A new approach to modeling the effects of temperature fluctuations on monthly electricity demand ☆

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ABSTRACT

We propose a novel approach to measure and analyze the short-run effect of temperature on monthly sectoral electricity demand. This effect is specified as a function of the density of temperatures observed at a high frequency with a functional coefficient, in contrast to conventional methods using a function of monthly heating and cooling degree days. Our approach also allows non-climate variables to influence the short-run demand response to temperature changes. Our methodology is demonstrated using Korean electricity demand data for residential and commercial sectors. In the residential sector, we do not find evidence that the non-climate variables affect the demand response to temperature. In contrast, we show conclusive evidence that the non-climate variables influence the demand response in the commercial sector. In particular, commercial consumers are less responsive to cold temperatures when controlling for the electricity price relative to city gas. They are more responsive to the price when temperatures are cold. The estimated effect of the time trend suggests that seasonality of commercial demand has increased in the winter but decreased in the summer.

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1. Introduction

In households and firms in modern economies, electricity is one of the most essential goods consumed. It is certainly no surprise that there is an extensive literature that seeks to explain the variability of electricity demand across markets or in a given market over time. There is a long tradition in this literature, going back at least to Engle et al. (1989), of modeling the long-run and short-run effects of economic covariates, such as price and income, using an error-correction model. See also Silk and Joutz (1997) and Beenstock et al. (1999), inter alia.

Because of the obvious effects of temperature on the demand for electricity in heating and cooling, these studies typically employ some temperature-based metric to control for short-run temperature-induced fluctuations in demand, which occur at seasonal and higher frequencies. Controlling instead for long-run influences on electricity demand, we focus on modeling these short-run (SR) demand fluctuations, which we may think of as the SR component of electricity demand. We may view the response of the SR demand component to temperature as a *temperature response function* (TRF).[1]

In modeling temperature effects, researchers have long recognized the inadequacy of temporally aggregated measures of temperature, such as a monthly average. A linear TRF based on a monthly average temperature suffers from at least two major well-known deficiencies: linearity fails to capture increased demand at both very high and very low temperatures, and the average over a month may not adequately reflect usage during periods of temperature extremes in a given month.

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[1] Our approach does not explicitly model a demand response from temperature fluctuations at periodicities longer than seasonal, because we do not differentiate between the distribution of temperatures in January of one year from that in January of another year.

The standard method for handling these deficiencies has been to employ heating degree days (HDD) and cooling degree days (CDD), which measure the number of degrees that the daily average temperatures in a given period – say, a month – fall below (for HDD) or rise above (for CDD) a threshold value, usually 18°C or 65°F (see, e.g., Gupta and Yamada, 1972; Al-Zayer and Al-Ibrahim, 1996; Sailor and Muñoz, 1997; Fan and Hyndman, 2011). Using these metrics in an otherwise linear model replaces a linear TRF with a piecewise linear TRF with a break point at the threshold temperature, addressing the first deficiency, while indirectly employing intra-monthly data (daily averages), addressing the second deficiency.

Of course, piecewise linearity of the TRF and an arbitrary specification of the threshold may still be inadequate, and there are a number of studies aimed at improving the functional form by way of more sophisticated nonlinear parametric methods or even nonparametric methods, including Engle et al. (1986), Filippini (1995), Pagá and Gürer (1996), Henley and Peirson (1998), Valor et al. (2001), Pardo et al. (2002), and Moral-Carcedo and Vicéns-Otero (2005).

The second deficiency, using a temporal aggregate, seems to have received less attention. Perhaps the indirect use of daily data by way of the HDD and CDD (H/CDD) metrics is viewed as adequate to capture intra-monthly fluctuations, and perhaps the lack of econometric methods to deal with data observed at different sampling frequencies has been an obstacle to using intra-monthly temperature data. Nonetheless, the fact that temporal aggregation may have a deleterious effect on inference is well known.

Two examples illustrate the inadequacy of using H/CDD data. First, suppose that two months have the same number of CDDs (20), but that one has 20 days on which the average temperature is 19°C with the remaining days at or below 18°C, but the other has one day on which the average temperature is 38°C but with the remaining days at or below 18°C. A deviation from the threshold of a single degree would not likely increase electricity usage much if at all, while a deviation of 20°C would very likely induce a massive increase in cooling. Introducing piecewise linearity into the TRF by way of CDDs cannot adequately capture this difference, because the number of CDDs is the same in both months.

As a second example, suppose that temperature fluctuations within a day are substantial, as may be the case in continental climates, such as the Midwestern US. On a given day, the average may show 18°C, while the fluctuation over the course of that day may be ±8°C.[2] Monthly measures of HDD and CDD would not count that day, even though automated thermostats may switch on the heat, the air conditioning, or even both during the course of that day.

There is a third – perhaps more subtle – deficiency of standard temperature response functions. A TRF based only on temperature does not take into account economic or other non-climate covariates, such as the price of electricity. The subtlety lies in the fact that demand models typically *do* include these covariates. However, controlling for short-run temperature fluctuations separately from these covariates means that the impact of cold weather, for example, must be the same regardless of the price of electricity. Since the price of electricity relative to an alternate heating source, such as city gas, may influence an economic agent's use of electricity at a given cold temperature, we should not expect the TRF to be stable as relevant economic covariates evolve.

Further, the effect of price in such models must be the same regardless of season. Nevertheless, if the electricity price is less expensive relative to rival fuels, demand for electricity in heating may increase during the winter time, even though the effect of

changes in price may be negligible during the spring and summer time when there is little demand for heating. Fan and Hyndman (2011) find differences in price elasticities between winter and summer.

In related research (Chang et al., 2014) focusing on time-varying coefficients in an error-correction model, we employ a semiparametric functional coefficient approach to the temperature response function that maps hourly and geographically disaggregated temperature observations onto a monthly measure of the seasonal component of electricity demand. This mixed sampling frequency functional coefficient approach easily addresses the first two deficiencies of the standard H/CDD approach mentioned above: the semiparametric specification allows for nonlinearity in the spirit of Engle et al. (1986), *inter alia*, while the functional coefficient explicitly utilizes intra-monthly temperature data.

In this paper, we focus only on the SR component of demand, and our main aim is to address the third deficiency in addition to the first two. In place of a TRF, we introduce a new concept: the cross-temperature response function (CTRF). The CTRF employs economic covariates directly in the component temperature response functions, both allowing the seasonal demand component to respond to non-climate variables and allowing the effects of non-climate variables to affect the response of the SR component of demand to temperature.

We decompose the effect of temperature on the SR component of electricity demand into three different components: a pure temperature effect, a price–temperature effect, and a time–temperature effect. We investigate the effect of temperature conditional on price and other factors proxied by time, so that the pure temperature effect can be identified.

We apply our model to Korean residential and commercial electricity demand, finding that non-climate variables have particularly substantial effects on changes in the temperature response function of the commercial sector.

The rest of the paper is organized as follows. In the next section, we introduce the TRF and CTRF, novel measures of seasonality using the entire intra-monthly temperature distribution for each month, and we show how they generalize extant measures of seasonality, average temperature and H/CDD data. We discuss data for our application to Korean electricity demand in Section 3 and our estimation results in Section 4. Section 5 concludes. An appendix contains some technical details of the derivations of the regression models in Section 2.

2. Measurement of the temperature effect

2.1. Temperature response function

The temperature response function was used by subsets of the present authors in previous work (Chang and Martinez-Chombo, 2003, and Chang et al., 2014). Because this concept is critical in developing our analysis, we provide here all of the details for the reader's convenience and in fact a more extensive discussion that supersedes the discussions of the temperature response function in those papers.

Consider a hypothetical measure y of the SR component of electricity demand. Such a SR measure abstracts from demand changes directly due to slowly evolving economic covariates, such as long-run income changes. We will refer to this component of demand simply as the *SR component*. Our main purpose is to estimate the mean of y conditional on temperature and economic covariates that may fluctuate frequently. Setting aside the possibility of economic covariates for now, we define the *temperature response function* (TRF) g to be a possibly nonlinear function that maps the temperature distribution (a distribution of stock variables observed over some period of

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[2] According to the US National Weather Service, http://www.srh.noaa.gov/ama/?n=50ranges, accessed October 10, 2014, average fluctuations of 30°F (16.68°C, or roughly ±8°C) are common for some parts of the Midwest (High Plains region) in March.

time) to a response of the SR component of demand (a flow variable measured over the same period of time).

Realistically, the measure of the SR component of electricity demand from a given economic sector is available only monthly, and we denote it by y_t for $t = 1, \ldots, T$.[3] The short run component can be constructed from monthly billing cycle data as described by Chang et al. (2014) . It is the observed seasonality of monthly demand, which we define to be the deviation of standardized monthly demand from its 12-month moving averages.[4] Letting f_t denote the density of temperature observations in month t, an estimator for the conditional mean of y_t given f_t is given by

$$\tau_t = \int f_t(r)g(r)dr, \tag{1}$$

where r is a dummy of integration, but we may think of r as representing intra-monthly temperature observations, and we are integrating over all temperatures in month t.

More formally, we may write $g(r_0) = \int \delta_{r_0}(r)g(r)dr$, where δ_{r_0} is the dirac delta function at r_0 — i.e., the function that has a spike at a point r_0 and integrates to 1. We may interpret the value of function g at r_0 as the temperature effect on the SR component of electricity demand when the temperature distribution is hypothetically concentrated at r_0 — i.e., when the temperature density is given by δ_{r_0}.

Note that τ_t captures both the inherent nonlinearity in the relationship by way of g and the available intra-monthly data by way of the functional approach. For a given TRF g, the relationship between the density f and temperature effect τ is linear, i.e., if the temperature densities f_1 and f_2 yield temperature effects τ_1 and τ_2, then the temperature effect of $c_1 f_1 + c_2 f_2$ becomes $c_1 \tau_1 + c_2 \tau_2$ for any constants c_1 and c_2. In this context, we may simply regard the TRF g as a functional coefficient of temperature density.

Suppose instead that we aggregate the temperature data into a single average temperature datum for month t, and then rely on a nonlinear function h to estimate the temperature effect. The average temperature in a given month is $\int r f_t(r)dr$, so that single-frequency parametric or nonparametric methods discussed in this literature could be used with $h(\int r f_t(r)dr)$ to estimate h. However, h is not the TRF — it does not estimate the response to temperature as g does, unless g and h are both (unrealistically) linear. Rather, h estimates the *aggregate* response to the *average* monthly temperature, and a temperature measurement of, say, 18°C means that demand must respond as if the temperature were constant at 18°C for the whole month.

Using H/CDD data in place of a monthly average improves the situation. These measures may be written as

$$HDD_t = \int h_H \left(\int r f_t(r)dr \right) ds \quad \text{and} \quad CDD_t = \int h_C \left(\int r f_t(r)dr \right) ds \tag{2}$$

where h_H and h_C are functions defined as $h_H(z) = max(18 - z, 0)$ and $h_C(z) = max(z - 18, 0)$ with the commonly used threshold temperature of 18°C, and where the integral across r denotes a daily average of intra-daily temperatures, while the integral across s denotes a monthly sum of daily h_H and h_C. H/CDD data are often used directly, or else $h(HDD_t, CDD_t)$ may be estimated. Because h_H and h_C are piecewise linear functions, it is possible to write τ_t as $b_1 HDD_t + b_2 CDD_t$ for constants b_1 and b_2 (linear h) for a piecewise linear V-shaped g. The coefficients c_1 and c_2 allow the desirable asymmetry of the V shape often discussed in the literature.

Both of the preceding examples, monthly average and H/CDD, are very special cases. The efforts to move away from linear functions h and/or g in favor of smooth functions – U-shaped instead of V-shaped – without a fixed threshold temperature clearly undermine the use of a monthly average and even undermine the use of a smooth nonlinear function of H/CDD data.

Using intra-monthly temperature data allows us to estimate Eq. (1) directly, more precisely estimating the response of monthly sectoral demand to the actual temperatures observed within a given month than can be done with a monthly measure of temperature.[5] Temporal aggregation tends to smooth variations in the data, so that high-frequency temperature data preserve variations that we cannot observe in the monthly or daily averages in the preceding examples. As a result, we can estimate g at temperatures more extreme than the minimum and maximum monthly or daily average, and hourly data provide roughly $24 \times 30 = 720$ times as many temperature observations as monthly.

To estimate the TRF g, we set

$$y_t = \tau_t + \varepsilon_t = \int f_t(r)g(r)dr + \varepsilon_t, \tag{3}$$

where ε_t is a mean-zero error term independent of f_t for $t = 1, \ldots, T$. We approximate the TRF g by a flexible Fourier functional (FFF) form, which decomposes the function g as a linear combination of a polynomial and pairs of trigonometric functions.[6]

For our subsequent analysis, we normalize the temperature so that the temperature densities (f_t) (and also the TRF g correspondingly) are defined on the unit interval $[0, 1]$. Though not absolutely necessary, the normalization will greatly simplify our presentation below. If the raw temperature r is observed in an interval given by $[a, b]$ for some constants a and b, the required normalization may be done by setting $s = (r - a)/(b - a)$ and making a change of variables from r to s. For our empirical analysis, we use $a = -20$ and $b = 40$ in degrees Celsius, because all of the temperatures in our data lie between $-20°$C and $40°$C.

To be more explicit, we momentarily denote the densities for raw and normalized temperatures respectively by (f_t^R) and (f_t^N), and the corresponding TRFs respectively by g_R and g_N. If the raw temperature density f_t^R is given, then we may easily obtain the corresponding density for normalized temperature as $f_t^N(s) = (b - a)f_t^R(a + (b - a)s)$ by the change of variables formula for each $t = 1, \ldots, T$. On the other hand, once we obtain the TRF g_N corresponding to (f_t^N) from our subsequent analysis, we may easily find the TRF g_R corresponding to (f_t^R) by $g_R(r) = g_N((r - a)/(b - a))$. Clearly, the temperature effects (τ_t) defined in Eq. (1) are not affected by our normalization here. In what follows, we will simply denote the normalized densities and

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[3] Hourly data are available for gross generation. Hourly data are also available for sectoral sales by automated meter reading, but these data constitute less than 5% of total sales in Korea. In order to use hourly data, we would either have to pool models of very different demand responses across sectors by using gross generation or else rely on a very small subsample to make inferences about the whole market. Not only does the demand function differ across sectors, but the pricing scheme also differs across sectors. In Korea, for example, the residential sector has a progressive pricing rate, whereas the commercial and industrial sectors have fixed rates. Monthly is an observation frequency that allows differentiation between demand responses in different sectors, is high enough to capture short-run demand fluctuations (especially seasonality), and is quite commonly used in the literature.

[4] Section 3 contains additional details of the available data that we employ. We construct the standardized monthly demand by considering workday equivalents to control the calendar effects and 21 different billing cycles of the sectoral electricity demands in Korea. A detailed explanation of the standardization is given in Sections 3.2 and 3.3 of Chang et al. (2014).

[5] If intra-monthly sales data for a given market were also available, we could model high-frequency demand features, such as time-of-day effects, that monthly data cannot easily explain.

[6] The FFF form (Gallant, 1981) is well known in semiparametric economic analysis, and has been used in the energy literature — e.g., by Serletis and Shahmoradi (2008) to model interfuel substitution in a full energy demand system for the US, by Park and Zhao (2010) to model gasoline demand, and more specifically by Chang et al. (2014) for electricity demand.

the normalized TRF simply by $f_t(s)$ and $g(s)$ instead of $f_t(r)$ and $g(r)$ for $t = 1, \ldots, T$. This notational convention should cause no confusion.

Under our normalization, the TRF g is defined on the unit interval $[0, 1]$ and therefore it can be approximated as

$$g(s) \simeq \sum_{i=0}^{p} c_i s^i + \sum_{j=1}^{q} \left[c_{1j} \cos(2\pi j s) + c_{2j} \sin(2\pi j s) \right], \quad (4)$$

where c_i, c_{1j} and c_{2j} are unknown coefficients and p and q are the orders of the polynomial and trigonometric terms in our approximation.[7] By substituting Eq. (4) into Eq. (3) (see the appendix for additional details), we derive the regression model

$$y_t = \sum_{i=0}^{p} c_i x_{it} + \sum_{j=1}^{q} \left[c_{1j} x_{1jt} + c_{2j} x_{2jt} \right] + \varepsilon_t^{pq}, \quad (5)$$

where $x_{it} = \int s^i f_t(s) ds$, $x_{1jt} = \int \cos(2\pi j s) f_t(s) ds$, $x_{2jt} = \int \sin(2\pi j s) f_t(s) ds$, and ε_t^{pq} differs from ε_t by an approximation error that vanishes as $p, q \to \infty$. Practical determination of p and q is discussed below. We refer to the regression model in Eq. (5) as the *TRF model*.

We may estimate the regression in Eq. (5) by the conventional least squares method. Of course, the regressors x_{it}, $i = 1, \ldots, p$, and x_{1jt} and x_{2jt}, $j = 1, \ldots, q$, are not directly observable. However, they can easily be computed numerically, once we obtain estimates $\hat{f}_t$ of the temperature densities f_t for $t = 1, \ldots, T$, which may be accomplished by the usual nonparametric kernel method (e.g., Li and Racine, 2007) using intra-monthly (e.g., hourly) temperature observations collected in each month t.

The TRF g can then be estimated from the least squares estimates $\hat{c}_i, \hat{c}_{1j}$ and $\hat{c}_{2j}$ of the regression coefficients c_i, c_{1j} and c_{2j} in Eq. (5) for $i = 0, \ldots, p$ and $j = 1, \ldots, q$ as

$$\hat{g}(s) = \sum_{i=0}^{p} \hat{c}_i s^i + \sum_{j=1}^{q} \left[\hat{c}_{1j} \cos(2\pi j s) + \hat{c}_{2j} \sin(2\pi j s) \right] \quad (6)$$

using the approximation of g in Eq. (4).

2.2. Cross-temperature response function

Naturally, we may expect that non-climate variables (economic covariates), such as energy price, preference, technology, and policy, affect not only energy demand but also the temperature effect on demand. These variables change over time.

We can modify the TRF accordingly by letting it vary over time, more generally modeling it as

$$g_t(s) = \sum_{k=0}^{m} w_t^k g^k(s), \quad (7)$$

where, by setting $w_t^0 \equiv 1$, g^0 signifies the time-invariant component of the TRF, and g^k denotes the TRF measuring the temperature-dependent effect of covariate w_t^k on electricity demand for $k = 1, \ldots, m$. We refer to g^0 as the *base TRF* and to g^k as the *TRF with respect to w_t^k*. More specifically, in our application using time and relative electricity prices, we refer to these as the *time TRF* and *price TRF* respectively. In general, we refer to $g_t(s)$ as the *cross-temperature response function* (CTRF).

With the CTRF in Eq. (7), the total temperature effect becomes

$$\int f_t(s) g_t(s) ds = \sum_{k=0}^{m} w_t^k \int f_t(s) g^k(s) ds. \quad (8)$$

In particular, if we set $f_t = \delta_{s_0}$, where as before δ_{s_0} denotes the dirac-delta function at s_0, then we have $\int \delta_{s_0}(s) g_t(s) ds = \sum_{k=0}^{m} w_t^k g^k(s_0)$, which shows the effect of a spike at temperature s_0 on energy demand. The corresponding temperature effect is therefore given by a linear function of the covariates w_t^k with coefficients given by $g^k(s_0)$ and an intercept of $g^0(s_0)$. Note that the intercept and coefficients are functions of temperature.

By approximating the TRF g^k similarly to Eq. (4) (see the appendix for more details), we may construct a regression given by

$$y_t = \sum_{k=0}^{m} \sum_{i=0}^{p_k} c_i^k x_{it}^k + \sum_{k=0}^{m} \sum_{j=1}^{q_k} \left[c_{1j}^k x_{1jt}^k + c_{2j}^k x_{2jt}^k \right] + \varepsilon_t^{p_k q_k}, \quad (9)$$

where $x_{it}^k = w_t^k \int s^i f_t(s) ds$, $x_{1jt}^k = w_t^k \int f_t(s) \cos(2\pi j s) ds$, and $x_{2jt}^k = w_t^k \int f_t(s) \sin(2\pi j s) ds$, similarly to the TRF model in Eq. (5). We refer to the regression model in Eq. (9) as the *CTRF model*.

As with the TRF model in Eq. (5), the CTRF model in Eq. (9) can be estimated by least squares, given orders p_k and q_k of the polynomial and trigonometric terms in the TRF with respect to the k-th covariate for $k = 0, \ldots, m$ and given estimates of the temperature densities f_t for $t = 1, \ldots, T$.

Once we fit the regression in Eq. (9), the TRF with respect to each covariate is readily estimated. Specifically, if we denote the resulting least squares estimates by $\hat{c}_i^k, \hat{c}_{1j}^k$ and $\hat{c}_{2j}^k$ for $k = 0, \ldots, m, i = 0, \ldots, p_k$ and $j = 1, \ldots, q_k$, then we may use

$$\hat{g}^k(s) = \sum_{i=0}^{p_k} \hat{c}_i^k s^i + \sum_{j=1}^{q_k} \left[\hat{c}_{1j}^k \cos(2\pi j s) + \hat{c}_{2j}^k \sin(2\pi j s) \right] \quad (10)$$

for $k = 0, \ldots, m$ to estimate the TRF g^k with respect to the k-th covariate in Eq. (6).

We may set the support of some TRF to be a proper subset of the unit interval $[0, 1]$. In fact, there is a good reason to restrict the support of the price TRF. The reason is that gas is used extensively in heating but not as much in cooling. Therefore, we do not expect the SR component of electricity demand to respond to the price of electricity relative to gas at temperatures warmer than some threshold $\bar{r}$. This implies that the normalized TRF has support contained in $[0, \bar{s}]$ with $\bar{s} = (\bar{r} - a)/(b - a) < 1$. With this restriction in place, we may estimate the price TRF using the terms

$$\left(\frac{s}{\bar{s}} \right)^i 1\{0 \le s \le \bar{s}\} \quad \text{and} \quad \left(\cos \left(\frac{2\pi j}{\bar{s}} s \right) 1\{0 \le s \le \bar{s}\}, \right.$$
$$\left. \sin \left(\frac{2\pi j}{\bar{s}} s \right) 1\{0 \le s \le \bar{s}\} \right), \quad (11)$$

instead of s^i and $(\cos(2\pi j s), \sin(2\pi j s))$, where $1\{0 \le s \le \bar{s}\}$ denote the indicator function taking value 1 if $0 \le s \le \bar{s}$ and 0 otherwise.

3. Data

Our temperature distribution and measure of the SR demand component are identical to those used in our previous work (Chang et al., 2014). We use distributions of hourly temperatures sampled from 5 geographically distributed cities in Korea. Because demand data are available only in 21 overlapping billing cycles, rather than monthly, the monthly national temperature density is given by $f_t(s) = \sum_{a=1}^{5} \sum_{b=1}^{21} w_{at} w_b f_{ab_t}(s)$, where w_{at} and w_b are weights assigned to each city and each billing cycle, and $f_{ab_t}(s)$ is the density for each city a in each billing cycle b ending in month t.

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[7] We may approximate the raw TRF g using the trigonometric pairs with frequencies $2\pi j/(b - a)$ for $j = 1, 2, \ldots$, in place of those with frequencies $2\pi j$ for $j = 1, 2, \ldots$ used to approximate the normalized TRF g in Eq. (4).

There are consequently 105 densities of hourly temperature observations for each month in the sample. Issues relating to the use of billing cycle data were discussed by Train et al. (1984), and our geographic weighting of temperature data is similar to that of Moral–Carcedo and Vicéns-Otero (2005) for Spain. However, our approach using temperature distributions is quite a bit different from these approaches.

We obtain Korean residential and commercial electricity sales in megawatt hours (MWh) from Korea Electric Power Corporation (KEPCO). The billing cycle issue naturally pertains to the construction of our measure of SR component of electricity demand, and the rather involved construction of this measure takes into account calendar effects from high-frequency cycles in a workday and throughout a week, but with different numbers of weeks and workdays in each billing cycle and in each month. The problems of different loads on such days in constructing demand measures have been addressed by Pardo et al. (2002) and Moral-Carcedo and Vicéns-Otero (2005), inter alia.

Once a monthly demand measure is constructed, we take natural logs and subtract out the 12-month moving average of the series in logs in order to eliminate any stochastic or deterministic trends and thus isolate the SR component. Note that detrending the data in this way accounts for any long-run effects, including those from price, income, substitution, etc., so that we subsequently focus only on short-run effects. The interested reader is referred to Chang et al. (2014) for a more complete discussion of how these series were constructed.

Fig. 1 shows the resulting SR components of electricity demand for the residential and commercial sectors. If the SR component was created in such a way to be uncorrelated with the long-run trends proxied by the 12-month moving average, we could interpret a unit change in the SR component as an approximation to a percentage change in monthly demand, because the demand measure is in logs. Instead, we interpret a unit change to be an approximation to a percentage change in the SR component. The range of monthly demand data in the figure does not exceed 0.3. However, we may predict short-run changes in demand in excess of 30%, because we are estimating the function g, which in contrast to h defined in the previous section is invariant with respect to the time scale, and because we use more volatile hourly temperature data to do so.

In our analysis of Korean electricity demand, we set $w_t^1 \equiv t/T$, so that the first covariate is given by time. We include time as a proxy for changes in preferences, technology, government energy policy, among other latent variables, as many previous authors have done, including Watts and Quiggin (1984), Jones (1994), Hunt et al. (2003), and Halicioglu (2007).

We consider price to be an extremely important signal to which consumers may respond, but the price of electricity in Korea is set by the government and has not changed very much in over two decades. The lack of variation makes it difficult to distinguish the effect of price from a constant. Instead, we consider the real price of electricity *relative* to a close substitute, city gas. An index of this relative price, which we denote by RP_t, has changed substantially over the sample period. The relative price is essentially just a measure of the real price of electricity, but with a numeraire given by city gas instead of Korean Won. Because the ease of switching to an alternative fuel source critically depends on the availability of infrastructure to utilize the fuel source, we also consider the penetration rate PR_t of city gas. Using these series, we consider $w_t^2 = PR_t \ln RP_t$ in our empirical application.[8]
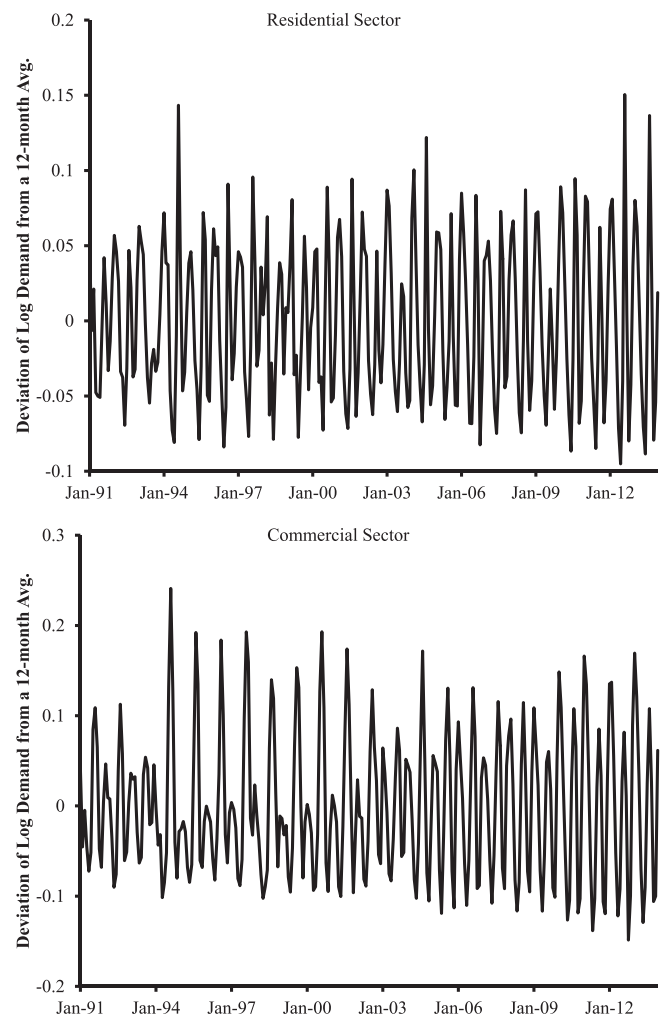

Fig. 1. SR component of electricity demand in the Korean residential and commercial sectors. Data constructed as deviations from a 12-month moving average of a measure of monthly national sectoral electricity demand (Chang et al., 2014).

In Korea, city gas is the closest substitute for electricity, so these variables are expected to play important roles in determining electricity demand. The functional form implies that if the price of electricity relative to city gas increases by 1%, for instance, the effect on electricity demand is given by the fraction of 1% equal to the penetration rate. If penetration rate goes up, then there will be higher substitutability in gas consumption (instead of electricity), so the effect of the relative price of electricity on the SR component of electricity demand should increase. However, because of the substitutability, the effect of cold temperatures on the SR component should decrease with both penetration and electricity price.

We obtain electricity and city gas price indices from the Korean Statistical Information Service (KOSIS) and relative price is constructed as the electricity price index divided by the gas price index. The penetration rate of city gas is from the Korea City Gas Association. These series are displayed in Fig. 2. City gas penetration relative to electricity has increased dramatically over the sample period, while the relative price of electricity has decreased dramatically.[9]

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[8] We estimated several alternative specifications, but we do not report the results because they did not increase explicative power over those reported. Specifically, we estimated a less parsimonious model with $w_t^2 = \ln RP_t$ and $w_t^3 = PR_t$ instead of $w_t^2 = PR_t \ln RP_t$. We also estimated models identical to those estimated below but (a) with income growth as a covariate and (b) with electricity price instead of $PR_t \ln RP_t$ in the price TRF.

[9] A referee astutely pointed out that if the increase and decrease were proportional, the product could have insufficient variation to distinguish it from a constant (the base TRF) or a linear trend (the time TRF). Fortunately, this is not the case, as evidenced by the significance of the coefficients of the respective TRFs in the commercial sector.
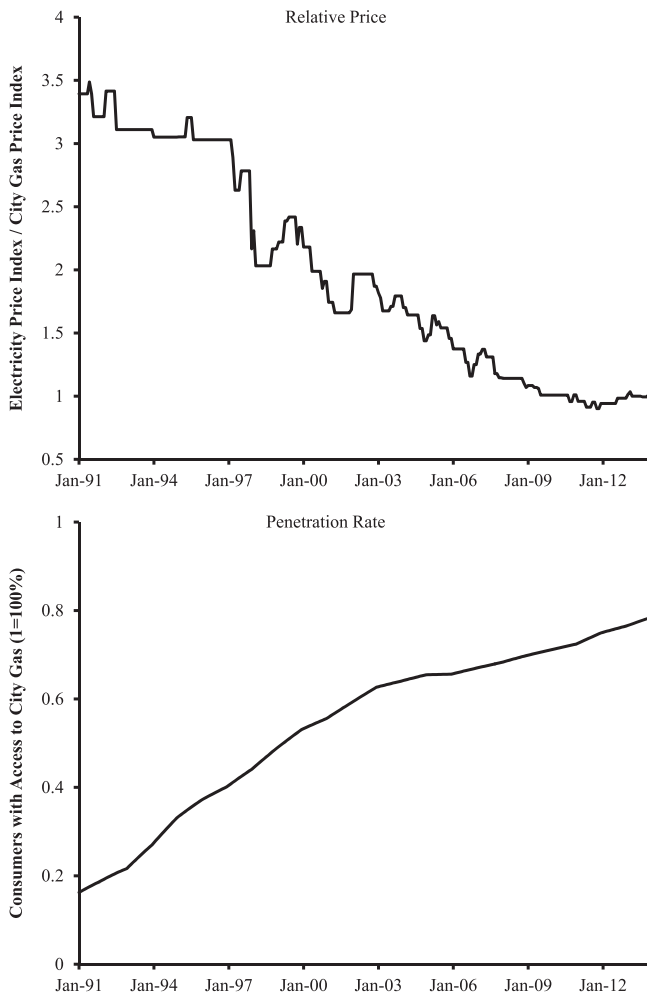
Fig. 2. Relative price of electricity (RP) and penetration rate of city gas (PR). RP is constructed as electricity price index divided by gas price index from the Korean Statistical Information Service (KOSIS). PR is from the Korea City Gas Association.

Table 1
Estimation results for the residential sector TRF model compared to a linear model using H/CDD. TRF estimates from least squares with robust standard errors on the regression in Eq. (5) with temperature densities estimated using a normal kernel with plug-in bandwidth.

	H/CDD model		TRF model	
	Est.	*t*-Value	Est.	*t*-Value
Coeff.	$b_1 HDD_t + b_2 CDD_t$		$g(s)$	
c_0	−0.0701	−20.1	0.699	2.219
b_1	0.0002	21.5		
b_2	0.0004	13.7		
c_1			−4.695	−2.776
c_2			5.404	3.202
c_{11}			−0.272	−2.370
c_{21}			0.225	16.880
R^2	0.479		0.825	
$\bar{R}^2$	0.475		0.825	

Gas cooling equipment is less efficient by 30–40% than electric cooling equipment, so gas cooling systems are currently used only for some public buildings to lower the summer peak of electricity demand in Korea. Therefore, we do not model any substitutional price effect in cooling demand.

Our final data set includes $T = 276$ monthly observations running from 1991:01 to 2013:12, since penetration rate data are available from 1991.

4. Estimation results

4.1. Residential temperature response function

We first analyze the temperature effect in residential electricity demand in Korea using the TRF model. To determine the orders p and q of the polynomial and trigonometric terms in our approximation of the TRF g in Eq. (4), we use the cross-validation criterion suggested by Burman et al. (1994) and choose p and q over the ranges of $p \in \{1, 2\}$ and $q \in \{0, 1\}$. The results suggest the choice of $p = 2$ and $q = 1$, i.e., the use of a second order polynomial with one pair of trigonometric functions.

The least squares estimates for the regression coefficients are reported in Table 1, and the corresponding estimate of the TRF is presented in Fig. 3. The estimated TRF has a shape that we normally

expect. It is U-shaped taking values that increase as the temperature gets below or above a comfortable range. The temperature effects caused by heating and cooling needs appear to be asymmetric, the latter generating substantially more demand than the former.

As a comparison, we also show results in Table 1 from fitting a simple model with just HDD, CDD, and a constant term. This imposes a V-shape instead of a U-shape. We again observe asymmetry of the magnitudes for heating and cooling, but the resulting fit of this model (in terms of R^2 and $\bar{R}^2$) is much weaker than that of the TRF model.

The estimate of the TRF can be useful in many different contexts. First, the TRF itself provides some useful information on the intensities of the heating and cooling energy demands. Because the TRF measures the demand response to a distribution of temperatures, we may interpret the value of the TRF at a specific temperature, say 28°C, as the demand response if the temperature stays constant at that temperature over some period (e.g., a month).

If we look at 18°C, 23°C, and 28°C, the estimated values of the TRF are −0.09, −0.05, and 0.10 respectively. A 5°C increase in temperature from 18°C to 23°C increases (the SR component of) demand by 0.04, or 4%. However, an increase in the same magnitude from 23°C to 28°C drives an increase in the SR component of demand of 15%. If the temperature instead drops from 13°C to 8°C, the increase is only 6%.

These examples show both the asymmetry in the slopes and the nonlinearity both above and below the threshold temperature. Clearly, demand responses to otherwise equal temperature changes
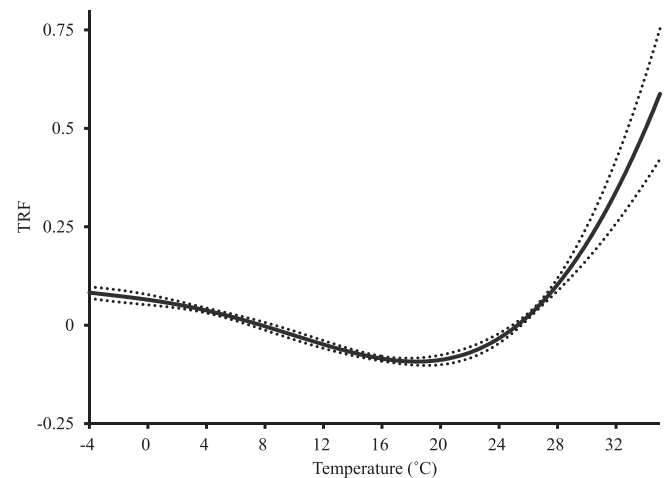

Fig. 3. Estimated residential sector TRF model with 95% confidence bands. Constructed with the coefficient estimates in Table 1 and the TRF in Eq. (6). Confidence bands are calculated according to Park (2010).

depend on the current temperature in a more complicated way than can be handled using H/CDD data.

The plotted TRF takes a maximum value of 0.59 at 35 °C and a minimum of −0.09 at about 18–19 °C, suggesting that the model could predict a 68% increase. To put the plausibility of such an increase into perspective, high-frequency demand data available for a subset of Korean residential consumers shows an increase of 69% during August 2012.

Second, we may identify the temperature effect as in Eq. (1) using the estimated TRF and temperature densities. Analysis of the temperature effect in energy demand is very critical in forecasting peak load and deciding how to optimally employ a mix of power plants in electricity supply.

Third, we may perform some informative counterfactual analysis on temperature-related electricity demand. For instance, we may forecast the temperature effect assuming the temperature distribution will be the same as the average of temperature distributions in past years, or we may predict the effect of an increase in temperature. If the temperature distribution at time t is shifted to the right by u units of normalized temperature, we would have an increase in the temperature effect of $\int f_t(s-u)g(s)ds - \int f_t(s)g(s)ds$. Note that $f_t(\cdot - u)$ denotes the temperature distribution with mean temperature increased by u, compared to the temperature distribution represented by $f_t(\cdot)$, since $\int s f_t(s-u)ds = \int (s+u)f_t(s)ds = \int s f_t(s)ds + u$.

We also estimate a CTRF for the residential sector using the methodology described below for the commercial sector (results not shown). We find that the confidence bands for the time TRF and price TRF contained a zero demand response for every temperature, suggesting that only the base TRF is useful in explaining the SR component of electricity demand. In light of the facts that residential electricity prices are kept artificially low and residential consumers are too small to warrant demand charges, the insignificance of the price TRF is not surprising. The residential time TRF exhibits a declining pattern similar to the commercial time TRF discussed below, but with much larger uncertainty.

4.2. Commercial cross-temperature response function

4.2.1. Estimation and empirical analysis of the CTRF model

We first estimate the TRF model to find the threshold temperature $\bar{r}$ to use in estimating the price TRF. We determine p and q using cross-validation for the TRF, and then we set $\bar{r} = 14.2\,°C$, where the estimated TRF is minimized. Note that $\bar{s} = (14.2 + 20)/60 = 0.57$ for the price TRF. Next, we choose p_k and q_k for each TRF. In doing so, we consider $p_k \in \{1, 2\}$ and fix $q_k = 1$, and the cross-validation criterion selects $p_0 = 2$, $p_1 = 1$, and $p_2 = 1$.

To compare the results of the TRF and CTRF models, we include a time trend and $PR_t lnRP_t$ as covariates alongside the TRF in the TRF model. In other words, to estimate the TRF, we are actually restricting the CTRF model by setting $p_1 = p_2 = 0$ and $q_1 = q_2 = 0$, with the convention that $q_k = 0$ means no trigonometric terms, but letting p_0 and q_0 (in the base TRF) exceed zero. Fixing $p_1 = p_2 = 0$ means that only a constant c_0^k is allowed in the TRFs with respect to time and price, and these constants become coefficients of these covariates in Eq. (9), since $\sum_{k=0}^{r} c_0^k x_0^k = \sum_{k=0}^{r} c_0^k w_t^k$. With the addition of the covariates, we refer to this as the *TRF+ model*.

The estimated results of TRF+ and CTRF models for commercial demand are summarized in Table 2, and the TRFs in the TRF+ and CTRF models are given in Figs. 4 and 5 respectively. A Wald test allows a formal comparison of the two models. Using the values of R^2 for each model in the table, a Wald test may be constructed as $(0.920 - 0.771)/(1 - 0.920) \times (276 - 13) = 489.84$, easily beating the χ_7^2 critical value of 14.07 for a size-0.05 test. The TRF+ model is thus rejected in favor of the CTRF model. As an additional comparison, we also show results from fitting a simple model with just HDD,

Table 2
Estimation results for the commercial TRF+ and CTRF models compared to a linear model using H/CDD. TRF+ and CTRF estimates from least squares with robust standard errors on the regression in Eq. (9) with temperature densities estimated using a normal kernel with plug-in bandwidth.

	H/CDD model		TRF+ model		CTRF model	
	Est.	t-Value	Est.	t-Value	Est.	t-Value
Coeff.	$b_1 HDD_t + b_2 CDD_t$		$g(s)$		$g^0(s)$	
c_0	−0.1106	−16.1	−0.339	−3.184	−2.549	−5.585
b_1	0.0003	10.3				
b_2	0.0009	24.3				
c_1			0.885	4.607	10.245	5.114
c_2					−7.966	−4.416
c_{11}			0.217	11.999	0.686	5.424
c_{21}			0.255	6.667	0.576	7.128
			t		$g^1(s)$	
c_0			−0.002	−0.131	0.662	2.912
c_1					−1.161	−2.757
c_{11}					0.123	3.115
c_{21}					−0.178	−1.942
			$PR_t \log RP_t$		$g^2(s)$	
c_0			0.027	0.962	0.004	6.689
c_1					−0.334	−3.775
c_{11}					0.300	6.795
c_{21}					−0.059	−0.580
R^2	0.558		0.771		0.920	
$\bar{R}^2$	0.554		0.767		0.917	

CDD, and a constant term. As in the case of the residential sector, the coefficients are statistically significant with meaningful signs and magnitudes, but the fit of this model is clearly inferior to that of the TRF+ and CTRF models.

The shapes of the estimated TRF in Fig. 4 and analogous base TRF in Fig. 5 are both U-shaped in the range of temperatures with the scale reflecting the fluctuations of the SR component of electricity demand. The only noticeable difference between the TRF and base TRF of the CTRF is that the latter appears to flatten out rather than continue to increase at the lowest temperatures.

As we can see in Table 2, the effect of time in the TRF+ model is estimated to be insignificant. Keeping in mind that the SR component of demand is detrended, this finding is not surprising. In the CTRF model, the effects of time are estimated to be *significant*. The time TRF and confidence intervals in Fig. 5 better illustrate the effects.
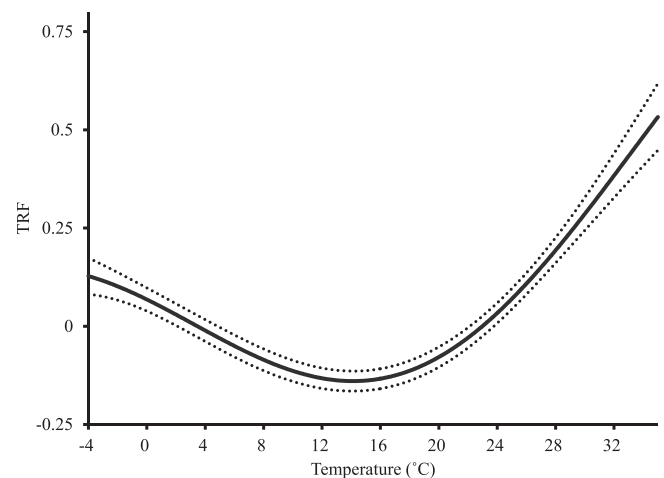

Fig. 4. Estimated commercial sector TRF with 95% confidence bands. Constructed from the coefficient estimates from the TRF+ model in Table 2 with the base TRF defined by $g^0(r)$ in Eq. (10). Confidence bands are calculated according to Park (2010).
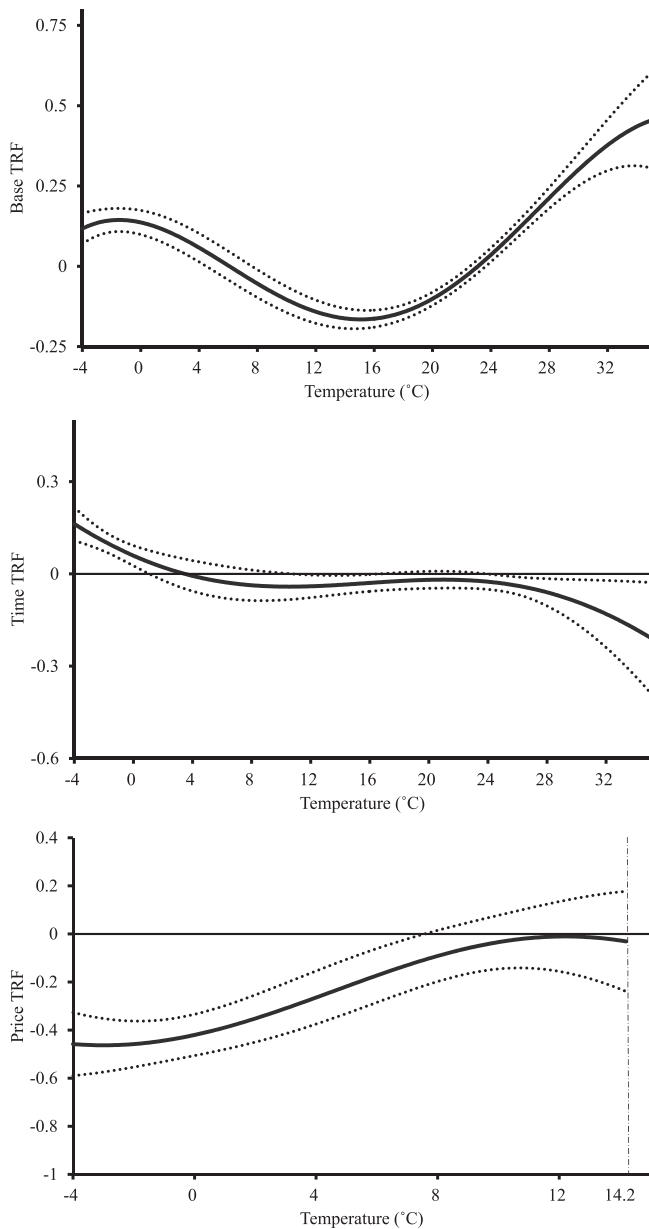
Fig. 5. Estimated commercial sector CTRF with 95% confidence bands. Constructed from the coefficient estimates from the CTRF model in Table 2 and TRFs defined by $g^k(r)$ in Eq. (10). Confidence bands are calculated according to Park (2010).

The time TRF takes positive values in the range of 1°C or less, close to zero in the range of 1–24°C, most of the temperature spectrum, and negative values in the range exceeding 24°C. Consider for example the temperatures of −4°C and 34°C, at which $\hat{g}^1$ is about 0.16 and −0.18. A change of ten years (a change in t/T of 120/276) increases the response of the SR component at −4°C by 7.0% but *decreases* the response at 34°C by 7.8%. These compare with base responses (from the base TRF) of 11.7% and 43.5% at −4° C and 34°C respectively.

These results suggest that, over a long span of time, the seasonality of commercial electricity demand in South Korea has increased in the winter time, but decreased in the summer. That is, the growth rate of heating demand has exceeded that of the average load, which is mainly due to a rapid increase in the supply of electric heating appliances in recent years so that consumers have switched their heating systems to electricity. However, the growth rate of the cooling demand is lower than that of the average load, which reflects the technical progress in electric cooling appliances so that consumers have replaced their cooling appliances by more energy efficient ones.

We can also see in Table 2 that the (short-run) price elasticity is estimated to be (insignificantly) positive in the TRF+ model — certainly the opposite sign of what we should expect. However, the CTRF model estimates a more sensible range of price elasticities. As shown in Fig. 5, the price TRF is estimated to be significantly negative at temperatures under approximately 7.5°C — that is, 95% confidence interval does not include zero below approximately 7.5°C. Above this temperature, electricity price relative to that of city gas has no significant impact on commercial consumption of electricity.

A more interesting result is that the magnitude of the price TRF increases as temperature decreases below 7.5°C, which means that the price effect in heating demand becomes clearer as the temperature becomes lower. Indeed, this result helps to explain the flattening of the base TRF discussed above: commercial consumers respond less to cold temperatures when accounting for the price of electricity relative to that of city gas.

In fact, electricity sales to the commercial sector of January 2012 increased by 39.0% compared with that of January 2006, whereas gas sales for the commercial sector grew by 0.7% during the same period. Meanwhile the electricity price index increased by 10.7% and gas price index grew by 61.5% between January 2006 and January 2012. The estimated price TRF clearly reflects this shift to electric heating from gas heating.

We illustrate in more detail how one can interpret the estimated price TRF in Fig. 5. The relative price elasticity of the SR component of demand is given by $\hat{g}^2 PR_t$. For example, if $PR_t = 1$ and we look at −4°C and 10°C, the estimated values of $\hat{g}^2 PR_t$ are approximately −0.46 and −0.03 respectively. Hence, if temperature changes from 10°C to −4°C, then the relative price elasticity of the SR component will change from nearly zero (completely price inelastic) to −0.46. Moreover, if the electricity price index decreases by 10% and the gas index is unchanged (a change in $PR_t lnRP_t$ of 0.10), the SR component of electricity demand for the commercial sector would increase by 4.6% at −4°C but only by 0.3% at 10°C, which shows quite different substitution patterns at the different temperature levels. These compare with base responses (from the base TRF) of 11.7% and −10.4% at −4°C and 10°C respectively.

Looking at the whole CTRF in the CTRF model as the sum of the individual TRFs, we can make another comparison with the TRF+ model. For example, at the counterfactual temperature of −4°C in January 2002 the sum of the base TRF and time-weighted time TRF is 11.7% + 7.0% × (133/276) = 15.1%. At a penetration rate of $PR_t = 1$, a relative price decrease of 10% increases the response of the SR component by an additional 4.6%, so that the total response is 19.7% more than that of the response at an average temperature with no price change. In contrast, the TRF+ model suggests a response of 12.8% − 0.2% × (133/276) = 12.7% at −4°C in January 2002, but that a relative price decrease of 10% *decreases* the response of the SR component by 0.3% (but not significantly). The aggregate response is therefore predicted by the TRF+ model to be only 12.4% above an average temperature with no price change.

4.2.2. Seasonal and temporal analyses

Fig. 6 shows the mean absolute error (MAE) of estimated residuals by months, and it shows that the CTRF model outperforms the TRF+ model in all months except April when MAEs for the two models are very close. The MAEs of January, February, March and August of CTRF model are 61%, 65%, 55% and 55% smaller than those of TRF+ model respectively, which shows rather clear price- and time-dependent temperature effects in the commercial sector in those months. We can deduce from our above results that time affects the temperature
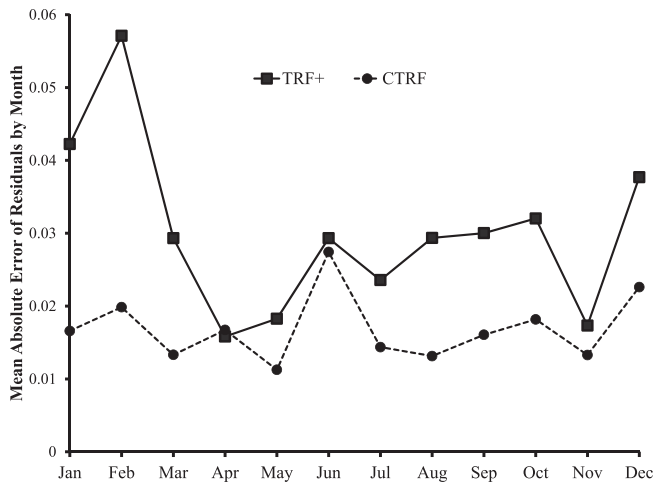
Fig. 6. Mean absolute error by months.

response in both winter and summer, while relative price also affects the temperature response in the winter.

The CTRF model enables us to decompose the monthly temperature effects into a price-dependent factor and a time trend-dependent factor, allowing us to better identify the aggregate changes in temperature effects due to time and relative price. The temperature effects in Eq. (8) may be written as

$$\int f_t(r)g_t(r)dr = \int f_t(r)g^0(r)dr + \frac{t}{T}\int f_t(r)g^1(r)dr + PR_t \ln RP_t \int f_t(r)g^2(r)dr$$

(12)

using our covariates $w_t^0 = 1$, $w_t^1 = t/T$ and $w_t^2 = PR_t \ln RP_t$.

Consider temperature effects for each month $M = 1, \ldots, 12$ constructed from this CTRF using:

1. TE_{M0}: the time index for month M in 1991 and the penetration rate and relative price for month M in 1991,
2. TE_{M1}: the time index for month M in 1991 and the penetration rate and relative price for month M in 2013, and
3. TE_{M2}: the time index for month M in 2013 and the penetration rate and relative price for month M in 2013.

The difference $TE_{M2} - TE_{M0}$ indicates the total change between 1991 and 2013. A component of the total change, $TE_{M1} - TE_{M0}$ is the change in the temperature effect due to the change in $PR_t logRP_t$ between 1991 and 2013 while holding constant other temporal drivers proxied by a time trend. Similarly, $TE_{M2} - TE_{M1}$ is the change caused by these other drivers, while holding the price covariate constant.

To estimate these effects, we estimate g^k and f_t as described above, except we pool observations in month M across all 23 years in the sample to estimate f_M for $M = 1, \ldots, 12$. By using the same temperature density for the same month in all years, the changes that we identify over time given by $TE_{M2} - TE_{M1}$ can be attributed to temporal drivers other than possible long-run temperature changes.

Table 3 shows the decompositions between 1991 and 2013 and their differences. The total change in temperature effect over the sample is positive in the winter months of December, January, February, and March, but negative in all other months.

The breakdown of the positive changes in the winter months are 66% price and 34% other factors proxied by time for December, 41% and 59% for January, 42% and 58% for February, and 76% and 24% for March. We may interpret this to mean that the overall increases in

the SR component of demand in winter months may be attributed to both increases due to price and penetration changes over the sample and those due to changes in other temporally varying non-climate, non-price variables. The increases due to price during these months given by $TE_{M1} - TE_{M0}$ have roughly the same magnitudes (4.3%–6.2 %), but the change from the other factors is less important in the warmer months of December and March (2.2% and 1.5%) than in January and February (9.1% and 7.9%).

Looking at the summer months of June, July, August and September, we see no effect on the SR component from price changes. This result is an artifact of our modeling strategy, since we set the price TRF to zero at temperatures above 14.2°C. Those four months are unlikely to have enough temperature observations below this threshold to make any substantial difference. We are essentially imposing that there will be no long-run effect of relative prices in summer months, except possibly through the time TRF if the price covariate has a time trend.

The remaining spring and fall months, April, May, October, and November show decreases in the temperature effect overall, but driven primarily by negative effects from non-climate, non-price factors and countervailed by *increases* due to price changes. In other words, relative price changes have led to only small increases (0.1%–2.1%) in the SR component of electricity demand in those months – due to decreases in the relative price from increases in gas prices – while other factors have driven more substantial decreases (2.3% –2.9 %).

5. Conclusions

In this paper, a general model is proposed in order to estimate and identify temperature effects in a short-run electricity demand function. We adopt a new approach using temperature densities to estimate a cross-temperature response function, which allows non-climate variables to have different effects on electricity demand at different temperatures. The CTRF and the TRF, the restricted version of the CTRF without non-climate variables, both allow us to exploit high-frequency temperature data in order to explain monthly sectoral electricity sales instead of relying on monthly aggregates of temperature, such as cooling and heating degree days. The availability of high-frequency temperature measurements can account for nonlinearity of demand responses to temperature fluctuations and for intra-monthly temperature fluctuations better than these monthly aggregates.

We fit our proposed models to Korean residential and commercial electricity demand data over 1991: 01–2013:12. The non-climate variables that we use appear to have little effect on the response of residential demand to temperature. The TRF that we estimate for the residential sector shows the expected asymmetric U-shape. We observe that a 5°C increase in temperature from 18°C to 23°C increases demand by 4%, while 5°C increase 23°C to 28°C increases demand by 15%, for example, illustrating the inadequacy of using cooling degree days, which would impose demand responses of the same magnitude for these temperature changes.

In contrast, the non-climate variables have substantial impacts on the response of commercial demand to temperature. Technical progress in electric appliances and changes in consumption habits, proxied by the time trend, have lowered the growth rate of the cooling demand and increased the growth rate of heating demand. For example, a change of ten years increases the short-run commercial demand response to a temperature of −4°C by 7% but decreases the response at a temperature of 34°C by 7.8%.

The effect of electricity price relative to its closest substitute is shown to significantly influence commercial electric heating demand, and vice versa. Specifically, demand for electric heating is less sensitive to colder temperatures when controlling for

Table 3
Decompositions of monthly temperature effects.

Month	TE_{M0}	TE_{M1}	TE_{M2}	$TE_{M2} - TE_{M0}$	$TE_{M1} - TE_{M0}$	$TE_{M2} - TE_{M1}$
January	−0.018	0.045	0.135	0.153	0.062	0.091
February	−0.013	0.044	0.124	0.137	0.058	0.079
March	−0.027	0.022	0.037	0.064	0.049	0.015
April	−0.085	−0.065	−0.088	−0.002	0.021	−0.023
May	−0.109	−0.105	−0.135	−0.026	0.004	−0.029
June	−0.046	−0.046	−0.075	−0.028	0.000	−0.028
July	0.044	0.044	0.010	−0.034	0.000	−0.034
August	0.128	0.128	0.080	−0.048	0.000	−0.048
September	0.047	0.047	0.012	−0.035	0.000	−0.035
October	−0.085	−0.083	−0.110	−0.025	0.001	−0.027
November	−0.102	−0.087	−0.112	−0.009	0.015	−0.024
December	−0.036	0.006	0.028	0.064	0.043	0.022

price — as evidenced by the flattening of the base TRF when a price TRF is estimated. Moreover, this demand is more price elastic at colder temperatures — as evidenced by a negative price TRF. Repeating our example above to illustrate the latter point, when temperature decreases from 10°C to −4°C, the relative price elasticity changes from almost completely inelastic to −0.46 (at a 100% penetration rate of city gas).

Over the whole sample period, increases due to price during the winter month of December–March have roughly the same magnitudes (4.3% –6.2 %), while the change from the other factors proxied by a linear trend is less important in December and March (2.2% and 1.5%) than in January and February (9.1% and 7.9%). Relative price changes have led to only small increases (0.1% –2.1 %) in the SR component of electricity demand in the spring and fall months of April, May, October, and November – due to decreases in the relative price from increases in gas prices – while non-climate, non-price factors have driven more substantial decreases (2.3% –2.9 %).

Appendix A. Derivations of the regression models

A.1. Temperature response function

Substituting Eq. (4) into the integral in Eq. (3), we get

$$\int f_t(s)g(s)ds = \sum_{i=0}^{p} c_i \int s^i f_t(s)ds + \sum_{j=1}^{q} \left[c_{1j} \int f_t(s)\cos(2\pi js)ds \right.$$
$$\left. + c_{2j} \int f_t(s)\sin(2\pi js)ds \right], \tag{A.1}$$

up to an approximation error. The TRF model in Eq. (5) follows from Eq. (3) with the error term re-defined to accommodate the approximation error.

A.2. Cross-temperature response function

Similarly to Eq. (4) for the TRF model, we may approximate

$$g^k(s) \simeq \sum_{i=0}^{p_k} c_i^k s^i + \sum_{j=1}^{q_k} \left[c_{1j}^k \cos(2\pi js) + c_{2j}^k \sin(2\pi js) \right] \tag{A.2}$$

for each TRF $k = 0, \ldots, m$. Substituting this approximation into the integral on the right-hand side of Eq. (8), we may write

$$\int f_t(s)g_t(s)ds = \sum_{k=0}^{m}\sum_{i=0}^{p_k} c_i^k w_t^k \int s^i f_t(s)ds + \sum_{k=0}^{m}\sum_{j=1}^{q_k} \left[c_{1j}^k w_t^k \int f_t(s)\cos(2\pi js)ds \right.$$
$$\left. + c_{2j}^k w_t^k \int f_t(s)\sin(2\pi js)ds \right] \tag{A.3}$$

up to an approximation error, similarly to Eq. (A.1). The CTRF model in Eq. (9) follows from Eq. (3) once again, but with the time-varying TRF $g_t(s)$ and a newly redefined error term.

Appendix B. Supplementary data

Supplementary data to this article can be found online at http://dx.doi.org/10.1016/j.eneco.2016.07.013.

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