

From Population Growth to TFP Growth

Hiroshi Inokuma
Bank of Japan

Juan M. Sánchez
St. Louis Fed

Indiana University
September 12, 2024

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Introduction

Motivation

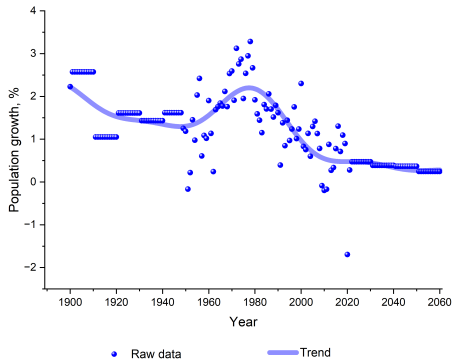
A growing concern in developed economies is that the slowing population growth may translate into lower economic growth.

Goal

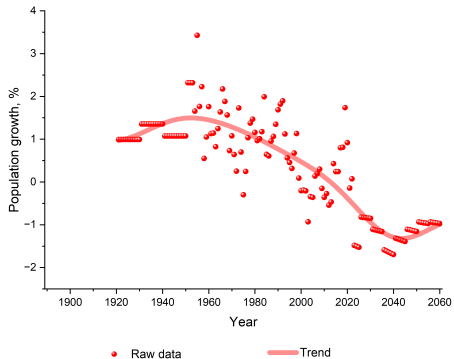
Study the impact of population growth on TFP growth.

Slowdown in population growth

United States

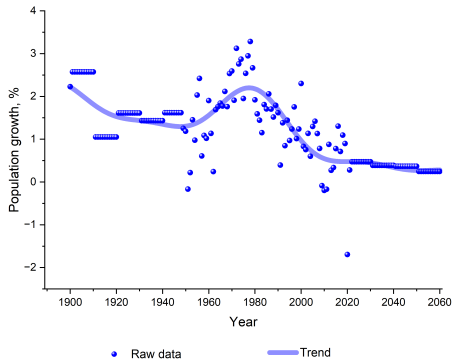


Japan

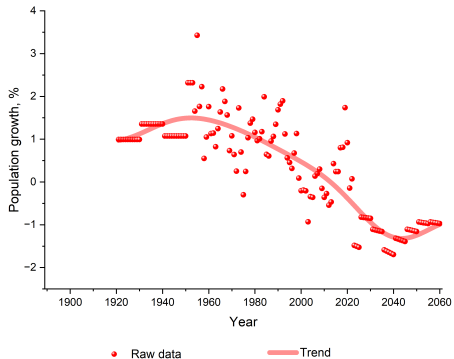


Slowdown in population growth

United States



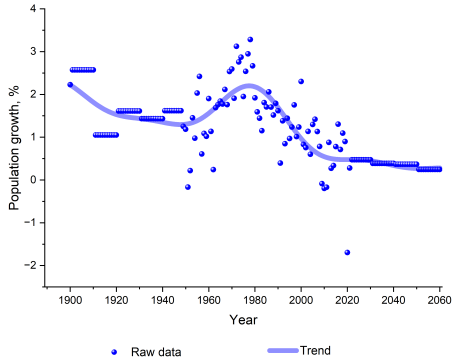
Japan



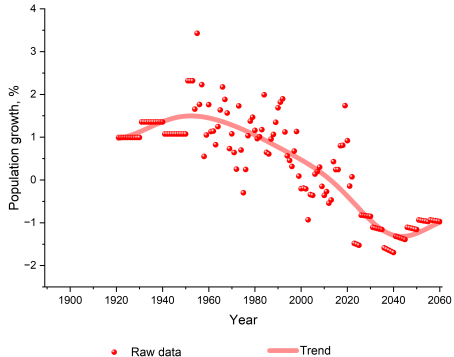
Trend population growth in the US was close to 2.2%, and it will be less than 0.3% in 2060.

Slowdown in population growth

United States



Japan



Trend population growth in the US was close to 2.2%, and it will be less than 0.3% in 2060.

In Japan, the trend population growth was close to 1.5% in 1950 and will be below -1.3% in 2040.

This paper: Theory

Extend Hopenhayn (1992)'s framework to incorporate endogenous growth.

- We purposely abstract away from scale effects (common in growth models) and idiosyncratic distortions (common in firm dynamics models).
- Focus on the **business demographics** channel for the impact of population growth on productivity growth.

Characterize the balanced growth path.

Find a “**sufficient statistic**” determining the sign and magnitude of this channel.

- The growth rate of the size of old surviving business.

This paper: Quantitative

Calibrate the balanced growth path (BGP) of the model to the US and Japan for the period 1980-1999.

Find a significant **long-run** impact of population growth on TFP growth:

- ▶ In the US, a drop in population growth as projected for 1970-2060, implies a long-run decline in productivity growth of about 0.3pp.
- ▶ In Japan, the predicted drop in population growth for 1950-2060 implies a long-run decline in productivity growth of about 0.6 pp.

Solve transitions driven only by population growth for the US and Japan to analyze the impact on the last 40 years.

Validate the model's quantitative predictions by comparing the results with local projections and IV regressions using US state-level data.

Related literature

Firm dynamics models

- ▶ Population growth slowdown reduces business dynamism
- ▶ Karahan, Pugsley and Sahin (2019), **Hopenhayn, Neira and Singhania (2022)**

Endogenous growth

- ▶ Long-run analysis of negative population growth and living standards (Jones, 2020, 2022)

Growth, business dynamism, population growth

- ▶ Engbom (2018), Akcigit and Ates (2019), **Alon, Berger, Dent and Pugsley (2018)** , **Peters and Walsh (2022)**, Kalyani (2022)

Other factors of production

- ▶ Cooley, Henriksen and Nusbaum (2019) study the impact through K and L
- ▶ Vandenbroucke (2021) analyzes the impact through H because of workers' age composition

The Economy

- ▶ Entrepreneurs start and run firms.
- ▶ Workers and entrepreneurs pool resources and live in a single household.
- ▶ This household solves

$$\max_{\{c_t\}, \{k_t\}} \sum_{t=1}^{\infty} \frac{(\beta g_{M_t})^{t-1} c_t^{1-\epsilon}}{1-\epsilon} \quad \text{s.t.} \quad c_t + g_{M_{t+1}} k_{t+1} = w_t + s_t + r_t k_t + (1-\delta)k_t, \quad (1)$$

where $k_t \equiv K_t/M_t$, $s_t \equiv S_t/M_t$, δ is the depreciation rate, and β is the discount factor.

Starting a business

- ▶ Entrepreneurs must pay a fixed cost $w_t c_E$ to get a project.
- ▶ They choose how far ahead of the pack the project is, \hat{x}/χ , where \hat{x} is the productivity aim, and χ is the average productivity of successful projects.
 - Innovators stand on the shoulders of previous innovations (Aghion and Howitt, 1992).

- ▶ The cost of research choosing $g = \hat{x}/\chi$ is

$$R(\hat{x}/\chi) = \frac{1}{z_R} \left(\frac{\hat{x}}{\chi} \right)^\iota, \quad \iota > 2.$$

- ▶ Then, innovators develop ideas to start their businesses. The probability of entering the market (σ) hinges on the amount of money spent on developing the project,

$$D(\sigma) = \frac{\sigma^2}{2z_D}.$$

Value of starting a project

Each firm in the economy operates a decreasing return to scale technology, hires labor, and rents capital, given its productivity x_i

$$S(x_i; w, r) = \max_{k_i, l_i} \{x_i^\zeta k_i^\alpha l_i^{1-\alpha-\zeta} - wl_i - rk_i\}. \quad (2)$$

The value of a project started with potential productivity \hat{x} is

$$I(\hat{x}; \{w_t\}, \{r_t\}) = \sum_{t=1}^{\infty} \hat{\beta}_t \mathbb{E}_{\hat{x}} [S(x_t; w_t, r_t) | \hat{x}],$$

where $\hat{\beta}_t$ is the market discount factor.

Choice of the step size of innovation

At the time of innovation, an entrepreneur chooses σ and \hat{x} to maximize its payoff,

$$V(\{w_t\}, \{r_t\}, \chi_t) = \max_{\sigma, \hat{x}_t} \underbrace{\sigma I(\hat{x}_t; \{w_t\}, \{r_t\})}_{\text{Revenue from project}} - \underbrace{w_t R(\hat{x}_t / \chi_t)}_{\text{Research cost}} - \underbrace{w_t D(\sigma)}_{\text{Development cost}} \quad (3)$$

The solution provides:

- ▶ The step size of innovation chosen by the entrepreneur g_t^* or \hat{x}_t^* / χ_t .
- ▶ The value for an entrepreneur of starting a firm, V_t .

Free-entry implies:

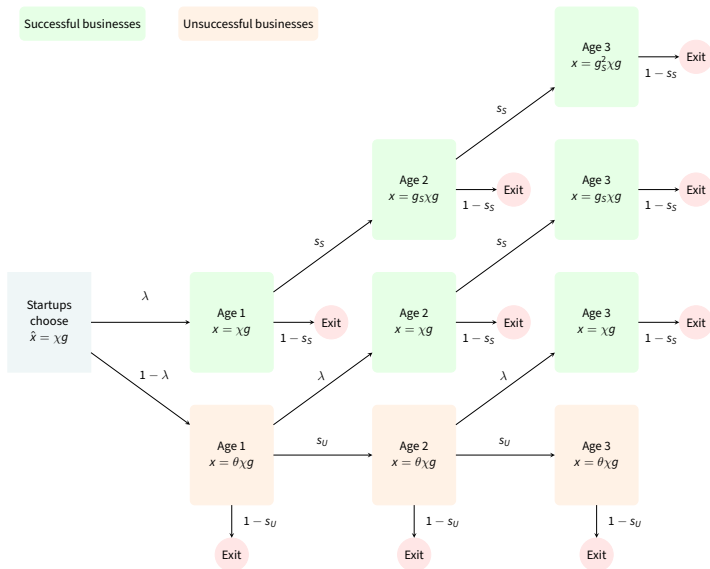
$$V_t = w_t c_E. \quad (4)$$

- ▶ The assumption that the entry cost increases one-to-one with wages makes the model tractable and is common in growth models (e.g. Klette and Kortum, 2004). The assumption is also supported by the data presented in Klenow and Li (2022).

A firm's life-cycle

Successful businesses

Unsuccessful businesses



Productivity and age

The reason for this simplified structure for productivity is that it allows us to construct some useful expressions for a business life-cycle.

The businesses born a years ago (i.e., those age a today) can be divided into businesses that today are (i) out of business, (ii) unsuccessful, and (iii) successful.

The age a unsuccessful businesses contribution to the average productivity relative to potential productivity \hat{x} is

$$\Lambda_{U,a} \equiv \theta(1 - \lambda)^a (s_U)^{a-1},$$

The same expression for successful businesses is

$$\Lambda_{S,a}(g_S) \equiv \sum_{j=1}^a [g_S^{j-1} (s_U)^{a-j} (1 - \lambda)^{a-j} \lambda (s_S)^{j-1}].$$

Productivity and age

This notation is useful because we can write the expected productivity at age a of business with potential productivity is \hat{x} as

$$\mathbb{E}[x_a|\hat{x}] = (\Lambda_{S,a}(g_S) + \Lambda_{U,a})\hat{x}, \quad (5)$$

and the survival probability up to age a is

$$\Lambda_{S,a}(1) + \Lambda_{U,a}/\theta. \quad (6)$$

Productivity and age

$$X_{1,t} = \frac{\text{sum prod of age-1 businesses}}{\text{number of age-1 businesses}} = \frac{\hat{x}_t (\Lambda_{S,1}(g_S) + \Lambda_{U,1}) n_t}{(\Lambda_{S,1}(1) + \Lambda_{U,1}/\theta) n_t} = \hat{x}_t (\lambda + (1 - \lambda)\theta).$$

Similarly, for age 2 businesses, average productivity is simply

$$X_{2,t} = \frac{\hat{x}_{t-1} (\Lambda_{S,2}(g_S) + \Lambda_{U,2}) n_{t-1}}{(\Lambda_{S,2}(1) + \Lambda_{U,2}/\theta) n_{t-1}}.$$

And the average productivity of the pool of age-1 and age-2 businesses as

$$\begin{aligned} X_{1-2,t} &= \frac{\text{sum prod of age-1 and age-2 businesses}}{\text{number of age-1 and age-2 businesses}} \\ &= \frac{\hat{x}_t (\Lambda_{S,1}(g_S) + \Lambda_{U,1}) n_t + \hat{x}_{t-1} (\Lambda_{S,2}(g_S) + \Lambda_{U,2}) n_{t-1}}{(\Lambda_{S,1}(1) + \Lambda_{U,1}/\theta) n_t + (\Lambda_{S,2}(1) + \Lambda_{U,2}/\theta) n_{t-1}}. \end{aligned}$$

Aggregate productivity

Following this logic, the average productivity of all businesses in the economy is

$$\chi_t = \frac{\sum_{a=1}^{\infty} \hat{\chi}_{t-a+1} (\Lambda_{S,a}(g_S) + \Lambda_{U,a}) n_{t-a+1}}{\sum_{a=1}^{\infty} (\Lambda_{S,a}(1) + \Lambda_{U,a}/\theta) n_{t-a+1}}. \quad (7)$$

and the average productivity of successful projects is

$$\chi_t = \frac{\sum_{a=1}^{\infty} \hat{\chi}_{t-a+1} \Lambda_{S,a}(g_S) n_{t-a+1}}{\sum_{a=1}^{\infty} \Lambda_{S,a}(1) n_{t-a+1}},$$

This equation is crucial to solving the model since it depends on two key equilibrium variables: potential productivity $\hat{\chi}_t$ and number of entrants n_t .

Equilibrium

Definition 1

An equilibrium, given a sequence of labor supply $\{M_t\}$, is a sequence of prices (w_t, r_t) , firm choices $(l_{i,t}, k_{i,t}, g_t)$, household choices (c_t, k_t) , a measure of entrants n_t , and the number of successful and unsuccessful firms, $N_{t,S}$ and $N_{t,U}$, such that

- ▶ c_t and k_t solve household's optimization problem (1).
- ▶ $l_{i,t}$ and $k_{i,t}$ solve the establishment's static problem (2).
- ▶ g_t is the choice of innovation that arises from problem (3).
- ▶ Free entry condition (4) is satisfied.
- ▶ $n_t, N_{t,S}$, and $N_{t,U}$ are consistent with the law of motion.

$$N_{t,U} = \sum_a n_{t-a+1} \Lambda_{U,a} / \theta, \quad N_{t,S} = \sum_a n_{t-a+1} \Lambda_{S,a} (1), \quad N_t = N_{t,U} + N_{t,S}.$$

- ▶ Labor, capital, and output market clear every period.

Equilibrium choice of innovation

Lemma 1

The step size of innovation is constant, $g^* = \left(\frac{2C_{EZR}}{\iota - 2} \right)^{\frac{1}{\iota}}$.

Thus, the productivity growth rate of young firms, which is determined by g^* , and the productivity growth rate of old firms, which is g_S , will be constant.

Insight

The key for this result is the free entry condition. Many essential features of the economy affect the level of income but not the size of innovation. It resembles the result in Atkeson and Burstein (2010).

Balanced growth path equilibrium

Lemma 2

Given the constant growth rate of the labor supply, $g_M > S_{S,\infty}$, there is BGP equilibrium where

- ▶ w, K, C, N, X , and χ grow at constant rates. i.e. $g_w, g_K, g_C, g_N, g_X, g_\chi$ are constant.
- ▶ $N/L, L/M, g^*, r$, and σ are constant.

In particular,

- ▶ $g_N = g_M$, $g_X = g_\chi$, $g_w = (g_X)^{(1-\alpha)/\zeta}$, $r = \frac{g_w^\epsilon}{\beta} - (1 - \delta)$, $g^* = \left(\frac{2C_{EZR}}{\iota - 2}\right)^{\frac{1}{\iota}}$

Solving the model

Recall that the average productivity of successful business is

$$\chi_t = \frac{\sum_{a=1}^{\infty} \hat{\chi}_{t-a+1} \Lambda_{S,a}(g_S) n_{t-a+1}}{\sum_{a=1}^{\infty} \Lambda_{S,a}(1) n_{t-a+1}},$$

On a balanced growth, it simplifies to

$$\chi = \frac{\hat{\chi}_1 \sum_{a=1}^{\infty} \left(\frac{1}{g_X}\right)^{a-1} \left(\frac{1}{g_N}\right)^{a-1} \Lambda_{S,a}(g_S)}{\sum_{a=1}^{\infty} \left(\frac{1}{g_N}\right)^{a-1} \Lambda_{S,a}(1)}.$$

where g_N and g_X are used to account for the increase in the number of businesses n_t and potential productivity $\hat{\chi}_t$ over time.

Solving the model

Since $g_M = g_N$, $\hat{x}_1 = g_X \hat{x}_0$ and $\hat{x}_0 = \chi g^*$, and we have a closed form for g^* , we can rewrite

$$\chi = \frac{\hat{x}_1 \sum_{a=1}^{\infty} \left(\frac{1}{g_X}\right)^{a-1} \left(\frac{1}{g_N}\right)^{a-1} \Lambda_{S,a}(g_S)}{\sum_{a=1}^{\infty} \left(\frac{1}{g_N}\right)^{a-1} \Lambda_{S,a}(1)}.$$

as an equation to solve for productivity growth rate g_X as a function of population growth g_M ,

$$\left(\frac{2C_{EZ_R}}{\iota - 2}\right)^{\frac{1}{\iota}} = g^* = \frac{\sum_{a=1}^{\infty} \left(\frac{1}{g_M}\right)^a \Lambda_{S,a}(1)}{\sum_{a=1}^{\infty} \left(\frac{1}{g_X g_M}\right)^a \Lambda_{S,a}(g_S)}. \quad (8)$$

It is clear in equation (8) that there are two sources of growth determining g_X : g_S and g .

We can also see there is a role for population growth g_M in determining productivity growth g_X .

Growth rate of the size of surviving old businesses

Lemma 3

In a balanced growth equilibrium, the employment growth rate of surviving businesses converges monotonically to g_S/g_X as the age $\rightarrow \infty$.

\Rightarrow Old surviving businesses' employment growth (or size growth) is g_S/g_X .

Sign of the impact of g_M on g_X

The role of the growth rate of the size of surviving old businesses

Lemma 4

In a balanced growth equilibrium, if the growth rate of the size of surviving old businesses is negative, then an increase in the labor force growth rate g_M raises productivity growth g_X ; i.e.,

$$\text{If } g_S/g_X < 1 \Rightarrow dg_X/dg_M > 0.$$

Magnitude of the impact of g_M on g_X

The role of the growth rate of the size of surviving old businesses

Lemma 5

Suppose there are two economies 1 and 2, with:

- ▶ $(g_X)_1 = (g_X)_2$ and $(g_M)_1 = (g_M)_2$, but
- ▶ $(g_S/g_X)_1 < (g_S/g_X)_2 < 1$.

Then $(dg_X/dg_M)_1 > (dg_X/dg_M)_2$

Intuition in a simple example

To gain intuition from a simple expression, assume all new businesses become successful at age 1 ($\lambda_0 = 1$). Note that in this case,

$$\text{share of incumbent} = \frac{s \times n/g_M + s^2 \times n/g_M^2 + s^3 \times n/g_M^3 + \dots}{\underbrace{n}_{\text{new}} + \underbrace{s \times n/g_M + s^2 \times n/g_M^2 + s^3 \times n/g_M^3 + \dots}_{\text{incumbent}}} = \frac{s}{g_M},$$

Then, total productivity growth is simply

$$g_X = g_S \times (s/g_M) + g^* \times (1 - s/g_M)$$

This equation immediately clarifies that:

- ▶ there are two sources of growth (g_S and g), and
- ▶ “composition” role of population growth (g_M) on productivity growth.

Toward a quantitative model

In the model developed so far, productivity growth depends on the exogenous parameters g_S and s , and on a constant value of g^*

Why a quantitative model?

- ▶ To obtain magnitudes for this channel once the model is fitted to data.
 - We use business dynamics data for US and Japan.
- ▶ To consider richer models with endogenous g_S , s , and g^* .
 - How general are the analytical results?
- ▶ To evaluate the importance of equilibrium effects.

Two features added for the benchmark quantitative model

(1) **Congestion:** The cost of entry is a function of the number of entrants

$$V_t = w_t c_E (n_t / M_t)^\phi,$$

with ϕ calibrated as in Karahan, Pugsley and Sahin (2019).

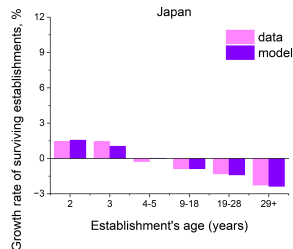
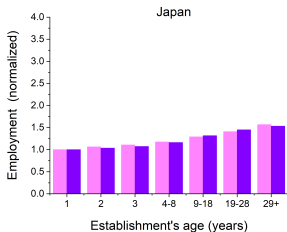
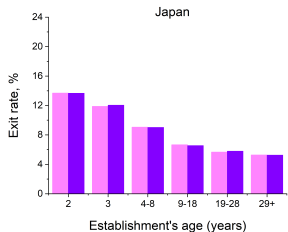
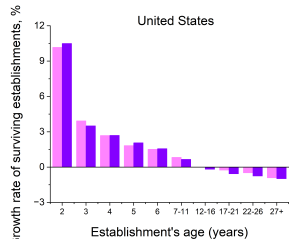
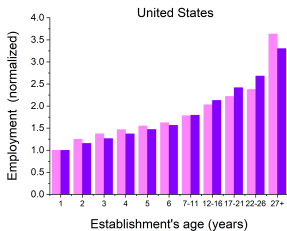
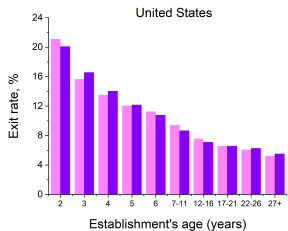
(2) **Spillovers:** The growth rate of old firms, g_s , is a function of the average productivity growth of successful firms.

$$g_{s_t} = \bar{g}_s + \gamma (g_{x_{t-1}} - \bar{g}_x),$$

Parameters calibrated to US and Japan

Parameter	Value	Basis
Entry cost, c_E	1	Normalization
Decreasing returns, ζ	0.2	Standard
Capital share, α	0.32	Standard
Depreciation rate, δ	0.07	Standard
Risk aversion, ϵ	2	Standard
Discount factor, β	0.96	Standard
Labor force growth rate, g_M	(1.0143, 1.0103)	Average g_M 1980-1999
Research cost exponent, ι	2.56	GHS
Convexity of aggregate entry cost, ϕ	0.55	KPS
Elasticity of g_S to g_X , γ	0.342	See Appendix.
Research cost slope, z_R	(0.933, 1.762)	Average prod. growth
Development cost slope, z_D	(2.413, 1.417)	Average estab. size
Jump of prod. at success, $1/\theta$	(16.5, 28.1)	Average size by age
Success probability, λ_a	See Appendix	Growth of estab.
Productivity growth of successful estab., \bar{g}_S	(1.054, 1.023)	Growth of old estab.
Survival of successful estab., s_S	(0.965, 0.973)	Exit rate of old estab.
Survival of unsuccessful estab., $s_{U,a}$	See Appendix	Life-cycle profile of exit rate

Fit of life-cycle profiles



Impact of population growth on TFP growth: BGPs

Recall that we have the model calibrated for the US and Japan for the period 1980-1999.

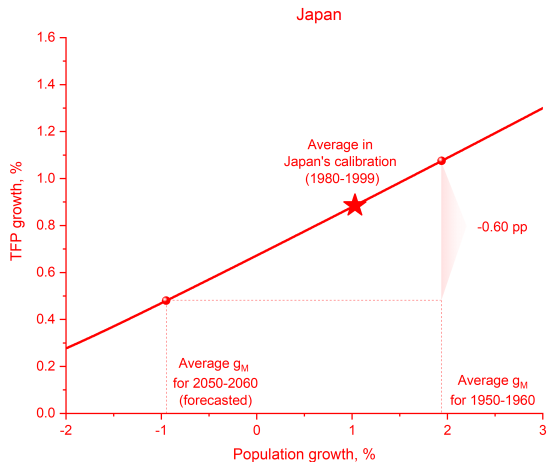
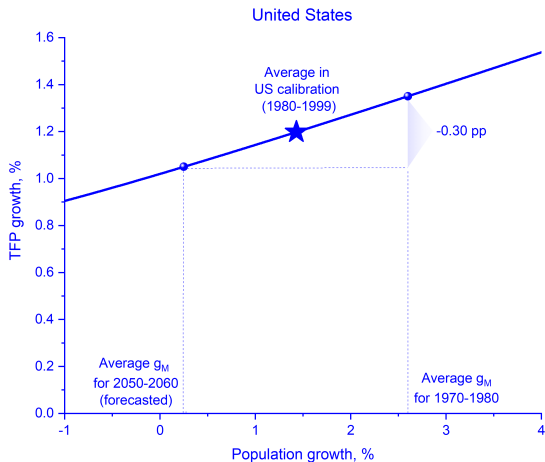
Now we take those models, and for a range of values of the exogenous variable, g_M , we report the productivity growth rate to which these economies would converge (a BGP for each g_M).

We report the growth in measured TFP, which in our model across BGP is simply

$$g_{TFP} = g_X^{\frac{\zeta(1-\tilde{\alpha})}{1-\alpha}}$$

(more on this later)

Impact of population growth on TFP growth: BGPs



Role of added features on the impact of g_M on g_{TFP}

Periods	Data's growth in labor force, %	Model's implied growth in TFP in the BGP, %			
		(A) Benchmark	(B) No congestion	(C) No spillover	(D) Simplest
United States					
1900-1910	2.60	1.35	1.31	1.32	1.29
1980-1999	1.43	1.20	1.20	1.20	1.20
2050-2060	0.25	1.05	1.08	1.08	1.10
Difference in pp	-2.35	-0.30	-0.23	-0.24	-0.19
Japan					
1950-1960	1.94	1.08	1.05	1.03	1.01
1980-1999	1.03	0.89	0.89	0.89	0.89
2050-2060	-0.95	0.48	0.53	0.58	0.62
Difference in pp	-2.89	-0.60	-0.52	-0.45	-0.39

Endogenous exit: Changes to the model

Incorporate a fixed cost shock for unsuccessful businesses.

A business with expected discounted profits l_a smaller than the fixed cost $c_a w \varepsilon$ exits.

The survival probability for unsuccessful businesses is given by

$$s_{u,a} = s_{u,\infty} \Pr(c_a w \varepsilon \leq l_a) = s_{u,\infty} \times F(l_a / (w c_a)),$$

where F is the distribution of ε . For tractability, we assume that $\varepsilon \sim \text{Weibull}(1, \vartheta)$.

Calibration:

- ▶ c_a to get the same age profile of the exit rate as the exogenous exit case.
- ▶ $\vartheta = 0.74$ such that as population growth declines from 2.66% to 0.78%, the exit rate declines by 0.88 percentage points as in Hopenhayn, Neira and Singhania (2022).
- ▶ As a reference, with exogenous exit, the composition effect due to firms getting older would imply a decline of 0.72 percentage points.

Endogenous innovation: Changes to the model

The problem of a successful business is

$$I_a(x, X, w) = S(x, X, w) + \frac{S}{1+r-\delta} \max_{g_S} [I_{a+1}(g_S x, g_X X, g_W w) - C_S(g_S)w],$$

where $C_S(g_S)$ is the cost function of achieving productivity growth of g_S .

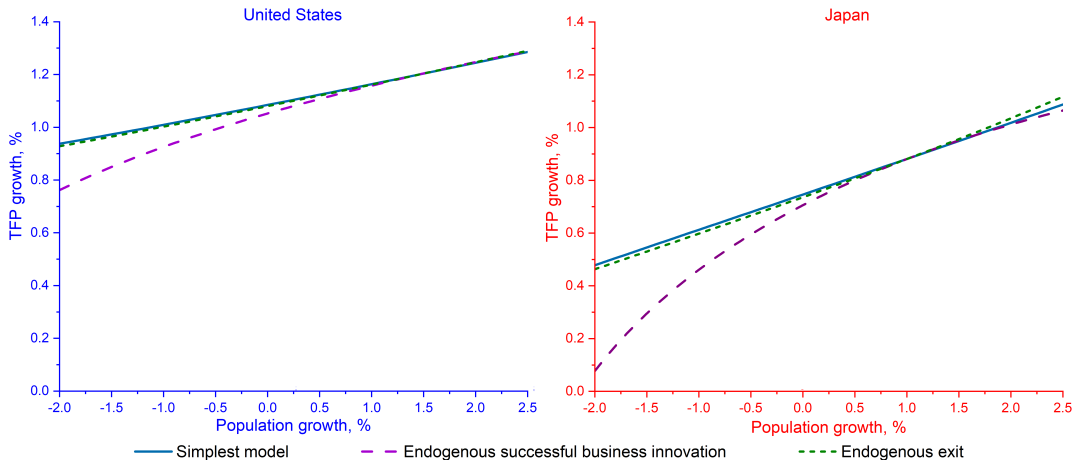
Assume

$$C_S(g_S) = c_S g_S^{\xi} \frac{X}{X^{\xi} X^{1-\xi}}.$$

Calibrate:

- ▶ c_S to have the same g_S in the reference period (1980-1999) as in our benchmark quantitative model
- ▶ ξ to have the elasticity of g_S to g_X as in the case of spillovers (around 0.3).

Endogenous exit and successful businesses innovation



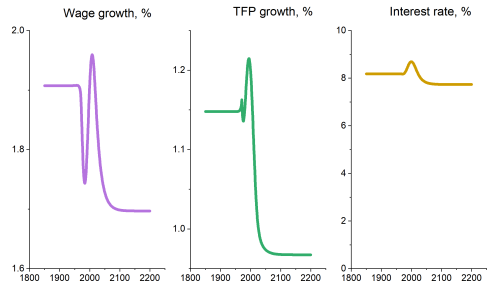
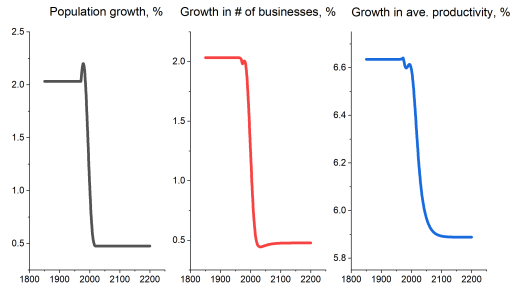
Impact on TFP growth in the last 40 years

How much of the decline in TFP growth over the last 40 years can be accounted for with population growth?

To answer this question, we compute transitions across two different BGPs.

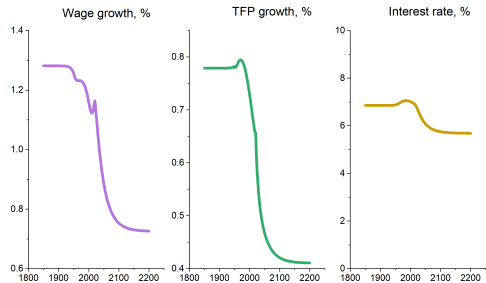
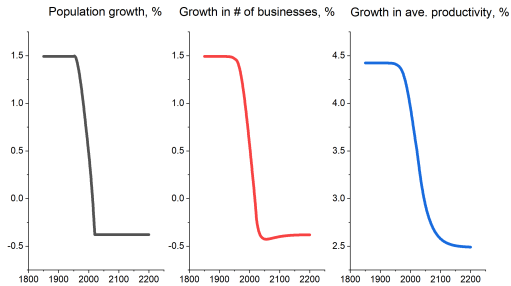
- ▶ The first values of g_M for US and Japan correspond to labor force growth in the first years there is a significant decline in its trend:
 - 1950 for Japan
 - 1970 for the US
- ▶ The final values of g_M are for the value of the trend in 2020 (alternatively, we consider forecasts for 2060).

Transition in the US



Year

Transition in Japan



Year

Significant impact on TFP growth the last 40 years

	United States		Japan	
	Change in g_{TFP} 1980-99 – 2000-19	Share accounted for	Change in g_{TFP} 1980-99 – 2000-19	Share accounted for
Data	0.185	—	0.418	—
Benchmark	0.088	47.5%	0.073	17.4%
No congestion	0.078	42.0%	0.064	15.4%
No spillover	0.073	39.3%	0.052	12.4%
Simplest	0.067	36.2%	0.046	11.0%

Expected future decline

Country	United States	Japan
<i>Benchmark</i>		
Between 2020 and 2050	-0.05	-0.19
Between 2020 and 2100	-0.05	-0.24
<i>Including forecast for g_M</i>		
Between 2020 and 2050	-0.05	-0.27
Between 2020 and 2100	-0.08	-0.35

Why? Sluggish response of measured TFP growth

Measured TFP is

$$TFP \equiv \frac{Y}{K^{\tilde{\alpha}} M^{1-\tilde{\alpha}}},$$

where the share of capital $\tilde{\alpha}$ is simply

$$\tilde{\alpha} \equiv 1 - \text{labor share of income} = 1 - \frac{wM}{Y}.$$

Note that $\tilde{\alpha}$ is different from α , and it is constant in a BGP but may vary along a transition and across BGPs. Starting from the definition of TFP, we can obtain

$$TFP = \left(\frac{\alpha}{r}\right)^{\frac{\alpha-\tilde{\alpha}}{1-\alpha}} \left(\frac{L}{M}\right)^{1-\tilde{\alpha}} \left(\frac{L}{N}\right)^{\frac{-\zeta(1-\tilde{\alpha})}{1-\alpha}} (X)^{\frac{\zeta(1-\tilde{\alpha})}{1-\alpha}}.$$

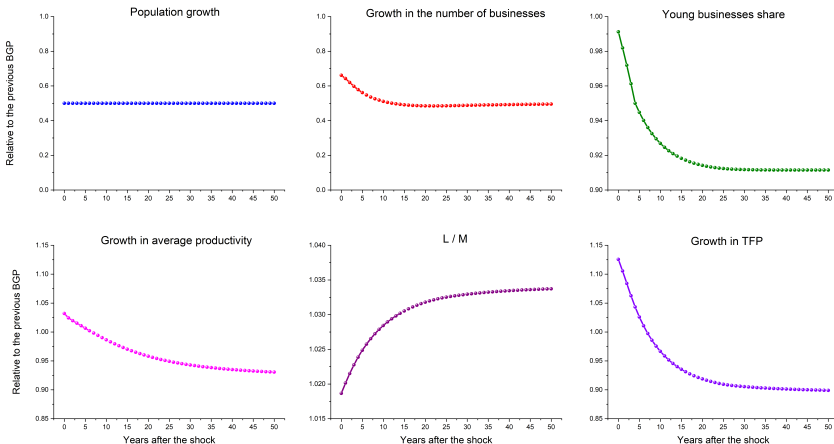
As mentioned before, given that in a BGP $g_M = g_L = g_N$ and $g_r = 0$, growth in TFP along a BGP is

$$g_{TFP} = g_X^{\frac{\zeta(1-\tilde{\alpha})}{1-\alpha}}.$$

Why? Sluggish response of g_{TFP} to changes in g_M

We carry out an experiment in the model calibrated for the US.

- ▶ A permanent fall in population growth from 2% to 1% in the year designated as zero.



Empirical analysis

The goal is to study the productivity dynamics after a change in g_M .

There are three challenges:

1. There are no long time series of state-level TFP.
2. As shown before, the response is non-monotonic.
3. Endogeneity.

Productivity dynamics after a change in g_M

- ▶ We deal with problem of lack of data by using labor productivity instead of TFP.
- ▶ To deal with the non-monotonic dynamics, we use local projections as in Jordà (2005).
- ▶ Our left-hand side variable is the change in TFP growth,

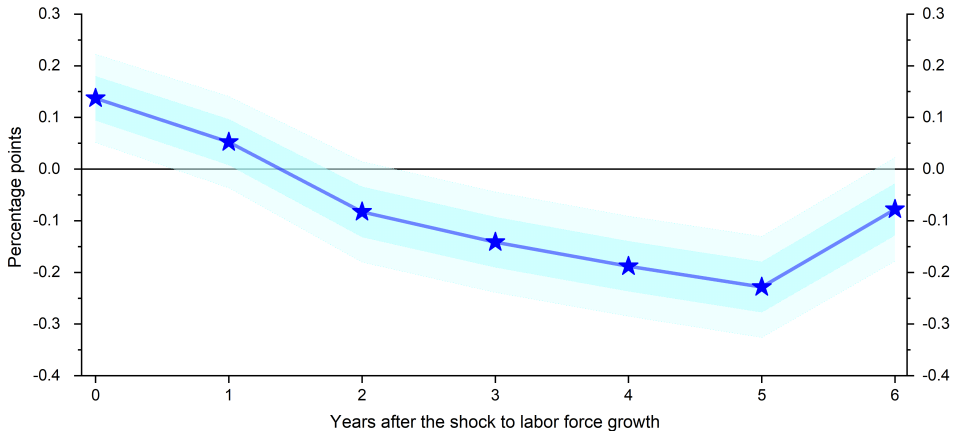
$$\Delta(g_{prod})_{t+i,t-1}^S = g_{prod,t+i}^S - g_{prod,t-1}^S$$

- ▶ Then, for each $i = 0, 1, \dots, 10$, we run the regression

$$\Delta(g_{prod})_{t+i,t-1}^S = \beta_0^i + \beta_1^i \times \Delta(g_M)_{t,t-1}^S + \text{controls.}$$

Productivity dynamics after a change in g_M

Change in labor productivity growth after a 1pp decline in labor force growth



Note: The shaded areas represent one (darker) and two (lighter) standard error bands.

Cross-sectional OLS and IV regressions for US states

- ▶ We obtain a similar coefficient than in periods 3, 4, and 5 in a cross-sectional (across state) regression of the average g_M on the average g_{prod} since 1999 to 2019.
- ▶ The advantage of this analysis is that we can follow Karahan, Pugsley and Sahin (2019) and use the birth rate lagged by 15 years as an instrument for the average labor force growth.

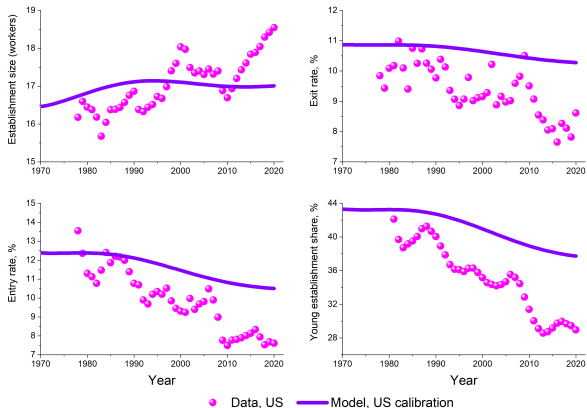
Cross-sectional OLS and IV regressions for US states

Dependent variable	OLS			IV, lagged birth rate		
	Equal	state's weight log(Population)	Population	Equal	state's weight log(Population)	Population
Average g_{prod}	0.182 (0.000)	0.190 (0.000)	0.232 (0.000)	0.202 (0.044)	0.194 (0.042)	0.243 (0.018)
log(Initial income pc)	0.007 (0.026)	0.007 (0.026)	0.006 (0.010)	0.007 (0.019)	0.007 (0.019)	0.006 (0.012)
log(Population)	-0.001 (0.239)	-0.000 (0.276)	-0.000 (0.814)	-0.001 (0.227)	-0.000 (0.267)	-0.000 (0.776)
R-squared	0.310	0.312	0.424	0.308	0.312	0.423
First-stage reg F stat	—	—	—	31.788	28.821	5.218
Observations	49	49	49	49	49	49

Note: There is also a constant in each regression, and the values in parenthesis are the p-values corresponding to robust standard errors.

Decline of business dynamism in the US

- ▶ The final validation exercise is to look at the statistics highlighted in the “business dynamism” literature and compare the predictions of the model and the data.



Conclusions

- ▶ The impact of population growth on TFP growth across BGPs depends on a sufficient statistic: the growth rate of the size of surviving old establishments.
- ▶ The calibrations for Japan and the US suggest a significant impact of population growth on TFP growth across BGPs.
- ▶ In the transition, population growth takes a long time to affect TFP growth because of two counterbalancing forces.
- ▶ Population growth may account for up to around 47% and 17% of the slowdown of TFP in the last 40 years in the US and Japan, respectively.
- ▶ The model predicts a significant decline in TFP growth in the rest of the century in both countries, even assuming population is stable after 2020.
- ▶ The findings in the model are validated with state-level and firm dynamics data for the US.

Our new mechanism vis-à-vis scale effects models

We have abstracted away from scale effects on growth up to this point by extending the firm dynamics model of Hopenhayn (1992).

Assume the technology for a project of productivity x_i , capital k_i and labor l_i is

$$y_i = x_i k_i^\alpha l_i^{1-\alpha}.$$

and the final consumption good is a CES combination of goods or varieties according to

$$Y = \left[\sum_{i=1}^N y_i^{\frac{\tilde{\sigma}-1}{\tilde{\sigma}}} \right]^{\frac{\tilde{\sigma}}{\tilde{\sigma}-1}},$$

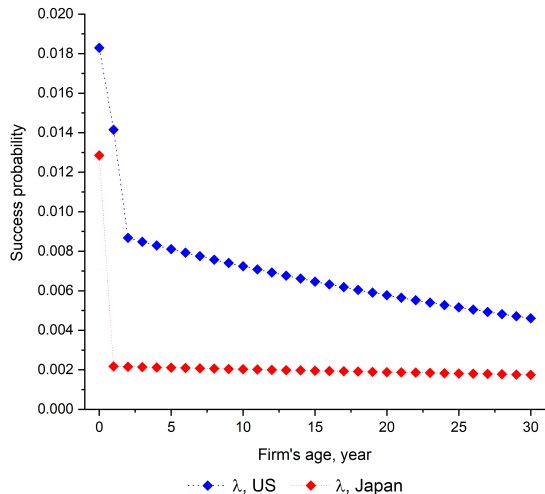
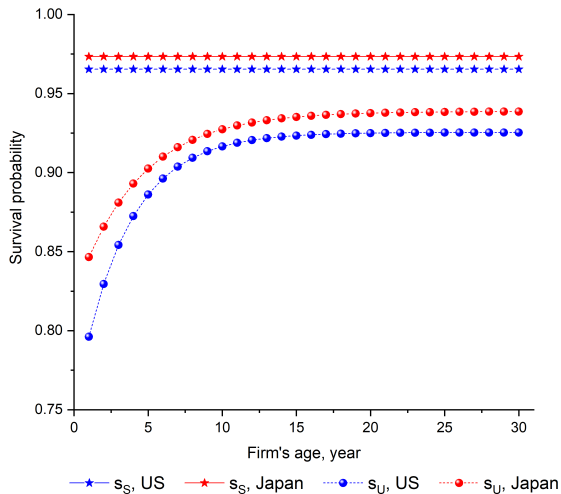
where N is the number of firms, each producing a different variety as in Peters and Walsh (2022).

Then, TFP growth is simply

$$g_{TFP} = g_{\tilde{x}} + \frac{1}{\tilde{\sigma} - 1} g_N.$$

Calibrating $\tilde{\sigma} = 4$ such that it is consistent with the “degree of diminishing returns” calibrated in Jones (2022), this equation says that for each one percentage point decline in population growth, there would be a 0.33 percentage point decline in productivity growth.

Probability of exit and success over the life-cycle



Calibration of spillovers

Regression for g_s	OLS		Instrumental Variables			
$g_{x,t-1}$	0.342*	0.304*	0.384*	0.387*	0.458**	0.466**
	(0.186)	(0.199)	(0.207)	(0.200)	(0.205)	(0.196)
Trend	no	yes	no	yes	no	yes
R squared	0.124	0.138	0.108	0.092	0.115	0.093
First stage statistic F	-	-	14.380	11.930	25.190	21.540
Hansen's χ^2 , p value	-	-	0.128	0.144	0.168	0.194
Instruments	-	-	VC	VC	VC & entry	VC & entry
Observations	23	23	23	23	23	23

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