O-Ring Production Networks

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Abstract

We study a production network where quality choices are interconnected across firm boundaries. High-quality firm sources high-quality inputs and sell to high-quality firms that value its output. Consistent with the theory, we document a novel assortative matching pattern of skills in the network of Turkish manufacturing firms. A trade shock that increases the relative demand for high-quality output increases the firm's skill intensity and shifts the firm toward skill-intensive partners. To evaluate the general equilibrium effect of the trade shocks, we develop and estimate a quantitative model with heterogeneous firms, endogenous quality choices, and network formation. Method of Simulated Moments estimates indicate strong complementarity of quality in production and a moderate directed search in relationship formation.

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1 Introduction

The space shuttle *Challenger* exploded because one of its innumerable components, the O-rings, malfunctioned during launch. Using this as a leading example, Kremer (1993) studies production processes, in which the value of output may dramatically decrease due to the failure of a single task. In his model, a product may founder from the mistake of a single unskilled worker, even if it aggregates the high value added of many skilled workers. To avoid such losses, a firm that produces complex, higher-quality products hires skilled workers for all its tasks.

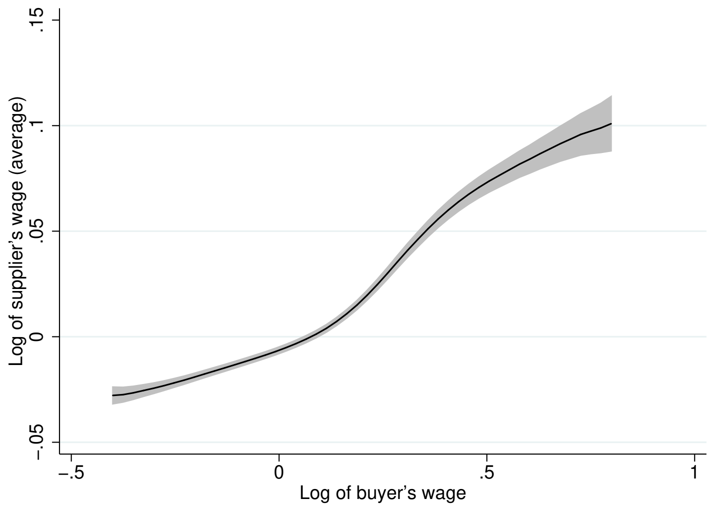
Extending this rationale *across* firm boundaries, the high-quality firm above will source high-quality inputs and sell to high-quality firms that value its output. So, skill-intensive firms match with each other in the network. A firm's decision to upgrade its quality depends critically on the willingness of its trading partners to also upgrade or on its ability to find new higher-quality partners. This rationale applies to the quality of products as well as to the other modern technologies of inventory controls, research and development, and internal communications. Improvements in these areas generally allow for greater product scope and for flexibility to respond to demand and supply shocks. A firm profits from them if its suppliers also offer scope and flexibility, and if its customers value these same improvements. Shocks to the quality of a few firms may then have large general effects on the quality and demand for skills in the network.

We study this interconnection in firms' quality levels theoretically and empirically. Our data comprise all formal Turkish manufacturing firms from 2011 to 2015. We merge value-added tax (VAT) data with matched employer-employee and customs data. We observe the value of trade between each buyer-seller pair of firms; exports by firm, product and destination, and the occupation and wage of each worker in each firm. We develop a quantitative model that accounts for the salient features of the data, structurally estimate it, and use counterfactuals to study general equilibrium effects of trade shocks.

We document a novel, strong assortative matching of skills in the network. As an example, Figure 1 graphs the relation between a firm's average log wage (adjusted for industry, region, size) against the average of its suppliers' wage.[1] The slope, 0.294 (standard error 0.013) is large. A typical firm has about eleven suppliers and the y-axis is the average over these suppliers. This increasing relation between buyer and supplier wage may arise from an extensive margin—high-wage firms match more with each other—or from an intensive margin—high-wage firms spend relatively more on their high-wage sup-

[1] The figure has only manufacturing firms, later used in our structural estimation but an equally strong pattern emerges in the corresponding figure with all sectors, in Appendix Figure A2.

Figure 1: Assortative Matching on Wages



Notes: Wage is defined as the average value of monthly payments per worker. Supplier wage is constructed as the unweighted average of monthly wages paid by all manufacturing suppliers of a firm. Both x- and y-axis variables are demeaned from 4-digit NACE industry and region means and adjusted for firm size, i.e. employment. The fitted curve is obtained from local polynomial regression with Epanechnikov kernel of (residual) wages. The shaded area shows the 95% confidence intervals.

pliers. In a decomposition exercise, we find that the extensive margin accounts for about 60% of the relation and the intensive margin accounts for the remainder 40%.

The cross-sectional relation in Figure 1 could arise from our proposed mechanism of quality choices as well as from firms' exogenous characteristics.[2]. We use shift-share regressions to provide evidence that firms endogenously respond to shocks. Consider a firm that exports a particular product category to a high-income country, say cotton table linens to Switzerland. An increase in the Swiss imports of these linens from countries *other than Turkey* is associated with an increase in the Turkish firm's average wage, and the average wage of its suppliers and customers. The new employees, suppliers and customers that the firm adds over the years, from 2011 to 2015, had on average higher wages than the firms' existing employees and partners in 2011. Our proposed mechanism may explain these facts: A shock that increases the demand for high-quality output increases the firm's skill intensity and shifts the firm toward skill-intensive trading partners in its production network.

As explained above, the interconnection in firms' quality choices implies that a relatively small (but non-negligible) shock may have a large general equilibrium effect. To

[2]In Burstein and Vogel (2017), for example, a firm's demand for skilled workers depends on its exogenous productivity

evaluate this claim, we develop a quantitative model with heterogeneous firms, endogenous quality choices and endogenous network formation. Like in Kremer (1993), a firm's quality determines its production function. We assume that higher-quality firms are more skill intensive and allow the marginal product of high-quality inputs to be higher in the production of high-quality output. Firms post costly ads to search for customers and suppliers. Firms may imperfectly direct their search toward customers of specific quality levels. A standard matching function aggregates these ads to form the network of firm-to-firm trade.

The model differs from previous network models (below) in two aspects. First is its use of log-supermodular shifters to generate assortative matching in the network. We follow Teulings (1995) and Costinot and Vogel (2010) for labor, Fieler et al. (2018) for material inputs and apply it anew to directed search.[3] Second, the network in the model is formed from a search and matching set up, typically used in labor.[4] This approach facilitates aggregation as the shares of profit, labor and materials in revenue are constant, and revenue is a log-linear function of the firm's productivity for a given quality.

We estimate the model to manufacturing data using the method of simulated moments. We exclude services because the shift-share regressions above, used in the estimation, applies only to tradable goods. The estimation matches well the joint distribution of firm sales and wages. Larger firms post more ads and have more customers and suppliers, a strong and well-documented empirical regularity.[5] In the data and in the model, the (endogenous) elasticity of sales with respect to number of suppliers and with respect to number of customers is about 0.5.

The model also matches well the patterns of assortative matching on wages. To capture differences in the matching patterns (extensive margin), the model predicts relatively little directed search. About 16 percent of the ads posted by buyers in the lowest quintile of wages are directed to high-wage suppliers. Differences in marginal productivity capturing the spending patterns (intensive margin), in turn, are large. The marginal product of an input in the 90^{th} percentile of the quality distribution is always larger than the marginal product of an input in the 10^{th} percentile. But the ratio of these marginal products is 1.48 when producing output in the 90^{th} percentile of the quality distribution, and the ratio is 1.13 when producing output in the 10^{th} percentile.

In the data, export intensity is generally higher among high-wage firms than among low-wage firms. This pattern holds in the estimated model because the relative demand

[3]See also Milgrom and Roberts (1990) and Costinot (2009) for earlier applications to economics and international trade.

[4]See Mortensen et al. (1986) and Rogerson et al. (2005) for surveys.

[5]See Bernard et al. (2019) and Lim (2019) for example.

for higher-quality is higher abroad. A firm that experiences a ten percent increase in its export demand upgrades quality, hires more skilled workers, and consequently increases its average wage by 0.4 percent. This response is in line with the shift-share regressions in the data.

The network literature has focused on Hicks-neutral shocks, while quality in our model by definition changes the types of inputs that firms use. To depart from Hicks-neutrality, we abstract from dynamics in Lim (2019) and Huneeus (2020) and from asymmetries in network centrality in Hulten (1978), Acemoglu et al. (2012), Baqaee and Farhi (2019). The model features roundabout production, technologies with constant elasticities of substitution, and each firm has a continuum of suppliers and customers. Some of these elements and our exploitation of shocks to international trade appear in open economy models as Lim (2019), Tintelnot et al. (2018), Bernard et al. (2019, 2020), Eaton et al. (2020), Huneeus (2020).

The estimated model is consistent with previous theories and well-established facts in the quality literature. Namely, the production of higher-quality is intensive in skilled labor, as in Schott (2004), Verhoogen (2008), Khandelwal (2010), and in higher-quality inputs, as in Kugler and Verhoogen (2012), Manova and Zhang (2012), and Bastos et al. (2018). Fieler et al. (2018) combines both of these elements to study, like us, the general equilibrium effect of international trade on demand for skills and quality. Our main novel fact, the assortative matching in wages in the network of firms, follows from the combination of these two elements. They complement previous findings on prices with direct information on the extent to which skill-intensive, high-wage firms trade with each other. In this sense, our fact is akin to Voigtländer (2014) who shows that skill-intensive sectors use intensively inputs from other skill-intensive sectors in the United States.

The rest of the paper is organized as follows. Section 2 describes our data and the novel empirical facts. Section 3 develops a closed-economy model of firm endogenous quality choices and network formation. We also lay out the basic model solution procedures and identification argument. Section 4 extends the model to a small open economy by which we implement our baseline estimation and connect to the empirical regressions in Section 2. Section 5 reports our estimation results and their quantitative implications.

2 Data and Empirical Facts

2.1 Data

In the empirical analysis, we combine five micro-level administrative datasets from Turkey, all of which are maintained by the Ministry of Industry and Technology (MoIT).[6] These are (1) domestic firm-to-firm trade transactions data; (2) firm-level balance sheet and income statement data; (3) firm registry; (4) linked employer-employee data; and (5) firm-product-destination level customs data. All five datasets use the same unique firm identifier. For most of the descriptive analysis and moments, we rely on cross-section data for the year 2015. In the rest of the analysis, we use panel data for the 2011-2015 period.

The first dataset is collected by the Ministry of Finance for the purpose of calculating and collecting the value added tax (VAT). It covers all domestic firm-to-firm transactions as long as the total value of transactions for a seller-buyer pair exceeds 5,000 Turkish Liras (TLs) (about $1,800 based on the average exchange rate in 2015) in a given year.

The second dataset that we use in the empirical analysis is detailed firm-level balance sheet and income statement data. For the purpose of our exercise, we use data on gross, domestic, and foreign sales of firms.

The firm registry informs us about the location (province level) and industry of operation of firms in the sample. Industries are reported according to the 4-digit NACE classification, which is the standard industry classification system used in the EU countries.

We merge the three firm-level datasets described above with linked employer-employee data collected by the Social Security Institution. This dataset informs us about quarterly wage payments received by each worker employed by a firm, as well as their occupations (according to 4-digit ISCO classification), age and gender. Each worker is assigned a unique identifier, allowing us to trace them across firms and over time.

Finally, the customs data available at MoIT reports the value of Turkish exports disaggregated by firm, destination country, and 10-digit Harmonized System product code. We aggregate the annual data at the level of firm, country, and 4-digit HS product code, and supplement it with annual data on bilateral trade flows at the same level of product disaggregation available from BACI.

We restrict our estimation sample to manufacturing firms and track all transactions between those firms. We aggregate their purchases from (and sales to) wholesalers, retail-

[6] The empirical analysis in this paper is based on confidential data accessed on the premises of MoIT. Access to these data requires a special permission involving a background check and the results can only be exported upon approval by the authorized staff.

ers and service firms into a single input category so that we do not track transactions that involve these three types of firms.[7] We drop firms that do not report balance sheets or income statements, as micro entities keep records using single-entry bookkeeping system. This leaves us about 78,000 manufacturing firms in 2015.

2.2 Descriptive Facts

As explained in the introduction, our paper is motivated by two empirical observations. First, the average wage paid by a firm is positively and strongly correlated with the average wage paid by its suppliers. Second, domestic trade links and trade values generated by high-wage firms are disproportionately destined for other high-wage firms.

As it is widely accepted in the literature, we use average wage paid by a firm to its workers a good proxy for firm's quality. We use two alternative measures of firm-level wages. First, we construct firm-level wage as firm's total monthly wage bill divided by the total number of workers, $wage_f$. An alternative firm-level wage variable is constructed using the linked employer-employee data. To do so, we calculate the average value of monthly wage received by each worker in a given firm ($wage_{ef}$). Next, we adjust $wage_{ef}$ for the worker's occupation, age and gender by running the following regression:

$$\ln wage_{ef} = \beta_1 Age_e + \beta_2 Gender_e + \alpha_o + e_{ef}, \tag{1}$$

where α_o denotes occupation fixed effects at the 1-digit ISCO level. We recover the residuals from the above regression (e_{ef}) and calculate its median within a firm.

Using the firm-level wage measure, we also construct average supplier and buyer wages. Denoting the set of suppliers of firm f by Ω_f^S, average supplier wage is defined as follows:

$$\ln wage_f^S = \sum_{\omega \in \Omega_f^S} \ln wage_\omega s_{\omega f}, \tag{2}$$

where ω indexes suppliers, and $s_{\omega f}$ is the share of f's purchases from supplier ω. Similarly, average buyer wage is defined as

$$\ln wage_f^B = \sum_{\omega \in \Omega_f^B} \ln wage_\omega b_{\omega f}, \tag{3}$$

where Ω_f^B denotes the set of buyers of firm f, and $b_{\omega f}$ is the share of f's domestic sales to

[7] We drop the following industries on both sides of transactions: finance, insurance, utilities and public services.

buyer ω. For completeness, we also construct unweighted averages of supplier and buyer wages for each firm.

A striking pattern in the domestic production network in Turkey is that high-wage manufacturing firms supply relatively more material inputs from other high-wage manufacturing firms. In other words, there is a strong positive assortative matching on wages.[8] To the extent that firm-level wage is a good proxy for firm's quality, Figure 1 presents a pattern in line with the conjecture that high-quality firms are more likely to partner with each other in the production network. The same pattern holds when we include manufacturing as well as service firms in the sample (see Figure A2). On the other hand, as presented in Figures A1 and A2, we do not observe a similar matching pattern on firm's sales or network size: while there is no systematic relationship between a firm's revenue and the revenue of its suppliers, there exists a weak negative relationship between the number of suppliers of a buyer and the average number of customers of its suppliers. The latter has also been reported by Bernard et al. (2019).

We also investigate the presence of positive assortative matching between buyer's wage and the average wage paid by its suppliers using regression analysis. In particular, we estimate variants of the following equation:

$$\ln wage_f^S = \beta \ln wage_f + \alpha_{sr} + e_f, \tag{4}$$

where the operator sr refers to industry (4-digit NACE level) and province pairs. Adding industry-province fixed effects controls for, among others, industry specific occupation composition and regional variations in wages. Results are presented in Table 1. Specification presented in the first column does not include any control variables. The estimate on buyer's wage is economically and statistically significant: a 10 percent increase in average buyer's wage is associated with an almost 3 percent increase in average supplier wages. Adding industry-province fixed effects in the second column leads to only a slight decrease in this estimate. In column (3), we control for the buyer's size. Assuming that larger firms are more likely to be high-productive, they could pay higher wages to their workers and supply higher-quality inputs. As expected, the estimate of the coefficient on the buyer's wage smaller than in the first two columns. However, it is still highly statistically and economically significant. Finally, the last column estimates the baseline specification for the full sample that includes manufacturing as well as service firms. The estimated

[8] Since wages are highly correlated with firm size, and may vary across industries and regions within a country, we use the residuals from the regression of the logarithm of wages (as well as the average value of supplier wages) on firm size (proxied by employment), province and industry fixed effects.

Table 1: Assortative Matching on Wages

Dependent variable: $\ln wage_f^S$

	Manufacturing firms			All firms
	(1)	(2)	(3)	(4)
$\ln wage_f$	0.294	0.259	0.188	0.241
	(0.013)	(0.012)	(0.009)	(0.013)
$\ln employment_f$			0.044	
			(0.003)	
R^2	0.095	0.173	0.199	0.150
N	77,418	77,418	77,418	410,608
Fixed effects		ind-prov	ind-prov	ind-prov

Notes: Wage is defined as the average value of monthly payments per worker. Denoting the set of suppliers of firm f by Ω_f^S, average supplier wage is defined as follows: $\ln wage_f^S = \sum_{\omega \in \Omega_f^S} \ln wage_\omega s_{\omega f}$, where ω indexes suppliers, and $s_{\omega f}$ is the share of f's purchases from supplier ω. Ind and prov refer to 4-digit NACE industries and provinces, respectively. Robust standard errors are clustered at 4-digit NACE industry level.

coefficient on buyer's wage is very close to the baseline estimate presented in column (2).[9]

Next, we decompose the estimated sorting coefficient into an extensive and intensive margin. In particular, we take the weighted average of supplier wages as defined in equation (2) and re-write it as the sum of the unweighted average of supplier wages and a residual term as follows:

$$\underbrace{\sum_{\omega \in \Omega_f} \frac{1}{|\Omega_f|} \ln wage_\omega}_{\text{Extensive margin}} + \underbrace{\sum_{\omega \in \Omega_f} (s_{\omega f} - 1/|\Omega_f|)(\ln wage_\omega - \sum_{\omega' \in \Omega_f} (1/|\Omega_f|) \ln wage_{\omega'})}_{\text{Intensive margin}} \tag{5}$$

The extensive margin will matter to the extent that supplier networks of high- and low-quality buyers differ in quality from each other. The intensive margin term will be positive if a high-quality firm buys disproportionately more from high-quality suppliers that pay above-average wages to their workers.

Table 2 presents the results of the decomposition exercise applied to the baseline specification in column (2) of Table 1. It shows that almost 60 percent of the positive sorting on wages between buyers and suppliers is explained by the extensive margin effect: high-quality firms, on average, match with high-quality suppliers in the production network. The remaining 40 percent is explained by an intensive margin effect: even if the set of suppliers was fixed across buyers, high-quality buyers would buy disproportionately more from high-quality suppliers. Our model presented in the following section captures

[9] Appendix Table A1 shows that using the alternative defintion of firm-level wages as explained in equation (1) produces very similar coefficient estimates.

Table 2: Assortative Matching on Wages: Decomposition

	total (A) $\ln wage_f^S$	extensive margin	intensive margin
$\ln wage_f$	0.259	0.152	0.107
	(0.012)	(0.007)	(0.007)
share of (A)		*59%*	*41%*
R^2	0.173	0.150	0.089
N	77,418	77,418	77,418
Fixed effects	ind-prov	ind-prov	ind-prov

Notes: Wage is defined as the average value of monthly payments per worker. Denoting the set of suppliers of firm f by Ω_f^S, average supplier wage is defined as follows: $\ln wage_f^S = \sum_{\omega \in \Omega_f^S} \ln wage_\omega s_{\omega f}$, where ω indexes suppliers, and $s_{\omega f}$ is the share of f's purchases from supplier ω. Ind and prov refer to 4-digit NACE industries and provinces, respectively. Decomposition is defined in equation (5). Robust standard errors are clustered at 4-digit NACE industry level.

the observed positive assortative matching on wages at both the intensive and extensive margins.[10]

We end the descriptive analysis by presenting concentration of sales and expenditures among high-wage firms in the raw data. To do so, we first group firms in our sample according to their monthly wage payments per worker. In particular, we sort firms in ascending order based on average monthly wage payments and group them into five equal-sized groups (i.e. quintiles). Next, we aggregate firm-to-firm trade links and values at the level of buyer and supplier quintiles. Figure 2 shows the share of trade links and values for each pair of quintiles from buyers' as well as suppliers' perspectives. Two patterns emerge from the data. First, firms in the top quintile of the wage distribution disproportionately supply from and sell to other firms in the top-quintile. Second, the majority of trade partners of top-quintile firms also belong to the top quintile of the wage distribution. These results are consistent with our earlier findings that there is strong positive assortative matching on wages at both the intensive and extensive margins.

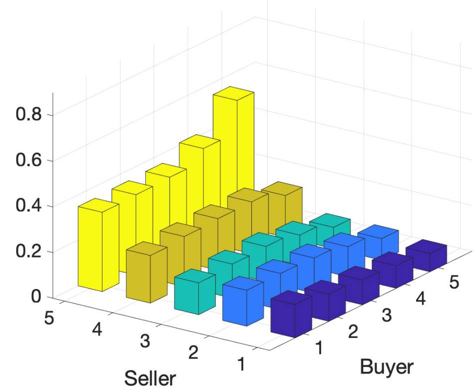
2.3 Effect of Shift-share Trade Shocks

While we control for a large set of fixed effects and firm size in the cross-section sorting regressions presented above, there is still a potentially large number of firm-level confounding unobserved factors (e.g. productivity) that would bias the estimate of the degree of sorting on wages (quality). A priory, given the potentially large number of such factors, it is difficult to predict the direction of the bias in the cross-section estimates.

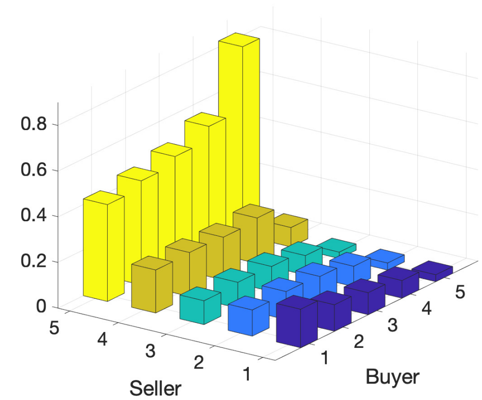
[10] Appendix Table A3 presents the results for matching at the extensive margin on market share and network size.

Figure 2: Firm-to-firm Trade Links and Values by Quintile

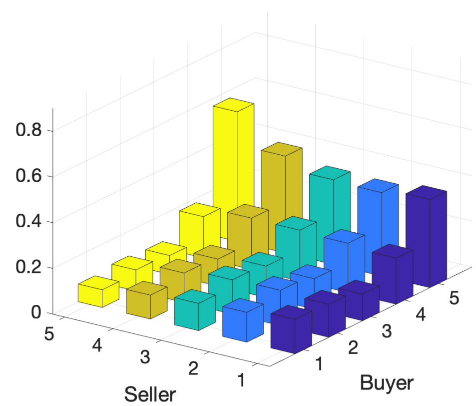
(a) Share of suppliers



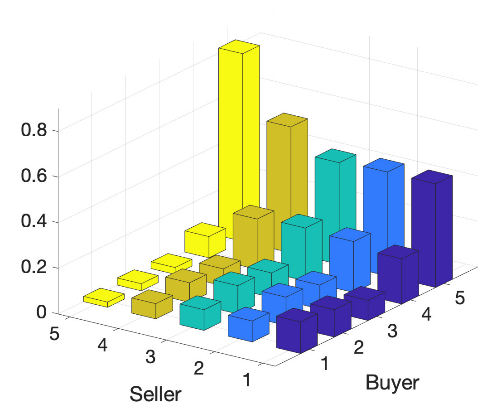
(b) Spending shares



(c) Share of buyers



(d) Sales shares



Notes: Sample includes manufacturing buyers and suppliers. Firms are sorted according to the average value of their monthly payments per worker, and grouped into five equal-sized groups. For each buyer (supplier) quintile, expenditures (sales) and number of suppliers (buyers) are aggregated at the level of supplier (buyer) quintile. Buyer and supplier quintiles are shown on x- and y-axis while z-axis shows the corresponding shares. For instance, in panel (a), values on the z-axis show for each buyer quintile on the x-axis the share of suppliers that belong to the wage quintiles on the y-axis.

To address the concerns discussed above, we need to instrument the firm-level wages. In particular, we need a variable that would capture some exogenous variation in the buyer's incentive to upgrade its quality, proxied by its wage in equation (4), which does not have a direct effect on the incentive of its suppliers to change their own quality. One such candidate is changes in relative global demand for quality in products, defined in terms of 4-digit HS product codes, which are already exported by the buyer. Our instrument relies on the exogenous variation in the growth of imports of a product by a given country (called a variety) from the rest of world, excluding Turkey as a supplier. Given the level of aggregation, it is very unlikely that Turkey has market power in the supply of any product category. To capture the quality bias in demand, we weight the rate of growth in variety-level imports by the per capita income of the destination country. This strategy can be justified by the findings of a large number of empirical papers (e.g. Hallak (2006)), which suggest that high-income countries import relatively more high-quality goods compared to low-income countries.

As in Adao et al. (2019), we start with a regression of shift-shares, which corresponds to the first-stage of the 2SLS model:

$$\Delta \ln y_f \quad = \quad \delta \underbrace{\left(\sum_{c=1}^{C} \sum_{k=1}^{K} x_{ckf} Z_{ck} \right)}_{\text{ExportShock}_f} + \alpha_{sr} + \epsilon_f, \tag{6}$$

where the dependent variable, $\Delta \ln y_f$, is the logarithmic change in firm-level wages or (domestic) sales between 2011-2012 and 2014-2015; c is a country that trades with Turkey, k is a 4-digit HS product category, and δ is the parameter of interest. The weights x_{ckf} are constructed as the share of firm f's exports of product k to importer c in its total sales in 2010. These weights do not add up to one since we do not include domestic sales. The instruments Z_{ck} are defined as:

$$Z_{ck} \quad = \quad \Delta(\ln \text{Imports}_{ck} * \ln \text{pcGDP}_c), \tag{7}$$

and they capture the logarithmic change in the value of country c's income-weighted imports of k from the rest of the world (excluding Turkey) between 2011-2012 and 2014-2015, where the importer's per capita income is measured in constant 2010 USD. To highlight the importance of capturing the quality bias in changes in world import demand, we also construct a version of Z_{ck} that does not adjust for the importer's per capita income, which corresponds to the world import demand shocks in Hummels et al. (2014).

The first two columns of Table 3 present the results from estimating equation (6).

Table 3: Effects of Export Shock

	$\Delta \ln \text{wage}_f$ (first stage) (1)	$\Delta \ln$ domestic sales_f (2)	Δexport intensity_f (3)	$\Delta \ln wage_f^S$ OLS (4)	$\Delta \ln wage_f^S$ IV (5)	$\Delta \ln \text{wage}_f$ (first stage) (6)
ExportShock_f	0.042	-0.026	0.0146			
	(0.006)	(0.022)	(0.0023)			
$\Delta \ln \text{wage}_f$				0.085	0.434	
(IV = ExportShock_f)				(0.008)	(0.185)	
ExportShock_f						0.021
(Unadjusted)						(0.006)
F-Stat	43.6	1.409				0.404
N	33,157	33,157	33,157	33,157	33,157	33,157
Fixed effects	ind-prov	ind-prov	ind-prov	ind-prov	ind-prov	ind-prov

Notes: Wage is defined as the average value of monthly payments per worker. Denoting the set of suppliers of firm f by Ω_f^S, average supplier wage is defined as follows: $\ln wage_f^S = \sum_{\omega \in \Omega_f^S} \ln wage_\omega s_{\omega f}$, where ω indexes suppliers, and $s_{\omega f}$ is the share of f's purchases from supplier ω. Δ operator denotes changes between 2011-2012 and 2014-2015. ExportShock_f is a weighted average of changes in (real per capita) income-adjusted imports at the country (c) and 4-digit HS product (k) level between 2011-2012 and 2014-2015, where weights are constructed as the share of firm f's exports of product k to importer c in its total sales in 2010. See equations (6) and (7) for details. ExportShock_f (Unadjusted) is defined similarly except that country-product level import values are not adjusted for the per capita GDP of the destination country. Ind and prov refer to 4-digit NACE industries and provinces, respectively. Robust standard errors are clustered at 4-digit NACE industry level.

The estimate for wages in column (1) implies that a 10 log-point increase in ExportShock leads to a 0.4 log-point increase in firm-level wages. Firms that receive a positive external demand shock that is biased towards high quality varieties upgrade the quality of their workforce.[11] The estimate is statistically significant and the F-statistic is sufficiently high, suggesting that ExportShock should be an informative instrument for wages. Alternatively, we construct our instrument without adjusting import values by importer's income. As the F-statistics presented in the last column of the table shows, this instrument is not informative about the changes in firm-level wages.[12]

Columns (2) and (3) of Table 3 show that while ExportShock does not have a discernable impact on receiving firm's domestic sales, it increases the firm's export intensity, defined as foreign sales as a share of total sales.

Finally, column (5) reports the results from the 2SLS regression, where buyer's wage is instrumented with ExportShock. The estimate is economically and statistically signif-

[11] The result is robust to the inclusion of additional controls such as firm's initial market share, size and export intensity (share of foreign sales in total sales).

[12] This result supports the underlying assumptions of the empirical setup in Hummels et al. (2014), where this shift-share variable is used as an instrument for firm-level exports when studying the effect of exports on wages.

icant: a 10 percent increase in buyer's wage leads to a 4.3 percent increase in (weighted) average of supplier wages. As a benchmark, column (4) reports the OLS estimate obtained from a long-differenced specification. Compared to this OLS estimate as well as the cross-section estimates presented earlier, the 2SLS estimate of wage sorting is noticeably larger, implying an even stronger positive assortative matching on quality in the production network.

To understand the mechanisms behind the 2SLS results, we now investigate the changes in the composition of worker quality at the firm level, as well as that of its suppliers and buyers. First, we check whether a firm that receives a quality-biased export demand shock replaces its low-quality workers with high-quality ones. To do so, we use the linked employer-employee data and compare the average wage received by the firm's new employees and the wage received by the firm's average worker. To alleviate endogeneity concerns, we compare wages of the two groups of workers before the shock. In particular, we identify workers hired by firm f after the shock and calculate the average value of the monthly wage paid to these workers by their former employers:

$$\frac{\sum wage_{e,t=0}}{\text{Number of new workers}},$$

where e indexes the workers hired by firm f after the shock (2014-2015). We compare this to the average wage paid by firm f before the shock, $wage_{f,t=0}$, or the average wage paid by the firm to its former workers, $wage^{\text{former workers}}_{f,t=0}$. First column of Table 4 presents strong evidence that firms that receive a larger quality-biased export demand shock upgraded the quality of their workers. The magnitude of the estimate reported in the lower panel suggests that the effect is primarily driven by replacing low-quality (low-wage) workers with high-quality (high-wage) ones.

In columns (2) and (3) of Table 4, we investigate whether ExportShock caused a similar change in the quality composition of the receiving firm's suppliers and buyers. To do so, we identify the newly matched suppliers and buyers of the firm after the shock. Then, we compare the average wages paid by these new matches before the shock to either the unweighted average of the firm's suppliers (buyers), or average wages paid by the firm's former suppliers (buyers) before the shock. The results suggest that firms that receive a larger quality-biased export demand shock shifted the composition of their suppliers and buyers towards higher-quality (higher-wage) firms.

Table 5 presents additional evidence on the quantitative importance of the impact of ExportShock on affected firms' input (or buyer) composition. In column (1), the equation in (6) is estimated with the dependent variable replaced with the share of newly hired

Table 4: Effects of Export Shock on Composition of Inputs

Log of	Average wage of new workers relative to all workers at $t = 0$	Average wage paid by new suppliers relative to all suppliers at $t = 0$	Average wage paid by new buyers relative to all buyers at $t = 0$
ExportShock$_f$	0.0189	0.0241	0.0303
	(0.010)	(0.007)	(0.009)
R^2	0.0531	0.0439	0.0434
Log of	Average wage of new workers relative to former workers at $t = 0$	Average wage paid by new suppliers relative to former suppliers at $t = 0$	Average wage paid by new buyers relative to former buyers at $t = 0$
ExportShock$_f$	0.0247	0.0220	0.0305
	(0.009)	(0.012)	(0.009)
R^2	0.0542	0.0662	0.0683
N	33157	33157	33157
Fixed effects	ind-prov	ind-prov	ind-prov

Notes: Wage is defined as the average value of monthly payments per worker. ExportShock$_f$ is a weighted average of changes in (real per capita) income-adjusted imports at the country (c) and 4-digit HS product (k) level between 2011-2012 and 2014-2015, where weights are constructed as the share of firm f's exports of product k to importer c in its total sales in 2010. Time $t = 0$ represents the period before the export shock, 2011-2012. Ind and prov refer to 4-digit NACE industries and provinces, respectively. Robust standard errors are clustered at 4-digit NACE industry level.

Table 5: Effects of Export Shock on Composition of Inputs: Additional evidence

Share of new	Workers with wages higher than f's avg. wage at $t = 0$	Suppliers with wages higher than f's avg. supplier wage at $t = 0$	Buyers with wages higher than f's avg. buyer wage at $t = 0$
ExportShock$_f$	0.421	0.152	0.169
	(0.154)	(0.0690)	(0.0657)
R^2	0.167	0.0403	0.0394
N	33157	33157	33157
Fixed effects	ind-prov	ind-prov	ind-prov

Notes: Wage is defined as the average value of monthly payments per worker. ExportShock$_f$ is a weighted average of changes in (real per capita) income-adjusted imports at the country (c) and 4-digit HS product (k) level between 2011-2012 and 2014-2015, where weights are constructed as the share of firm f's exports of product k to importer c in its total sales in 2010. Time $t = 0$ represents the period before the export shock, 2011-2012. Ind and prov refer to 4-digit NACE industries and provinces, respectively. Robust standard errors are clustered at 4-digit NACE industry level.

workers after the shock, who received higher monthly wages than the firm's average worker before the shock. The coefficient on ExportShock is estimated to be positive and highly statistically significant, concurring with the results reported in Table 4 that quality-biased demand shock leads to a quality upgrading of the receiving firm's workforce. Results presented in columns (2) and (3) suggest that the shock generates similar compositional changes in the receiving firm's network of suppliers and buyers.

2.4 Other Characteristics of the Network

We use the 2015 cross-section of manufacturing firms to generate data moments for the structural estimation of the model. The sample covers almost 77,500 firms, and we track all transactions between those firms. We aggregate their purchases from (and sales to) wholesale, retail and service firms into a single input category and do not track transactions that involve them. On average, almost half of domestic sales and purchases of firms in our sample are accounted for by trade partners operating in wholesale, retail and service industries.[13]

Almost a third of manufacturing firms in our sample are exporters. On average, foreign sales account for about a quarter of their total sales. As expected, high-wage firms are more likely to be exporters: while only 8 percent of firms in the bottom quintile of the wage distribution are exporters, it increases to 57 percent in the top quintile.

In our manufacturing-to-manufacturing network, distributions of number of buyers and suppliers are (i) highly skewed, and (ii) increasing in wages. For instance, the ratio of average number of buyers (and suppliers) for firms in the top quintile of the wage distribution is almost four times higher than the corresponding average in the bottom quintile.

While the number of network connections increases with firm-level wages, their most important determinant is firm size (measured in terms of total sales) . Table 6 reports the elasticity of buyer and supplier connections with respect to sales. Three important points are in order here. First, firm size itself explains more than a third of variation in the number of buyers, and more than 60 percent of variation in the number of suppliers (columns (1) and (4)). Second, the result is not driven by the industry composition of firms as adding 4-digit industry fixed effects does not notably change the estimate of the elasticity or the R^2 of the regression (columns (2) and (5)). Finally, controlling for firm size, wages do not have a significant explanatory power for the number of network connections (columns (3) and (6)).

[13]Our data also inform us about firm-level imports. However, in the sample, the average share of imported inputs in total material purchases is only 0.05, and the median is zero.

Table 6: Firm Sales and Network Connections

Number of	Customers			Suppliers		
	(1)	(2)	(3)	(4)	(5)	(6)
$\ln Sales_f$	0.440	0.462	0.459	0.577	0.593	0.590
	(0.016)	(0.013)	(0.013)	(0.011)	(0.009)	(0.009)
$\ln Wage_f$			0.278			0.208
			(0.211)			(0.175)
R^2	0.328	0.472	0.472	0.609	0.645	0.645
N	77,418	77,418	77,418	77,418	77,418	77,418
Fixed effects		Ind	Ind		Ind	Ind

Notes: Wage is defined as the average value of monthly payments per worker. All variables are in logarithms. Ind refers to 4-digit NACE industries and provinces. Robust standard errors are clustered at 4-digit NACE industry level.

3 The Closed-Economy Model

The model captures positive assortative matching, at the intensive and extensive margins, in a network endogenously formed through search and matching. To highlight these novel features, we present a closed economy.

There are two sectors: Services and manufacturing. The service sector is perfectly competitive. It produces a homogeneous good with constant returns to scale using manufacturing inputs. The manufacturing sector has heterogeneous firms and free entry.

Like in Kremer (1993), a manufacturing firm chooses its production function, which determines the marginal product of its labor and material inputs. Here, the choice is from a line segment $Q \subset \mathbb{R}_+$ and we refer to it as the firm's quality. All tasks performed in a firm of quality $q \in Q$ are also indexed by q. For example, if q is associated with management practices or an integrated computer software, all workers in production or not need to abide by such practices and use the software. Earnings per worker and the marginal product of higher-q inputs may be higher in the production of higher-q output.

Manufacturing firms post ads to find suppliers and customers and are matched to form the firm-to-firm network. Firms may imperfectly direct these ads to other firms' quality levels. Like Lim (2019), each firm is matched with a continuum of suppliers and customers, and it charges the monopolistic-competition markup.

The manufacturing sector is in Section 3.1. Section 3.1.1 sets up the firm's problem, and Section 3.1.2 aggregates firm choices to form the network. The service sector is in section 3.2, and the equilibrium is in section 3.3. Whenever convenient, we assume functions are continuous, differentiable, and integrable. Parametric assumptions in the estimation ensure these conditions.

3.1 Manufacturing

3.1.1 Entry and the Firm's Problem

The revenue of a firm with quality q, price p and a mass v of ads to find customers (v stands for visibility) is

$$p^{1-\sigma} v D(q) \tag{8}$$

where $\sigma > 1$ is the elasticity of substitution between manufacturing varieties and $D(q)$ is an endogenous demand shifter.

The cost of a bundle of inputs to produce quality q when the firm posts a measure m of ads to find manufacturing suppliers is

$$C(m, q) = w(q)^{1-\alpha_m-\alpha_s} P_s^{\alpha_s} [m^{1/(1-\sigma)} c(q)]^{\alpha_m} \tag{9}$$

where $\alpha_m, \alpha_s > 0$ are Cobb-Douglas weights with $\alpha_m + \alpha_s \in (0,1)$, P_s is the price of the service good, $w(q)$ is the wage rate per efficiency unit of task q, and $c(q)$ is the cost of a bundle of manufacturing inputs when the firm posts a measure one of ads to find suppliers. The marginal cost of the firm is $C(m, q)/z$ where z is her productivity.

The cost of posting v ads to find customers and m ads to find suppliers is respectively

$$\begin{aligned} &w(q) f_v \frac{v^{\beta_v}}{\beta_v} \\ &w(q) f_m \frac{m^{\beta_m}}{\beta_m} \end{aligned} \tag{10}$$

where f_m, f_v, β_m, and β_v are positive parameters with $\beta_v > 1$, $\beta_m > \alpha_m$.

From (8), the firm charges markup $\sigma/(\sigma-1)$ over marginal cost. Given q, she chooses v, m to maximize profit:

$$\max_{v,m} \frac{v m^{\alpha_m}}{\sigma} \left[\frac{\sigma}{\sigma-1} \frac{C(1,q)}{z}\right]^{1-\sigma} D(q) - w(q) f_v \frac{v^{\beta_v}}{\beta_v} - w(q) f_m \frac{m^{\beta_m}}{\beta_m} \tag{11}$$

Rearranging the first order conditions, the firm's revenue x, mass of ads to find customers

v and to find suppliers m, and price p are functions of productivity z and quality q:

$$
\begin{aligned}
x(z,q) &= \Pi(q) z^{\gamma(\sigma-1)} \\
v(z,q) &= \left(\frac{x(z,q)}{\sigma f_v w(q)}\right)^{1/\beta_v} \\
m(z,q) &= \left(\frac{x(z,q)}{\sigma f_m w(q)/\alpha_m}\right)^{1/\beta_m} \\
p(z,q) &= \frac{\sigma}{\sigma-1}\frac{C(m(z,q),q)}{z}
\end{aligned}
\tag{12}
$$

where

$$
\begin{aligned}
\Pi(q) =& [\sigma w(q)]^{1-\gamma}\left[D(q)\left(\frac{\sigma}{\sigma-1}C(1,q)\right)^{1-\sigma}\left(\frac{f_m}{\alpha_m}\right)^{-\alpha_m/\beta_m} f_v^{-1/\beta_v}\right]^{\gamma} \\
\gamma =& \frac{\beta_v\beta_m}{\beta_v(\beta_m-\alpha_m)-\beta_m} > 1.
\end{aligned}
\tag{13}
$$

The elasticity of revenue $x(z,q)$ with respect to productivity z is $\gamma(\sigma-1)$. It is greater than $(\sigma-1)$ because more productive firms post more ads m and v.

Entry and Technology Choice A large mass of entrepreneurs may pay f units of the service good to create a new variety. Upon entry, each entrepreneur draws, independently from a common distribution, a random variable ω that determines her productivity at each $q \in Q$ through a function $z(q,\omega)$. We parameterize $\omega = (\omega_0, \omega_1) \in \mathbb{R}^2$ and

$$
z(q,\omega) = \exp\left\{\omega_0 + \omega_1 \log(q) + \overline{\omega}_2[\log(q)]^2\right\}
\tag{14}
$$

where $\overline{\omega}_2$ is a parameter common to all firms. Since profit (11) is a share $1/(\gamma\sigma)$ of revenue, firm ω chooses q to maximize revenue:

$$
q(\omega) = \arg\max_{q\in Q}\left\{x(z(q,\omega),q)\right\}.
\tag{15}
$$

Function $\Pi(q)$ is by construction (below) continuous in q so that (15) is the maximization of a continuous function in a compact set Q.

Let N be the equilibrium mass of firms, and take total manufacturing absorption as the numeraire. Then, average sales per firm is $1/N$ and free entry implies

$$
N = (\gamma\sigma f P_s)^{-1}.
\tag{16}
$$

3.1.2 Manufacturing firm-to-firm trade

Firm choices above give rise to the measure

$$J(z,q) = N\text{Prob}\left\{\omega : z(q(\omega),\omega) \leq z \quad \text{and} \quad q(\omega) \leq q\right\}. \tag{17}$$

Assume J has a density, denoted with $j(z,q)$. Next we put structure in the model to derive the endogenous terms in $\Pi(q)$ as functions of J and firm outcomes in (12). In this section, manufacturing firm-to-firm trade determines the input cost $c(q)$ and the component of demand $D(q)$ that comes from manufacturing.

Production Function Following Fieler et al. (2018), a firm of quality q matched with a set of suppliers Ω aggregates its manufacturing inputs with a constant elasticity of substitution (CES) function:

$$Y(q,\Omega) = \left[\int_{\omega\in\Omega} y(\omega)^{(\sigma-1)/\sigma}\phi_y(q,q(\omega))^{1/\sigma} d\omega\right]^{\sigma/(\sigma-1)} \tag{18}$$

where $y(\omega)$ is the quantity of input ω and function $\phi_y(q,q')$ governs the productivity of an input of quality q' when producing an output of quality q. We parameterize

$$\phi_y(q,q') = \frac{\exp(q'-\nu_y q)}{1+\exp(q'-\nu_y q)}, \tag{19}$$

which is increasing in input quality and decreasing in output quality if $\nu_y > 0$. It is also log-supermodular if $\nu_y > 0$. Then, the ratio of the firm's demand for any two inputs 1 and 2 with prices $p(1)$ and $p(2)$ and qualities $q(1) > q(2)$,

$$\frac{y(1)}{y(2)} = \left(\frac{p(1)}{p(2)}\right)^{-\sigma} \frac{\phi_y(q,q(1))}{\phi_y(q,q(2))}, \tag{20}$$

is strictly increasing in the producing firm's quality q. Higher-quality firms spend relatively more on higher-quality firms for any set of input suppliers.

Network We introduce directed search. Buyers can only see the selling ads that are directed to their own q. The ads posted by a seller with quality q' are distributed across buyers' qualities $q \in Q$ according to function $\phi_v(q;\mu(q'))$ which we parameterize as the density of a normal distribution with variance parameter ν_v and mean $\mu(q') \in Q$ chosen by the seller posting the ads. Below, this choice depends only on the seller's quality, justifying the notation $\mu(q)$.[14]

This set up implies that there's a continuum of matching submarkets, one for each buyer quality. In the submarket of buyers with quality $q \in Q$, the total measure of ads posted by buyers and sellers is respectively:

$$M(q) = \int_Z m(z,q)j(z,q)dz \tag{21}$$

$$V(q) = \int_Q \phi_v(q,\mu(q'))\overline{V}(q')dq' \tag{22}$$

where $\overline{V}(q)$ is the measure of ads posted by sellers of quality q:

$$\overline{V}(q) = \int_Z v(z,q)j(z,q)dz.$$

A standard matching function (Petrongolo and Pissarides (2001)) determines measure of matches with buyers of quality q:

$$\tilde{M}(q) = V(q)\left[1 - \exp(-\kappa M(q)/V(q))\right]. \tag{23}$$

where parameter $\kappa > 0$ captures the efficiency in the matching market. The success rate of ads is $\theta_v(q) = \tilde{M}(q)/V(q)$ for sellers and $\theta_m(q) = \tilde{M}(q)/M(q)$ for buyers.

Input Costs and Demand Using (22), for each ad posted by a buyer of quality q, the probability of finding a supplier of with productivity-quality (z',q') is

$$\theta_m(q)\frac{\phi_v(q,\mu(q'))v(z',q')j(z',q')}{V(q)} \tag{24}$$

Combining with the CES price associated with production function (18), a bundle of manufacturing inputs used by a firm of quality q posting a measure one of ads to find suppliers costs:

$$c(q) = \left[\frac{\theta_m(q)}{V(q)}\int_Q \phi_y(q,q')\phi_v(q,\mu(q'))P(q')^{1-\sigma}dq'\right]^{1/(1-\sigma)} \tag{25}$$

where

$$P(q) = \left[\int_Z p(z,q)^{1-\sigma}v(z,q)j(z,q)dz\right]^{1/(1-\sigma)} \tag{26}$$

[14] One dimension of directed search, whether from buyers or sellers, is enough to generate assortative matching at the extensive margin.

takes into account the greater visibility of firms that post more selling ads $v(z, q)$.

We now turn to demand. A firm with quality q posts price p and v selling ads centered around $\mu \in Q$. From (21), the measure of buyers with (z', q') matched to the firm is

$$v\theta_v(q')\phi_v(q', \mu)\frac{m(z', q')j(z', q')}{M(q')}$$

Conditional on the match, the firm's sales to a buyer with (z', q') is

$$\phi_y(q', q)\left(\frac{p}{c(q')}\right)^{1-\sigma}\frac{\alpha_m(\sigma-1)}{\sigma}\frac{x(z', q')}{m(z', q')}$$

Multiplying these last two expressions and summing over buyers (z', q'), the sales of the firm to other manufacturing firms is

$$p^{1-\sigma}v\tilde{D}(q, \mu)$$

$$\text{where}\quad \tilde{D}(q, \mu) = \int_Q \frac{\theta_v(q')}{M(q')}\phi_y(q', q)\phi_v(q', \mu)c(q')^{\sigma-1}X_m(q')dq', \tag{27}$$

$$X_m(q) = \frac{\alpha_m(\sigma-1)}{\sigma}\int_Z x(z, q)j(z, q)dz \tag{28}$$

$X_m(q)$ is the total absorption of manufacturing inputs by buyers of quality q.[15]

The firm's direction of search $\mu(q)$ maximizes the demand component associated with sales to other manufacturing firms:

$$D_m(q) = \max_{\mu \in Q}\{\tilde{D}(q, \mu)\}. \tag{29}$$

3.2 Service Sector and Final Demand

Service firms aggregate manufacturing inputs into a homogeneous good sold in a perfect market. Their production function is given by $Y(0, \Omega)$ in (18). There's a fixed set of

[15] We may also derive $\tilde{D}_m(q, \mu)$ from (25). The share of spending on materials by buyers of quality q' allocated to a supplier with price p, quality q, and v ads targeted at buyers of quality μ is

$$\theta_m(q')\frac{\phi_y(q', q)\phi_v(q', \mu)vp^{1-\sigma}}{V(q)c(q')^{1-\sigma}}.$$

Multiplying by domestic spending on materials $X_m(q')$ and integrating over buyers q', demand is

$$vp^{1-\sigma}\int_Q \frac{\theta_m(q')}{V(q')}\phi_y(q', q)\phi_v(q', \mu)c(q')^{\sigma-1}X_m(q')dq'$$

which is the expression above since $\theta_m(q)/V(q) = \theta_v(q)/M(q)$.

service firms, each endowed with a fixed measure of $\overline{m}$ of manufacturing suppliers.[16] The probability that a service firm matches with a supplier with productivity-quality (z, q) is proportional to the measure of selling ads:

$$\frac{v(z,q)j(z,q)}{V_T}$$
$$\text{where} \quad V_T = \int_Q \overline{V}(q)dq \tag{30}$$

Then, the price index of the service good Y_s is

$$P_s = \left[\frac{\overline{m}}{V_T}\int_Q \phi_y(0,q)P(q)^{1-\sigma}dq\right]^{1/(1-\sigma)} \tag{31}$$

Total sales to the service sector by a manufacturing firm with price p, quality q, posting v ads in Home to find customers is:

$$\frac{v}{V_T}\left(\frac{p}{P_s}\right)^{1-\sigma}\overline{m}\phi_y(0,q)X_s$$

where X_s is total absorption of services. Using (31), these sales are

$$p^{1-\sigma}vD_s(q) \tag{32}$$
$$\text{where} \quad D_s(q) = \phi_y(0,q)\left[\int_Q \phi_y(0,q')P(q')^{1-\sigma}dq'\right]^{-1} X_s$$

which does not depend on $\overline{m}$.

Households consume only the service good. Then service absorption X_s is the share of manufacturing absorption in (11) allocated to labor income plus profits:

$$X_s = 1 - \frac{(\sigma-1)}{\sigma}\alpha_m.$$

3.3 Equilibrium

The demand shifter faced by a manufacturing firm in (8) is the sum of demand from service (32) and manufacturing firms (29):

$$D(q) = D_m(q) + D_s(q). \tag{33}$$

[16]Parameter $\overline{m}$ preserves the log linear form of demand in (8). Ads posted by sellers v would be irrelevant if service firms observed all varieties. Making the service sector more symmetric to manufacturing, with imperfect competition, and costly matches, would complicate the model without new insights.

We take the supply of efficiency units of labor to produce task q to be an exogenous function $L(q, w)$ where w is the whole wage schedule, $w(q)$ for all $q \in Q$. Labor markets clear if

$$L(q, w) = \frac{1}{w(q)\sigma} \left[(1 - \alpha_m - \alpha_s)(\sigma - 1) + 1 - \frac{1}{\gamma} \right] \int_Z x(z, q) j(z, q) dz \tag{34}$$

where the constant is the labor share in manufacturing production in (11). In our empirical application, we assume that average earnings per firm is strictly increasing in q. Using a Roy (1951) model, Teulings (1995) provides a micro foundation for $L(q, w)$ and for this estimation assumption (see Appendix A).[17]

Definition An **equilibrium** is a mass of firms N, a measure function $J(z, q)$, and functions $w(q)$, $\theta_m(q)$, $\theta_v(q)$, $c(q)$, $D(q)$ satisfying the following conditions:

1. Free entry (16).
2. Labor market clearing (34).
3. Firms maximize profits. Firm ω chooses $q(\omega)$ in (15) and has productivity $z(\omega) = z(q(\omega), \omega)$ at the optimal. Its sales, measure of ads, and prices are $x(z(\omega), q(\omega))$, $m(z(\omega), q(\omega))$, $v(z(\omega), q(\omega))$, and $p(z(\omega), q(\omega))$ in (12). The direction of selling ads $\mu(q(\omega))$ solves (29).
4. The measure $J(z, q)$ is consistent with firm choices (17).
5. The success rate of ads $\theta_m(q) = \tilde{M}(q)/M(q)$ and $\theta_v(q) = \tilde{M}(q)/V(q)$ where $M(q)$, $V(q)$ and $\tilde{M}(q)$ are in (21), (22) and (23). Functions $c(q)$ and $D(q)$ satisfy (25) and (33).

3.4 Properties of the Network

The model is designed to capture the key features of the data in Section 2.1. Under the assumption that earnings per worker is increasing in firm quality, assortative matching in the model's network arises through buyers' and sellers' quality levels.

For a firm with quality q, the measure of its input suppliers of quality q_1 relative to input suppliers of quality q_2 is (integrating (24)):

$$\frac{\phi_v(q, \mu(q_1))\overline{V}(q_1)}{\phi_v(q, \mu(q_2))\overline{V}(q_2)} \tag{35}$$

[17] See also Costinot and Vogel (2010) for an application of Teulings (1995) to international trade.

The firm's average spending on its suppliers of quality q_1 relative to its suppliers of quality q_2 is (integrating (20)):

$$\frac{\phi_y(q,q_1)}{\phi_y(q,q_2)}\left(\frac{P(q_1)}{P(q_2)}\right)^{1-\sigma}\frac{\overline{V}(q_2)}{\overline{V}(q_1)} \tag{36}$$

Multiplying these expressions (or using equation (25)), the ratio of the firm's total spending on the two qualities is:

$$\frac{\phi_v(q,\mu(q_1))}{\phi_v(q,\mu(q_2))}\frac{\phi_y(q,q_1)}{\phi_y(q,q_2)}\left(\frac{P(q_1)}{P(q_2)}\right)^{1-\sigma} \tag{37}$$

These expressions summarize the extensive (35), intensive (36) and total (37) assortative matching in the network. Since the terms $\overline{V}(q)$ and $P(q)$ are common to all buyers, functions ϕ_y and ϕ_v govern assortative matching in the network. By definition, a function ϕ is log-supermodular if $\phi(q,q_1)/\phi(q,q_2)$ is increasing in q whenever $q_1 > q_2$ or equivalently $d^2\log(\phi(q,q'))/dqdq' > 0$. If ϕ_y is log-supermodular ($\nu_y > 0$ in (19)), then higher quality firms spend relatively more on each of its higher quality suppliers (36). This difference drives higher quality suppliers to search more for higher quality customers so that $\mu(q)$ is increasing in q. Recall, ϕ_v is the density of a normal random variable with mean $\mu(q)$ and standard deviation ν_v. Then, $d^2\log(\phi_v(q,\mu(q')))/dqdq' = \mu'(q')/\nu_v > 0$, and higher-quality firms have relatively more higher-quality suppliers in (35).

Larger firms have more trading partners in the data, Table 6. In the model, measure of suppliers $\theta_m(q)m(z,q)$ and customers $\theta_v(q)v(q,z)$ increase with firm sales with elasticities $1/\beta_v$ and $1/\beta_m$ in (12) for a given quality.

3.5 Estimation Strategy of the Closed Economy

We calibrate some parameters and propose a two-stage estimation in Sections 3.5.1 and 3.5.1. We modify the procedure and implement it only in the open economy. An economy is defined by parameters $\{\alpha_m, \alpha_s, \sigma, f_m, f_v, \beta_m, \beta_v, f, \overline{m}, \kappa, \nu_y, \nu_v, \overline{\omega}_2\}$, the bivariate distribution of firms' productivity parameters (ω_0, ω_1) in (14), and labor supply $L(q,w)$.

We calibrate $\{\alpha_m, \alpha_s, \sigma, f_m, f_v, \beta_m, \beta_v, f, \overline{m}\}$. We set $\alpha_m = 0.33$ and $\alpha_s = 0.38$ in (9) to the cost shares of services and manufacturing in the Turkish manufacturing sector. The elasticity of substitution $\sigma = 5$ following Broda and Weinstein (2006). Since search efforts are not observable, we cannot separately identify the cost of a mass one of ads, f_m and f_v, from the matching efficiency κ in (23). We then set $f_m = f_v = 1$. We set $\beta_m = 1/0.59$ and $\beta_v = 1/0.46$ to match the endogenous elasticity of number of suppliers and number of customers with respect to firm sales in Table 6.[18] Parameter $\overline{m}$ is not identified because it governs the theoretical price index P_s in (31) but not the observable sales of manufacturing to service firms and consumers in (32). We pick $\overline{m}$ so that equilibrium $P_s = 1$. We observe worker earnings, but not endowments or wage per efficiency unit of labor. In a cross-section we can set $w(q) = 1$ for all q by judiciously picking the measure of efficiency units of labor. We normalize the equilibrium mass of firms $N = 1$ so that each firm in the data corresponds to a weight $1/1e+05$ of firms in the model, where $1e+05$ is the number of firms in the data. With $N = P_s = 1$, the entry cost in (16) is $f = (\gamma\sigma)^{-1} = 0.069$.

3.5.1 First Stage of Estimation Procedure

We estimate κ, ν_y, ν_v, and the equilibrium distribution of productivity-quality $J(z, q)$, and labor supply $L(q, w)$ at the equilibrium w using the method of simulated moments. We parameterize $J(z, q)$ as follows. Let $Q = [0, 10]$. The marginal distribution of q has a log-normal distribution, truncated in Q, with mean parameter zero and variance parameter ς. The distribution of z conditional on q is log-normal with mean parameter $a_1 \log(q)$ and standard deviation parameter a_2. We simulate the economy for each guess of these six parameters $\{\kappa, \nu_y, \nu_v, a_1, a_2, \varsigma\}$ and iterate over these guesses to match 30 moments from the data.

Simulation procedure Discretize the quality space Q into a grid of T=200 points of equal mass given ς, as suggested by Judd (1998).[19] Start with an initial guess of $c(q) > 0$ and $D(q) > 0$ for all q in the grid and follow steps 1-4:

1. Calculate $C(1, q)$ in (9) and $\Pi(q)$ in (13).

2. Use firm outcomes (12) to calculate aggregate mass of ads $M(q)$ and $V(q)$ in (21) and (22), the mass of matches $\tilde{M}(q)$ in (23) and get the success rates $\theta_m(q)$ and $\theta_v(q)$. Calculate spending on materials $X_m(q)$ in (28) and price indices $P(q)$ in (26).

3. Update the guesses of $c(q)$ and $D(q)$ using (25) and (33).

4. Repeat steps 1-3 until functions $c(q)$ and $D(q)$ converge.

[18] In the data and model, sales are the largest indicator of a firm's number of trading partners so that ignoring wages (or q) provides a good approximation.

[19] Grid points q_i satisfy $F(q_i) = (2i-1)/(2*T)$ for $i = 1, ..., 200$ where F is the cumulative distribution of the truncated log normal.

With the log-normal parametrization of $J(z,q)$ all integrals over z have closed form, including functions $X_m(q)$, $M(q)$ and $\overline{V}(q)$ in step 2. We sum over the grid to estimate numerically integrals over q.[20]

After the simulations, we specify the labor supply $L(q,w)$ to exactly match the distribution of average earnings per worker in the data at the equilibrium wage level $w(q)=1$ for all $q \in Q$. The simulation yields the total demand for efficiency units of labor for all $q \in Q$ in (34). We pick the total supply $L(q,w)$ to match demand for each q, and the endowment per worker in firms with quality q to match the earnings per worker in a firm with wage rank in the data equal to the quality rank of q. See Appendix A for this procedure in the Roy model of Teulings (1995).

Moments We match 30 moments. The coefficients in the extensive and intensive margin regressions in Table 2 (2 moments) We rank firms according to their average wage per worker. By quintile of firm wage, we match:

1. The mean number of suppliers (5 moments) and mean number of customers (5 moments)
2. The share in total network sales (5 moments) and the standard deviation of sales (5 moments).
3. Average of log-wage of suppliers, unweighted (4 moments) and weighted by spending shares (4 moments).

Identification Although all parameters are estimated jointly, the parameters are associated to some moments more closely. The average number of trading partners per firm identifies κ, the efficiency in transforming ads into matches in (23). Total sales and standard deviation by quintile of quality identifies the parameters a_1 and a_2 of the log-normal distributions $J(z,q|q)$.

The third set of moments summarize nonparametrically the total and extensive margins of assortative matching in the network in Table 2 As described in Section 3.4 parameters ν_y and ς govern the intensive margin in (36) through the log-supermodularity of ϕ_y. Parameter ν_v governs the extensive margin (35) through the log-supermodularity of ϕ_v.

[20] For example, from (12) and (28),

$$\begin{aligned} X_m(q) &= \frac{\alpha_m(\sigma-1)}{\sigma}\Pi(q)\int_Z z^{\gamma(\sigma-1)}dJ(z,q|q) \\ &= \frac{\alpha_m(\sigma-1)}{\sigma}\Pi(q)\exp\left(\gamma(\sigma-1)a_1 q + [\gamma(\sigma-1)a_2]^2/2\right) \end{aligned}$$

In principle, our estimation allows for a less parametric approach to inverting the joint distribution of sales and ranking of wages to get $J(z,q)$. We pursued our approach for two reasons. First, the log-normal form of $J(z,q|q)$ yields closed-form integrals that make the simulation very fast, less than 0.1 seconds in Matlab for each parameter guess. Second, we do well in capturing key features of the data (below) even with the parametric restrictions.

3.5.2 Second Stage of Estimation Procedure

In the second stage, we estimate the bivariate distribution of (ω_0, ω_1) given the measure $J(z,q)$ from the first stage and $\overline{\omega}_2$. We discuss the identification of $\overline{\omega}_2$ below.

Recall that we parameterize firm productivity in equation (14) as

$$\log z(q,\omega) = \omega_0 + \omega_1 \log(q) + \overline{\omega}_2[\log(q)]^2$$

where $\overline{\omega}_2$ is a parameter and ω_0 and ω_1 are firm specific. Substituting $z(q,\omega)$ into the firm's quality choice in (15), we have

$$q(\omega) = \arg\max_{q\in Q} \left\{ \gamma(\sigma-1) \left[\omega_0 + \omega_1 \log(q) + \overline{\omega}_2[\log(q)]^2\right] + \log\Pi(q) \right\}$$

Consider any productivity-quality pair (z^*, q^*) with q^* in the interior of Q. The firm ω^* that corresponds to such pair satisfies $z(q^*, \omega^*) = z^*$ and the first order condition:

$$\exp\left[\omega_0^* + \omega_1^* \log(q^*) + \overline{\omega}_2[\log(q^*)]^2\right] = z^* \tag{38}$$

$$\gamma(\sigma-1)\left[\omega_1^* + 2\overline{\omega}_2 \log(q^*)\right] + \frac{\partial \log \Pi(q^*)}{\partial \log(q^*)} = 0 \tag{39}$$

The second order sufficient conditions are

$$2\gamma(\sigma-1)\overline{\omega}_2 - \frac{\partial^2 \log\Pi(q)}{\partial(\log(q))^2} \leq 0 \qquad \text{for all } q. \tag{40}$$

So for any $\overline{\omega}_2$ satisfying (40) and any (z^*, q^*), we can find (ω_0^*, ω_1^*) that satisfies (38) and (39). So, firm ω^* produces quality q^* and productivity z^* in equilibrium.

We estimate the distribution of (ω_0, ω_1) as follows. First, we approximate the term $\partial^2 \log\Pi(q)/\partial(\log(q))^2$ in (40) using the estimate of $\Pi(q)$ in the first stage, and ensure that our choice of $\overline{\omega}_2$ satisfies (40) for all $q \in Q$. Second, for each q in the quality grid of Q, we find the corresponding ω_1 that satisfies (39). The density of the marginal distribution of ω_1 equals the density of the marginal distribution of q in measure $J(z,q)$. The distribution

$J(z,q|q)$ is log normal with parameters a_1 and a_2 estimated in the first stage. Then, the distribution of ω_0 conditional on ω_1 is normal with mean $[(a_1-\omega_1)\log(q)-\overline{\omega}_2(\log(q))^2]$ and standard deviation a_2. This distribution of (ω_0,ω_1) exactly matches the joint distribution of sales and ranking of wages in the simulated data of the first stage.

Parameter $\overline{\omega}_2$ is thus not identified with the cross-sectional distribution of sales and wages. It captures the elasticity of firms choices of q with respect to shocks to the economy. Denote the parameters of the economy as Θ and consider a shock that affects an element Θ_i for a single firm ω. The first order condition (39) implicitly defines the firm's optimal choice $q(\omega)$ as a function of parameter Θ_i:

$$\frac{\partial \log q(\omega)}{\partial \Theta_i} = -\frac{\frac{\partial^2 \log \Pi(q(\omega))}{\partial \log q \partial \Theta_i}}{2\gamma(\sigma-1)\overline{\omega}_2 - \frac{\partial^2 \log \Pi(q(\omega))}{\partial(\log(q))^2}} \tag{41}$$

where the denominator is the second order condition (40) evaluated at the optimal $q(\omega)$. The firm is infinitely elastic to the shock if the second order condition holds with equality and infinitely inelastic as it approaches negative infinity. In the open economy, we interpret the Bartik shocks in Table 3 as such partial equilibrium shocks. We use the regression coefficients to estimate $\partial \log q(\omega)/\partial\Theta_i$ and our estimated economy to get the derivatives of $\Pi(q)$. We can then use (41) to estimate $\overline{\omega}_2$. A key assumption is that the shock does not affect other firms. Otherwise, would affect Π not only directly in the firm's problem, but through other firm's choices in measure J.

4 Open Economy

We embed the model above into a small open economy. The distinctions arise mainly in the manufacturing firm's problem below. Section 4.1 sketches the estimation procedure. Appendix B presents the full model and details the estimation procedure.

Given our empirical focus on exports, we do not model imports of manufactures. Manufacturing firms may export by paying a fixed cost, posting ads abroad and facing an exogenous foreign demand. The service good may be traded with no frictions. The cost of the foreign service good, in terms of the domestic numeraire, is eP^* where P^* is exogenous and e is the real exchange rate. Equilibrium in trade implies $P_s = eP^*$ and imports of services equal exports of manufactures.

Manufacturing firms A large mass of entrepreneurs may pay a fixed cost f to create a new manufacturing variety. Upon entry, an entrepreneur draws (ω_0,ω_1) determining her productivity $z(q,\omega)$ in (14). The entrepreneur chooses $q \in Q$ and then draws a random fixed export cost f_E units of the service good from a common distribution. She then decides her export status $E \in \{0,1\}$, posts ads to search for domestic suppliers, for domestic customers and for foreign customers if $E = 1$. We introduce randomness in the fixed cost because firms in the data with similar size and wages have different export status. And the timing of the information flow makes the two-stage estimation viable, as explained below.

The export revenue of a firm with quality q, price p and v ads to find customers in foreign is

$$p^{1-\sigma} v e^{\sigma} D_F(q) \tag{42}$$

where $D_F(q)$ is an exogenous demand function. The cost of posting v ads in foreign is the same as the domestic cost in (10), $w(q) f_v v^{\beta_v}/\beta_v$. Assuming the same curvature β_v is important to maintain the log linearity in the firm's problem. We assume f_v only to simplify notation since f_v is not identified (Section 3.5).

A firm with quality q, productivity z and export status $E \in \{0,1\}$ chooses the mass of ads to find suppliers m, the mass of ads to find customers v and the share $r_v \in [0,1]$ of the selling ads that are posted domestically:

$$\begin{aligned} \max_{m,v,r_v} \frac{v m^{\alpha_m}}{\sigma} \left[\frac{\sigma}{\sigma-1} \frac{C(1,q)}{z}\right]^{1-\sigma} & [r_v D_H(q) + (1-r_v) E e^{\sigma} D_F(q)] \\ - w(q) f_v [r_v^{\beta} + (1-r_v)^{\beta}] \frac{v^{\beta_v}}{\beta_v} - w(q) f_m \frac{m^{\beta_m}}{\beta_m} \end{aligned} \tag{43}$$

where $C(1,q)$ is the input cost in (9) and $D_H(q)$ is the endogenous domestic demand shifter, denoted with $D(q)$ in the closed economy (equation (8)). The optimal share of ads does not depend on productivity z:

$$\frac{1-r_v(q,E)}{r_v(q,E)} = \left(\frac{E e^{\sigma} D_F(q)}{D_H(q)}\right)^{1/(\beta_v-1)} \tag{44}$$

Given the optimal r_v, problem (43) differs from the closed economy (11) only in the level of demand and cost of posting selling ads v. Then, the profit, labor and input shares are the same as in the closed economy, and the relationship between sales, ads and prices take the form of (12). Total sales is

$$x(z,q,E) = \Pi(q,E) z^{\gamma(\sigma-1)} \tag{45}$$

where

$$\Pi(q,E) = [\sigma w(q)]^{1-\gamma}\left[D(q,E)\left(\frac{\sigma}{\sigma-1}C(1,q)\right)^{1-\sigma}\left(\frac{f_m}{\alpha_m}\right)^{-\alpha_m/\beta_m} f_v^{-1/\beta_v}\right]^{\gamma} \tag{46}$$
$$D(q,E) = \left[D_H(q)^{\beta_v/(\beta_v-1)} + E(e^{\sigma}D_F(q))^{\beta_v/(\beta_v-1)}\right]^{(\beta_v-1)/\beta_v}.$$

Exporting increases a firm's profit more than proportionately to the export demand shifter ($D(q,1) > D_H(q) + e^{\sigma}D_F(q)$) because it increases the firm's incentives to search for suppliers, which lowers its price and in turn increases the firm's incentives to search for domestic customers.

The firm exports if its fixed cost parameter $f_E \leq \overline{f}_E(z,q)$ where

$$\overline{f}_E(z,q) = \frac{z^{\gamma(\sigma-1)}}{\gamma\sigma P_s}\left[\Pi(q,1) - \Pi(q,0)\right]. \tag{47}$$

Denote with Φ the cumulative distribution function of f_E. After observing its productivity $z(q,\omega)$ but before observing f_E, the firm chooses its quality:

$$q(\omega) = \arg\max_{q\in Q}\left\{\frac{z(q,\omega)^{\gamma(\sigma-1)}}{\gamma\sigma}\left[\Pi(q,1)\Phi\left(\overline{f}_E(z(q,\omega),q)\right) + \Pi(q,0)\left[1-\Phi\left(\overline{f}_E(z(q,\omega),q)\right)\right]\right]\right.$$
$$\left. - P_s\mathbb{E}(f_E|f_E \leq \overline{f}_E(z(q,\omega),q))\right\} \tag{48}$$

Aggregation, Network, Equilibrium Appendix B makes exactly the same assumptions on production and network formation as in the closed economy. The only difference is that, because sales, mass of ads and prices depend on export status, aggregation in $M(q)$, $\overline{V}(q)$, $X_m(q)$, and $P(q)$ is over two measure functions:

$$\tilde{J}(z,q,1) = J(z,q)\Phi\left(\overline{f}_E(z,q)\right)$$
$$\tilde{J}(z,q,0) = J(z,q)\left[1-\Phi\left(\overline{f}_E(z,q)\right)\right] \tag{49}$$

where $J(z,q)$ is the measure in (17). The equilibrium is also similarly defined with the additional equilibrium variable e, real exchange rate, and condition $P_s = eP^*$.

4.1 Estimation of the Open Economy

Appendix C presents a two-stage estimation procedure similar to Section 3.5, where the first stage estimates measure $J(z,q)$ and the second stage estimates the distribution of (ω_0,ω_1) determining $z(q,\omega)$. This two-stage procedure is viable due to the timing of information on the fixed export costs. The exporting threshold in (47) is used to derive measures $\tilde{J}(z, q, E)$ in (49), which are used to aggregate firm outcomes and generate the general equilibrium functions $c(q)$ and $D(q)$ in the first stage. If firms knew their fixed cost of exporting before choosing quality, then (47) would not hold because the exporting decision would depend not only on the firm's (z, q) at the optimal but also on (z, q) conditional on changing export status. This requires knowledge of the curvature of the whole schedule $z(q, \omega)$ which is only estimated in the second stage.

The calibrated parameters $\{\alpha_m, \alpha_s, \sigma, f_m, f_v, \beta_m, \beta_v, f, \overline{m}\}$, wage $w(q) = 1$, and labor supply $L(q, \omega)$ are set as in Section 3.5. The export market adds to the definition of the economy the foreign price of services P^*, foreign demand $D_F(q)$, parameters of the distribution of exporting costs, and equilibrium real exchange rates e. The real exchange rate e is not separately identified from foreign demand in (42). We thus set $e = P^* = 1$. We parameterize the distribution of fixed export cost f_E units of the service good from a log-normal distribution with mean and standard deviation parameters μ_E and σ_E. We parameterize

$$D_F(q) = b_1 q^{b_2}$$

where b_1 and b_2 are parameters to be estimated.

First stage We use the method of simulated moments to estimate $\{\kappa, \nu_y, \nu_v, a_1, a_2, \varsigma\}$ (as before) and the additional export-related parameters $\{b_1, b_2, \mu_E, \sigma_E\}$. Computationally, the open and closed economy are similar because, with the log normal parametrization of fixed costs, integrals over productivity z are still in closed-form. We add seven moments to the estimation: The share of firms exporting for each quintile of wage (5 moments), the average export intensity for exporting firms, and the export intensity at the upper quintile of wages. In all, there are 10 parameters and 37 moments in the first stage. Our results below are preliminary, and for simplicity, we do not iterate on the optimal direction of search in (29) and assume that $\mu(q) = q$.

Intuitively, parameter b_1 governs the level of export intensity while b_2 governs how export intensity changes across quintile of firm average wages. If b_2 is large, $D_F(q)/D_H(q)$ is increasing in q and export intensity increases with quintile of wages. Parameter μ_E governs the share of firms exporting and σ_E governs how this share changes across quintiles. If σ_E is large, then the share of firms exporting does not vary much across quintiles because it depends more on firm f_E draws than on wages and sales.

Second stage We estimate the distribution of (ω_0, ω_1) given $J(z, q)$ from the first stage. We follow the basic strategy of exploiting the level and first order condition of Section

3.5.2 (equations (38) and (39)). But in the closed economy, a firm's optimal quality depended only on how its productivity changed with quality, on parameter ω_1 . Here, it depends also on ω_0. If $D_F(q)/D_H(q)$ is increasing as in the estimated model, then firms with a large ω_0 are more likely to export and choose higher quality for a given ω_1.

We estimate $\overline{\omega}_2$ using the Bartik regressions of Table 3. Fix a guess of $\overline{\omega}_2$ and the corresponding estimated distribution of (ω_0, ω_1) from the second stage estimation. A shock that increases a single firm's export demand $D_F(q)$ by, say 10 percent, in general changes the firm's optimal quality $q(\omega)$ in (48). In particular, if $D_F(q)/D_H(q)$ were increasing in quality as in the estimated first stage, the firm increases $q(\omega)$. Since each quality in the grid is associated with an average earnings per worker in the data (the ranking is the same), the change in quality is also associated with a change in the firm's average earnings per worker, denoted with $\Delta^{\text{Bartik}}(\omega)$.

We sample firms and estimate the expected effect from the Bartik shocks in the model as the average $\Delta^{\text{Bartik}}(\omega)$ weighted by firms' export probabilities. In the data, a 10 percent increase in a export demand increases the average wage per worker at the firm by 0.4 percent (Column (1) Table 3). We iterate over guesses of $\overline{\omega}$ to match this 0.4 percentage change. For each guess of $\overline{\omega}_2$ we estimate the distribution of (ω_0, ω_1).

5 Estimation Results

5.1 First Stage Estimation Results

In this section, we report the results of quantitative analysis of our model. Recall that there are three sets of the parameters that govern the firm behaviors and equilibrium outcomes. The first set is the degree of directed search ν_v, the complementarity of input-output qualities ν_y, and the matching friction parameter κ. ν_v controls the precision of firm's effort in directing their search for the targeted quality segment. Our estimated value is 2.94. Combined with our parametric assumption of normal density, it indicates that despite more search ads ends up in suppliers' own quality segment, it is far from perfect. For instance, buyers of the lowest quintile in the quality space still gets 16.1% of the search ads from the sellers in top quintile. Intuitively, this parameter is disciplined by the wage sorting at the extensive margin. We were able to match both the sorting regression coefficient at the extensive margin as well as the unweighted average log wage of the matched suppliers for customer firms using this single parameter. The estimated complementarity parameter ν_y is 0.84. To interpret this parameter, thinking of a firm at the top output quality quintile: its demand for the top quality quintile supplier is 5.8

Table 7: Parameter Estimates

	Parameters	Estimates
Matching friction	κ	0.00086
Directed search	ν_v	2.94
Complementarity	ν_y	0.84
Sd of quality distribution	ς	0.92
Mean of z conditional distribution	a_1	0.065
Sd of z conditional distribution	a_2	0.12
Mean of log export cost	μ_E	-3.70
Sd of log export cost	σ_E	2.41
Foreign demand shifter	b_1	110
Foreign demand curvature	b_2	0.51
Objective function value	0.7554	

Notes: This table summarizes the estimated parameters from the first-stage estimation. The first set of parameters are the matching friction parameter (κ), the degree of directed search (ν_v), and the complementarity of input-output qualities (ν_y). The second set are parameters of the joint distribution of quality choices and firms' realized productivity, i.e. the standard deviation of marginal quality distribution (ς), the conditional mean and standard deviation of productivity distribution (a_1, a_2). The last set are export market parameters including the mean and standard deviation of log export cost (μ_E, σ_E), and the foreign demand shifter and curvature parameter (b_1, b_2). In particular, these parameters are estimated in our first stage using the method of simulated moments.

times more than the lowest quality quintile supplier even if both charge the same price. In contrast, a firm at the bottom output quality quintile would also prefer the top quality supplier. But her demand for top quintile supplier will be only 0.8 times more than the bottom quintile supplier. This parameter helps us to match the wage sorting at the intensive margin and the input-weighted average log wage of the matched suppliers for customer firms. κ is estimated to be 0.086%, which indicates a low success rate of finding a business partner given the search effort. This is not surprising given that the mean number of supplier and customer in our data ranges from 5 to 25, a tiny fraction of all the potential partners out there in the manufacturing industry. The matching function overall did a good job of capturing the key features of the in-degree and out-degree distribution in our data. Our search cost specification (10) provides a tight connection of the number of business partners and firm sales. It is re-assuring that we were also able to match the rising number of suppliers and customers based on wage quintiles, a feature emerging naturally from the market tightness at each quality segment.

The second set of parameters ς, a_1, a_2 determine the joint distribution of quality choices and firms' realized productivity. As we explained in Section 3.5.2, this joint distribution should be interpreted more as a parametrized version of firm's optimal policy function in the data equilibrium. In the data, firms of the highest wage quintile accounts for 76% of sales made in the whole production network, indicating large heterogeneity of quality chosen by the firms. The resulting parameter $\varsigma = 0.92$ helps to explain this dispersion of sales in the data. The parameter $a_1 = 0.065$, implying a positive correlation of the realized productivity z and firm's quality q. This positive relationship fits the overall distribution of network sales across wage quintiles well. Meanwhile, the standard deviation of the conditional distribution ($a_2 = 0.12$) does a good job fitting the standard deviation of log sales within each wage quintile. The recovered joint distribution of (z, q) will be used in our second stage estimation to quantify firm's cost of quality upgrading.

The third set of parameters $\mu_E, \sigma_E, b_1, b_2$ are related to the export market performance of firms. The random log export cost has a mean parameter $\mu_E = -3.70$ and standard deviation $\sigma_E = 2.41$. The export distribution is right skewed, which fits the fraction of exporters among firms in each wage quintiles well. Finally the foreign demand parameters $b_1 = 110, b_2 = 0.51$ explain the overall export intensity and that of the top firms'.

Table 8: Targeted Moments

Moments		Q1	Q2	Q3	Q4	Q5
Mean Number of Supplier	Data	5.8	6.7	5.8	11.4	25.8
	Model	2.1	3.7	6.1	10.7	28.1
Mean Number of Customer	Data	5.6	7.0	6.7	11.7	25.1
	Model	2.9	4.8	7.3	11.9	23.8
Sd of Log Sales	Data	1.37	1.34	1.37	1.52	1.79
	Model	1.43	1.40	1.43	1.47	1.63
Share of Total Network Sales	Data	0.03	0.04	0.04	0.10	0.78
	Model	0.01	0.03	0.06	0.15	0.76
Fraction of Exporters	Data	0.08	0.18	0.16	0.34	0.57
	Model	0.05	0.13	0.22	0.34	0.55
Unwgt. Average Log Wage of Suppliers	Data	0.00	0.01	0.01	0.04	0.14
(Q1 normalized)	Model	0.00	0.01	0.02	0.04	0.14
Wgt. Average Log Wage of Suppliers	Data	0.00	0.02	0.02	0.07	0.23
(Q1 normalized)	Model	0.00	0.02	0.04	0.08	0.22
		Ext.	Int.			
Wage Sorting Regression Coefficients	Data	0.15	0.11			
	Model	0.17	0.10			
		Mean	Q5			
Export Intensity of Exporters	Data	0.24	0.26			
	Model	0.22	0.28			

Notes: This table shows the targeted moments used in the first stage estimation and compares our simulated moments to that from the data. Firms are ranked according to their average wage per worker. We match the following moments by quintile of firm wage: the mean number of suppliers (5 moments), the mean number of customers (5 moments), the share in total network sales (5 moments), the standard deviation of sales (5 moments), the fraction of exporters (5 moments), the unweighted average log-wage of suppliers (4 moments), and the average log-wage of suppliers weighted by spending share (4 moments), where the latter two are normalized with respect to the first quintile. Besides, we also match the mean and the top quintile export intensity of exporters (2 moments), and the two coefficients in the extensive and intensive margin wage sorting regressions (2 moments).

Overall, our model moments generated from the estimated parameters fits the data moments very well. As a further validation, we check some of the out-of-sample fit using the firm-to-firm trade patterns that were documented in Section 2.2 Figure 2. In Appendix E.2 Figure A3 and A4, we report the model counterpart of these figures. Our model was able to match the extent of how high/low quintile supplier and buyers disproportionately transact among themselves. This qualitative pattern is directly implied by the fact that our model matches the wage sorting regressions. Nevertheless, it is particularly reassuring that our model also *quantitatively* fits the aggregate firm-to-firm trade pattern well.

5.2 Second Stage Estimation Results

We then utilize the quality and productivity realizations in the data equilibrium to recover the fundamental firm heterogeneity. The important insight is that given a specific value of $\bar{\omega}_2$, we can find a joint density of (ω_0, ω_1) to rationalize the observed empirical distribution $J(z, q)$ as firm's optimal quality choices[21]. We conduct this inversion non-parametrically on grids of 200 quality and 100 productivity levels.

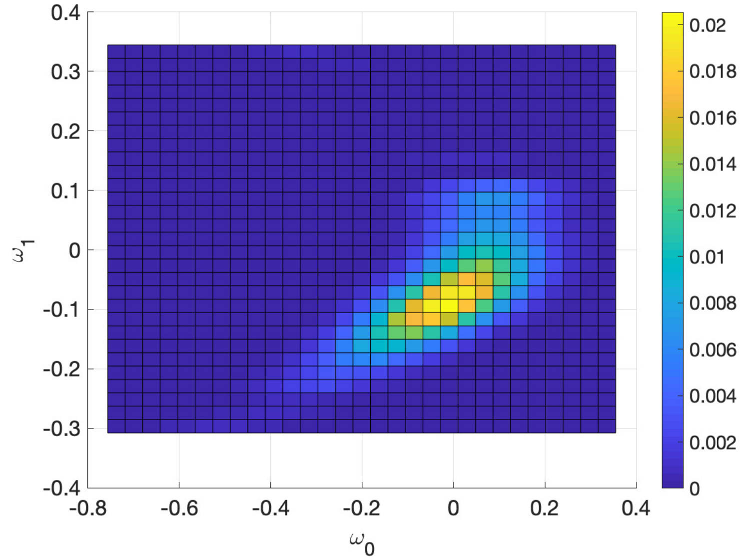
However, as argued in Section 3.5.2, the cross-sectional moments are not sufficient to identify the curvature of quality upgrading costs $\bar{\omega}_2$. We rely on the variation in idiosyncratic export shock reported in Table 3 to identify $\bar{\omega}_2$. Our empirical estimates implies that a 10% export shock induces a wage increase of 0.42% of an exporting firm in expectation. We search for the value of $\bar{\omega}_2$ such that firms in our model respond by the same magnitude on average[22]. This gives us the estimate of $\bar{\omega}_2 = -0.595$. Intuitively, ω_1 determines the sensitivity of firm production cost with respect to higher quality. Firms with more favorable ω_1 will choose higher quality. ω_0, on the other hand, unconditionally varies a firm's physical efficiency. The negative $\bar{\omega}_2$ estimate implies that when firms upgrade their quality, the cost in terms of efficiency loss is increasing such that the optimal choice of quality is bounded, even for firms with very favorable draw of technology opportunity ω_1.

In Figure 3, we report the estimated empirical distribution of ω_0 and ω_1. To easily summarize the density, we put the estimated values in 30 bins in each dimension. The color bar illustrates the fraction of firms ending up in each cell of ω_0, ω_1 combinations. Overall, we can see that a large fraction (51.5%) of the firms are concentrated in the range of $[-0.15, -0.04]$ for ω_1 and $[-0.18, 0.09]$ for ω_0. In addition, the firms' ability of

[21] The intuition is similar to the empirical auction literature, which infer bidders' valuation from their observed bids and the first order condition of bidder optimal response in the equilibrium.

[22] Given each guess of $\bar{\omega}_2$, we will need to re-estimate the empirical density of ω_0, ω_1 so that it is consistent with $J(z, q)$.

Figure 3: Joint Density of (ω_0, ω_1)



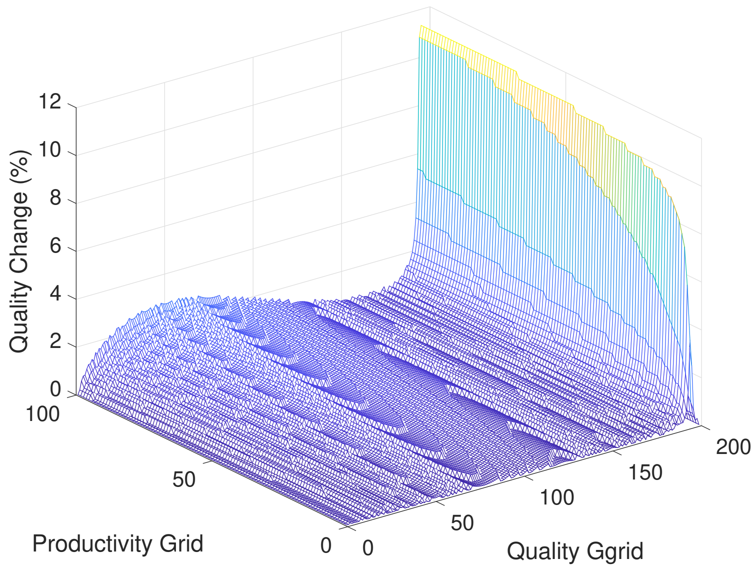
Notes: This figure reports the estimated empirical distribution of fundamental firm heterogeneity (ω_0, ω_1). Estimated values of ω_0 (on the x-axis) and ω_1 (on the y-axis) are grouped into 30 bins in each dimension, and the color bar illustrates the fraction of firms ending up in each cell of ω_0, ω_1 combinations.

upgrading quality ω_1 and productive efficiency ω_0 seem to positively correlated. We find a correlation of 0.60 for these estimates.

To further digest these estimates and illustrate how they shape up the average response of the firms to an idiosyncratic trade shock, Figure 4 reports the percentage change of firm quality choice to a 10% trade shock. We observe that the response is overall larger in high quality firms, reflecting the convexity of payoff function at the upper end of quality. Meanwhile, conditional on quality, firm's productivity also plays a non-trivial role. This is due to the rising export probability and thus the potential of benefiting from export demand shocks.

Equipped with these second stage estimates, we will be able to investigate the general equilibrium effect of an export demand shock through our O-Ring production network.

Figure 4: Quality Response to Trade Shock



Notes: This figure displays the percentage change of firm quality choice to a 10% export demand shock. The new optimal quality after the trade shock is solved using grid search. The quality response is reported on (z, q) grids of 200 quality (on the x-axis) and 100 productivity (on the y-axis) levels.

6 Conclusion

TBD

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Contents (Appendix)

A Roy Model of Labor Supply

In the main text, the supply of efficiency units of labor of task q is $L(q, w)$, an exogenous function of the task quality q and the full equilibrium wage schedule $w(q')$ for all $q' \in Q$. This appendix provides a micro foundation for labor supply based on the Roy model in Teulings (1995). It provides sufficient conditions for the ranking of average earnings per firm to equal the ranking of task quality q (also in Teulings (1995)), and it shows that we can construct a set of worker endowments such that labor markets clear and the distribution of earnings per worker across firms exactly matches the data. These claims hold for any fixed continuous and differentiable w, assumptions which hold in the estimation where $w(q) = 1$ for all $q \in Q$.

A measure H of workers have heterogeneous skills, indexed with $s \in [0, 1]$, and distributed in [0,1] according to a density $h(s)$. A worker with skill s is endowed with $e(q, s)$ efficiency units of labor if she works at a firm of quality q. She observes the wage schedule $w(q)$ and chooses task quality q to maximize earnings:

$$\max_{q \in Q} \{w(q)e(q, s)\} \tag{50}$$

Let $s^*(q)$ be the set of skills that choose quality q. To ease notation, assume that $s^*(q)$ is a function or the empty set.[23] The mass of workers supplying task q is $h(s^*(q))$ where we define $h(\emptyset) = 0$.

[23] Correspondence $s^*(q)$ is a function in the interior or Q assuming that functions $w(q)$ and $h(q)$ are continuous and differentiable, and that $e(q, s)$ is continuous, differentiable and strictly log supermodular.

Then, the supply of efficiency units of labor of task q is

$$L(q, \omega) = Hh(s^*(q))e(q, s^*(q))$$

where we can define $e(q, s^*(q)) = 0$ if $s^*(q) = \emptyset$. Earnings per worker in firms of task q is $w(q)e(q, s^*(q))$.

In the estimation, we assume that earnings per worker is strictly increasing in q. This assumption holds if $e(q, s)$ is increasing in s and strictly log-supermodular. Given this monotonicity, each q in the model is associated with an earnings per worker y in the data where y is such that the share of firms with qualities smaller than or equal to q in the model is equal to the share of firms with earnings per worker less than or equal to y in the data. To show that we can construct a set of endowments $e(q, s)$ that clear the labor market and that deliver the data's distribution of average earnings across firms, it suffices to show that for any quality-earnings pair $(q^*, y^*) \in Q \times \mathbb{R}_{++}$, we can find an endowment function $e(q, s^*)$ such that q^* is the choice and y^* is the maximum in problem (50) when the worker skill is s^*. We parameterize

$$e(q, s^*) = \exp(s_0^* + s_1^* \log(q) + \overline{s}_2[\log(q)]^2)$$

where $\overline{s}_2$ and $(s_0^*, s_1^*) \in \mathbb{R}^2$ are specific to skill s^*. Sufficient conditions for $e(q, s^*)$ are:

$$\begin{aligned}
y^* &= w(q^*) \exp(s_0^* + s_1^* \log(q^*) + \overline{s}_2[\log(q^*)]^2) \\
0 &= \frac{d \log[w(q^*)]}{d \log(q)} + s_1^* + 2\overline{s}_2[\log(q^*)] \\
0 &> \frac{d^2 \log[w(q)]}{d[\log(q)]^2} + 2\overline{s}_2 \qquad \text{for all } q \in Q.
\end{aligned}$$

These conditions are analogous to the construction of firm productivity in the second stage of the estimation. The lack of identification of $\overline{s}_2$ is the same as that of $\overline{\omega}_2$.

B Open Economy Model

We present the parts of the model that were missing from Section 4. A manufacturing firm with productivity z, quality q and export status E has the following sales x, a measure of ads v to find customers (domestic and abroad) and m to find suppliers, and price:

$$\begin{aligned}
x(z,q,E) &= \Pi(q,E) z^{\gamma(\sigma-1)} \\
v(z,q,E) &= \left(\frac{x(z,q,E)}{\sigma f_v w(q)}\right)^{1/\beta_v} \\
m(z,q,E) &= \left(\frac{x(z,q,E)}{\sigma f_m w(q)/\alpha_m}\right)^{1/\beta_m} \\
p(z,q,E) &= \frac{\sigma}{\sigma-1}\frac{C(m(z,q,E),q)}{z}
\end{aligned} \tag{51}$$

where

$$\begin{aligned}
\Pi(q,E) &= [\sigma w(q)]^{1-\gamma}\left[D(q,E)\left(\frac{\sigma}{\sigma-1}C(1,q)\right)^{1-\sigma}\left(\frac{f_m}{\alpha_m}\right)^{-\alpha_m/\beta_m} f_v^{-1/\beta_v}\right]^{\gamma} \\
D(q,E) &= \left[D_H(q)^{\beta_v/(\beta_v-1)} + E(e^{\sigma}D_F(q))^{\beta_v/(\beta_v-1)}\right]^{(\beta_v-1)/\beta_v}.
\end{aligned} \tag{52}$$

With the fixed exporting cost, profit is no longer a constant share of revenue. The expected profit of a firm that draws a productivity parameter ω upon entry is (equation (48)):

$$\begin{aligned}
\pi(\omega) = \max_{q\in Q}\Bigg\{ &\frac{z(q,\omega)^{\gamma(\sigma-1)}}{\gamma\sigma}\left[\Pi(q,1)\Phi\left(\overline{f}_E(z(q,\omega),q)\right) + \Pi(q,0)\left[1-\Phi\left(\overline{f}_E(z(q,\omega),q)\right)\right]\right] \\
&- P_s\mathbb{E}(f_E|f_E \le \overline{f}_E(z(q,\omega),q))\Bigg\}
\end{aligned}$$

Free entry implies

$$P_s f = \mathbb{E}_{\omega}(\pi(\omega)) \tag{53}$$

The firm choices give rise to the measure functions:

$$\begin{aligned}
\tilde{J}(z,q) &= N\text{Prob}\left\{\omega : z(q(\omega),\omega) \le z \quad \text{and} \quad q(\omega) \le q\right\} \\
J(z,q,1) &= \tilde{J}(z,q)\Phi\left(\overline{f}_E(z,q)\right) \\
J(z,q,0) &= \tilde{J}(z,q)\left[1-\Phi\left(\overline{f}_E(z,q)\right)\right]
\end{aligned} \tag{54}$$

$J(z,q,E)$ is the measure of functions with export status $E \in \{0,1\}$ and productivity-quality pairs less than or equal to (z,q). Denote the density of J as $j(z,q,E)$ for $E = 0,1$.

The production function (18) and network formation are the same as in the closed economy, only expressions for some aggregate variables change. The mass of ads posted by firms of quality q to find suppliers and sellers is respectively:

$$M(q) = \sum_{E=0,1} \int_Z m(z,q,E)j(z,q,E)dz \tag{55}$$

$$\overline{V}(q) = \sum_{E=0,1} r_v(q,E) \int_Z v(z,q,E)j(z,q,E)dz \tag{56}$$

The mass of ads directed at buyers of quality q, $V(q)$, and the mass of matches $\tilde{M}(q)$ are in (22) and (23). The success rate of ads is $\theta_v(q) = \tilde{M}(q)/V(q)$ for sellers and $\theta_m(q) = \tilde{M}(q)/M(q)$ for buyers, as before.

Cost function $c(q)$ and demand functions $\tilde{D}(q,\mu)$ and $D_m(q)$ are in equations (25), (27) and (29) respectively, where now the price index $P(q)$ and spending on manufacturing inputs $X_m(q)$ are:

$$P(q) = \left[\sum_{E=0,1} r_v(q,E) \int_Z p(z,q,E)^{1-\sigma} v(z,q,E)j(z,q,E)dz\right]^{1/(1-\sigma)} \tag{57}$$

$$X_m(q) = \frac{\alpha_m(\sigma-1)}{\sigma} \sum_{E=0,1} \int_Z x(z,q,E)j(z,q,E)dz. \tag{58}$$

We also maintain the assumptions on consumer demand and the service sector. If we take gross manufacturing production as the numeraire as before, domestic production of services is

$$X_s = 1 - \frac{\alpha_m(\sigma-1)}{\sigma} - B$$

where B is imports of services. With trade balance it equals exports of manufactures:

$$B = \int_{q\in Q} r_v(q,1)e^{\sigma} D_F(q) \left[\int_z p(z,q,1)^{1-\sigma} v(z,q,1)j(z,q,1)dz\right] dq.$$

The demand for manufacturing goods stemming from service firms and total demand shifter $D(q)$ are in equations (32) and (33). The price index in services P_s is in (31).

Labor markets clear if

$$L(q,w) = \frac{1}{w(q)\sigma} \left[(1-\alpha_m-\alpha_s)(\sigma-1) + 1 - \frac{1}{\gamma}\right] \left[\sum_{E=0,1} \int_Z x(z,q,E)j(z,q,E)dz\right] \tag{59}$$

An **equilibrium** is a mass of firms N, an exchange rate e, measure functions $J(z,q,1)$ and $J(z,q,0)$, and functions $w(q)$, $\theta_m(q)$, $\theta_v(q)$, $c(q)$, $D(q)$ satisfying the following conditions:

1. Frictionless trade in services, $P_s = eP_s^*$.

2. Free entry (53).

3. Labor market clearing (59).

4. Firms maximize profits. Firm ω chooses $q(\omega)$ in (48) and has productivity $z(\omega) = z(q(\omega), \omega)$ at the optimal. The firm export status is $E = 1$ if its fixed cost of exporting is less than $\overline{f}_E(q(\omega), z(q, \omega))$, and $E = 0$ otherwise. Its sales, measure of ads, and prices are $x(z(\omega), q(\omega), E)$, $m(z(\omega), q(\omega), E)$, $v(z(\omega), q(\omega), E)$, and $p(z(\omega), q(\omega), E)$ in (51). The direction of selling ads $\mu(q(\omega))$ solves (29).

5. For $E = 0, 1$, the measures $J(z, q, E)$ are consistent with firm choices (54).

6. The success rate of ads $\theta_m(q) = \tilde{M}(q)/M(q)$ and $\theta_v(q) = \tilde{M}(q)/V(q)$ where $\tilde{M}(q)$ is in (23), $V(q)$ is in (22), and $M(q)$ and $\overline{V}(q)$ are in (55) and (56). Cost $c(q)$ satisfies (25) and $D(q)$ satisfies (33), where $P(q)$ and $X_m(q)$ are in (57) and (58).

C Estimation of the Open Economy

D Moments in the Closed Economy

This appendix presents the expressions for calculating the moments in the estimation procedure of the closed economy. Denote with q one of the $T = 200$ points in the quality grid Q. We write $q \in q$ whenever the quality q is in quintile q of qualities. In the simulated model, using the expression for $x(z, q)$ and the parametric distribution of $J(z, q)$, the mean and variance of sales of firms in grid point q is respectively:

$$
\begin{aligned}
E(x|q) &= \Pi(q) \exp\left(\gamma(\sigma-1)a_1 \log(q) + [\gamma(\sigma-1)a_1]^2\right) \\
V(x|q) &= \Pi(q)]^2 \exp\left(2\gamma(\sigma-1)a_1 \log(q) + [\gamma(\sigma-1)a_1]^2\right)\left(\exp([\gamma(\sigma-1)a_1]^2) - 1\right)
\end{aligned}
$$

The mean and variance of sales for quintile $q = 1, ..., 5$ of the quality grid is respectively

$$
\begin{aligned}
E(x|q \in q) &= \frac{5}{T} \sum_{q \in q} E(x|q) \\
V(x|q \in q) &= \left\{ \frac{5}{T} \sum_{q \in q} \left[V(x|q) + E(x|q)^2\right] \right\} - [E(x|q \in q)]^2
\end{aligned}
\tag{60}
$$

The share of suppliers in quintile q_s among all suppliers to buyers in quintile q_b is

$$\frac{\sum_{q_b \in q_b} \frac{\theta_m(q_b)}{V(q_b)} \sum_{q_s \in q_s} \phi_v(q_b, \mu(q_s)) \overline{V}(q_s)}{\sum_{q_b \in q_b} \frac{\theta_m(q_b)}{V(q_b)} \sum_{q_s \in Q} \phi_v(q_b, \mu(q_s)) \overline{V}(q_s)} \tag{61}$$

The share of these suppliers in the material purchases of buyers in q_b implied in $c(q)$ is

$$\frac{\sum_{q_b \in q_b} \frac{\theta_m(q_b)}{V(q_b)} \sum_{q_s \in q_s} \phi_v(q_b, \mu(q_s)) \phi_y(q_b, q_s) P(q_s)}{\sum_{q_b \in q_b} \frac{\theta_m(q_b)}{V(q_b)} \sum_{q_s \in Q} \phi_v(q_b, \mu(q_s)) \phi_y(q_b, q_s) P(q_s)}. \tag{62}$$

In the calculations above, functions $\Pi(q)$, $V(q)$, $\theta_m(q)$ and $\theta_m(q)$ are taken from the simulation procedure in the maintext. Function $\phi_v(q, q')$ is the density of a normal variable evaluated at q with mean parameter q' and variance ν_v. Function

$$\phi_y(q, q') = \frac{\exp(q' - \nu_y q)}{1 + \exp(q' - \nu_y q)}$$

D.1 Identification of $\overline{\omega}_2$ and Labor

Parameter $\overline{\omega}_2$ cannot be identified from the joint distribution of sales and wages used in the first and second stages. It captures the elasticity of firms choices of q with respect to shocks to the economy. Denote with Θ all parameters defining the economy and with $J(\Theta)$ the equilibrium measure of productivity-quality pairs in (17), which we have now made explicit, is itself a function of the parameters of the economy. The first order condition (39) implicitly defines a firm's quality choice as a function of the firm's ω_1 and of parameters Θ. Define

$$g(q; \omega, \Theta, J(\Theta)) = \gamma(\sigma - 1)\left[\omega_1 + 2\overline{\omega}_2 \log(q)\right] + \frac{\partial \log \Pi(q; \Theta, J(\Theta))}{\partial \log q}$$

We consider two types of shocks. A general equilibrium shock that changes Θ for all firms, and a partial equilibrium shock that changes Θ for only one firm. From the implicit function theorem, the derivative of the firm's choice $\tilde{q}$ with respect to an element Θ_i of the parameter vector is

$$\frac{d\tilde{q}}{d\Theta_i} = -\frac{dg(\tilde{q}; \tilde{\omega}, \Theta, J(\Theta))/d\Theta_i}{dg(\tilde{q}; \tilde{\omega}, \Theta, J(\Theta))/d\tilde{q}} \tag{63}$$

$$\frac{\partial\tilde{q}}{\partial\Theta_i} = -\frac{\partial g(\tilde{q}; \tilde{\omega}, \Theta, J(\Theta))/\partial\Theta_i}{dg(\tilde{q}; \tilde{\omega}, \Theta, J(\Theta))/d\tilde{q}} \tag{64}$$

in the general and partial equilibrium shocks, respectively. In both equations, the denominator is the firm's second order conditions in (40) evaluated at the optimal $\tilde{q}$. If these conditions hold near equality, the firm's choice is infinitely elastic, $\frac{d\tilde{q}}{d\Theta_i} \to \infty$ and as $\overline{\omega}_2 \to -\infty$, then $\frac{\tilde{q}}{d\Theta_i} \to 0$ and the firm's choice is fixed.

The difference between (63) and (64) is in the numerator. The total derivative of function g with respect to Θ_i, in (63), depends on changes in all firm's choices through measure function J. That is, it depends on $\overline{\omega}$ through $\frac{dq(\omega)}{d\Theta_i}$ for all firms ω. In contrast, the partial derivative in (64) is known from the structure of the estimated model. It is the partial derivative of Π with respect to a fundamental parameter.

Then, we can estimate $\overline{\omega}_2$ using (64) if we observed a partial equilibrium Θ_i and the ensuing change in the choice of q of the firm undergoing the shock. The $dq(\omega)/d\Theta_i$ in (64) may be approximated by the change in the firm's q divided by the change in Θ_i. Together with the estimated $\partial g(\tilde{q}; \tilde{\omega}, \Theta, J(\Theta))/\partial\Theta_i$, we can pin down a value of the denominator (the expression in (40)) and use it to back out $\overline{\omega}_2$.

We interpret the Bartik shocks in the open economy as such observable partial equilibrium shocks. The Bartik regressions give us an estimate of the change in a firm's quality choice (ranking of wages) given a percentage change in the firm's export demand without affecting other firms choices, without affecting J. And we use the estimated open-economy model from the first stage to estimate how the firm's problem changes (Π changes) given the observed foreign demand shock.

E Additional Tables and Figures

E.1 Tables

Table A1: Assortative Matching on Wages: Alternative definition of wages

Dependent variable: $\ln wage_f^S$

	Manufacturing firms			All firms
	(1)	(2)	(3)	(4)
$\ln wage_f$	0.300	0.262	0.190	0.258
	(0.011)	(0.010)	(0.007)	(0.010)
$\ln employment_f$			0.044	
			(0.003)	
R^2	0.092	0.163	0.183	0.128
N	77,418	77,418	77,418	410,608
Fixed effects		ind-prov	ind-prov	ind-prov

Notes: Firm-level wage is calculated as the within-firm median value of the residuals obtained from the following regression:

$$\ln wage_{ef} = \beta_1 Age_e + \beta_2 Gender_e + \alpha_o + e_{ef},$$

where $wage_{ef}$ denotres the average value of monthly wage received by each worker in a given firm, and α_o occupation fixed effects at the 1-digit ISCO level. Denoting the set of suppliers of firm f by Ω_f^S, average supplier wage is defined as follows: $\ln wage_f^S = \sum_{\omega \in \Omega_f^S} \ln wage_\omega s_{\omega f}$, where ω indexes suppliers, and $s_{\omega f}$ is the share of f's purchases from supplier ω. Ind and prov refer to 4-digit NACE industries and provinces, respectively. Robust standard errors are clustered at 4-digit NACE industry level.

Table A2: Assortative Matching on Other Variables

	$\ln \text{market share}_f^S$		$\ln \text{outdegree}_f^S$	
	manuf	all	manuf	all
	(1)	(2)	(3)	(4)
$\ln \text{market share}_f$	0.175	0.154		
	(0.013)	(0.029)		
$\ln \text{indegree}_f$			0.0985	-0.034
			(0.012)	(0.063)
R^2	0.11	0.14	0.09	0.14
N	77,418	410,608	77,418	410,608
Fixed effects	ind-prov	ind-prov	ind-prov	ind-prov

Notes: Market share is the share of a firm's sales in total sales of its 4-digit NACE industry, and indegree is the number of domestic suppliers of a firm. Both variables are in logarithms. Denoting the set of suppliers of firm f by Ω_f^S, average supplier market share is defined as follows: $\ln \text{market share}_f^S = \sum_{\omega \in \Omega_f^S} \ln \text{market share}_\omega s_{\omega f}$, where ω indexes suppliers, and $s_{\omega f}$ is the share of f's purchases from supplier ω. $\ln \text{outdegree}_f^S$ is defined similarly using the number of buyers (outdegree) of firm f's each supplier. Ind and prov refer to 4-digit NACE industries and provinces, respectively. Robust standard errors are clustered at 4-digit NACE industry level.

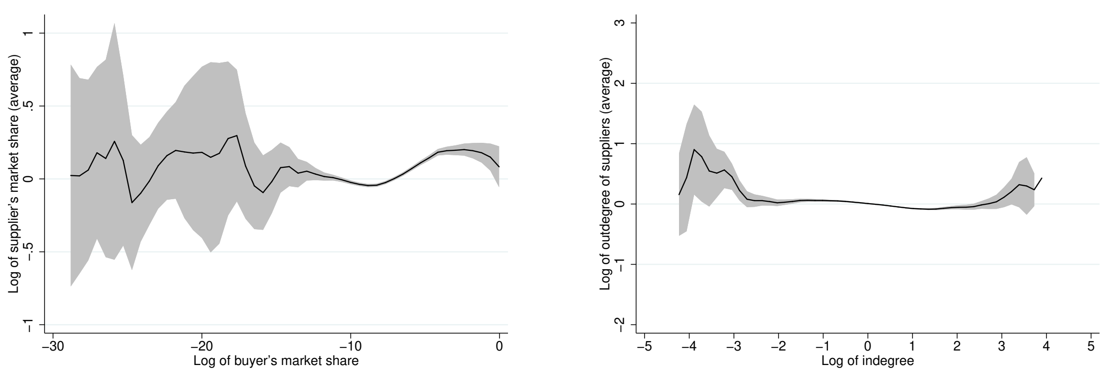
Table A3: Assortative Matching on Other Variables (Extensive margin)

	$\ln \text{market share}_f^S$		$\ln \text{outdegree}_f^S$	
	manuf	all	manuf	all
	(1)	(2)	(3)	(4)
$\ln \text{market share}_f$	0.042	0.009		
	(0.009)	(0.025)		
$\ln \text{indegree}_f$			0.009	-0.131
			(0.009)	(0.060)
R^2	0.07	0.12	0.08	0.13
N	77,418	410,608	77,418	410,608
Fixed effects	ind-prov	ind-prov	ind-prov	ind-prov

Notes: Market share is the share of a firm's sales in total sales of its 4-digit NACE industry, and indegree is the number of domestic suppliers of a firm. Both variables are in logarithms. Denoting the set of suppliers of firm f by Ω_f^S, unweighted average of supplier market share is defined as follows: $\ln \text{market share}_f^S = \sum_{\omega \in \Omega_f^S} \ln \text{market share}_\omega (1/|\Omega_f|)$, where ω indexes suppliers. $\ln \text{outdegree}_f^S$ is defined similarly using the number of buyers (outdegree) of firm f's each supplier. Ind and prov refer to 4-digit NACE industries and provinces, respectively. Robust standard errors are clustered at 4-digit NACE industry level.

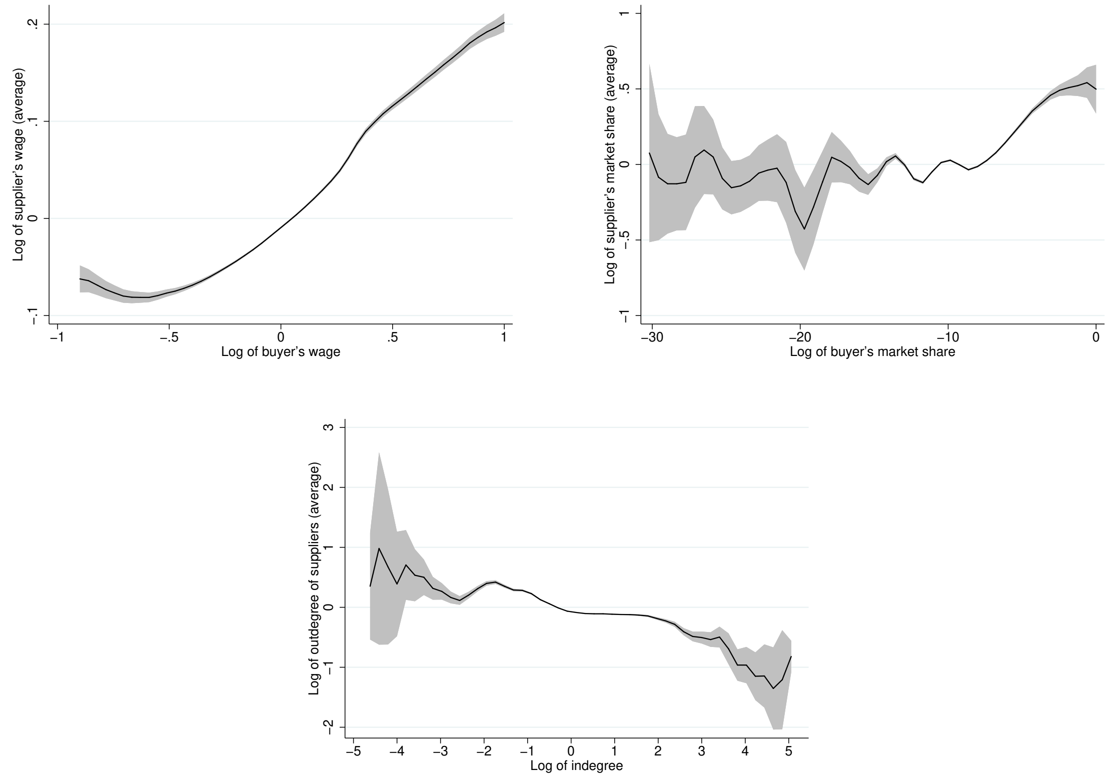
E.2 Figures

Figure A1: Matching on Sales and Network Size (Manufacturing firms)



Notes: Sample includes manufacturing firms on both sides of the transaction. Market share is defined as the firm's share in gross sales of its respective 4-digit NACE industry. Indegree and outdegree refer to a firm's number of suppliers and buyers, respectively. Both x- and y-axis variables are demeaned from 4-digit NACE industry averages. The fitted curves are obtained from local polynomial regression with Epanechnikov kernel of the (residual) x-axis variables. The shaded areas show the respective 95% confidence intervals.

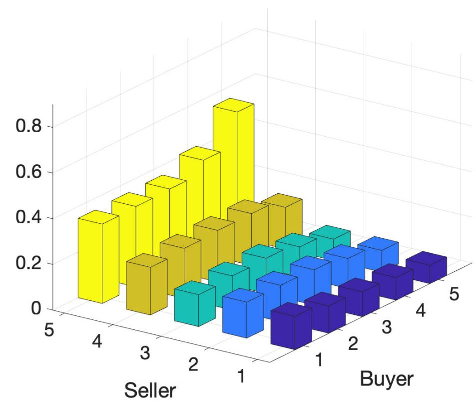
Figure A2: Matching on Wages, Sales and Network Size (All firms)



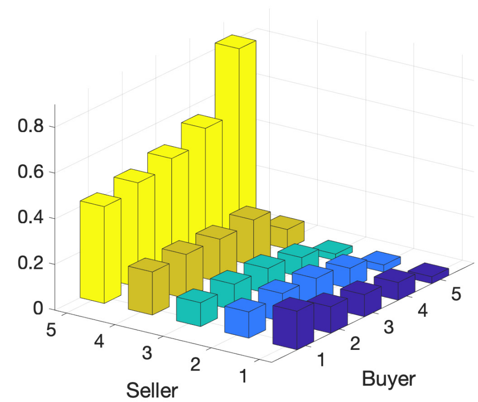
Notes: Sample includes manufacturing and service firms on both sides of the transaction. Wage is the average value of monthly payments per worker. Both buyer and supplier wages are demeaned from their respective industry (4-digit NACE) and region means and adjusted for firm size, i.e. employment. Market share is defined as the firm's share in gross sales of its respective 4-digit NACE industry. Indegree and outdegree refer to a firm's number of suppliers and buyers, respectively. Both x- and y-axis variables are demeaned from 4-digit NACE industry averages. The fitted curves are obtained from local polynomial regression with Epanechnikov kernel of the (residual) x-axis variables. The shaded areas show the respective 95% confidence intervals.

Figure A3: Untargeted Firm-to-firm Trade Moments for Buyers

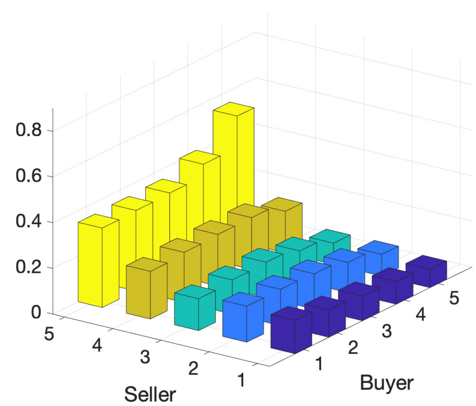
(a) Share of Suppliers (Data)



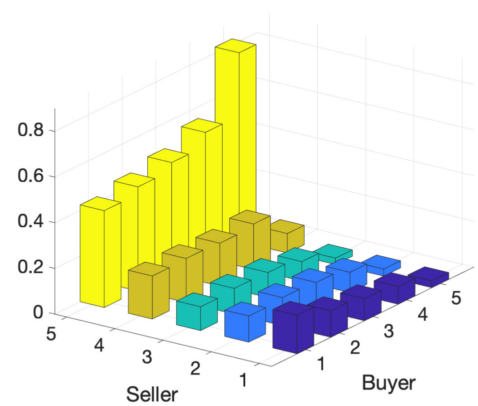
(b) Spending Shares (Data)



(c) Share of Suppliers (Model)



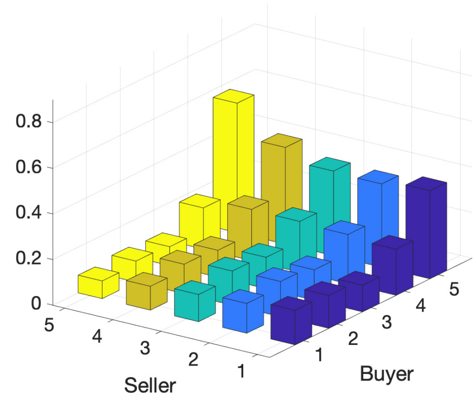
(d) Spending Shares (Model)



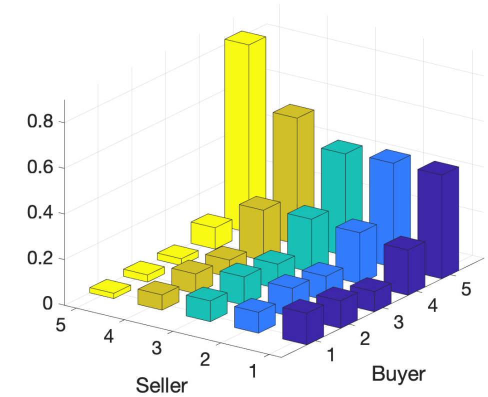
Notes: This figure compares the data moments (top panels) to the untargeted moments implied by the model (bottom panels). Firms are ranked according to their average wage per worker. For each buyer quintile, number of suppliers and expenditures are aggregated at the level of supplier quintile. Buyer and supplier quintiles are shown on x- and y-axis while z-axis shows the corresponding shares. For instance, in panel (a), values on the z-axis show for each buyer quintile on the x-axis the share of suppliers that belong to the wage quintiles on the y-axis.

Figure A4: Untargeted Firm-to-firm Trade Moments for Suppliers

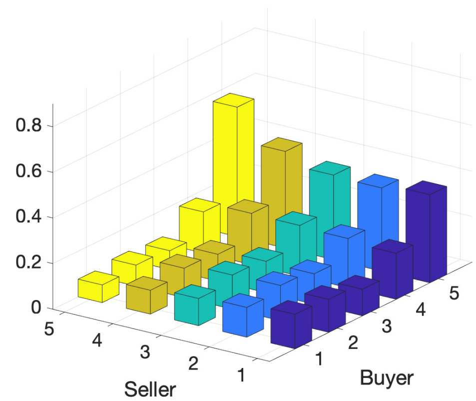
(a) Share of Buyers (Data)



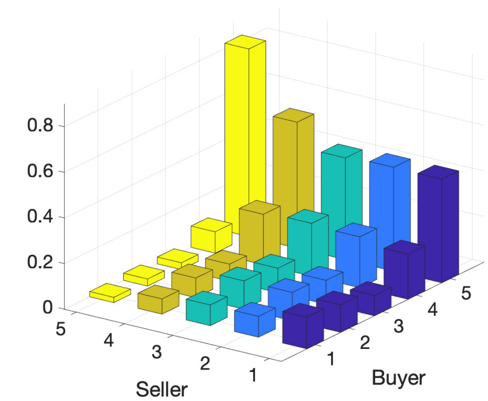
(b) Sales Shares (Data)



(c) Share of Buyers (Model)



(d) Sales Shares (Model)



Notes: This figure compares the data moments (top panels) to the untargeted moments implied by the model (bottom panels). Firms are ranked according to their average wage per worker. For each supplier quintile, number of buyers and sales are aggregated at the level of buyer quintile. Buyer and supplier quintiles are shown on x- and y-axis while z-axis shows the corresponding shares. For instance, in panel (a), values on the z-axis show for each supplier quintile on the y-axis the share of buyers that belong to the wage quintiles on the x-axis.