Comparative Analysis of Simultaneous Ascending Auctions and Combinatorial Auctions

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1 Introduction

In 1994, the United States Federal Communications Commission (FCC) began auctioning off blocks of spectrum to telecommunications companies to be used in emerging wireless telephone markets. Shortly after, the United Kingdom held the first European spectrum auction in 2000, distributing licenses for the up and coming third generation (3G) mobile telephone market. Other European countries followed, including Germany, France, the Netherlands, and Denmark, among others. Many of these auctions were successful, raising large revenues for the respective governments, allocating licenses to the firms who valued them the most, and creating an efficient post-auction market between winning firms. However, this was not the case for every auction, and it is the choice of auction design that set the failures apart from the successes. This paper seeks to analyze the different auction types that were chosen and bring to light their advantages and disadvantages.

In the past couple decades, many different auction designs have been considered and utilized in distributing spectrum. These include, but are not limited to, simultaneous ascending multi-round auctions, combinatorial (or "package bidding") auctions, combinatorial clock auctions, as well as others. Here we will focus on the first two. A simultaneous ascending multi-round (SMR) auction is one in which there are numerous items up for sale with multiple bidders. The auction is conducted in rounds, and bidders are typically able to bid on a set number of items individually within a given round. The auction ends if a round passes in which no new bids are placed, and the items are allocated to the highest bidder for each item. Combinatorial auctions are also conducted in rounds, but bidders are allowed to bid on packages of items rather than being restricted to bidding on individual items. Similar to the SMR auction, the auction ends when a round passes in which no new bids are placed, and the items are allocated in such a way that the particular packages chosen maximize the seller's revenue.

The two main goals of auction design are typically efficiency and revenue maximization (Klemperer). However, it is not always clear whether the focus of an auction should lie more heavily in the former or the latter. Even less clear is which auction type will achieve the outcome that the auction designers are trying to achieve. Every auction scenario is different, and should be tailored to fit the scenario. In Section 2, formal yet simple models of the auction designs are presented in order to make clear the strategies of the bidders and the logic behind using these particular designs. Section 3 compares the two auction designs in terms of efficiency and revenue, and discusses when each respective auction obtains better results than the other. Section 4 analyzes the use of the auction designs in practice. Specifically, the UK 3G spectrum auction and the 2008 US 700 Mhz band auction (Auction 73) are considered, and we apply some of the results of the previous sections to the outcomes of these auctions. Section 5 concludes.

2 The Auctions and Their Models

As stated before, the goal of any auction is to tailor the design to fit the scenario [1]. Though both SMR and combinatorial auctions are relatively efficient and bring in sizeable revenues for the government, bidders behave differently in each auction. Differences in the outcomes can be traced back to what the auction is designed to accomplished. In some cases, the combinatorial auction yields more efficient results and collects significantly higher revenues than an SMR auction would. In other cases, the SMR auction trumps the combinatorial auction in its outcomes. We begin by looking at the combinatorial auction, followed by the SMR auction.

2.1 The Combinatorial Auction Model

When auctioning off a large number of items, there are often complementarities between items. This can cause valuations on packages of items to be higher than the sum of the values of the individual items. In the context of spectrum auctions, it is more profitable for a firm to operate in areas that are nearby each other. Therefore, when licenses are split between geographic regions, firms would prefer to purchase licenses that are close to each other. This allows them to spend less money on capital and infrastructure build out, and in turn realize economies of scale, making consumers better off.

We present here part of Rothkopf's model [2], and report some of the consequential theorems that follow. We assume there are n licenses being auctioned, and we denote the set of these licenses by A. Let $C \subset A$ be a permittable combination, and let $\mathcal{P}$ be the set of all permittable combinations, i.e., $\mathcal{P} = \{C \subset A : C \text{ is permittable }\}$. Since not all combinations are permittable, the cardinality of $\mathcal{P}$ must be less than or equal to $2^{|A|}$, where equality occurs when the combinations include all possible licenses. A combination is deemed "permittable" when it satisfies a given set of qualities. Often, these qualities will differ between auctions, such as combinations being restricted to geographical location or restricted in number. These different types will be discussed later in this section.

In any combinatorial auction, resulting outcomes must not have any overlap. This is clear, since it would not make sense for two firms to operate under the same license. However, this is where the auction becomes difficult to analyze. Often there are multiple packages that contain at least one license in common, and thus it is not clear who should be the one to win the auction. To begin the analysis of this issue, we let $\mathcal{W}$ be a set of permittable packages, or an *outcome*, and define

$$\Omega_{\mathcal{P}} = \{\mathcal{W} \subset \mathcal{P} : C_1, C_2 \in \mathcal{W} \Rightarrow C_1 \cap C_2 = \emptyset\}.$$

where $\Omega_{\mathcal{P}}$ is the set of all possible outcomes.

In this model, we assume that the goal of the auction is to maximize revenue. Though this is not always the case in practice, larger revenues are typically associated with an efficient allocation of the items, as it points towards who values the items the most. Extending the model to meet this goal, let $b(C)$ be the highest bid on the package C. By convention, $b(C) = 0$ if there are no bids on that particular package. If $\mathcal{W}$ is an outcome, define

$$\text{rev}(\mathcal{W}) = \sum_{C \in \mathcal{W}} b(C)$$

as the revenue received by the seller of the licenses. By our assumption, the seller must try and find some optimal outcome such that revenue is maximized. Formally, if $\mathcal{W}_{opt}$ is the optimal outcome, the seller seeks to determine

$$\mathcal{W}_{opt} = \max\{\text{rev}(\mathcal{W}) : \mathcal{W} \in \Omega_{\mathcal{P}}\}.$$

To do so, the seller must solve a linear programming problem, namely

$$\max \sum_{C \in \mathcal{P}} b(C) x_C$$

subject to

$$\begin{aligned} \forall C \in \mathcal{P}, &\qquad x_C \in \{0, 1\} \\ \forall i \in A, &\qquad \sum_{C \ni i} x_c \leq 1 \end{aligned}$$

where x_C are indicator variables ($x_C = 1$ if and only if C is contained in a particular outcome). The first constraint ensures that each license in an outcome is accounted for, and the ones that are not vanish. The second constraint restricts each license to only one permittable combination C in an outcome. For instance, if license i were contained in two separate combinations $C, C' \in \mathcal{W}$, there would be no way to determine who gets the license. The programming problem determines which combinations will maximize the revenue received, given the highest bids on each combination.

After defining the maximization problem, the seller must decide what rules he wants to implement in order to make the problem solvable in polynomial time. Though the simplex method does not always guarentee this, there exist particular sets of rules and algorithms that restrict the number of packages to a small enough number that do, which are shown in [2]. For instance, tailoring the rules to only allow combinations with synergetic (or closeby) licenses will significantly reduce the overall number of potential outcomes, and will aid in making the auction computationally manageable. This method was used by the United States in Auction 73 for a particular section of the 700 MHz band, which will be discussed in Section 5.

In addition to making the auction computationally manageable for the seller, it is important to also make it computationally manageable for the bidders. The problem faced by the bidders is known as the winning bid problem. The challenge is to find a minimal bid on a permitted combination C such that $b_{min}(C)$ is the smallest bid with C becoming a winning combination, assuming that all $C' \neq C \in \mathcal{P}$ remain unchanged.

In order to ensure that the licenses will in fact be allocated efficiently, it is important for the bidders to be able to be certain that their bids truly reflect their valuations. Should bidders be unable to compute these bids in a reasonable amount of time, it could lead to overbidding (the winner's curse) or drastic underbidding, in which case bidders with lower valuations may end up with the licenses, creating an inefficiency. Specific bidding strategies will not be discussed here, as the number and complexity of these strategies lie beyond the scope of this paper. The following proposition allows us to focus on the auction design from the seller's perspective without neglecting the bidder behaviour completely [2].

Proposition 1. *If the problem of finding revenue maximizing outcome is solvable in polynomial time, then, given $C \in \mathcal{P}$, the minimum winning bid problem is also solveable in polynomial time.*

There are three main design structures that will be discussed. A *nested tree structure* begins with a fixed number of mutually exclusive areas, where packages can only be assembled within their respective regions. Bids may be placed on any number of licenses within a specific region, including a package that contains the region as a whole. Assume there are K regions, with m_k licenses available for each of the k regions, and let $b(C)$ be the highest winning bid on each package such that $\forall C, C' \in \mathcal{P}_k$, where $\mathcal{P}_k$ is the set of all possible combinations in region k, $C' \cap C = \emptyset$. Also, let B_k be the highest bid on the region as a whole. The winning combination of bids in region k will then be

$$\max\left\{B_k, \sum_{C\in\mathcal{P}_k} b(C)\right\}$$

Rothkopf presents the algorithm for tree structures is presented in Appendix B [2], and proves that tree structures can be solved in $O(n^2)$ time (or, in other words, in a time proportional to a polynomial of degree 2 with the argument being the number of licenses). However, they may not be optimal to utilize when synergies lie outside the confinements of each area, even if they are relatively close. When this is the case, it may be preferable to use a *geometric structure*. These structures are useful when complementarities exist between licenses that are geographically adjacent. If there is a line connecting the regions, with each one given an integer label, then a package can only consist of integers that are all contained within a closed interval. To give a concrete example, imagine that California, Arizona, and New Mexico are three geographic regions, labeled 1, 2, and 3 respectively. Then the packages $\{1,2\},\{2,3\}$ and $\{1,2,3\}$ would be allowed, as well as all singletons, but the package $\{1,3\}$ would not be since California and New Mexico (and hence, the numbers 1 and 3) do not share a border. Restricting package bids to have this geometric structure allows the problem to be solved in $O(n^2)$ time, which is also shown in Appendix B of Rothkopf [2].

The last structure emphasizes the number of items in each package. This structure is less interesting, as there is a fine line between the problem being computationally manageable and being NP-complete (not solvable in polynomial time). This is formalized in the following theorem [2].

Theorem 1. *If $\mathcal{P} \subset \{C \subset A : |C| \leq 2\}$ then $\mathcal{W}_{opt}$ can be determined in $O(n^3)$ time. If $\mathcal{P} \subset \{C \subset A : |C| \leq 3\}$, then finding $\mathcal{W}_{opt}$ is NP-complete.*

Since with a large number of licenses, it is typically optimal to include more than 2 in a package, this structure is not of much help.

2.2 The Simultaneous Ascending Auction Model

When complementarities between items are small, or when bidders are only interested in a small number of items, the simultaneous ascending auction may be used instead of the combinatorial auction. In theoretical models, bidders may be allowed to bid on any number of items that are being auctioned, as this is efficient in terms of price discovery. In practice, however, this number is often limited in order to create competition in the post auction market [3]. Here we consider only the extreme scenarios, namely when demand for items is unrestricted, and when it is restricted to a single item.

2.2.1 Unrestricted Demand for Items

To begin the model, assume again that there are n items being auctioned, and that there are $k > n$ bidders. Each has a valuation vector $\mathbf{v}_k = (v_1, \ldots, v_n)^t$, where v_i is the bidder's valuation for the i^{th} item. We assume the values are nonnegative, and are 0 if the bidder has no interest in the item. Let $\mathbf{p}_t = (p_{1,t}, \ldots, p_{n,t})^t$ be the vector consisting of the price of each item in round t of bidding. Each bidder has a strictly increasing vector-valued bidding function $\mathbf{b}_{k,t}(\mathbf{v}_k) = (b_{1,k,t}(v_1), \ldots, b_{n,k,t}(v_n))^t$ corresponding to their bids on each item, where $b_{i,k,t}(v_i) > p_{i,t}$ for any time t. We let $b_i(v_i) = 0$ when $v_i = 0$, and if $v_i - p_{i,t} \leq 0$ for any item and at any time t, we set $b(v_{i,t}) = 0$, as bidding a positive amount over the current price results in negative profits for the bidder (assuming there does not exist synergy between the items).

If a bidder has the high bid on any item at time t, we assume that their bid on that item in that time period is 0, as it would not make sense to bid again over their previous bid. Given these assumptions, the bidder must then try to maximize the payoff function

$$\pi(\mathbf{v}_k) = \sum_{k=1}^{n} \mathbf{v}_k - \mathbf{b}_{k,t}(\mathbf{v}_k)$$

for every time period. In the absence of minimal bid increments, it is then optimal for each bidder to bid as small of an amount possible on each item that they do not currently have the high bid on, as this will be what maximizes the expression above. In doing so, this creates a Walrasian pricing process that leads to a competitive equilibrium between bidders [3]. In practice, it is standard to implement minimum bid increments in each round in order to speed up the auction process. Though this does not lead to a perfect pricing environment, it can be quite close if bidders consistently bid the minimum bid increment in each round.

2.2.2 Restriction to Unitary Demand

The analysis of the SMR auction when bidders are only allowed to bid on a single item is not much different than the previous scenario. An auction practitioneer may decide to limit the bidders to this case whenever the number of items are sufficiently small. An example of this will be discussed in Section 5. To determine optimal bidding behaviour of the bidders, we simplify the previous maximization problem to finding, trivially,

$$\max_t \{v_1 - b_{1,t}(v_1), \ldots, v_n - b_{n,t}(v_n)\} \text{ for all } t.$$

When minimum bid increments are imposed, the bidder must find the greatest difference between each of his valuations and the current minimum bids allowed on each item. Note that if a bidder has the high bid at any given time, they would not be better off by placing a bid on some other object. In the previous time period we can assume they solved the maximization problem and bid accordingly. Then inductively, since bids are monotone increasing, it follows that

$$\max_{t-1} \{v_j - b_{j,t-1}\} \geq v_i - b_{i,t-1} \geq v_i - b_{i,t}$$

for any i and t, where we assume $b_{j,0} = 0$. Therefore we conclude that the bidder is content to maintain his current position.

3 Comparison of Auction Designs

As stated above, two of the most important indicators of a good auction design are their resulting efficiency and the revenue they bring in. Though the two often correlate relatively well, there are instances in which one auction design will achieve one of the goals but not the other while the second auction design achieves both. We will first discuss how each auction fares in terms of efficiency and revenue separately, and then compare the auctions together when both are being considered.

3.1 Efficiency

An allocation of a set of items is said to be efficient when they are successfully paired with the bidders who value them the most. Auctions have had a large amount of success in creating efficiency. This is due to the fact that bids are typically cast as a function of the valuations of the items to the bidders. Since it is always optimal for bidders with higher valuations to bid larger amounts that other bidders with lower valuations, the high value bidders often end up with the items. Therefore an auction design that encourages bidders to reveal their true values through their bids will lead to the most efficient outcome.

Another measure of efficiency that should be considered is what happens after the auction, assuming that the items are allocated efficiently in the auction setting. In situations where items are consumed by individuals with no externalities, the efficiency is more appropriately measured during the auction. However, if the use of the items does have a noticable effect on society, the allocation of items should also support efficiency in the post-auction market. This can be applied to a number of examples, including spectrum auctions, oil leasing auctions, etc. Ideally, the auction should promote a competitive market in order to maximize social gains.

In terms of spectrum auctions, the licenses should be allocated such that the 3G (or any other generation) mobile phone market is less concentrated, incentivizing firms to compete via lower prices and higher levels of innovation. We will look at the efficiency of these auctions in both situations.

3.1.1 Efficiency in the Auction Market

In the absence of complementarities between items, or at least when these complementarities are small, the SMR auction typically achieves this efficiency as shown above. The ability to easily determine what bid will maximize revenue leads us to this conclusion. On the other hand, in a combinatorial auction, this can be a challenge to bidders. When the number of items is sufficiently large, the optimal bid may not be found with complete certainty, and in fact may not even be computable depending on the auction rules. Additionally, bidders experience the threshold problem, where the efficient allocation is not met due to the nature of package bidding [4]. To see why, consider the following example. Suppose A and B are two spectrum licenses, and there are two bidders. Bidder 1 has the valuation vector $\mathbf{v}_1 = (100, 85)^t$ and bidder 2's vector is $\mathbf{v}_2 = (90, 90)^t$. The optimal allocation is clearly for A to go to bidder 1 and B to go to bidder 2. However, if bidder 1 bids over \$180 on the package with both licenses, this will outweigh bidder 2's bid on either individual license, and bidder 1 will receive both licenses resulting in an inefficiency.

When licenses are complementary, the situation changes. In this case, bidders can experience what is known as the exposure problem [4]. This problem arises when there exists "synergy" between licenses, meaning that the bidders value combinations of licenses more than the sum of the individual licenses themselves. Because bidders are only allowed to submit bids on individual licenses in SMR auctions, this can lead to the participants encountering this problem. It is possible for one bidder, say bidder 1, to demand a single license, while another, bidder 2, would prefer that license in addition to other complementary licenses. The final allocation might cause bidder 2 to overbid for the licenses he actually wins, since without the last license, the synergy disappears. Consider our previous example. Suppose bidder 2 has synergy between licenses A

and B, so that his valuations on each license is the same but he values the package at 190. Since he can only bid on the licenses individually, in order to obtain both items, he must pay up to 190 split in some way between the items. For simplicity, assume he bids 95 on each license. Since no synergy between the licenses exists for bidder 1, he will bid up to his valuation on each individual license. Then the resulting allocation gives A to bidder 1 and B to bidder 2, but bidder 2 experiences negative profits since he overpaid, expecting to receive both licneses. The efficient allocation would have been for both licenses to go to bidder 2, since his valuation of the two licenses together is greater than the sum of the resulting valuations.

Using a combinatorial auction can help to rid the participants of the exposure problem. Had package bidding been allowed in our example, bidder 2 could have bid anywhere between 185 and 190 and won both licenses with a nonnegative profit, which would have been efficient.

3.1.2 Efficiency in the Post-Auction Market

Though items may be allocated to bidders with the highest valuation, this does not necessarily mean that the market directly created from this allocation will be socially efficient. When selling spectrum licenses, an auction should avoid distributing them to a small number of bidders. This will prevent the market power of any individual firm from being too large, and the resulting competition will lead to lower prices for consumers accessing the network. In this sense, it is important for the auction to encourage entry. When licenses are allocated as such, it will induce competition between the firms, leading to lower prices. This will likely encourage firms to innovate at a faster pace in order to overcome the higher level of competition, making consumers of mobile phone industries better off [3].

On the other hand, allocating licenses to smaller firms may reduce the market power of incumbents, but may put the entrants in an unstable position depending on the amount spent on the licenses. Though the licenses are typically viewed as a sunk cost to the new firms, the cost of building infrastructure, building a network, and actually competing with existing technology may be overwelming. In markets where incumbent firms are few and have large market shares, this leaves little room for the new firms to operate and make enough revenue to sustain these initial buildout costs [3]. Additionally, it is possible that assigning licenses to non-incumbent firms may actually inhibit innovation. As incumbent firms have adapted to the cost structure of the industry, they are more likely to be able to absorb the costs of the research and development that goes into innovating new products than a new firm [5]. It follows from this that, since prices tend to fall when the number of firms in an industry increases, the profits gained by the incumbent firm will also fall and reduce their ability to invest in development.

With these considerations in mind, we turn to the individual auction designs. The SMR auction, as will be seen in the next section, does well in assigning licenses to entrant firms when the rules of the auction are specifically designed to do so. Depending on the number of licenses being offered, this can result in many or few firms gaining licenses, depending on the pre-existing market and the decisions of the seller. When the number of licenses outnumber incumbent firms by a small amount, however, it may lead an entrant firm to overpay for the license. This is what is expected to have happened in the UK 3G auction in 2000 [6]. The entrant firm, TIW, paid roughly £4.39 billion (≈ $7.09 billion in 2000 pounds and dollar amounts) for a license, though most estimates both before and after the auction projected the value of the license at about £2.6 billion [6]. Despite this, it can be argued that this happened due to the UK auctions being the first in Europe to occur. This may have caused firms to overestimate the values of the licenses. TIW was then bought out by Hutchison, a large Hong Kong based company, relatively soon after the end of the auction. This was in part due to the two companies forming the joint venture "the MVN Operator", with Hutchison owning roughly 90 percent of the company [7]. However, TIW also state on their website that they were "in a consolidated market dominated by a small number of big players and very few new investment opportunities" [8], suggesting that perhaps this auction design was not entirely efficient.

This outcome may also be a result of the number of licenses and the structure of the industry that was being created. Having a small number of licenses almost guarentees a more concentrated market, and hence makes it inherently more difficult for new firms to adjust to their new market position properly. Increasing the number of licenses for sale may produce an entirely different outcome, and may even attract more entry to the auction itself. In this sense, it could be unfair to place the blame entirely on the SMR auction design, as a combinatorial auction could easily have had the same result. However, in a combinatorial auction, the ability to win more than one license and to potentially win complementary packages of licenses helps to lower future costs, which may help new firms in their starting years in the industry.

3.2 Revenue

It is important to encourage entry to auctions not only to make the allocation efficient, but also to help create a more efficient and competitive market that follows. Entry is also a very important factor in determining the revenue brought in by the auction. In an indirect sense, these two issues are related in that higher revenues typically correspond to higher valuations for items. If designed poorly, however, the revenue gained from auctions can be very small and will often indicate low efficiency [1].

When the number of licenses being offered outnumber the incumbent firms, it creates a competitive auction in which firms find it optimal to bid closer to their true values. The multi-round aspect of the SMR auction is partially responsible for this, as firms are consistently given new information on the valuations of other firms and can adjust appropriately. Additionally, this rids the firm of the uncertainty faced in single round auctions, as they do not have to account for how others will bid within a specific time frame, but rather how they will bid as the auction progresses. A firm loses nothing by bidding the smallest amount, as if they do not have the high bid come the next round, they can simply continue doing so until the current high bid on any license is greater than their valuation. This raises prices to levels associated with firms' valuations, and thus creates an efficient pricing process. The same can be said about combinatorial auctions, though the pattern of which licenses to bid on is usually not as clear.

This outcome is unlikely to result when the number of licenses is less than or equal to the number of incumbent firms. It suffices here to only consider the case where the numbers are equal, as restricting the number of licenses to be less than the number of incumbents may result in a competitive bidding process, but would result in a more concentrated industry, which largely defeats the purpose of the auction. Here, firms have an incentive to bid up to the reserve prices of the licenses and then stop, as once each firm is the high bidder of at least one license, the payoffs of bidding on more will likely be small. This has a negative effect on entry to the auction, as smaller firms will find participation unattractive if they do not think they can outbid any of the incumbents. It is difficult to compare the SMR and combinatorial auctions in this case, as the problem stems from an issue independent of the specifics of each auction design.

Collusion is another major issue to consider in either auction. In this case, the combinatorial auction seems a more likely candidate to combat collusion than the SMR auction. This is largely due to the complexity of the auction, since it is difficult enough for firms to bid optimally in a reasonable amount of time, let alone collude with several other firms that are trying to do the same thing. In an SMR auction, on the other hand, bidding is much simpler and signals can be used to convey messages between firms without having to worry about the computational difficulties that arise in combinatorial auctions. In this sense, combinatorial auctions are more likely to generate higher revenues than the SMR auction.

Overall, it is difficult to compare the revenues brought in by both auctions. There are many factors that contribute to the amount of revenue received, and there are often not enough parallels between these auctions in practice to properly compare them. However, laboratory experiments conducted by the FCC shed a little light on the matter [9]. Carried out in 2006, these experiments randomly assigned license values to participants and observed their bidding behaviour in both combinatorial and non-combinatorial auction settings. The results of the experiments confirmed many of the theoretical implications of the auctions discussed so far. Specifically, it was found that combinatorial auctions distributed licenses more efficiently than the SMR auction in the midst of complementarities, but less so when these complementarities were small. Additionally, the combinatorial auction resulted in higher revenues in situations where participants experienced partially overlapping complementarities between items, but were lower when the complementarities overlapped fully. With these results in mind, we will look at specific auctions and determine whether or not these implications hold in practice.

4 Applications

In this section, we will look at a few different auctions and analyze the results in terms of efficiency and revenue. First, we take a look at what defines spectrum and characterize it. Doing so gives insight as to why firms may potentially bid how they do. Our analysis of the auction designs begins with the European 3G auctions. The success of the UK will specifically be considered, followed by a discussion of Switzerland's poor auction results and what they did wrong. After that we turn to Auction 73 in the United States, where it was held partially as an SMR auction and partially as a combinatorial auction.

4.1 Spectrum

Before discussing auction designs, it is important to understand exactly what spectrum is and how it is used. Radio signals, which can be used for a variety of telecommunication purposes including broadband television, mobile telephone devices, and internet, are sent and received on specific wavelengths corresponding to different frequencies. For instance, a signal sent out at a frequency of 700 Megahertz can only be received on wavelengths close by. Therefore, it is in a firm's best interest to acquire different blocks of spectrum that are relatively close to one another in order to reduce operating costs.

Spectrum can be used in "paired" or "unpaired" bands. Paired spectrum consists of two different blocks, which allows for signals to be sent and received on different wavelengths. This is known as FDD-LTE. In contrast, firms operating on unpaired spectrum have to use the same spectrum to send and receive signals. Unpaired spectrum is typically less valuable to firms for this reason.

Licenses for spectrum can be concentrated on specific geographical areas or across a nation as a whole. US licenses usually consist of the former type, where there are over a hundred different "economic areas" that different licenses cover (FCC website). This allows the industry to be less concentrated, and allows smaller firms to operate in less populous areas without having to defend themselves against industry giants.

4.2 The European 3G Auctions

After witnessing the success of the US spectrum auctions, which began taking place in 1994, Europe decided to hold spectrum auctions of their own beginning in the year 2000. The specifics of these auctions are covered extensively in [1], which is the basis of the following discussion. The UK was the first of the European countries to hold an auction, which likely contributed to its overwhelming success and unexpectedly high revenues. The pre-existing 2G market in the UK consisted of four incumbents - Orange, Vodafone, One-2-One, and Cellnet. It was expected that these four firms would inevitably win a license in the upcoming 3G spectrum auction, so to encourage entry and garner a competitive auction environment, it was decided that the auction would be for 5 licenses of varying size. The auction designers decided to go with a simultaneous ascending design, with restriction to unitary demand for licenses, as they felt it would "[insulate them] against the problems with collusion that arose in America" [1].

In order to further attract new firms, the largest of these licenses, A, was exclusively reserved for new entrants. It consisted of 2×15 Mhz blocks of paired spectrum, and one 5 Mhz block of unpaired spectrum. In contrast, the second largest license did not include the unpaired block, and the remaining three licenses (which were all of equal size) had only 2×10 Mhz blocks of paired spectrum, and also included the 5 Mhz block. As discussed in the beginning, the second largest license, B, was valued more than the remaining three due to its extra paired capacity.

The auction was a huge success, raising £22.5 billion ($34 billion) for the UK government, despite projections that were less than half that (Klemperer). This result can largely be credited to the auction design, as it successfully incentivized bidders to genuinely compete. The fact that the number of licenses outweighed the number of incumbent firms attracted 9 additional bidders to the auction, which created a competitive environment for all participants. According to Klemperer, a designer of the auction, the results appeared to be efficient [1] in terms of the sums of the overall valuations of the licenses.

Later in 2000, Switzerland also held a spectrum auction, utilizing an almost identical form of the SMR auction used by the UK. However, the Swiss were only offering four licenses for sale instead of five. Additionally, the auction allowed for last minute joint-bidding agreements between firms. Predictably, the auction was a flop, as lack of licenses and poor auction structure caused the number of bidders to shrink from 9 to 4 days before the auction took place. As a result, each of the four licenses sold for their very low reserve price, raising a mere €20 per capita compared to €650 per capita raised by the UK.

4.3 Auction 73 - The US 700 Mhz Band Auction

The US has held many spectrum auctions since 1994, but one that stands out in particular is their 2008 auction, Auction 73, for sections of the 700 Mhz band of spectrum. Unlike the European 3G auctions in which only a small number of licenses were sold, Auction 73 held nearly 1,100 licenses for sale at once, each corresponding to a specific block of spectrum. The blocks were then subdivided into individual licenses, each in a specific category of geographical area. The first of these, "economic areas", were broadly defined and covered a sizable portion of land. "Cellular market areas" were also used, which were more concentrated areas and were split between metropolitan and rural areas. The "regional economic area groupings" (REAG's) divided the country and various territorial areas into 12 large blocks, and the "nationwide area" covered the US as a whole [10].

The spectrum blocks were split as follows: Two blocks, A and E, consisted of 176 economic area licenses each. A was seen as more valuable, as it gave access to 2×6 Mhz of paired spectrum, where E licenses contained $6Mhz$ licenses for unpaired spectrum. Block B consisted of cellular market area licenses, each covering 2×6 Mhz of paired spectrum. Block C contained the 12 regional economic area licenses, covering 2×11 Mhz of paired spectrum, and Block D covered 2×5 Mhz of paired spectrum nationwide.

What sets this auction apart from others is that while the FCC used a SMR auction for the majority of these license, they restricted bidding on block C licenses to package bidding [11]. However, the rules for block C were not very liberating. The FCC split the 12 licenses into three different groups; the first contained 8 licenses that covered only the 50 States; the second, the "Atlantic Package", packaged together Puerto Rico and the Virgin Islands (REAG 10) and the Gulf of Mexico (REAG 12); the third "Pacific Package" consisted of Guam and the Northern Marina Islands (REAG 9) and American Samoa (REAG 11). Firms were allowed to place bids on any of these three packages or any number of individual licenses, but were not allowed to create their own packages. This took care of the computational issues that often accompany combinatorial auctions, and also made sure to account for the exposure problem by packaging the licenses close to others in their relative area.

It is possible that these packages still did not maximize the firms payoffs in terms of complementarities, as the resulting allocation gave Verizon Wireless seven of the eight 50 States licenses. Specifically, the Verizon acquired all but Alaska, which was instead bought by Triad 700, LLC for $1.738 million, a mere fraction of the amount paid by Verizon for the other licenses. There are a couple potential explanations for this. One is that Verizon did not desire all 8 states, and would have been better off designing their own package. The other is that perhaps Verizon did bid on the package, but also on each of the states individually. We know from the discussion of the combinatorial model that the accepted winning bids are the ones that maximize the sellers' revenue. Therefore, it is possible that the sum of Verizon's bid on individual licenses for the 50 States licenses and Triad 700's bid on Alaska outweighed a package bid by Verizon on all eight licenses. However, as Triad 700's winning bid was only 4.8% of Verizon's smallest winning bid (Hawaii, $36.138 million), the first case seems more likely.

The auction raised approximately $19.1 billion in revenue for the US government, and allocated licenses across the country to 101 different firms. Needless to say, the auction succeeded in attracting enough bidders to make the auction competitive, and distributed licenses to a large enough number of firms to ideally make the post-auction market competitive and socially efficient. The only license that was not sold was the D block license, due to the highest bid not exceeding the reserve price. However, this was resolved in the succeeding Auction 76, where the license went up for resale and was sold above its reserve price [10].

5 Conclusion

We analyzed the combinatorial auction and simultaneous multi-round (SMR) ascending auction using very simplified models that certainly do not reflect actual auction scenarios perfectly. For instance, in many auctions, there are specific rules that differentiate bidders strategies, such as allowing discounted bids for smaller firms, rules on bidding eligibility per round, and constraints on the number of items won, among others. Additionally, both auctions may be further analyzed by incorporating budget constraints for bidders rather than allowing them to bid up to some arbitrary finite value. However, this may prove to be a difficult task, as combinatorial auctions without budget constraints are already computationally complex. Last, there does not appear to be a great deal of literature on what happens after auctions are over. When does an auction allocate items efficiently during the auction itself, yet result in a more concentrated or socially inefficient market afterwards? When and if this is the case, what kind of policies, laws, or changes in auction design could prevent this in the future?

Despite these uncertainties, auctions are exceedingly practical and can lead to efficient allocations of resources that might not be reached if left to the free market. In particular, if a free market for such resources could not exist, as is the case with spectrum, auctions create such a market. However, it is important to understand how to pair an auction with an appropriate situation, as not doing so can lead to detrimental results. Good and bad examples of auction design have been observed over the years in the form of government spectrum auctions, some which were tremendously successful and some that were not. In some cases, the SMR auction prevailed over the combinatorial auction, whereas in other situations where licenses had synergy, package bidding was more efficient. The results of our analysis point to the same conclusion made by Paul Klemperer - that auctions are indeed not "one size fits all," but rather they "depend on the details of the context" [1].

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