Fixed-smoothing Asymptotic Theory and HAR Inference for Quantile Regression*

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Abstract

In this paper, we develop fixed-smoothing asymptotic theory for quantile regression (QR) in time series in the presence of unknown form of weak temporal dependence. By formulating the long-run variance (LRV) estimator using the demeaned QR process, our asymptotic theory effectively controls the estimation uncertainty inherent in the non-differentiable QR score process. We show that the LRV estimate in quantile regression (QR) weakly converges to a random matrix scaled by the true LRV, and corresponding QR-Wald statistics converge to non-standard limits. Also, Wald and t-tests using orthonormal series long-run variance (OS-LRV) estimators can be conducted with standard F and t asymptotic critical values. For the practical implementation of heteroskedasticity and autocorrelation robust (HAR) inference in QR, we extend HAR literature in mean regression and propose an optimal smoothing parameter selection rule based on the Neyman-Pearson principle. Monte Carlo simulation results confirm that the our QR-HAR procedure with a data-driven testing-optimal smoothing parameter significantly reduces size distortions in finite samples, particularly in scenarios with moderate sample sizes, stronger temporal dependence, and multiple parameters in the joint null hypothesis.

JEL Classification: C13, C14, C21, C23;

Keywords: Quantile regression, Heteroskedasticity and autocorrelation robust, Long-run variance, Alternative asymptotics, Testing-optimal smoothing parameter choice

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