizens

Citizen

We are again interested in a sample from each type of preference: ordered, aversion, and affinity. First, ordered:

Two Other Citizens

Citizen

|  | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 1 | 1 | 1 | 0 | 0 | -1 |
| $B$ | 1 | 0 | 0 | 0 | 0 | -1 |
| C | 1 | 0 | -1 | 0 | -1 | -1 |

Aversion:

Affinity:

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We are once again unsurprised to find that picking the favorite is a weakly dominant strategy for both ordered and affinity preferences, and not picking the averse option is a weakly dominant strategy for an aversion preference.

## $3.4 \quad n$ Citizens

We are finally able to extend our analysis to model a single citizen's choice in a population of $n$, faced with three options.

The outcomes that could possibly be presented to Citizen are categorized into bins:
(1) Enough fellow citizens pick $A$ that $A$ will prevail, regardless of Citizen's choice.
(2) Enough fellow citizens pick $B$ that $B$ will prevail, regardless of Citizen's choice.
(3) Enough fellow citizens pick $C$ that $C$ will prevail, regardless of Citizen's choice.
(4) $A$ and $B$ are tied for the lead, $C$ is greater than one vote behind.
(5) $A$ and $C$ are tied for the lead, $B$ is greater than one vote behind.
(6) $B$ and $C$ are tied for the lead, $A$ is greater than one vote behind.
(7) $A$ and $B$ are tied for the lead, $C$ is one vote behind.
(8) $A$ and $C$ are tied for the lead, $B$ is one vote behind.
(9) $B$ and $C$ are tied for the lead, $A$ is one vote behind.
(10) $A$ leads $B$ and $C$ by one vote.
(11) $B$ leads $A$ and $C$ by one vote.
(12) $C$ leads $A$ and $B$ by one vote.
(13) $A$ leads $B$ by one vote, $C$ is behind.
(14) $A$ leads $C$ by one vote, $B$ is behind.
(15) $B$ leads $A$ by one vote, $C$ is behind.
(16) $B$ leads $C$ by one vote, $A$ is behind.
(17) $C$ leads $A$ by one vote, $B$ is behind.
(18) $C$ leads $B$ by one vote, $A$ is behind.

The outcome matrix yielded will be unwieldy, so we deviate from above notation, and simply write $\frac{1}{3}$ for boxes where vote is a three way split, and $.5 A B$, if the vote is split between two options, in this case $A$ and $B$ :
$n-1$ Other Citizens

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $A$ | $A$ | $B$ | C | A | A | . $5 B C$ | $A$ | A | $\frac{1}{3}$ | $A$ | . $5 A B$ | . $5 A C$ | A | A | . $5 A B$ | $B$ | . $5 A C$ | C |
| $B$ | $A$ | $B$ | C | $B$ | . $5 A C$ | B | $B$ | $\frac{1}{3}$ | $B$ | . $5 A B$ | $B$ | . $5 B C$ | . $5 A B$ | A | $B$ | B | C | . $5 B C$ |
| C | $A$ | B | C | . $5 A B$ | C | C | $\frac{1}{3}$ | C | C | . $5 A C$ | . 5 BC | C | A | . 5 AC | B | . $5 B C$ | C | C |

We analyze the payoffs for each type of preference, beginning with ordered:

|  | 1 | $n-1$ Other Citizens |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| $A$ | 1 | 0 | -1 | 1 | 1 | -. 5 | 1 | 1 | 0 | 1 | . 5 | 0 | 1 | 1 | . 5 | 0 | 0 | -1 |
| $B$ | 1 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | . 5 | 0 | -. 5 | . 5 | 1 | 0 | 0 | -1 | -. 5 |
| C | 1 | 0 | -1 | . 5 | -1 | -1 | 0 | -1 | -1 | 0 | -. 5 | -1 | 1 | 0 | 0 | -. 5 | -1 | -1 |

Aversion:

|  | 1 | $n-1$ Other Citizens |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| $A$ | 0 | 0 | -1 | 0 | 0 | -. 5 | 0 | 0 | $-\frac{1}{3}$ | 0 | 0 | -. 5 | 0 | 0 | 0 | 0 | -. 5 | -1 |
| $B$ | 0 | 0 | -1 | 0 | -. 5 | 0 | 0 | $-\frac{1}{3}$ | 0 | 0 | 0 | -. 5 | 0 | 0 | 0 | 0 | -1 | -. 5 |
| C | 0 | 0 | -1 | 0 | -1 | -1 | $-\frac{1}{3}$ | -1 | -1 | -. 5 | -. 5 | -1 | 0 | -. 5 | 0 | -. 5 | -1 | -1 |

Affinity:

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | $\frac{1}{3}$ | 1 | . 5 | . 5 | 1 | 1 | . 5 | 0 | . 5 | 0 |
| $B$ | 1 | 0 | 0 | 0 | . 5 | 0 | 0 | $\frac{1}{3}$ | 0 | . 5 | 0 | 0 | . 5 | 1 | 0 | 0 | 0 | 0 |
| $C$ | 1 | 0 | 0 | . 5 | 0 | 0 | $\frac{1}{3}$ | 0 | 0 | . 5 | 0 | 0 | 1 | . 5 | 0 | 0 | 0 | 0 |

These payoff matrices are perhaps more interesting. In looking at ordered preferences, it can be noted that simply choosing $A, B$, or $C$ is not a dominant strategy, but picking $C$ is a weakly dominated strategy, so $C$ should never be chosen.

Unsurprisingly, choosing $C$ is also a weakly dominated strategy if Citizen has an aversion preference for $C$.

Finally, for an affinity preference, it is always a weakly dominant strategy to choose the option for which Citizen has an affinity.

## 4 Incorporating Campaigning

Thus far, this analysis has considered only a citizen's ability to vote as a means of affecting the outcome of the electoral decisions. This simplification has allowed us to view each citizen as equally represented and pivotal in the outcome, and has limited the outcome space to the simple combinations of aggregate voting behavior. While the previous analysis will be useful in moving forward, it fails to provide any interesting insight, and furthermore fails to properly represent the real world, even as a theoretical model.

In reality, individuals can influence the outcome of an election through campaigning. In this context, campaigning is defined as any action through which
an individual incurs a cost in an effort to promote their preferences in any way other than merely voting. These costs may be pecuniary, non-pecuniary, or most likely any combination of the two. A pecuniary campaign includes using money to pay for furthering their preferred option, and non-pecuniary campaigning includes any expense of time or even damage to personal reputation or relationship. For example, attending a gala fundraiser to support an option is pecuniary, because the gala will require payment from attendees, but it is also non-pecuniary, as it will take time out of a citizen's day, and may strain familial or professional relationships.

A final important point to make is that this new modeling perspective will inherently mean unequal representation in the election. This disparity can easily be demonstrated by noting that distribution of resources is usually not equal, and therefore those with more resources will have more say in the outcome of the electoral decision. These resources are not merely financial however, and human capital can often be what gives individuals the most resources for campaigning. This principle applies to celebrities and any citizen whose choices are in the public eye, and can affect the aggregate decision. If a key leader in a society supports a particular option, many may be swayed to support her, and therefore the option. This effect strengthens with the popularity or perceived wisdom of the leader.

### 4.1 Remainder of the Population's Choice

It would not be pragmatic to try and assign preferences and payouts to the aggregate population, as this analysis would force us to say one choice is better or worse for the population than another, which is not what this analysis sets out to do. Instead, the remaining populace is seen as a player which will not alter its behavior based on Citizen's choice, but will move simultaneously. Thus, Citizen is playing a strategic game where they cannot predict the move of the population, but the population is merely making the move, without considering Citizen's choice.

In actuality, Citizen may have a good idea of how their fellow populace will vote, but we wish to consider all possibilities, so we begin by assuming Citizen has no prior knowledge of how the rest of the population will behave (other than of course knowing that they will certainly be presented with one of the ten options below, since the ten options are exhaustive). We defer a more discriminatory breakdown of outcomes until Section 6, as we wish to derive as general a result as possible, and only include restrictions if necessary to reach conclusions.

The population has ten possible options to give to Citizen. These options will appear quite similar to the options given to Citizen above in the pure voting model. However, the introduction of the concept of campaigning allows broader definitions of options 1 and $6-10$ below:

Notation: $A=B=C$ means that all the options are in a dead heat, and whichever one Citizen campaigns for will win. $A \geq B$ means that $A$ is marginally
winning, and if Citizen campaigns for $A$ or $C, A$ will win, but if Citizen campaigns for $B, B$ will win. $C \approx 0$ does not mean $C$ has no votes, but rather that $C$ is so far behind that Citizen's campaigning will not help.
(1) $A=B=C($ Dead heat $)$
(2) $B=C \approx 0$ (A triumphant)
(3) $A=C \approx 0$ (B triumphant)
(4) $A=B \approx 0$ (C triumphant)
(5) $A \geq B, C \approx 0$ (A ahead, C hopeless)
(6) $A \geq C, B \approx 0$ (A ahead, B hopeless)
(7) $B \geq A, C \approx 0$ (B ahead, C hopeless)
(8) $B \geq C, A \approx 0$ (B ahead, A hopeless)
(9) $C \geq A, B \approx 0$ (C ahead, B hopeless)
(10) $C \geq B, A \approx 0$ (C ahead, A hopeless)

### 4.2 The Matrix for Candidate Victory

The $3 \times 10$ outcome matrix will always yield the same results, and the population and Citizen will always have the same choices, regardless of preference, so the possible election outcomes will be universal across all preferences, and only utility of outcomes will change:

| Citizen | $n-1$ Other Citizens |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|  | A | A | $A$ | $B$ | C | $A$ | A | $A$ | $B$ | A | C |
|  | $B$ | $B$ | $A$ | $B$ | C | $B$ | A | $B$ | $B$ | C | $B$ |
|  | C | C | $A$ | $B$ | C | A | C | $B$ | C | C | C |

### 4.3 Split Campaigning

It is not immediately apparent why a citizen would choose to only campaign for a single option, especially when she may have an aversion preference. However, we are able to see that it would indeed be unwise for a citizen to ever split campaign support between two candidates, given they know the scenario presented to them by the remainder of the populace.

We first consider the canonical ordered preference $A>B>C$. For scenario 1 , supporting any two of the options may not ensure a victory for $A$, the most preferred option, and thus campaigning should not be split between two options. In any of scenarios $2-4$, campaigning is irrelevant to the outcome, so there is no reason a citizen would choose to campaign at all, let alone split their campaigning. In scenarios 5 and 6 , campaigning for any option other than $A$ will
only increase the chances of $B$ or $C$ prevailing, so only $A$ should be campaigned for. Again in scenario 7, campaigning for anything other than $A$, will only increase the chances of $B$ winning, whereas campaigning for $A$ only increases the chances of $A$ winning. Analogously in 8 , campaigning for $A$ will have no effect, and therefore should be abandoned, and splitting between $B$ and $C$ will only increase the chances of $C$ winning over $B$, whereas as campaigning solely for $B$ will ensure a $B$ victory. Lastly in 9 and 10 , supporting $C$ will increase the chances of the worst possible outcome, and campaigning for the option at approximately zero will have no effect, thus only the option marginally behind should be be allocated campaigning. We conclude that for ordered preferences split campaigning is never a dominant strategy.

Now we turn to aversion preference $A=B>C$, and again analyze each scenario. For scenario 1 , of course Citizen should not campaign for $C$, but splitting campaigning between $A$ and $B$ will only increase the chances that $C$ will surpass either of them, and therefore only one of $A$ or $B$ should be arbitrarily chosen and campaigned for. Campaigning is irrelevant for scenarios 2, 3, 4, 5, and 7. In scenario 6 , campaigning for $B$ is pointless, and therefore only $A$ should be supported. This same logic applies to 8,9 , and 10 , thus split campaigning is never a dominant strategy for an aversion preference.

Finally, we consider affinity preference $A>B=C$. For scenario 1, diverting any campaigning away from $A$ will only decrease the chances of $A$ winning. In scenarios 2,3 , and 4, campaigning is irrelevant. For scenarios 5, 6, 7, and 9, diverting campaigning away from $A$ only decreases the chances of $A$ winning, and for scenarios 8 and 10 , campaigning will be irrelevant to the utility of the outcome. Therefore splitting campaigning is never a dominant strategy for an affinity preference.

Now we have (somewhat tediously) proven that if a citizen were able to perfectly predict what scenario they would be given by the aggregate populace, not only would they have a complete strategy, but furthermore that strategy would not include split campaigning. Unfortunately, this result is rather weak, as we wish to generalize Citizen's strategic behavior as generally as possible, and most likely she will not have perfect information about what scenario she will be dealt, so we analyze her potential strategies as generally as possible, adding additional assumptions only when absolutely necessary to obtain results.

## 5 Preferences and Payoffs

Again, it becomes pertinent to clarify the reasoning in only analyzing the preferences of Citizen. Any individual citizen may campaign for $A, B$, or $C$, for any variety of ideological or ethical reasons. The preference of the remainder of the population, however, may favor one candidate over another in any of the ten scenarios listed above, but it is assumed that the population will make its choice without regard to what any specific citizen chooses. Thus, the population is essentially revealing and maximizing its utility through the option chosen, and the Citizen is left to campaign for options in such a way as to maximize their
own personal utility.
Due to the symmetry of the situation, there need not be analysis of all 13 preferences, but rather one analysis of each set of preferences, which will then hold for each unique preference. The first set will be preferences $1-6$, which are ordered preferences:

### 5.1 Ordered Preferences: Pref. 1 ( $A$ affinity, $C$ aversion)

| Citizen | $n-1$ Other Citizens |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|  | A | 1 | 1 | 0 | -1 | 1 | 1 | 1 | 0 | 1 | -1 |
|  | $B$ | 0 | 1 | 0 | -1 | 0 | 1 | 0 | 0 | -1 | 0 |
|  | C | -1 | 1 | 0 | -1 | 1 | -1 | 0 | -1 | -1 | -1 |

Regardless of the choice by the remainder of the population, choosing $C$ will be a weakly dominated strategy. So the payoff table can be simplified by removing $C$ :

Citizen

From this matrix, neither strategy is dominant for Citizen, and the game can only be further analyzed through extending assumptions about the remainder of population behavior, which is done in Section 6 .

### 5.2 Aversion Preferences: Pref. 7 ( $C$ aversion)

Citizen


Unsurprisingly, $C$ is again a weakly dominated strategy, so it can be removed:

Citizen

|  | $n-1$ Other Citizens |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| A | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | -1 |
| $B$ | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | -1 | 0 |

Again, neither strategy is dominant for Citizen, and the game can only be further analyzed through extending assumptions, as done in Section 6.

### 5.3 Affinity Preferences: Pref. 10 ( $A$ affinity)

Choosing $A$ is a weakly dominant strategy.

### 5.4 Indifference Preference: Pref. 13 (Indifference)

All strategies yield the same payoff, so strategy choice is irrelevant. The indifference preference is wholly uninteresting, and will not receive much attention in this analysis.

## 6 Extending Assumptions

Previously, this analysis has sought to remove weakly dominated strategies, and ascertain dominant strategies. For affinity preferences (Pref. 10-12) a weakly dominant strategy was found, and for the indifference preference (Pref. 13) strategy is irrelevant to payoff. However, for ordered preferences (Pref. 1-6) and aversion preferences (Pref. 7-9) a strategy cannot be determined without additional assumptions about the remainder of the population's choice.

A mixed strategy is unappealing for a few reasons. First, the game is only strategic from Citizen's perspective. If payoffs for the population were introduced, a mixed strategy might exist, but this would contradict the heretofore parameters of this analysis, and without a probabilistic knowledge of the population's choice, it is simply not possible to solve for a mixed strategy for Citizen. If, however, Citizen were able to ascertain the probabilities of each scenario being dealt, a mixed strategy might be a viable candidate. We will complete this analysis when relevant, but this concession will only further abstract our conclusions from reality, making them more academically interesting, but less useful in application.

Our purpose now becomes deriving the simplest assumptions that can be added so that a strategy will exist for ordered or aversion preferences.

### 6.1 Ordered Preferences

Below is the simplified matrix from above ordered preferences analysis:

| Citizen | $n-1$ Other Citizens |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|  | A | 1 | 1 | 0 | -1 | 1 | 1 | 1 | 0 | 1 | -1 |
|  | $B$ | 0 | 1 | 0 | -1 | 0 | 1 | 0 | 0 | -1 | 0 |

The aggregate population choices for which Citizen is indifferent between $A$ and $B$ can also be ignored, as they will not affect the strategy outcome. Here is the further simplified matrix:

| Citizen |  | $n-1$ Other Citizens |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 5 | 7 | 9 | 10 |
|  | A | 1 | 1 | 1 | 1 | -1 |
|  | $B$ | 0 | 0 | 0 | -1 | 0 |

The matrix above gives powerful insight which can be generalized due to the symmetry of the model. Additionally, we try to find a mixed strategy. We assign probabilities $p, q, r$, and $t$, to scenarios $1,5,7$, and 9 respectively, and assign probability $1-p-q-r-t$ to scenario 10 . Thus the expected utility of purely campaigning for $A$ is $2 p+2 q+2 r+2 t-1$, and the expected utility of purely campaigning for $B$ is $-t$. In a mixed strategy, Citizen will choose proportions of campaigning for each option such that the expected utility is constant. If we assign probability $a$ to $A$ and $1-a$ to $B$, we are left with

$$
a(2 p+2 q+2 r+2 t-1)=(1-a)(-t)
$$

Lemma 6.1. Let $I=[0,1) \in \mathbb{R}$, and let $p, q, r$, and $t$ each be in $I$ such that $p+q+r+t$ is also in $I$. Then $\exists a \in I$ such that $M=a(2 p+2 q+2 r+2 t-$ $1)=(1-a)(-t), M>2 p+2 q+2 r+2 t-1$, and $M>-t$ if and only if $(p+q+r+t) \in\left[0, \frac{1}{2}\right)$.

Proof. Start with the assumption $(p+q+r+t) \in\left[0, \frac{1}{2}\right)$.
Solving for $a$ in the equality expression shows: $a=\frac{-t}{2 p+2 q+2 r+t-1}=\frac{-t}{2(p+q+r)+t-1}$. Since $p, q$, and $r$ are symmetric in this expression, we group them together and make the change of variables $x=p+q+r$, so that $a=\frac{-t}{2 x+t-1}$. Optimizing $a$ with an unconstrained first order condition yields $t=0, x=\frac{1}{2}$, which is undefined and on the boundary of the open side of the interval in the assumption, but by L'Hopital's rule:

$$
\left.\lim _{x \rightarrow \frac{1}{2}} \frac{-t}{2 x+t-1}\right|_{t=0}=0
$$

So $a$ will be zero anywhere in the interval where $t=0$. Now, constraining along the same boundary, $x+t=\frac{1}{2}$,

$$
L(t, x, \lambda)=\frac{-t}{2 x+t-1}+\lambda\left(x+t-\frac{1}{2}\right)
$$

it is again found that $t=0, x=\frac{1}{2}$ is the only critical point. Lastly, we check the limit of $t$ approaching the boundary:

$$
\lim _{t \rightarrow \frac{1}{2}} \frac{-t}{2 x+t-1}=\left.\frac{-t}{t-1}\right|_{t=\frac{1}{2}}=1
$$

Thus $a \in[0,1)$, and $a$ satisfies the equality containing $M$. Finally, we note $2 p+2 q+2 r+2 t-1=2(p+q+r+t)-1<2(.5)-1=0$, and $-t<0$, thus multiplying either of these expressions by a scalar between 0 and 1 will increase their value, proving the inequalities containing $M$.

Now, begin with the assumptions that $\exists a \in I$ such that $M=a(2 p+2 q+$ $2 r+2 t-1)=(1-a)(-t), M>2 p+2 q+2 r+2 t-1$, and $M>-t$. Then directly we show:

$$
\begin{gathered}
a(2 p+2 q+2 r+2 t-1)>2 p+2 q+2 r+2 t-1 \\
\Rightarrow 2 p+2 q+2 r+2 t-1<0 \\
\Rightarrow p+q+r+t \in\left[0, \frac{1}{2}\right)
\end{gathered}
$$

Thus a mixed strategy will yield the highest expected utility if and only if the probability of $p+q+r+t$ is less than one-half, in which case the best strategy would be to pick $A$ with probability $a$, and $B$ with probability ( $1-a$ ). Again, this would be hard to ever apply in practice, unless Citizen were fairly confident about predicting probability $t$, and the sum of probabilities $p, q$, and $r$.

The final results for ordered preferences can be summarized as follows:

For a Citizen with ordered preference $A>B>C$ :
(1) With no information or belief about how the $n-1$ other citizens will vote, the strategy of $C$ is weakly dominated, and should never be chosen.
(2) If there is information or belief that the $n-1$ other citizens will NOT choose $C \geq B, A \approx 0$ ( C ahead, A hopeless), then A is a weakly dominant strategy.
(3) If there is information or belief that the $n-1$ other citizens will NOT choose ANY of $A=B=C$ (Dead heat), $A \geq B, C \approx 0$ (A ahead, C hopeless), $B \geq A, C \approx 0$ ( B ahead, C hopeless), $C \geq A, B \approx 0$ (C ahead, B hopeless), then B is a weakly dominant strategy.
(4) A mixed strategy is only sometimes defined, and when defined it would be difficult to apply unless Citizen were fairly certain about the probabilities of rest of the population's choices.

### 6.2 Aversion Preferences

Below is the simplified matrix from above aversion preferences analysis:

|  |  | $n-1$ Other Citizens |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Citizen | A | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | -1 |
|  | $B$ | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | -1 | 0 |

The aggregate population choices for which the Citizen is indifferent between $A$ and $B$ can also be ignored, as they will not affect the strategy outcome. Here is the further simplified matrix:

$$
\begin{aligned}
& \text { n-1 Other Citizens } \\
& \text { Citizen }
\end{aligned}
$$

Again, the matrix above gives powerful insight which can generalized due to the symmetry of the model. A mixed strategy leads to a more interesting result here. We assign relative probabilities $p$ and $1-p$ to scenarios 9 and 10 respectively. Purely campaigning for $A$ yields expected utility of $p-1$, and for $B$ yields $-p$. A mixed strategy will imply

$$
\begin{gathered}
a(p-1)=(1-a)(-p) \\
\Rightarrow a=p,(1-a)=(1-p)
\end{gathered}
$$

We also can note that because $0 \leq p \leq 1$, we have $p(p-1) \geq p-1$ and $p(p-1) \geq-p$, thus the expected utility of this mixed strategy is greater than or equal to the expected value of each pure strategy, for all $p$ !

The final results for aversion preferences can be summarized as follows:
For a Citizen with aversion preference $A=B>C$ :
(1) With no information or belief about how the $n-1$ other citizens will vote, the strategy of $C$ is weakly dominated, and should never be chosen.
(2) If there is information or belief that the $n-1$ other citizens will NOT choose $C \geq B, A \approx 0$ (C ahead, A hopeless), then A is a weakly dominant strategy.
(3) If there is information or belief that the $n-1$ other citizens will NOT choose $C \geq A, B \approx 0$ ( C ahead, B hopeless), then B is a weakly dominant strategy.
(4) If the relative probabilities of scenarios 9 and 10 occurring were known, with probability of 9 occurring $p$, it would be a weakly dominant strategy to choose $A$ with probability $p$.

## 7 Summary of Strategic Results

For any given preference and amount of information, the analysis presented above has proven the definitive existence or nonexistence of a strategy, depending on the combination. These results can be summarized:

For a Citizen with ordered preference $A>B>C$ :
(1) With no information or belief about how the $n-1$ other citizens will vote, the strategy of $C$ is weakly dominated, and should never be chosen.
(2) If there is information or belief that the $n-1$ other citizens will NOT choose $C \geq B, A \approx 0$ (C ahead, A hopeless), then A is a weakly dominant strategy.
(3) If there is information or belief that the $n-1$ other citizens will NOT choose ANY of $A=B=C$ (Dead heat), $A \geq B, C \approx 0$ (A ahead, C hopeless), $B \geq A, C \approx 0$ ( B ahead, C hopeless), $C \geq A, B \approx 0$ (C ahead, B hopeless), then B is a weakly dominant strategy.
(4) If we assign relative probabilities $p, q, r$, and $t$, to scenarios $1,5,7$, and 9 respectively, assign relative probability $1-p-q-r-t$ to scenario 10 , and Citizen estimates $p+q+r+t<\frac{1}{2}$, then Citizen should choose $A$ with probability $\frac{-t}{2 p+2 q+2 r+t-1}$.

For a Citizen with aversion preference $A=B>C$ :
(1) With no information or belief about how the $n-1$ other citizens will vote, the strategy of $C$ is weakly dominated, and should never be chosen.
(2) If there is information or belief that the $n-1$ other citizens will NOT choose $C \geq B, A \approx 0$ ( C ahead, A hopeless), then A is a weakly dominant strategy.
(3) If there is information or belief that the $n-1$ other citizens will NOT choose $C \geq A, B \approx 0$ ( C ahead, B hopeless), then B is a weakly dominant strategy.
(4) If the relative probabilities of scenarios 9 and 10 occurring were known, with probability of 9 occurring $p$, it would be a weakly dominant strategy to choose $A$ with probability $p$.

For a Citizen with affinity preference $A>B=C$ :
(1) Choosing A is always a weakly dominant strategy.

For a Citizen with the indifference preference $A=B=C$ :
(1) Choice of strategy is irrelevant to payoff.

## 8 Applications

This section seeks to demonstrate the strength of these results in practical application to the United States 2016 Presidential Election. Three theoretical citizen situations are examined: current President Barack Obama, Louise Mensch, and Penn Jillette.

### 8.1 President Barack Obama

President Obama has vocally endorsed Democratic nominee Hillary Clinton (Bradner, 2016), opposed Republican nominee Donald Trump (Liptak and Collinson, 2016), and thus far remained silent on libertarian candidate Gary Johnson. Therefore, his preferences may reasonably be assumed to be $A>B>C$, where $A$ is Clinton, $B$ is Johnson, and $C$ is Trump. Since his preference is ordered, he should not vote for Trump under any circumstances.

Obama may also make some additional assumptions about U.S. voting behavior. For the sake of this analysis, the president has little faith in Johnson's chances, so rules out options 1 (Dead heat), 3 (Johnson triumphant), 5 (Clinton ahead, Trump hopeless), 7 (Johnson ahead, Trump hopeless), 8 (Johnson ahead, Clinton hopeless), and 10 (Trump ahead, Clinton hopeless). President Obama's simplified matrix is then:


Thus Obama should always support Clinton under these preferences and assumptions.

### 8.2 Louise Mensch

Louise Mensch is a social media user and member of the \#NeverTrump movement (MSNBC, 2016), which aims to prevent Donald Trump from attaining the presidency. While Mensch may also have a preference between Johnson and Clinton, the argument presented here will make the assumption that she is indifferent between the two, and therefore has aversion preference $A=B>C$, where $A$ is Clinton, $B$ is Johnson, and $C$ is Trump. Immediately, the strategy of supporting Trump can be eliminated.

Perhaps Mensch has a slightly different set of assumptions about United States behavior than Obama, and she rules out options 3 (Johnson triumphant), 5 (Clinton ahead, Trump hopeless), 7 (Johnson ahead, Trump hopeless), and 10 (Trump ahead, Clinton hopeless). Louise Mensch then has a simpified matrix:


Although the preferences and assumptions of Mensch differed from President Obama, she too should support Clinton.

### 8.3 Penn Jillette

Renowned magician Penn Jillette is also known for his political beliefs, strongly emphasizing the importance of personal liberties. Additionally, he has publicly stated that he supports Gary Johnson, and strongly dislikes both Clinton and Trump (Burningham, 2016). Although he has said he would support Clinton over Trump, for the sake of example this analysis assumes he is indifferent between the two, and therefore has preference $A>B=C$, where $A$ is Johnson, $B$ is Clinton, and $C$ is Trump. Without any further analysis, it can be deduced from above reasoning that this version of Jillette should vote for Johnson, regardless of his assumptions about the United States' choice.

## 9 Conclusion

The analysis presented above achieves the goal of constructing an algorithmic and logical approach to supporting a candidate in an election where three viable candidates are considered. The two parameters that must be considered are preferences of the Citizen, and choice of the United States, or simply the option that is ultimately given to the Citizen.

If a Citizen were able to perfectly deduce what the United States' choice would be, the Citizen would always simply pick the candidate which yields the highest payoff to the Citizen. However, in most real applications, a Citizen will never know for sure what decision they will be presented with, and must thereby
determine what set of decisions they believe will plausibly be given to them. This analysis proves that there need not be perfect information for a Citizen to eliminate supporting one candidate as a weakly dominated strategy, and in some cases enough eliminations can be made so that supporting a particular candidate is a weakly dominant strategy, with at least some probability.

As is always the case, the ideas presented here could be expanded further. Our key goal was to introduce and construct, in the most logically elegant way, a groundwork theoretical model for both voting and campaigning from an individual strategic perspective. We chose to expand our model to the case with $n$ citizens and three options, but of course further analysis could seek models for 4,5 , or potentially even $n$ options.

Additionally, we have not fully explored the range of the individual citizen's support possibilities. We here assumed that citizens could only campaign in support of options, and we thus neglected the opportunity for campaigning against options. Furthermore, the possibility of collusion was not discussed and therefore assumed irrelevant, which may be a fair assumption for a "large" population, such as the aggregate United States, but may break down with smaller populations.

## 10 Bibliography

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