How to Make an Action *Better*

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Product Design

Two car sellers A and B; products a, b.

Each firm's product yields (random) monetary rewards/costs to consumers.

Heterogeneous populace with (risk-averse) utilities in money and beliefs.

A contemplating replacing a with a new alternative, $\hat{a}$.

Question: What are the properties of $\hat{a}$ that make it no worse versus b than a?

What robustly improves upon a versus b?

Bilateral Trade I

Two parties: Buyer (B) and Seller (S).

Asset (initially with seller) pays out random reward. State, upon which reward depends, is contractible.

What we know:

B and S are rational EU maximizers.

Status quo arrangement, *in money*.

B's outside option (S's too).

What we don't know:

Beliefs of B and S.

Risk preferences (utility functions) of B and S–maybe we know the shape.

Aggregate risk.

Bilateral Trade II

Question: What other contracts must B and S be willing to accept?

That is, given what we know.

Answer: Soon (relatively).

High-level research question

What does one choice robustly tell us about another?

Start with two actions, then general (finite).

The Formal Setting

Setup I

Unknown state of the world, $\theta \in \Theta$, compact, metrizable.

Agent (decision-maker, DM) is EU maximizer with belief $\mu \in \Delta \equiv \Delta(\Theta)$.

Two actions, $\{a, b\}$.

Action: bounded, (Borel-)measurable function from Θ to $\mathbb{R}$ (money). Write a_θ, b_θ.

No action is dominated.

Agent has a continuous, strictly increasing, and concave utility function $u : \mathbb{R} \to \mathbb{R}$.

Replacing (transforming) a

Definition. Action $\hat{a}$ is *b-Superior* to action a if

$$\mathbb{E}_\mu u(a_\theta) \geq \mathbb{E}_\mu u(b_\theta) \Rightarrow \mathbb{E}_\mu u(\hat{a}_\theta) \geq \mathbb{E}_\mu u(b_\theta),$$

and

$$\mathbb{E}_\mu u(a_\theta) > \mathbb{E}_\mu u(b_\theta) \Rightarrow \mathbb{E}_\mu u(\hat{a}_\theta) > \mathbb{E}_\mu u(b_\theta),$$

for any strictly increasing, concave, and continuous u.

The Question

When is $\hat{a}$ b-superior to a?

Keep in mind,

Results in terms of (deterministic) state-dependent payoffs.

Not lotteries.

Road map

1. Characterize b-superiority in terms of state-dependent payoffs.
2. Modify class to just strictly increasing and continuous us? Not just risk averse.
3. Multiple alternatives? Set of bs.
4. Shutting down a dimension.
5. Applications.
6. A calibration result.
7. Rational inattention (perfect for the end of the talk).

Characterization

The result

Definition. A *Mixture* of actions a and b is an action a^λ that yields payoff $a^\lambda_\theta := \lambda a_\theta + (1-\lambda) b_\theta$ in each state $\theta \in \Theta$ for some $\lambda \in [0,1]$.

$$\mathcal{A} := \{\theta \in \Theta : a_\theta > b_\theta\}, \quad \mathcal{B} := \{\theta \in \Theta : a_\theta < b_\theta\}, \quad \mathcal{C} := \{\theta \in \Theta : a_\theta = b_\theta\}.$$

Main Theorem. Fix a and b. Action $\hat{a}$ is b-superior to action a if and only if $\hat{a}_\theta > b_\theta$ for all $\theta \in \mathcal{A}$ and $\hat{a}$ (weakly) dominates a mixture of a and b.

Sufficiency

Assume $\hat{a}$ is a mixture of a and b.

Let L_a, L_b, and $L_{\hat{a}}$ be lotteries induced by DM's belief $\mu \in \Delta$.

Suppose $L_a \succeq L_b$.

Independence $\Rightarrow$ for any $\lambda \in [0,1]$, $\tilde{L} := \lambda L_a + (1-\lambda) L_b \succeq L_b$.

Mixture $\Rightarrow$ there exists λ s.t. $\lambda a_\theta + (1-\lambda) b_\theta = \hat{a}_\theta$ for all $\theta \Rightarrow \tilde{L}$ is an MPS of $L_{\hat{a}}$

Risk Aversion $\Rightarrow L_{\hat{a}} \succeq \tilde{L} \succeq L_b$.

Alternative Perspective: understand action $a \in \mathbb{R}^\Theta$ and represent preferences by concave and monotone $V\colon \mathbb{R}^\Theta \to \mathbb{R}$. Concavity of V is sufficient! Quasi-concavity almost.

Necessity, Step 1

Definition. $\hat{a}$ *pairwise-dominates* a collection of mixtures of a and b if for any pair $(\theta, \theta') \in \mathcal{A} \times \mathcal{B}$, there exists a $\lambda_{\theta,\theta'} \in [0,1]$ such that

$$\hat{a}_\theta \geq \lambda_{\theta,\theta'} a_\theta + (1 - \lambda_{\theta,\theta'}) b_\theta \quad \text{and} \quad \hat{a}_{\theta'} \geq \lambda_{\theta,\theta'} a_{\theta'} + (1 - \lambda_{\theta,\theta'}) b_{\theta'}.$$

Lemma. If $\hat{a}$ pairwise-dominates a collection of mixtures of a and b, $\hat{a}_{\theta^\dagger} \geq a_{\theta^\dagger}$ for all $\theta^\dagger \in \mathcal{C}$, and $\hat{a}_\theta \geq b_\theta$ for all $\theta \in \mathcal{A}$, then $\hat{a}$ dominates a mixture of a and b.

Necessity, Step 2

Lemma. $\hat{a}$ is b-superior to a only if i. for any $\theta^\dagger \in \mathcal{C}$, $\hat{a}_{\theta^\dagger} \geq a_{\theta^\dagger}$; ii. for any $\theta \in \mathcal{A}$, $\hat{a}_\theta > b_\theta$; and iii. $\hat{a}$ pairwise-dominates a collection of mixtures of a and b.

Prove this by contraposition: pairwise domination allows focus on just two states.

Construct "very concave" (locally) u that moves belief in right (wrong) direction!

Could appeal to revealed-preference results...**if they existed**. None (to my knowledge) are suitable for this, with SEU + risk-aversion.

Shorter direct proof (following Jewitt '86), but lemmas will be useful later.

The Innocuity of EU

For concave (BAD TERM) DM, $a \succeq b \Rightarrow \hat{a} \succeq b$ if and only if $\hat{a}$ dominates a mixture of a and b.

For SEU DM, $a \succeq b \Rightarrow \hat{a} \succeq b$ if and only if $\hat{a}$ dominates a mixture of a and b.

SEU has little bite: concavity includes variational, max-min, mean-variance...

Road map

1. ~~Characterize b-superiority in terms of state-dependent payoffs.~~
2. Modify class to just strictly increasing and continuous us? Not just risk averse.
3. Multiple alternatives? Set of bs.
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Just Monotone?

New definition

Definition. Action $\hat{a}$ is *b-Better* than action a if

$$\mathbb{E}_\mu u(a_\theta) \geq \mathbb{E}_\mu u(b_\theta) \implies \mathbb{E}_\mu u(\hat{a}_\theta) \geq \mathbb{E}_\mu u(b_\theta),$$

and

$$\mathbb{E}_\mu u(a_\theta) > \mathbb{E}_\mu u(b_\theta) \implies \mathbb{E}_\mu u(\hat{a}_\theta) > \mathbb{E}_\mu u(b_\theta),$$

for any strictly increasing, ~~concave,~~ and continuous u.

The Question redux

When is $\hat{a}$ b-better to a?

The result

Theorem. Action $\hat{a}$ is b-better than action a if and only if $\hat{a}$ dominates a or b, and for all $\theta \in \mathcal{A}$ $\hat{a}_\theta > b_\theta$.

Idea: risk-averse agent, need rotations of a "toward" b; risk-loving agent, away.

Multiple Alternatives (and A Benchmark)

One versus many

Now finite set of actions ($|A| = m + 1 \geq 2$).

For any $a \in A$, $B := A \setminus \{a\}$

First, known beliefs.

Second, back to both dimensions.

How to Make a~~n Action~~ **Lottery** *Better*

Lotteries

For any fixed $\mu \in \Delta$, each $a \in A$ induces a lottery L_a.

Definition. Lottery $L_{\hat{a}}$ B-Improves upon lottery L_a if

$$L_a \succ L_b \text{ for all } b \in B \implies L_{\hat{a}} \succeq L_b \text{ for all } b \in B,$$

for all utilities in the specified class.

Classes: risk-averse, monotone, even *experiments*.

Notation

For a fixed $a \in A$, enumerate the lotteries in B: $L_1, \ldots, L_m$.

$\lambda \in \mathbb{R}^m$ is a convex weight if $0 \leq \lambda_j \leq 1$ for all $j \in \{1, \ldots, m\}$ and $\sum_{j=1}^{m} \lambda_j \leq 1$.

$\trianglerighteq$: the dominance relation in the specified class.

For risk aversion, $L_a \trianglerighteq L_b$ means L_a SOSD L_b.

For monotone, $L_a \trianglerighteq L_b$ means L_a FOSD L_b.

How to make a lottery better

Theorem. Lottery $L_{\hat{a}}$ B-improves upon lottery L_a if and only if for all $b \in \{1, \ldots, m\}$ there exists a convex weight λ such that

$$\sum_{i=1}^{m} \lambda_i L_i + \left(1 - \sum_{i=1}^{m} \lambda_i\right) L_{\hat{a}} \trianglerighteq \sum_{i=1}^{m} \lambda_i L_a + \left(1 - \sum_{i=1}^{m} \lambda_i\right) L_b.$$

Proof

Step 1. Revealed preference reformulation. $L_{\hat{a}}$ B-improves upon lottery $L_a \Leftrightarrow$ there does not exist a utility function in the specified class such that

$$L_a \succ L_b \text{ for all } b \in B, \text{ and } L_b \succ L_{\hat{a}} \text{ for some } b \in B.$$

Step 2. Appeal to our betters. By Corollary 3 in Fishburn '75, this holds if and only if for all $b \in B$

$$\sum_{i=1}^{m} \lambda_i L_i + \left(1 - \sum_{i=1}^{m} \lambda_i\right) L_{\hat{a}} \trianglerighteq \sum_{i=1}^{m} \lambda_i L_a + \left(1 - \sum_{i=1}^{m} \lambda_i\right) L_b,$$

for some convex weight λ. ■

Idea: linearity $\Rightarrow$ separating hyperplane!

Back to Making Actions Better

More structure

Restore both dimensions of uncertainty–knowing utilities but not beliefs is boring–both beliefs and utilities.

Two assumptions on A and Θ (recalling $B := A \setminus \{a\}$):

1. **Rich:** for each $a \in A$, there exists at least one $\theta \in \Theta$ such that $a_\theta > \max_{b \in B} b_\theta$.
2. **Single-Peaked:** for each $a \in A$ and any pair $\theta, \theta' \in \Theta$, either
   - 2.1 there exists $\lambda \in [0,1]$ such that

$$\lambda a_\theta + (1-\lambda) a_{\theta'} \geq \max_{b \in B} \{\lambda b_\theta + (1-\lambda) b_{\theta'}\}, \quad \text{or}$$

   - 2.2 there exists $b \in B$ such that $b_\theta \geq a_\theta$ and $b_{\theta'} \geq a_{\theta'}$.

Satisfied by "quadratic loss" w/ $\Theta = [0,1]$, $A \subset [0,1]$, and $a_\theta = -(a-\theta)^2$.

Robust improvements versus many

Definition. Action $\hat{a}$ is *B-Superior* to action a if

$$\mathbb{E}_\mu u(a_\theta) \geq \max_{b \in B} \mathbb{E}_\mu u(b_\theta) \implies \mathbb{E}_\mu u(\hat{a}_\theta) \geq \max_{b \in B} \mathbb{E}_\mu u(b_\theta),$$

and

$$\mathbb{E}_\mu u(a_\theta) > \max_{b \in B} \mathbb{E}_\mu u(b_\theta) \implies \mathbb{E}_\mu u(\hat{a}_\theta) > \max_{b \in B} \mathbb{E}_\mu u(b_\theta),$$

for any strictly increasing, concave, and continuous u.

Want a to improve versus entire set of actions B.

The result

Letting $\mathcal{A}_B := \{\theta \in \Theta \colon a_\theta > \max_{b \in B} b_\theta\}$,

> **Proposition.** Fix a and B. Action $\hat{a}$ is B-superior to action a if and only if i) $\hat{a}_\theta > \max_{b \in B} b_\theta$ for all $\theta \in \mathcal{A}_B$; and ii) for all $b \in B$, $\hat{a}$ (weakly) dominates a mixture of a and b.

But what about just a "lower" action (not necessarily a) after a transformation?

Robust comparative statics exercise doable?

Yes! Another paper: robust CS exercise for SEU agent.

Lower action after transformation if and only if* everything "tilts up."

Proof sketch

Sufficiency: almost immediate from earlier Theorem (just an aggregation).

Necessity: more involved.

Fix a and use notation

$$\mathcal{A}_B := \left\{\theta \in \Theta : a_\theta > \max_{b \in B} b_\theta\right\} \quad \text{and} \quad \mathcal{B}_B := \left\{\theta \in \Theta : a_\theta < \max_{b \in B} b_\theta\right\},$$

and, for a fixed b,

$$\mathcal{A}_b := \{\theta \in \Theta : a_\theta > b_\theta\} \quad \text{and} \quad \mathcal{B}_b := \{\theta \in \Theta : a_\theta < b_\theta\}.$$

Difficulty is $\mathcal{A}_B \subseteq \mathcal{A}_b, \mathcal{B}_b \subseteq \mathcal{B}_B$. But not necessarily equal!

Proof sketch continued

Extra structure: Battigalli, Cerreia-Vioglio, Maccheroni, and Marinacci ′16 and Weinstein ′16 ⇒ increased risk aversion can only increase (in a set-inclusion sense) the justifiable set. "Falling Tide."

So, on each edge of simplex, nothing new becomes justifiable as we vary (risk-averse) utility.

Moreover, "relevant" actions on edges do not change.

Ultimately, proof via contraposition: move to an edge and show pair-wise dominance fails for a "relevant" action.

Applications

Politics

Two political parties, each running candidate: a versus b.

Unknown state $\theta \in \Theta$, candidate a (b) produces wealth a_θ (b_θ) in state θ for voters.

Voters known to be risk averse, but vary in beliefs and precise utility functions.

Party running a contemplating replacing him with $\hat{a}$.

Politics

Remark. $\hat{a}$ can do no worse versus b than a if and only if $\hat{a}$ dominates a mixture of a and b.

Moderation is robust.

Bilateral trade

Buyer (B) and seller (S). State $\theta \in \Theta$.

S has asset: pays out v_θ in state θ. State is contractible.

Status quo trade agreement: transfer $\gamma_\theta \in \mathbb{R}$ from B to S.

B's outside option is the sure-thing 0. S's is asset.

We know neither beliefs nor utilities. Both are risk averse and **would accept status quo.**

What other arrangements must they accept?

Characterization

Some accounting: for all $\theta \in \Theta$ and for any $\lambda \in [0,1]$,

$$\lambda(v_\theta - \gamma_\theta) = v_\theta - \lambda\gamma_\theta - (1-\lambda)v_\theta.$$

So, transfer that produces mixture is

$$\lambda\gamma_\theta + (1-\lambda)v_\theta,$$

precisely a mixture for S.

Adding in budget balance $\Rightarrow$

Remark. A new trade agreement, $(\hat{\gamma}_\theta)_{\theta\in\Theta}$ must be acceptable if and only if there exists some $\lambda \in (0,1]$ such that $\hat{\gamma}_\theta = \lambda\gamma_\theta + (1-\lambda)v_\theta$ for all $\theta \in \Theta$.

High-level implication

Take an environment with risk: e.g., driving, long-run environment.

Take an acceptable policy that lowers risk: insurance, abatement.

The only other robustly acceptable policies involve *more* risk!

Less insurance,

More pollution.

Calibration

Rabin (2000)

Identifies an absurdity implied by expected utility by "calibrating a relationship between risk attitudes over small and large stakes."

Striking example: "suppose from any initial wealth level, a person turns down gambles where she loses $100 or gains $110, each with 50% probability. Then she will turn down 50-50 bets of losing $1,000 or gaining *any* sum of money."

But these insights persist beyond EU (Safra & Segal '08): "...leaves us with the choice between several controversial conclusions: People do not reject small risks, people reject excellent large risk, or, explanations that seem to be more likely, people are not globally risk averse or people do not utilize just one preference relation."

Objective versus subjective

Commonality between calibration papers: *Objective probabilities.*

What if "PROBABILITY DOES NOT EXIST?" (De Finetti '74).

SEU gives a behavioral definition of probability: "...rate at which an individual is willing to bet on the occurrence of an event" (Nau '01).

Can we calibrate an SEU agent?

Calibrating the subjective

Two binary-action menus, $A = \{s, r\}$ and $\hat{A} = \{s, \hat{r}\}$.

Two states, $\Theta = \{0, 1\}$.

Safe s gives 0 in each state;

Risky r gives $\alpha > 0$ in state 1 and $-\beta < 0$ in state 0; and

Risky $\hat{r}$ gives $\hat{\alpha} > 0$ in state 1 and $-\hat{\beta} < 0$ in state 0.

DM has initial wealth $w \in \mathbb{R}$ and has (risk-averse) utility over terminal wealths.

Say *The safe option must remain optimal* if $s \succeq r$ for all $w \in \mathbb{R} \Rightarrow s \succeq \hat{r}$ for all $w \in \mathbb{R}$.

Obviously worse actions become worse, and that's it

Say *The risky option becomes worse* if $\hat{\beta} \geq \beta$, and an *Actuarial worsening* transpires: $\frac{\alpha}{\beta} \geq \frac{\hat{\alpha}}{\hat{\beta}}$.

Corollary. The safe option must remain optimal if and only if the risky option becomes worse.

Necessity requires some work.

Information Acquisition

Endogenizing beliefs

When a DM **acquires** information before taking a choice, what changes to a lead to it being chosen **more frequently** (versus b)?

Start with a prior $\mu_0 \in \Delta^\circ$ (full support)

Now the DM (given her menu) acquires info *flexibly*: chooses any feasible distribution over posteriors $F \in \mathcal{F}_{\mu_0}$.

Benefit of information is the value function, in belief μ,

$$V(\mu) := \max_{\tilde{a} \in \{a,b\}} \mathbb{E}_\mu u(\tilde{a}_\theta).$$

More setup

With menu $\{a, b\}$ ($\{\hat{a}, b\}$), the DM solves

$$\max_{F \in \mathcal{F}(\mu_0)} \int_{\Delta} V(\mu)\, dF(\mu) - D(F) \qquad \left(\max_{\hat{F} \in \mathcal{F}(\mu_0)} \int_{\Delta} \hat{V}(\mu)\, d\hat{F}(\mu) - D\left(\hat{F}\right) \right),$$

where D is a uniformly posterior-separable (UPS) cost.

Any solution F^* ($\hat{F}^*$) produces an optimal choice probability of action a ($\hat{a}$) p ($\hat{p}$).

Definition. $\hat{a}$ is selected more than a if for any UPS D, prior $\mu_0 \in \operatorname{int} \Delta$, and strictly increasing, concave u; for any optimal p, there exists an optimal choice probability $\hat{p} \geq p$.

Two states is special

Proposition. If there are two states, $\hat{a}$ is selected more than a if and only if $\hat{a}$ dominates a or b.

Sufficiency: straightforward and mechanical.

Key to necessity: mixture is like increasing the cost of information acquisition $\Rightarrow$ Leaves only dominance improvements.

Three or more states

$\hat{a}$ dominating a insufficient for $\hat{a}$ to be selected more.

Idea: can come up with a "twisted" UPS cost.

Troubling. An unambiguous improvement to $a \Rightarrow \hat{a}$ is chosen strictly less!

Summary

How to make an action better (versus others)?

If agent is risk averse, make it more like the others.

Otherwise, make it unambiguously better.

Some implications: assuming risk aversion,

1. moderation is very robust,
2. for an acceptable policy that lowers risk, only other acceptable policies also lower risk but less,
3. force encouraging conformity in product design,
4. avoiding risk only necessitates avoiding worse risk.

With endogenous info, some obvious improvements aren't.

Thank you!