Is Software Eating the World?

Sangmin Aum[1] Yongseok Shin[2]

[1]Kyung Hee University

[2]Washington University and Federal Reserve Bank of St. Louis

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Software is eating the world.

— Marc Andreessen (Wall Street Journal, 2011)

Labor Income Share

- Labor share: long-run stability (Keynes, 1939) $\rightarrow$ pervasive decline since the 1980s (Karabarbounis and Neiman, 2013).

- Three leading explanations on the labor share decline:
  (1) Capital-labor substitution (Karabarbounis and Neiman, 2013)
  (2) Intangible capital (Koh, Santaeulàlia-Llopis and Zheng, 2020)
  (3) Reallocation to firms with low or falling labor share (Autor, Dorn, Katz, Patterson and Van Reenen, 2020; Kehrig and Vincent, 2021)

This Paper

1. Reconcile conflicting views on the capital-labor elasticity of substitution (Macro vs. micro elasticity redux)
   - $\sigma > 1$ (Karabarbounis and Neiman, 2013; Piketty, 2014; Hubmer, 2021)
   - $\sigma < 1$ (Antras, 2004; Raval, 2019; Oberfield and Raval, 2021)

2. Point to the connection among the three leading explanations via a common factor: the rise of software
   - Software innovation $\rightarrow$ substitution, intangibles, and reallocation (+ markup) $\rightarrow$ labor share decline

Capital-Labor Substitution: Software and Equipment

- The key element is the elasticity of substitution b/w capital and labor (σ).
  - prices $\rightarrow$ labor share, depending on $\sigma := \frac{\partial \ln(L/K)}{\partial \ln(r/w)}$

- Capital goods are heterogeneous and workers differentially use equipment and software (Aum, 2020). Figure
  - $\rightarrow$ Software may interact with labor in a way different from how equipment does (i.e., $\sigma_s \neq \sigma_e$)

- Software (and intangibles) is becoming increasingly important as an embodiment of technological progress (e.g., various service delivered over the internet, platform business, AI).

- Micro elasticity $\neq$ macro elasticity (reallocation)

What We Do

- For a 3-factor production function, estimate both σ_s and σ_e using establishment-level data from Korea (Economic Census)

- Derive the aggregate elasticity using model and data, extending Oberfield and Raval (2021)

- Quantify the role of software-embodied technological change in the decline of labor share

Overview

1. Empirical Facts on Labor Share and Software

2. Model - **Micro and Macro Elasticities with Three Factors**

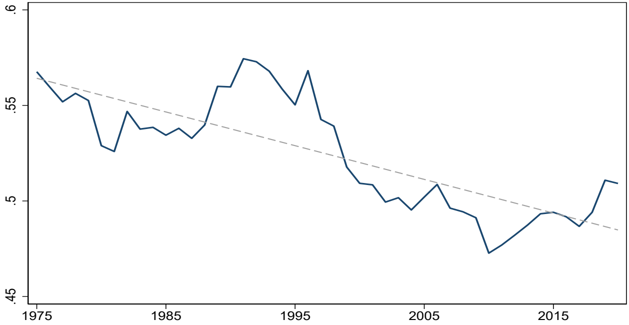
3. Estimation and Aggregation

4. Decomposition - **Impacts on the Labor Share**

Empirical Facts

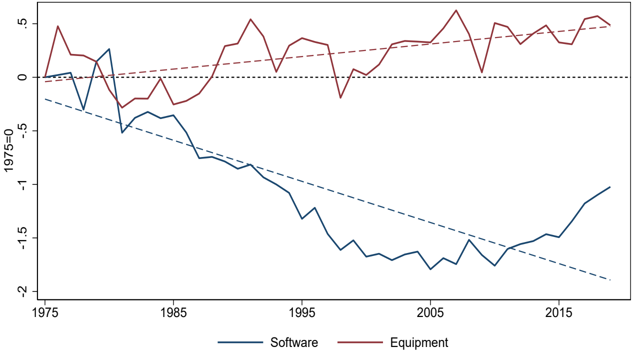
Labor Share

- Estimate the labor income of the proprietors: $NLS = \frac{CE + NLS \times PI}{GDP - CFC}$
- Then compute $LS = \frac{CE + NLS \times PI}{GDP}$



Labor Share and Two Types of Capital

- From $LS = \frac{wL}{\mu(wL+\sum_j R^j K^j)}$, impute $R^j K^j$ from NA (detail), compare $\ln wL/(R^j K^j)$ for $j \in \{\text{software, equipment}\}$. Other



(A similar pattern holds in the US data.)

Software Intensity and Labor Share

- Software/value-added (s) and labor share at the firm level

$$\Delta LS_{i,t} = a_i + c_t + b_1 s_{i,t-1} + b_2 e_{i,t-1} + \varepsilon_{i,t}$$

	ΔLS	Non-prod.	Production
s	-1.122***	-0.726***	-0.455***
	(0.132)	(0.094)	(0.076)
e	-0.008	-0.005	-0.003
	(0.006)	(0.003)	(0.003)
N	42225	42048	39093
R^2	0.191	0.215	0.191

SE in parentheses, SE clustered by industry.

* $p < .1$, ** $p < .05$, *** $p < .01$

Software Intensity and Sales Growth

- Software/value-added (s) and sales growth at the firm level

$$\Delta \ln pY_{i,t} = a_i + c_t + b_1 s_{i,t-1} + b_2 e_{i,t-1} + \varepsilon_{i,t}$$

	$\Delta \ln pY$
s	1.473***
	(0.329)
e	0.028*
	(0.014)
N	42217
R^2	0.289

SE in parentheses, SE clustered by industry.

* $p < .1$, ** $p < .05$, *** $p < .01$

Software Intensity and Productivity

- Software/value-added (s) and TFP at the firm level

$$\Delta \ln z_{i,t} = a_i + c_t + b_1 s_{i,t-1} + b_2 e_{i,t-1} + \varepsilon_{i,t}$$

	$\Delta \ln z$
s	0.897**
	(0.390)
e	0.033*
	(0.019)
N	16868
R^2	0.300

SE in parentheses, SE clustered by industry.

$^{*}\ p < .1$, $^{**}\ p < .05$, $^{***}\ p < .01$

Software Intensity and Markup

- Software/value-added (s) and markup at the firm level Details

$$\Delta \ln \mu_{i,t} = a_i + c_t + b_1 s_{i,t-1} + b_2 e_{i,t-1} + \varepsilon_{i,t}$$

	PF	Lerner	UC
s	0.344***	0.388***	0.287***
	(0.074)	(0.088)	(0.085)
e	0.017***	0.003*	0.022**
	(0.005)	(0.002)	(0.009)
N	38369	40762	36757
R^2	0.246	0.248	0.250

SE in parentheses, SE clustered by industry.

* $p < .1$, ** $p < .05$, *** $p < .01$

Software Intensity and Concentration

- Software/value-added (s) and concentration measures by 2-digit industry (j), using the firm-level data

$$concen_{j,t} = a + b_1 s_{j,t} + b_2 e_{j,t} + \varepsilon_{j,t}$$

	HHI	CR4	CR8
s	0.747***	0.384***	0.353***
	(0.000)	(0.000)	(0.000)
e	0.058***	0.045***	0.031***
	(0.000)	(0.000)	(0.000)
N	1143	1143	1143
R^2	0.035	0.019	0.016

Robust SE in parentheses.

* $p < .1$, ** $p < .05$, *** $p < .01$

Summary of Facts

- Labor share fell relative to software share, not equipment share.

- Firms with higher software intensity exhibit

  1. more decline of labor share,
  2. more productivity growth, and
  3. more markup increase.

- Industries with higher software intensity are associated with higher concentration.

Model

Environment

- Establishments' production function is

$$Y_i = \left(\left[A_i^L (L_i)^{\frac{\sigma_e - 1}{\sigma_e}} + A_i^e (K_i^e)^{\frac{\sigma_e - 1}{\sigma_e}} \right]^{\frac{\sigma_e(\sigma_s - 1)}{(\sigma_e - 1)\sigma_s}} + A_i^s (K_i^s)^{\frac{\sigma_s - 1}{\sigma_s}} \right)^{\frac{\sigma_s}{\sigma_s - 1}},$$

- The demand system is given by a Kimball aggregator

$$\sum_i H\left(\frac{Y_i}{Y}\right) = 1,$$

where $H(\cdot)$ is a smooth, increasing, and concave function.

- $K^{m\prime} = (1 - \delta^m)K^m + X^m$, for $m \in \{e, s\}$, and $Y = C + X^e / M^e + X^s / M^s$.

- M^m's are technological change specific to factor m.

$$r = \frac{(1 + r^n)}{M^e_{-1}} - \frac{1 - \delta^e}{M^e}, \text{ and } q = \frac{(1 + r^n)}{M^s_{-1}} - \frac{1 - \delta^s}{M^s}$$

Elasticity of Substitution

- Various definitions on the elasticity of substitution with three factors.

- Allen-Uzawa elasticity of substitution ($\sigma_{xy} = CC_{xy}/[C_x C_y]$) $\rightarrow \sigma_{LE} = \sigma_e$, and $\sigma_{LS} = \sigma_s$ in the nested CES.

- Micro elasticity of substitution satisfies

$$\sigma_e = 1 + \frac{d \ln e_i/(1-e_i)}{d \ln w/r}, \quad \sigma_s = 1 + \frac{d \ln s_i/(1-s_i)}{(1-e_i) d \ln w/q + e_i d \ln r/q},$$

where $e_i \equiv rK_i^e/(wL_i + rK_i^e)$, $s_i \equiv qK_i^s/(wL_i + rK_i^e + qK_i^s)$.

Macro elasticity of substitution

- Define the aggregate elasiticity of substitution as

$$\bar{\sigma}_e^w \equiv 1 + \frac{d \ln e/(1-e)}{d \ln w}, \ \bar{\sigma}_e^r \equiv 1 + \frac{d \ln (1-e)/e}{d \ln r}$$
$$\bar{\sigma}_s^w \equiv 1 + \frac{d \ln s/(1-s)}{(1-e) d \ln w}, \ \bar{\sigma}_s^r \equiv 1 + \frac{d \ln s/(1-s)}{e d \ln r},$$
$$\bar{\sigma}_s^q \equiv 1 + \frac{d \ln (1-s)/s}{d \ln q}$$

where $e \equiv rK^e/(wL + rK^e)$, $s \equiv qK^s/(wL + rK^e + qK^s)$.

- With three factors, need to define $\bar{\sigma}$ separately for the changes in w, r, and q.

Micro vs. Macro Elasticity

- Price change + Different intensities $\rightarrow$ Reallocation (Micro$\neq$Macro)

e.g., A fall in the price of equipment, r
$\rightarrow$ Establishments with higher equipment share become effectively more productive $\rightarrow$ Their size grows
$\rightarrow$ They have higher equipment share to begin with
$\rightarrow$ Equipment demand in the aggregate increases further

Two vs. Three Factors

- Two factors: Plants w/ a higher K share always have a lower L share.

1. An increase in w and a decrease in r have the same effect.
2. If $\epsilon > \sigma \rightarrow$ Macro $\bar{\sigma}$ is always greater than micro σ.

- Three factors: Plants w/ a higher S share may or may not have a lower E share.

1. An increase in r and a decrease in q have different effects.
2. $\epsilon > \sigma \rightarrow$ Macro $\bar{\sigma}$ need not be greater than micro σ.

  * For example, plants that benefit more from r reduction need not have a lower S share.

Homogeneous vs. Heterogeneous Markups

- Homogeneous

1. A change in price is always proportional to a change in MC, and hence factor share (Shephard's lemma).
2. The degree of reallocation depends on the common elasticity of demand.

- Heterogeneous

1. More productive i faces a smaller elasticity of demand and,
2. charges lower prices and higher markups.
3. The reallocation depends on how factor shares and the elasticity of demand are distributed.

Micro and Macro Elasticity

Proposition

The relationship between micro and aggregate elasticity is

$$\bar{\sigma}_e^n = (1-\chi)\sigma_e + \chi[\zeta^n \sigma_s + (1-\zeta^n)\bar{\epsilon}_e^n], \; n \in \{w, r\},$$
$$\bar{\sigma}_s^n = (1-\xi^n)\sigma_s + \xi^n \bar{\epsilon}_s^n, \; n \in \{w, r, q\},$$

where ℓ, e, and s are labor, equipment, and software income share, respectively. Also,

- Weignt parameters (χ, ξ_j^n) depend on the variance (or covariance) of factor shares,
- The reallocation parameter ($\bar{\epsilon}_j^n$) is a weighted average of ϵ_i,
- ζ^n is a weigted average of s_i.

Micro and Macro Elasticity

- χ and ξ^q depend on the dispersions of equipment or software share, and always lie between 0 and 1.
  - $\chi \propto var(e_i)$ and $\xi^q \propto var(s_i)$
- ξ^r (ξ^w) can be negative, when equipment (labor) share is positively correlated with software share.
  - $\xi^r \propto -cov(k_i, s_i)$ and $\xi^w \propto -cov(\ell_i, s_i)$
- $\bar{\epsilon}_s^q$ is smaller when the makup is larger near the tail of software share distribution.
  - $\bar{\epsilon}_s^q \approx \frac{\sum_i (s_i - s)^2 \omega_i \epsilon_i b_i}{\sum_i (s_i - s)^2 \omega_i}$ with $b_i = \frac{d \ln p_i}{d \ln mc_i}$.

Aggregate Markup

- Changes in factor prices alter markup distribution and hence the aggregate markup.

- A change in the aggregate markup also consists of within and between changes,
  where the weight depends on the covariance between the markup size and factor intensity:

$$\frac{d \ln \mu}{s \cdot d \ln q} = (1 - \eta^q)\bar{b}^q + \eta^q \bar{\epsilon}^q_\mu - 1,$$

  where $\eta^q \propto -cov(\mu_i, s_i)$ and $\bar{b}^q$ and $\bar{\epsilon}^q_\mu$ are weighted averages of b_i and $\epsilon_i b_i$, respectively.

Estimation and Aggregation

Data

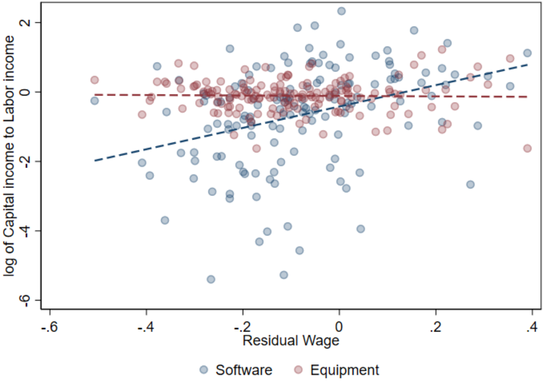
1. Economic Census 2015 (manufacturing plants)
   - K^s: software; K^e: equipment; wL_i: annual wage bill

2. Regional Employment Survey 2015
   - w^j: regional average of a residual hourly wage, controlling for gender, age, education, experience

3. National Accounts: rate of return on capital by type (r, q)
   - $R^m = (1 + r)p^m_{-1} - (1 - \delta^k)p^m, \; m \in \{e, s\}$

Relation between wage and factor share

- Software



(Each point is an administrative region.)

Estimation Strategy

- Estimate

$$\ln \frac{rK_i}{wL_i} = (\sigma_e - 1) \ln \hat{w}^j + \text{Controls} + \varepsilon_i$$

$$\frac{1}{1-e_i} \ln \frac{qS_i}{wL_i + rK_i} = (\sigma_s - 1) \ln \hat{w}^j + \text{Controls} + \nu_i$$

Controls include industry, firm type, and firm age dummies.

- Identification assumptions:
  1. In a given period, the price of capital is same across regions but wage differs.
  2. Regional wage is exogenous to individual establishment.

- Bartik (1991) instrument details

Micro Elasticity

- Software and labor are substitutes; equipment and labor are complements: $\sigma_e < 1, \sigma_s > 1$

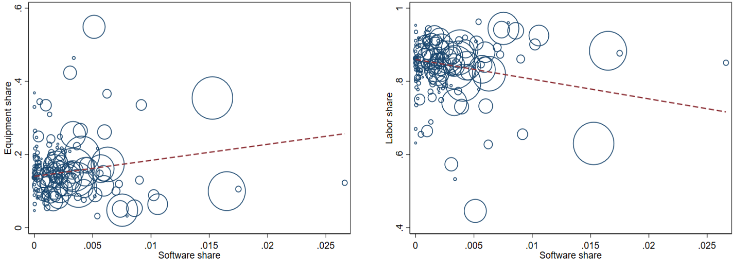
	OLS	Bartik	BGS
σ_e	0.661 ***	0.493 ***	0.274
	(0.084)	(0.153)	(0.220)
σ_s	1.124 ***	1.697 ***	2.522 ***
	(0.119)	(0.229)	(0.350)

- $\sigma_e < 1$ and $\sigma_s > 1$ robust for (i) estimation with $K_i^s > 0$ obs. only, (ii) alternative ordering in the nested CES (i.e., (L+S)+E), (iii) broader capital coverage (Tan/Intan), and (iv) alternative wage

Robustness

Relation between factor shares

- Software



(a) Software & Equipment (b) Software & Labor

(Aggregated to region; size of the circle is regions' value added share.)

Aggregation

- Distributional parameters

	χ	ξ^q	ξ^w	ξ^r	
Reallocation-wgt.	0.1337	0.1705	0.1986	-0.4286	
	$\bar{\epsilon}_e^w$	$\bar{\epsilon}_e^r$	$\bar{\epsilon}_s^w$	$\bar{\epsilon}_s^r$	$\bar{\epsilon}_s^q$
Reallocation-btwn.	4.2062	4.2087	1.8472	6.3139	1.345
	$\bar{b}^w - 1$	$\bar{b}^r - 1$	$\bar{b}^q - 1$		
Markup-within	-0.2232	-0.2202	-0.2770		
	η^w	η^r	η^q		
Markup-wgt.	-0.0004	0.0153	-0.2661		
	$\bar{\epsilon}_\mu^w - 1$	$\bar{\epsilon}_\mu^r - 1$	$\bar{\epsilon}_\mu^q - 1$		
Markup-btwn,	-12.5805	-6.5113	1.0461		

- Equipment and software shares are positively correlated (negative ξ^r)

Aggregatation

- Aggregate elasticities: $\bar{\sigma}_j^n$

	Equipment		Software		
	$\bar{\sigma}_e^w$	$\bar{\sigma}_e^r$	$\bar{\sigma}_s^q$	$\bar{\sigma}_s^w$	$\bar{\sigma}_s^r$
Aggregate Elasticity	0.9904	0.9890	1.6369	1.7268	-0.2817

- Markup elasticities: $\frac{d \ln \mu}{d \ln mc}\Big|_{-n} = (1 - \eta^n)\bar{b}^n + \eta^n \bar{\epsilon}_\mu^n - 1$

	Wage	Equipment	Software
Markup Elasticity	-0.2183	-0.3166	-0.6292

Decomposition

Changes in the Labor Share

- Consider exogenous changes in the price of capital (q or r).
- A discrete approximation of the impact on the labor share is

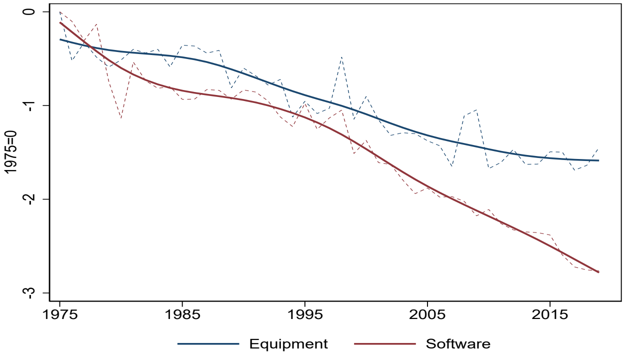
$$LS_t - LS_{t-1} = \overline{LS}_t \times \Big[-\bar{s}_t \left((\bar{\sigma}_s^q - 1) - (\bar{\sigma}_\mu^q - 1) \right) \ln \frac{1/q_t}{1/q_{t-1}}$$
$$-\bar{e}_t \Big((\bar{\sigma}_e^r - 1) - \bar{s}_t (\bar{\sigma}_s^r - 1) - (1 - \bar{s}_t)(\bar{\sigma}_\mu^r - 1) \Big) \ln \frac{1/r_t}{1/r_{t-1}} \Big],$$

where $\bar{x}_t = \frac{x_t + x_{t-1}}{2}$ and $\bar{\sigma}_\mu^n - 1 = \frac{d \ln \mu}{d \ln mc}\Big|_{-n}$.

- Note: $\sigma_s > 1$ but $\bar{\sigma}_s^r < 1$

Capital Embodied Technological Change

- $\ln R^s$ ($\ln q$) declines faster than $\ln R^e$ ($\ln r$). Comparison



Decomposition Results

- Since 1990, software-embodied technological change led to 3.0 p.p. decline of the labor share in Korea (61% of total).
  - 13% of this effect comes from reallocation.

- Overall capital-embodied technological change still lowers the labor share (42% of total). O-R comparison

	LS	$\Delta \ln 1/q$				$\Delta \ln 1/r$
		Total	Within	Reallocation	Markup	
Changes	-0.049	-0.030	-0.009	-0.006	-0.015	+0.009
(% of total)		(61.1)	(17.6)	(12.8)	(30.6)	(-19.3)

Conclusion

Main Takeaways

- The (micro and macro) elasticity of substitution between labor and equipment is below 1 but between labor and software is above 1.

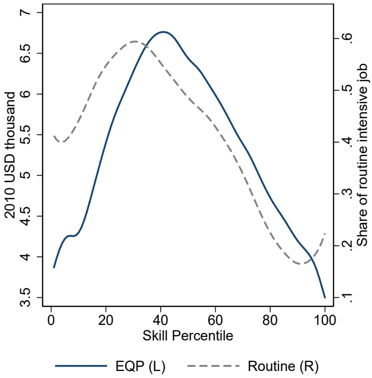
- Labor share declined because software (intangible) substitutes for labor and reallocates factors to low labor share firms.

- Including reallocation to high markup firms, software-embodied technological changes can explain at least 61 percent of the labor share decline in Korea.

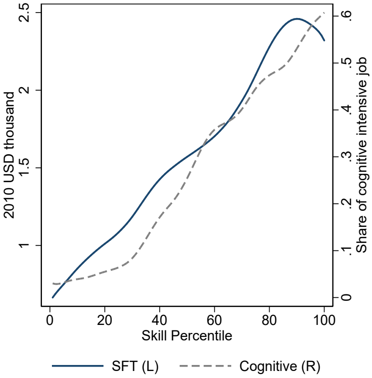
Thank You!

Appendix

Capital Usages by Occupation



(a) Equipment



(b) Software

Go back

Labor and Capital Income

- No arbitrage implies

$$R^j = (1+r)p^j_{-1} - (1-\delta^j)p^j,$$

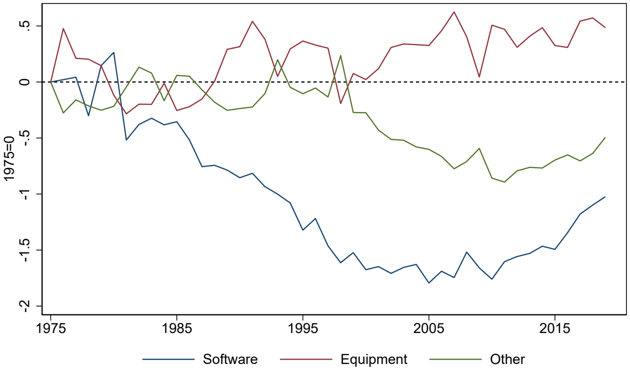
- Assuming homogeneous of degree one production function,

$$LS = \frac{wL}{\mu(wL + \sum_j R^j K^j)}$$

- Get wL, K^j, p^j, δ^j from NA. Get μ from firm-level financial data. Then impute r from above equations.

Back

Labor and Capital Income



Back

Bartik (1991) instrument

- Following Oberfield and Raval (2021), consider Bartik instrument with service industries as labor supply shock to manufacturing plants.
  - $Z_r = \sum_{i \in N_s} \omega_{r,i,0} \log(L_{i,t}/L_{i,0})$
  - Rise of services $\rightarrow$ Reduce available workers who might have worked in manufacturing plants.

- Two most important industries in terms of Rotemberg weights (Goldsmith-Pinkham, Sorkin and Swift, 2020) are research & development and business support services.
  - Research & development and business support services account for 80% of positive weights and 93% of overall weights.

  - Suggests the validity of the instrument in the sense that these industries share relatively common labor supply pools with manufacturing.

Go back

Robustness

- $\sigma_e < 1$ and $\sigma_s > 1$ robust for (i) estimation with $K_i^s > 0$ obs. only, (ii) alternative ordering in the nested CES (i.e., (L+S)+E), (iii) alternative capital coverage (tangible vs intangible), and (iv) alternative wage Back

	Benchmark	Positive obs.	Alt. order	Tan/Intan	Alt. wage
Equipment (σ_e)	0.493 ***	0.547 *	0.491 ***	0.654 ***	0.521 ***
	(0.153)	(0.317)	(0.153)	(0.162)	(0.144)
Software (σ_s)	1.697 ***	1.471 ***	1.155 ***	2.815 ***	1.659 ***
	(0.229)	(0.417)	(0.197)	(0.434)	(0.217)

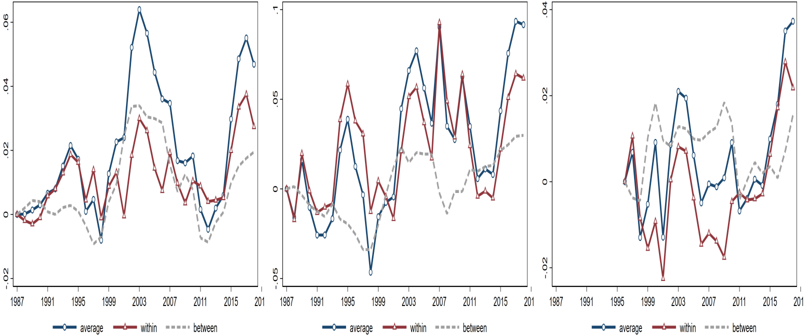
Markups

- Accounting profit:
  $1 - 1/\mu^{AP} =$ Operating Profit / Sales

- User cost of capital:
  $1 - 1/\mu^{UC} =$ (Operating Income $-(r+\delta)$K) / Sales

- Production function:
  $\mu^{PF} = (\partial \log F / \partial \log \text{COGS})/(\text{COGS}/\text{ Sales})$ Go back 1 Go back 2

Markups

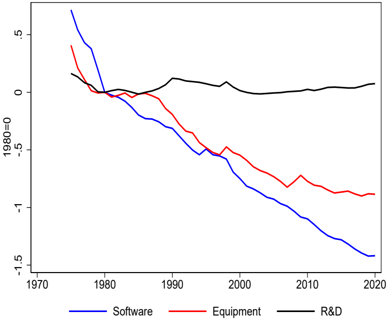


(a) Accounting Profit (b) User Cost Capital (c) Production Function

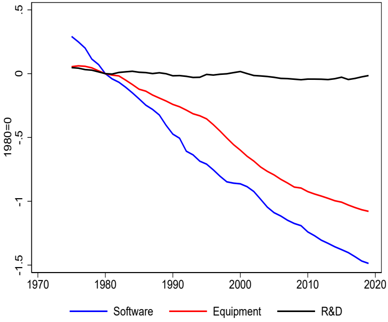
Go back 1 Go back 2

Capital Embodied Technological Change

- Relative price of investment across capital types.



(d) Korea

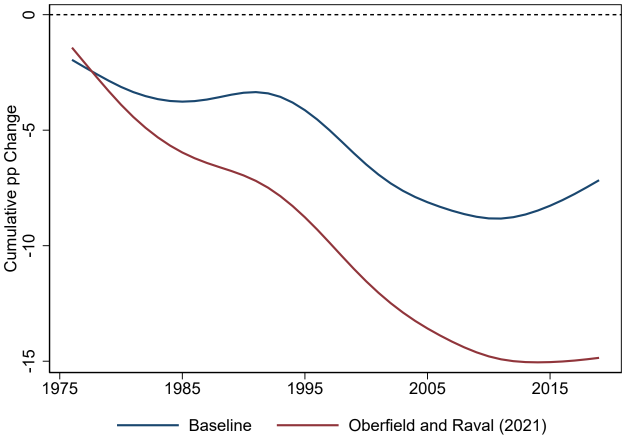


(e) US

Back

Required Factor Bias of Technical Change

- $dLS = LS(\sigma - 1)\, d \ln \text{factor prices} + \text{factor bias}$ Back



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