GLOBALIZATION, TRADE IMBALANCES AND LABOR MARKET ADJUSTMENT*

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Abstract

We argue that modeling trade imbalances is crucial to understanding transitional dynamics in response to globalization shocks. We build and estimate a general equilibrium, multi-country, multi-sector model of trade with two key ingredients: (a) endogenous trade imbalances arising from households' consumption and saving decisions; (b) labor market frictions across and within sectors. We use our model to perform several empirical exercises. We find that the "China shock" accounted for 25% of the decline in US manufacturing between 2000 and 2014—twice the magnitude predicted from a model imposing balanced trade. A concurrent rise in US service employment led to a negligible aggregate unemployment response. We then benchmark our model's predictions for the gains from trade against the popular "ACR" sufficient statistics approach. We find that our predictions for the long-run gains from trade and consumption dynamics significantly diverge. *JEL* Code: F16.

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"One major contrast between most economic analyses of globalization's impact and those of the broader public ... is the focus, or lack thereof, on trade imbalances. The public tends to see trade surpluses or deficits as determining winners and losers; the general equilibrium trade models that underlay the 1990s' consensus gave no role to trade imbalances at all. The economists' approach is almost certainly right for the long run ... Yet in the long run we are all dead, and rapid changes in trade balances can cause serious problems of adjustment ..."

Paul Krugman, "Globalization: What Did We Miss?"[1]

1 Introduction

A large body of evidence shows that globalization can lead to significant labor market disruption. For instance, Autor et al. (2013) show that American workers in regions facing steeper import competition from China are less likely to work in manufacturing and more likely to be unemployed.[2] This work has generated considerable interest and research in understanding, modeling, and quantifying the adjustment process in response to globalization shocks.[3] Yet, this literature has abstracted from modeling trade imbalances, and has been silent on how they could influence the labor market adjustment process.

This gap is puzzling in light of the size and persistence of trade imbalances in the last three decades, coupled with an increased discomfort among American policy makers towards trade deficits. Indeed, there is a pervasive concern among policy makers and the public that trade deficits crowd out domestic production, reducing jobs and hurting workers.[4] When trade is balanced, equilibrium forces ensure that a contraction of import-competing sectors is met with a simultaneous expansion of export-oriented sectors. On the other hand, if globalization shocks induce countries to run trade imbalances, these shifts are no longer synchronized, affecting the dynamics of reallocation. Hence, the behavior of trade imbalances can influence the dynamics of job losses and gains, especially in the presence of unemployment and labor market frictions.

In this paper, we study how endogenizing trade imbalances influences the labor market adjustment process in response to globalization. Does ignoring trade imbalances when we investigate the labor market consequences of trade shocks matter at all? How much insight do we lose in doing so? To address these questions, we build on existing models of globalization and labor market adjustment and develop a quantitative, general equilibrium, multi-country, multi-sector model with three key ingredients: (i) Consumption-saving decisions in each country are determined by the optimizing behavior of representative households, leading to endogenous trade imbalances; (ii) Labor market

[1] See Krugman (2019).

[2] Other recent papers tying globalization shocks to labor market disruptions include Pierce and Schott (2016), Costa et al. (2016), Dix-Carneiro and Kovak (2017, 2019), Dauth et al. (2018), Utar (2018), among many others.

[3] See Artuç et al. (2010), Dix-Carneiro (2014), Traiberman (2019), and Caliendo et al. (2019).

[4] For examples of recent policy discussions, see Scott (1998), Bernanke (2005), and Navarro (2019).

frictions across and within sectors lead to unemployment dynamics, and sluggish transitions to shocks; and (iii) Ricardian comparative advantage forces promote trade but geographical barriers inhibit it.

In our model, trade imbalances arise from country-level representative households making consumption and savings decisions. These decisions are made under perfect foresight of aggregate variables and give rise to an Euler Equation that dictates how countries smooth consumption over time in response to shocks in productivity, trade costs, and inter-temporal preferences. Our approach relies neither on ad hoc rules for imbalances nor on specifying the path of imbalances exogenously, which are common in the international trade literature. Instead, our perspective builds on the workhorse model of imbalances in international macroeconomics, providing a natural benchmark for understanding how they shape the labor market adjustment process.[5]

Turning to production and the labor market, each household is comprised of individual workers. These workers choose in which sector to work, taking into account how their choices affect the household's maximizing problem. Similarly, firms choose in which sector to produce, maximizing expected discounted profits. Together, a firm and worker produce goods that can be traded across countries. Labor markets feature two sources of frictions: (i) switching costs to moving across sectors à la Artuç et al. (2010); and (ii) matching frictions within sectors à la Mortensen and Pissarides (1994). In particular, our framework allows for job creation and destruction to respond to trade shocks, leading to rich unemployment dynamics and speaking to a key concern of the public's anxiety over globalization.[6]

We estimate our model using a simulated method of moments and data from the World Input Output Database as well as several sources of microdata around the world. To ensure tractability of the estimation procedure, we assume the economy is in steady state and we match data moments for the year 2000. The procedure conditions on the observed trade shares and allows us to estimate our parameters country by country, greatly simplifying the process.

To understand the main mechanisms at play in our model, we first consider a hypothetical situation where China's productivity steadily grows for fifteen years before reaching a plateau. In this case, China smooths consumption by consuming over production in the short run—generating trade deficits—and then below in the long run—generating a permanent trade surplus. These patterns in trade imbalances lead to non-monotonic patterns of adjustment. In the short run, China expands its non-tradable sectors and contracts its tradable sectors. However, in the long run, it pays off its debt by permanently expanding its tradable sectors above their initial steady-

[5]See Obstfeld and Rogoff (1995) for a survey of this approach to imbalances in international macroeconomics. More recent work on global imbalances builds on the standard consumption savings model by adding financial frictions (e.g., Caballero et al. (2008) and Mendoza et al. (2009)), or demographics (e.g., Barany et al. (2018)).

[6]Pavcnik (2017) reviews survey data showing that only 20% of Americans believe trade creates jobs, while 50% believe it destroys them.

state levels.

These non-monotonic patterns of adjustment contrast with predictions of the model if trade is imposed to balance for all countries in all periods—the typical approach in the International Trade literature. In this scenario, sectors gradually and monotonically expand or contract until the new steady state is reached. Importantly, we observe considerably less reallocation in this scenario, both in the short and long runs. This exercise shows that the behavior of trade imbalances closely dictates the pattern and the magnitude of sectoral reallocation. Next, we show that the exact path of shocks affecting the global economy—and not just their initial and final levels—is critical for the evolution of trade imbalances and their long-run consequences. Relevant for the policy debate, trade surpluses (deficits) do not necessarily lead to lower (higher) unemployment.

We revisit China's rise as a major international trade player through the lens of our model. This event has generated much attention in academic and policy circles, which are mainly concerned with the effect of China's trade surplus on the labor market and on the manufacturing sector in the United States. We consider changes in Chinese productivity and trade costs with the rest of the world, as well as shocks to China's saving rate—the so-called "savings glut." We first estimate that these changes in the Chinese economy explain 36% of the deterioration on the US trade deficit between 2000 and 2014. Next we find that the economic shocks China experienced over this period accounted for 25% of the decline in American manufacturing. Our model predicts fast job creation in services of the same magnitude, leading to a zero effect on unemployment. If balanced-trade is imposed, we would estimate that China accounted for 13% of the decline of US manufacturing. As before, we also have simultaneous job creation in other sectors, leading to a muted unemployment response. However, the balanced-trade model predicts a much smaller expansion in services, and a much larger one in Agriculture.

We estimate that shocks to Chinese productivity were responsible for the bulk of China's effect on the size of US employment in manufacturing. China's savings glut had a significant short-run negative effect, but this effect was completely undone by 2014. Finally, we find that the effect of the "China shock" on US consumption was positive. Although small in absolute terms, these consumption gains are larger than previously-estimated effects of large trade shocks such as NAFTA and the US-China trade war (Caliendo and Parro, 2022).

Next, we study the implications of trade imbalances and labor market frictions for the gains from trade, typically computed using the sufficient-statistics approach of Arkolakis et al. (2012) and extended by Costinot and Rodríguez-Clare (2014). Differences in predicted long-run consumption effects of trade are significant, with both imbalances and labor market frictions playing important roles in these discrepancies. We also evaluate the relative performance of these approaches over the transition path. We find that the discrepancies are smaller once we focus on the comparison of net present values of consumption. Nevertheless, our model generates large swings in consumption, whereas the formula in Costinot and Rodríguez-Clare (2014) implies flatter dynamics.

As a final exercise, we compare outcomes of our model with an alternative approach to modelling trade imbalances assumed in many quantitative general equilibrium models of trade. In this approach, trade imbalances do not arise from economic decisions. Rather, each countries' profits are pooled into a global portfolio and redistributed back to countries according to country-specific shares that are calibrated to match initial observed cross-sectional imbalances (Caliendo and Parro, 2022). We show that this approach leads to different patterns for the evoltion of trade imbalances across countries. In turn, this leads to distinct behavior of reallocation and unemployment.

Our paper speaks to a large literature that investigates the labor market consequences of globalization, both empirically and quantitatively. We make two contributions to this literature by incorporating both involuntary unemployment and trade imbalances into the state-of-the-art Ricardian trade model of Caliendo and Parro (2015). Broadly speaking, quantitative trade models based on Eaton and Kortum (2002) have only allowed for a non-employment option (i.e., voluntary unemployment) or have focused on steady-state analyses, ignoring transitional dynamics. Caliendo et al. (2019) is an important example of a dynamic quantitative trade model in which workers make a labor supply decision and face mobility frictions across sectors and regions. However, their model does not feature job losses and unemployment. On the other end, Carrère et al. (2020) and Guner et al. (2020) incorporate search frictions and unemployment into multi-sector extensions of Eaton and Kortum (2002), but do not study out-of-steady-state dynamics. In a recent exception, Rodriguez-Clare et al. (2020) incorporates wage rigidity into the model of Caliendo et al. (2019) to investigate the unemployment effects of the China Shock on local labor markets in the United States.[7]

Importantly, though, none of these papers model trade imbalances. We do so by incorporating the workhorse model of imbalances used in the international macroeconomics literature allowing for savings decisions by means of an international bonds market as in Reyes-Heroles (2016).[8] In that regard, our paper is closely related to Kehoe et al. (2018) who explore the implications of the increase in the United States trade deficit for the secular decline in manufacturing labor over the last four decades. However, their paper does not incorporate sluggish labor market adjustment nor unemployment dynamics.[9]

[7] In addition to these papers based on the Eaton and Kortum model, Helpman and Itskhoki (2010) add labor market frictions to a two-country Melitz model, and Heid and Larch (2016) add labor market frictions to an Armington model of trade. Coşar et al. (2016) incorporate search frictions and unemployment to a quantitative small open economy Melitz model with firm dynamics, but focus on steady-state analyses. Ruggieri (2019) extends that model to study the transition in response to trade shocks. Similarly, Helpman and Itskhoki (2015) also analyze the dynamic behavior of a two-country Melitz model with labor market frictions. Finally, Kambourov (2009), Artuç et al. (2010), Dix-Carneiro (2014), and Traiberman (2019) also study transitional dynamics, but through the lens of small open economy models.

[8] A few papers have analyzed the consequences of current account rebalancing on labor reallocation and unemployment by considering changes in imbalances as exogenous, *e.g.* Obstfeld and Rogoff (2005), Dekle et al. (2007), and Eaton et al. (2013).

[9] In International Macroeconomics, Kehoe and Ruhl (2009), Meza and Urrutia (2011), and Ju et al. (2014) are

This paper is structured as follows. Section 2 outlines our model. Section 3 describes the data we use and our estimation procedure. In section 4, we present a detailed discussion of our model's mechanisms. Section 5 studies a series of counterfactual experiments, including an analysis of the impact of the China shock on the US labor market, and comparisons between predictions of our model and those from other popular approaches in the literature. We conclude and discuss future research in section 6.

2 Model

Our model builds on existing workhorse models of globalization, trade imbalances and labor market adjustment. Trade imbalances are modeled according to the inter-temporal approach of Obstfeld and Rogoff (1995), and the trade block is based on Caliendo and Parro (2015). We adopt the framework in Artuç et al. (2010) to model labor mobility frictions across sectors and the structure in Mortensen and Pissarides (1994) to model search frictions and job creation and destruction. Sections 2.1 through 2.8 formalize our model showing how these different frameworks fit together.

The economy consists of $i = 1, ..., N$ countries, each with a constant labor endowment given by a continuum of workers with mass $\overline{L}_i$. Three different types of goods are available in the economy: a non-tradable final good, K non-tradable sectoral composite intermediate goods, and tradable intermediate varieties. All agents have perfect foresight over all aggregate variables, and we do not consider aggregate uncertainty.

2.1 Technology

We start by describing the technologies available in every period t to produce the different types of goods. The final non-tradable good is produced by identical, perfectly competitive firms in each country. Its output is given by a Cobb-Douglas aggregate of the K sector-specific composite intermediate goods. We denote country i's share of expenditure on sector k goods by $\mu_{k,i}$.

Sector-specific composite goods are produced by identical, perfectly competitive firms operating in each sector k of each country i. Total output of sector k is given by a constant elasticity of substitution (CES) aggregate over the output of a sector-specific, continuum of varieties indexed by $j \in [0, 1]$. These sector-specific goods are solely used as intermediate inputs for the production of the final good or as intermediates in the production of varieties. Like the final good, these composites are non-traded.

Units of variety $j \in [0, 1]$ for a particular sector k are produced by firms that combine the labor of one single worker with composite intermediate inputs purchased from all sectors. For a given variety j, a firm-worker pair engaged in production is associated with a particular productivity x

examples of the scarce work studying the the interaction between the current account and labor market reallocation.

that we refer to as a match-specific productivity. In addition to the match-specific productivity, firms producing variety j in sector k and country i at time t have access to a common technology with productivity $z^t_{k,i}(j)$. Total output by a firm producing variety j in sector k with match-specific productivity x, and employing composite intermediate inputs $\left\{M^t_{\ell,i}\right\}_{\ell=1}^{K}$ at time t, is given by:

$$y^t_{k,i}\left(j,x\right) = z^t_{k,i}\left(j\right) x^{\gamma_{k,i}} \left(\prod_{\ell=1}^{K} \left(M^t_{\ell,i}\right)^{\nu_{k\ell,i}}\right)^{\left(1-\gamma_{k,i}\right)}, \tag{1}$$

where $\gamma_{k,i} \in (0,1)$, $\nu_{k\ell,i} > 0$, and $\sum_{\ell=1}^{K} \nu_{k\ell,i} = 1$.

2.2 Labor Markets

Workers and single-worker firms producing varieties engage in a costly search process. Firms post vacancies, but not all of them are filled. Workers search for a job, but not all of them are successful, leading to involuntary unemployment. We assume that labor markets are segmented by sector—firms posting vacancies in sector k in period t can only match with workers searching in that sector in that period, and vice versa. More precisely, denote the sector-specific unemployment rate by $u^t_{k,i}$, and the vacancy posting rate as $v^t_{k,i}$. Both variables are expressed as a fraction of the labor force $L^t_{k,i}$, measured as the sum of employed and unemployed workers in sector k in country i at time t. In every period, the fraction of the labor force that matches with a firm is determined by a function, $m_i\left(u^t_{k,i}, v^t_{k,i}\right)$, which is homogenous of degree 1, and strictly increasing and concave in each argument. Given the homogeneity assumption, we can recast the matching process in terms of labor market tightness, defined as:

$$\theta^t_{k,i} \equiv \frac{v^t_{k,i}}{u^t_{k,i}}. \tag{2}$$

We denote the probability that a firm matches with a worker as $q_i(\theta^t_{k,i}) \equiv m_i\left((\theta^t_{k,i})^{-1}, 1\right)$. Consequently, the probability that an unemployed worker matches with a firm is $\theta^t_{k,i} q_i(\theta^t_{k,i})$. After matching, firms and workers draw a match productivity, x, and firms choose in which variety j to operate. We detail the choice of j in section 2.4.1. Before doing so, we describe the household's problem and the timing of events.

2.3 Households

Countries are organized into representative families, each with a household head that chooses individual consumption, the allocation of workers across sectors, and aggregate savings to maximize aggregate utility. We first describe the utility function and budget constraint of the household head. Next, we outline the timing of events in the labor market. Finally, we obtain optimal decision rules for each household head. For ease of notation, we temporarily omit the country subscript i and let ℓ index individuals.

2.3.1 Utility and Budget Constraint

The household head aggregates individual-level utilities, $\mathcal{U}_\ell^t$, across a continuum of workers/family members of mass $\overline{L}$ and maximizes its expected net present value given by:

$$E_0 \left\{ \sum_{t=0}^{\infty} (\delta)^t \phi^t \int_0^{\overline{L}} \mathcal{U}_\ell^t d\ell \right\}, \tag{3}$$

where δ is the discount factor, which we assume to be common across countries, and ϕ^t is a country-specific inter-temporal preference shifter that the household head experiences in period t. As will become clear later, inter-temporal preference shifters will be important for matching the observed time-series behavior of final expenditures across countries.[10] Given that agents have perfect foresight with respect to all aggregate variables, E_0 denotes expectations with respect to matching probabilities, exogenous match destruction, match-specific productivity draws, and future worker-level idiosyncratic shocks. Some of these events are described below. For future reference, we implement our model at a quarterly frequency, so that each period corresponds to a quarter.

The utility for worker ℓ at time t depends on her consumption level, c_ℓ^t, employment status, $e_\ell^t \in \{0, 1\}$ (with 1 denoting employment), her current sector, k_ℓ^t (determined in period $t-1$), and her future sector of choice, k_ℓ^{t+1} (determined in period t). More specifically, the utility for worker ℓ at time t is given by:

$$\mathcal{U}_\ell^t \equiv \mathcal{U}\left(c_\ell^t, e_\ell^t, k_\ell^t, k_\ell^{t+1}, \boldsymbol{\omega}_\ell^t\right) = u\left(c_\ell^t\right) + e_\ell^t \eta_{k_\ell^t} + \left(1 - e_\ell^t\right)\left(-C_{k_\ell^t, k_\ell^{t+1}} + b_{k_\ell^{t+1}} + \omega_{k_\ell^{t+1},\ell}^t\right). \tag{4}$$

All workers enjoy utility from consumption according to the strictly increasing and concave utility function u. Employed workers ($e_\ell^t = 1$) enjoy an additional non-pecuniary sector-specific benefit, $\eta_{k_\ell^t}$.[11] Unemployed workers in sector k_ℓ^t can switch to sector k_ℓ^{t+1}, so that mobility cost $C_{k_\ell^t, k_\ell^{t+1}}$, utility of unemployment $b_{k_\ell^{t+1}}$ and idiosyncratic shock $\omega_{k_\ell^{t+1},\ell}^t$ are incurred (during period t). Mobility costs $C_{k_\ell^t, k_\ell^{t+1}}$ capture different frictions workers face to switch across sectors, and include sector-specific human capital investments, geographical mobility costs (reflecting that sectors can be concentrated in different regions), and information frictions. Importantly, these parameters

[10]The use of these shifters is common in the international macroeconomics literature (Stockman and Tesar, 1995; Bai and Ríos-Rull, 2015). The fact that these shifters lead to wedges in Euler equations implies that they can also be viewed as generated by asset markets frictions. Nevertheless, these parameters do not respond to shocks in the model.

[11]These preference terms, also known as compensating differentials, primarily serve a quantitative purpose—as shown in Artuç and McLaren (2015), matching observed wage differentials without these sector-specific non-pecuniary benefits is difficult, and can lead to implausibly large estimated mobility costs.

are critical to generating realistic inter-sectoral transition rates of employment. The utility of unemployment $b_{k_\ell^{t+1}}$ reflects a taste for leisure or distaste for the unemployment status. As we describe later, this parameter is an important driver of workers' outside option, and consequently, of workers' reservation wages. $\boldsymbol{\omega}_\ell^t = \left(\omega_{1,\ell}^t, ..., \omega_{K,\ell}^t\right)$ are idiosyncratic sector-specific preference shocks received by unemployed workers in period t. These shocks are assumed to be *iid* across individuals and over time and play two important roles. First, they generate gross aggregate flows across sectors, in excess of net flows, allowing the model to generate realistic worker transition rates. Second, they generate smooth aggregate labor supply curves across sectors.[12] One can also interpret $C_{k_\ell^t, k_\ell^{t+1}} - \omega_{k_\ell^{t+1},\ell}^t$ as an individual-level mobility cost.

In addition to consumption and employment decisions, the household head has access to international financial markets by means of buying and selling one-period riskless bonds that are available in zero net supply around the world. One can think of international bond markets in period t as spot markets in which a family buys a piece of paper with face value of B^{t+1} in exchange for a bundle of goods with the same value, and the piece of paper represents a promise to receive goods in period $t+1$ with a value equal to $R^{t+1}B^{t+1}$. International bond markets are frictionless, so that the nominal returns, R^{t+1}, are equalized across countries. The budget constraint faced by the household head is given by:

$$P^{F,t} \int_0^{\overline{L}} c_\ell^t d\ell + B^{t+1} \leq \Upsilon^t + \Pi^t + R^t B^t, \tag{5}$$

where $P^{F,t}$ denotes the price of one unit of the final good, Υ^t represents aggregate wages across all workers, and Π^t stands for aggregate profits across all firms—all measured at time t. In words, equation (5) states that the family can purchase consumption goods or bonds for next period using wage income and profits, net of interest payments (or collections) on past bonds.

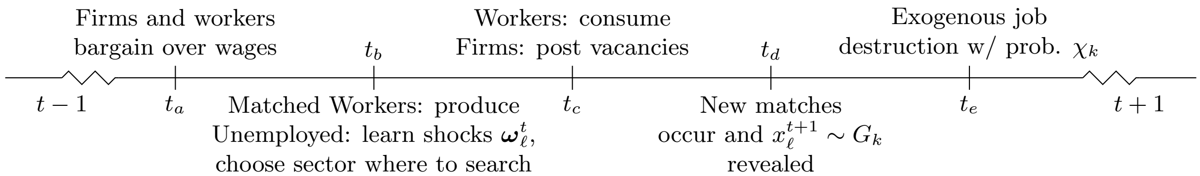
2.3.2 Timing of Events

Figure 1 details the timing of the model. If a worker ended period $t-1$ unemployed in sector k she realizes her vector of preference shocks, $\boldsymbol{\omega}_\ell^t$ (at interim period t_b). At this point, the household decides whether the worker should search in sector k at time t (at no additional cost), or incur the moving cost, $C_{kk'}$, and search in sector k'. Following Artuç et al. (2010), we assume the $\omega_{k,\ell}^t$ shocks are *iid* across individuals, sectors and time, and are distributed according to a Gumbel distribution with mean 0 and shape parameter ζ.

Workers who decide to search in sector k match with a firm with probability $\theta_k^t q(\theta_k^t)$ (at interim

[12] Given these attractive properties, these idiosyncratic preference shocks have been adopted in a variety of papers modeling the labor market response to trade shocks. For example, see Artuç et al. (2010), Dix-Carneiro (2014), Traiberman (2019), and Caliendo et al. (2019).

Figure 1: Timing of the Model



period t_d). We follow Mortensen and Pissarides (1994) and assume that once a worker and a firm match at t, a match-specific productivity for $t+1$ production, x_ℓ^{t+1}, is randomly drawn from a distribution G_k with $[0, \infty)$ support. This productivity is constant over time from then on. At this point, the household head or the firm can break a match if it is not profitable. Finally, at the end of every period (interim subperiod t_e), there is an exogenous probability χ_k that existing matches dissolve (excluding new ones). Successful matches that occur at time t only start to produce at $t+1$. Workers employed in sector k at time t are paid wages denoted by $w_k^t\left(x_\ell^t\right)$ and enjoy the non-pecuniary benefit, η_k.[13] Section 2.5 describes the wage bargaining process that occurs at t_a and section 2.4.2 describes the decision of firms to post vacancies at time t_c.

2.3.3 Household's and Workers' Decisions

The allocation of workers follows a controlled stochastic process: while the household head can choose workers' sectors given knowledge of mobility costs and idiosyncratic preference shocks, employment status is a probabilistic outcome given the matching and exogenous job destruction processes. Given an initial level of bond holdings, B_0, the head of the household chooses the path of consumption allocations, c_ℓ^t, the path of sectoral choices, k_ℓ^t, the path of job continuation decisions, and the path of bond holdings, B^{t+1}, to maximize (3) subject to the budget constraint (5) and the stochastic process governing employment status. The head of the household has perfect foresight over all aggregate variables, and takes both the path of prices and aggregate profits as given. Appendix A formalizes this problem.

To characterize the solution to this problem, let $\widetilde{\lambda}^t$ be the Lagrange multiplier on the family's budget constraint (5).[14] The optimality condition with respect to c_ℓ^t is $u'\left(c_\ell^t\right) = \widetilde{\lambda}^t P^{F,t}$, so that individual consumption is equalized across individuals within the household: $c_\ell^t = c^t \;\forall \ell$. Henceforth, we will refer to c^t as per capita consumption. Armed with this observation, we show in Appendix A that the labor supply decisions solving the household head's problem can be decentralized and written recursively for unemployed and employed workers. We now turn to this recursive formula-

[13] In an abuse of notation, we have preemptively assumed that the wage will depend on the sector and match productivity, but not the variety, j, that the firm and worker produce. Below we will verify that this will be the case.

[14] In an abuse of terminology we will continue to refer to $\widetilde{\lambda}^t$ as the Lagrange multiplier. However, the correct shadow price associated with period's t budget constraint is given by $(\delta)^t \phi^t \widetilde{\lambda}^t$.

tion.

From here on, we return to indexing countries by i. Moreover, since workers are symmetric up to x and η in each sector and country, we stop indexing individual workers. We denote by $\widetilde{U}_{k,i}^{t}(\boldsymbol{\omega}^t)$ the value of unemployment in sector k, country i at time t conditional on individual shocks $\boldsymbol{\omega}^t$, and by $W_{k,i}^{t}(x)$ the value of employment conditional on match-specific productivity x. If we define $\widehat{\phi}_i^{t+1} \equiv \frac{\phi_i^{t+1}}{\phi_i^t}$, the sector choice, $k' \equiv k^{t+1}$ solves:

$$\widetilde{U}_{k,i}^{t}(\boldsymbol{\omega}^t) = \max_{k'} \left(\begin{array}{c} -C_{kk',i} + \omega_{k'}^{t} + b_{k',i} \\ +\theta_{k',i}^{t} q\left(\theta_{k',i}^{t}\right) \delta \widehat{\phi}_i^{t+1} \int_0^\infty \max\left\{W_{k',i}^{t+1}(x), U_{k',i}^{t+1}\right\} dG_{k',i}(x) \\ +\left(1-\theta_{k',i}^{t} q\left(\theta_{k',i}^{t}\right)\right) \delta \widehat{\phi}_i^{t+1} U_{k',i}^{t+1}, \end{array} \right), \tag{6}$$

and

$$W_{k,i}^{t}(x) = \widetilde{\lambda}_i^t w_{k,i}^{t}(x) + \eta_{k,i} + \delta \widehat{\phi}_i^{t+1}(1-\chi_{k,i}) \max\left\{W_{k,i}^{t+1}(x), U_{k,i}^{t+1}\right\} + \delta \widehat{\phi}_i^{t+1} \chi_{k,i} U_{k,i}^{t+1}. \tag{7}$$

In equation (6), $U_{k,i}^{t} \equiv E_{\boldsymbol{\omega}}\left(\widetilde{U}_{k,i}^{t}(\boldsymbol{\omega}^t)\right)$ is the expected value of $\widetilde{U}_{k,i}^{t}(\boldsymbol{\omega}^t)$, integrated over $\boldsymbol{\omega}^t$. The first line in this equation corresponds to the costs of switching sectors, $-C_{kk',i} + \omega_{k'}^{t}$, and the value of being unemployed in that sector, $b_{k',i}$. The second line is the probability of finding a match $\theta_{k',i}^{t} q\left(\theta_{k',i}^{t}\right)$ multiplied by the discounted expected value of the match. Note that for low values of $W_{k',i}^{t+1}(x)$, the household head dissolves the match so that the worker obtains $U_{k',i}^{t+1}$. Finally, the third line is the discounted expected value of being unemployed next period if the worker fails to successfully match. If we integrate the left hand side of (6) with respect to $\boldsymbol{\omega}$, we obtain a Bellman equation in $U_{k,i}^{t}$.

With Gumbel distributed $\boldsymbol{\omega}$ shocks, the optimal policy for sectoral choices can be aggregated into a multinomial logit transition matrix, $s_{\ell k,i}^{t,t+1}$. This matrix measures transition rates from unemployment in sector ℓ to search in sector k between t and $t+1$. According to the timing described in Figure 1, only unemployed workers are allowed to move across sectors, implying that inter-sectoral transitions can only occur through an unemployment spell. To be able to generate realistic yearly *employment* transition rates across sectors, we implement our model at the quarterly frequency.

In equation (7), $w_{k,i}^{t}(x)$ is the wage paid by a firm with match productivity x. Note that it is multiplied by the household head's Lagrange multiplier on the budget constraint $\widetilde{\lambda}_i^t$. To understand the role of the Lagrange multiplier, note that $\widetilde{\lambda}_i^t w_{k,i}^{t}(x) = u'(c_t) \times \left(w_{k,i}^{t}(x)/P_i^{F,t}\right)$, which is the marginal utility accrued to the whole household from the additional consumption brought in by a worker employed in sector k with match productivity x. Therefore, individual workers internalize the effect of their labor supply decisions on the whole family's utility, allowing us to decentralize the problem of the household. The second term of equation (7) is the non-pecuniary benefit of working in sector k. The final two terms are continuation values: with probability $(1-\chi_{k,i})$ the match does not exogenously dissolve and the worker can choose whether to continue; with probability $\chi_{k,i}$ the match exogenously breaks and the worker exits to unemployment in sector k.

Our formulation, where household heads make consumption and aggregate savings decisions, is attractive as it leads to a tractable numerical solution of the model. If we were to model individual consumption and savings decisions, we would need to keep track of the evolution of the full distribution of savings across individuals in the economy, greatly complicating the computation of the equilibrium. The trade-off we face is that the formulation we adopt here leads to the equalization of consumption across individuals, so that we cannot study how globalization and trade imbalances impact consumption inequality.

2.4 Firms

2.4.1 Incumbents

Perfectly competitive firms produce according to (1) by combining the labor of one single worker with composite intermediate inputs purchased from all sectors. Let $p_{k,i}^{t}(j)$ denote the price paid for a unit of production of variety j in sector k and country i at time t. A firm producing variety j with match-specific productivity x obtains revenue $Y_{k,i}^{t}(j,x) \equiv p_{k,i}^{t}(j)\, y_{k,i}^{t}(j,x)$.

Firms are price takers in product and intermediate-input markets. They choose intermediates, $\left\{M_{\ell,i}^{t}\right\}_{\ell=1}^{K}$, to solve:

$$S_{k,i}^{t}(j,x) = \max_{\left\{M_{\ell,i}^{t}\right\}} p_{k,i}^{t}(j)\, z_{k,i}^{t}(j)\, x^{\gamma_{k,i}} \left(\prod_{\ell=1}^{K} \left(M_{\ell,i}^{t}\right)^{\nu_{k\ell,i}}\right)^{(1-\gamma_{k,i})} - \sum_{\ell=1}^{K} P_{\ell,i}^{I,t} M_{\ell,i}^{t}, \tag{8}$$

where $S_{k,i}^{t}(j,x)$ denotes the revenue net of intermediate input payments generated by the match between a firm and a worker with productivity x producing variety j, and $P_{\ell,i}^{I,t}$ is the price of one unit of sector ℓ's composite intermediate good. One can show that:

$$S_{k,i}^{t}(j,x) = \widetilde{w}_{k,i}^{t}(j)\, x, \tag{9}$$

where

$$\widetilde{w}_{k,i}^{t}(j) \equiv \gamma_{k,i}\left(1-\gamma_{k,i}\right)^{\frac{1-\gamma_{k,i}}{\gamma_{k,i}}} \left(P_{k,i}^{M,t}\right)^{\frac{\gamma_{k,i}-1}{\gamma_{k,i}}} \left(p_{k,i}^{t}(j)\, z_{k,i}^{t}(j)\right)^{\frac{1}{\gamma_{k,i}}}, \tag{10}$$

and $P_{k,i}^{M,t} \equiv \prod_{\ell=1}^{K} \left(\frac{P_{\ell,i}^{I,t}}{\nu_{k\ell,i}}\right)^{\nu_{k\ell,i}}$ is the price of one unit of the Cobb-Douglas bundle of intermediate goods.

We assume that in any period t, both new entrants and incumbent firms are free to costlessly choose what variety j to produce within their sector. We refer to this property as *costless variety switching*. With this assumption, no arbitrage across varieties will ensure that $\widetilde{w}_{k,i}^{t}(j) = \widetilde{w}_{k,i}^{t}(j')$ and $p_{k,i}^{t}(j)z_{k,i}^{t}(j) = p_{k,i}^{t}(j')z_{k,i}^{t}(j')$ for all pairs j, j' of varieties produced in country i. Therefore, $\widetilde{w}_{k,i}^{t}$ and $p_{k,i}^{t}z_{k,i}^{t}$ do not depend on the specific variety that is produced. This symmetry across varieties allows us to drop the index j identifying individual varieties. Given the expression in, (9), we will henceforth refer to $\widetilde{w}_{k,i}^{t}$ as *sectoral surpluses*. As will become clearer in section 2.6, these sectoral surpluses will play the same role as wages do in Caliendo and Parro (2015).

We can now write the value function for incumbent firms, $J_{k,i}^{t}(x)$, as:

$$J_{k,i}^{t}(x) = \widetilde{\lambda}_{i}^{t}\left(\widetilde{w}_{k,i}^{t}x - w_{k,i}^{t}(x)\right) + (1-\chi_{k,i})\,\delta\widehat{\phi}_{i}^{t+1}\max\left\{J_{k,i}^{t+1}(x), 0\right\}. \tag{11}$$

The first term is the firm's current profit, and the second is the firm's continuation value of the match.[15] If $J_{k,i}^{t}(x) < 0$ the firm does not produce and exits.

2.4.2 New Entrants

Potential entrants can match with a worker by posting vacancies in sector k. We assume that posting a vacancy costs $\kappa_{k,i}$ units of the final good, and so amounts to total cost $\kappa_{k,i}P_{i}^{F,t}$. Vacancies are posted at the interim period t_c as illustrated in Figure 1. If a firm successfully matches with a worker at t, production starts at $t+1$. If we denote the expected value of an open vacancy by $V_{k,i}^{t}$, then:

$$V_{k,i}^{t} = -\widetilde{\lambda}_{i}^{t}\kappa_{k,i}P_{i}^{F,t} + \delta\widehat{\phi}_{i}^{t+1}\left[\begin{array}{c} q_i\left(\theta_{k,i}^{t}\right)\int_{0}^{\infty}\max\left\{J_{k,i}^{t+1}(s), 0\right\}dG_{k,i}(s) \\ +\left(1-q_i\left(\theta_{k,i}^{t}\right)\right)\max\left\{V_{k,i}^{t+1}, 0\right\} \end{array}\right]. \tag{12}$$

The first term on the right hand side is the cost of posting vacancies scaled by the Lagrange multiplier $\widetilde{\lambda}_{i}^{t}$. The second term says that in the next period entrants find a match with probability $q_i\left(\theta_{k,i}^{t}\right)$ and obtain the expected value of $\max\left\{J_{k,i}^{t+1}, 0\right\}$ starting in the next period. If they do not find a match, they can post another vacancy. To close the model, we impose free entry so that $V_{k,i}^{t} \leq 0\ \forall k, i, t$.[16]

2.5 Wages and Labor Market Dynamics

The surplus of a match between a worker and a firm, in a given sector k, is defined as the utility generated by the match in excess of the parties' outside options. The firms' outside option is to post

[15] Firm profits are multiplied by the multiplier on the family's budget constraint in order to keep the units, utils, consistent between the firm's and worker's problem. However, if one divides $J_{k,i}^{t}(x)$ by $\widetilde{\lambda}_{i}^{t}$, then from the Euler Equation we derive below, it is clear that this formulation is equivalent to a risk neutral firm discounting profits using the nominal returns R^{t+1}.

[16] In the equilibria we consider in this paper, we verify that this condition holds with equality, both in steady state and along transition paths.

another vacancy, which is zero under free entry. The worker's is $U^t_{k,i}$, the value of search in sector k. Hence, the surplus of the match with productivity x is given by $\mathcal{S}^t_{k,i}(x) \equiv J^t_{k,i}(x) + W^t_{k,i}(x) - U^t_{k,i}$. If a match with positive surplus is formed, we assume that firms and workers engage in Nash bargaining over this surplus, with the workers' bargaining weight given by $\beta_{k,i}$. The resulting wage equation is:

$$w^t_{k,i}(x) = \beta_{k,i}\widetilde{w}^t_{k,i}x + (1-\beta_{k,i})\frac{\left(U^t_{k,i} - \delta\widehat{\phi}^{t+1}_i U^{t+1}_{k,i} - \eta_{k,i}\right)}{\widetilde{\lambda}^t_i}. \tag{13}$$

This is similar to the standard wage equation in search models: the wage is a weighted average between value added and a function of their outside option. By integrating wages across all individuals in the economy at time t, we obtain the family's total wage income Υ^t.

Equations (7) and (11) imply that the surplus function is strictly increasing in x. This observation paired with the Nash bargaining assumption, implies that matches only remain active at t if $x > \underline{x}^t_{k,i}$, where $\underline{x}^t_{k,i}$ solves:

$$\mathcal{S}^t_{k,i}\left(\underline{x}^t_{k,i}\right) = J^t_{k,i}\left(\underline{x}^t_{k,i}\right) = W^t_{k,i}\left(\underline{x}^t_{k,i}\right) - U^t_{k,i} = 0. \tag{14}$$

Note that $\underline{x}^t_{k,i}$ can respond to contemporaneous as well as future anticipated aggregate shocks, leading to endogenous job creation and destruction and dynamics in the labor market. In the remainder of this section we describe these dynamics in detail.

Since workers can switch sectors between periods t_a and t_c, the sector-specific unemployment rates differ at these two points in time within the same period. To this end, we first define the beginning of period t sector-specific unemployment rate as $\widetilde{u}^{t-1}_{k,i}$, and labor force as $L^{t-1}_{k,i}$. After workers switch sectors, (measured before matching at t_d), we define $u^t_{k,i}$ to be the share of sector-k workers searching for a job. It is given by:

$$u^t_{k,i} = \frac{\sum_{\ell=1}^{K} L^{t-1}_{\ell,i}\widetilde{u}^{t-1}_{\ell,i}s^{t,t+1}_{\ell k,i}}{L^t_{k,i}}, \tag{15}$$

where $s^{t,t+1}_{\ell k,i}$ denotes the transition rate from unemployment in sector ℓ to search in sector k between t and $t+1$—it aggregates the individual-level solutions of equation (6) across all unemployed workers at t. $L^t_{k,i}$ is the number of workers in sector k at t (more precisely at t_c) and is equal to:

$$L^t_{k,i} = L^{t-1}_{k,i} + \underbrace{\sum_{\ell\neq k} L^{t-1}_{\ell,i}\widetilde{u}^{t-1}_{\ell,i}s^{t,t+1}_{\ell k,i}}_{\text{Inflow}} - \underbrace{L^{t-1}_{k,i}\widetilde{u}^{t-1}_{k,i}\left(1 - s^{t,t+1}_{kk,i}\right)}_{\text{Outflow}}, \tag{16}$$

where the second term on the right hand side is the flow of unemployed workers into sector k, and the third term is the flow of unemployed workers out of sector k.

Only firms with $x \geq \underline{x}_{k,i}^{t+1}$ produce at $t+1$. Therefore, the *number* of jobs created in sector k is given by:

$$JC_{k,i}^{t} = L_{k,i}^{t} u_{k,i}^{t} \theta_{k,i}^{t} q_i \left(\theta_{k,i}^{t}\right) \left(1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right). \tag{17}$$

The rate at which unemployed workers find new jobs depends on two endogenous objects. First, it depends on labor market tightness, $\theta_{k,i}^{t}$, which determines the probability of a match. Second, job creation depends on $\underline{x}_{k,i}^{t+1}$, which determines the probability a match is successful.

In turn, the number of jobs destroyed is given by:

$$JD_{k,i}^{t} \equiv \left(\chi_{k,i} + (1-\chi_{k,i}) \Pr\left(\underline{x}_{k,i}^{t} \leq x < \underline{x}_{k,i}^{t+1} | \underline{x}_{k,i}^{t} \leq x\right)\right) L_{k,i}^{t-1} \left(1 - \widetilde{u}_{k,i}^{t-1}\right), \tag{18}$$

where $\Pr\left(\underline{x}_{k,i}^{t} \leq x < \underline{x}_{k,i}^{t+1} | \underline{x}_{k,i}^{t} \leq x\right)$ is the share of active firms above the productivity threshold at t but below at $t+1$. After accounting for job destruction and creation, the rate of unemployment at the end of period t, is given by:

$$\widetilde{u}_{k,i}^{t} = \frac{L_{k,i}^{t} u_{k,i}^{t} - JC_{k,i}^{t} + JD_{k,i}^{t}}{L_{k,i}^{t}}. \tag{19}$$

Equations (15)-(19) describe the evolution of labor market stocks over time. In any given period, these stocks are bound by the labor market clearing condition:

$$\sum_{k=1}^{K} L_{k,i}^{t} = \overline{L}_i. \tag{20}$$

Before discussing market clearing and equilibrium, we briefly discuss the value in our approach to labor markets. In our model, the aggregate unemployment rate equals the labor-force weighted average of sectoral unemployment rates. Consider, as an example, a shock to US manufacturing that shifts labor demand to services. What will happen to aggregate unemployment? In equilibrium, two forces act in tandem to determine the net effect. First, there will be reallocation across sectors, which can differ in their unemployment rates. Second, there will be job destruction in manufacturing, but there will also be job creation in services. The net effect of job creation and job destruction will be mediated by the size of the change in labor demand in each sector, and the ease with which workers can move across sectors. Notice that the same forces come into play if the shock had originated in US services. Succinctly, since both labor reallocation and search take time, sectoral shocks—positive or negative—can have ambiguous impacts on unemployment. In sections 4 and 5, we demonstrate the quantitative significance of this interaction between labor reallocation, job destruction, and job creation in the aggregate unemployment response to trade shocks.

2.6 International Trade

Our model of international trade closely follows Caliendo and Parro (2015). Varieties are traded across countries, and given perfect competition and iceberg trade costs, the cost of variety j from sector k produced in country o can be purchased in country i at a price $p^t_{k,o}(j)\, d^t_{k,oi}$, where the first term is the price of variety j in country o and the second term is the iceberg trade cost of shipping from country o to country i at time t. From equation (10) and costless variety switching we can write:

$$p^t_{k,i}(j) = \frac{c^t_{k,i}}{z^t_{k,i}(j)}, \tag{21}$$

for each variety j, where $c^t_{k,i} \equiv \left(\frac{\widetilde{w}^t_{k,i}}{\gamma_{k,i}}\right)^{\gamma_{k,i}} \left(\frac{P^{M,t}_{k,i}}{1-\gamma_{k,i}}\right)^{1-\gamma_{k,i}}$ acts like the unit cost in Caliendo and Parro (2015).

We assume that in any country i, sector k and period t, the productivity component $z^t_{k,i}(j)$ is independently drawn from a Frechet distribution with scale parameter $A^t_{k,i}$—which is country, sector, and time specific—and time-invariant shape parameter, λ.[17] Consumers buy the lowest cost variety across countries, treating the same variety from different origins as perfect substitutes. Define $\Phi^t_{k,i} \equiv \sum_{o=1}^N A^t_{k,o}\left(c^t_{k,o} d^t_{k,oi}\right)^{-\lambda}$. With this notation in hand, Caliendo and Parro (2015) show that under our assumptions, $P^{I,t}_{k,i} = \Gamma_{k,i}\left(\Phi^t_{k,i}\right)^{-1/\lambda}$ and $P^{F,t}_i = \Xi_{k,i}\prod_{k=1}^K \left(P^{I,t}_{k,i}\right)^{\mu_{k,i}}$, where $\Gamma_{k,i}$ and $\Xi_{k,i}$ are constants. Moreover, within-sector trade shares take the form:

$$\pi^t_{k,oi} \equiv \frac{E^t_{k,oi}}{E^t_{k,i}} = \frac{A^t_{k,o}\left(c^t_{k,o} d^t_{k,oi}\right)^{-\lambda}}{\Phi^t_{k,i}}, \tag{22}$$

where $E^t_{k,oi}$ is the total expenditure of country i on sector k varieties produced by country o and $E^t_{k,i} = \sum_{o=1}^N E^t_{k,oi}$ is the total expenditure of country i on sector k varieties. Market clearing requires that total revenue $Y^t_{k,o}$ coming from the production of varieties in sector k and country o must be equal to sales to all countries $i = 1, ..., N$, and so:

$$Y^t_{k,o} = \sum_{i=1}^N E^t_{k,oi} = \sum_{i=1}^N \pi^t_{k,oi} E^t_{k,i}. \tag{23}$$

Define $E^{C,t}_i \equiv \overline{L}_i P^{F,t}_i c^t_i$ as total expenditure on final goods, and let $E^{V,t}_{k,i}$ be the total expenditure of sector k in country i on vacancy posting costs. We can write $E^t_{k,i}$ as:

$$E^t_{k,i} = \mu_{k,i} E^{C,t}_i + \mu_{k,i}\sum_{\ell=1}^K E^{V,t}_{\ell,i} + \sum_{\ell=1}^K \left(1-\gamma_{\ell,i}\right)\nu_{\ell k,i} Y^t_{\ell,i}. \tag{24}$$

[17] The CDF for the Frechet is given by $F^t_{k,i}(z) = \exp\left(-A^t_{k,i} \times z^{-\lambda}\right)$.

The right hand side represents total expenditure on sector k goods used in final consumption, vacancy posting costs, and as intermediate inputs, respectively. In turn, let I_i^t denote total disposable income in country i, which is given by the portion of revenue that is not devoted to intermediate good payments minus vacancy posting costs, that is, $I_i^t \equiv \sum_{\ell=1}^K \left(\gamma_{\ell,i} Y_{\ell,i}^t - E_{\ell,i}^{V,t}\right)$. Net exports are then given by $NX_i^t \equiv I_i^t - E_i^{C,t}$, and we can rewrite (24) as:

$$E_{k,i}^t = \mu_{k,i} \left(\sum_{\ell=1}^K \gamma_{\ell,i} Y_{\ell,i}^t - NX_i^t \right) + \sum_{\ell=1}^K \left(1 - \gamma_{\ell,i}\right) \nu_{\ell k,i} Y_{\ell,i}^t. \tag{25}$$

Finally, labor market clearing dictates that total revenues coming from the production of varieties in sector k and country i is given by:

$$\gamma_{k,i} Y_{k,i}^t = \widetilde{w}_{k,i}^t L_{k,i}^{t-1} \left(1 - \widetilde{u}_{k,i}^{t-1}\right) \int_{\underline{x}_{k,i}^t}^{\infty} \frac{s}{1 - G_{k,i}\left(\underline{x}_{k,i}^t\right)} dG_{k,i}\left(s\right). \tag{26}$$

2.7 Trade Imbalances

Note that equations (23), (25) and (26) can be solved for any given values of $\left\{NX_i^t\right\}$, such that $\sum_{i=1}^N NX_i^t = 0$. However, these are not necessarily consistent with the household's optimal dynamic behavior. To this end, we turn to the determination of net exports $\left\{NX_i^t\right\}$ in equilibrium. The solution to the household head's problem described in section 2.3.1 must satisfy the following Euler equation:

$$\frac{u'(c_i^t)/P_i^{F,t}}{u'(c_i^{t+1})/P_i^{F,t+1}} = \delta \widehat{\phi}_i^{t+1} R^{t+1}, \tag{27}$$

and financial and goods markets in each country are linked according to:

$$NX_i^t = I_i^t - E_i^{C,t} = B_i^{t+1} - R^t B_i^t. \tag{28}$$

Finally, to close this part of the model, we impose that bonds are in zero net supply, $\sum_{i=1}^N B_i^t = 0$, and that the initial distribution of bonds is given by $\left\{B_i^0\right\}$. If the model is initially in steady state, it is easy to verify that $R^0 = \frac{1}{\delta}$.

To understand how trade imbalances arise in our model, it is helpful to impose $u(c) = \log(c)$—which we do in our quantitative analyses in subsequent sections. To simplify the exposition, assume further that there are no inter-temporal preference shocks, and so $\widehat{\phi}_i^t = 1$ for all i and t. In this case, equation (27) implies that $E_i^{C,t+1} = \delta R^{t+1} E_i^{C,t}$ for all i over the transition path. Normalizing $\sum_{i=1}^N E_i^{C,t} = 1$—so that all nominal variables are expressed as a fraction of world expenditure on final goods—we obtain that $R^t = 1/\delta$ for all t. In turn, this implies that individual countries' expenditures on final goods are constant as a share of world expenditure following a shock. Therefore, for any path of shocks, countries immediately smooth final expenditures as a share of global expenditures. To fix ideas, suppose that China realizes that it will gradually become more productive and richer. In this case, our model predicts that China will consume above production in the short run and then below in the long run, leading to short-run trade deficits and long-run trade surpluses. Nonetheless, in the data, we rarely observe this stark version of expenditure smoothing we have just discussed. The inter-temporal preference shocks $\widehat{\phi}_i^t = 1$ are wedges that reconcile our model with the observed data.

It is also important to emphasize that our model can generate persistent trade deficits and trade surpluses, even if the global economy is initialized at balanced trade across all countries. To see this, start from an initial steady state. Suppose that at time $t = 1$, the economy unexpectedly experiences a series of shocks that end in finite time. In this case, the limiting behavior of the final steady-state value of deficits is given by:

$$NX_i^{\infty} = -\frac{1-\delta}{\delta} \times \lim_{T\to\infty} \left(B_i^0 \times \prod_{\tau=1}^{T-1} R^{\tau} + \sum_{t=1}^{T-1} \left(\prod_{\tau=t+1}^{T-1} R^{\tau} \right) NX_i^t \right). \tag{29}$$

This equation shows that the behavior of long run imbalances is determined by initial wealth allocations $\left\{B_i^0\right\}$ and the short-run behavior of net exports $\left\{NX_i^t\right\}$. This second piece is key in our model: if a country runs a series of trade deficits in the short run, *even if they begin with a zero bond position*, they may run trade surpluses in perpetuity.[18] In other words, given a positive interest rate and an infinite horizon, debts that are accumulated in the short run can be rolled over in perpetuity, leading to a persistent trade surplus. Our quantitative analyses show that these persistent trade imbalances can be economically important.

2.8 Equilibrium

An equilibrium in this model is a set of initial steady-state allocations $\{L_{k,i}^0, \underline{x}_{k,i}^0, B_i^0,\}$, a final steady-state allocations $\{L_{k,i}^{\infty}, \underline{x}_{k,i}^{\infty}, B_i^{\infty},\}$ and sequences of policy functions for workers/firms $\{s_{kk',i}^t, \underline{x}_{k,i}^t, w_{k,i}^t(x)\}$, value functions for workers/firms $\{U_{k,i}^t, W_{k,i}^t, J_{k,i}^t\}$, labor market tightnesses $\{\theta_{k,i}^t\}$, bond decisions by the households $\left\{B_i^t\right\}$, bond returns $\left\{R^t\right\}$, allocations $\{L_{k,i}^t, u_{k,i}^t\}$, profits and household consumption $\{\Pi_i^t, C_i^t\}$, trade shares $\left\{\pi_{k,io}^t\right\}$, sectoral surpluses $\{\widetilde{w}_{k,i}^t\}$, and price indices $\left\{P_{k,i}^{I,t}, P_{k,i}^{F,t}\right\}$ such that: (a) Worker and firms' value functions solve (6), (7), and (11); (b) Consumption and bonds decisions solve (3) subject to (5); (c) The free entry condition holds in each country and sector: $V_{k,i}^t = 0\ \forall k, i, t$; (d) The wage equation solves the Nash bargaining problem and is given by (13). (e) Allocations and unemployment rates evolve according to (15), (16), (19); (f) Prices are set competitively and goods markets clear: (22)-(24); (g) Labor markets clear:

[18] We invoke a transversality condition that $\lim_{T\to\infty}\left[\prod_{s=1}^{T} R^s\right]^{-1} B_i^T \to 0\ \forall i$. Hence, running a surplus or deficit in perpetuity would still involve paying down interest, while rolling over (or very gradually adjusting) the principal.

$\sum_{k=1}^{K} L_{k,i}^{t} = \overline{L}_i$; (h) Bonds market clears: $\sum_{i=1}^{N} B_i^t = 0$; and (i) The initial and final steady-state equilibria satisfy equations (B.1)-(B.22) in Online Appendix B.

3 Calibration and Data

3.1 Calibration

We calibrate our model to a global economy with six sectors and six countries. We consider a world comprised of the United States, China, and four country aggregates: Europe, Asia/Oceania, the Americas, and the Rest of the World. Each country's economic activity consists of six sectors: Agriculture; Low-, Mid- and High-Tech Manufacturing; Low- and High-Tech Services. Appendix Tables C.1 and C.2 detail these divisions.

Table I summarizes the parameters we need to numerically solve the model. We split them into three categories: (i) parameters that are fixed at values previously reported in the literature, as they are difficult to identify given available data (Panel A); (ii) parameters that can be determined without having to solve the model (Panel B); and (iii) parameters that are estimated by the method of simulated moments. We calibrate our model using a variety of datasets for year 2000 or closest year available.

We start by discussing parameters fixed according to values reported in the literature, which are listed in Panel A. First, we calibrate the model at the quarterly frequency. In this case, annual steady-state international bonds' returns are given by $1/\delta^4$, so we set $\delta^4 = 0.95$ implying annual returns of 5%.[19] The estimation of the dispersion of $\boldsymbol{\omega}$ shocks typically requires panel data and instrumental variable strategies. As a result, we impose this parameter to be common across countries and set $\zeta_i = 6.52$ based on the estimate Artuç and McLaren (2015) obtained using US data.[20] Next, we parameterize the matching function according to den Haan et al. (2000): $q_i(\theta) = \left(1 + \theta^{\xi_i}\right)^{1/\xi_i}$. Flinn (2006) discusses the difficulty in identifying the parameters of matching functions without relying on data on vacancies, and the challenge in estimating the bargaining power parameters without firm-level data. To this end, we impose US estimates from den Haan et al. (2000), $\xi_i = 1.27$, for all countries. In addition, we follow a standard practice in the search literature setting $\beta_{k,i} = 0.5$ (for example, see Mortensen and Pissarides (1999)). The Frechet scale parameter $\lambda = 4$ comes from Simonovska and Waugh (2014). Finally, we assume individuals have log utility over consumption, $u(c) = \log(c)$, and that match-specific productivities x are drawn from a log-normal distribution with standard deviation $\sigma_{k,i}$. That is, $G_{k,i} \sim \log\mathcal{N}(0, \sigma_{k,i}^2)$.

[19] This choice is based on the fact that both the Federal Funds and T-Bill rates in 1999-2000 were between 5% and 6%: https://fred.stlouisfed.org/series/FEDFUNDS and https://fred.stlouisfed.org/series/DTB1YR.

[20] One note of caution is that their estimate considers an annual model. In Appendix D, we argue that their estimate must be multiplied by 4.05 to be suitable for a quarterly model. Therefore, we set $\zeta_i = 4.05 \times 1.61 = 6.52$ for all countries. This number is close to the estimate of 5.34 obtained by Caliendo et al. (2019) in a similarly quarterly model.

Table I: Summary of Parameters

Panel A. Fixed According to the Literature			
Parameter	Value	Description	Source
δ	0.9873	Discount factor	5% annual interest rate
ζ_i	6.52	Dispersion of $\boldsymbol{\omega}$ shocks	Artuç and McLaren (2015)
ξ_i	1.27	Matching Function	den Haan et al. (2000)
λ	4	Frechet Scale Parameter	Simonovska and Waugh (2014)
$\beta_{k,i}$	0.5	Worker Bargaining Power	Standard

Panel B. Estimated Outside of the Model		
Parameter	Description	Source
$\mu_{k,i}$	Final Expenditure Shares	WIOD
$\gamma_{k,i}$	Labor Expenditure Shares	WIOD
$\nu_{k\ell,i}$	Input-Output Matrix	WIOD

Panel C. Estimated by Method of Simulated Moments	
Parameter	Description
$\widetilde{\kappa}_{k,i}$	Vacancy Costs
$\chi_{k,i}$	Exogenous Job Destruction Rates
$\sigma^2_{k,i}$	$G_{k,i}$, distribution of x
$C_{kk'}$	Mobility Costs
$\eta_{k,i}$	Sector-Specific Utility
$b_{k,i}$	Value of Unemployment

Notes: Artuç and McLaren (2015) estimate $\zeta = 1.61$ for the US using an annual model. The quarterly version of their model requires a correction of $4.05 \times 1.61 = 6.52$, which is the value we use here. The matching function is parameterized as $q_i(\theta) = \left(1+\theta^{\xi_i}\right)^{1/\xi_i}$. As discussed in the text, we estimate $\widetilde{\kappa}_{k,i} \equiv \frac{\kappa_{k,i}P_i^F}{\widetilde{w}_{k,i}}$. The distribution of match-specific productivities is imposed to follow a log-normal distribution $G_{k,i} \sim \log\mathcal{N}(0, \sigma^2_{k,i})$.

Turning to Panel B, we can directly calibrate final expenditure shares $\mu_{k,i}$, labor expenditure shares $\gamma_{k,i}$, and input-output shares $\nu_{k\ell,i}$, without having to solve the model. To that aim, we employ the World Input Output Database (WIOD), which compiles data from national accounts combined with bilateral international trade data for a large collection of countries. These data cover 56 sectors and 44 countries, including a Rest of the World aggregate, between 2000 and 2014. We refer the reader to our Data Appendix for details on how these different parameters are computed.

We estimate the parameters described in Panel C using the method of simulated moments (MSM). Let $\Theta = (\Theta_1, ..., \Theta_N)$ be the vector of these country-specific parameters. Our estimation procedure assumes that the economy is in steady state in 2000 and conditions on observed trade shares $\pi_{k,oi}^{Data}$ and net exports NX_i^{Data}—so these moments are perfectly matched.

A convenient aspect of our approach is that, by conditioning on observed trade shares and trade imbalances, and normalizing total world revenues $\sum_k \sum_i Y_{k,i} = 1$, we can solve for sector-country revenues $\{Y_{k,i}\}$ independently of Θ. Specifically, equations (23), (25), and the normalization lead to a system of equations in $\{Y_{k,i}\}$, which can be solved before starting the estimation procedure. Consequently, the sector- and country-specific labor demand side of the model is fixed throughout the estimation procedure, allowing the labor supply side in each country to be solved in isolation. To see this, note that equation (26) contains revenues on the left hand side, and the right hand side *only* depends on country-specific sectoral variables and parameters. Therefore, in steady state, observed trade flows and trade imbalances are sufficient statistics for international linkages. This property allows us to estimate the model country by country, greatly simplifying the estimation procedure.[21]

Another convenient aspect of conducting the estimation conditional on the observed trade shares is that we do not have to estimate the technology parameters $A_{k,i}$ and trade costs $d_{k,oi}$. We develop algorithms to perform counterfactual responses to shocks to technology parameters and trade costs relying on the exact hat algebra approach in Dekle et al. (2007), Dekle et al. (2008) and Caliendo and Parro (2015).

However, because the estimation algorithm does not recover $A_{k,i}$ or $d_{k,oi}$, we cannot recover $\kappa_{k,i}$ directly. Instead, we only recover the initial steady state value of $\widetilde{\kappa}_{k,i} \equiv \frac{\kappa_{k,i} P_i^F}{\widetilde{w}_{k,i}}$ and use exact hat algebra to update $\widetilde{\kappa}_{k,i}$ in response to shocks. The complete definition of the steady-state equilibrium and the full estimation algorithm is described in the online appendices B and J.1.

For a given guess of Θ, we solve for the steady-state equilibrium, conditional on $\pi_{k,oi}^{Data}$ and NX_i^{Data}, to generate: (a) aggregate unemployment rates across countries; (b) the quarterly persistence rate in unemployment in the US; (c) labor market tightness across countries; (d) employment allocations and average wages across sectors and countries; (e) yearly worker transition rates between sectors across countries; and (f) cross-sectional wage dispersion across countries. We obtain data counterparts of these objects using several datasets, which we describe in the next section.

3.2 Data and Identification

We use data from the Current Population Survey (CPS) in the United States to obtain unemployment rates, quarterly persistence in unemployment, and employment allocations. Labor market

[21]The method of simulated moments objective function is highly non-linear and non-convex, so that global optimization routines, such as Simulated Annealing, must be applied. Breaking a large parameter vector into smaller subsets of parameters that can be estimated separately greatly simplifies the estimation procedure.

tightness in the United States is obtained from the Federal Reserve Economic Data (FRED).[22] For the remaining countries, we obtain unemployment rates from ILOSTAT and employment allocations from WIOD. Average wages across countries and sectors are similarly drawn from the WIOD.[23] Given the difficulty in finding data on labor market tightness for the remaining countries in the sample, we target the US's labor market tightness value for all countries. As we discuss below, targeting labor market tightness is important in identifying the various parameters of the model.

To be able to identify mobility costs, job destruction rates and the dispersion of the worker-firm matches, we make use of micro-data from several countries. Except for the US and China, all the remaining countries are country aggregates. In these cases, we select one country or set of countries as "representative" for which we measure yearly worker transition rates across sectors, and the cross-sectional coefficient of variation of wages. Panel A in Table II lists the representative countries and the datasets we have used to obtain these statistics and Panel B lists all the remaining moments used in our estimation procedure.[24]

We impose a series of parameter restrictions either because they are needed for identification given our data, or because they reduce the parameter space, simplifying the estimation procedure. The non-pecuniary benefits $\eta_{k,i}$ can only be identified relative to a sector of reference. Therefore we set $\eta_{k_0,i} = 0$ for $k_0 = Agriculture$. Next, we impose that $C_{kk,i} = 0$ for all k and i, that is, it is costless for workers to remain in their current sector. We allow $C_{kk',US}$ to be fully flexible for $k \neq k'$, but we impose that the mobility cost matrix for other countries is a re-scaled version of the US's: $C_{kk',i} = \psi_i C_{kk',US}$ for $i \neq US$. The scale factor ψ_i is estimated targeting transition rates for the various additional countries. Finally, we save on the number of parameters to be estimated by imposing the following equality across sectors within countries: $b_{k,i} = b_i$, $\widetilde{\kappa}_{k,i} = \widetilde{\kappa}_i$, and $\sigma_{k,i} = \sigma_i$ $\forall k$.

We conclude this section by discussing identification. Data on wage premia relative to Agriculture identify the non-pecuniary benefits $\eta_{k,i}$'s. Yearly sector-specific worker transition rates conditional on wage differentials and ζ_i identify mobility costs—see Artuç et al. (2010) for a precise discussion. The identification of the remaining parameters ($\chi_{k,i}$, $\widetilde{\kappa}_i$, σ_i, b_i) is discussed in detail in Appendix E. There, we show that there is a clear mapping from the data moments at the quarterly frequency to each of these parameters in a one-sector version of model.

[22] Specifically, we make use of the "Total Unfilled Job Vacancies for the United States" and "Unemployment Level" series.

[23] Given that workers are homogeneous in our model, we adjust the wage data from WIOD to control for differences in skill composition across sectors. We also adjust wages for differences in industrial composition across countries in each of our four country aggregates. Our Data Appendix provides the details behind this procedure.

[24] The Brazilian *Relação Anual de Informações Sociais* and the Turkish *Entrepreneur Information System* (EIS) are administrative datasets. See Dix-Carneiro (2014) and Demir et al. (2021) for a description of these data. We are extremely grateful to Wei Huang and Banu Demir for their very generous help with China's Urban Household Survey and with Turkey's EIS data, respectively.

Table II: Summary of Statistics Used in the MSM Procedure

Panel A: Yearly Worker Transition Rates and Coefficient of Variances of Wages		
Country Aggregate (Representative Country)	Source	Year
United States	Current Population Survey (CPS)	1999-2000
China	Urban Household Survey	2004
Europe (United Kingdom)	Labour Force Survey	1999-2001
Asia/Oceania (Korea, Australia)	Korean Labor and Income Panel Study	1999-2000
	Household, Income and Labour Dynamics in Australia	2001-2002
Americas (Brazil)	Relação Anual de Informações Sociais	1999-2000
Rest of World (Turkey)	Entrepreneur Information Survey	2014

Panel B: Remaining Statistics	
Statistic	Source
Trade shares	WIOD
Net exports	WIOD
Unemployment rates	ILOSTAT and CPS
Quarterly persistence in unemployment (US)	CPS
Labor market tightness (US)	FRED
Employment allocations	WIOD and CPS
Average wages	WIOD

Notes: For Asia/Oceania, we target the population-weighted average of transition rates and coefficient of variation of wages for South Korea and Australia. We were not able to gather information for the year 2000 for all the datasets we employ. In these cases, we selected the closest possible year for which the relevant data are available.

In the multi-sector case with yearly data, the proof is more complicated, but the essence remains. Sector-specific worker persistence rates pin down $\chi_{k,i}$. In turn, the coefficient of variation of wages, persistence rate in unemployment and labor market tightness $\theta_{k,i}$ (which we target directly) are informative about the dispersion of productivity draws σ_i and about the equilibrium cutoffs $\underline{x}_{k,i}$. To see this, note that the quarterly persistence rate in sector k-unemployment is $\theta_{k,i}q(\theta_{k,i})(1 - G_{k,i}(\underline{x}_{k,i};\sigma_i))$, which can be be inverted to obtain $\underline{x}_{k,i}$, conditional on $G_{k,i}$. Through the free entry condition (i.e., steady-state version of equation (12) with $V^t_{k,i} = 0$), $\chi_{k,i}$, $\theta_{k,i}$ and $\underline{x}_{k,i}$ pin down $\widetilde{\kappa}_i$. As for b_i, note that, in steady state, $\widetilde{\lambda}_i \widetilde{w}_{k,i}\underline{x}_{k,i} = (1-\delta)\,U_{k,i} - \eta_{k,i}$. The value of the left-hand side is restricted by the moments we target, as we discuss in in Appendix E. In turn, the value of $\eta_{k,i}$ is informed by wage differentials in the data. Finally, b_i determines $U_{k,i}$, so it must adjust to ensure that the equality holds.[25]

3.3 Estimates and Model Fit

The collection of all estimated parameters can be found in the Online Appendix F, in Tables F.1 through F.8. We first discuss the parameters that are obtained outside of the model. Table F.1 displays the final expenditure shares $\mu_{k,i}$. We can separate the countries in this table in two groups

[25]This connection is clearer in the one sector model as, in this case, $(1-\delta)U = b + \theta\widetilde{\kappa}\widetilde{\lambda}\widetilde{w}\frac{\beta}{1-\beta}$. Appendix E elaborates on this argument.

with similar expenditure shares: (1) United States, Europe, Asia/Oceania, and Americas; and (2) China and Rest of the World. The most striking difference between these two countries is that China and the Rest of the World spend a much larger share of their disposable income on agricultural goods and significantly lower share on High-Tech Services. The large Chinese share of expenditures in Agriculture will drive some of the results we report on section 4.

Table F.2 displays the share of revenues devoted to labor payments. This share varies mostly within countries and across sectors and ranges between 0.24 (in High-Tech Manufacturing in China) and 0.68 (in High-Tech Services in Rest of the World). Finally, Table F.3 displays the average across countries of the input-output matrices. As is well known, the diagonal elements tend to be larger than the off-diagonal elements.

Estimates of mobility costs in the US are displayed in Table F.4. Given that we estimate our model at the quarterly frequency, we report the values of mobility costs as a fraction of ζ, the dispersion of idiosyncratic preference shocks for sectors. This allows us to contrast our estimates with those in the literature, which were typically extracted from annual models. In addition, to make our estimates more directly comparable to those in Artuç et al. (2010) and Artuç and McLaren (2015), we express C_{US}/ζ relative to $\widetilde{\lambda}_{US} \times \overline{w}_{US}$—as these papers normalize the average wage in the US $\overline{w}_{US} = 1$ and have $\widetilde{\lambda}_{US} = 1$. Table F.4 shows that values of individual components of $C_{US}/(\widetilde{\lambda}_{US} \times \overline{w}_{US} \times \zeta)$ are uniformly below 3.5, and, for the most part, between 0 and 2. We find that it is typically costly to move into High-Tech Services and away from Agriculture. Comparing those values with estimates from Artuç and McLaren (2015), which gravitate around 4, we obtain lower mobility costs, but our numbers are in the same order of magnitude. These differences are in large part accounted for by search frictions, which are absent in their model.[26] Table F.5 compares mobility costs around the world, appropriately normalized, $C_i/(\widetilde{\lambda}_i \overline{w}_i)$, to those in the US. It shows that, with the exception of China and Asia/Oceania, (normalized) mobility costs are estimated to be similar across countries. In China, however, normalized mobility costs are estimated to be almost twice those in the US, whereas in Asia/Oceania they are half as large.

Sector-specific utilities are shown in Table F.6. The role of these parameters is to help the model fit wage differentials across sectors, and more precisely, wage premia relative to Agriculture. Given that workers choose sectors based on wages scaled by the Lagrange multiplier $\widetilde{\lambda}_i$ (see equations (6) and (7)) we compare our estimates of $\eta_{k,i}$ to the model-implied values of $\widetilde{\lambda}_i \times \overline{w}_i$ across countries, where $\overline{w}_i$ is the average wage in country i. We find that $\eta_{k,i}$ typically falls between -0.6 and 0.3 times $\widetilde{\lambda}_i \times \overline{w}_i$. The large negative values of $\eta_{k,i}$ are needed for the model to fit the often large wage premia associated with High-Tech Manufacturing that is observed in poorer countries (China, Americas and Rest of the World).

Our estimates of exogenous job destruction rates $\chi_{k,i}$ range from 0.003 to 0.08—see Table F.7.

[26] We confirmed this conclusion by re-estimating our model without search frictions—see Appendix I.

Usually, these parameters are comfortably pinned down by sector-specific employment persistence rates. However, for poor countries with a very large agricultural sector, the model struggles to fit the size of that sector given typically observed persistence rates. In poor countries such as China and the Rest of the World, the model faces a trade off between fitting sector-specific persistence rates, the size of the agricultural sector and wage differentials. By assigning low job destruction rates to Agriculture, the sector becomes relatively more attractive to workers, which partly compensates the low wages paid in the sector. This allows the model to better match the large size of these sectors in China and the Rest of the World, at the cost of a poorer fit of the persistence rate in Agriculture.

Table F.8 shows that the values of unemployment b_i are estimated to be negative across all countries. Negative values of unemployment are not uncommon in the search literature. Typically, a negative value of unemployment is necessary to generate the magnitudes of wage dispersion typically found in the data (e.g., Hornstein et al., 2011; Meghir et al., 2015).[27] In our model, negative values of unemployment are important to rationalize a series of moments in the data, including the persistence in unemployment and the unemployment rates in the data—the identification of this parameter is discussed in greater detail in Appendix E. Table F.8 also shows the estimates for the dispersion of match-specific productivities σ_i. These typically range from 0.5 to 1 and are, unsurprisingly, in the same order of magnitude as the coefficient of variation of wages across countries. Finally, the bottom row of Table F.8 display estimates of $\widetilde{\kappa}_i$, which imply that vacancy costs $\kappa_i \times P_i^F$ range between 4 and 8 times average wages across countries and sectors (with the Americas having the largest estimates).

Figure 2 shows the model fit for the various moments we target: (a) average wages relative to Agriculture across sectors and countries; (b) the cross-sectional coefficient of variation of wages across countries; (c) labor market tightness across countries; (d) yearly worker transition rates across sectors for all countries and quarterly persistence rate in unemployment; (e) employment shares across sectors and countries; and (f) unemployment rates across countries.

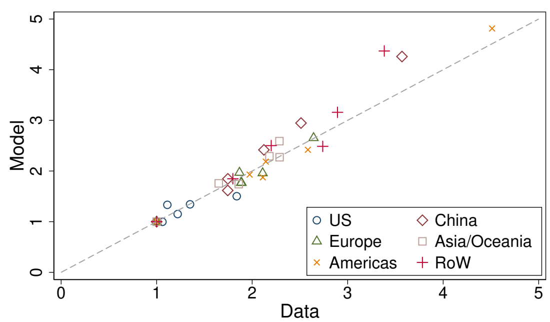
Overall, the model provides a good fit of the data. However, there is a non-trivial tension between fitting employment persistence rates across sectors, the aggregate unemployment rate and labor market tightness. To see this, note that persistence rates in a sector pin down exogenous job destruction rates χ: larger destruction rates will lead to lower employment persistence across sectors. In turn, steady-state unemployment rates directly depend on χ, and on the job finding rate $\theta q(\theta)(1 - G(\underline{x}))$ (see equation (B.7)). Conditional on χ, for the model to be able to generate relatively low unemployment rates, the job finding rate must be relatively large. The larger χ is, the larger the job finding rate must be. However, the job finding rate cannot be larger than $\theta q(\theta)$,

[27] It is hard to directly compare the magnitude of our estimates of b to those in the search literature, as the full value of being unemployed in our model also depends on switching costs, C, and cost shocks, ω.

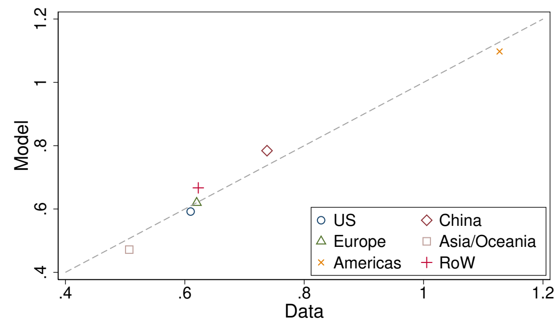
which we target in the estimation by trying to match labor market tightness. This means that in countries where persistence rates are low (large χ) and unemployment rates are also low, there will be a tradeoff between matching the unemployment rate and labor market tightness. This explains why we tend to both overestimate labor market tightness and the unemployment rate for many countries: a larger θ would produce a lower unemployment rate, but we are simultaneously trying to anchor labor market tightness to its 2000 value of 0.86.

Figure 2: Model Fit

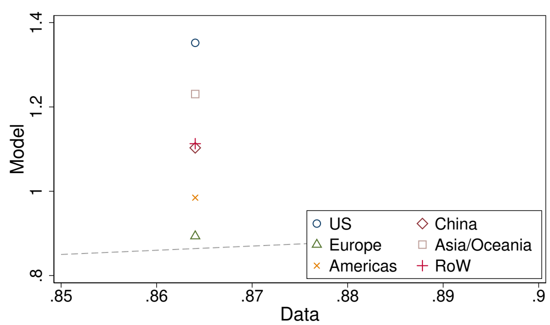
(a) Average Wages Relative to Agriculture



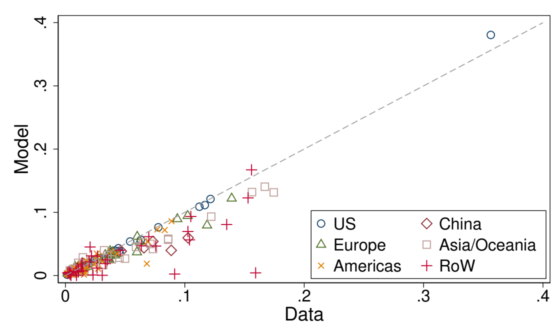
(b) Coefficient of Variation of Wages



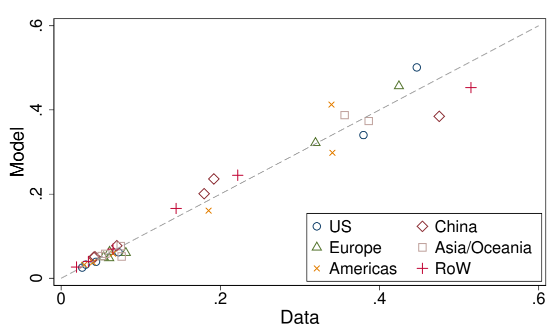
(c) Labor Market Tightness



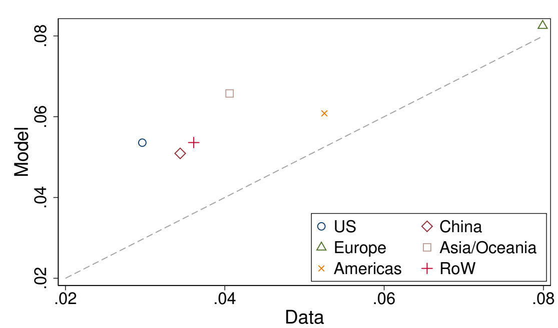
(d) Transition Rates Across Sectors



(e) Employment Shares



(f) Unemployment Rates

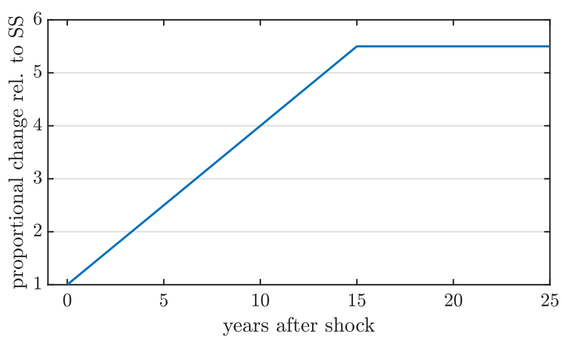


4 Mechanisms

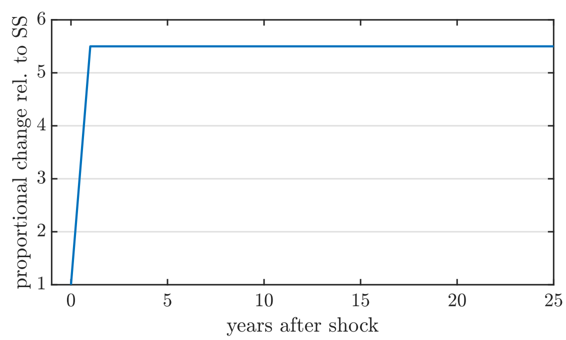
In order to understand the rich mechanisms at play in our model, we study its behavior in response to two types of shocks. First, we simulate a slow linear increase in Chinese productivity $A_{k,China}$, uniform across sectors, reaching a plateau of a 5.5 times increase after 15 years. The magnitude of this shock is in line with the size of actual changes in Chinese productivity that we recover in section 5.1. Next, to illustrate that the exact path of shocks fed to the model is consequential not only for short-run responses, but also for long-run outcomes, we feed the model with a 5.5 times once-and-for-all increase in $A_{k,China}$ at $t = 1$. These shocks are illustrated in Figure 3.[28] In both cases, the global economy is initially in steady state. The shocks are unanticipated at $t = 0$, but their paths are revealed at $t = 1$ and, from then on, fully anticipated. To highlight the quantitative and qualitative importance of modeling trade imbalances, we study the behavior of the complete model with international bonds as well as the behavior of the same model without these bonds, where trade is balanced in every period—that is $NX_i^t = 0\ \forall i, t$. All remaining parameters are fixed at calibrated values and $\widehat{\phi}_i^t = \widehat{d}_{k,oi}^t = 1$ for all k, i and t. From now on we focus on patterns in the US and China to streamline the exposition. Appendix G contains figures for all countries.

Figure 3: Shocks to Chinese Productivity $\widehat{A}_{k,China}^t \equiv \frac{A_{k,i}^t}{A_{k,i}^0}$ – Uniform Across Sectors

(a) Slow Growth



(b) Once-And-For-All

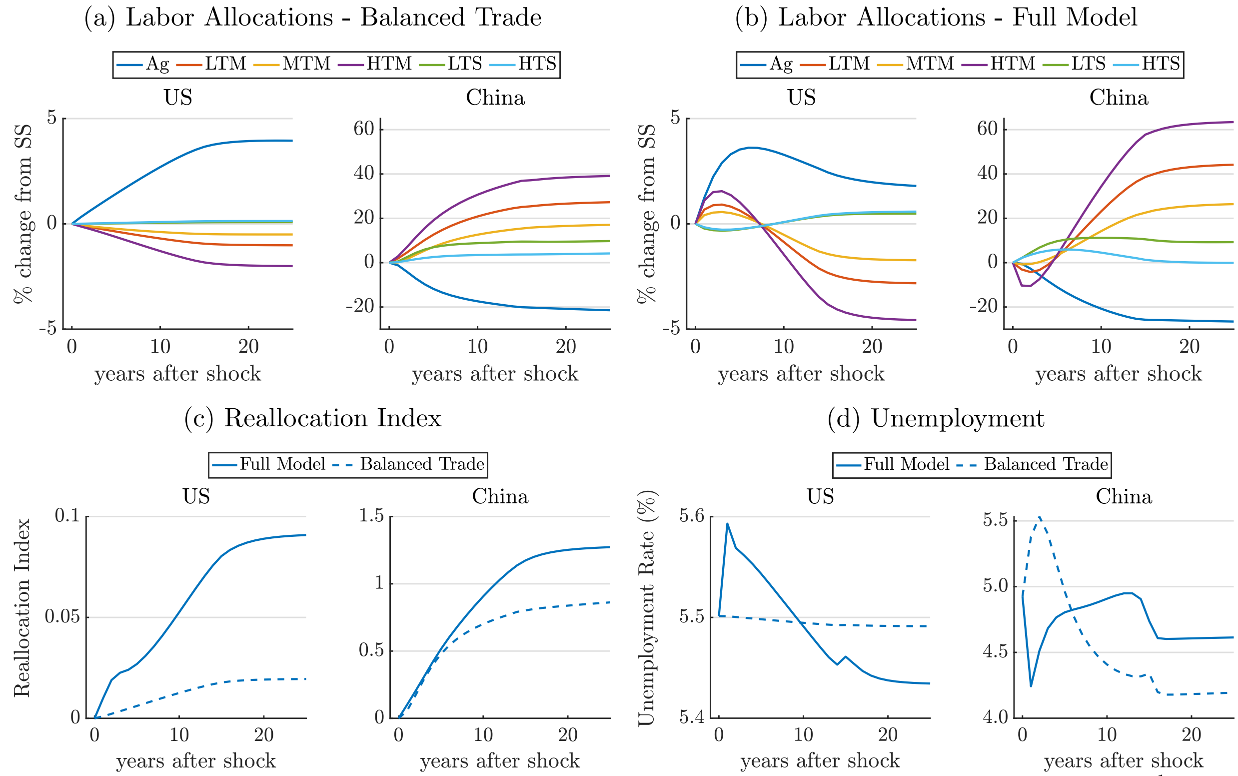


We start with the slow moving shock depicted in Figure 3a. Before we study the behavior of the full model with trade imbalances, it is instructive to start with an economy where trade is balanced every period, that is, with no access to an international bond market. Figure 4a shows a smooth and monotonic reallocation process towards a new long-run steady state, which is shaped by static trade forces including differences in technology and preferences. There are two salient features of this new steady state: a strong expansion of manufacturing in China and a similarly strong contraction of Agriculture. Patterns in the remaining countries mirror these: they increase specialization in Agriculture and downsize their manufacturing sectors. As is well known, these forces interact in complex and often nuanced ways in multi-sector Ricardian models of trade.[29] However, we highlight two features that can help us understand this pattern of specialization across countries in response to the shock. First, China becomes richer and that tilts world production towards its consumption

[28]We assume that the shock is unveiled at $t = 0$, but between t_c and t_d—see Figure 1. That is, the shock occurs **after** production, after unemployed workers' decisions of where to search and after firms post vacancies at $t = 0$.

[29]See, for example, Costinot and Rodríguez-Clare (2014) and Caliendo and Parro (2015).

Figure 4: Labor Market Dynamics in Response to Slow Productivity Growth in China (Figure 3a)

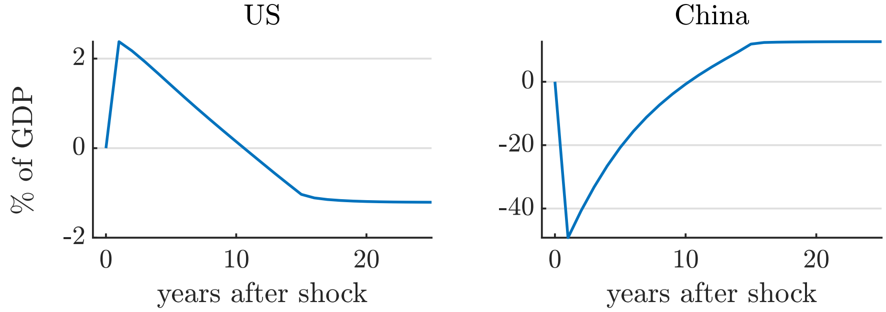


Notes: We summarize the extent of reallocation with the following index: $\mathit{Reallocation}_i^t = \frac{1}{2}\sum_{s=1}^{t}\sum_{k=1}^{J}\left|\frac{L_{i,k}^{s}}{\bar{L}_i} - \frac{L_{i,k}^{s-1}}{\bar{L}_i}\right|$, which accumulates yearly changes in sectoral employment shares over time. Ag: Agriculture; LTM = Low-Tech Manufacturing; MTM: Mid-Tech Manufacturing; HTM: High-Tech Manufacturing; LTS: Low-Tech Services; HTS: High-Tech Services.

basket, which is heavily skewed towards Agriculture (see Table F.1). Second, China has initially low revealed comparative advantage (Balassa, 1965) in Agriculture, which becomes even lower after the shock. Put together, world production of Agriculture must increase to satisfy Chinese demand, but China is relatively better in other activities and specializes accordingly.

With the above discussion as our comparison point, we turn to our full model with imbalances. First, we consider the behavior of net exports, which are illustrated for China and the US in Figure 5. Given perfect foresight, the growth path of productivity is fully anticipated by the Chinese households, who internalize that their long-run income will greatly exceed their short-run income. They respond by smoothing consumption, substituting future expenditures (when they are relatively rich) towards increased expenditures in the short run (when they are relatively poor). In doing so, they sustain trade deficits in the short run by borrowing from the rest of the world—selling bonds. In the long run, China runs a permanent trade surplus as they must pay interest on their accumulated debt—see the discussion following equation (29). Meanwhile, all other countries' trade imbalances mirror China's: they finance the Chinese short-run consumption boom by running trade surpluses (purchasing bonds from China). This leads them to sustain permanent trade deficits in the long run as they enjoy returns on their bond holdings.

Figure 5: Net Exports Over GDP in Response to Slow Productivity Growth in China (Figure 3a)



These movements in trade imbalances lead to substantially different reallocation patterns compared to the model with balanced trade, as can be seen by comparing Figures 4a and 4b. Most striking are the non-monotonic patterns of reallocation that arise in the full model with imbalances. To understand these patterns, note that consumption smoothing in China implies an immediate increase in its expenditure above current production. Because preferences are homothetic, Chinese expenditures expand proportionally in *all* sectors. Since trade in services typically faces larger costs, Chinese households respond by quickly reallocating labor towards these sectors. This expansion in services is amplified relative to the case without deficits, and must be accompanied by a contraction in employment in physical goods sectors—which are easier to import. Consequently, there is a short-run expansion in services above the final long-run level, and an initial decline in all of the remaining sectors.

In the long run, China must repay its debt. To do so, China expands production (and exports) in easy-to-trade goods, such as manufacturing, which occurs through the contraction of the previously expanded services sectors. The need to pay its debt, alongside the aforementioned forces that guide the balanced-trade long-run steady state, shape China's final patterns of production. Thus, manufacturing sectors expand while Agriculture contracts.

The behavior of reallocation in the remaining countries is symmetric. In the short run, other countries lend to China by increasing their shipments of relatively tradable goods, causing reallocation towards those sectors. In the long run, as China repays its debt, the other countries contract their manufacturing sectors, consuming over production. This leads to an expansion of services, as expenditures increase proportionally in *all* sectors, and services are most cheaply provided by local labor.

The behavior of trade imbalances have important implications for the extent of reallocation in the economy—as Figure 4c shows. First, it leads to non-monotonic patterns of adjustment, so that short run reallocation is undone in the long run. Second, there are permanent shifts in consumption driven by long-run imbalances, which amplify the magnitude of reallocation in the long run relative to a world without imbalances. For example, US employment in High-Tech Manufacturing contracts by 5% in the long run in the model with imbalances but only by 2% in model with balanced trade.

In China, High-Tech Manufacturing expands by 61% compared to 40% when balanced trade is imposed. With these short and long run differences in mind, we now turn to the implications for aggregate unemployment.

Figure 4d shows rich dynamic responses that are quite different between the full and the balanced-trade models. Importantly, it shows that Chinese unemployment spikes up in the short run if balanced trade is imposed, but declines in the full model with trade imbalances. To better understand these differences, it is useful to introduce the following decomposition in changes in aggregate unemployment:

$$\Delta u_i^t = \underbrace{\sum_k u_{k,i}^0 \frac{\Delta L_{k,i}^t}{\bar{L}_i}}_{\text{Reallocation}} + \underbrace{\sum_k \frac{L_{k,i}^0}{\bar{L}_i} \Delta u_{k,i}^t}_{\text{Job Creation/Destruction}} + \underbrace{\sum_k \frac{\Delta L_{k,i}^t}{\bar{L}_i} \Delta u_{k,i}^t}_{\text{Covariance}}, \tag{30}$$

where Δ refers to changes between time t and initial steady-state values (indexed by time 0), and u_i^t is the aggregate unemployment rate in country i at time t. Aggregate unemployment responds to shocks because labor is reallocated across sectors with different initial levels of unemployment $u_{k,i}^0$ (Reallocation Channel), because sector-specific unemployment rates respond due to within-sector job creation or destruction (Job Creation/Destruction Channel), or because of a residual term that interacts changes in sector-specific unemployment with changes in employment shares.

Figure 6 plots the decomposition in equation (30) for China. To understand the Reallocation Channel, it is important to highlight that, in our model, sector-specific unemployment rates tend to be larger in manufacturing sectors than in service sectors. This difference is partly driven by relatively lower wages and exogenous separation rates in services.[30] Note that for both the full and balanced-trade models, the Reallocation Channel tends to increase unemployment as labor is reallocated to high-unemployment manufacturing sectors.

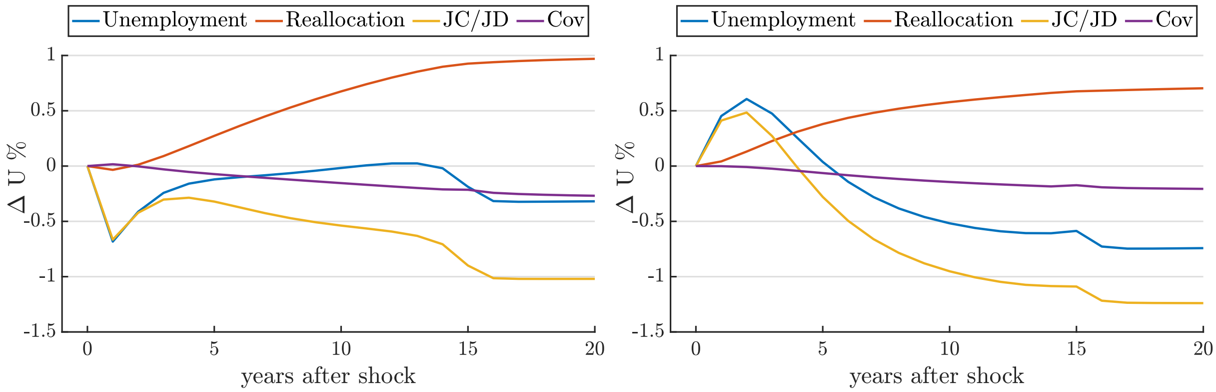
On the other hand, the contribution of the Job Creation/Destruction Channel differs markedly across the two models, especially in the short run. To understand the Job Creation/Destruction Channel, it helps to consider two opposing forces that come into play after a shock. First, shocks triggering reallocation across sectors tend to contribute to short-run increases in unemployment as jobs are destroyed and workers must spend time searching for new opportunities. Second, positive demand shocks tend to lead to a surge in vacancy posting, tightening labor markets and contributing to a decline in unemployment.[31]

[30] Intuitively, lower wages make sectors less attractive to workers, which tends to reduce the number of searchers; and lower separation rates lead to lower flows to unemployment. Both forces tend to lead to lower unemployment. High-Tech Services pay a large wage premium in the US, yet its unemployment rate is relatively low. This is explained by high mobility costs into that sector, which tends to increase labor market tightness and reduce the unemployment rate in that sector.

[31] In addition to our decomposition to better understand the forces driving unemployment dynamics, we have re-estimated our model (a) removing mobility costs; and (b) removing search frictions. We find that removing mobility

Figure 6: Unemployment Decompositions for China

(a) Full Model (b) Trade Balance



Turning to the shock under consideration, in both models, there is substantial reallocation across sectors, and this tends to increase unemployment in the short run. However, in the full model with trade imbalances, the second force highlighted in the previous paragraph dominates the first. In response to the shock, expenditures in China immediately jump up, leading to a very rapid expansion of vacancies (especially in services), and to a *reduction* in unemployment in the short run. In contrast, in the balanced-trade model, consumption in China responds more gradually over time as there is no consumption smoothing mechanism. In turn, vacancies also respond gradually, and do not offset the short-run increase in unemployment driven by reallocation. In the long run, both models have similar predictions for unemployment, albeit the magnitude is slightly different (with a difference of 0.5%). China is under a strong growth path, which tends to reduce the productivity threshold for production, contributing to lower unemployment.

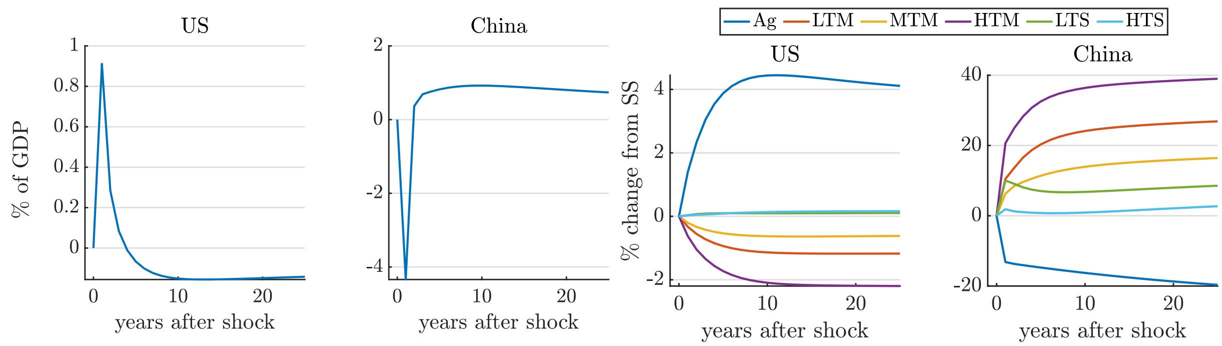
Having described how the global economy adjusts to slow productivity growth in China, we turn to its behavior in response to a sudden productivity boost of 5.5 times at once at $t = 1$. These two shocks have the same long-run values of productivity, yet they have different implications for how the global economy responds both in the short and long runs. In the wake of a sudden permanent shock, Chinese households are immediately and perennially richer and so want to instantly increase consumption of all sectors. Absent reallocation frictions, output would immediately jump to its new steady state and households would have no incentives to trade bonds. However, labor market frictions lead to a slow convergence to the new optimal level of output. To smooth consumption, Chinese households borrow from other countries—especially those with lower labor market frictions such as the US. As Figure 7a shows, the response of trade imbalances is much more modest than in the case of slow productivity growth. However, these responses are not negligible: China experiences a trade deficit of over 4% of GDP in the short run and sustains a trade surplus of 0.7% after 25

costs leads to amplified unemployment responses, while removing search frictions leads to a dampened response that also tends to go in the opposite direction of our findings in this section. More details and explanations can be found in Appendix I.

Figure 7: Outcomes Following Once-and-for-all Shock in Figure 3b:
Full Model with Trade Imbalances

(a) Evolution of Trade Imbalances

(b) Labor Allocations



years. This exercise shows that the exact path of shocks influences the equilibrium path of bonds, long-run trade imbalances, and consequently, long-run outcomes.

Turning to a comparison of reallocation patterns, Figure 7b shows that the effects of the once-and-for-all shock do not feature the non-monotonic patterns documented in Figure 4b. In addition, these shocks also have distinct implications for the final allocation of labor across sectors in the long run. High-Tech Manufacturing in China expands by 40% (61%) in the long run following the once-and-for-all (slow growth) shock. In the US, High-Tech Manufacturing contracts by 2% (5%) in the long run following the once-and-for-all (slow growth) shock. According to the same reallocation index as in Figure 4c, there is 3 times more cumulative reallocation in the US and 20% more in China in response to the slow productivity growth shock relative to the once-and-for-all shock.

To sum up, the main takeaways from this section are as follows. First, the exact path of globalization shocks is key for the behavior of trade imbalances and long-run outcomes. Second, the behavior of trade imbalances closely dictates patterns of sectoral reallocation, and can significantly amplify the amount of reallocation in the economy. Finally, unemployment responses are rich and nuanced. Importantly, trade surpluses (deficits) do not necessarily lead to lower (higher) unemployment. This point is more comprehensively illustrated in Figures G.2 and G.1 in the Appendix. Specifically, all countries experience lower long-run unemployment rates, irrespective of the sign of their net exports. In addition, all countries sustaining trade surpluses in the short run go through temporary increases in unemployment.[32]

[32]We have also simulated the impact of temporary productivity growth in China, which reverts to its initial level. Importantly, we find that even temporary shocks can have permanent effects on labor allocations. This is because temporary shocks can lead to permanent trade imbalances and, consequently, to permanent changes in labor allocation.

5 Counterfactuals

Section 4 showed that the exact path of shocks shape the magnitude and evolution of trade imbalances over time, directly influencing long-run outcomes through changes in the long-run global distribution of bond holdings. For this reason, we conduct an empirical exercise in which we extract the various shocks the global economy experienced between 2000 and 2014. Given the interest on the impacts of the "China shock" on the US's trade deficit and labor market, we use our extracted shocks to study this event through the lens of our model. We also use these shocks to compare the consumption gains in response to changes in trade costs in our model to those obtained in standard models of trade, as summarized by the sufficient statistic approach developed by Arkolakis et al. (2012). Finally, we revisit the slow productivity growth shock in Figure 3a to compare predictions of our model relative to another popular approach in the International Trade literature to modeling trade imbalances.

5.1 Extracting Shocks from the Data

Relying on the model's structure and data from the WIOD, we extract three main sets of shocks affecting the global economy between 2000 and 2014: changes in trade costs $\widehat{d}^t_{k,oi}$, productivity shocks $\widehat{A}^t_{k,i}$, and inter-temporal preference shocks $\widehat{\phi}^t_i$. We measure changes in trade costs and productivity relative to 2000 (which we label $t = 0$): $\widehat{d}^t_{k,oi} = \frac{d^t_{k,oi}}{d^0_{k,oi}}$, $\widehat{A}^t_{k,i} = \frac{A^t_{k,i}}{A^0_{k,i}}$. On the other hand, shocks to inter-temporal preferences are relative to the previous period: $\widehat{\phi}^{t+1}_i \equiv \frac{\phi^{t+1}_i}{\phi^t_i}$. In addition to these shocks, we consider changes over time in parameters driving preferences ($\mu^t_{k,i}$) and technology ($\gamma^t_{k,i}$ and $\nu^t_{k\ell,i}$).

In essence, we make use of the gravity structure of the model to obtain shocks to productivity and trade costs—the procedure we employ is similar to Head and Ries (2001) and Eaton et al. (2016).[33] For inter-temporal preference shocks, we follow Reyes-Heroles (2016) and back out $\widehat{\phi}^t_i$ using the Euler equation and time-series data on aggregate expenditures. We leave the details of the implementation to Appendices H and J.6.

The rest of this section summarizes the main time-series patterns in these shocks. First, Figure H.4a shows increases in productivity all over the world. In particular, China has experienced strong growth in productivity across all sectors, but especially in manufacturing sectors.[34] Other emerging economies—which comprise the bulk of the Americas and the Rest of the World aggregate—also

[33] We impose $\widehat{A}^t_{k,i} = \widehat{A}^{T_{Data}}_{k,i}$ and $\widehat{d}^t_{k,oi} = \widehat{d}^{T_{Data}}_{k,oi}$ for all $t > T_{Data}$, where T_{Data} is the last period for which we have data ($T_{Data} = 4 \times 14$ quarters and refers to December 2014).

[34] While we plot changes in the productivity location parameters $\widehat{A}^t_{k,i}$, this is not directly comparable to productivity in the classic sense of a Solow Residual. In order to make sense of the magnitudes, note that TFP growth, defined as $\widehat{c}^t_{k,i}/\widehat{P}^{I,t}_{k,i}$, can be expressed as $(\widehat{A}^t_{k,i}/\widehat{\pi}^t_{k,ii})^{1/\lambda}$. Therefore, using our recovered values for $\widehat{A}^t_{k,i}$, data on changes in trade shares, and imposing $\lambda = 4$, the magnitude for actual annualized TFP growth in China ranges from 2.0 to 3.4% per year, depending on the sector—which is in line with growth accounting estimates discussed in Zhu (2012).

experienced impressive productivity growth, while growth was more muted for advanced economies.

Turning to trade costs, Figure H.4b shows that import trade costs decline over our sample period for the United States and Asia, and are approximately flat in Europe (with some heterogeneity across sectors). Perhaps surprisingly, starting after the 2007-2008 financial crisis, initially falling trade costs begin to flatten out or revert in most countries. This more recent behavior of trade costs likely reflect the slowdown in global trade that occurred following the financial crisis (Bems et al., 2013). The sources for these increasing frictions are myriad, and include policy changes in countries like China, as well as changes in supply chain management, and other reasons.

Finally, we turn to our measure of shocks to inter-temporal preferences, which are presented in Figure H.5. The most striking patterns are found in China, the Americas, and the aggregated remaining countries (Rest of the World), which exhibit persistent shocks to their inter-temporal preferences. These persistent deviations are often referred to as the "global savings glut" (Bernanke, 2005). It is important to recognize that there are rich dynamics to consumption in the real world, reflecting preferences, frictions, and other factors. We are agnostic on the exact theory, instead summarizing the effect of these channels with the $\widehat{\phi}_i^t$ shocks.

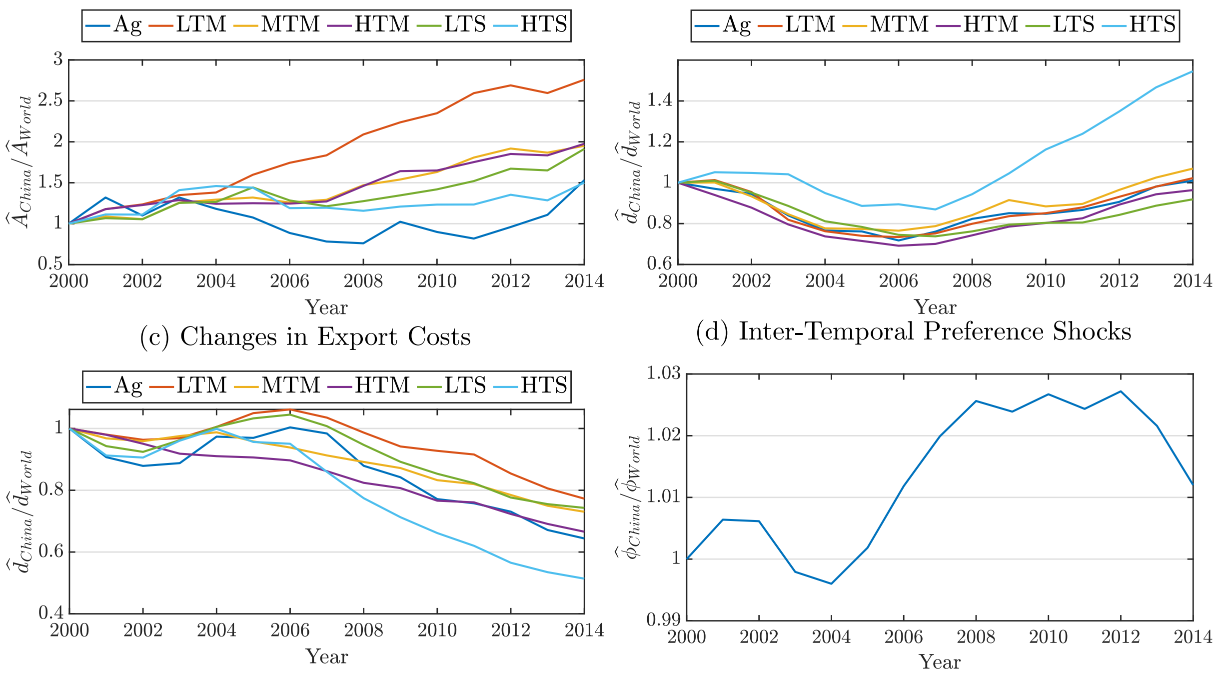
5.2 The China Shock

The impact of China's emergence as a key international trade player on the US economy has attracted much academic interest since the work of Autor et al. (2013) and Pierce and Schott (2016). Armed with the various shocks that the global economy experienced between 2000 and 2014, we investigate the role of the "China Shock" on the adjustment of the American labor market through the lens of our model. However, before proceeding, we need to take a stand on how to measure the "China Shock."

The constellation of shocks extracted in section 5.1 characterizes the world "With the China Shock." As for the counterfactual world "Without the China Shock," one possibility is to neutralize all Chinese shocks to productivity, trade costs and inter-temporal preferences and set $\widehat{A}_{k,China}^t = \widehat{d}_{China,d,k}^t = \widehat{d}_{o,China,k}^t = \widehat{\phi}_{China}^t = 1$ for all sectors and periods. However, this counterfactual would be too extreme because all countries in the world experience strong productivity growth in almost all sectors over that period, as we show in Figure H.4a. It is therefore unreasonable to pursue a counterfactual world where China experienced no changes in fundamentals and at the same time keep strong growth in productivity in the remaining countries. This would mean that China would be becoming significantly poorer than the rest of the world over the period we consider. Consequently, we define our counterfactual "Without the China Shock" as the constellation of *all* of the globalization shocks we recovered in section 5.1, with the exception of China's. For China, we set productivity ($\widehat{A}_{k,China}^t$), trade cost ($\widehat{d}_{k,i,China}^t$ and $\widehat{d}_{k,China,i}^t$) and inter-temporal preference shocks ($\widehat{\phi}^t_{China}$) to be equal to the average of shocks experienced by the remaining countries.[35] Therefore, this section quantifies the impact of the shocks accrued to China over this period above those accrued to the "average country"—excluding China—over the 2000-2014 period. We refer to the consequences of these excess shocks as impacts of the "China Shock."

Figure 8: China Shocks Relative to World Average Shocks

(a) Productivity Growth (b) Changes in Import Costs



(c) Changes in Export Costs (d) Inter-Temporal Preference Shocks

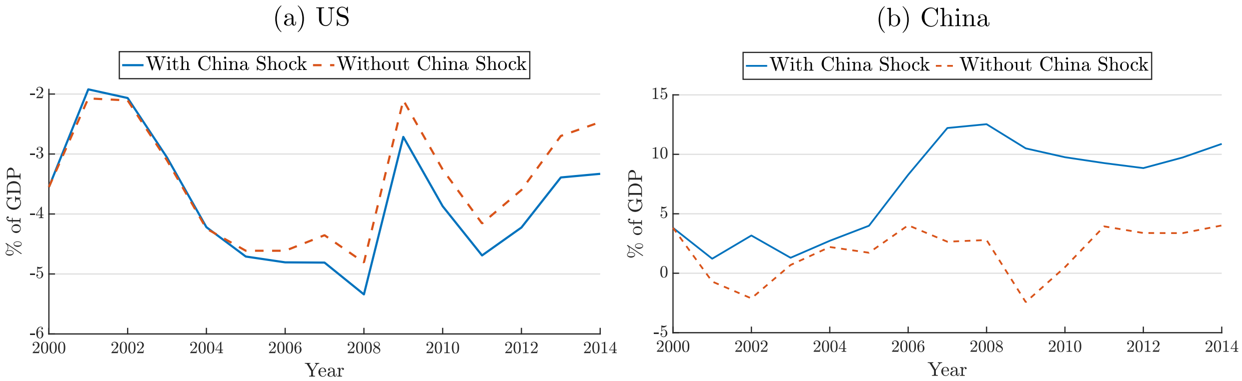
Figure 8 shows realized shocks to China relative to the rest of the world's average. Chinese productivity growth exceeds that of the average country in all sectors, but this pattern stands out for manufacturing sectors, and most strongly in Low-Tech Manufacturing. Relative import costs are relatively flat during the period we consider, although they first decline before recovering. In contrast, export costs strongly decline over that period, highlighting an asymmetrical behavior of trade costs. Finally, China experiences large inter-temporal preference shocks relative to the rest of the world, reflecting the salient savings glut we discussed in the previous section.[36]

We start by investigating the effect of the China Shock on trade imbalances. Figure 9a shows that the observed evolution of Chinese fundamentals (productivity, trade costs and inter-temporal preferences) contributed significantly to the deterioration of the US trade deficit over the 2000-2014 period. If Chinese fundamentals had followed the average path of the rest of the world, the US

[35]Technology and preference parameters $\mu^t_{k,i}$, $\gamma^t_{k,i}$, and $\nu^t_{k\ell,i}$ vary over time but are imposed to be the same across the two simulations and equal to the values obtained in section 5.1. All the remaining parameters are fixed at calibrated values.

[36]The large trade surplus that China has been running since the early 2000s is a puzzle for models in which the main driving forces are productivity shocks. For instance, as argued by Song et al. (2011), financial frictions within China are key drivers of the Chinese savings glut. Our inter-temporal preference shocks constitute a reduced-form way to allow the model to match the time series behavior of Chinese aggregate expenditures and the rest of the world.

Figure 9: The China Shock: Net Exports



Notes: The solid blue line ("With China Shock") depicts the evolution of Net Exports once we feed the model with all recovered shocks from section 5.1. The dashed red line ("Without China Shock") depicts the evolution of Net Exports if we feed the model with all recovered shocks but the productivity ($\widehat{A}_{k,i}^{t}$), trade cost ($\widehat{d}_{k,oi}^{t}$) and inter-temporal preference shocks (ϕ_i^t) to China are imposed to be equal to the average of the shocks received by all other countries. The evolution of preference ($\mu_{k,i}^t$) and technology parameters ($\gamma_{k,i}^t$ and $\nu_{k\ell,i}^t$) is imposed to be the same across the two counterfactuals.

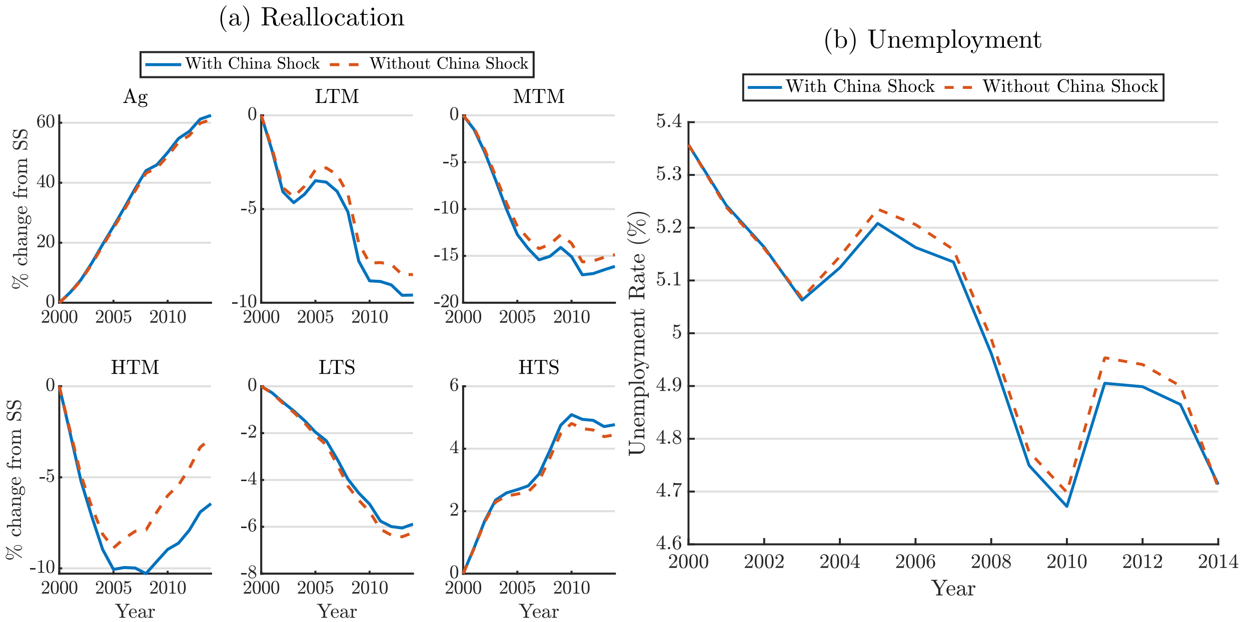
trade deficit would have been of 2.5% of GDP in 2014 (red dashed line) as opposed to 3.4% (blue solid line). This implies that the China Shock, as we define it, led to a deterioration of 36% of the US trade deficit between 2000 and 2014. In parallel, China's surplus would similarly be much more modest by the end of 2014 (4% against 11% of GDP).

Autor et al. (2016) hypothesize that the behavior of trade imbalances could have significantly influenced the American labor market response to changes in Chinese fundamentals. Specifically, in a balanced-trade environment, a surge in imports must be synchronized with an offsetting expansion of exports, leading to significant reallocation within tradable sectors. On the other hand, if the import surge is concomitant with a deterioration of the trade deficit, there are no equilibrium forces propelling export-oriented industries. Instead, labor displaced from import-competing industries are reallocated to non-tradable sectors or remain idle in unemployment—at least in the short run. We use our model to rigorously examine these hypotheses.

Figure 10a investigates the impact of the China Shock on the American labor market and on the decline of manufacturing. We observe a reduction in all manufacturing sectors—the solid blue line is consistently below the red dashed line across all these sectors. To quantify the effect of the China Shock on the decline of manufacturing, we first estimate that the global shocks (including the China Shock) led to a total of 1,917k manufacturing jobs lost over this period. Next, the first row of Table III computes the decline in manufacturing "With the China Shock" minus the decline in manufacturing "Without the China Shock" and shows that the China Shock accounted for 451k/1,917k=23% of the manufacturing decline over that period. However, this decline in manufacturing was mirrored by an offsetting expansion in services, as we show in Figure 10a and Table III.[37] More precisely, Low-Tech Services expand by 184k jobs and High-Tech Services expand

[37] Bloom et al. (2019) find similar reallocation patterns towards services, induced by the China shock, using US firm-level data.

Figure 10: The China Shock: Labor Allocations and Unemployment in the US



Notes: The solid blue line ("With China Shock") depicts the evolution of allocations / unemployment once we feed the model with all recovered shocks from section 5.1. The dashed red line ("Without China Shock") depicts the evolution of labor allocations / unemployment if we feed the model with all recovered shocks but the productivity ($\widehat{A}_{k,i}^{t}$), trade cost ($\widehat{d}_{k,oi}^{t}$) and inter-temporal preference shocks (ϕ_i^t) to China are imposed to be equal to the average of the shocks received by all other countries. The evolution of preference ($\mu_{k,i}^t$) and technology parameters ($\gamma_{k,i}^t$ and $\nu_{k\ell,i}^t$) is imposed to be the same across the two counterfactuals. Ag: Agriculture; LTM: Low-Tech Manufacturing; MTM: Mid-Tech Manufacturing; HTM: High-Tech Manufacturing; LTS: Low-Tech Services; HTS: Hgh-Tech Services.

by 240k jobs. This implies that 424k/451k=94% of the destroyed manufacturing jobs were re-created in services, and the China shock had a virtually zero impact on unemployment, as we illustrate in Figure 10b.[38]

To gauge the importance of trade imbalances for the estimates above, we reconduct the exercise imposing balanced trade in our model. The second row of Table III shows that such a model predicts 264k jobs lost in manufacturing caused by the China Shock over the period we consider, and therefore the China Shock would explain only 13% of the decline in manufacturing.[39] Similarly to what we reported in the model with trade imbalances, we find that the China shock also had a negligible impact on unemployment. However, the pattern of reallocation is broadly in line with the predictions of Autor et al. (2016): 172k/264k=65% of the jobs destroyed in manufacturing are created in services (mostly in Low-Tech), and the rest of the jobs are reallocated to other tradable sectors—Agriculture.

The lessons we draw thus far from this exercise are fourfold: (a) Shocks to Chinese fundamentals led to a quantitatively important deterioration of the US trade deficit between 2000 and 2014; (b)

[38]In the long run, though, the size of services does not respond to the China shock. The bulk of reallocation is from manufacturing to agriculture, a pattern that is also consistent with the hypothesis put forward by Autor et al. (2016).

[39]The model with balanced trade predicts a decline of manufacturing of 1,991k jobs in response to all global shocks—a similar decline we obtain with the model with trade imbalances.

China accounted for a quarter of the decline in American manufacturing over that period; (c) this estimate is halved in a balanced-trade world, which underestimates reallocation to services; (d) unemployment did not respond to the China shock.

Table III: Effect of the China Shock on Employment in the US (2000-2014)

Employment Change in '000s	Agric.	Manuf.	Services
Full Model	51.9	-450.6	423.5
Balanced Trade	106.1	-264.1	171.5

Notes: Effects of the China Shock computed between 2000 and 2014 as the change in employment "With China Shock" (all extracted shocks) minus the change in employment "Without China Shock" (all extracted shocks but China receives average world shocks). The total 2000-2014 predicted change in manufacturing employment is -1,917.4k jobs in our full model with trade imbalances and -1,991.6k jobs in our model with balanced trade.

The "China Shock" we have studied in the previous paragraphs reflects changes in productivity, trade costs and inter-temporal preferences. We now use our model to evaluate the relative contribution of these shocks for the decline in US's manufacturing sector. However, it is important to keep in mind that the impacts of these shocks interact in complex ways, so our procedure does provide an exact decomposition of effects. We compute changes in manufacturing employment "With the China Shock" minus changes in manufacturing employment under three different scenarios (keeping everything else constant): (a) productivity growth in China is set to the world average ("Without $\widehat{A}_{China}$"); (b) changes in trade costs from and to China are set to the world average ("Without $\widehat{d}_{China}$"); (c) shocks to inter-temporal preferences in China are set to the world average ("Without $\widehat{\phi}_{China}$"). Our results are shown in Table IV. We find that changes in productivity appear to have played the most important role for the decline in US manufacturing over 2000 and 2014, followed by changes in trade costs. Interestingly, we find a modest role for the Chinese savings glut. However this small effect masks quite a sizable negative impact in the short run.

Having analyzed the labor market, we now study the welfare impacts of the China Shock. The household's utility in equation (4) can be decomposed into (1) consumption, $u(c) = \log(c)$, and (2) parameters and shocks driving labor supply b, C, η and ω. We focus on the first piece as it is the typical object of study in the International Trade literature. In order to take account of transitional dynamics, we thus define the *consumption gains* in country i, $\widehat{W}_i$, as the ratio between (a) the level of constant consumption that yields the same net present value of consumption utility along the transition path and (b) the initial steady-state consumption. Mathematically:

$$\widehat{W}_i \equiv \exp\left\{(1-\delta)\sum_{t=0}^{\infty}\delta^t \log(C_i^t) - \log(C_i^{SS_0})\right\}, \tag{31}$$

Table IV: Effect of the China Shock on Manufacturing Employment in the US (2000-2014): Contribution of Different Shocks

	Change in Employment in '000s			
	LTM	MTM	HTM	Total
Without China Shock	-63.1	-59.7	-327.7	-450.6
Without $\widehat{A}_{China}$	-80.4	-24.9	-245.7	-351.0
Without $\widehat{d}_{China}$	-4.3	-46.4	-230.9	-281.5
Without $\widehat{\phi}_{China}$	17.3	6.9	29.8	54.1

Notes: Effects of the China Shock computed between 2000 and 2014 as the change in employment "With China Shock" minus the change in employment "Without China Shock", "Without $\widehat{A}_{China}$", "Without $\widehat{d}_{China}$", or "Without $\widehat{\phi}_{China}$". See text for details. LTM: Low-Tech Manufacturing; MTM: Mid-Tech Manufacturing; HTM: High-Tech Manufacturing.

where $C_i^{SS_0}$ is the level of consumption in country i in the initial steady state, before any shocks. We compute the gains from the China shock as $\frac{\widehat{W}_i^{\text{With China Shock}}}{\widehat{W}_i^{\text{Without China Shock}}}$, where $\widehat{W}_i^{\text{With China Shock}}$ measures the consumption effects of global shocks including the China shock, and $\widehat{W}_i^{\text{Without China Shock}}$ measures the consumption effects of global shocks excluding the China shock. Table V displays these gains. Consonant with the effects reported by Caliendo et al. (2019) for the US, we find modest welfare effects of the China shock for the US (gain of 0.17%) and around the world (the effects are positive, but uniformly below 0.7%).

The gains from trade in the class of trade models we build on are often small in magnitude (Arkolakis et al., 2012). Therefore, it is more fruitful to make comparison across models, rather than interpreting our estimates in an absolute sense. This needs to be done with some caution, as different models contain different ingredients, and so we focus on models that contain the input-output and gravity structure that our model exhibits. With this in mind, Caliendo and Parro (2015) find that NAFTA led to US consumption gains of 0.08%, while Caliendo and Parro (2022) find that the US-China trade war lowered US real incomes by only 0.01%. Therefore, our model suggests that the consequences of the China Shock for US consumption were larger than those from NAFTA and the recent trade war.

As a final word of caution, we stress that our model is best suited to discussing *aggregate* welfare, given our assumptions implying within-country consumption equalization. There may yet be distributional consequences to the "China Shock" that are different in a world with and without endogenous imbalances. Given the quantitative importance of trade imbalances we have documented so far, we believe this is fertile ground for future work.

Table V: Consumption Gains of the China Shock (2000-2014) in %

US	Europe	Asia/Oceania	Americas	RoW
0.169	0.128	0.440	0.167	0.701

Notes: Consumption gains computed as $100 \times \left(\frac{\widehat{W}_i^{\text{With China Shock}}}{\widehat{W}_i^{\text{Without China Shock}}} - 1\right)\%$.

5.3 Comparison with Existing Approaches

5.3.1 Sufficient Statistic Approach to Gains from Trade

This section studies the implications of both trade imbalances and labor market frictions for the consumption gains from trade, and for how these compare with the widely used sufficient-statistics approach based on Arkolakis et al. (2012), henceforth ACR. ACR show that, across a large class of International Trade models, the consumption gains from trade can be computed based on just two statistics: changes in the share of expenditure on domestically-produced goods and the elasticity of trade.[40] This conclusion led to an explosion in the use of the ACR formula to assess the consumption gains from trade in a variety of contexts. Our model nests the Ricardian model considered in ACR, but violates two of their key assumptions: (i) no labor market frictions; and (ii) no trade imbalances.

We consider the changes in trade costs $\widehat{d}_{k,oi}^t$ between 2000 and 2014 that we obtained in section 5.1, purging the model of shocks to inter-temporal preferences and to productivity ($\widehat{A}_{k,i}^t = \widehat{\phi}_i^t = 1$ for all k, i and t). Similarly, we impose that preference and technology parameters are fixed at the calibrated values in Tables F.1, F.2 and F.3. In this case, the implied consumption gains from trade following Costinot and Rodríguez-Clare (2014), who extend the ACR formula to allow for input-output linkages, are given by:

$$\widehat{W}_i^{\text{ACR, Static}} = \prod_{j=1}^{K}\prod_{k=1}^{K} \widehat{\pi}_{k,ii}^{-\mu_{j,i}\aleph_{jk,i}/\lambda}, \tag{32}$$

where all of the changes (denoted by hats) are between final and initial steady states, and $\aleph_{jk,i}$ is the j,k^{th} element of the Leontief Inverse of the input-output matrix in country i. We obtain $\widehat{\pi}_{k,ii}$ solving our full model, which starts from a balanced-trade steady state ($NX_i^0 = 0$ for all i). To perform the comparison between consumption gains in our model and those using the ACR formula, we first focus on the change in steady-state consumption given by our framework. Given the static nature of ACR's framework, this is a more direct comparison.

Figure 11a demonstrates the differences between the long-run ACR gains in consumption (blue

[40] This class of models includes the workhorse frameworks in modern International Trade, including Eaton and Kortum (2002), Melitz (2003), and the Armington model.

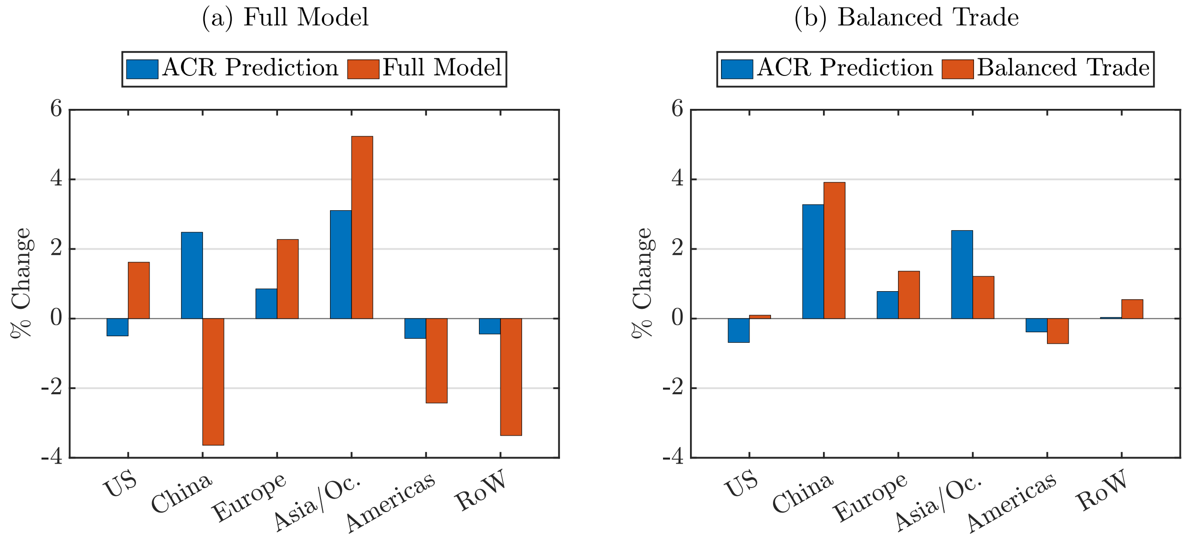
bars) and the long-run gains from our model (red bars). For example, the ACR formula predicts a 0.5% decline in long-run consumption in the US, whereas our model predicts that the US experiences a long-run gain of 1.6%. Our conclusions differ starkly in China, where the ACR formula predicts a gain of 2.5%, but our model predicts a long-run loss of 3.7%. To give a better sense of the magnitude of these discrepancies, we compute the mean (maximum) of the absolute value of the deviation in predictions between our full model and ACR's prediction: 2.8 (6.1) percentage points. These deviations are large if compared to the mean absolute value of consumption gains across countries predicted by the ACR formula: 1.3%.[41]

These numbers differ on account of both labor market frictions and long-run trade imbalances that arise in our model. As we discussed in section 4, long-run trade imbalances, and thus long-run consumption levels, depend on the full path of shocks fed into the model, and not just on the initial and final levels of trade costs. In contrast, the ACR formula is based on a static model so that the exact path of shocks is irrelevant for the (long-run) gains from trade.

We plot the long-run imbalances resulting from our model in Figure 12. They are particularly large in China and the Rest of the World, who sustain long-run trade surpluses exceeding 4% of GDP. These large long-run trade surpluses imply long-run levels of consumption that are substantially lower than the initial ones, explaining some of the losses in Figure 11a. This long-run comparison masks the fact that our model predicts strong consumption growth (and trade deficits) in these countries in the short run, as we illustrate in the red dashed line of Figure 13b—a finding we revisit a few paragraphs below.

[41] The ACR formula predicts long-term losses in some countries, as shown in Figure 11a. Note that, as shown in Figure H.4b, some countries are becoming more protected, which can negatively affect their own welfare but also the welfare of their trading partners. In addition, changes in trade costs across the globe can increase competition among countries with similar comparative advantage patterns, leading to some negative effects. Finally, the relatively large long-run losses predicted by our model for China, the Americas, and the Rest of the World are explained by a trade off between short-run increases and long-run declines in consumption, as is illustrated in Figure 13b.

Figure 11: Shocks in Trade Costs and the Long-Run Consumption Gains from Trade

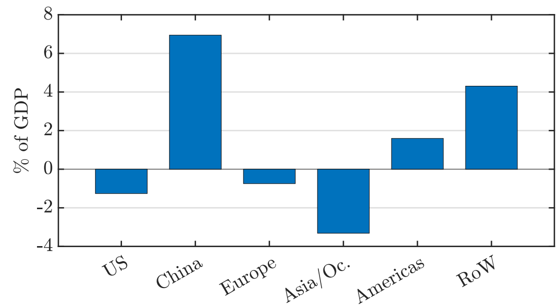


Notes: In Panel (a), the blue bars, "ACR Prediction," refer to the consumption gains computed using equation (32); the red bars, "Full Model," refer to the change in steady-state consumption given by our full model with trade imbalances. $\widehat{\pi}_{k,ii}$ is obtained by simulating our full model initialized with $NX_i^t = 0 \ \forall i$ at $t = 0$. In Panel (b), the blue bars, "ACR Prediction," refer to the consumption gains computed using equation (32); the red bars, "Balanced Trade," refer to the change in steady-state consumption given by our model imposing Balanced Trade. $\widehat{\pi}_{k,ii}$ is obtained by simulating our model with Balanced Trade ($NX_i^t = 0 \ \forall i$ for all t). Blue bars differ across panels because $\widehat{\pi}_{k,ii}$ differs across scenarios.

Figure 11b investigates the separate role of trade imbalances and labor market frictions behind the discrepancies for long-run consumption. Specifically, we simulate our model under balanced trade, removing one source of discrepancy between our model and the framework leading to the ACR formula. We still find significant differences between the gains predicted by our model (under balanced trade) and those by the ACR formula, although the magnitude of these discrepancies is now smaller. To quantify the extent of these discrepancies, we again compute the mean (maximum) of the absolute value of the deviation in predictions between our model under balanced trade and ACR's formula: 0.7 (1.3) percentage points. These deviations are still economically important if compared to the mean absolute value of consumption gains predicted by the ACR formula: 1.3%. We conclude from these exercises that trade imbalances and labor market frictions both contribute substantially to the divergences we document.

An alternative way of comparing our welfare predictions with those from ACR's formula, is to take the full transition path into account. We compute the present value of consumption gains implied by our model as described in equation (31). Similarly, we can calculate "ACR Dynamic" gains from trade, by taking the net present value of the static gains calculated by (32) in every period. More precisely, the static ACR formula implies predicted changes in consumption between

Figure 12: Steady-State Changes in Net Exports in Response to Shocks in Trade Costs



period 0 and period t given by:

$$\frac{C_i^t}{C_i^0} = \prod_{j=1}^{K}\prod_{k=1}^{K} (\widehat{\pi}_{k,ii}^t)^{-\mu_{j,i}\aleph_{jk,i}/\lambda}, \tag{33}$$

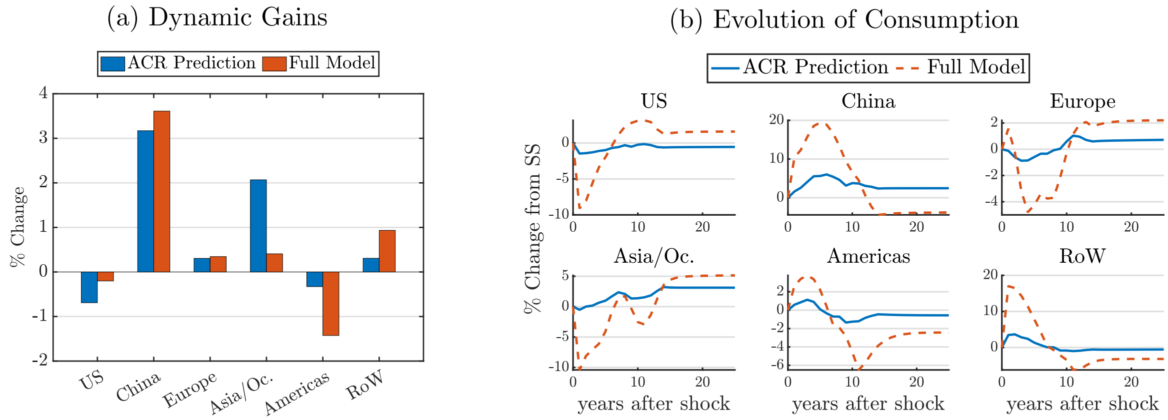
where $\widehat{\pi}_{k,ii}^t$ is the change in trade shares between periods 0 and t, which is computed using our full model. Applying equation (31) to this path of consumption we obtain the following formula for the "ACR Dynamic" gains from trade:

$$\widehat{W}_i^{\text{ACR, Dynamic}} = \exp\left\{(1-\delta)\sum_{t=0}^{\infty}\delta^t \log\left(\prod_{j=1}^{K}\prod_{k=1}^{K}(\widehat{\pi}_{k,ii}^t)^{-\mu_{j,i}\aleph_{jk,i}/\lambda}\right)\right\}. \tag{34}$$

Figure 13a compares the consumption gains predicted by the "ACR Dynamic" formula (34) to the dynamic gains computed according to our model (31). Although predictions are now similar for China and Europe, they are still different in the remaining countries. For example, Asia/Oceania enjoys a consumption gain of 2.1% according to the dynamic ACR formula, whereas our model predicts a gain of 0.4%. Also noteworthy, the Americas lose by 1.4% according to our model, but by less than 0.3% according to the dynamic ACR formula. The mean (maximum) of the absolute value of the deviation in predictions between our model under balanced trade and ACR's prediction is given by 0.7 (1.1) percentage points. These magnitudes are still important compared to the average absolute value of growth in consumption: 1.7%.

We conclude pointing out that the similarity in the dynamic gains calculations (compared to long-run comparisons) mask differences between the short- and long- run behavior of consumption in our model compared to ACR's prediction (33). These patterns are illustrated in Figure 13b, which show that the evolution of consumption predicted by ACR's formula is relatively stable. On the other hand, our model generates large swings in consumption around the ACR prediction. The net effect is that the dynamic ACR gains are closer to our model's dynamic predictions than the long-run predictions. These results suggest that if a researcher is interested in quantifying consumption changes between two points in time, our model will deliver different predictions compared to ACR's formula, which are economically important. However, this discrepancy will not be as large—although still important—if the researcher is only interested in computing the net present value of gains over the full transition.

Figure 13: Shocks in Trade Costs and Dynamics Consumption Gains from Trade



Notes: In Panel (a), the blue bars, "ACR Prediction," refer to the present value of gains calculated by using equation (34); the red bars, "Full Model," refer to the present value of gains over the transition in the full model with trade imbalances, using equation (31). In both cases, $\widehat{\pi}^t_{k,ii}$ is obtained by simulating our full model initialized with $NX^t_i = 0\ \forall i$ at $t = 0$. In Panel (b), we depict the evolution of consumption implied by our full model and the evolution of consumption as implied by the ACR formula perid by period (33).

5.3.2 Trade Imbalances Resulting from a System of Transfers

A commonly used approach to model trade imbalances in the literature is to create a system of transfers across locations to match observed imbalances at a given point in time.[42] We implement this approach in the context of our model by imposing that all firm profits of each country, net of vacancy posting costs, are sent to a global portfolio. This global portfolio is then redistributed proportionally to countries in a way that matches the initial observed trade imbalances in the data. Formally, net exports are given by:

$$NX^t_i = \Pi^t_i - \iota_i \Pi^t_{World}, \tag{35}$$

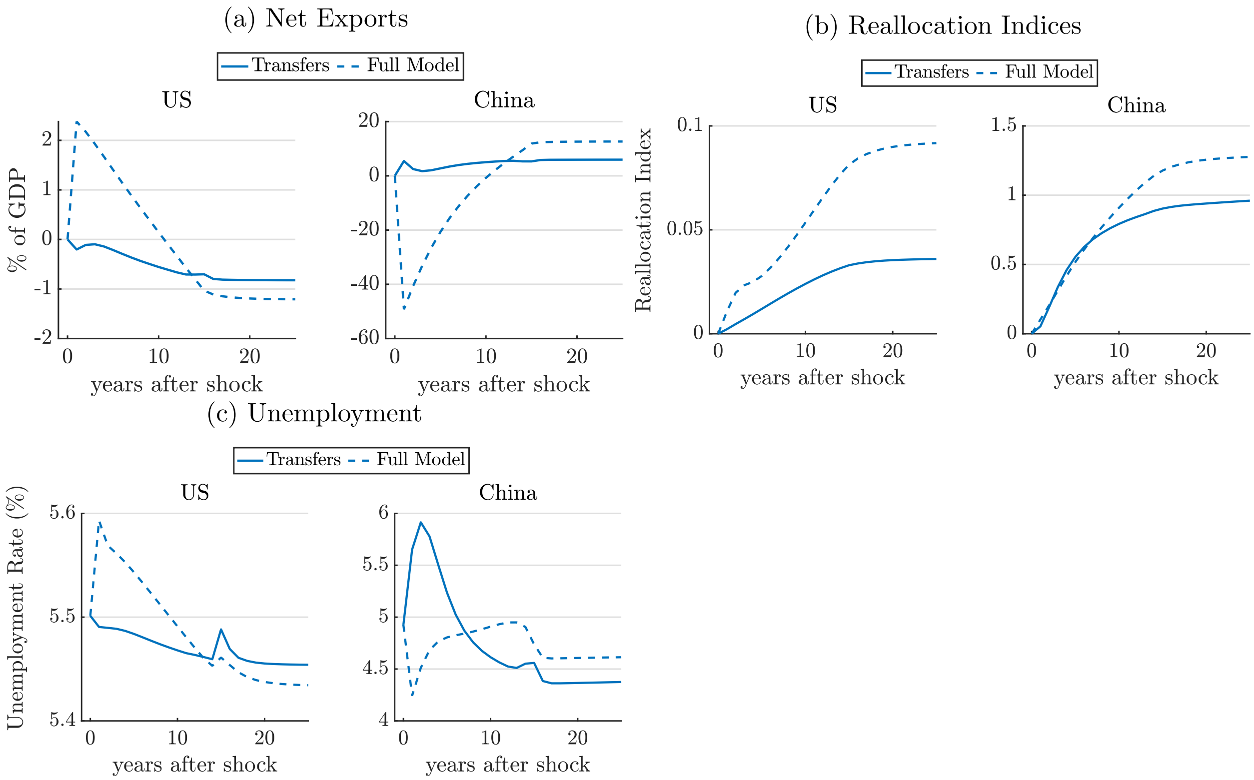
where Π^t_i aggregates profits across all firms in country i at time t and Π^t_{World} aggregates all profits all over the world. The share ι_i is calibrated to match the initial level of trade imbalances in the data.

To compare our approach with this popular alternative, we revisit the counterfactual we studied in Section 4 and subject the global economy with the slow Chinese productivity growth depicted in

[42]Prominent examples of this approach include Caliendo et al. (2017), Fajgelbaum et al. (2018), Caliendo et al. (2019), Fajgelbaum and Gaubert (2020), and Caliendo et al. (2021).

Figure 3a. We then compare predictions that arise from our complete model with trade imbalances to those that arise following the procedure described in equation (35).

Figure 14: Comparing Outcomes Across Models

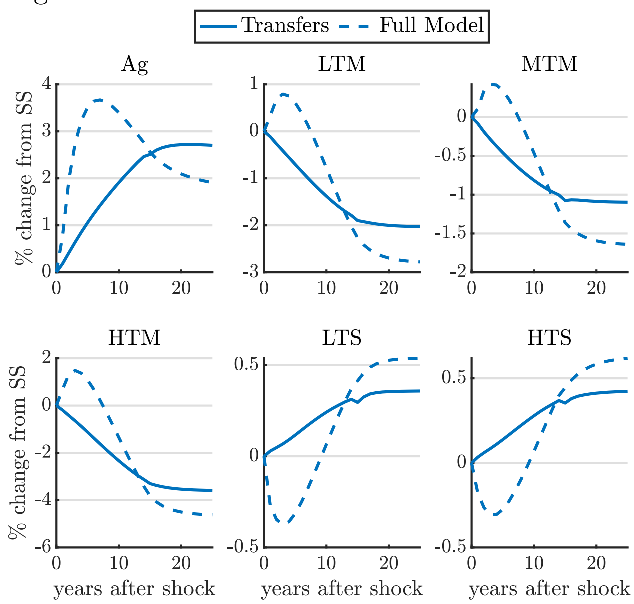


Notes: Responses to slow productivity growth in China (see shock in Figure 3a). Comparison between predictions of our "Full Model," and "Transfers"—model with imbalances given by equation (35). Reallocation index is given by $Reallocation_i^t = \frac{1}{2}\sum_{s=1}^{t}\sum_{k=1}^{J}\left|\frac{L_{i,k}^{s}}{L_i} - \frac{L_{i,k}^{s-1}}{L_i}\right|$, which accumulates yearly changes in sectoral employment shares over time.

Figure 14a demonstrates sharp differences in the behavior of trade imbalances across specifications. In particular, the model following equation (35) predicts that China runs a trade surplus every period, different from the large short-run trade deficit implied by our model. In turn, our model predicts a twice as large trade surplus for China in the long run. This behavior of trade imbalances has implications for the amount of reallocation in response to the slow productivity growth in China and for unemployment responses. Figure 14b shows that our model leads to more reallocation than the system of transfers model—more than 2 times more in the US and 20% more in China. Figure 14c shows that the behavior of unemployment dictated by this alternative procedure is different from our full model's predictions. In fact, it is similar to the behavior we would have obtained with a balanced-trade model—see Figure 4d. However, both models have similar implications for long-run unemployment.

Finally, we compare the implications of both models for the dynamics of reallocation. Figure 15 shows considerably distinct US labor market dynamics across sectors. Under the system of transfers, we predict a long-run decline of manufacturing in the US of 528k jobs—25% smaller than the predicted 697k jobs lost under our baseline model.

Figure 15: Reallocation Across Sectors in the US



Notes: Labor market responses to slow productivity growth in China (see shock in Figure 3a). Comparison between predictions of our "Full Model," and "Transfers"—model with imbalances given by equation (35). Ag: Agriculture; LTM: Low-Tech Manufacturing; MTM: Mid-Tech Manufacturing; HTM: High-Tech Manufacturing; LTS: Low-Tech Services; HTS: High-Tech Services.

6 Concluding Remarks

Our work shows that carefully modeling trade imbalances can have quantitatively important implications for the adjustment process in response to globalization shocks and opens important questions for future work. Given the importance of imbalances for the reallocation process, it is natural to extend the model to allow for heterogeneous workers and speak to the inequality effects of trade within this framework. Incorporating endogenous capital accumulation is an equally important extension: aside from its role in shaping global imbalances (Jin, 2012), capital can have important implications for inequality through capital-skill complementarity (Parro, 2013; Reyes-Heroles et al., 2020).

An additional valuable extension of our framework would allow workers to make borrowing and savings decisions at the individual level, which will aggregate into global imbalances. Even though this is a hard problem, especially regarding estimation, we believe that our method of simulated moments that can be performed country by country (conditional on trade shares and imbalances) can be applied to this situation. Finally, our model imposed perfect foresight on aggregate variables. This approach is appropriate to study the consequences of long-run trends in various fundamentals. However, it would be worthwhile investigating a version of our model with aggregate uncertainty and study the role of precautionary savings in trade imbalances.

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Online Appendix, Not for Publication

A Decentralizing the Labor Supply Decision in the Household Problem

Section 2.3.3 states that the allocation of workers follows a controlled stochastic process. Indeed, while the household head can choose workers' sectors given knowledge of switching costs and shocks, employment itself remains a probabilistic outcome. To this end, let $\widetilde{e}_k^t\left(x_\ell^{t+1}\right) \in \{0,1\}$ indicate whether the household head continues on with a match at time t given a match productivity of x_ℓ^{t+1} in sector k. In this case, the probability that worker ℓ is employed in sector k at time $t+1$, conditional on match productivity x_ℓ^{t+1} and time t information $\left(k_\ell^t, e_\ell^t\right)$ is given by:

$$\begin{aligned}\Pr\left(k_\ell^{t+1}=k, e_\ell^{t+1}=1 | x_\ell^{t+1}, k_\ell^t, e_\ell^t\right) = {} & \mathcal{I}\left(k_\ell^t=k\right) e_\ell^t \left(1-\chi_k\right) \widetilde{e}_k^t\left(x_\ell^{t+1}\right) \\ & + \left(1-e_\ell^t\right) \mathcal{I}\left(k_\ell^{t+1}=k\right) \theta_k^t q\left(\theta_k^t\right) \widetilde{e}_k^t\left(x_\ell^{t+1}\right).\end{aligned} \tag{A.1}$$

In words, if $\mathcal{I}\left(k_\ell^t=k\right) e_\ell^t = 1$, then worker ℓ is employed in sector k at time t and the match survives with probability $(1-\chi_k)$ if the family planner decides to keep the match ($\widetilde{e}_k^t\left(x_\ell^{t+1}\right)=1$). If $e_\ell^t=0$, that is, the worker is unemployed at t, and the planner chooses $k_\ell^{t+1}=k$, then the worker is employed in sector k at time $t+1$ with probability $\theta_k^t q\left(\theta_k^t\right) \widetilde{e}_k^t\left(x_\ell^{t+1}\right)$. Importantly, workers' sector and employment status at $t+1$, k_ℓ^{t+1} and e_ℓ^{t+1}, are determined by actions taken at t.

We are now ready to formalize the problem that the household head solves. The household head chooses the path of consumption, c_ℓ^t, the path of sectoral choices, k_ℓ^t, continuation decisions, $\widetilde{e}_k^t(x)$, and bonds, B^{t+1}, to solve:

$$\max_{\left\{k_{\ell,}^t, \widetilde{e}_k^t(.), B^{t+1}, c_\ell^t\right\}} E_0\left\{\sum_{t=0}^{\infty} (\delta)^t \phi^t \int_0^{\overline{L}} \mathcal{U}_\ell^t d\ell\right\}, \tag{A.2}$$

subject to the budget constraint (5), equation (A.1) and conditional on B^0. We show that the solution to this problem can be decentralized to individual workers solving equations (6) and (7).

Given that consumption is optimally set to be equal across ℓ (see discussion in section 2.3.3), the Lagrangian of problem (A.2), (5) and (A.1) can be written as:

$$\mathcal{L} = E_0\left\{\begin{array}{c} \sum\limits_{t=0}^{\infty}\left[(\delta)^t \phi^t \left(\overline{L}u\left(c^t\right) - \widetilde{\lambda}^t\left(\overline{L}P^{F,t}c^t + B^{t+1} - \Pi^t - R^t B^t\right)\right)\right] + \\ \int_0^{\overline{L}}\left[\sum\limits_{t=0}^{\infty}\left[(\delta)^t \phi^t \left(\begin{array}{c} \left(1-e_\ell^t\right)\left(-C_{k_\ell^t, k_\ell^{t+1}} + \omega_{\ell, k_\ell^{t+1}}^t + b_{k_\ell^{t+1}}\right) + \\ e_\ell^t \eta_{k_\ell^t} + \widetilde{\lambda}^t\left(\sum\limits_{k=1}^{K} \mathcal{I}\left(k_\ell^{t+1}=k\right) e_\ell^t w_k^t\left(x_\ell^t\right)\right)\end{array}\right)\right]\right] d\ell \end{array}\right\} \tag{A.3}$$

Because each worker is infinitesimal, and the allocation of one worker does not interfere with the allocation/utility of other individual workers (conditional on aggregates), maximizing

$$\int_0^{\overline{L}} \left[\sum_{t=0}^{\infty} (\delta)^t \phi^t \left(\begin{array}{c} \left(1-e_\ell^t\right)\left(-C_{k_\ell^t,k_\ell^{t+1}} + \omega_{\ell,k_\ell^{t+1}}^t + b_{k_\ell^{t+1}}\right) + \\ e_\ell^t \eta_{k_\ell^t} + \widetilde{\lambda}^t \left(\sum\limits_{k=1}^{K} \mathcal{I}\left(k_\ell^{t+1} = k\right) e_\ell^t w_k^t \left(x_\ell^t\right) \right) \end{array} \right) \right] d\ell \tag{A.4}$$

means maximizing each individual term. Therefore, the planner solves, for each individual, the recursive problem:

$$\mathcal{L}_W^t\left(k_\ell^t, e_\ell^t, x_\ell^t, \boldsymbol{\omega}_\ell^t\right) = \max_{\left\{k_\ell^{t+1}, \widetilde{e}_k^{t+1}(.)\right\}} \left\{ \begin{array}{c} \left(1-e_\ell^t\right)\left(-C_{k_\ell^t,k_\ell^{t+1}} + \omega_{\ell,k_\ell^{t+1}}^t + b_{k_\ell^{t+1}}\right) + e_\ell^t \eta_{k_\ell^t} \\ +\widetilde{\lambda}_t \sum\limits_{k=1}^{K} \mathcal{I}\left(k_\ell^t = k\right) e_\ell^t w_k^t \left(x_\ell^t\right) + \\ \delta \widehat{\phi}^{t+1} E_t \mathcal{L}_W^{t+1}\left(k_\ell^{t+1}, e_\ell^{t+1}, x_\ell^{t+1}, \boldsymbol{\omega}_\ell^{t+1}\right) \end{array} \right\}, \tag{A.5}$$

where $\widehat{\phi}^{t+1} \equiv \frac{\phi^{t+1}}{\phi^t}$.

Denote by $\mathcal{F}^t$ the set of information at t. So, $E_t(.) = E\left(.|\mathcal{F}^t\right)$. For an unemployed worker in sector k at time t, $k_\ell^t = k$, $e_\ell^t = 0$:

$$\mathcal{L}_W^t\left(k_\ell^t = k, e_\ell^t = 0, x_\ell^t, \boldsymbol{\omega}_\ell^t\right) = \max_{k', \left\{\widetilde{e}_k^{t+1}(.)\right\}} - C_{kk'} + \omega_{\ell,k'}^t + b_{k'} + \delta \widehat{\phi}^{t+1} E_t \mathcal{L}_W^{t+1}\left(k', e_\ell^{t+1}, x_\ell^{t+1}, \boldsymbol{\omega}_\ell^{t+1}\right). \tag{A.6}$$

Using the law of iterated expectations we obtain:

$$\begin{aligned}
&\mathcal{L}_W^t\left(k_\ell^t = k, e_\ell^t = 0, x_\ell^t, \boldsymbol{\omega}_\ell^t\right) = \max_{k', \left\{\widetilde{e}_k^{t+1}(.)\right\}} - C_{kk'} + \omega_{\ell,k'}^t + b_{k'} \\
&+ \delta \widehat{\phi}^{t+1} E\left\{ E\left[\mathcal{L}_W^{t+1}\left(k', 1, x_\ell^{t+1}, \boldsymbol{\omega}_\ell^{t+1}\right) | x_\ell^{t+1}, \mathcal{F}^t\right] \times \Pr\left(k_\ell^{t+1} = k', e_\ell^{t+1} = 1 | x_\ell^{t+1}, \mathcal{F}^t\right) | \mathcal{F}^t \right\} \\
&+ \delta \widehat{\phi}^{t+1} E\left\{ E\left[\mathcal{L}_W^{t+1}\left(k', 0, x_\ell^{t+1}, \boldsymbol{\omega}_\ell^{t+1}\right) | x_\ell^{t+1}, \mathcal{F}^t\right] \times \Pr\left(k_\ell^{t+1} = k', e_\ell^{t+1} = 0 | x_\ell^{t+1}, \mathcal{F}^t\right) | \mathcal{F}^t \right\} \\
&= \max_{k', \left\{\widetilde{e}_k^{t+1}(.)\right\}} - C_{kk'} + \omega_{\ell,k'}^t + b_{k'} \\
&+ \delta \widehat{\phi}^{t+1} \theta_{k'}^t q\left(\theta_{k'}^t\right) E\left\{ \mathcal{L}_W^{t+1}\left(k', 1, x_{\ell'}^{t+1}, \boldsymbol{\omega}_\ell^{t+1}\right) \widetilde{e}_{k'}^{t+1}\left(x_\ell^{t+1}\right) | \mathcal{F}^t \right\} \\
&+ \delta \widehat{\phi}^{t+1} E\left\{ \left(1 - \theta_k^t q\left(\theta_{k'}^t\right) \widetilde{e}_{k'}^{t+1}\left(x_\ell^{t+1}\right)\right) \mathcal{L}_W^{t+1}\left(s', 0, x_\ell^{t+1}, \boldsymbol{\omega}_\ell^{t+1}\right) | \mathcal{F}^t \right\}
\end{aligned} \tag{A.7}$$

For an employed worker in sector k, $k_\ell^t = k$, $e_\ell^t = 1$:

$$\begin{aligned}
\mathcal{L}_W^t\left(k_\ell^t = k, e_\ell^t = 1, x_\ell^t, \boldsymbol{\omega}_\ell^t\right) &= \max_{\left\{\widetilde{e}_k^{t+1}(.)\right\}} \widetilde{\lambda}^t w_k^t\left(x_\ell^t\right) + \eta_k + \delta\widehat{\phi}^{t+1} E_t \mathcal{L}_W^{t+1}\left(k, e_\ell^{t+1}, x_\ell^t, \omega_\ell^{t+1}\right) \\
&= \max_{\left\{\widetilde{e}_k^{t+1}(.)\right\}} \widetilde{\lambda}^t w_k^t\left(x_\ell^t\right) + \eta_k \\
&+ \delta\widehat{\phi}^{t+1} E \left\{ \begin{array}{c} E\left[\mathcal{L}_W^{t+1}\left(k, 1, x_\ell^t, \boldsymbol{\omega}_\ell^{t+1}\right) | k_\ell^{t+1} = k, , e_\ell^{t+1} = 1, x_\ell^{t+1}, \mathcal{F}^t\right] \times \\ \Pr\left(k_\ell^{t+1} = k, e_\ell^{t+1} = 1 | x_\ell^{t+1}, \mathcal{F}^t\right) | \mathcal{F}^t \end{array} \right\} \\
&+ \delta\widehat{\phi}^{t+1} E \left\{ \begin{array}{c} E\left[\mathcal{L}_W^{t+1}\left(k, 0, x_\ell^t, \boldsymbol{\omega}_\ell^{t+1}\right) | k_\ell^{t+1} = k, e_\ell^{t+1} = 0, x_\ell^{t+1}, \mathcal{F}^t\right] \times \\ \Pr\left(k_\ell^{t+1} = k, e_\ell^{t+1} = 0 | x_\ell^{t+1}, \mathcal{F}^t\right) | \mathcal{F}^t \end{array} \right\} \\
&= \max_{\left\{\widetilde{e}_k^{t+1}(.)\right\}} \widetilde{\lambda}^t w_k^t\left(x_\ell^t\right) + \eta_k \\
&+ \delta\widehat{\phi}^{t+1}\left(1 - \chi_k\right) E \left[\begin{array}{c} \widetilde{e}_k^{t+1}\left(x_\ell^t\right) \mathcal{L}_W^{t+1}\left(k, 1, x_\ell^t, \boldsymbol{\omega}_\ell^{t+1}\right) \\ + \left(1 - \widetilde{e}_k^{t+1}\left(x_\ell^t\right)\right) \mathcal{L}_W^{t+1}\left(k, 0, x_\ell^t, \boldsymbol{\omega}_\ell^{t+1}\right) | \mathcal{F}^t \end{array} \right] \\
&+ \delta\widehat{\phi}^{t+1}\chi_k E\left[\mathcal{L}_W^{t+1}\left(k, 0, x_\ell^t, \boldsymbol{\omega}_\ell^{t+1}\right) | \mathcal{F}^t\right] \qquad \text{(A.8)}
\end{aligned}$$

Make the following definitions

$$\begin{aligned}
\widetilde{U}_k^t\left(\boldsymbol{\omega}_\ell^t\right) &\equiv \mathcal{L}_W^t\left(k_\ell^t = k, e_\ell^t = 0, x_\ell^t, \boldsymbol{\omega}_\ell^t\right), \text{ and} \\
W_k^t(x) &\equiv \mathcal{L}_W^t\left(k_\ell^t = k, e_\ell^t = 1, x, \boldsymbol{\omega}_\ell^t\right). \qquad \text{(A.9)}
\end{aligned}$$

$\widetilde{U}_k^t\left(\boldsymbol{\omega}_\ell^t\right)$ is the value of unemployment in sector k, conditional on the preference shocks $\boldsymbol{\omega}_\ell^t$, and $W_k^t(x)$ is the value of a job with match productivity x. Note that $\mathcal{L}_W^t\left(k_\ell^t = k, e_\ell^t = 0, x_\ell^t, \boldsymbol{\omega}_\ell^t\right)$ does not depend on x_ℓ^t and $\mathcal{L}_W^t\left(k_\ell^t = k, e_\ell^t = 1, x, \boldsymbol{\omega}_\ell^t\right)$ does not depend on $\boldsymbol{\omega}_\ell^t$. Rewrite $\widetilde{U}_k^t\left(\boldsymbol{\omega}_\ell^t\right)$ as

$$\begin{aligned}
\widetilde{U}_k^t\left(\boldsymbol{\omega}_\ell^t\right) = &\max_{k', \left\{\widetilde{e}_k^{t+1}(.)\right\}} - C_{kk'} + \omega_{\ell,k'}^t + b_{k'} \\
&+ \delta\widehat{\phi}^{t+1}\theta_{k'}^t q\left(\theta_{k'}^t\right) \int W_{k'}^{t+1}(x)\, \widetilde{e}_{k'}^{t+1}(x)\, dG_{k'}(x) \\
&+ \delta\widehat{\phi}^{t+1}\left(1 - \theta_{k'}^t q\left(\theta_{k'}^t\right) \Pr\left(\widetilde{e}_{k'}^{t+1}\left(x_\ell^{t+1}\right) = 1\right)\right) E_\omega\left(\widetilde{U}_{k'}^{t+1}\left(\boldsymbol{\omega}_\ell^{t+1}\right)\right), \qquad \text{(A.10)}
\end{aligned}$$

and so:

$$\begin{aligned}
\widetilde{U}_k^t\left(\boldsymbol{\omega}_\ell^t\right) = & \max_{k',\left\{\widetilde{e}_k^{t+1}(.)\right\}} - C_{kk'} + \omega_{\ell,k'}^t + b_{k'} \\
& + \delta\widehat{\phi}^{t+1}\theta_{k'}^t q\left(\theta_{k'}^t\right)\int\left(\begin{array}{c} W_{k'}^{t+1}\left(x\right)\widetilde{e}_{k'}^{t+1}\left(x\right) + \\ E_\omega\left(\widetilde{U}_{k'}^{t+1}\left(\omega_\ell^{t+1}\right)\right)\left(1-\widetilde{e}_{k'}^{t+1}\left(x\right)\right)\end{array}\right)dG_{k'}\left(x\right) \\
& + \delta\widehat{\phi}^{t+1}\left(1-\theta_{k'}^t q\left(\theta_{k'}^t\right)\right)E_\omega\left(\widetilde{U}_{k'}^{t+1}\left(\boldsymbol{\omega}_\ell^{t+1}\right)\right). \qquad \text{(A.11)}
\end{aligned}$$

Now, wewrite $W_k^t\left(x\right)$ as:

$$\begin{aligned}
W_k^t\left(x\right) = & \max_{\left\{\widetilde{e}_k^{t+1}(.)\right\}}\widetilde{\lambda}^t w_k^t\left(x\right) + \eta_k \\
& + \delta\widehat{\phi}^{t+1}\left(1-\chi_k\right)\widetilde{e}_k^{t+1}\left(x\right)W_k^{t+1}\left(x\right) \\
& + \delta\widehat{\phi}^{t+1}\left(1-\left(1-\chi_k\right)\widetilde{e}_k^{t+1}\left(x\right)\right)E\left(\widetilde{U}_k^{t+1}\left(\boldsymbol{\omega}_\ell^{t+1}\right)\right), \qquad \text{(A.12)}
\end{aligned}$$

and so:

$$\begin{aligned}
W_k^t\left(x\right) = & \max_{\left\{\widetilde{e}_k^{t+1}(.)\right\}}\widetilde{\lambda}^t w_k^t\left(x_\ell^t\right) + \eta_k \\
& + \delta\widehat{\phi}^{t+1}\left(1-\chi_k\right)\left(\widetilde{e}_k^{t+1}\left(x\right)W_k^{t+1}\left(x\right) + \left(1-\widetilde{e}_k^{t+1}\left(x\right)\right)E_\omega\left(\widetilde{U}_k^{t+1}\left(\boldsymbol{\omega}_\ell^{t+1}\right)\right)\right) \\
& + \delta\widehat{\phi}^{t+1}\chi_k E_\omega\left(\widetilde{U}_k^{t+1}\left(\boldsymbol{\omega}_\ell^{t+1}\right)\right). \qquad \text{(A.13)}
\end{aligned}$$

It is now clear that the optimal policy $\widetilde{e}_k^{t+1}\left(.\right)$ is:

$$\widetilde{e}_k^{t+1}\left(x\right) = \left\{\begin{array}{c} 1 \text{ if } W_k^{t+1}\left(x\right) > E_\omega\left(\widetilde{U}_k^{t+1}\left(\boldsymbol{\omega}_\ell^{t+1}\right)\right) \\ 0 \text{ otherwise}\end{array}\right\}. \qquad \text{(A.14)}$$

Define $U_k^t \equiv E_\omega\left(\widetilde{U}_k^t\left(\boldsymbol{\omega}_\ell^t\right)\right)$. We conclude that equations (A.11), (A.13) and (A.14) imply equations (6) and (7).

B Steady State Equilibrium

In this section we derive the equations characterizing the steady state equilibrium. The key conditions that we impose is that variables are constant over time, inflows of workers into each sector equal outflows, and job destruction rates equal job creation rates. We also impose that the preference shifters $\left\{\widehat{\phi}_i^t\right\}$ are constant and equal to 1 in the long run.

Wage Equation

$$w_{k,i}(x) = \beta_{k,i}\widetilde{w}_{k,i}x + \frac{(1-\beta_{k,i})(1-\delta)U_{k,i} - (1-\beta_{k,i})\eta_{k,i}}{\widetilde{\lambda}_i} \tag{B.1}$$

Firms' value function

$$J_{k,i}(x) = \frac{1-\beta_{k,i}}{1-(1-\chi_{k,i})\delta}\widetilde{\lambda}_i\widetilde{w}_{k,i}\left(x - \underline{x}_{k,i}\right) \tag{B.2}$$

Probability of filling a vacancy

$$q_i(\theta_{k,i}) = \frac{\kappa_{k,i}P_i^F}{\widetilde{w}_{k,i}} \times \frac{1-\delta(1-\chi_{k,i})}{\delta(1-\beta_{k,i})I_{k,i}\left(\underline{x}_{k,i}\right)} \tag{B.3}$$

where

$$I_{k,i}\left(\underline{x}_{k,i}\right) \equiv \int_{\underline{x}_{k,i}}^{\infty}\left(s - \underline{x}_{k,i}\right)dG_{k,i}(s) \tag{B.4}$$

Unemployed workers' Bellman equation

$$U_{k,i} = \zeta_i \log\left(\sum_{k'}\exp\left\{\frac{-C_{kk',i} + b_{k',i} + \theta_{k',i}\frac{\kappa_{k',i}P_i^F}{\widetilde{w}_{k',i}} \times \widetilde{\lambda}_i\widetilde{w}_{k',i}\frac{\beta_{k',i}}{\left(1-\beta_{k',i}\right)} + \delta U_{k',i}}{\zeta_i}\right\}\right) \tag{B.5}$$

Transition rates

$$s_{k\ell,i} = \frac{\exp\left\{\frac{-C_{k\ell,i}+b_{\ell,i}+\theta_{\ell,i}\frac{\kappa_{\ell,i}P_i^F}{\widetilde{w}_{\ell,i}}\times\widetilde{\lambda}_i\widetilde{w}_{\ell,i}\frac{\beta_{\ell,i}}{\left(1-\beta_{\ell,i}\right)}+\delta U_{\ell,i}}{\zeta_i}\right\}}{\sum_{\overline{k}}\exp\left\{\frac{-C_{k\overline{k},i}+b_{\overline{k},i}+\theta_{\overline{k},i}\frac{\kappa_{\overline{k},i}P_i^F}{\widetilde{w}_{\overline{k},i}}\times\widetilde{\lambda}_i\widetilde{w}_{\overline{k},i}\frac{\beta_{\overline{k},i}}{\left(1-\beta_{\overline{k},i}\right)}+\delta U_{\overline{k},i}}{\zeta_i}\right\}} \tag{B.6}$$

Steady-state unemployment rates

$$u_{k,i} = \frac{\chi_{k,i}}{\theta_{k,i} q_i\left(\theta_{k,i}\right)\left(1 - G_{k,i}\left(\underline{x}_{k,i}\right)\right) + \chi_{k,i}} \tag{B.7}$$

Trade + Price System

Input Bundle Price

$$P_{k,i}^{M} = \prod_{\ell=1}^{K}\left(\frac{P_{\ell,i}^{I}}{\nu_{k\ell,i}}\right)^{\nu_{k\ell,i}} \tag{B.8}$$

Domestic Sectoral Output Price

$$c_{k,i} = \left(\frac{\widetilde{w}_{k,i}}{\gamma_{k,i}}\right)^{\gamma_{k,i}}\left(\frac{P_{k,i}^{M}}{1-\gamma_{k,i}}\right)^{1-\gamma_{k,i}} \tag{B.9}$$

Price of Composite Sector-Specific Intermediate Good

$$P_{k,i}^{I} = \Gamma_{k,i}\left[\sum_{j=1}^{N}\frac{A_{k,j}}{\left(c_{k,j} d_{k,ji}\right)^{\lambda}}\right]^{-1/\lambda} \tag{B.10}$$

where $\Gamma_{k,i}$ is a sector and country specific constant.

Price of Final Consumption Good

$$P_{i}^{F} = \prod_{k=1}^{K}\left(\frac{P_{k,i}^{I}}{\mu_{k,i}}\right)^{\mu_{ki}} \tag{B.11}$$

Trade Shares

$$\pi_{k,oi} = \frac{A_{k,o}\left(c_{k,o} d_{k,oi}\right)^{-\lambda}}{\Phi_{k,i}}, \tag{B.12}$$

where

$$\Phi_{k,i} = \sum_{o=1}^{N} A_{k,o}\left(c_{k,o} d_{k,oi}\right)^{-\lambda}. \tag{B.13}$$

Zero net flows condition

$$\left(L_i.u_i\right) = \left(\sum_{\ell=1}^{K} s_{\ell k,i} L_{\ell,i} u_{\ell,i}\right)_{k=1}^{K} = s_i'\left(L_i.u_i\right) \tag{B.14}$$

Product market clearing

Gross Output

$$\gamma_{k,o}Y_{k,o} = \widetilde{w}_{k,o}L_{k,o}\left(1-u_{k,o}\right)\int_{\underline{x}_{k,o}}^{\infty}\frac{s}{1-G_{k,i}\left(\underline{x}_{k,o}\right)}dG_{k,o}\left(s\right) \tag{B.15}$$

$$= \widetilde{w}_{k,o}\widetilde{L}_{k,o} \tag{B.16}$$

Expenditure with Vacancies

$$E^{V}_{k,o} = \kappa_{k,o}P^{F}_{o}\theta_{k,o}u_{k,o}L_{k,o} \tag{B.17}$$

Market Clearing System

$$Y_{k,o} = \sum_{i=1}^{N}\pi_{k,oi}E_{k,i} \tag{B.18}$$

$$E_{k,i} = \mu_{k,i}\left(\sum_{\ell=1}^{K}\gamma_{\ell,i}Y_{\ell,i}\right) + \sum_{\ell=1}^{K}\left(1-\gamma_{\ell,i}\right)\nu_{\ell k,i}Y_{\ell,i} - \mu_{k,i}NX_{i} \tag{B.19}$$

Normalization: World total revenue is the numeraire

$$\sum_{i=1}^{N}\sum_{k=1}^{K}Y_{k,i} = 1 \tag{B.20}$$

Final Good Consumption Expenditure

$$E^{C}_{i} = \sum_{k=1}^{K}\gamma_{k,i}Y_{k,i} - \sum_{k=1}^{K}E^{V}_{k,i} - NX_{i} \tag{B.21}$$

Lagrange multipliers

$$\widetilde{\lambda}_{i} = \frac{\overline{L}_{i}}{E^{C}_{i}} \tag{B.22}$$

C Country and Sector Definitions

Table C.1 displays how we divide the world according to the country divisions in the World Input Output Database. Table C.2 details how we define the six sectors we consider in our quantitative exercises.

Table C.1: Country Definitions

1	USA
2	China
3	Europe
4	Asia/Oceania
5	Americas
6	Rest of the World (ROW)

Notes: Asia/Oceania = {Australia, Japan, South Korea, Taiwan}, Americas = {Brazil, Canada, Mexico}, Rest of the World ={Indonesia, India, Russia, Turkey, Rest of the World}. This partition of the world was dictated by data availability from the World Input Output Database.

Table C.2: Sector Definitions

1	Agriculture/Mining	Agriculture, Forestry and Fishing; Mining and quarrying
2	Low-Tech Manufacturing	Wood products; Paper, printing and publishing; Coke and refined petroleum; Basic and fabricated metals; Other manufacturing
3	Mid-Tech Manufacturing	Food, beverage and tobacco; Textiles; Leather and footwear; Rubber and plastics; Non-metallic mineral products
4	High-Tech Manufacturing	Chemical products; Machinery; Electrical and optical equipment; Transport equipment
5	Low Tech Services	Utilities; Construction; Wholesale and retail trade; Transportation; Accommodation and food service activities; Activities of households as employers
6	Hi Tech Services	Publishing; Media; Telecommunications; Financial, real estate and business services; Government, education, health

D Dispersion of Idiosyncratic Preference Shocks

The model in Artuç et al. (2010) implies the following steady-state relationship:

$$\log\left(\frac{s^{ij}}{s^{ii}}\right) - \delta \log\left(\frac{s^{ij}}{s^{jj}}\right) = \frac{1-\delta}{\zeta} C^{ij} + \frac{\delta}{\zeta}\left(w^i - w^j\right)$$

Where w_i is the wage in sector i, s^{ij} is the share of workers employed in sector i in period t who choose to be employed in sector j in period $t+1$, C^{ij} is the mobility cost between sectors i and j, ζ measures the dispersion in idiosyncratic preferences for sectors, and δ is the discount rate at the QUARTERLY frequency.

Approximate the transition rate matrix s as:

$$s = I + \Delta s,$$

where the Δs terms are small. This is a low-mobility approximation, meaning that the diagonal of matrix s is close to 1 and that off-diagonal elements are close to 0.

In that case, the matrix for ANNUAL transition rates $\widetilde{s}$ can be approximated as:

$$\widetilde{s} = s^4 \approx I + 4\Delta s \approx I + 4\left(s - I\right) = 4s - 3I.$$

Therefore, $\widetilde{s}^{ij} \approx 4s^{ij}$ if $i \neq j$ and $\widetilde{s}^{ii} \approx 4s^{ii} - 3 \approx 1$. In turn, $\log\left(\frac{s^{ij}}{s^{ii}}\right) \approx \log\left(\frac{\widetilde{s}^{ij}}{4}\right)$, $\log\left(\frac{s^{ij}}{s^{jj}}\right) \approx \log\left(\frac{\widetilde{s}^{ij}}{4}\right)$ and:

$$\begin{aligned}\log\left(\frac{s^{ij}}{s^{ii}}\right) - \delta \log\left(\frac{s^{ij}}{s^{jj}}\right) &\approx (1-\delta)\log\left(\frac{\widetilde{s}^{ij}}{4}\right)\\ &= \frac{1-\delta}{\zeta} C^{ij} + \frac{\delta}{\zeta}\left(w^i - w^j\right).\end{aligned}$$

This implies:

$$\begin{aligned}(1-\delta)\log\left(\widetilde{s}^{ij}\right) &\approx (1-\delta)\log(4) + \frac{1-\delta}{\zeta} C^{ij} + \frac{\delta}{\zeta}\left(w^i - w^j\right)\\ \Rightarrow \log\left(\widetilde{s}^{ij}\right) &\approx \log(4) + \frac{1}{\zeta} C^{ij} + \frac{1}{\zeta}\frac{\delta}{1-\delta}\left(w^i - w^j\right).\end{aligned}$$

Given that the annual discount rate is δ^4 we multiply both sides by $\left(1-\delta^4\right)$:

$$\begin{aligned}
\left(1-\delta^4\right)\log\left(\tilde{s}^{ij}\right) &\approx \left[\left(1-\delta^4\right)\log(4)+\frac{\left(1-\delta^4\right)}{\zeta}C^{ij}\right]+\frac{1}{\zeta}\frac{\delta}{1-\delta}\left(1-\delta^4\right)\left(w^i-w^j\right)\\
&\approx \left[\left(1-\delta^4\right)\log(4)+\frac{\left(1-\delta^4\right)}{\zeta}C^{ij}\right]+\frac{\delta}{1-\delta}\frac{\left(1-\delta^4\right)}{\zeta}\left(w^i-w^j\right).
\end{aligned}$$

So, the coefficient on wage differentials at the yearly frequency is $\frac{\delta}{\zeta}\frac{\left(1-\delta^4\right)}{1-\delta}$ compared with $\frac{\delta}{\zeta}$ at the quarterly frequency. In turn, the ACM coefficient on wage differentials is given by: $\beta^{ACM}=\frac{\delta}{\zeta}\frac{\left(1-\delta^4\right)}{1-\delta}$. This implies that ζ – at the quarterly frequency – is $\zeta=\frac{\delta}{1-\delta}\frac{\left(1-\delta^4\right)}{\beta^{ACM}}$

In ACM, $\zeta^{Annual}=\frac{\delta^4}{\beta^{ACM}}$. As we saw above, $\zeta^{quart}=\frac{\delta}{1-\delta}\frac{\left(1-\delta^4\right)}{\beta^{ACM}}$. And so $\zeta^{quart}=\frac{\delta}{1-\delta}\frac{\left(1-\delta^4\right)}{\delta^4}\zeta^{Annual}$.

With $\delta^4=0.97$, the value used in Artuç et al. (2010), we get $\zeta^{quart}=4.05\times\zeta^{Annual}$.

E Discussion of Identification

To obtain intuition about identification of the various parameters in the model, we focus on a simplified one-sector model. To further simplify the exposition, assume we match quarterly transitions. Finally, given that our estimation procedure allows the estimation to be conducted country by country, we focus on a single country and omit the country index. Consider the following data: labor market tightness, θ^{Data}; (quarterly) persistence in unemployment, p_{UU}^{Data}; (quarterly) transition rate from employment to unemployment, p_{EU}^{Data}; coefficient of variation of wages, $\left(\sigma_w^2/\overline{w}\right)^{Data}$. We will show that the model implies a mapping from these data to the job destruction rate χ, vacancy costs $\widetilde{\kappa}$, dispersion of match-specific productivities σ and unemployment value b. In the one sector model, inter-sectoral mobility costs and sector-specific utilities are absent, so the current discussion is not relevant for the identification of this set of parameters.

In the one-sector model, quarterly transitions from employment to unemployment is given by: $\Pr\left(E \to U\right) = \chi$. Therefore, we can recover χ directly from from the data: $\chi = p_{EU}^{Data}$.

The model implies that quarterly transitions from unemployment to unemployment are given by: $\Pr(U \to U) = 1 - \theta q\left(\theta\right)\left(1 - G\left(\underline{x};\sigma\right)\right)$. Therefore, data on labor market tightness and persistence rate in unemployment pin down $\underline{x}$, conditional on σ. That is, $\underline{x} = \widetilde{f}_1\left(\theta^{Data}, p_{UU}^{Data}, \sigma\right)$.

The coefficient of variation in the model is given by $\sigma_w/\overline{w} = \widetilde{f}_3\left(\underline{x}, \sigma\right) = \widetilde{f}_3\left(\widetilde{f}_1\left(\theta^{Data}, p_{UU}^{Data}, \sigma\right), \sigma\right)$— see sections J.2.3 and J.2.4. This implies that the dispersion of shocks can be pinned down by the coefficient of variation in the data, labor market tightness and the persistence rate in unemployment:

$$\sigma = f_3\left(\theta^{Data}, p_{UU}^{Data}, \left(\frac{\sigma_w}{\overline{w}}\right)^{Data}\right).$$

Plugging this back on the equation determining $\underline{x}$, we obtain:

$$\underline{x} = \widetilde{f}_1\left(\theta^{Data}, p_{UU}^{Data}, f_3\left(\theta^{Data}, p_{UU}^{Data}, \left(\frac{\sigma_w}{\overline{w}}\right)^{Data}\right)\right) = f_1\left(\theta^{Data}, p_{UU}^{Data}, \left(\frac{\sigma_w}{\overline{w}}\right)^{Data}\right).$$

In turn, the Free Entry Condition dictates that $\widetilde{\kappa} = q\left(\theta\right)\frac{\delta(1-\beta)}{1-\delta(1-\chi)} I\left(\underline{x}\right)$. This implies that we can recover $\widetilde{\kappa}$ given data on labor market tightness, the job destruction rate and knowledge of $\underline{x}$. However, as we have shown above, there is a mapping from the data to each of these objects, leading to the following chain of equalities:

$$\widetilde{\kappa} = \widetilde{f}_2\left(\theta^{Data}, \chi, \underline{x}\right) = \widetilde{f}_2\left(\theta^{Data}, p_{EU}^{Data}, f_1\left(\theta^{Data}, p_{UU}^{Data}, \left(\frac{\sigma_w}{\overline{w}}\right)^{Data}\right)\right) = f_2\left(\theta^{Data}, p_{EU}^{Data}, p_{UU}^{Data}\right).$$

In other words, $\widetilde{\kappa}$ is pinned down given data on labor market tightness, the transition rate from employment to unemployment, and the persistence rate in unemployment.

The conclusion so far is that there is a mapping from θ^{Data}, p_{UU}^{Data}, p_{EU}^{Data}, $\left(\frac{\sigma_{w}^{2}}{\overline{w}}\right)^{Data}$ to χ, $\widetilde{\kappa}$, and σ. b is then pinned down by the following equilbrium relationship:[43]

$$\underline{x} = \frac{b}{\widetilde{\lambda}\widetilde{w}} + \theta\widetilde{\kappa}\frac{\beta}{1-\beta}.$$

This is because $\underline{x}$ is pinned down by $\theta^{Data}, p_{UU}^{Data}$, $\widetilde{\kappa}$ is pinned down by data, and we argue below that $\widetilde{\lambda}\widetilde{w}$ is also determined by data.

According to Steps 7 and 8 of the estimation algorithm (Appendix section J.1), $\widetilde{w} = \frac{\gamma Y}{\widetilde{L}} = \frac{\gamma Y}{L(1-u)g(\underline{x},\sigma)} = h\left(\underline{x},\sigma,\theta\right)$—as u is a function of θ and $\underline{x}$, and Y is fixed outside of the estimation procedure. Remember that there is a mapping from data to $\underline{x}$, $\widetilde{\kappa}$ and σ, and so there is a mapping from data to $\widetilde{w}$. In turn, $\widetilde{\lambda} = \frac{L}{E^{C}} = f\left(\widetilde{\kappa},\theta,\widetilde{w},u\right)$—see Steps 9 and 10 of the estimation algorithm—and again, there is a mapping from data to $\widetilde{\lambda}$ and $\widetilde{\lambda}\widetilde{w}$.

[43] Note that the steady-state version of the wage equation (13) and $W(\underline{x}) = U$ imply $\widetilde{\lambda}\widetilde{w}\underline{x} = (1-\delta)U - \eta$. Then, note that in a one-sector model, the present value of unemployment given by equation (B.5) becomes $(1-\delta)U = b + \theta\widetilde{\kappa}\widetilde{\lambda}\widetilde{w}\frac{\beta}{1-\beta}$.

F Parameter Estimates

In this section, we display the complete set of parameter estimates we discuss in section 3. Specifically, those highlighted in Panels B and C of Table I.

Table F.1: Final Expenditure Shares $\mu_{k,i}$

	Country					
Sector	US	China	Europe	Asia/Oc.	Americas	RoW
Agr.	0.01	0.12	0.02	0.01	0.02	0.09
LT Manuf.	0.03	0.02	0.04	0.03	0.04	0.03
MT Manuf.	0.05	0.11	0.08	0.07	0.10	0.11
HT Manuf.	0.10	0.15	0.11	0.10	0.11	0.10
LT Serv.	0.30	0.35	0.34	0.38	0.33	0.39
HT Serv.	0.51	0.25	0.41	0.41	0.40	0.29

Table F.2: Labor Shares in Production $\gamma_{k,i}$

	Country					
Sector	US	China	Europe	Asia/Oc.	Americas	RoW
Agr.	0.45	0.58	0.56	0.54	0.62	0.67
LT Manuf.	0.37	0.25	0.32	0.35	0.28	0.27
MT Manuf.	0.33	0.28	0.31	0.37	0.32	0.28
HT Manuf.	0.39	0.24	0.33	0.32	0.31	0.25
LT Serv.	0.61	0.37	0.49	0.54	0.56	0.48
HT Serv.	0.62	0.55	0.63	0.67	0.67	0.68

Table F.3: Input-Output Table – Averages Across Countries $\frac{1}{N}\sum_{i=1}^{N}\nu_{k\ell,i}$, Standard Dev. across Countries in Parentheses.

User ↓ Supplier →	Agr.	LT Manuf.	MT Manuf.	HT Manuf.	LT Serv.	HT Serv.
Agr.	0.27	0.08	0.12	0.14	0.26	0.14
	(0.05)	(0.02)	(0.03)	(0.03)	(0.05)	(0.06)
LT Manuf.	0.19	0.38	0.04	0.08	0.22	0.08
	(0.04)	(0.06)	(0.01)	(0.01)	(0.04)	(0.04)
MT Manuf.	0.22	0.07	0.29	0.11	0.22	0.09
	(0.03)	(0.02)	(0.04)	(0.02)	(0.05)	(0.04)
HT Manuf.	0.02	0.16	0.07	0.46	0.18	0.11
	(0.02)	(0.02)	(0.01)	(0.05)	(0.04)	(0.05)
LT Serv.	0.06	0.14	0.10	0.10	0.34	0.26
	(0.04)	(0.03)	(0.02)	(0.03)	(0.07)	(0.10)
HT Serv.	0.01	0.08	0.03	0.11	0.27	0.51
	(0.00)	(0.02)	(0.02)	(0.06)	(0.06)	(0.16)

Table F.4: Mobility Costs in the US – $C_{US}/(\widetilde{\lambda}_{US} \times \overline{w}_{US} \times \zeta)$

From ↓ To →	Agr.	LT Manuf.	MT Manuf.	HT Manuf.	LT Serv.	HT Serv.
Agriculture	0	3.22	3.43	2.96	2.17	3.35
LT Manufacturing	1.09	0	0.55	0.02	1.19	2.20
MT Manufacturing	1.81	0.04	0	0.22	0.52	2.13
HT Manufacturing	1.32	0.59	0.94	0	1.20	1.39
LT Services	0.00	0.67	0.64	0.40	0	0.00
HT Services	0.61	1.38	1.61	0.47	0.00	0

Notes: Estimates of mobility costs in the literature, such as Artuç et al. (2010) and Artuç and McLaren (2015), estimate C_{US} (a) at the annual frequency; (b) normalize the average wage in the US $\overline{w}_{US} = 1$; and (c) have $\widetilde{\lambda}_{US} = 1$. To be able to compare our estimates to those, we express C_{US} as a fraction of $\widetilde{\lambda}_{US} \times \overline{w}_{US} \times \zeta$.

Table F.5: Mobility Costs Around the World Relative to the US's $\frac{C_i/(\widetilde{\lambda}_i \overline{w}_i)}{C_{US}/(\widetilde{\lambda}_{US}\overline{w}_{US})} = \psi_i \times \frac{\widetilde{\lambda}_{US}\overline{w}_{US}}{\widetilde{\lambda}_i \overline{w}_i}$

	Country					
	US	China	Europe	Asia/Oc.	Americas	RoW
$\psi_i \times \frac{\widetilde{\lambda}_{US}\overline{w}_{US}}{\widetilde{\lambda}_i \overline{w}_i}$	1	1.79	0.67	0.46	1.12	0.82

Notes: Remember that we impose $C_{kk',i} = \psi_i \times C_{kk',US}$. This table reports $\frac{C_i/(\widetilde{\lambda}_i \overline{w}_i)}{C_{US}/(\widetilde{\lambda}_{US}\overline{w}_{US})} = \psi_i \times \frac{\widetilde{\lambda}_{US}\overline{w}_{US}}{\widetilde{\lambda}_i \overline{w}_i}$ so that we are better able to compare estimated mobility costs relative to the US. $\overline{w}_i$ is the average wage in country i.

Table F.6: Sector-Specific Utility $\eta_{k,i}/(\widetilde{\lambda}_i \times \overline{w}_i)$

Sector	US	China	Europe	Country Asia/Oc.	Americas	RoW
Agriculture	0	0	0	0	0	0
LT Manufacturing	0.09	-0.18	-0.26	-0.20	0.03	-0.12
MT Manufacturing	0.16	0.13	-0.05	-0.08	0.10	0.05
HT Manufacturing	0.04	-0.67	-0.50	-0.27	-0.82	-0.58
LT Services	0.26	0.27	-0.09	-0.05	0.21	-0.07
HT Services	0.08	0.31	-0.21	-0.34	-0.04	-0.24

Notes: Workers decide in what sector to search partly based on wages scaled by $\widetilde{\lambda}_i$. To aid the interpretation of the magnitude of the estimates of $\eta_{k,i}$, we express them as a fraction of $\widetilde{\lambda}_i \times \overline{w}_i$, where $\overline{w}_i$ is the average wage in country i. $\eta_{Agriculture} = 0$.

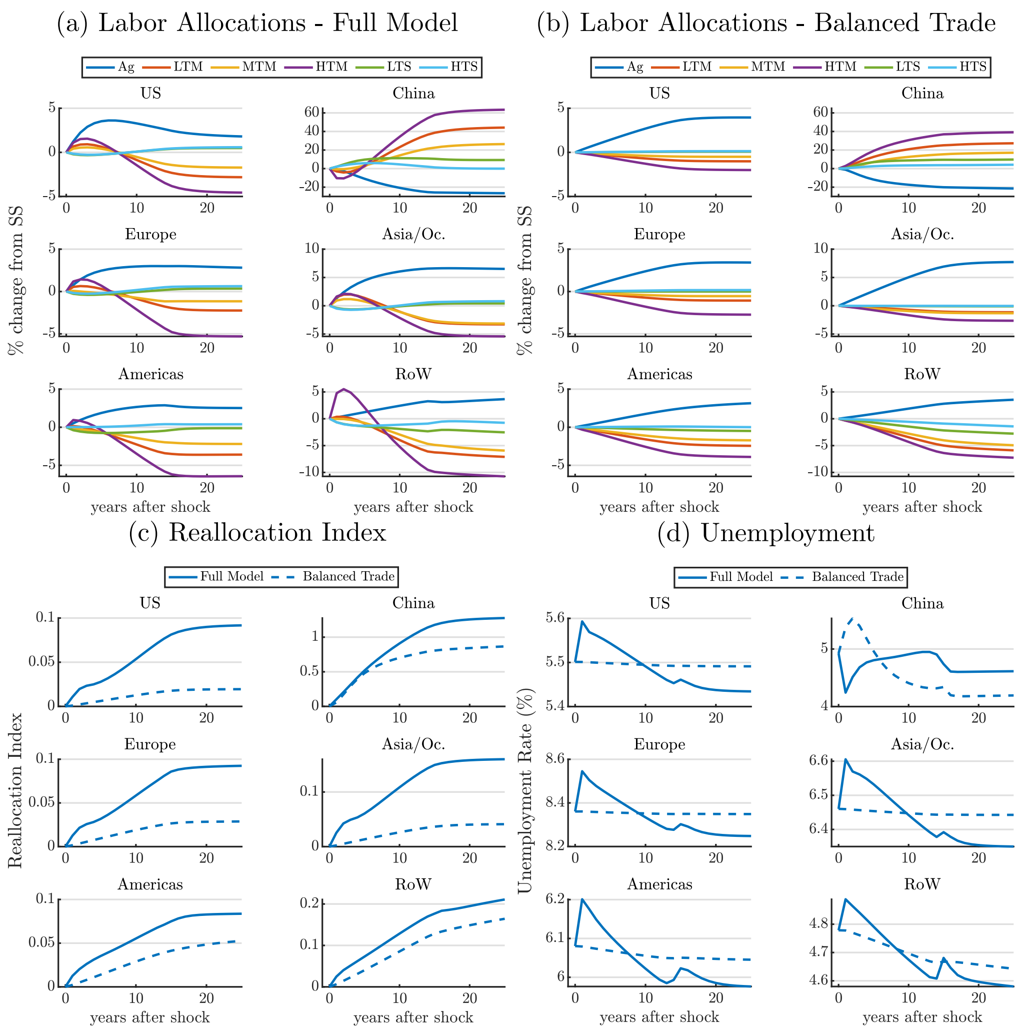
Table F.7: Exogenous Job Destruction Rates $\chi_{k,i}$

Sector	US	China	Europe	Country Asia/Oc.	Americas	RoW
Agr.	0.039	0.003	0.049	0.046	0.014	0.003
LT Manuf.	0.058	0.055	0.055	0.070	0.053	0.068
MT Manuf.	0.060	0.055	0.071	0.066	0.046	0.054
HT Manuf.	0.057	0.055	0.051	0.067	0.040	0.085
LT Serv.	0.035	0.045	0.045	0.039	0.041	0.050
HT Serv.	0.029	0.041	0.032	0.029	0.026	0.056

Table F.8: All Remaining Parameters: σ_i, b_i, and $\widetilde{\kappa}_i$

	US	China	Europe	Country Asia/Oc.	Americas	RoW
Match Prod. Dispersion σ_i	0.66	0.67	0.73	0.56	0.99	0.53
Value of Unemp. b_i	-13.45	-12.26	-12.35	-13.89	-13.56	-13.42
Vacancy Costs $\widetilde{\kappa}_i$	4.59	4.54	4.83	3.53	8.22	3.22

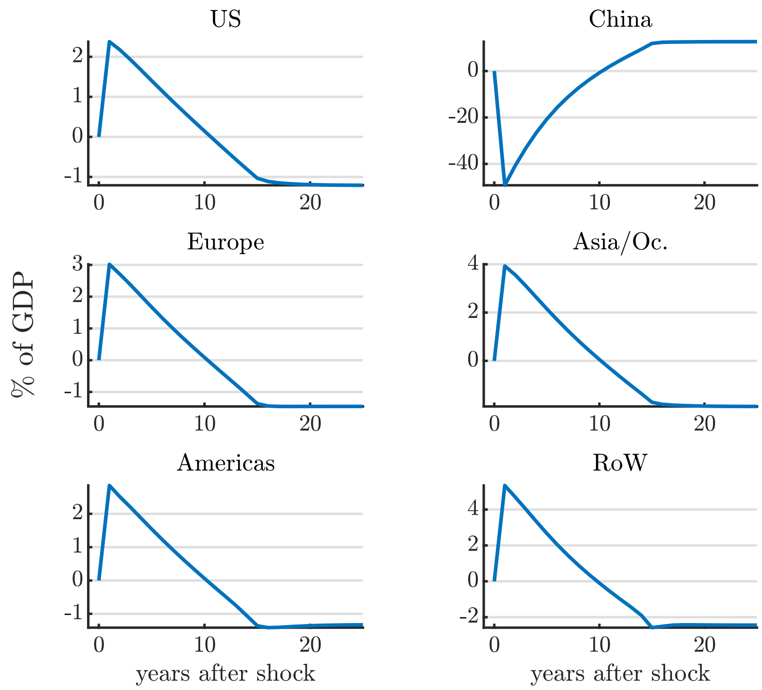
Figure G.1: Labor Market Dynamics in Response to Slow Productivity Growth in China (Figure 3a)



G Mechanisms: Details

In section 4 we conduct a counterfactual exercise where Chinese productivity slowly increases over time until it reaches a plateau—see Figure 3a. To streamline the exposition in the main text, we only showed how different variables in the US and China responded to this shock. For completeness, Figures G.1 and G.2 show versions of Figures 4 and 5 across all countries.

Figure G.2: Net Exports Over GDP in Response to Slow Productivity Growth in China (Figure 3a)



H Extracting Shocks from the Data: Details

H.1 Procedure and Results

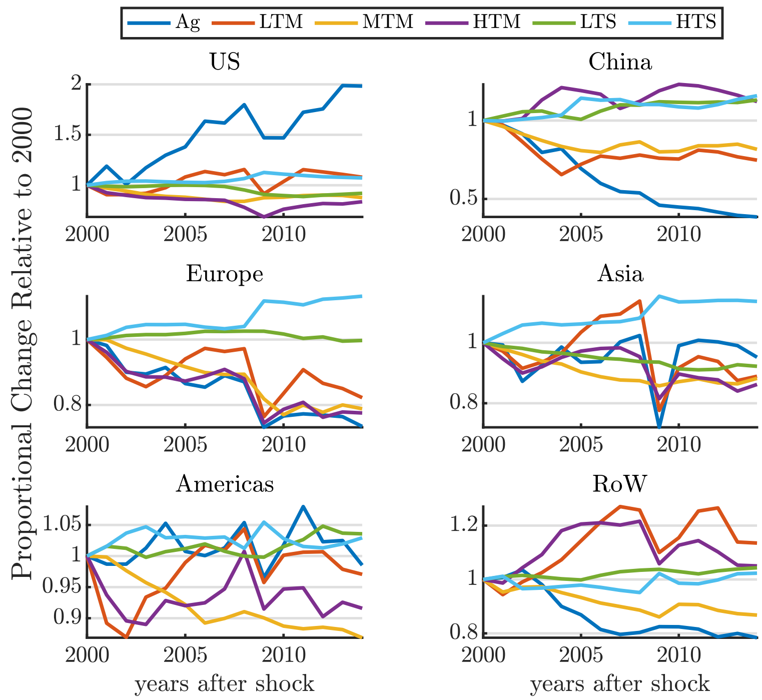
This section obtains the time series for three sets of shocks affecting the global economy between December of 2000 and December of 2014: changes in trade costs $\left\{\widehat{d}^t_{k,oi}\right\}$, productivity shocks $\left\{\widehat{A}^t_{k,i}\right\}$, and inter-temporal preference shocks $\left\{\widehat{\phi}^t_i\right\}$. We measure changes in trade costs and productivity relative to December of 2000 (which we label $t = 0$): $\widehat{d}^t_{k,oi} = \frac{d^t_{k,oi}}{d^0_{k,oi}}$, $\widehat{A}^t_{k,i} = \frac{A^t_{k,i}}{A^0_{k,i}}$. On the other hand, shocks to inter-temporal preferences are relative to the previous period: $\widehat{\phi}^{t+1}_i \equiv \frac{\phi^{t+1}_i}{\phi^t_i}$.[44]

As we recover these three sets of shocks, we also allow parameters driving preferences ($\left\{\mu^t_{k,i}\right\}$) and technology ($\left\{\gamma^t_{k,i}\right\}$ and $\left\{\nu^t_{k\ell,i}\right\}$) to evolve over time. Figures H.1, H.2 and H.3 show the evolution of these parameters over the 2000-2014 period. Importantly, we observe a widespread decline in the labor shares in the production of manufacturing goods, with the exception of High-Tech Manufacturing in the US and Europe, where it is approximately stable. Also noteworthy is the large decline in expenditure shares in Agriculture in China, and large increase in the US (although small in absolute terms, as the initial share of expenditure in Agriculture in the US is

[44] We impose $\widehat{A}^t_{k,i} = \widehat{A}^{T_{Data}}_{k,i}$ and $\widehat{d}^t_{k,oi} = \widehat{d}^{T_{Data}}_{k,oi}$ for all $t > T_{Data}$, where T_{Data} is the last period for which we have data ($T_{Data} = 4 \times 14$ quarters and refers to December 2014).

0.01).

Figure H.1: Evolution of Final Expenditure Shares $\mu_{k,i}^t$



Notes: This figure plots the evolution of $\mu_{k,i}^t/\mu_{k,i}^{2000}$ across countries and sectors.

We use WIOD data to construct time series of trade shares $\left\{\pi_{k,oi}^t\right\}$, sectoral price indices $\left\{P_{k,i}^{I,t}\right\}$, final good expenditures $\left\{E_i^{C,t}\right\}$, expenditure shares $\left\{\mu_{k,i}^t\right\}$, labor shares in production $\left\{\gamma_{k,i}^t\right\}$, and input-output matrices $\left\{\nu_{k\ell,i}^t\right\}$ between December of 2000 and December of 2014. Armed with these data, we can exploit the gravity structure of the trade block of the model, as in Head and Ries (2001) and Eaton et al. (2016), to recover the changes in bilateral trade costs combining $P_{k,i}^{I,t} = \Gamma_{k,i}\left(\Phi_{k,i}^t\right)^{-1/\lambda}$ and equation (22):

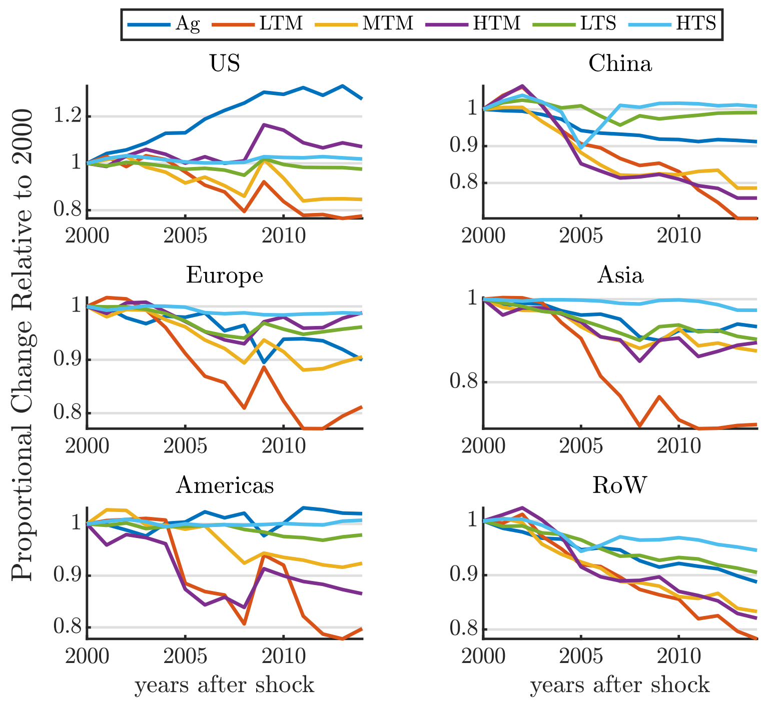
$$\widehat{d}_{k,oi}^t = \frac{\widehat{P}_{k,i}^{I,t}}{\widehat{P}_{k,o}^{I,t}}\left(\frac{\widehat{\pi}_{k,oo}^t}{\widehat{\pi}_{k,oi}^t}\right)^{1/\lambda}. \tag{H.1}$$

In turn, we rely on the Euler equation (27) and normalize $\widehat{\phi}_{US}^t = 1\ \forall t$, as in Reyes-Heroles (2016), to recover the inter-temporal preference shocks:

$$\widehat{\phi}_i^{t+1} = \frac{E_i^{C,t+1}}{E_i^{C,t}}\frac{E_{US}^{C,t}}{E_{US}^{C,t+1}} \text{ for } t = 1, ..., T_{Data} - 1, \tag{H.2}$$

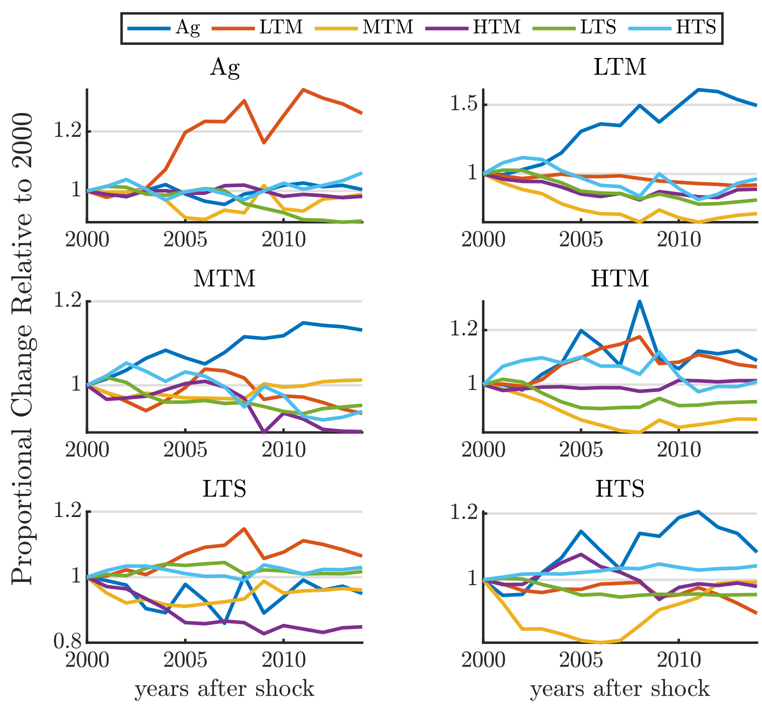
where T_{Data} is the last period for which we have data, which refers to December of 2014. Note that

Figure H.2: Evolution of Labor Shares in Production $\gamma_{k,i}^t$



Notes: This figure plots the evolution of $\gamma_{k,i}^t/\gamma_{k,i}^{2000}$ across countries and sectors.

Figure H.3: Evolution of Input-Output Tables—Averages Across Countries $\overline{\nu}_{k,\ell}^t = \frac{1}{N}\sum_{i=1}^N \nu_{k\ell,i}^t$



Notes: This figure plots the evolution of $\overline{\nu}_{k,\ell}^t/\overline{\nu}_{k,\ell}^{2000}$. Each panel corresponds to a "user" sector. Within each panel, we plot changes in the share of expenditures for a given "user" sector.

we still need to determine $\widehat{\phi}_i^t$ for $t > T_{Data}$, but we will need to use the structure of the model to do so, as this value depends on the model-implied steady-state value for final good expenditures $E_i^{C,\infty}$. Given equilibrium steady-state aggregate expenditures $E_i^{C,\infty}$, we impose that expenditures $E_i^{C,t}$ evolve linearly between $t = T_{Data}$ and $t = \widetilde{T} >> T_{Data}$ and is constant from then on, that is, $E_i^{C,t} = E_i^{C,\infty}$ for all $t > \widetilde{T}$. $\widehat{\phi}_i^t$ for $t > T_{Data}$ is then obtained using the Euler equation and this extended path for $E_i^{C,t}$. Note that this formulation sets $\widehat{\phi}_i^t = 1$ for all $t > \widetilde{T}$.

Finally, we recover the productivity shocks $\left\{\widehat{A}_{k,i}^t\right\}$ using:

$$\widehat{A}_{k,i}^t = \frac{\widehat{\pi}_{k,ii}^t}{\left(\widehat{P}_{k,i}^{I,t}\right)^\lambda} \left(\widehat{c}_{k,i}^t\right)^\lambda. \tag{H.3}$$

Given that $\widehat{c}_{k,i}^t$ depends on $\widetilde{w}_{k,i}^t$, which has no data counterpart, we need to use the full structure of the model to recover the sequence of productivity shocks. Online Appendix J.6 details the algorithm to recover the shocks $\left\{\widehat{A}_{k,i}^t\right\}$ as well as $\left\{\widehat{\phi}_i^t\right\}_{t=T_{Data}+1}^\infty$ using the full structure of the model. To be able to recover the full set of shocks the economy experienced between 2000 and 2014, we assume the economy faces no additional shocks after 2015. That is, the values for $\left\{A_{k,i}^t\right\}$ and $\left\{d_{k,oi}^t\right\}$ are imposed to be constant from 2015 onwards.[45] The same assumption is imposed on preferences and technology parameters.

Figure H.4a shows an increase in productivity all over the world. In particular, China has experienced large increases in productivity, especially in manufacturing sectors.[46] Other emerging economies—which comprise the bulk of the Americas and the Rest of the World aggregate—also experienced impressive productivity growth, while growth was more muted for advanced economies.

Turning to trade costs, we first construct a summary statistic to capture this large object. We focus on the average import cost for each country-sector pair, weighted by their initial steady state import shares:

$$\overline{d}_{k,i}^t = \sum_{o \neq i} \frac{\pi_{k,oi}^0}{1 - \pi_{k,ii}^0} \widehat{d}_{k,oi}^t. \tag{H.4}$$

Figure H.4b plots this index for each country and sector. In general, import trade costs are declining for the United States and Asia, and approximately flat in Europe (with some heterogeneity across sectors). Perhaps surprisingly, starting after the 2008 financial crisis and concurrent collapse in trade, initially falling import trade costs in China begin to revert and are actually larger by the

[45] We also assume the economy is in steady state in 2000 and fully anticipates the full set of current and future shocks in 2001.

[46] While we plot changes in the productivity location parameters $\left\{\widehat{A}_{k,i}^t\right\}$, this is not directly comparable to productivity in the classic sense of a Solow Residual. In order to make sense of the magnitudes, note that TFP growth, defined as $\widehat{c}_{k,i}^t / \widehat{P}_{k,i}^{I,t}$, can be expressed as $(\widehat{A}_{k,i}^t / \widehat{\pi}_{k,ii}^t)^{1/\lambda}$. Therefore, using our recovered values for $\widehat{A}_{k,i}^t$, data on changes in trade shares, and imposing $\lambda = 4$, the magnitude for actual annualized TFP growth in China ranges from 3 to 5% per year, depending on the sector—which is in line with growth accounting estimates discussed in Zhu (2012).

end of the sample. This estimate of changes in trade costs reflects the fall in the share of trade in output, as documented in Bems et al. (2013). The sources for these increasing frictions are myriad, and include policy changes in countries like China, as well as changes in supply chain management, and other reasons. That said, our measures of frictions are a standard, straightforward, measure of the implied barriers to trade.

Finally, we turn to our measure of shocks to inter-temporal preferences, which are presented in Figure H.5. The shocks in the US are normalized to 1 in every period. In Europe and Asia (except China), the discount factor shocks fluctuate around 1, suggesting little persistent deviations in consumption behavior from what would be expected with a simple consumption smoothing model. On the other hand, China, the Americas, and the aggregated remaining countries (Rest of the World) exhibit persistent shocks to their inter-temporal preferences, suggesting increased patience over the period we consider. These persistent deviations are often referred to as the "global savings glut."[47] It is important to recognize that there are rich dynamics to consumption in the real world, reflecting preferences, frictions, and other factors. We are agnostic on the exact theory, instead summarizing the effect of these channels with the $\widehat{\phi}_i^t$ shocks. This is useful because it allows us to ask counterfactual questions about the dynamics of globalization shocks without the global savings glut, without having to specify what policy or change in deep parameters to achieve this—a useful benchmark to compare against the usual assumption in trade of no consumption smoothing whatsoever.

H.2 Comparison Between Model and Data

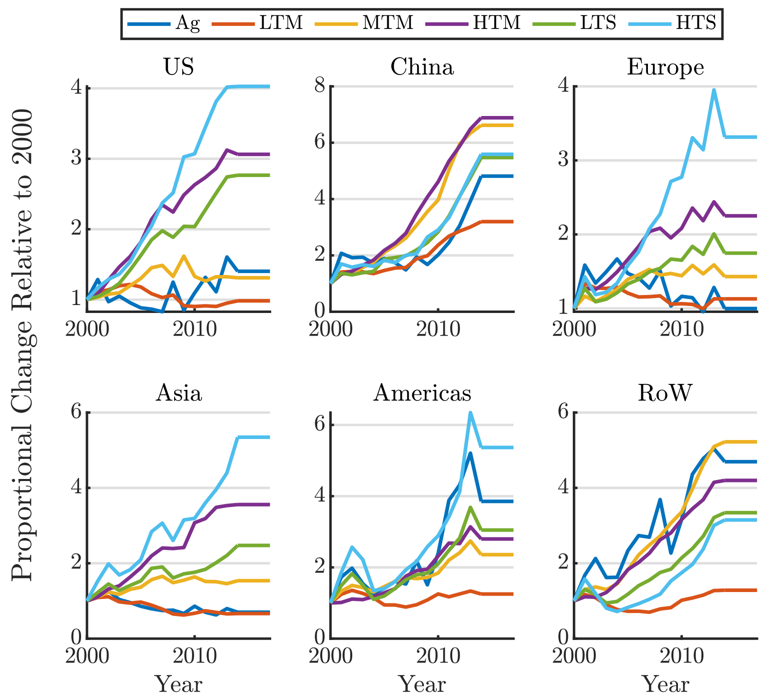
We compare the evolution of trade imbalances, labor allocation and gross output once we feed the model with the shocks recovered in section H.1 to the observed evolution of these variables. Figures H.6, H.7, and H.8 focus this comparison to the US and China.

First, Figure H.6 shows that the resulting evolution of net exports in the US and China closely mimics those observed in the data. The main distinction is that our model predicts larger surpluses for China closer to the end of the period. Second, our model very closely replicates the evolution of gross output in the US. Finally, our model generates time paths for labor allocations that replicate the main features observed in the data. In particular, our model predicts a large decline in manufacturing employment (although not as large as the one observed in the data), and a relatively large increase in employmen in High-Tech Services. Perhaps more visible in the figure is the large increase in Agriculture employment. However, note that this increase is large in proportional terms, but small in absolute terms as the US starts with 2.5% of the labor force in Agriculture.

[47] The large trade surplus that China has been running since the early 2000s is a puzzle for models in which the main driving forces are productivity shocks. For instance, as argued by Song et al. (2011), financial frictions within China are key drivers of the Chinese savings glut. Our inter-temporal preference shocks constitute a reduced-form way to allow the model to match the time series behavior of Chinese aggregate expenditures and the rest of the world.

Figure H.4: Extracted Globalization Shocks

(a) Productivity Shocks $\widehat{A}_{k,i}^{t}$



(b) Trade-Weighted Import Costs $\overline{d}_{k,i}^{t}$ (See Equation (H.4))

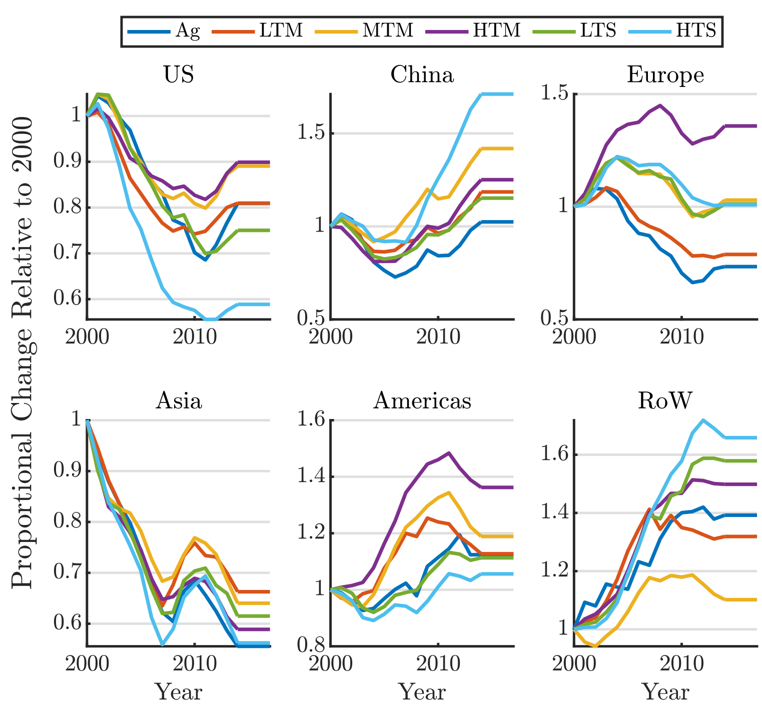
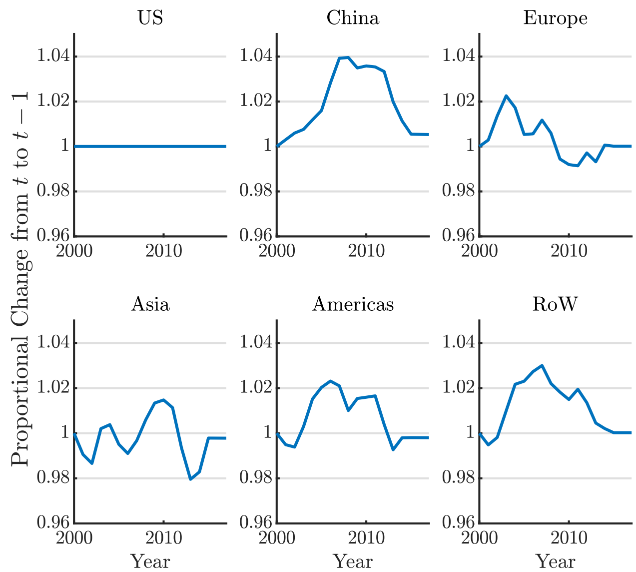


Figure H.5: Extracted Inter-Temporal Preference Shocks $\widehat{\phi}_i^t$



Our model replicates the evolution of gross output over the period we consider. Therefore, discrepancies between observed labor allocations and those implied by the model must be explained by how we back out changes in productivities $\widehat{A}_{k,i}^t$ in equation (H.3). For example, the large predicted increase in US Agriculture employment reflect (a) its increase in gross output value relative to the rest of the world shown in Figure H.7, but (b) a concurrent "somewhat slow" predicted productivity growth $\widehat{A}_{Agriculture,US}^t$ shown in Figure H.4a. Our model predicts slow productivity growth in Agriculture because *observed* US agricultural prices $\widehat{P}_{Agriculture,US}^{I,t}$ don't decline as much as they would need to for our model to match the evolution of labor in Agriculture.

Figure H.6: Evolution of Net Exports in the US and China over 2000-2014: Data and Model

(a) Data (b) Model

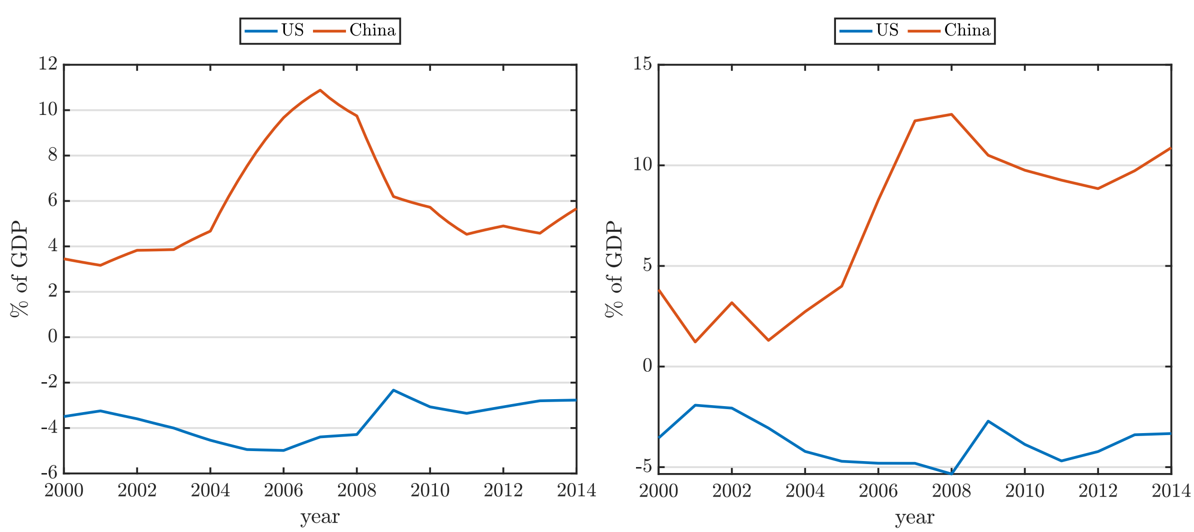
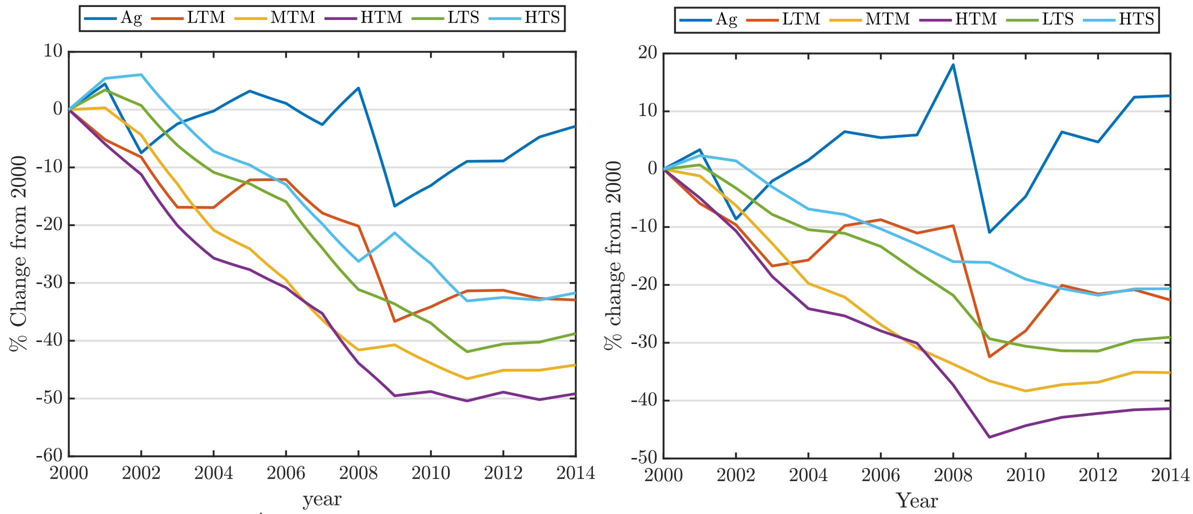


Figure H.7: Evolution of Gross Output in the US over 2000-2014

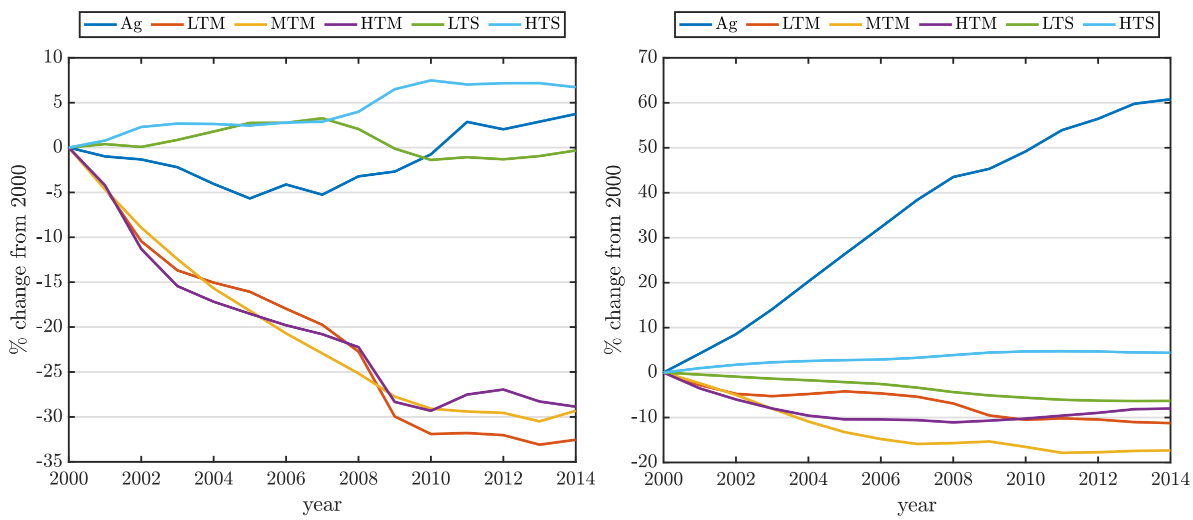
(a) Data (b) Model



Notes: We impose $\sum_i \sum_k Y_{k,i}^t = 1$ in every period.

Figure H.8: Evolution of Labor Allocations in the US over 2000-2014

(a) Data (b) Model



I Alternative Labor Market Structures

This appendix studies the behavior of our model with trade imbalances under alternative labor market structures. First, we study the implications of our model after shutting down inter-sectoral mobility costs. Next, we study our model with mobility costs but without search frictions.

I.1 No Mobility Costs

In the model without mobility costs, we set both costs $C_{kk',i}$ and idiosyncratic shocks $\boldsymbol{\omega}^t$ to zero. We re-estimate the model with these restrictions and target the same moments displayed in Figure 2, with the exception of transitions rates across sectors.[48]

Table I.1: Sector-Specific Utility $\eta_{k,i}/(\widetilde{\lambda}_i \times \overline{w}_i)$
Model w/o Mobility Costs

	Country					
Sector	US	China	Europe	Asia/Oc.	Americas	RoW
Agriculture	0	0	0	0	0	0
LT Manufacturing	-0.19	-0.27	-0.37	-0.07	-0.34	-0.00
MT Manufacturing	0.00	-0.06	-0.34	0.05	-0.36	-0.24
HT Manufacturing	-0.43	-0.73	-0.23	-0.05	-0.33	-0.31
LT Services	0.04	0.00	-0.13	-0.32	-0.28	-0.69
HT Services	0.15	-0.39	0.03	-0.41	0.08	-0.88

Notes: Workers decide in what sector to search partly based on wages scaled by $\widetilde{\lambda}_i$. To aid the interpretation of the magnitude of the estimates of $\eta_{k,i}$, we express them as a fraction of $\widetilde{\lambda}_i \times \overline{w}_i$, where $\overline{w}_i$ is the average wage in country i. $\eta_{Agriculture} = 0$.

Tables I.1 to I.3 contain the results of the estimation under this assumption. A few patterns emerge. Vacancy costs are higher, and job destruction rates are generally lower than in the baseline model. For example, the average log difference in $\chi_{k,i}$ between our full model and that without mobility costs is 1.1 log points. A comparison between Tables I.1 and F.6 shows significant differences in the sector-specific utilities $\eta_{k,i}$ (normalized by $\widetilde{\lambda}_i \overline{w}_i$). In particular, we observe that these tend to be "more negative" in the model without mobility costs. Since we normalize $\eta_{k,i}$ in Agriculture to be equal to zero in each country, this means that the relative non-pecuniary benefits in Agriculture are now larger than those estimated in the full model. Additionally, the dispersion in $\eta_{k,i}/(\widetilde{\lambda}_i \times \overline{w}_i)$ tends to increase within countries. For example, in the US, the gap between the largest and smallest value of $\eta_{k,i}/(\widetilde{\lambda}_i \times \overline{w}_i)$ is 0.26 in our full model, but it is 0.58—more than twice

[48] In practice, we use the same estimation and solution algorithms as the full model, but we set mobility costs C and the dispersion of idiosyncratic shocks ζ to be both very small, with C/ζ being reduced by a factor of 100 relative to estimates of the full model.

Table I.2: Exogenous Job Destruction Rates $\chi_{k,i}$
Model w/o Mobility Costs

	Country					
Sector	US	China	Europe	Asia/Oc.	Americas	RoW
Agr.	0.005	0.003	0.003	0.003	0.003	0.003
LT Manuf.	0.006	0.041	0.015	0.042	0.021	0.062
MT Manuf.	0.014	0.041	0.003	0.038	0.003	0.024
HT Manuf.	0.003	0.041	0.055	0.045	0.051	0.074
LT Serv.	0.009	0.034	0.040	0.006	0.009	0.006
HT Serv.	0.022	0.003	0.058	0.017	0.046	0.013

Table I.3: All Remaining Parameters: σ_i, b_i, and $\widetilde{\kappa}_i$
Model w/o Mobility Costs

	Country					
	US	China	Europe	Asia/Oc.	Americas	RoW
Match Prod. Dispersion σ_i	0.72	0.72	0.71	0.71	1.06	0.75
Value of Unemp. b_i	-7.46	-6.75	-4.30	-4.66	-5.81	-3.89
Vacancy Costs $\widetilde{\kappa}_i$	10.03	6.69	4.75	6.51	9.49	5.48

as large—in the model without mobility costs.

To gain some intuition for why this is the case, note that the absence of switching costs equalizes the value of being unemployed ($U_{k,i}$) across sectors. Since we constrain b_i to be the same across sectors within a country, this amounts to equalizing the net present value of an employment relationship. In the presence of mobility costs, this is not the case: different sectors have different values of unemployment, $U^t_{k,i}$.[49] These differences in the continuation value of a sector can help rationalize wages and labor shares by offering an additional reason some sectors may be more or less valuable than others. With these differences shut down, the $\eta_{k,i}$ terms must play a more prominent role in fitting the data. In our data, the size of agriculture is too large to be rationalized by wages alone, and so the non-pecuniary value of this sector must rise to compensate. In general, the net present value of employment in a sector depends on the the direct payoff (which depends on wages and $\eta_{k,i}$), the probability of a match (which is determined by $\theta_{k,i}$ and σ_i), the variance in match quality (which depends on σ_i), and the exit rate from a match ($\chi_{k,i}$). The exact way in which the moments trade off these parameters after setting $C_{kk',i}$ and ζ_i to zero is difficult to pin down precisely.

Turning to results, Figure I.1 shows unemployment and consumption dynamics across different versions of our model in response to the slow Chinese productivity growth shock of section 4. The blue line depicts the evolution of different outcomes for the baseline model, while the red line plots these outcomes in the absence of mobility frictions. All outcomes are relative to their initial steady-state values. There are clear differences in both cases. First, ignoring mobility costs leads to much larger spikes in unemployment in many (but not all) countries. For example, after impact, US unemployment rises by just under 2% in the baseline model, but over 3% with no mobility costs. More starkly, our model predicts a near 15% decline in the unemployment rate in China. On the other hand, the model without mobility costs predicts an increase of over 20% in Chinese unemployment. These deviations also persist for many countries—up to 25 years in many cases.

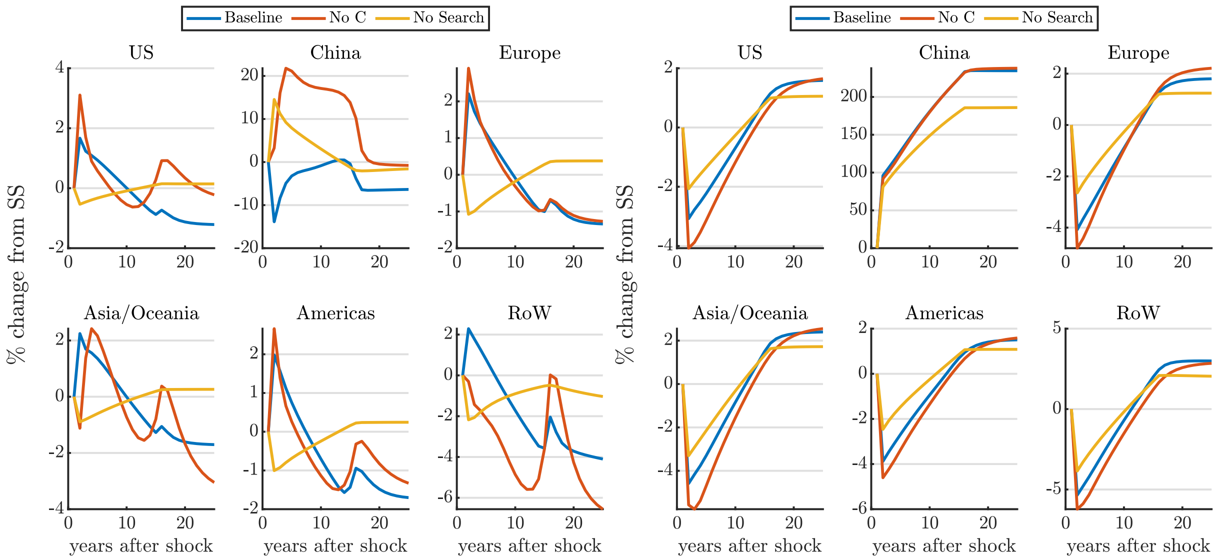
Consumption dynamics follow each other more closely, but with some key differences. The red line, which follows the model without mobility costs, tends to be lower in every country except for China—where the shock occurred—in the short run. In the long run, the lines catch up and begin to reverse their order in the very last period (earlier for Europe). This suggests that without mobility costs, workers face more consumption volatility, at least for the shock under consideration.

[49] In the absence of search frictions, there is a very tight link between moving costs, transition rates, and the option value of being in a sector (see Artuç et al. (2010) for a complete discussion).

Figure I.1: Comparing Labor Market Structures: Responses to Slow Productivity Growth in China (See Shock in Figure 3a)

(a) Unemployment

(b) Consumption

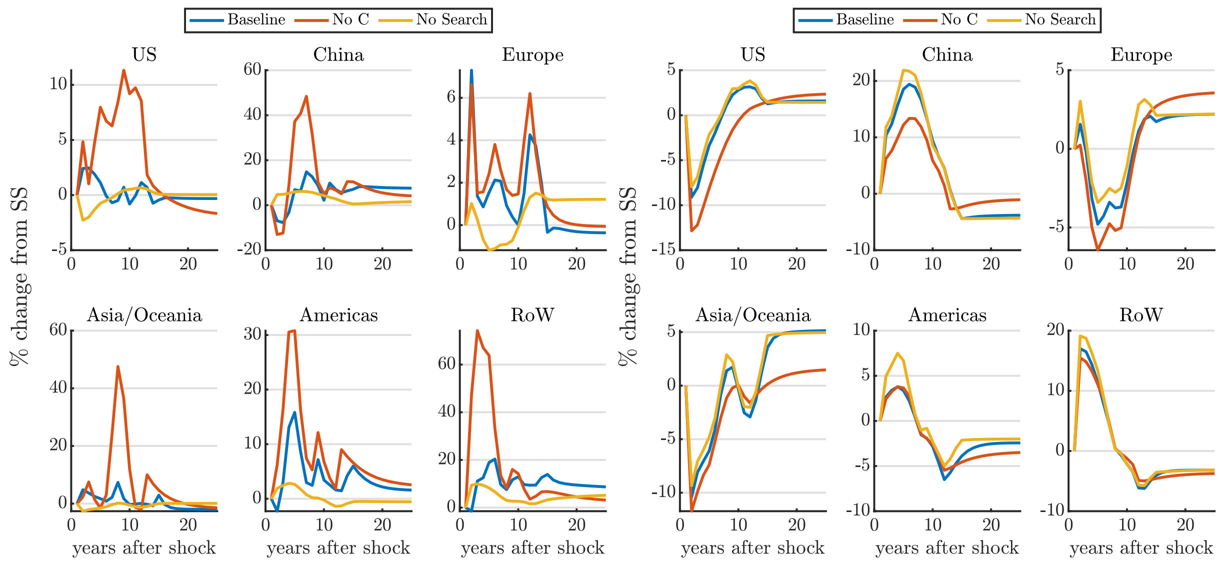


Notes: The blue line, “Baseline,” plots outcomes for the Baseline Model, estimated in the main text. The red line, “No C,” plots outcomes for the modelre-estimated with no mobility costs. The yellow line, “No Search,” plots outcomes for the model re-estimated without search and matching frictions. All outcomes are relative to their initial steady-state values.

These differences are not specific to the slow moving Chinese shock that we considered in section 4. Figure I.2 shows similar patterns of unemployment spikes when we consider the trade costs shocks analyzed in Section 5.3.1 (see Figure H.4b). In this example, the spikes are very large—as much as a 50% increase in the unemployment rate in China, and a nearly 80% increase in the Rest of the World. These numbers are up to four times larger than in the baseline model. Consumption patterns are more similar in magnitude. However, there are still substantial deviations between our baseline model and the model without mobility costs—for example, the consumption spike in China is 20% in the baseline model, but only around 12% in the model without mobility costs.

Figure I.2: Comparing Labor Market Structures:
Responses to Extracted Trade Costs (See section H)

(a) Unemployment (b) Consumption



Notes: The blue line, "Baseline," plots outcomes for the Baseline Model, estimated in the main text. The red line, "No C," plots outcomes for the modelre-estimated with no mobility costs. The yellow line, "No Search," plots outcomes for the model re-estimated without search and matching frictions. All outcomes are relative to their initial steady-state values.

Our two exercises suggest that the unemployment response to shocks is larger in the absence of intersectoral mobility costs. Moreover, consumption is more volatile, albeit the consumption differences are much smaller than the unemployment differences. This suggests that adding mobility costs on top of search frictions acts to temper large swings in job creation and destruction.

I.2 No Search and Matching

In this version of the model, we adopt a setup closer to Artuç and McLaren (2015), but with the addition of a non-employment sector as in Dix-Carneiro (2014) or Traiberman (2019). There are now $k = 0, 1, ..., K$ sectors, where $i = 0$ corresponds to non-employment. In this case, we have a single Bellman equation:

$$\widetilde{W}^t_{k,i}(\omega^t) = \widetilde{\lambda}^t_i w^t_{k,i} + \eta_{k,i} + \max_{k'} \left\{ \delta \widehat{\phi}^{t+1}_i E_\omega \left[\widetilde{W}^{t+1}_{k,i}(\omega^{t+1}) \right] - C_{kk',i} + \omega^t_{k'} \right\}, \tag{I.1}$$

with the convention that $w^t_{0,i} = 0 \ \forall i, t$, and mobility costs to unemployment are set to 0. This leads to the following Bellman equation for the integrated value function:

$$W^t_{k,i} = \widetilde{\lambda}^t_i w^t_{k,i} + \eta_{k,i} + \zeta_i \log \left(\sum_{k'} \exp \left(\frac{\delta \widehat{\phi}^{t+1}_i W^{t+1}_{k,i} - C_{kk',i}}{\zeta_i} \right) \right), \tag{I.2}$$

where $W_{k,i}^t \equiv E_\omega \left[\widetilde{W}_{k,i}^t(\omega^t)\right]$. The solution to equation (I.1) yields a similar multinomial logit expression for transition rates, $s_{kk',i}^{t,t+1}$ as in the main model. The difference is that this transition matrix now applies to *all* workers, not just to those who are unemployed. The allocation of workers across sectors evolves according to:

$$L_{k,i}^{t+1} = \sum_{\ell=0}^{K} L_{\ell,i}^t s_{\ell k,i}^{t,t+1}. \tag{I.3}$$

In this setup, workers are both ex-ante and ex-post homogenous. Firms do not post vacancies and there is no match-specific productivity. Instead, perfectly competitive firms can produce varieties as in Eaton and Kortum (2002) and Caliendo and Parro (2015), using a Cobb-Douglas aggregate of labor and intermediate inputs. The expressions characterizing trade and goods markets are the same as in section 2.6 except that $\widetilde{w}_{k,i}^t$, the sectoral surplus, is replaced with $w_{k,i}^t$, which is the sector-specific wage. Value-added in sector k in country i is thus given by:

$$\gamma_{k,i} Y_{k,i}^t = w_{k,i}^t L_{k,i}^t. \tag{I.4}$$

With these modifications, there are no longer $\underline{x}_{k,i}^t$ or $\theta_{k,i}^t$ terms in the model. The solution algorithm is largely the same as before, with the steps for calculating $\underline{x}_{k,i}^t$ or $\theta_{k,i}^t$ removed. Details of the modified algorithms are available upon request.

Table I.4: Mobility Costs in the US – $C_{US}/(\widetilde{\lambda}_{US} \times \overline{w}_{US} \times \zeta)$
Model w/o Search Frictions

From ↓ To →	Agr.	LT Manuf.	MT Manuf.	HT Manuf.	LT Serv.	HT Serv.
Unemployment	7.12	6.49	7.13	5.11	4.63	3.76
Agriculture	0	7.27	6.79	6.33	4.55	9.15
LT Manufacturing	6.78	0	5.02	4.02	4.26	5.50
MT Manufacturing	7.97	5.48	0	4.71	4.27	8.85
HT Manufacturing	8.36	6.47	6.26	0	5.52	9.31
LT Services	7.75	7.56	6.72	6.57	0	5.11
HT Services	9.18	22.93	7.90	7.36	5.00	0

Notes: Estimates of mobility costs in the literature, such as Artuç et al. (2010) and Artuç and McLaren (2015), estimate C_{US} (a) at the annual frequency; (b) normalize the average wage in the US $\overline{w}_{US} = 1$; and (c) have $\widetilde{\lambda}_{US} = 1$. To be able to compare our estimates to those, we express C_{US} as a fraction of $\widetilde{\lambda}_{US} \times \overline{w}_{US} \times \zeta$. Mobility costs to unemployment are set to 0.

In order to estimate this version of the model, we target the same moments depicted in Figure 2, except for wage dispersion and labor market tightness. Tables I.4 to I.6 contain the estimates from this version of the model. Without the addition of search and matching frictions, the model requires

Table I.5: Mobility Costs Around the World Relative to the US's
$\frac{C_i/(\widetilde{\lambda}_i \overline{w}_i)}{C_{US}/(\widetilde{\lambda}_{US} \overline{w}_{US})} = \psi_i \times \frac{\widetilde{\lambda}_{US} \overline{w}_{US}}{\widetilde{\lambda}_i \overline{w}_i}$
Model w/o Search Frictions

			Country			
	US	China	Europe	Asia/Oc.	Americas	RoW
$\psi_i \times \frac{\widetilde{\lambda}_{US} \overline{w}_{US}}{\widetilde{\lambda}_i \overline{w}_i}$	1	1.07	0.99	0.91	1.12	1.15

Notes: Remember that we impose $C_{kk',i} = \psi_i \times C_{kk',US}$. This table reports $\frac{C_i/(\widetilde{\lambda}_i \overline{w}_i)}{C_{US}/(\widetilde{\lambda}_{US} \overline{w}_{US})} = \psi_i \times \frac{\widetilde{\lambda}_{US} \overline{w}_{US}}{\widetilde{\lambda}_i \overline{w}_i}$ so that we are better able to compare estimated mobility costs relative to the US. $\overline{w}_i$ is the average wage in country i.

Table I.6: Sector-Specific Utility $\eta_{k,i}/(\widetilde{\lambda}_i \times \overline{w}_i)$
Model w/o Search Frictions

			Country			
Sector	US	China	Europe	Asia/Oc.	Americas	RoW
Unemployment	-5.89	-4.46	-3.14	-6.25	-3.39	-3.32
Agriculture	0	0	0	0	0	0
LT Manufacturing	-0.29	-1.42	-0.60	-0.79	-0.92	-1.05
MT Manufacturing	-0.28	-1.00	-0.61	-0.54	-0.66	-0.68
HT Manufacturing	-0.61	-2.36	-1.16	-1.04	-1.95	-2.49
LT Services	-0.03	-0.79	-0.55	-0.44	-0.67	-1.05
HT Services	-0.26	-0.73	-0.73	-0.73	-0.82	-1.46

Notes: Workers decide in what sector to search partly based on wages scaled by $\widetilde{\lambda}_i$. To aid the interpretation of the magnitude of the estimates of $\eta_{k,i}$, we express them as a fraction of $\widetilde{\lambda}_i \times \overline{w}_i$, where $\overline{w}_i$ is the average wage in country i. $\eta_{Agriculture} = 0$.

much larger mobility costs to rationalize the transition matrix. The median log difference across the off-diagonal elements of Tables I.4 and F.4 (excluding mobility costs from unemployment) is 2.00—a nearly 7.5 fold increase in mobility costs. The values of ψ_i are compressed closer to 1, suggesting such large costs are required of most countries in the world. The value of the unemployment sector is very negative, similar to the value of b_i in the full model. Finally, the $\eta_{k,i}$ parameters for the production sectors tend to have become negative. The model needs to make agriculture more attractive to workers to be able to simultaneously match its relatively large size and relatively low wage. As in the model without mobility costs, the spread in η's also increases relative to the baseline. For example, in the US, the gap between the largest and smallest of these terms (excluding unemployment and normalized by the mean wage times the Lagrange multiplier), is 0.61, relative to 0.26 in baseline. This suggests that heterogeneity in labor market tightness (which is absent in this version of the model) and match productivity are important ingredients to explain the allocation of labor across sectors that cannot be explained only by mean wage differences.

Turning to the simulation results, Figure I.1 plots unemployment and consumption paths in response to slow productivity growth in China for the model with No Search, represented by the yellow line, and for the Baseline model, which is depicted in blue. Again, outcomes are shown relative to their initial steady-state values. In this case, there are stark differences in unemployment dynamics. First, unemployment changes are usually smaller in magnitude in the No Search model compared to our Baseline model. Second, without search, unemployment tends to go in the opposite direction of its behavior in the Baseline model. To understand the pattern of unemployment in the No Search scenario, consider the log relative probability of entering unemployment versus staying in one's sector:

$$\log\left(\frac{s^{t,t+1}_{k0,i}}{s^{t,t+1}_{kk,i}}\right) = \delta\left(W^{t+1}_{0,i} - W^{t+1}_{k,i}\right)/\zeta_i. \tag{I.5}$$

To analyze this term, it is easiest to expand $\delta\left(W^{t+1}_{0,i} - W^{t+1}_{k,i}\right)/\zeta_i$ using equation (I.2) and focus on the $-\widetilde{\lambda}^{t+1}_i w^{t+1}_{k,i}/\zeta_i$ component, ignoring continuation values for intuition's sake (remember that $w^{t+1}_{0,i} \equiv 0$).

Remember our discussion at the end of section 2.7: with log utility of consumption, country-specific final goods expenditures (when expressed as a share of world total expenditures) are equalized over time. Consequently, as soon as China and the world realize that China is gradually becoming more productive and richer, China immediately starts to command a larger share of global final goods expenditures. If we normalize $\sum_{i=1}^{N} E^{C,t}_i = 1$, this implies that $E^{C,t}_i$ increases in China and decreases in all of the remaining countries. In turn, $\widetilde{\lambda}^t_i$ immediately and permanently increases all over the world, but declines in China (recall that $\widetilde{\lambda}^t_i = u'(c_t)/P^{F,t}_i = \overline{L}_i/E^{C,t}_i$).

To understand the behavior of wages, we first note that in the model without search frictions, country-specific GDP is given by the sum of final goods expenditures, E^C_i, and net exports, NX_i.

Both of these terms go up in China in the long run (consumption smoothing in China dictates that it will run a trade deficit in the short run and a trade surplus in the long run). Because net exports sum to zero in the global economy, global GDP must sum to one. Given this normalization, China commands a larger share of GDP in the long run, and therefore wages go up in China and down in the rest of the world. However, because of the slow moving shock and the reinforcing behavior of trade imbalances, wages around the world evolve gradually toward this new steady state. In the short run, the increase in $\widetilde{\lambda}_i^t$ dominates outside of China, reducing unemployment. However, in the long run, declines in wages offset the increase in the Lagrange multiplier, and unemployment increases outside of China.

We now aim to understand why unemployment changes are usually smaller in magnitude in the No Search model compared to our Baseline model. The intuition for this result can also be obtained by inspecting equation (I.5). Simplifying to a steady-state version for one country (so dropping the t superscript and i subscript), and again treating any feedback into continuation value differences as second order for the purpose of illustration, the elasticity of s_{k0}/s_{kk} with respect to $\widetilde{\lambda}w_k$ is given by:

$$\frac{\partial \log\left(s_{k0}/s_{kk}\right)}{\partial \log\left(\widetilde{\lambda}w_k\right)} = -\delta\widetilde{\lambda}w_k/\zeta. \tag{I.6}$$

In words, changes in transitions to unemployment are mainly determined by $\widetilde{\lambda}w_k$, which in practice is a small number. More precisely, its value averaged across countries, sectors, and time, is 0.16. To ballpark this number, it suggests that a 10% change in the value of $\widetilde{\lambda}w_k$, would change flows into unemployment by 1.6%. Since flows to unemployment are small in the first place ($\approx 2\%$ in the US), we conclude that typical globalization shocks are unlikely to yield to a large increases in unemployment. In contrast, our baseline model, which has within-sector job creation and destruction, allows for richer unemployment dynamics which are not solely tied to real wages.

Consumption dynamics are once again relatively similar across models. In the slow moving shock, there seems to be a systematic smaller consumption response in countries besides China. However, in the trade costs shock exercise, there is very little difference.

We draw two conclusions from having estimated and simulated the model without search frictions. First, search frictions play an important role in explaining the mobility costs across sectors and the non-pecuniary value of sectors. Our evidence for this is the increase in the magnitudes of the mobility cost and sector-specific utility terms in this version of the model relative to the baseline estimation. Second, search frictions lead to very different predictions for the responses of unemployment to shocks. Unlike the removal of mobility costs, which seemed mostly to amplify the unemployment response, the removal of search and matching frictions changes the direction and dampens the unemployment response to trade shocks. The sign differences stem from the fact that absent job creation and destruction, changes in the real wage alone largely dictate the unemployment response. The dampening is because without job creation and destruction, the magnitude of the unemployment response is, to a first order, governed by $\widetilde{\lambda}_i w_{k,i}/\zeta_i$—a value that is empirically small.

J Solution Methods

This Section presents the different algorithms we developed to estimate the model and to perform counterfactual simulations. Section J.1 details the estimation algorithm and section J.2 obtains expressions for simulated moments. Section J.3 outlines an exact hat algebra algorithm to compute changes in the steady state equilibrium in response to shocks in trade costs, productivities or net exports. Section J.4 develops the algorithm solving for the transition path of our complete model with trade imbalances. Section J.5 adapts this algorithm to the case where we have exogenous deficits. Finally, section J.6 outlines the procedure we use in section 5.1 to extract the shocks in trade costs, productivities and inter-temporal shocks.

J.1 Estimation Algorithm

Define $I_{k,i}(x) \equiv \int_x^\infty (s-x)\, dG_{k,i}(s)$. Imposing $G_{k,i} \sim \log \mathcal{N}\left(0, \sigma_{k,i}^2\right)$ and a bit of algebra leads to:

- $G_{k,i}(x) = \Phi\left(\frac{\ln x}{\sigma_{k,i}}\right)$
- $I_{k,i}(x) = \exp\left(\frac{\sigma_{k,i}^2}{2}\right)\Phi\left(\sigma_{k,i} - \frac{\ln x}{\sigma_{k,i}}\right) - x\Phi\left(-\frac{\ln x}{\sigma_{k,i}}\right)$
- $I_{k,i}(0) = \exp\left(\frac{\sigma_{k,i}^2}{2}\right)$
- $\int_{\underline{x}_{k,i}}^\infty \frac{s}{1-G_{k,i}\left(\underline{x}_{k,i}\right)} dG_{k,i}(s) = \exp\left(\frac{\sigma_{k,i}^2}{2}\right)\frac{\Phi\left(\sigma_{k,i} - \frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}$

Note: The estimation procedure we describe takes trade shares $\pi_{k,oi}^{Data}$ and net exports NX_i^{Data} as given.

Step 1: Solve for $\{Y_{k,i}\}$ using:

$$Y_{k,o} = \sum_{i=1}^N \sum_{\ell=1}^K \pi_{k,oi}^{Data} \left(\mu_{k,i}\gamma_{\ell,i} + (1-\gamma_{\ell,i})\,\nu_{\ell k,i}\right) Y_{\ell,i} - \sum_{i=1}^N \pi_{k,oi}^{Data} \mu_{k,i} NX_i$$

$$\sum_{o=1}^N \sum_{k=1}^K Y_{k,o} = 1$$

The rest of the procedure conditions on these values of $\{Y_{k,i}\}$.

Step 2: Guess model parameters Ω. We treat $\widetilde{\kappa}_{k,i} \equiv \frac{\kappa_{k,i} P_i^F}{\widetilde{w}_{k,i}}$ as parameters to be estimated.

Step 3: Define

$$\varpi_{k,i} \equiv \frac{\left(1-\left(1-\chi_{k,i}\right)\delta\right)\widetilde{\kappa}_{k,i}}{\delta\left(1-\beta_{k,i}\right)}$$

If $\frac{\left(1-\left(1-\chi_{k,i}\right)\delta\right)\widetilde{\kappa}_{k,i}}{\delta\left(1-\beta_{k,i}\right)I_{k,i}(0)} = \frac{\varpi_{k,i}}{I_{k,i}(0)} \geq 1$, the free entry condition cannot be satisfied—$I_{k,i}$ is decreasing. Abort the procedure and highly penalize the objective function.

Step 4: Find $\underline{x}_{k,i}^{ub}$ such that $\frac{\left(1-\left(1-\chi_{k,i}\right)\delta\right)\widetilde{\kappa}_{k,i}}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}_{k,i}^{ub}\right)} = 1 \iff I_{k,i}\left(\underline{x}_{k,i}^{ub}\right) = \varpi_{k,i}$. If along the algorithm $\underline{x}_{k,i}$ goes above $\underline{x}_{k,i}^{ub}$, we update it to be equal to $\underline{x}_{k,i}^{ub}$ (minus a small number).

Step 5: Guess $\{L_{k,i}\}$, and $\left\{\underline{x}_{k,i}\right\}$

Step 6: Compute $I_{k,i}\left(\underline{x}_{k,i}\right)$, $G_{k,i}\left(\underline{x}_{k,i}\right)$, $\theta_{k,i}$ and $u_{k,i}$.

- $\theta_{k,i} = q_i^{-1}\left(\frac{\varpi_{k,i}}{I_{k,i}\left(\underline{x}_{k,i}\right)}\right)$ where $q_i^{-1}(y) = \left(\frac{1-y^{\xi_i}}{y^{\xi_i}}\right)^{1/\xi_i}$
- $u_{k,i} = \frac{\chi_{k,i}}{\theta_{k,i} q_i\left(\theta_{k,i}\right)\left(1-G_{k,i}\left(\underline{x}_{k,i}\right)\right)+\chi_{k,i}}$

Step 7: Compute $\left\{\widetilde{L}_{k,i}\right\}$

$$\begin{aligned}
\widetilde{L}_{k,i} &\equiv L_{k,i}\left(1-u_{k,i}\right)\int_{\underline{x}_{k,i}}^{\infty}\frac{s}{1-G_{k,i}\left(\underline{x}_{k,i}\right)}dG_{k,i}(s) \\
&= L_{k,i}\left(1-u_{k,i}\right)\exp\left(\frac{\sigma_{k,i}^2}{2}\right)\frac{\Phi\left(\sigma_{k,i}-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}
\end{aligned}$$

Step 8: Compute $\{\widetilde{w}_{k,i}\}$

$$\widetilde{w}_{k,i} = \frac{\gamma_{k,i} Y_{k,i}}{\widetilde{L}_{k,i}}$$

Step 9: Compute $\left\{E_{k,i}^V\right\}$

$$E_{k,i}^V = \widetilde{\kappa}_{k,i}\widetilde{w}_{k,i}\theta_{k,i}u_{k,i}L_{k,i}$$

Step 10: Compute $\left\{E_i^C\right\}$

$$E_i^C = \sum_{k=1}^{K} \gamma_{k,i} Y_{k,i} - \sum_{k=1}^{K} E_{k,i}^V - NX_i$$

Step 11: Compute $\left\{\widetilde{\lambda}_i\right\}$

$$\widetilde{\lambda}_i = \frac{\overline{L}_i}{E_i^C}$$

Step 12: Obtain $\{U_{k,i}\}$.

- Step 12a: Guess $\left\{U_{ki}^0\right\}$
- Step 12b: Compute until convergence

$$U_{k,i}^{g+1} = \zeta_i \log\left(\sum_{\ell=1}^{K} \exp\left\{\frac{-C_{k\ell,i} + b_{\ell,i} + \theta_{\ell,i}\widetilde{\kappa}_{\ell,i}\widetilde{\lambda}_i\widetilde{w}_{\ell,i}\frac{\beta_{\ell,i}}{(1-\beta_{\ell,i})} + \delta U_{\ell,i}^g - \delta U_{k,i}^g}{\zeta_i}\right\}\right) + \delta U_{k,i}^g$$

Step 13: Update $\{L_{k,i}\}$.

- Step 13a: Given knowledge of $\{U_{k,i}\}$, compute transition rates $s_{k\ell,i}$.

$$s_{k\ell,i} = \frac{\exp\left\{\frac{-C_{k\ell,i}+b_{\ell,i}+\theta_{\ell,i}\widetilde{\kappa}_{\ell,i}\widetilde{\lambda}_i\widetilde{w}_{\ell,i}\frac{\beta_{\ell,i}}{1-\beta_{\ell,i}}+\delta U_{\ell,i}}{\zeta_i}\right\}}{\sum_{\overline{k}} \exp\left\{\frac{-C_{k\overline{k},i}+b_{\overline{k},i}+\theta_{\overline{k},i}\widetilde{\kappa}_{\overline{k},i}\widetilde{\lambda}_i\widetilde{w}_{\overline{k},i}\frac{\beta_{\overline{k},i}}{1-\beta_{\overline{k},i}}+\delta U_{\overline{k},i}}{\zeta_i}\right\}}$$

- Step 13b: Find y_i such that

$$\left(I - s_i'\right) y_i = 0$$

- Step 13c: Find allocations $L_{k,i}$

$$L_{k,i} u_{k,i} = \varphi y_{k,i}$$

$$\Rightarrow L_{k,i} = \varphi \underbrace{y_{k,i}/u_{k,i}}_{\widetilde{y}_{k,i}}$$

$$\Rightarrow L_i' 1_{K\times 1} = \varphi \widetilde{y}_{k,i}' 1_{K\times 1} = \overline{L}_i$$

$$\Rightarrow \varphi = \frac{\overline{L}_i}{\widetilde{y}_{k,i}' 1_{K\times 1}}$$

$$(L_{k,i})' = \varphi \widetilde{y}_{k,i}$$

$$L_{k,i}^{new} = (1-\lambda_L)\, L_{k,i} + \lambda_L\, (L_{k,i})'$$

Step 14: Update $\left\{\underline{x}_{k,i}\right\}$.

Note that in equilibrium:

$$\widetilde{\lambda}_i \widetilde{w}_{k,i} \underline{x}_{k,i} = (1-\delta)\, U_{k,i} - \eta_{k,i} \tag{J.1}$$

So, we update $\underline{x}_{k,i}$ according to:

$$\left(\underline{x}_{k,i}\right)' = \frac{(1-\delta)\, U_{k,i} - \eta_{k,i}}{\widetilde{\lambda}_i \widetilde{w}_{k,i}}$$

$$\underline{x}_{k,i}^{new} = \min\left\{(1-\lambda_x)\, \underline{x}_{k,i} + \lambda_x \left(\underline{x}_{k,i}\right)', \underline{x}_{k,i}^{ub}\right\}$$

Step 15: Armed with $L_{k,i}^{new}$ and $\underline{x}_{k,i}^{new}$ go to Step 6 until $\left\|\left\{L'_{k,i} - L_{k,i}\right\}\right\| \to 0$ and $\left\|\left\{\underline{x}'_{k,i} - \underline{x}_{k,i}\right\}\right\| \to 0$.

Note that $\left\|\left\{\underline{x}_{k,i}^{new} - \underline{x}_{k,i}\right\}\right\| \to 0$ does not imply that (J.1) is satisfied. Therefore, we penalize deviations from (J.1) in the objective function.

Step 16: Generate moments, compute Loss Function, guess new parameter set Ω and go to Step 3, until objective function is minimized.

Note: Given that we condition on the trade shares $\pi_{k,oi}^{Data}$, we can estimate the model country by country, separately. However, in practice, we will first estimate the model for the US and obtain all of the US specific parameters. Next, armed with US-specific mobility costs $C_{kk'}$ and sector-specific exogenous exit components χ_k (we will impose $\chi_{k,i} = \chi_i + \chi_k$), we estimate the remaining countries' parameters separately, in parallel.

J.2 Expressions for Simulated Moments

J.2.1 Employment Shares

$$emp_{k,i} = \frac{L_{k,i}\,(1-u_{k,i})}{\sum\limits_{k=1}^{K} L_{k,i}\,(1-u_{k,i})}$$

J.2.2 National Unemployment Rate

$$unemp_i = \frac{\sum_{k=1}^{K} L_{k,i} u_{k,i}}{\sum_{k=1}^{K} L_{k,i}}$$

J.2.3 Sector-Specific Average Wages

$$w_{k,i}(x) = (1 - \beta_{k,i}) \widetilde{w}_{k,i} \underline{x}_{k,i} + \beta_{k,i} \widetilde{w}_{k,i} x$$

$$\begin{aligned}
\overline{w}_{k,i} &= \frac{\int_{\underline{x}_{k,i}}^{\infty} w_{k,i}(s)\, dG_{k,i}(s)}{1 - G_{k,i}\left(\underline{x}_{k,i}\right)} \\
&= (1 - \beta_{k,i}) \widetilde{w}_{k,i} \underline{x}_{k,i} + \beta_{k,i} \widetilde{w}_{k,i} \int_{\underline{x}_{k,i}}^{x_{\max}} \frac{s}{1 - G_{k,i}\left(\underline{x}_{k,i}\right)} dG_{k,i}(s) \\
&= (1 - \beta_{k,i}) \widetilde{w}_{k,i} \underline{x}_{k,i} + \beta_{k,i} \widetilde{w}_{k,i} \exp\left(\frac{\sigma_{k,i}^2}{2}\right) \frac{\Phi\left(\sigma_{k,i} - \frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}
\end{aligned}$$

J.2.4 Sector-Specific Variance of Wages

$$\begin{aligned}
\sigma_{w,k,i}^2 &= \frac{\int_{\underline{x}_{k,i}}^{\infty} \left(w_{k,i}(s) - \overline{w}_{k,i}\right)^2 dG_{k,i}(s)}{1 - G_{k,i}\left(\underline{x}_{k,i}\right)} \\
&= \left(\beta_{k,i} \widetilde{w}_{k,i}\right)^2 \times \frac{\int_{\underline{x}_{k,i}}^{\infty} \left(s - \exp\left(\frac{\sigma_{k,i}^2}{2}\right) \frac{\Phi\left(\sigma_{k,i} - \frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}\right)^2 dG_{k,i}(s)}{1 - G_{k,i}\left(\underline{x}_{k,i}\right)} \\
&= \left(\beta_{k,i} \widetilde{w}_{k,i}\right)^2 \times \left(\exp\left(2\sigma_{k,i}^2\right) \frac{\Phi\left(2\sigma_{k,i} - \frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)} - \exp\left(\sigma_{k,i}^2\right) \left(\frac{\Phi\left(\sigma_{k,i} - \frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}\right)^2\right)
\end{aligned}$$

J.2.5 Transition Rates

Note that the transition rates $s_{kk^{prime},i}^{t,t+1}$ denote transitions from unemployment in sector k to search in sector k' within period t. There are no data counterfactuals for this variable. However, we can construct a matrix with transition rates between all possible (model) states between time t and time $t+N$ (where N is even)—where variables are measured at the t_a stage (which is the production stage). From this matrix, we can obtain N-period transition rates between all states observed in the data (employment in each of the sectors and unconditional unemployment). First, we obtain the one-year transition matrix $\widetilde{s}^{t,t+1}$ between states $\{\widetilde{u}_1, ..., \widetilde{u}_K, 1, ..., K\}$. Here, we abuse notation to mean $\widetilde{u}_k$ as sector-k unemployment at the very beginning of a period.

The one-year transition rate between sector-ℓ unemployment and sector-k unemployment is given by:

$$\widetilde{s}_{\widetilde{u}_\ell \widetilde{u}_k,i}^{t,t+1} = s_{\ell k,i}^{t,t+1}\left(1-\theta_{k,i}^t q_i\left(\theta_{k,i}^t\right)\left(1-G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right)\right), \tag{J.2}$$

that is, a share $s_{\ell k,i}^t$ of individuals starting period t unemployed in sector ℓ choose to search in sector k. A fraction $\left(1-\theta_{k,i}^t q_i\left(\theta_{k,i}^t\right)\left(1-G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right)\right)$ of those do not find a match that survives until $t+1$. Similarly, the one-year transition rate between sector-ℓ unemployment and sector-k employment is given by:

$$\begin{aligned}\widetilde{s}_{\widetilde{u}_\ell k,i}^{t,t+1} &= s_{\ell k,i}^{t,t+1}\theta_{k,i}^t q_i\left(\theta_{k,i}^t\right)\left(1-G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right)\\ &= s_{\ell k,i}^{t,t+1}-\widetilde{s}_{\widetilde{u}_\ell \widetilde{u}_k,i}^{t,t+1}.\end{aligned} \tag{J.3}$$

According to the timing assumptions of the model, the one-year transition rate between employment in sector k and employment in sector k' is zero if $k \neq k'$. However, the persistence rate of employment in sector k is given by the probability that a match does not receive a death shock times the probability that the match is not dissolved because the threshold for production increases in the following period:

$$\widetilde{s}_{kk',i}^{t,t+1} = \begin{cases} 0 \text{ if } k \neq k' \\ \left(1-\chi_{k,i}\right)\Pr\left(x \geq \underline{x}_{k,i}^{t+1} | x \geq \underline{x}_{k,i}^t\right) \text{ if } k = k' \end{cases}. \tag{J.4}$$

Finally, the one-year transition rate between sector-k employment and unemployment in sector ℓ is given by:

$$\widetilde{s}_{k\widetilde{u}_\ell,i}^{t,t+1} = \begin{cases} 0 \text{ if } k \neq \ell \\ \chi_{k,i}+\left(1-\chi_{k,i}\right)\Pr\left(x < \underline{x}_{k,i}^{t+1} | x \geq \underline{x}_{k,i}^t\right) \text{ if } k = \ell \end{cases}. \tag{J.5}$$

That is, if a worker is employed in sector k at t, she cannot start next period unemployed in sector ℓ if $k \neq \ell$. Otherwise, workers transition between sector k employment to sector k unemployment if their match is hit with a death shock or if their employer's productivity goes below the threshold for production at $t+1$.

We can now write the N-period transition matrix as:

$$\widetilde{s}^{t,t+N} = \widetilde{s}^{t+k-1,t+k} \times \ldots \times \widetilde{s}^{t+1,t+2} \times \widetilde{s}^{t,t+1}, \tag{J.6}$$

and we can write transition rates between unemployment $\widetilde{u}$ and sector-k employment between t

and $t+N$ as:

$$\widetilde{s}_{\widetilde{u},k,i}^{t,t+N} = \frac{\sum_{\ell=1}^{K} L_{\ell,i}^{t-1} \widetilde{u}_{\ell,i}^{t-1} \widetilde{s}_{\widetilde{u}_\ell,k}^{t,t+N}}{\sum_{\ell=1}^{K} L_{\ell,i}^{t-1} \widetilde{u}_{\ell,i}^{t-1}}. \tag{J.7}$$

Finally, we can write transition rates between sector-k employment and unemployment $\widetilde{u}$ as:

$$\widetilde{s}_{k,\widetilde{u},i}^{t,t+N} = 1 - \sum_{k'=1}^{K} \widetilde{s}_{k,k',i}^{t,t+N}. \tag{J.8}$$

1-period transition rates

$$\widetilde{s}_{\widetilde{u}_\ell \widetilde{u}_k,i} = s_{\ell k,i} \left(1 - \theta_{k,i} q_i \left(\theta_{k,i}\right) \left(1 - G_{k,i} \left(\underline{x}_{k,i}\right)\right)\right)$$

$$\widetilde{s}_{\widetilde{u}_\ell k,i} = s_{\ell k,i} \theta_{k,i} q_i \left(\theta_{k,i}\right) \left(1 - G_{k,i} \left(\underline{x}_{k,i}\right)\right)$$

$$\widetilde{s}_{\ell k,i} = \begin{cases} 0 \text{ if } \ell \neq k \\ \left(1 - \chi_{\ell,i}\right) \text{ if } \ell = k \end{cases}$$

$$\widetilde{s}_{\ell \widetilde{u}_k,i} = \begin{cases} 0 \text{ if } \ell \neq k \\ \chi_{k,i} \text{ if } \ell = k \end{cases}$$

N-period transition rates from and to unconditional unemployment: $\widetilde{s}^N$

$$\widetilde{s}_{\widetilde{u},k,i}^{N} = \frac{\sum_{\ell=1}^{K} L_{\ell,i} u_{\ell,i} \widetilde{s}_{\widetilde{u}_\ell,k,i}^{N}}{\sum_{\ell=1}^{K} L_{\ell,i} u_{\ell,i}}$$

$$\widetilde{s}_{k,\widetilde{u},i}^{N} = 1 - \sum_{\ell=1}^{K} \widetilde{s}_{k,\ell,i}^{N}.$$

J.3 Algorithm: Steady-State Equilibrium Following Shock

Consider shocks $\left\{A^0_{k,i}\right\} \to \left\{A^1_{k,i}\right\}$, $\left\{d^0_{k,oi}\right\} \to \left\{d^1_{k,oi}\right\}$, $\left\{NX^0\right\} \to \left\{NX^1_i\right\}$

We will be using 0 superscripts to denote the initial steady state, and 1 superscripts to denote the final steady state.

Start from estimated Steady State: $\left\{L^0_{k,i}\right\}$, $\left\{\underline{x}^0_{k,i}\right\}$, $\left\{\widetilde{w}^0_{k,i}\right\}$, $\left\{\pi^0_{k,oi}\right\}$

Note that $\pi^0_{k,oi} = \pi^{Data}_{k,oi}$

We also have $\widetilde{\kappa}^0_{k,i} = \frac{\kappa_{k,i} P^{F,0}_i}{\widetilde{w}^0_{k,i}}$, but we do not know $\left\{P^{F,0}_i\right\}$

Denote relative changes in variable a by $\widehat{a} = \frac{a^1}{a^0}$

Step 1: Guess $\left\{L^1_{k,i}\right\}$ and $\left\{\underline{x}^1_{k,i}\right\}$

Step 2: Guess $\left\{\widetilde{w}^1_{k,i}\right\}$

- Step 2a: Compute $\widehat{\widetilde{w}}_{k,i} = \frac{\widetilde{w}^1_{k,i}}{\widetilde{w}^0_{k,i}}$ and iteratively solve for $\widehat{P}^I_{k,i}$ and $\widehat{c}_{k,i}$ using the system

$$\widehat{c}_{k,i} = \left(\widehat{\widetilde{w}}_{k,i}\right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}^I_{\ell,i}\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}}$$

$$\widehat{P}^I_{k,i} = \left(\sum_{o=1}^{N} \pi^0_{k,oi} \widehat{A}_{k,o} \left(\widehat{c}_{k,o}\widehat{d}_{k,oi}\right)^{-\lambda}\right)^{-1/\lambda}$$

- Step 2b: Compute $\widehat{P}^F_{k,i}$:

$$\widehat{P}^F_i = \prod_{k=1}^{K} \left(\widehat{P}^I_{k,i}\right)^{\mu_{ki}}$$

- Step 2c: Compute $\widehat{\pi}_{k,oi}$:

$$\widehat{\pi}_{k,oi} = \widehat{A}_{k,o} \left(\frac{\widehat{c}_{k,o}\widehat{d}_{k,oi}}{\widehat{P}^I_{k,i}}\right)^{-\lambda}$$

- Step 2d: Compute
  - $\pi^1_{k,oi} = \pi^0_{k,oi}\widehat{\pi}_{k,oi}$
  - $\widetilde{\kappa}^1_{k,i} \equiv \frac{\kappa_{k,i}P_i^{F,1}}{\widetilde{w}^1_{k,i}} = \frac{\kappa_{k,i}P_i^{F,0}}{\widetilde{w}^0_{k,i}}\frac{P_i^{F,1}}{P_i^{F,0}}\frac{\widetilde{w}^0_{k,i}}{\widetilde{w}^1_{k,i}} = \widetilde{\kappa}^0_{k,i}\frac{\widehat{P}_i^F}{\widehat{\widetilde{w}}_{k,i}}$

Step 3: If $\widetilde{\kappa}^1_{k,i} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}^1_{k,i}\right)} \geq 1$ abort, set $\underline{x}^1_{k,i}$ such that $\widetilde{\kappa}^1_{k,i} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}^1_{k,i}\right)} = 1-\varepsilon$ and go back to Step 1 with this new guess. If $\widetilde{\kappa}^1_{k,i} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}^1_{k,i}\right)} < 1$, proceed to Step 4.

Step 4: Compute

$$q_i\left(\theta^1_{k,i}\right) = \widetilde{\kappa}^1_{k,i} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}^1_{k,i}\right)}$$

$$\theta^1_{k,i} = q_i^{-1}\left(\widetilde{\kappa}^1_{k,i} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}^1_{k,i}\right)}\right)$$

$$u^1_{k,i} = \frac{\chi_{k,i}}{\theta^1_{k,i}q_i\left(\theta^1_{k,i}\right)\left(1-G_{k,i}\left(\underline{x}^1_{k,i}\right)\right)+\chi_{k,i}}$$

Step 5: Solve system in $\left\{Y^1_{k,o}\right\}$

$$Y^1_{k,o} = \sum_{i=1}^{N}\pi^1_{k,oi}\left(\mu_{k,i}\left(\sum_{\ell=1}^{K}\gamma_{\ell,i}Y^1_{\ell,i}\right)+\sum_{\ell=1}^{K}\left(1-\gamma_{\ell,i}\right)\nu_{\ell k,i}Y^1_{\ell,i}\right)-\sum_{i=1}^{N}\pi^1_{k,oi}\mu_{k,i}NX^1_i$$

$$\sum_{i=1}^{N}\sum_{k=1}^{K}Y^1_{k,i} = 1$$

Step 6: Compute $\left\{\widetilde{L}^1_{k,i}\right\}$

$$\begin{aligned}\widetilde{L}^1_{k,i} &\equiv L^1_{k,i}\left(1-u^1_{k,i}\right)\int_{\underline{x}^1_{k,i}}^{x_{\max}}\frac{s}{1-G_{k,i}\left(\underline{x}^1_{k,i}\right)}dG_{k,i}\left(s\right)\\ &= L^1_{k,i}\left(1-u^1_{k,i}\right)\exp\left(\frac{\sigma^2_{k,i}}{2}\right)\frac{\Phi\left(\sigma_{k,i}-\frac{\ln\underline{x}^1_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln\underline{x}^1_{k,i}}{\sigma_{k,i}}\right)}\end{aligned}$$

Step 7: Update $\left\{\widetilde{w}^1_{k,i}\right\}$

$$\left(\widetilde{w}^1_{k,i}\right)^{new} = \frac{\gamma_{k,i}Y^1_{k,i}}{\widetilde{L}^1_{k,i}}$$

Go back to Step 2a and repeat until converegence of $\left\{\widetilde{w}_{k,i}^1\right\}$.

Step 8: Compute

$$E_{k,i}^{V,1}=\widetilde{\kappa}_{k,i}^1\widetilde{w}_{k,i}^1\theta_{k,i}^1u_{k,i}^1L_{k,i}^1$$

$$E_i^{C,1}=\sum_{k=1}^{K}\gamma_{k,i}Y_{k,i}^1-\sum_{k=1}^{K}E_{k,i}^{V,1}-NX_i^1$$

Step 9: Obtain Lagrange Multipliers

$$\widetilde{\lambda}_i^1=\frac{\overline{L}_i}{E_i^{C,1}}$$

Step 10: Compute Bellman Equations

$$U_{k,i}^1=\zeta_i\log\left(\sum_{k'}\exp\left\{\frac{-C_{kk',i}+b_{k',i}+\theta_{k',i}^1\widetilde{\kappa}_{k',i}^1\widetilde{\lambda}_i^1\widetilde{w}_{k',i}^1\frac{\beta_{k',i}}{\left(1-\beta_{k',i}\right)}+\delta U_{k',i}^1}{\zeta_i}\right\}\right)$$

Step 11: Update $\left\{L_{k,i}^1\right\}$.

- Step 11a: Given knowledge of $\left\{U_{k,i}^1\right\}$, compute transition rates $s_{k\ell,i}^1$.

$$s_{k\ell,i}^1=\frac{\exp\left\{\frac{-C_{k\ell,i}+b_{\ell,i}+\theta_{\ell,i}^1\widetilde{\kappa}_{\ell,i}^1\widetilde{\lambda}_i^1\widetilde{w}_{\ell,i}^1\frac{\beta_{\ell,i}}{\left(1-\beta_{\ell,i}\right)}+\delta U_{\ell,i}^1}{\zeta_i}\right\}}{\sum_{\overline{k}}\exp\left\{\frac{-C_{k\overline{k},i}+b_{\overline{k},i}+\theta_{\overline{k},i}^1\widetilde{\kappa}_{\overline{k},i}^1\widetilde{\lambda}_i^1\widetilde{w}_{\overline{k},i}^1\frac{\beta_{\overline{k},i}}{\left(1-\beta_{\overline{k},i}\right)}+\delta U_{\overline{k},i}^1}{\zeta_i}\right\}}$$

- Step 11b: Find y_i such that

$$\left(I-\left(s_i^1\right)^T\right)y_i=0$$

- Step 11c: Find allocations $L_{k,i}$

$$L_{k,i}^1u_{k,i}^1=\varphi y_{k,i}$$

$$\Rightarrow L_{k,i}^{1}=\varphi\underbrace{y_{k,i}/u_{k,i}^{1}}_{\widetilde{y}_{k,i}}$$

$$\Rightarrow \left(L_{i}^{1}\right)^{T}1_{K\times 1}=\varphi\widetilde{y}_{k,i}^{T}1_{K\times 1}=\overline{L}_{i}$$

$$\Rightarrow \varphi=\frac{\overline{L}_{i}}{\widetilde{y}_{k,i}^{T}1_{K\times 1}}$$

$$\left(L_{k,i}^{1}\right)'=\varphi\widetilde{y}_{k,i}$$

$$\left(L_{k,i}^{1}\right)^{new}=\left(1-\lambda_{L}\right)L_{k,i}^{1}+\lambda_{L}\left(L_{k,i}^{1}\right)'$$

Step 12: Update $\left\{\underline{x}_{k,i}^{1}\right\}$.

Note that in equilibrium:

$$\widetilde{\lambda}_{i}^{1}\widetilde{w}_{k,i}^{1}\underline{x}_{k,i}^{1}=\left(1-\delta\right)U_{k,i}^{1}-\eta_{k,i}$$

So, we update $\underline{x}_{k,i}^{1}$ according to:

$$\left(\underline{x}_{k,i}^{1}\right)'=\frac{\left(1-\delta\right)U_{k,i}^{1}-\eta_{k,i}}{\widetilde{\lambda}_{i}^{1}\widetilde{w}_{k,i}^{1}}$$

$$\left(\underline{x}_{k,i}^{1}\right)^{new}=\min\left\{\left(1-\lambda_{x}\right)\underline{x}_{k,i}^{1}+\lambda_{x}\left(\underline{x}_{k,i}^{1}\right)',\underline{x}_{k,i}^{ub}\right\}$$

Step 13: Armed with $\left(L_{k,i}^{1}\right)^{new}$ and $\left(\underline{x}_{k,i}^{1}\right)^{new}$ go to Step 2 until $\left\|\left\{\left(L_{k,i}^{1}\right)'-L_{k,i}^{1}\right\}\right\|\rightarrow 0$ and $\left\|\left\{\left(\underline{x}_{k,i}^{1}\right)'-\underline{x}_{k,i}^{1}\right\}\right\|\rightarrow 0$.

J.4 Algorithm: Out-of-Steady-State Transition

Inner Loop: conditional on paths for expenditures $\left\{E_i^{C,t}\right\}$—determined in the Outer Loop below.

Consider paths $\left\{A_{k,i}^t\right\}_{t=0}^{T_{SS}}$ and $\left\{d_{o,i,k}^t\right\}_{t=0}^{T_{SS}}$ with $A_{k,i}^0 = 1$ and $d_{o,i,k}^0 = 1$. Also, consider paths $\left\{\phi_i^t\right\}_{t=0}^{T_{SS}}$ with $\phi_i^0 = 1$ and $\widehat{\phi}_i^t = 1$ for $T \leq t \leq T_{SS}$, for some $T << T_{SS}$.

Step 0: Given paths $\left\{E_i^{C,t}\right\}$, compute paths $\left\{\widetilde{\lambda}_i^t\right\}$: $\widetilde{\lambda}_i^t = \frac{\overline{L}_i}{E_i^{C,t}}$

Step 1: Guess paths $\left\{\widetilde{w}_{k,i}^t\right\}_{t=1}^{T_{SS}}$ for each sector k and country i.

Step 2: Compute $\underline{x}_{k,i}^{T_{SS}}$ consistent with $\widetilde{w}_{k,i}^{T_{SS}}$ and $\widetilde{\lambda}_i^{T_{SS}}$. Obtain $\theta_{k,i}^{T_{SS}}$, $U_{k,i}^{T_{SS}}$, $s_{k\ell,i}^{T_{SS},T_{SS}+1}$ and $\pi_{k,oi}^{T_{SS}}$.

- Step 2a: Compute $\widehat{\widetilde{w}}_{k,i} = \frac{\widetilde{w}_{k,i}^{T_{SS}}}{\widetilde{w}_{k,i}^0}$, $\widehat{A}_{k,i} = \frac{A_{k,i}^{T_{SS}}}{A_{k,i}^0}$ and $\widehat{d}_{k,i} = \frac{d_{o,i,k}^{T_{SS}}}{d_{o,i,k}^0}$. Iteratively solve for $\widehat{P}_{k,i}^I$ and $\widehat{c}_{k,i}$ using the system

$$\widehat{c}_{k,i} = \left(\widehat{\widetilde{w}}_{k,i}\right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}_{\ell,i}^I\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}}$$

$$\widehat{P}_{k,i}^I = \left(\sum_{o=1}^{N} \pi_{k,oi}^0 \widehat{A}_{k,o} \left(\widehat{c}_{k,o}\widehat{d}_{k,oi}\right)^{-\lambda}\right)^{-1/\lambda}$$

- Step 2b: Compute $\widehat{P}_{k,i}^F$:

$$\widehat{P}_i^F = \prod_{k=1}^{K} \left(\widehat{P}_{k,i}^I\right)^{\mu_{ki}}$$

- Step 2c: Compute

$$\widehat{\pi}_{k,oi} = \widehat{A}_{k,o} \left(\frac{\widehat{c}_{k,o}\widehat{d}_{k,oi}}{\widehat{P}_{k,i}^I}\right)^{-\lambda}$$

  And obtain $\pi_{k,oi}^{T_{SS}} = \pi_{k,oi}^0 \widehat{\pi}_{k,oi}$

- Step 2d: Compute

  - $\widetilde{\kappa}_{k,i}^{T_{SS}} = \widetilde{\kappa}_{k,i}^0 \frac{\widehat{P}_i^F}{\widehat{\widetilde{w}}_{k,i}}$

- Step 2e: Guess $\left\{\underline{x}_{k,i}^{T_{SS}}\right\}$

- Step 2f: Compute

$$\theta_{k,i}^{T_{SS}} = q_i^{-1}\left(\widetilde{\kappa}_{k,i}^{T_{SS}} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right) I_{k,i}\left(\underline{x}_{k,i}^{T_{SS}}\right)}\right)$$

- Step 2g: Compute Bellman Equations

$$U_{k,i}^{T_{SS}} = \zeta_i \log\left(\sum_{k'} \exp\left\{\frac{-C_{kk',i} + b_{k',i} + \theta_{k',i}^{T_{SS}}\widetilde{\kappa}_{k',i}^{T_{SS}}\widetilde{\lambda}_i^{T_{SS}}\widetilde{w}_{k',i}^{T_{SS}}\frac{\beta_{k',i}}{\left(1-\beta_{k',i}\right)} + \delta U_{k',i}^{T_{SS}}}{\zeta_i}\right\}\right)$$

- Step 2h: Compute

$$\left(\underline{x}_{k,i}^{T_{SS}}\right)' = \frac{\left(1-\delta\right) U_{k,i}^{T_{SS}} - \eta_{k,i}}{\widetilde{\lambda}_i^{T_{SS}}\widetilde{w}_{k,i}^{T_{SS}}}$$

- Step 2i: Update $\underline{x}_{k,i}^{T_{SS}} = \left(1-\lambda_x\right)\underline{x}_{k,i}^{T_{SS}} + \lambda_x\left(\underline{x}_{k,i}^{T_{SS}}\right)'$, for a small step size λ_x, and go back to Step 2d until convergence.

- Step 2j: Compute $s_{kk'}^{T_{SS},T_{SS}+1}$

$$s_{k\ell,i}^{T_{SS},T_{SS}+1} = \frac{\exp\left\{\frac{-C_{k\ell,i}+b_{\ell,i}+\theta_{\ell,i}^{T_{SS}}\widetilde{\kappa}_{\ell,i}^{T_{SS}}\widetilde{\lambda}_i^{T_{SS}}\widetilde{w}_{\ell,i}^{T_{SS}}\frac{\beta_{\ell,i}}{1-\beta_{\ell,i}}+\delta U_{\ell,i}^{T_{SS}}}{\zeta_i}\right\}}{\sum_{\bar{k}}\exp\left\{\frac{-C_{k\bar{k},i}+b_{\bar{k},i}+\theta_{\bar{k},i}^{T_{SS}}\widetilde{\kappa}_{\bar{k},i}^{T_{SS}}\widetilde{\lambda}_i^{T_{SS}}\widetilde{w}_{\bar{k},i}^{T_{SS}}\frac{\beta_{\bar{k},i}}{1-\beta_{\bar{k},i}}+\delta U_{\bar{k},i}^{T_{SS}}}{\zeta_i}\right\}}$$

Step 3: Obtain series $\left\{\pi_{k,oi}^t\right\}_{t=0}^{T_{SS}}$, $\left\{\widetilde{\kappa}_{k,i}^t\right\}_{t=0}^{T_{SS}}$. Define $\widehat{x}^t \equiv \frac{x^t}{x^0}$.

- Step 3a: For $t = 1, ..., T_{SS}-1$ compute $\widehat{\widetilde{w}}_{k,i}^t = \frac{\widetilde{w}_{k,i}^t}{\widetilde{w}_{k,i}^0}$ and iteratively solve for $\widehat{P}_{k,i}^{I,t}$ and $\widehat{c}_{k,i}^t$ using the system

$$\widehat{c}_{k,i}^t = \left(\widehat{\widetilde{w}}_{k,i}^t\right)^{\gamma_{k,i}}\prod_{\ell=1}^{K}\left(\widehat{P}_{\ell,i}^{I,t}\right)^{\left(1-\gamma_{k,i}\right)\nu_{k\ell,i}}$$

$$\widehat{P}_{k,i}^{I,t} = \left(\sum_{o=1}^{N}\pi_{k,oi}^0\widehat{A}_{k,o}^t\left(\widehat{c}_{k,o}^t\widehat{d}_{k,oi}^t\right)^{-\lambda}\right)^{-1/\lambda}$$

- Step 3b: Compute $\widehat{P}_{k,i}^{F,t}$ for $t = 1, ..., T_{SS}-1$:

$$\widehat{P}_i^{F,t} = \prod_{k=1}^{K}\left(\widehat{P}_{k,i}^{I,t}\right)^{\mu_{ki}}$$

- Step 3c: Compute $\widehat{\pi}^t_{k,oi}$ for $t = 1, ..., T_{SS} - 1$:

$$\widehat{\pi}^t_{k,oi} = \widehat{A}^t_{k,o} \left(\frac{\widehat{c}^t_{k,o} \widehat{d}^t_{k,oi}}{\widehat{P}^{I,t}_{k,i}} \right)^{-\lambda}$$

- Step 3d: Compute or $t = 1, ..., T_{SS} - 1$:
  - $\pi^t_{k,oi} = \pi^0_{k,oi} \widehat{\pi}^t_{k,oi}$
  - $\widetilde{\kappa}^t_{k,i} \equiv \frac{\kappa_{k,i} P^{F,t}_i}{\widetilde{w}^t_{k,i}} = \frac{\kappa_{k,i} P^{F,0}_i}{\widetilde{w}^0_{k,i}} \frac{P^{F,t}_i}{P^{F,0}_i} \frac{\widetilde{w}^0_{k,i}}{\widetilde{w}^t_{k,i}} = \widetilde{\kappa}^0_{k,i} \frac{\widehat{P}^{F,t}_i}{\widehat{\widetilde{w}}^t_{k,i}}$

Step 4: Given knowledge of $\widetilde{w}^{T_{SS}}_{k,i}$, $\widetilde{\lambda}^{T_{SS}}_i$ and $\underline{x}^{T_{SS}}_{k,i}$ (and therefore $J^{T_{SS}}_{k,i}(s)$), start at $t = T_{SS} - 1$ and sequentially compute (backwards) for each $t = T_{SS} - 1, ..., 1$

- Step 4a: Given $\widetilde{w}^t_{k,i}$, $\underline{x}^{t+1}_{k,i}$, $\widetilde{\kappa}^t_{k,i}$, $\widetilde{\lambda}^t_i$ and $J^{t+1}_{k,i}(s)$ compute $\theta^t_{k,i}$.

If $\frac{\widetilde{\lambda}^t_i \widetilde{\kappa}^t_{k,i} \widetilde{w}^t_{k,i}}{\delta \widehat{\phi}^{t+1}_i \int_{\underline{x}^{t+1}_{k,i}}^{x_{\max}} J^{t+1}_{k,i}(s) dG_{k,i}(s)} \leq 1$ then

$$\theta^t_{k,i} = q_i^{-1} \left(\frac{\widetilde{\lambda}^t_i \widetilde{\kappa}^t_{k,i} \widetilde{w}^t_{k,i}}{\delta \widehat{\phi}^{t+1}_i \int_{\underline{x}^{t+1}_{k,i}}^{x_{\max}} J^{t+1}_{k,i}(s)\, dG_{k,i}(s)} \right)$$

If $\frac{\widetilde{\lambda}^t_i \widetilde{\kappa}^t_{k,i} \widetilde{w}^t_{k,i}}{\delta \widehat{\phi}^{t+1}_i \int_{\underline{x}^{t+1}_{k,i}}^{x_{\max}} J^{t+1}_{k,i}(s) dG_{k,i}(s)} > 1$, it is not possible to satisfy $V^t_{k,i} = 0$, so that $V^t_{k,i} < 0$ and

$$\theta^t_{k,i} = 0.$$

- Step 4b: Given $\underline{x}^{t+1}_{k,i}$, $W^{t+1}_{k,i}(x) = \frac{\beta_{k,i}}{1-\beta_{k,i}} J^{t+1}_{k,i}(x) + U^{t+1}_{k,i}$ (for $x \geq \underline{x}^{t+1}_{k,i}$), $\theta^t_{k,i}$, $U^{t+1}_{k,i}$ compute $U^t_{k,i}$. Notice that $\int_{\underline{x}^{t+1}_{k,i}}^{x_{\max}} W^{t+1}_{k,i}(s)\, dG_{k,i}(s) = \frac{\beta_{k,i}}{1-\beta_{k,i}} \int_{\underline{x}^{t+1}_{k,i}}^{x_{\max}} J^{t+1}_{k,i}(s)\, dG_{k,i}(s) + \left(1 - G_{k,i}\left(\underline{x}^{t+1}_{k,i}\right)\right) U^{t+1}_{k,i}$ so that:

$$U^t_{k,i} = \zeta_i \log \left(\sum_{k'} \exp \left\{ \frac{\begin{array}{c} -C_{kk',i} + b_{k',i} \\ + \delta \widehat{\phi}^{t+1}_i \theta^t_{k',i} q_i\left(\theta^t_{k',i}\right) \frac{\beta_{k,i}}{1-\beta_{k,i}} \int_{\underline{x}^{t+1}_{k,i}}^{x_{\max}} J^{t+1}_{k,i}(s)\, dG_{k,i}(s) + \delta \widehat{\phi}^{t+1}_i U^{t+1}_{k',i} \end{array}}{\zeta_i} \right\} \right)$$

- Step 4c: Given $J^{t+1}_{k,i}(x)$, $\widetilde{w}^t_{k,i}$, $U^t_{k,i}$, $U^{t+1}_{k,i}$ and $\underline{x}^{t+1}_{k,i}$ compute $J^t_{k,i}(x)$

$$\begin{aligned} J^t_{k,i}(x) = {} & (1 - \beta_{k,i}) \widetilde{\lambda}^t_i \widetilde{w}^t_{k,i} x + (1 - \beta_{k,i}) \eta_{k,i} \\ & - (1 - \beta_{k,i}) \left(U^t_{k,i} - \delta \widehat{\phi}^{t+1}_i U^{t+1}_{k,i} \right) + (1 - \chi_{k,i}) \delta \widehat{\phi}^{t+1}_i \max\left\{ J^{t+1}_{k,i}(x), 0 \right\} \end{aligned}$$

- Step 4d: Solve for $\underline{x}_{k,i}^{t}$: $J_{k,i}^{t}\left(\underline{x}_{k,i}^{t}\right)=0$

Step 5: Compute transition rates $\left\{s_{kk',i}^{t,t+1}\right\}_{t=1}^{T_{SS}-1}$ for all countries i according to:

$$s_{kk',i}^{t,t+1}=\frac{\exp\left\{\begin{array}{c}-C_{kk',i}+b_{k',i}+\\ \delta\widehat{\phi}_i^{t+1}\theta_{k',i}^{t}q(\theta_{k',i}^{t})\frac{\beta_{k',i}}{1-\beta_{k',i}}\int_{\underline{x}_{k',i}^{t+1}}^{x_{\max}}J_{k',i}^{t+1}\left(x\right)dG_{k',i}(x)+\delta\widehat{\phi}_i^{t+1}U_{k',i}^{t+1}\end{array}\right\}}{\sum_{k''}\exp\left\{\begin{array}{c}-C_{kk'',i}+b_{k'',i}+\\ \delta\widehat{\phi}_i^{t+1}\theta_{k'',i}^{t}q(\theta_{k'',i}^{t})\frac{\beta_{k'',i}}{1-\beta_{k'',i}}\int_{\underline{x}_{k'',i}^{t+1}}^{x_{\max}}J_{k'',i}^{t+1}\left(x\right)dG_{k'',i}(x)+\delta\widehat{\phi}_i^{t+1}U_{k'',i}^{t+1}\end{array}\right\}}.$$

Step 6: Start loop over t going forward ($t=0$ to $t=T_{SS}-1$)

Initial conditions: we know $\widetilde{u}_{k,i}^{t=-1}=u_{k,i}^{t=0}$, $L_{k,i}^{t=-1}=L_{k,i}^{t=0}$, and $\theta_{k,i}^{t=0}$ from the initial steady state computation. Obtain $\widetilde{u}_{k,i}^{t}$ and $L_{k,i}^{t}$ using flow conditions and sequences $\left\{\theta_{k,i}^{t}\right\}$, $\left\{\underline{x}_{k,i}^{t}\right\}$.

- Step 6a: Compute

$$JC_{k,i}^{t}=L_{k,i}^{t}u_{k,i}^{t}\theta_{k,i}^{t}q_i\left(\theta_{k,i}^{t}\right)\left(1-G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right)$$

$$JD_{k,i}^{t}=\left(\chi_{k,i}+(1-\chi_{k,i})\max\left\{\frac{G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)-G_{k,i}\left(\underline{x}_{k,i}^{t}\right)}{1-G_{k,i}\left(\underline{x}_{k,i}^{t}\right)},0\right\}\right)L_{k,i}^{t-1}\left(1-\widetilde{u}_{k,i}^{t-1}\right)$$

$$\widetilde{u}_{k,i}^{t}=\frac{L_{k,i}^{t}u_{k,i}^{t}-JC_{k,i}^{t}+JD_{k,i}^{t}}{L_{k,i}^{t}}$$

- Step 6b: Compute

$$L_{k,i}^{t+1}=L_{k,i}^{t}+IF_{k,i}^{t+1}-OF_{k,i}^{t+1},$$

where

$$IF_{k,i}^{t+1}=\sum_{\ell\neq k}L_{\ell,i}^{t}\widetilde{u}_{\ell,i}^{t}s_{\ell k,i}^{t+1,t+2},$$

and

$$OF_{k,i}^{t+1}=L_{k,i}^{t}\widetilde{u}_{k,i}^{t}\left(1-s_{kk,i}^{t+1,t+2}\right).$$

- Step 6c: Compute

$$u_{k,i}^{t+1}=\frac{\sum_{\ell=1}^{K}L_{\ell,i}^{t}\widetilde{u}_{\ell,i}^{t}s_{\ell k,i}^{t+1,t+2}}{L_{k,i}^{t+1}}$$

- Step 6d: Compute

$$\widetilde{L}_{k,i}^{t+1} = L_{k,i}^{t}\left(1-\widetilde{u}_{k,i}^{t}\right)\int_{\underline{x}_{k,i}^{t+1}}^{\infty}\frac{s}{1-G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)}dG_{k,i}\left(s\right)$$
$$= L_{k,i}^{t}\left(1-\widetilde{u}_{k,i}^{t}\right)\exp\left(\frac{\sigma_{k,i}^{2}}{2}\right)\frac{\Phi\left(\sigma_{k,i}-\frac{\ln \underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}$$

- Step 6e: Compute expenditure with vacancies

$$E_{k,i}^{V,t+1} = \widetilde{\kappa}_{k,i}^{t+1}\widetilde{w}_{k,i}^{t+1}\theta_{k,i}^{t+1}u_{k,i}^{t+1}L_{k,i}^{t+1}$$

- Step 6f: Solve for $\left\{Y_{k,i}^{t+1}\right\}$ in the system

$$E_{k,i}^{t+1} = \mu_{k,i}E_{i}^{C,t+1} + \sum_{\ell=1}^{K}\left(\mu_{k,i}E_{\ell,i}^{V,t+1} + \left(1-\gamma_{\ell,i}\right)\nu_{\ell k,i}Y_{\ell,i}^{t+1}\right).$$

$$Y_{k,o}^{t+1} = \sum_{i=1}^{N}\pi_{k,oi}^{t+1}E_{k,i}^{t+1}.$$

- Step 6g: Compute $\left(\widetilde{w}_{k,i}^{t+1}\right)' = \frac{\gamma_{k,i}Y_{k,i}^{t+1}}{\widetilde{L}_{k,i}^{t+1}}$

Step 7: Compute distance $dist\left(\left\{\widetilde{w}_{k,i}^{t}\right\},\left\{\left(\widetilde{w}_{k,i}^{t}\right)'\right\}\right)$

- Step 7b: Update $\widetilde{w}_{k,i}^{t} = \left(1-\lambda_{w}\right)\widetilde{w}_{k,i}^{t} + \lambda_{w}\left(\widetilde{w}_{k,i}^{t}\right)'$ $t = 1, ..., T_{SS}$, for a small step size λ_w.
- Step 7c: At this point, we have a new series for $\left\{\widetilde{w}_{k,i}^{t}\right\}$ – go back to Step 2 until convergence of $\left\{\widetilde{w}_{k,i}^{t}\right\}$.

Step 8: Compute disposable income $\left\{I_{i}^{t}\right\}_{t=1}^{T_{SS}}$

$$I_{i}^{t} = \sum_{\ell=1}^{K}\left(\gamma_{\ell,i}Y_{\ell,i}^{t} - E_{\ell,i}^{V,t}\right)$$

Outer Loop: iteration on $\left\{NX_{i}^{t}\right\}$

Step 0: Impose a change in a subset of parameters that happens at $t = 0$, but between t_c and t_d. That is, the shock occurs **after** production, workers' decisions of where to search and after firms post vacancies at $t = 0$. Impose a large value for T_{SS}. Assume that for $t \geq T_{SS}$ the system will have converged to a new steady state. World expenditure with final goods $\sum\limits_{i=1}^{I} E_i^{C,t}$ is normalized to 1 for every t.

Step 1: Start with estimated state equilibrium at $t = 0$. Remember that we used the normalization $\sum\limits_{i=1}^{I}\sum\limits_{k=1}^{K} Y_{k,i} = 1$ during the estimation procedure. **Change the normalization** from $\sum\limits_{i=1}^{I}\sum\limits_{k=1}^{K} Y_{k,i} = 1$ to $\sum\limits_{i=1}^{I} E_i^C = 1$. Nominal variables to be renormalized: $\left\{Y_{k,i}^0\right\}$, $\left\{\widetilde{w}_{k,i}^0\right\}$, $\left\{E_i^{C,0}\right\}$, $\{NX_i^0\}$.

Step 2: Obtain B_i^0 with respect to the normalization $\sum\limits_{i=1}^{I} E_i^C = 1$. Equation (28) gives us:

$$B_i^0 = \frac{NX_i^0}{(1 - \frac{1}{\delta})}$$

Step 3: Make initial guess for $NX_i^{T_{SS}}$ (with respect to the normalization $\sum\limits_{i=1}^{I} E_i^C = 1$).

Step 4: Compute steady state equilibrium at T_{SS}, conditional on $NX_i^{T_{SS}}$, and the change in parameter values.

- Step 4a: Notice that the steady-state algorithm uses the normalization $\sum\limits_{i=1}^{I}\sum\limits_{k=1}^{K} Y_{k,i} = 1$. Normalize $NX_i^{T_{SS}}$ with respect to normalization $\sum\limits_{i=1}^{I}\sum\limits_{k=1}^{K} Y_{k,i} = 1$. To perform such normalization, use revenue $\left\{Y_{k,i}^{T_{SS}}\right\}$ obtained in the initial steady state if this is the first outer loop iteration, otherwise use revenue $\left\{Y_{k,i}^{T_{SS}}\right\}$ obtained in Step 6 below.

- Step 4b: After computing the final steady state, **change the normalization** from $\sum\limits_{i=1}^{I}\sum\limits_{k=1}^{K} Y_{k,i} = 1$ to $\sum\limits_{i=1}^{I} E_i^C = 1$ using $\left\{E_i^C\right\}$ obtained in Step 3a. Nominal variables to be renormalized: $\left\{Y_{k,i}^{T_{SS}}\right\}$, $\left\{\widetilde{w}_{k,i}^{T_{SS}}\right\}$, $\left\{E_i^{C,T_{SS}}\right\}$, $\left\{NX_i^{T_{SS}}\right\}$.

Step 5: Start at $t = T_{SS} - 1$ and go backward until $t = 1$ and sequentially compute:

$$R^{t+1} = \frac{1}{\delta}\frac{\sum_{i=1}^{N} \frac{E_i^{C,t+1}}{\widehat{\phi}_i^{t+1}}}{\sum_{i=1}^{N} E_i^{C,t}} = \frac{1}{\delta}\sum_{i=1}^{N} \frac{E_i^{C,t+1}}{\widehat{\phi}_i^{t+1}},$$

$$E_i^{C,t} = \frac{E_i^{C,t+1}}{\delta \widehat{\phi}_i^{t+1} R^{t+1}}$$

to obtain paths for $\left\{R^t\right\}$ and $\left\{E_i^{C,t}\right\}$. Note that, because B_i^1 is decided at $t = 0$, before the shock, $R^1 = R^0 = \frac{1}{\delta}$.

Step 6: Solve for the out-of-steady-state dynamics conditional on aggregate expenditures $\left\{E_i^{C,t}\right\}$.

Step 7: Using the path for disposable income $\left\{I_i^t\right\}_{t=1}^{T_{SS}}$ obtained in Step 6 and equation (5) compute:

$$\left(NX_i^t\right)' = I_i^t - E^{C,t} \text{ for } 1 \leq t < T_{SS}$$

$$\left(NX_i^{T_{SS}}\right)' = -\frac{1-\delta}{\delta} \frac{1}{\left(\prod_{\tau=1}^{T_{SS}-1} \left(R^\tau\right)^{-1}\right)} \left(B_i^0 + \sum_{t=1}^{T_{SS}-1} \left(\prod_{\tau=1}^{t} \left(R^\tau\right)^{-1}\right) \left(NX_i^t\right)'\right)$$

Step 8: Compute

$$dist\left(\left\{NX_i^{T_{SS}}\right\}, \left\{\left(NX_i^{T_{SS}}\right)'\right\}\right)$$

Step 9: Update $NX_i^{T_{SS}}$

$$NX_i^{T_{SS}} = \left(1 - \lambda_o\right) NX_i^{T_{SS}} + \lambda_o \left(NX_i^{T_{SS}}\right)',$$

for a small step size λ_o Go back to Step 4 until convergence of $\left\{NX_i^{T_{SS}}\right\}$.

J.5 Algorithm: Out-of-Steady-State Transition, Exogenous Deficits (No Bonds)

Consider paths $\left\{A_{k,i}^{t}\right\}_{t=0}^{T_{SS}}$ and $\left\{d_{o,i,k}^{t}\right\}_{t=0}^{T_{SS}}$ with $A_{k,i}^{0}=1$ and $d_{k,oi}^{0}=1$. Also, consider paths $\{\phi_i^t\}_{t=0}^{T_{SS}}$ with $\phi_i^0=1$ and $\widehat{\phi}_i^t=1$ for $T\leq t\leq T_{SS}$, for some $T<<T_{SS}$.

We condition on an exogenous path for $\{NX_i^t\}_{t=1}^{T_{SS}}$.

Step 1: Guess paths $\left\{\widetilde{\lambda}_i^t\right\}_{t=1}^{T_{SS}}$ for each country i.

Step 2: Guess paths $\left\{\widetilde{w}_{k,i}^t\right\}_{t=1}^{T_{SS}}$ for each sector k and country i.

Step 3: Compute $\underline{x}_{k,i}^{T_{SS}}$ consistent with $\widetilde{w}_{k,i}^{T_{SS}}$ and $\widetilde{\lambda}_i^{T_{SS}}$. Obtain $\theta_{k,i}^{T_{SS}}$, $U_{k,i}^{T_{SS}}$, $s_{k\ell,i}^{T_{SS},T_{SS}+1}$ and $\pi_{k,oi}^{T_{SS}}$.

- Step 3a: Compute $\widehat{\widetilde{w}}_{k,i}=\frac{\widetilde{w}_{k,i}^{T_{SS}}}{\widetilde{w}_{k,i}^{0}}$, $\widehat{A}_{k,i}=\frac{A_{k,i}^{T_{SS}}}{A_{k,i}^{0}}$ and $\widehat{d}_{k,i}=\frac{d_{o,i,k}^{T_{SS}}}{d_{k,oi}^{0}}$. Iteratively solve for $\widehat{P}_{k,i}^{I}$ and $\widehat{c}_{k,i}$ using the system

$$\widehat{c}_{k,i}=\left(\widehat{\widetilde{w}}_{k,i}\right)^{\gamma_{k,i}}\prod_{\ell=1}^{K}\left(\widehat{P}_{\ell,i}^{I}\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}}$$

$$\widehat{P}_{k,i}^{I}=\left(\sum_{o=1}^{N}\pi_{k,oi}^{0}\widehat{A}_{k,o}\left(\widehat{c}_{k,o}\widehat{d}_{k,oi}\right)^{-\lambda}\right)^{-1/\lambda}$$

- Step 3b: Compute $\widehat{P}_{k,i}^{F}$:

$$\widehat{P}_{i}^{F}=\prod_{k=1}^{K}\left(\widehat{P}_{k,i}^{I}\right)^{\mu_{ki}}$$

- Step 3c: Compute

$$\widehat{\pi}_{k,oi}=\widehat{A}_{k,o}\left(\frac{\widehat{c}_{k,o}\widehat{d}_{k,oi}}{\widehat{P}_{k,i}^{I}}\right)^{-\lambda},$$

  and obtain $\pi_{k,oi}^{T_{SS}}=\pi_{k,oi}^{0}\widehat{\pi}_{k,oi}$

- Step 3d: Compute

  – $\widetilde{\kappa}_{k,i}^{T_{SS}}=\widetilde{\kappa}_{k,i}^{0}\frac{\widehat{P}_i^F}{\widehat{\widetilde{w}}_{k,i}}$

- Step 3e: Guess $\left\{\underline{x}_{k,i}^{T_{SS}}\right\}$

- Step 3f: Compute

$$\theta_{k,i}^{T_{SS}}=q_i^{-1}\left(\widetilde{\kappa}_{k,i}^{T_{SS}}\times\frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}_{k,i}^{T_{SS}}\right)}\right)$$

- Step 3g: Compute Bellman Equations

$$U_{k,i}^{T_{SS}} = \zeta_i \log \left(\sum_{k'} \exp \left\{ \frac{-C_{kk',i} + b_{k',i} + \theta_{k',i}^{T_{SS}} \widetilde{\kappa}_{k',i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{k',i}^{T_{SS}} \frac{\beta_{k',i}}{(1-\beta_{k',i})} + \delta U_{k',i}^{T_{SS}}}{\zeta_i} \right\} \right)$$

- Step 3h: Compute

$$\left(\underline{x}_{k,i}^{T_{SS}} \right)' = \frac{(1-\delta) U_{k,i}^{T_{SS}} - \eta_{k,i}}{\widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{k,i}^{T_{SS}}}$$

- Step 3i: Update $\underline{x}_{k,i}^{T_{SS}} = (1-\lambda_x) \underline{x}_{k,i}^{T_{SS}} + \lambda_x \left(\underline{x}_{k,i}^{T_{SS}} \right)'$, for a small step size λ_x, and go back to Step 2d until convergence.

- Step 3j: Compute $s_{kk'}^{T_{SS},T_{SS}+1}$

$$s_{k\ell,i}^{T_{SS},T_{SS}+1} = \frac{\exp \left\{ \frac{-C_{k\ell,i} + b_{\ell,i} + \theta_{\ell,i}^{T_{SS}} \widetilde{\kappa}_{\ell,i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{\ell,i}^{T_{SS}} \frac{\beta_{\ell,i}}{1-\beta_{\ell,i}} + \delta U_{\ell,i}^{T_{SS}}}{\zeta_i} \right\}}{\sum_{\overline{k}} \exp \left\{ \frac{-C_{k\overline{k},i} + b_{\overline{k},i} + \theta_{\overline{k},i}^{T_{SS}} \widetilde{\kappa}_{\overline{k},i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{\overline{k},i}^{T_{SS}} \frac{\beta_{\overline{k},i}}{1-\beta_{\overline{k},i}} + \delta U_{\overline{k},i}^{T_{SS}}}{\zeta_i} \right\}}$$

Step 4: Obtain series $\left\{ \pi_{k,oi}^t \right\}_{t=0}^{T_{SS}}$, $\left\{ \widetilde{\kappa}_{k,i}^t \right\}_{t=0}^{T_{SS}}$. Define $\widehat{x}^t \equiv \frac{x^t}{x^0}$.

- Step 4a: For $t = 1, ..., T_{SS} - 1$ compute $\widehat{\widetilde{w}}_{k,i}^t = \frac{\widetilde{w}_{k,i}^t}{\widetilde{w}_{k,i}^0}$ and iteratively solve for $\widehat{P}_{k,i}^{I,t}$ and $\widehat{c}_{k,i}^t$ using the system

$$\widehat{c}_{k,i}^t = \left(\widehat{\widetilde{w}}_{k,i}^t \right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}_{\ell,i}^{I,t} \right)^{(1-\gamma_{k,i})\nu_{k\ell,i}}$$

$$\widehat{P}_{k,i}^{I,t} = \left(\sum_{o=1}^{N} \pi_{k,oi}^0 \widehat{A}_{k,o}^t \left(\widehat{c}_{k,o}^t \widehat{d}_{k,oi}^t \right)^{-\lambda} \right)^{-1/\lambda}$$

- Step 4b: Compute $\widehat{P}_{k,i}^{F,t}$ for $t = 1, ..., T_{SS} - 1$:

$$\widehat{P}_i^{F,t} = \prod_{k=1}^{K} \left(\widehat{P}_{k,i}^{I,t} \right)^{\mu_{ki}}$$

- Step 4c: Compute $\widehat{\pi}_{k,oi}^t$ for $t = 1, ..., T_{SS} - 1$:

$$\widehat{\pi}_{k,oi}^t = \widehat{A}_{k,o}^t \left(\frac{\widehat{c}_{k,o}^t \widehat{d}_{k,oi}^t}{\widehat{P}_{k,i}^{I,t}} \right)^{-\lambda}$$

- Step 4d: Compute or $t = 1, ..., T_{SS} - 1$:

  - $\pi^t_{k,oi} = \pi^0_{k,oi}\widehat{\pi}^t_{k,oi}$
  - $\widetilde{\kappa}^t_{k,i} \equiv \frac{\kappa_{k,i}P^{F,t}_i}{\widetilde{w}^t_{k,i}} = \frac{\kappa_{k,i}P^{F,0}_i}{\widetilde{w}^0_{k,i}}\frac{P^{F,t}_i}{P^{F,0}_i}\frac{\widetilde{w}^0_{k,i}}{\widetilde{w}^t_{k,i}} = \widetilde{\kappa}^0_{k,i}\frac{\widehat{P}^{F,t}_i}{\widehat{\widetilde{w}}^t_{k,i}}$

Step 5: Given knowledge of $\widetilde{w}^{T_{SS}}_{k,i}$, $\widetilde{\lambda}^{T_{SS}}_i$ and $\underline{x}^{T_{SS}}_{k,i}$ (and therefore $J^{T_{SS}}_{k,i}(s)$), start at $t = T_{SS} - 1$ and sequentially compute (backwards) for each $t = T_{SS} - 1, ..., 1$

- Step 5a: Given $\widetilde{w}^t_{k,i}$, $\widetilde{\lambda}^t_i$, $\underline{x}^{t+1}_{k,i}$, $\widetilde{\kappa}^t_{k,i}$ and $J^{t+1}_{k,i}(s)$ compute $\theta^t_{k,i}$.

If $\frac{\widetilde{\lambda}^t_i\widetilde{\kappa}^t_{k,i}\widetilde{w}^t_{k,i}}{\delta\widehat{\phi}^{t+1}_i\int^{x_{\max}}_{\underline{x}^{t+1}_{k,i}} J^{t+1}_{k,i}(s)dG_{k,i}(s)} \leq 1$ then

$$\theta^t_{k,i} = q^{-1}_i\left(\frac{\widetilde{\lambda}^t_i\widetilde{\kappa}^t_{k,i}\widetilde{w}^t_{k,i}}{\delta\widehat{\phi}^{t+1}_i\int^{x_{\max}}_{\underline{x}^{t+1}_{k,i}} J^{t+1}_{k,i}(s)\,dG_{k,i}(s)}\right)$$

If $\frac{\widetilde{\lambda}^t_i\widetilde{\kappa}^t_{k,i}\widetilde{w}^t_{k,i}}{\delta\widehat{\phi}^{t+1}_i\int^{x_{\max}}_{\underline{x}^{t+1}_{k,i}} J^{t+1}_{k,i}(s)dG_{k,i}(s)} > 1$, it is not possible to satisfy $V^t_{k,i} = 0$, so that $V^t_{k,i} < 0$ and

$$\theta^t_{k,i} = 0.$$

- Step 5b: Given $\underline{x}^{t+1}_{k,i}$, $W^{t+1}_{k,i}(x) = \frac{\beta_{k,i}}{1-\beta_{k,i}}J^{t+1}_{k,i}(x) + U^{t+1}_{k,i}$ (for $x \geq \underline{x}^{t+1}_{k,i}$), $\theta^t_{k,i}$, $U^{t+1}_{k,i}$ compute $U^t_{k,i}$. Notice that $\int^{x_{\max}}_{\underline{x}^{t+1}_{k,i}} W^{t+1}_{k,i}(s)\,dG_{k,i}(s) = \frac{\beta_{k,i}}{1-\beta_{k,i}}\int^{x_{\max}}_{\underline{x}^{t+1}_{k,i}} J^{t+1}_{k,i}(s)\,dG_{k,i}(s) + \left(1 - G_{k,i}\left(\underline{x}^{t+1}_{k,i}\right)\right)U^{t+1}_{k,i}$ so that:

$$U^t_{k,i} = \zeta_i \log\left(\sum_{k'}\exp\left\{\frac{\begin{array}{c}-C_{kk',i} + b_{k',i}\\ +\delta\widehat{\phi}^{t+1}_i\theta^t_{k',i}q_i\left(\theta^t_{k',i}\right)\frac{\beta_{k,i}}{1-\beta_{k,i}}\int^{x_{\max}}_{\underline{x}^{t+1}_{k,i}} J^{t+1}_{k,i}(s)\,dG_{k,i}(s) + \delta\widehat{\phi}^{t+1}_i U^{t+1}_{k',i}\end{array}}{\zeta_i}\right\}\right)$$

- Step 5c: Given $\widetilde{\lambda}^t_i$, $J^{t+1}_{k,i}(x)$, $\widetilde{w}^t_{k,i}$, $\theta^t_{k,i}$, δ, $U^t_{k,i}$, $U^{t+1}_{k,i}$ and $\underline{x}^{t+1}_{k,i}$ compute $J^t_{k,i}(x)$

$$\begin{aligned} J^t_{k,i}(x) &= (1-\beta_{k,i})\widetilde{\lambda}^t_i\widetilde{w}^t_{k,i}x + (1-\beta_{k,i})\eta_{k,i} \\ &\quad - (1-\beta_{k,i})\left(U^t_{k,i} - \delta\widehat{\phi}^{t+1}_i U^{t+1}_{k,i}\right) + (1-\chi_{k,i})\delta\widehat{\phi}^{t+1}_i\max\left\{J^{t+1}_{k,i}(x), 0\right\}\end{aligned}$$

- Step 5d: Solve for $\underline{x}^t_{k,i}$: $J^t_{k,i}\left(\underline{x}^t_{k,i}\right) = 0$

Step 6: Compute transition rates $\left\{s^{t,t+1}_{kk',i}\right\}_{t=1}^{T_{SS}-1}$ for all countries i according to:

$$s^{t,t+1}_{kk',i} = \frac{\exp\left\{\begin{array}{c} -C_{kk',i} + b_{k',i} + \\ \delta\widehat{\phi}_i^{t+1}\theta^t_{k',i}q(\theta^t_{k',i})\frac{\beta_{k',i}}{1-\beta_{k',i}}\int_{\underline{x}^{t+1}_{k',i}}^{x_{\max}} J^{t+1}_{k',i}(x)\,dG_{k',i}(x) + \delta\widehat{\phi}_i^{t+1}U^{t+1}_{k',i}\end{array}\right\}}{\sum_{k''}\exp\left\{\begin{array}{c} -C_{kk'',i} + b_{k'',i} + \\ \delta\widehat{\phi}_i^{t+1}\theta^t_{k'',i}q(\theta^t_{k'',i})\frac{\beta_{k'',i}}{1-\beta_{k'',i}}\int_{\underline{x}^{t+1}_{k'',i}}^{x_{\max}} J^{t+1}_{k'',i}(x)\,dG_{k'',i}(x) + \delta\widehat{\phi}_i^{t+1}U^{t+1}_{k'',i}\end{array}\right\}}.$$

Step 7: Start loop over t going forward ($t = 0$ to $t = T_{SS} - 1$)

Initial conditions: we know $\widetilde{u}^{t=-1}_{k,i} = u^{t=0}_{k,i}$, $L^{t=-1}_{k,i} = L^{t=0}_{k,i}$, and $\theta^{t=0}_{k,i}$ from the initial steady state computation. Obtain $\widetilde{u}^t_{k,i}$ and $L^t_{k,i}$ using flow conditions and sequences $\left\{\theta^t_{k,i}\right\}$, $\left\{\underline{x}^t_{k,i}\right\}$.

- Step 7a: Compute

$$JC^t_{k,i} = L^t_{k,i}u^t_{k,i}\theta^t_{k,i}q_i\left(\theta^t_{k,i}\right)\left(1 - G_{k,i}\left(\underline{x}^{t+1}_{k,i}\right)\right)$$

$$JD^t_{k,i} = \left(\chi_{k,i} + (1-\chi_{k,i})\max\left\{\frac{G_{k,i}\left(\underline{x}^{t+1}_{k,i}\right) - G_{k,i}\left(\underline{x}^t_{k,i}\right)}{1 - G_{k,i}\left(\underline{x}^t_{k,i}\right)}, 0\right\}\right) L^{t-1}_{k,i}\left(1 - \widetilde{u}^{t-1}_{k,i}\right)$$

$$\widetilde{u}^t_{k,i} = \frac{L^t_{k,i}u^t_{k,i} - JC^t_{k,i} + JD^t_{k,i}}{L^t_{k,i}}$$

- Step 7b: Compute

$$L^{t+1}_{k,i} = L^t_{k,i} + IF^{t+1}_{k,i} - OF^{t+1}_{k,i},$$

where

$$IF^{t+1}_{k,i} = \sum_{\ell\neq k} L^t_{\ell,i}\widetilde{u}^t_{\ell,i}s^{t+1,t+2}_{\ell k,i},$$

and

$$OF^{t+1}_{k,i} = L^t_{k,i}\widetilde{u}^t_{k,i}\left(1 - s^{t+1,t+2}_{kk,i}\right).$$

- Step 7c: Compute

$$u^{t+1}_{k,i} = \frac{\sum_{\ell=1}^{K} L^t_{\ell,i}\widetilde{u}^t_{\ell,i}s^{t+1,t+2}_{\ell k,i}}{L^{t+1}_{k,i}}$$

- Step 7d: Compute

$$\widetilde{L}_{k,i}^{t+1} = L_{k,i}^{t}\left(1-\widetilde{u}_{k,i}^{t}\right)\int_{\underline{x}_{k,i}^{t+1}}^{\infty}\frac{s}{1-G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)}dG_{k,i}\left(s\right)$$
$$= L_{k,i}^{t}\left(1-\widetilde{u}_{k,i}^{t}\right)\exp\left(\frac{\sigma_{k,i}^{2}}{2}\right)\frac{\Phi\left(\sigma_{k,i}-\frac{\ln \underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}$$

$$\text{and } Y_{k,i}^{t+1} = \widetilde{w}_{k,i}^{t+1}\widetilde{L}_{k,i}^{t+1}$$

- Step 7e: Compute expenditure with vacancies

$$E_{k,i}^{V,t+1} = \widetilde{\kappa}_{k,i}^{t+1}\widetilde{w}_{k,i}^{t+1}\theta_{k,i}^{t+1}u_{k,i}^{t+1}L_{k,i}^{t+1}$$

- Step 7f: Compute $E_{i}^{C,t+1} = \frac{\overline{L}_{i}}{\widetilde{\lambda}_{i}^{t+1}}$

- Step 7g: Solve for $\left\{Y_{k,i}^{t+1}\right\}$ in the system

$$E_{k,i}^{t+1} = \mu_{k,i}E_{i}^{C,t+1} + \sum_{\ell=1}^{K}\left(\mu_{k,i}E_{\ell,i}^{V,t+1} + \left(1-\gamma_{\ell,i}\right)\nu_{\ell k,i}Y_{\ell,i}^{t+1}\right).$$

$$Y_{k,o}^{t+1} = \sum_{i=1}^{N}\pi_{k,oi}^{t+1}E_{k,i}^{t+1}.$$

- Step 7h: Normalize $\left\{Y_{k,i}^{t+1}\right\}$ to make sure it sums to 1 across sectors and countries.

- Step 7i: Compute $\left(\widetilde{w}_{k,i}^{t+1}\right)' = \frac{\gamma_{k,i}Y_{k,i}^{t+1}}{\widetilde{L}_{k,i}^{t+1}}$

Step 8: Compute $dist\left(\left\{\widetilde{w}_{k,i}^{t}\right\}_{t=1}^{T_{SS}}, \left\{\left(\widetilde{w}_{k,i}^{t}\right)'\right\}_{t=1}^{T_{SS}}\right)$

Step 9: Update $\widetilde{w}_{k,i}^{t} = \left(1-\alpha_{w}\right)\widetilde{w}_{k,i}^{t} + \alpha_{w}\left(\widetilde{w}_{k,i}^{t}\right)'$ for $t = 1, ..., T_{SS}$, for a small step size α_{w}, and go back to Step 3 until convergence of $\left\{\widetilde{w}_{k,i}^{t}\right\}$

Step 10: Compute disposable income $\left\{I_{i}^{t}\right\}_{t=1}^{T_{SS}}$

$$I_{i}^{t} = \sum_{\ell=1}^{K}\left(\gamma_{\ell,i}Y_{\ell,i}^{t} - E_{\ell,i}^{V,t}\right)$$

Step 11: Update $\left\{E_i^{C,t}\right\}_{t=1}^{T_{SS}}$ using

$$E_i^{C,t} = I_i^t - NX_i^t$$

Step 12a: Compute $\left(\widetilde{\lambda}_i^t\right)' = \frac{\overline{L}_i}{E_i^{C,t}}$ for all $t = 1, ..., T_{SS}$

- Step 12b: Compute $dist\left(\left\{\widetilde{\lambda}_i^t\right\}_{t=1}^{T_{SS}}, \left\{\left(\widetilde{\lambda}_i^t\right)'\right\}_{t=1}^{T_{SS}}\right)$
- Step 12c: Update $\widetilde{\lambda}_i^t = (1-\alpha_\lambda)\,\widetilde{\lambda}_i^t + \alpha_\lambda \left(\widetilde{\lambda}_i^t\right)'$ for $t = 1, ..., T_{SS}$, for a small step size α_λ, and go back to Step 2 until convergence of $\left\{\widetilde{\lambda}_i^t\right\}$.

J.6 Algorithm: Recovering Shocks

Important: We will need to keep track of two periods. Let T_{Data} denote the last period for which we have data. Let $\widetilde{T} > T_{Data}$ be the period after which there are no more shocks AND $E_i^{C,t}$ is assumed to be constant across countries (according to the $\sum_{i=1}^{N} E_i^{C,t} = 1$ normalization).

Inner Loop: conditional on paths for expenditures $\left\{E_i^{C,t}\right\}_{t=1}^{T_{SS}}$, net exports $\left\{NX_i^t\right\}_{t=1}^{T_{SS}}$ and shocks $\left\{\widehat{\phi}_i^t\right\}_{t=2}^{T_{SS}}$ and $\left\{\widehat{d}_{k,oi}^t\right\}_{t=1}^{T_{SS}}$, which are determined in the Outer Loop below.

As before, we denote changes relative to $t = 0$ by $\widehat{x}^t = \frac{x^t}{x^0}$. This loop **conditions** on data on $\left\{\widehat{\pi}_{k,oi}^t\right\}_{t=1}^{T_{Data}}$ and $\left\{\widehat{P}_{k,i}^{I,t}\right\}_{t=1}^{T_{Data}}$. We assume the state of the global economy at $t = 0$ is given by the estimated steady state. Define $\widehat{d}_{k,oi} \equiv \frac{d_{k,oi}^{T_{SS}}}{d_{k,oi}^0}$ and $\widehat{A}_{k,i} \equiv \frac{A_{k,i}^{T_{SS}}}{A_{k,i}^0}$.

Step 1: Given paths $\left\{E_i^{C,t}\right\}_{t=1}^{T_{SS}}$, compute paths $\left\{\widetilde{\lambda}_i^t\right\}_{t=1}^{T_{SS}}$: $\widetilde{\lambda}_i^t = \frac{\overline{L}_i}{E_i^{C,t}}$.

Step 2: Guess paths $\left\{\widetilde{w}_{k,i}^t\right\}_{t=1}^{T_{SS}}$ for each sector k and country i.

Step 3: For each $t = 1, ..., T_{Data}$ compute $\widehat{\widetilde{w}}_{k,i}^t \equiv \frac{\widetilde{w}_{k,i}^t}{\widetilde{w}_{k,i}^0}$, and obtain $\widehat{c}_{k,i}^t$:

$$\widehat{c}_{k,i}^t = \left(\widehat{\widetilde{w}}_{k,i}^t\right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}_{\ell,i}^{I,t}\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}}.$$

Compute:

$$\widehat{A}_{k,i}^t = \frac{\widehat{\pi}_{k,ii}^t}{\left(\widehat{P}_{k,i}^{I,t}\right)^{\lambda}} \left(\widehat{c}_{k,i}^t\right)^{\lambda}.$$

For $t \geq T_{Data} + 1$ impose:

$$\widehat{A}_{k,i}^t = \widehat{A}_{k,i}^{T_{Data}}.$$

Step 4: Compute $\underline{x}_{k,i}^{T_{SS}}$ consistent with $\widetilde{w}_{k,i}^{T_{SS}}$ and $\widetilde{\lambda}_i^{T_{SS}}$. Obtain $\theta_{k,i}^{T_{SS}}$, $U_{k,i}^{T_{SS}}$, $s_{k\ell,i}^{T_{SS},T_{SS}+1}$ and $\pi_{k,oi}^{T_{SS}}$.

- Step 4a: Compute $\widehat{\widetilde{w}}_{k,i} = \frac{\widetilde{w}_{k,i}^{T_{SS}}}{\widetilde{w}_{k,i}^0}$. Iteratively solve for $\widehat{P}_{k,i}^I$ and $\widehat{c}_{k,i}$ using the system

$$\widehat{c}_{k,i} = \left(\widehat{\widetilde{w}}_{k,i}\right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}_{\ell,i}^I\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}}$$

$$\widehat{P}_{k,i}^I = \left(\sum_{o=1}^{N} \pi_{k,oi}^0 \widehat{A}_{k,o} \left(\widehat{c}_{k,o}\widehat{d}_{k,oi}\right)^{-\lambda}\right)^{-1/\lambda}$$

- Step 4b: Compute $\widehat{P}^F_{k,i}$:

$$\widehat{P}^F_i = \prod_{k=1}^{K} \left(\widehat{P}^I_{k,i}\right)^{\mu_{ki}}$$

- Step 4c: Compute

$$\widehat{\pi}_{k,oi} = \widehat{A}_{k,o} \left(\frac{\widehat{c}_{k,o}\widehat{d}_{k,oi}}{\widehat{P}^I_{k,i}}\right)^{-\lambda}$$

  And obtain $\pi^{T_{SS}}_{k,oi} = \pi^0_{k,oi}\widehat{\pi}_{k,oi}$

- Step 4d: Compute

  – $\widetilde{\kappa}^{T_{SS}}_{k,i} = \widetilde{\kappa}^0_{k,i}\frac{\widehat{P}^F_i}{\widehat{\widetilde{w}}_{k,i}}$

- Step 4e: Guess $\left\{\underline{x}^{T_{SS}}_{k,i}\right\}$

- Step 4f: Compute

$$\theta^{T_{SS}}_{k,i} = q_i^{-1}\left(\widetilde{\kappa}^{T_{SS}}_{k,i} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right) I_{k,i}\left(\underline{x}^{T_{SS}}_{k,i}\right)}\right)$$

- Step 4g: Compute Bellman Equations

$$U^{T_{SS}}_{k,i} = \zeta_i \log\left(\sum_{k'} \exp\left\{\frac{-C_{kk',i} + b_{k',i} + \theta^{T_{SS}}_{k',i}\widetilde{\kappa}^{T_{SS}}_{k',i}\widetilde{\lambda}^{T_{SS}}_i\widetilde{w}^{T_{SS}}_{k',i}\frac{\beta_{k',i}}{\left(1-\beta_{k',i}\right)} + \delta U^{T_{SS}}_{k',i}}{\zeta_i}\right\}\right)$$

- Step 4h: Compute

$$\left(\underline{x}^{T_{SS}}_{k,i}\right)' = \frac{\left(1-\delta\right)U^{T_{SS}}_{k,i} - \eta_{k,i}}{\widetilde{\lambda}^{T_{SS}}_i\widetilde{w}^{T_{SS}}_{k,i}}$$

- Step 4i: Update $\underline{x}^{T_{SS}}_{k,i} = \left(1-\lambda_x\right)\underline{x}^{T_{SS}}_{k,i} + \lambda_x\left(\underline{x}^{T_{SS}}_{k,i}\right)'$, for a small step size λ_x and go back to Step 4e until convergence.

- Step 4j: Compute $s^{T_{SS},T_{SS}+1}_{kk'}$

$$s^{T_{SS},T_{SS}+1}_{k\ell,i} = \frac{\exp\left\{\frac{-C_{k\ell,i}+b_{\ell,i}+\theta^{T_{SS}}_{\ell,i}\widetilde{\kappa}^{T_{SS}}_{\ell,i}\widetilde{\lambda}^{T_{SS}}_i\widetilde{w}^{T_{SS}}_{\ell,i}\frac{\beta_{\ell,i}}{1-\beta_{\ell,i}}+\delta U^{T_{SS}}_{\ell,i}}{\zeta_i}\right\}}{\sum_{\overline{k}}\exp\left\{\frac{-C_{k\overline{k},i}+b_{\overline{k},i}+\theta^{T_{SS}}_{\overline{k},i}\widetilde{\kappa}^{T_{SS}}_{\overline{k},i}\widetilde{\lambda}^{T_{SS}}_i\widetilde{w}^{T_{SS}}_{\overline{k},i}\frac{\beta_{\overline{k},i}}{1-\beta_{\overline{k},i}}+\delta U^{T_{SS}}_{\overline{k},i}}{\zeta_i}\right\}}$$

Step 5: Obtain series $\left\{\pi_{k,oi}^{t}\right\}_{t=T+1}^{T_{SS}}$ and $\left\{\widetilde{\kappa}_{k,i}^{t}\right\}_{t=1}^{T_{SS}}$.

- Step 5a: For $t = T_{Data}+1, ..., T_{SS}$ do:

Compute $\widehat{\widetilde{w}}_{k,i}^{t} = \frac{\widetilde{w}_{k,i}^{t}}{\widetilde{w}_{k,i}^{0}}$ and iteratively solve for $\widehat{P}_{k,i}^{I,t}$ and $\widehat{c}_{k,i}^{t}$ using the system

$$\widehat{c}_{k,i}^{t} = \left(\widehat{\widetilde{w}}_{k,i}^{t}\right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}_{\ell,i}^{I,t}\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}},$$

$$\widehat{P}_{k,i}^{I,t} = \left(\sum_{o=1}^{N} \pi_{k,oi}^{0} \widehat{A}_{k,o} \left(\widehat{c}_{k,o}^{t} \widehat{d}_{k,oi}\right)^{-\lambda}\right)^{-1/\lambda}.$$

- Step 5b: Compute $\widehat{P}_{k,i}^{F,t}$ for $t = 1, ..., T_{SS}-1$ (remember $\widehat{P}_{k,i}^{I,t}$ is data for $t = 1, ..., T_{Data}$):

$$\widehat{P}_{i}^{F,t} = \prod_{k=1}^{K} \left(\widehat{P}_{k,i}^{I,t}\right)^{\mu_{ki}}$$

- Step 5c: Compute $\widehat{\pi}_{k,oi}^{t}$ and $\pi_{k,oi}^{t}$ for $t = T_{Data}+1, ..., T_{SS}-1$.

For $t = 1, ..., T_{SS}-1$ do:
First Case: If $t \leq T_{Data}$ then $\widehat{\pi}_{k,oi}^{t}$ is data, so do:

$$\pi_{k,oi}^{t} = \pi_{k,oi}^{0} \widehat{\pi}_{k,oi}^{t}$$

End of First Case
Second Case if $t \geq T_{Data}+1$ do:

$$\widehat{\pi}_{k,oi}^{t} = \left(\widehat{A}_{k,o}^{t}\right)' \left(\frac{\widehat{c}_{k,o}^{t} \widehat{d}_{k,oi}^{t}}{\widehat{P}_{k,i}^{I,t}}\right)^{-\lambda}$$

$$\pi_{k,oi}^{t} = \pi_{k,oi}^{0} \widehat{\pi}_{k,oi}^{t}$$

End of Second Case

- Step 5d: Compute for $t = 1, ..., T_{SS}-1$

  $$-\ \widetilde{\kappa}_{k,i}^{t} \equiv \frac{\kappa_{k,i} P_{i}^{F,t}}{\widetilde{w}_{k,i}^{t}} = \frac{\kappa_{k,i} P_{i}^{F,0}}{\widetilde{w}_{k,i}^{0}} \frac{P_{i}^{F,t}}{P_{i}^{F,0}} \frac{\widetilde{w}_{k,i}^{0}}{\widetilde{w}_{k,i}^{t}} = \widetilde{\kappa}_{k,i}^{0} \frac{\widehat{P}_{i}^{F,t}}{\widehat{\widetilde{w}}_{k,i}^{t}}$$

Step 6: Given knowledge of $\widetilde{w}_{k,i}^{T_{SS}}$, $\widetilde{\lambda}_{i}^{T_{SS}}$ and $\underline{x}_{k,i}^{T_{SS}}$ (and therefore $J_{k,i}^{T_{SS}}(s)$), start at $t = T_{SS}-1$ and sequentially compute (backwards) for each $t = T_{SS}-1, ..., 1$

- Step 6a: Given $\widetilde{w}_{k,i}^{t}$, $\underline{x}_{k,i}^{t+1}$, $\widetilde{\kappa}_{k,i}^{t}$, $\widetilde{\lambda}_{i}^{t}$ and $J_{k,i}^{t+1}(s)$ compute $\theta_{k,i}^{t}$.

If $\frac{\widetilde{\lambda}_i^t \widetilde{\kappa}_{k,i}^t \widetilde{w}_{k,i}^t}{\delta \widehat{\phi}_i^{t+1} \int_{\underline{x}_{k,i}^{t+1}}^{x_{\max}} J_{k,i}^{t+1}(s) dG_{k,i}(s)} \leq 1$ then

$$\theta_{k,i}^{t} = q_i^{-1}\left(\frac{\widetilde{\lambda}_i^t \widetilde{\kappa}_{k,i}^t \widetilde{w}_{k,i}^t}{\delta \widehat{\phi}_i^{t+1} \int_{\underline{x}_{k,i}^{t+1}}^{x_{\max}} J_{k,i}^{t+1}(s)\, dG_{k,i}(s)}\right)$$

If $\frac{\widetilde{\lambda}_i^t \widetilde{\kappa}_{k,i}^t \widetilde{w}_{k,i}^t}{\delta \widehat{\phi}_i^{t+1} \int_{\underline{x}_{k,i}^{t+1}}^{x_{\max}} J_{k,i}^{t+1}(s) dG_{k,i}(s)} > 1$, it is not possible to satisfy $V_{k,i}^{t} = 0$, so that $V_{k,i}^{t} < 0$ and

$$\theta_{k,i}^{t} = 0.$$

- Step 6b: Given $\underline{x}_{k,i}^{t+1}$, $W_{k,i}^{t+1}(x) = \frac{\beta_{k,i}}{1-\beta_{k,i}} J_{k,i}^{t+1}(x) + U_{k,i}^{t+1}$ (for $x \geq \underline{x}_{k,i}^{t+1}$), $\theta_{k,i}^{t}$, $U_{k,i}^{t+1}$ compute $U_{k,i}^{t}$. Notice that $\int_{\underline{x}_{k,i}^{t+1}}^{x_{\max}} W_{k,i}^{t+1}(s)\, dG_{k,i}(s) = \frac{\beta_{k,i}}{1-\beta_{k,i}} \int_{\underline{x}_{k,i}^{t+1}}^{x_{\max}} J_{k,i}^{t+1}(s)\, dG_{k,i}(s) + \left(1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right) U_{k,i}^{t+1}$ so that:

$$U_{k,i}^{t} = \zeta_i \log\left(\sum_{k'} \exp\left\{\frac{\begin{array}{c} -C_{kk',i} + b_{k',i} \\ +\delta \widehat{\phi}_i^{t+1} \theta_{k',i}^{t} q_i\left(\theta_{k',i}^{t}\right) \frac{\beta_{k,i}}{1-\beta_{k,i}} \int_{\underline{x}_{k,i}^{t+1}}^{x_{\max}} J_{k,i}^{t+1}(s)\, dG_{k,i}(s) + \delta \widehat{\phi}_i^{t+1} U_{k',i}^{t+1} \end{array}}{\zeta_i}\right\}\right)$$

- Step 6c: Given $J_{k,i}^{t+1}(x)$, $\widetilde{w}_{k,i}^{t}$, $U_{k,i}^{t}$, $U_{k,i}^{t+1}$ and $\underline{x}_{k,i}^{t+1}$ compute $J_{k,i}^{t}(x)$

$$\begin{aligned} J_{k,i}^{t}(x) &= (1-\beta_{k,i}) \widetilde{\lambda}_i^t \widetilde{w}_{k,i}^t x + (1-\beta_{k,i}) \eta_{k,i} \\ &\quad - (1-\beta_{k,i})\left(U_{k,i}^{t} - \delta \widehat{\phi}_i^{t+1} U_{k,i}^{t+1}\right) + (1-\chi_{k,i}) \delta \widehat{\phi}_i^{t+1} \max\left\{J_{k,i}^{t+1}(x), 0\right\} \end{aligned}$$

- Step 6d: Solve for $\underline{x}_{k,i}^{t}$: $J_{k,i}^{t}\left(\underline{x}_{k,i}^{t}\right) = 0$

Step 7: Compute transition rates $\left\{s_{kk',i}^{t,t+1}\right\}_{t=1}^{T_{SS}-1}$ for all countries i according to:

$$s_{kk',i}^{t,t+1} = \frac{\exp\left\{\begin{array}{c} -C_{kk',i} + b_{k',i} + \\ \delta \widehat{\phi}_i^{t+1} \theta_{k',i}^{t} q(\theta_{k',i}^{t}) \frac{\beta_{k',i}}{1-\beta_{k',i}} \int_{\underline{x}_{k',i}^{t+1}}^{x_{\max}} J_{k',i}^{t+1}(x)\, dG_{k',i}(x) + \delta \widehat{\phi}_i^{t+1} U_{k',i}^{t+1} \end{array}\right\}}{\sum_{k''} \exp\left\{\begin{array}{c} -C_{kk'',i} + b_{k'',i} + \\ \delta \widehat{\phi}_i^{t+1} \theta_{k'',i}^{t} q(\theta_{k'',i}^{t}) \frac{\beta_{k'',i}}{1-\beta_{k'',i}} \int_{\underline{x}_{k'',i}^{t+1}}^{x_{\max}} J_{k'',i}^{t+1}(x)\, dG_{k'',i}(x) + \delta \widehat{\phi}_i^{t+1} U_{k'',i}^{t+1} \end{array}\right\}}.$$

Step 8: Start loop over t going forward ($t = 0$ to $t = T_{SS} - 1$)

Initial conditions: we know $\widetilde{u}_{k,i}^{t=-1} = u_{k,i}^{t=0}$, $L_{k,i}^{t=-1} = L_{k,i}^{t=0}$, and $\theta_{k,i}^{t=0}$ from the initial steady state computation. Obtain $\widetilde{u}_{k,i}^{t}$ and $L_{k,i}^{t}$ using flow conditions and sequences $\left\{\theta_{k,i}^{t}\right\}$, $\left\{\underline{x}_{k,i}^{t}\right\}$.

- Step 8a: Compute

$$JC_{k,i}^{t} = L_{k,i}^{t} u_{k,i}^{t} \theta_{k,i}^{t} q_i \left(\theta_{k,i}^{t}\right) \left(1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right)$$

$$JD_{k,i}^{t} = \left(\chi_{k,i} + (1-\chi_{k,i}) \max\left\{\frac{G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right) - G_{k,i}\left(\underline{x}_{k,i}^{t}\right)}{1 - G_{k,i}\left(\underline{x}_{k,i}^{t}\right)}, 0\right\}\right) L_{k,i}^{t-1}\left(1 - \widetilde{u}_{k,i}^{t-1}\right)$$

$$\widetilde{u}_{k,i}^{t} = \frac{L_{k,i}^{t} u_{k,i}^{t} - JC_{k,i}^{t} + JD_{k,i}^{t}}{L_{k,i}^{t}}$$

- Step 8b: Compute

$$L_{k,i}^{t+1} = L_{k,i}^{t} + IF_{k,i}^{t+1} - OF_{k,i}^{t+1},$$

where

$$IF_{k,i}^{t+1} = \sum_{\ell \neq k} L_{\ell,i}^{t} \widetilde{u}_{\ell,i}^{t} s_{\ell k,i}^{t+1,t+2},$$

and

$$OF_{k,i}^{t+1} = L_{k,i}^{t} \widetilde{u}_{k,i}^{t}\left(1 - s_{kk,i}^{t+1,t+2}\right).$$

- Step 8c: Compute

$$u_{k,i}^{t+1} = \frac{\sum_{\ell=1}^{K} L_{\ell,i}^{t} \widetilde{u}_{\ell,i}^{t} s_{\ell k,i}^{t+1,t+2}}{L_{k,i}^{t+1}}$$

- Step 8d: Compute

$$\begin{aligned}
\widetilde{L}_{k,i}^{t+1} &= L_{k,i}^{t}\left(1 - \widetilde{u}_{k,i}^{t}\right) \int_{\underline{x}_{k,i}^{t+1}}^{\infty} \frac{s}{1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)} dG_{k,i}(s) \\
&= L_{k,i}^{t}\left(1 - \widetilde{u}_{k,i}^{t}\right) \exp\left(\frac{\sigma_{k,i}^{2}}{2}\right) \frac{\Phi\left(\sigma_{k,i} - \frac{\ln \underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}
\end{aligned}$$

- Step 8e: Compute expenditure with vacancies

$$E_{k,i}^{V,t+1} = \widetilde{\kappa}_{k,i}^{t+1} \widetilde{w}_{k,i}^{t+1} \theta_{k,i}^{t+1} u_{k,i}^{t+1} L_{k,i}^{t+1}$$

- Step 8f: Solve for $\left\{Y_{k,i}^{t+1}\right\}$ in the system

$$E_{k,i}^{t+1} = \mu_{k,i} E_i^{C,t+1} + \sum_{\ell=1}^{K} \left(\mu_{k,i} E_{\ell,i}^{V,t+1} + (1-\gamma_{\ell,i}) \nu_{\ell k,i} Y_{\ell,i}^{t+1} \right).$$

$$Y_{k,o}^{t+1} = \sum_{i=1}^{N} \pi_{k,oi}^{t+1} E_{k,i}^{t+1}.$$

- Step 8g: Compute $\left(\widetilde{w}_{k,i}^{t+1}\right)' = \frac{\gamma_{k,i} Y_{k,i}^{t+1}}{\widetilde{L}_{k,i}^{t+1}}$

Step 9: Compute distance $dist\left(\left\{\widetilde{w}_{k,i}^{t}\right\}, \left\{\left(\widetilde{w}_{k,i}^{t}\right)'\right\}\right)$

- Step 9b: Update $\widetilde{w}_{k,i}^{t} = (1-\lambda_w)\, \widetilde{w}_{k,i}^{t} + \lambda_w \left(\widetilde{w}_{k,i}^{t}\right)'$ $t = 1, ..., T_{SS}$, for a small step size λ_w.
- Step 9c: At this point, we have a new series for $\left\{\widetilde{w}_{k,i}^{t}\right\}$ – go back to Step 3 until convergence of $\left\{\widetilde{w}_{k,i}^{t}\right\}$.

Step 10: Compute disposable income $\left\{I_i^t\right\}_{t=1}^{T_{SS}}$

$$I_i^t = \sum_{\ell=1}^{K} \left(\gamma_{\ell,i} Y_{\ell,i}^{t} - E_{\ell,i}^{V,t} \right)$$

Outer Loop: iteration on $\left\{NX_i^t\right\}$

Step 0: Compute changes in trade costs $\left\{\widehat{d}_{k,oi}^{t}\right\}_{t=1}^{T_{Data}}$:

$$\widehat{d}_{k,oi}^{t} = \left(\frac{\widehat{\pi}_{k,oo}^{t}}{\widehat{\pi}_{k,oi}^{t}} \right)^{1/\lambda} \frac{\widehat{P}_{k,i}^{I,t}}{\widehat{P}_{k,o}^{I,t}}.$$

Set $\widehat{d}_{k,oi}^{t} = \widehat{d}_{k,oi}^{T_{Data}}$ for $t > T_{Data}$.

Step 1: Start with the estimated state equilibrium at $t = 0$. Remember that we used the normalization $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$ during the estimation procedure. **Change the normalization** from $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$ to $\sum_{i=1}^{I} E_i^C = 1$. Nominal variables to be renormalized: $\left\{Y_{k,i}^0\right\}$, $\left\{\widetilde{w}_{k,i}^0\right\}$, $\left\{E_i^{C,0}\right\}$, $\left\{NX_i^0\right\}$.

Step 2: Compute $E_i^{C,t} = \frac{E_i^{C,0}}{\sum\limits_{i=1}^{N} E_i^{C,0}} \frac{\left(E_i^{C,t}\right)_{Data}}{\left(E_i^{C,0}\right)_{Data}}$ for $t = 1, ..., T_{Data}$ where $E_i^{C,0}$ is aggregate consumption expenditure in the estimated steady state, and $\left(E_i^{C,t}\right)_{Data}$ comes directly from the data. Normalize $E_i^{C,t}$ to ensure that $\sum\limits_{i=1}^{N} E_i^{C,t} = 1$ in every period.

Step 3: Normalize $\widehat{\phi}_{US}^t = 1$ for all $t = 1, ..., T_{SS}$. This yields:

$$R^{t+1} = \frac{E_{US}^{C,t+1}}{\delta E_{US}^{C,t}} \text{ for } t = 1, ..., T_{Data} - 1.$$

Obtain remaining shocks $\left\{\widehat{\phi}_i^t\right\}_{t=2}^{T_{Data}}$ using:

$$\widehat{\phi}_i^{t+1} = \frac{E_i^{C,t+1}}{\delta E_i^{C,t} R^{t+1}} \text{ for } t = 1, ..., T_{Data} - 1.$$

Step 4: Obtain B_i^0 with respect to the normalization $\sum\limits_{i=1}^{I} E_i^C = 1$. Equation (28) gives us:

$$B_i^0 = \frac{NX_i^0}{\left(1 - \frac{1}{\delta}\right)}.$$

Step 5: Make initial guess for $NX_i^{T_{SS}}$ (with respect to the normalization $\sum\limits_{i=1}^{I} E_i^C = 1$).

Step 6: Compute steady state equilibrium at T_{SS}, conditional on $NX_i^{T_{SS}}$, $\widehat{A}_{k,i}^{T_{SS}}$ and $\widehat{d}_{k,oi}^{T_{SS}}$. If this is the first iteration of the outer loop, impose $\widehat{A}_{k,i}^{T_{SS}} = 1$. Otherwise, feed $\widehat{A}_{k,i}^{T_{SS}}$ resulting from Step 9 below.

- Step 6a: Notice that the steady-state algorithm uses the normalization $\sum\limits_{i=1}^{I}\sum\limits_{k=1}^{K} Y_{k,i} = 1$. Normalize $NX_i^{T_{SS}}$ with respect to normalization $\sum\limits_{i=1}^{I}\sum\limits_{k=1}^{K} Y_{k,i} = 1$. To perform such normalization, use revenue $\left\{Y_{k,i}^{T_{SS}}\right\}$ obtained in the initial steady state if this is the first outer loop iteration, otherwise use revenue $\left\{Y_{k,i}^{T_{SS}}\right\}$ obtained in Step 9 below.

- Step 6b: After computing the final steady state, **change the normalization** from $\sum\limits_{i=1}^{I}\sum\limits_{k=1}^{K} Y_{k,i} = 1$ to $\sum\limits_{i=1}^{I} E_i^C = 1$ using $\left\{E_i^C\right\}$ obtained in Step 3a. Nominal variables to be renormalized: $\left\{Y_{k,i}^{T_{SS}}\right\}$, $\left\{\widetilde{w}_{k,i}^{T_{SS}}\right\}$, $\left\{E_i^{C,T_{SS}}\right\}$, $\left\{NX_i^{T_{SS}}\right\}$.

Step 7: Impose $E_i^{C,t} = \begin{cases} E_i^{C,T_{Data}} + \frac{E_i^{C,T_{SS}} - E_i^{C,T_{Data}}}{\widetilde{T} - T_{Data}} (t - T_{Data}) \text{ for } t = T_{Data} + 1, ..., \widetilde{T} \\ E_i^{C,T_{SS}} \text{ for } t > \widetilde{T} \end{cases}$.

That is, $E_i^{C,t}$ evolves linearly between T_{Data} and $\widetilde{T}$ when it reaches its steady state value determined in Step 6.

Step 8: Compute

$$R^{t+1} = \frac{E_{US}^{C,t+1}}{\delta E_{US}^{C,t}} \text{ for } t \geq T_{Data}.$$

And obtain remaining shocks $\left\{\widehat{\phi}_i^t\right\}_{t=T_{Data}+1}^{T_{SS}}$ using

$$\widehat{\phi}_i^{t+1} = \frac{E_i^{C,t+1}}{\delta E_i^{C,t} R^{t+1}} \text{ for } t \geq T_{Data}.$$

Step 9: Solve for the out-of-steady-state dynamics conditional on aggregate expenditures $\left\{E_i^{C,t}\right\}_{t=0}^{T_{SS}}$, on preference shifters $\left\{\widehat{\phi}_i^t\right\}_{t=2}^{T_{SS}}$ and trade cost shocks $\left\{\widehat{d}_{k,oi}^t\right\}_{t=1}^{T_{SS}}$.

Step 10: Using the path for disposable income $\left\{I_i^t\right\}_{t=1}^{T_{SS}}$ obtained in Step 9 and equation (5) compute:

$$\left(NX_i^t\right)' = I_i^t - E^{C,t} \text{ for } 1 \leq t < T_{SS}$$

$$\left(NX_i^{T_{SS}}\right)' = -\frac{1-\delta}{\delta} \frac{1}{\left(\prod_{\tau=1}^{T_{SS}-1} \left(R^\tau\right)^{-1}\right)} \left(B_i^0 + \sum_{t=1}^{T_{SS}-1} \left(\prod_{\tau=1}^{t} \left(R^\tau\right)^{-1}\right) \left(NX_i^t\right)'\right)$$

Step 11: Compute

$$dist\left(\left\{NX_i^{T_{SS}}\right\}, \left\{\left(NX_i^{T_{SS}}\right)'\right\}\right).$$

Step 12: Update $NX_i^{T_{SS}}$

$$NX_i^{T_{SS}} = \left(1 - \lambda_o\right) NX_i^{T_{SS}} + \lambda_o \left(NX_i^{T_{SS}}\right)'$$

for a small step size λ_o.

Go back to Step 6 until convergence of $\left\{NX_i^{T_{SS}}\right\}$.