# Evaluating the Impact of Regulatory Policies on Social Welfare in Diff-in-Diff Settings

Dalia Ghanem
*UC Davis*

Désiré Kédagni
*UNC at Chapel Hill*

Ismaël Mourifié
*WashU & NBER*

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# Motivation

- Many regulatory policies impose thresholds or minimum/maximum standards on outcomes of interest:
  - minimum wages
  - minimum/maximum working time
  - minimum safety standards
  - minimum energy efficiency standards
  - reporting and action thresholds in pollution monitoring

- These policies tend to induce behavioral responses, such as bunching, that lead to mixed outcome distributions, e.g. Cengiz et al (2019, QJE)

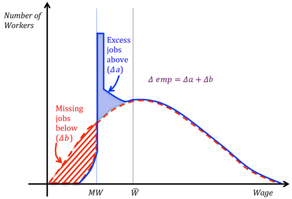



FIGURE I

- Since these policies target specific parts of the outcome distribution, identifying the impact on the distribution and specifically the parts targeted by the policy is crucial to properly assess its impact

# Motivation

- Existing (fully nonparametric) approaches to identify counterfactual distributions in DiD settings cannot allow for mixed outcomes, nonrandom selection into treatment and time variability in the potential outcome distribution for the same subpopulation

- Distributional DiD requires one of the following (Roth and Sant'Anna 2023):
  - random assignment
  - time homogeneity
  - mixture of the two (one subpopulation that obeys random assignment and another that obeys time homogeneity)

- Changes-in-changes (Athey and Imbens 2006) allows for nonrandom selection into treatment and time heterogeneity in their identification results for continuous and discrete outcomes, but their approach does not apply to mixed outcome distributions

# Contributions

- We propose a unifying (partial) identification approach:
  - applies to any type of distribution: mixed, continuous, discrete
  - does not restrict variability of marginal distribution across time
  - invariant to monotonic transformations
  - applies to repeated cross-sections or panel data
- Our bounding approach is valid under a novel assumption: "Copula Stability"
  - stability of dependence between treatment assignment and the untreated potential outcome over time
- We also propose social welfare treatment effect parameters suitable for quantifying the impact on social welfare and inequality in the lower or upper tail of the distribution

# Related Literature

- **Classical and recent DiD Literature**

  e.g. Ashenfelter (1978), Ashenfelter and Card (1985), Heckman and Robb (1985), Card and Krueger (1994), Callaway and Sant'Anna (2021), deChaisemartin and d'Hautefoeulille (2020), Goodman-Bacon (2021), Sun and Abraham (2021), Borusyak, Jaravel and Spiess (2023)

- **Parallel trends assumption**

  invariance to monotonic transformations (e.g. Athey and Imbens 2006, Roth and Sant'Anna 2023)

  connection to selection into treatment (e.g. Ghanem, Sant'Anna and Wuthrich 2023, Marx, Tamer and Tang 2023)

- **Alternatives to parallel trends assumptions**

  identifying counterfactual distributions (e.g. Athey and Imbens 2006, Bonhomme and Sauder 2011, Botosaru and Muris 2017, Callaway and Li 2019, Gunsilius 2023)

  bounds based on parallel trends violations (e.g. Manski and Pepper 2018, Rambachan and Roth 2023, Ban and Kédagni 2023)

  combining parallel trends and restrictions on assignment (e.g. Arkhangelsky, Imbens, Lei and Luo 2021, Arkhangelsky and Imbens 2022)

- **Copulas in policy evaluation**

  e.g. Rothe (2012), Mourifie (2015), Arellano and Bonhomme (2017), Callaway and Li (2019)

# Roadmap

Setup and Notation

Main Identifying Assumption

Partial Identification Result

Social Welfare Treatment Effect Parameters

Empirical Illustration

## Setup and Notation

- Consider 2-period/2-group POM:

$$\begin{cases} Y_0 &= Y_{00} \\ Y_1 &= Y_{11}D + Y_{10}(1-D) \end{cases}$$

where $Y_{td}$ denotes the potential outcome in period $t \in \{0,1\}$ with treatment status $d \in \{0,1\}$

- $q \equiv \mathbb{P}(D=0), \quad p \equiv \mathbb{P}(D=1) = 1-q$
- $\mathbb{X} = SuppX$
- $F_X(x) = \mathbb{P}(X \leq x)$, $F_X(x-) \equiv \mathbb{P}(X < x)$
- $Q_X^{\mathbb{T},-}(u) \equiv \inf\{x \in \mathbb{T} : F_X(x) \geq u\}$
- $Q_X^{\mathbb{T},+}(u) \equiv \sup\{x \in \mathbb{T} \cup \{-\infty\} : F_X(x) \leq u\}$
- $RanH \equiv \{H(y) : y \in \mathbb{R}\}$

# Setup and Notation: Copula

For random variables $Y_{t0}$ and $D$,

$$F_{Y_{t0},D}(y,d) = C_{Y_{t0},D}(F_{Y_{t0}}(y), F_D(d))$$

- A (sub)copula is the "link" between the joint and marginal distributions of $Y_{t0}$ and $D$
- As such, it is a scale-free measure of dependence

Background

# Setup and Notation: Copula

**Gaussian Example with Roy-style Selection**

$$\begin{cases} Y_0 & = & Y_{00} \\ Y_1 & = & \eta D + Y_{10} \\ D & = & \mathbb{1}\{\eta \geq 0\} \end{cases}$$

where

$$\begin{pmatrix} Y_{t0} \\ \eta \end{pmatrix} \sim N\left(0, \begin{pmatrix} \sigma_t^2 & \rho_t \sigma_t \\ \rho_t \sigma_t & 1 \end{pmatrix}\right)$$

$$F_{Y_{t0},D}(y,0) = \underbrace{C_{Y_{t0},D}}_{\Phi_2(\Phi^{-1}(\cdot),\Phi^{-1}(\cdot);\rho_t)} \left( \underbrace{F_{Y_{t0}}(y)}_{\Phi\left(\frac{y}{\sigma_t}\right)}, \underbrace{F_\eta(0)}_{\Phi(0)} \right), \quad y \in \mathbb{R}$$

- $C_{Y_{t0},D}(u,v)$ depends on $\rho_t$, the scale-free correlation coefficient between $Y_{t0}$ and $\eta$

# Main Identifying Assumption: Copula Stability

- Copula Stability (CS): Time-invariant horizontal copula between $Y_{t0}$ and $D$ at $q$

$$C_{Y_{00},D}(u,q) \quad = \quad C_{Y_{10},D}(u,q), \ \ \forall u \in [0,1]$$

# Main Identifying Assumption: Copula Stability

- Copula Stability (CS): Time-invariant horizontal copula between $Y_{t0}$ and $D$ at $q$

$$C_{Y_{00},D}(u,q) \quad = \quad C_{Y_{10},D}(u,q), \ \ \forall u \in [0,1]$$

- CS is crucial for our identification goals:
  - No restrictions on the type of $F_{Y_{t0}}$ ($\rightarrow$ unifying result)
  - No restrictions on variability in $F_{Y_{t0}}$ across time
  - Invariance to monotonic transformations

$$C_{g(Y_{t0}),D}(u,q) = C_{Y_{t0},D}(u,q) \ \ \forall g \text{ strictly increasing \& right-continuous}$$

Distinct from Callaway and Li (2019)

# Main Identifying Assumption: Intuition

By definition, for $t = 0, 1$,

$$F_{Y_{t0}|D=0}(y) = \delta_t(F_{Y_{t0}}(y)), \quad \text{where } \delta_t(\cdot) = C_{Y_{t0},D}(\cdot, q)/q$$

Then, we have

$$F_{Y_{t0}|D=1}(y) = \frac{1}{1-q}\left(\delta_t^{-1}\left(F_{Y_{t0}|D=0}(y)\right) - F_{Y_{t0}|D=0}(y)\right)$$

$$\underbrace{F_{Y_{t0}|D=1}(y)}_{Unob'counterfactual} = H_t\left(\underbrace{F_{Y_{t0}|D=0}(y)}_{factual}\right)$$

where

**Copula Stability**: $\delta(\cdot) \equiv \delta_0(\cdot) = \delta_1(\cdot) \iff H_0(\cdot) = H_1(\cdot)$.

**Intuition**: The relationship between the rank of the factual and its corresponding unob' counterfactual remains stable over time.

Parallel to PT

# Main Identifying Assumption: CS vs. PT

**Comparison to parallel trends (PT) for a given transformation $g$:**

$$\mathbb{E}[g(Y_{10}) - g(Y_{00})|D = 1] = \mathbb{E}[g(Y_{10}) - g(Y_{00})|D = 0]$$

$$\Updownarrow$$

$$Cov(g(Y_{00}), D) = Cov(g(Y_{10}), D) \quad \textit{(Covariance Stability)}$$

**Remarks**

- Our CS assumption can be interpreted as a dependence version of PT
- CS is invariant to monotonic transformations and does not restrict the marginals
- In general, PT and CS are nonnested, e.g. in Gaussian example
  - copula stability: $\rho_0 = \rho_1$
  - parallel trends on $Y_{t0}$: $\rho_0\sigma_0 = \rho_1\sigma_1$

more details

# Roadmap

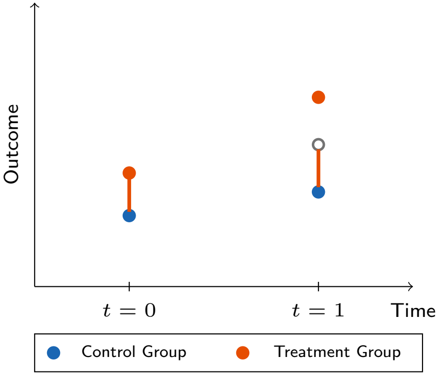
Setup and Notation ✓

Main Identifying Assumption ✓

Partial Identification Result

Social Welfare Treatment Effect Parameters

Empirical Illustration




To identify counterfactual expectation $\mathbb{E}[Y_{10}|D = 1]$

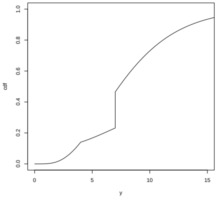
**Step 1**: Use Period 0 to recover $\Delta$ between T&C

**Step 2**: Transport difference to Period 1

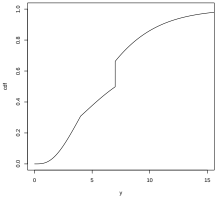
# (Partial) Identification through Copula Stability: Intuition

**Minimum Wage (MW) Example** DGP

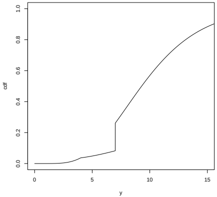
$F_{Y_{00}|D=0}$



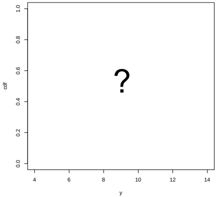

$F_{Y_{10}|D=0}$




$F_{Y_{00}|D=1}$




$F_{Y_{10}|D=1}$




Pre-treatment Period $(t = 0)$

Post-treatment Period $(t = 1)$

# (Partial) Identification through Copula Stability: Intuition

**Step 1**: Use Period 0 to recover the dependence structure from the observable $F_{Y_{00}}$ and $F_{Y_{00}|D=0}$

$$F_{Y_{00}|D=0}(y) = \frac{1}{q} F_{Y_{00},D}(y,q) = \frac{1}{q} C_{Y_{00},D}(F_{Y_{00}}(y), q)$$

**Sklar's Theorem**: There exists a unique subcopula $C : \overline{Ran F_{Y_{00}}} \times \{0, q, 1\} \rightarrow [0,1]$:

$$F_{Y_{00},D}(y,0) \quad = \quad C_{Y_{00},D}(F_{Y_{00}}(y), q), y \in [-\infty, \infty].$$

# (Partial) Identification through Copula Stability: Intuition

**Step 1**: Use Period 0 to recover the dependence structure from the observable $F_{Y_{00}}$ and $F_{Y_{00}|D=0}$

**MW Example**

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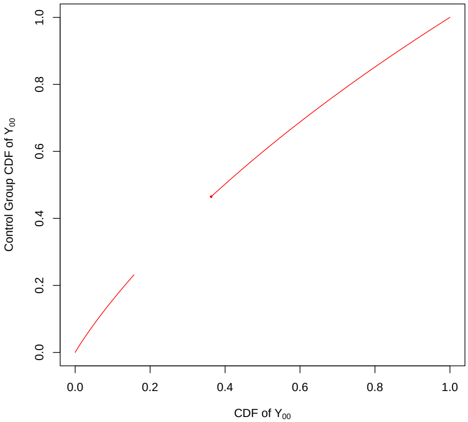

# (Partial) Identification through Copula Stability: Intuition

**Step 2**: Invoke copula stability, transport the dependence structure to Period 1 to bound $F_{Y_{10}|D=1}$

$$\overbrace{F_{Y_{10}|D=0}(y)}^{\text{Observed in Period 1}}$$

# (Partial) Identification through Copula Stability: Intuition

**Step 2**: Invoke copula stability, transport the dependence structure to Period 1 to bound $F_{Y_{10}|D=1}$

$$\overbrace{F_{Y_{10}|D=0}(y)}^{\text{Observed in Period 1}} \overset{Skar}{=} \frac{1}{q} C_{Y_{10},D}(F_{Y_{10}}(y), q)$$

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$$\overset{CS}{=} \frac{1}{q} C_{Y_{00},D}(F_{Y_{10}}(y), q),$$

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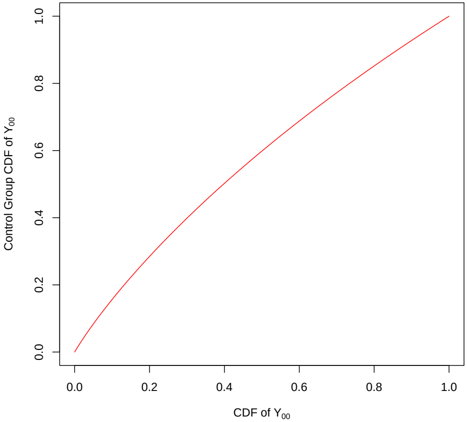
$$\overset{CS}{=} \frac{1}{q} C_{Y_{00},D}(F_{Y_{10}}(y), q),$$

- If $RanF_{Y_{00}} = [0,1]$, point-identification follows by inverting the copula to recover $F_{Y_{10}}(y) \Rightarrow F_{Y_{10}|D=1}(y)$

**Continuous Outcome:** $RanF_{Y_{00}} = [0,1]$



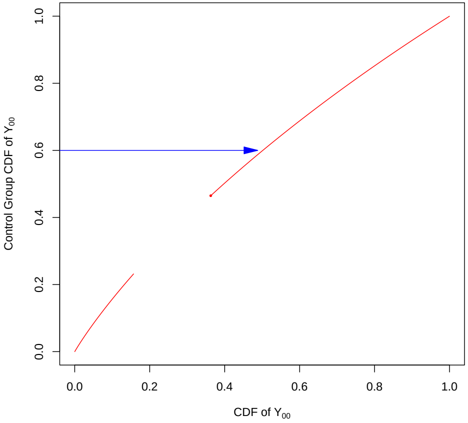
# (Partial) Identification through Copula Stability: Intuition

**Step 2**: Invoke copula stability, transport the dependence structure to Period 1 to bound $F_{Y_{10}|D=1}$

**MW Example:** $RanF_{Y_{00}} \subset [0,1]$

$$\overbrace{F_{Y_{10}|D=0}(y)}^{\text{Observed in Period 1}} \overset{Skar}{=} \frac{1}{q} C_{Y_{10},D}(F_{Y_{10}}(y), q) \overset{CS}{=} \frac{1}{q} C_{Y_{00},D}(F_{Y_{10}}(y), q),$$

- If $RanF_{Y_{00}} = [0,1]$, point-identification follows by inverting the copula to recover $F_{Y_{10}}(y) \Rightarrow F_{Y_{10}|D=1}(y)$
- If $RanF_{Y_{00}} \subset [0,1]$, one-to-one transport technique works only over $RanF_{Y_{00}|D=0} \cap RanF_{Y_{10}|D=0}$



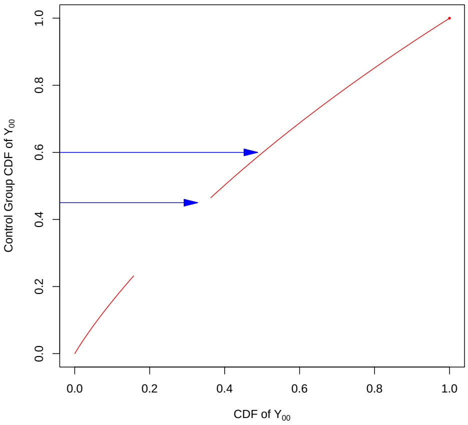
# (Partial) Identification through Copula Stability: Intuition

**Step 2**: Invoke copula stability, transport the dependence structure to Period 1 to bound $F_{Y_{10}|D=1}$

**MW Example:** $RanF_{Y_{00}} \subset [0,1]$

$$\overbrace{F_{Y_{10}|D=0}(y)}^{\text{Observed in Period 1}} \overset{Skar}{=} \frac{1}{q} C_{Y_{10},D}(F_{Y_{10}}(y), q)$$

$$\overset{CS}{=} \frac{1}{q} C_{Y_{00},D}(F_{Y_{10}}(y), q),$$




- If $RanF_{Y_{00}} = [0,1]$, point-identification follows by inverting the copula to recover $F_{Y_{10}}(y) \Rightarrow F_{Y_{10}|D=1}(y)$
- If $RanF_{Y_{00}} \subset [0,1]$, one-to-one transport technique works only over $RanF_{Y_{00}|D=0} \cap RanF_{Y_{10}|D=0}$
- Outside $RanF_{Y_{00}|D=0} \cap RanF_{Y_{10}|D=0}$, we can extend the copula

# (Partial) Identification through Copula Stability: Intuition

**Step 2**: Invoke copula stability, transport the dependence structure to Period 1 to bound $F_{Y_{10}|D=1}$

$$\overbrace{F_{Y_{10}|D=0}(y)}^{\text{Observed in Period 1}} \overset{Skar}{=} \frac{1}{q} C_{Y_{10},D}(F_{Y_{10}}(y), q)$$
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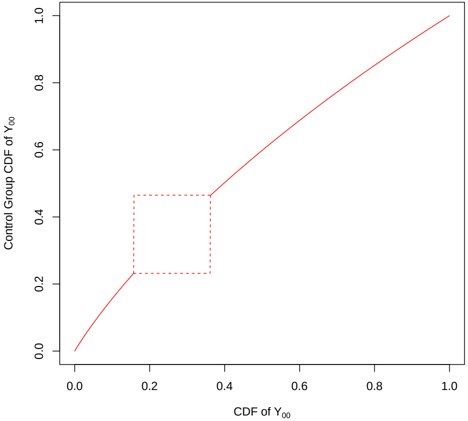
- If $RanF_{Y_{00}} \subset [0,1]$, one-to-one transport technique works only over $RanF_{Y_{00}|D=0} \cap RanF_{Y_{10}|D=0}$
- Outside $RanF_{Y_{00}|D=0} \cap RanF_{Y_{10}|D=0}$, we can extend the copula
  $\Rightarrow$ Set identification due to multiple extensions

*Note*: Our bounds assume strict monotonicity of copula

**MW Example:** $RanF_{Y_{00}} \subset [0,1]$




Proof Sketch

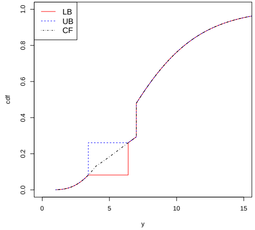
# (Partial) Identification through Copula Stability: Main Result

Theorem: Under the CS assumption as well as regularity conditions, the bounds on $F_{Y_{10}|D=1}(.)$ are:

$$\lim_{\tilde{y}\downarrow y} \sup\left\{F_{Y_0|D=1}\left(Q^{\mathbb{R},+}_{Y_0|D=0}\left(F_{Y_1|D=0}(t)\right)-\right) : t \leq \tilde{y} \;\&\; t \in \mathbb{Y}_{10|0} \cup \{-\infty\}\right\}$$

$$\leq \quad F_{Y_{10}|D=1}(y) \;\leq\; \lim_{\tilde{y}\downarrow y} \sup\left\{F_{Y_0|D=1}\left(Q^{\mathbb{R},-}_{Y_0|D=0}\left(F_{Y_1|D=0}(t)\right)\right) : t \leq \tilde{y} \;\&\; t \in \mathbb{Y}_{10|0} \cup \{-\infty\}\right\},\; y \in \mathbb{R}.$$

MW Ex: Bounds on $F_{Y_{10}|D=1}$



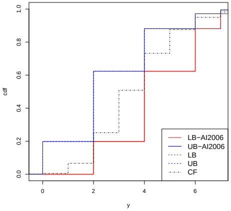

*Notes*: $CF$ denotes $F_{Y_{10}|D=1}$. $LB/UB$ denote the CS LB/UB.

**Remarks**

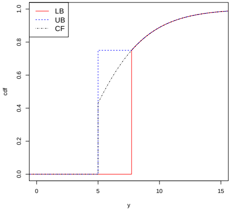
- The structure of the lower and upper bound is meant to guarantee their right-continuity
- The bounds are sharp assuming $RanF_{Y_{00}}$ is closed

# (Partial) Identification Result: Numerical Examples

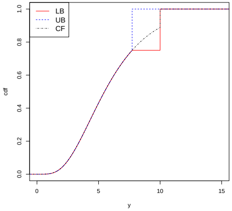
## Discrete (Poisson)



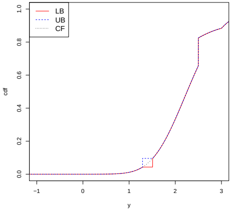

## Bottom-coding




## Top-coding




## Bunching




DGP | Poisson | Bottom-coding ($c_0 \neq c_1$) | Top-coding | Bunching

## Theorem

*If the CS assumption and regularity conditions hold for multiple pre-treatment periods* $t \in \{-T_0, \ldots, 0\}$*, then the bounds for* $F_{Y_{10}|D=1}(y)$ *for* $y \in \mathbb{R}$ *are given by:*

$$\begin{aligned} & \limsup_{\tilde{y} \downarrow y} \left\{ \max_{t \in \{-T_0,\ldots,0\}} F_{Y_t|D=1}\left(Q^{\mathbb{R},+}_{Y_t|D=0}\left(F_{Y_{1|D=0}}(s)\right) -\right) : s \leq \tilde{y} \ \& \ s \in \mathbb{Y}_{10|0} \cup \{-\infty\} \right\} \\ \leq \quad & F_{Y_{10}|D=1}(y) \\ \leq \quad & \limsup_{\tilde{y} \downarrow y} \left\{ \min_{t \in \{-T_0,\ldots,0\}} F_{Y_t|D=1}\left(Q^{\mathbb{R},-}_{Y_t|D=0}\left(F_{Y_{1|D=0}}(s)\right)\right) : s \leq \tilde{y} \ \& \ s \in \mathbb{Y}_{10|0} \cup \{-\infty\} \right\}, \end{aligned}$$

Skip

# (Partial) Identification through Copula Stability: Multiple Pre-treatment Periods

## Theorem

*If the CS assumption and regularity conditions hold for multiple pre-treatment periods $t \in \{-T_0, \dots, 0\}$, then the bounds for $F_{Y_{10|D=1}}(y)$ for $y \in \mathbb{R}$ are given by:*

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Skip

## Corollary (Testable Restriction)

*If the CS assumption and regularity conditions hold for multiple pre-treatment periods $t \in \{-T_0, \dots, 0\}$, then the following inequalities must be satisfied:*

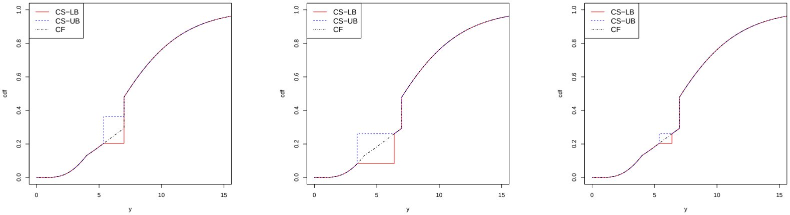
$$\Delta(y) \leq 0 \quad \forall y \in \mathbb{Y}_{10|0}, \text{ where}$$

$$\Delta(y) \equiv \max_{t \in \{-T_0,\dots,0\}} F_{Y_t}\left( Q^{\mathbb{R},+}_{Y_t|D=0}\left( F_{Y_{1|D=0}}(y) \right) - \right) - \min_{t \in \{-T_0,\dots,0\}} F_{Y_t}\left( Q^{\mathbb{Y}_{t|0},-}_{Y_t|D=0}\left( F_{Y_{1|D=0}}(y) \right) \right).$$

- If CS holds for multiple pre-treatment periods $\Rightarrow$ Identification Gain!

**CS bounds in MW Example with CS holding for $t \in \{-1, 0, 1\}$**

A. Using $t \in \{-1, 1\}$ B. Using $t \in \{0, 1\}$ C. Using $t \in \{-1, 0, 1\}$

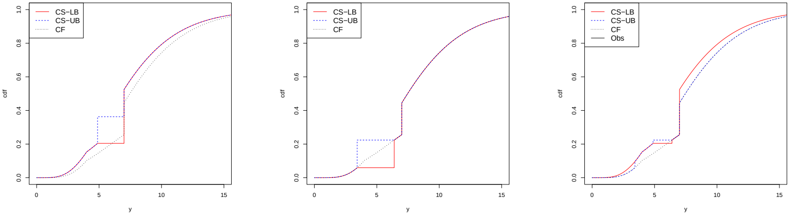


# (Partial) Identification through Copula Stability: Multiple Pre-treatment Periods

- If CS does not hold for all pre-treatment periods $\Rightarrow$ Testable Implication

**CS bounds in MW Example with CS holding for $t \in \{0, 1\}$ only**

A. Using $t \in \{-1, 1\}$ B. Using $t \in \{0, 1\}$ C. Using $t \in \{-1, 0, 1\}$



# (Partial) Identification Result: Connection to CiC (Athey and Imbens, 2006)

- For **continuous** cdfs, bounds simplify to point-identification case of CiC

$$F_{Y_{10}|D=1}(y) = F_{Y_0|D=1}\left(Q^{\mathbb{R},-}_{Y_0|D=0}\left(F_{Y_1|D=0}(y)\right)\right) \quad y \in \mathbb{R}$$

  CS ⇔ CiC conditions for continuous, strictly increasing cdfs

- For **discrete** outcomes, CS can be compatible with multi-dimensional heterogeneity

- For **mixed** outcomes, CiC point estimand will coincide with the CS upper bound

equivalence | binary outcome | mixed outcome

## Back to Roadmap

Setup and Notation ✓

Main Identifying Assumption ✓

Partial Identification Result ✓

Social Welfare Treatment Effect Parameters

Empirical Illustration

# Social Welfare Treatment Effect on the Treated (SWTT)

**Broad class of SWTT parameters**

$$\begin{aligned} SWTT_\omega &\equiv SW_\omega(F_{Y_{11}|D=1}) - SW_\omega(F_{Y_{10}|D=1}), \\ &= \int_0^1 \omega(\tau)\left(Q^{\mathbb{R},-}_{Y_{11}|D=1}(\tau) - Q^{\mathbb{R},-}_{Y_{10}|D=1}(\tau)\right) d\tau \\ &\text{where } \; SW_\omega(F_X) = \int_0^1 \omega(\tau) Q^{\mathbb{R},-}_X(\tau) d\tau \end{aligned}$$

**Examples: Overall SWTT**

- Utilitarian SWTT (ATT): $\omega(\tau) = 1$
  $\longrightarrow SW_\omega(F_X) = E[X]$
- Gini SWTT: $\omega(\tau) = 2(1-\tau)$
  $\longrightarrow SW_\omega(F_X) = E[X](1 - I_{Gini}(F_X))$

Related Literature: Mehran 1976, Weymark 1981, Aaberge, Havnes and Mogstad 2013, Kitagawa and Tetenov 2021

**Examples: Lower-tail SWTT**

Define $X^u = Q_X^{\mathbb{R},-}(V)$, where $V \sim \mathcal{U}[0,u]$ for $(u,1]$

- Lower-tail ATT(u): $\omega(\tau) = \mathbb{1}\{\tau \leq u\}/u$

  $\longrightarrow SW_\omega(F_X) = E[X^u]$

- Lower-tail Gini SWTT(u): $\omega(\tau) = 2(u-\tau)\mathbb{1}\{\tau \leq u\}/u^2$

  $\longrightarrow SW_\omega(u)(F_X) = E[X^u](1 - I_{Gini}(F_{X^u}))$

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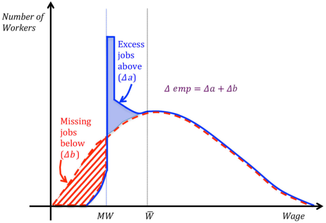



FIGURE I

**Remark**: Extension to any interquantile SWTT ($\longrightarrow$ paper)

# Back to Roadmap

Setup and Notation ✓

Main Identifying Assumption ✓

Partial Identification Result ✓

Social Welfare Treatment Effect Parameters ✓

Empirical Illustration

Skip

# Empirical Illustration

Cengiz et al (2019, *QJE*) examine the impact of 138 state-level minimum wage (MW) changes:

- using individual-level NBER Merged Outgoing Rotation Group of the CPS for 1979-2016
- conducting their analysis on the quarterly-state-level distribution of hourly wages

To keep our illustration succinct, we consider a subsample of their data:

- two years: 2010 ($t = 0$) and 2015 ($t = 1$)
- treatment group ($D = 1$): increase in minimum wage by at least \$0.25
- subsample: states with pre-treatment (2010) MW $\geq$\$8 (remaining subsample $\rightarrow$ paper)

| | Pre-treatment (2010) | | | Post-treatment (2015) | | |
|---|---|---|---|---|---|---|
| | Mean | S.D. | # Obs | Mean | S.D. | # Obs |
| States with Pre-Treatment Minimum Wage $\geq$ \$8 | | | | | | |
| Control | 20.12 | 13.96 | 4,737 | 22.30 | 15.48 | 4,454 |
| Treatment | 23.13 | 17.42 | 19,877 | 25.83 | 18.74 | 18,039 |

Wage Bins

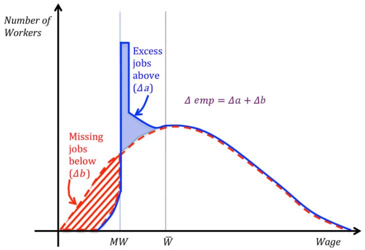
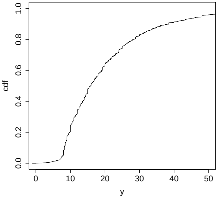


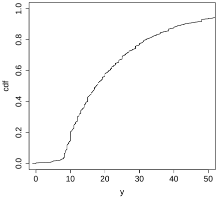

FIGURE I

# Empirical Illustration: Observed Distributions

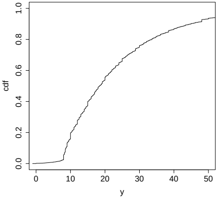
$F_{Y_{00}|D=0}$



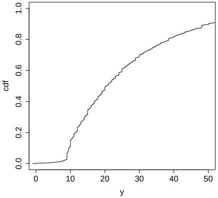

$F_{Y_{10}|D=0}$




$F_{Y_{00}|D=1}$




$F_{Y_{11}|D=1}$




Pre-treatment Period $(2010)$

Post-treatment Period $(2015)$

# Empirical Illustration

In the following, we will be comparing CS bounds with distributional DiD in terms of the following:

- Counterfactual Distribution
- ATT and Gini SWTT
- Lower-tail ATT and Gini SWTT
- Parameters from Cengiz et al (2019) measuring employment changes around new MW

# Empirical Illustration

In the following, we will be comparing CS bounds with distributional DiD in terms of the following:

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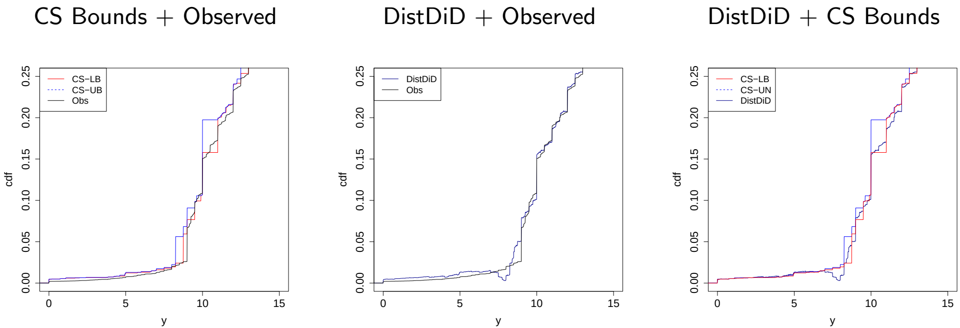
Recall that under the distributional DiD assumption,

$$F_{Y_{10}|D=1}(y) = F_{Y_0|D=1}(y) + F_{Y_1|D=0}(y) - F_{Y_0|D=0}(y)$$

$\Rightarrow$ Testable restriction: monotonicity of $F_{Y_{10}|D=1}$ (Roth and Sant'Anna 2023)

# Empirical Illustration: Counterfactual Distribution




*Notes*: *Obs* denotes $F_{Y_{11}|D=1}$. *CF-LB*/*CF-UB* denote the CS LB/UB on $F_{Y_{10}|D=1}$. *DistDiD* denotes the distributional DID estimate of $F_{Y_{10}|D=1}$. *CiC-PE* denotes the CiC point estimand that assumes continuity of the observed distribution.

**Remarks**:

- Distributional DiD exhibits clear monotonicity violation around the pre-treatment MW
- CiC point estimaotr equals the CS upper bound as expected

Skip    CiC figure

# Empirical Illustration: Bounds on Gini SWTT

Decomposing Gini SWTT:

$$SWTT_\omega = \underbrace{ATT(1 - I_{Gini}(F_{Y_{11|D=1}}))}_{\text{Mean Component } (\Delta_M)} - \underbrace{\mathbb{E}[Y_{10}|D=1](I_{Gini}(F_{Y_{11|D=1}}) - I_{Gini}(F_{Y_{10|D=1}}))}_{\text{Inequality Component } (\Delta_I)}$$

ATT and Gini SWTT Results

| | $\hat{F}_{Y_1\,|\,D=1}$ | SWTT | | | | |
|---|---|---|---|---|---|---|
| | | CS-LB | CS-UB | DiD | DistDiD | CiC-PE |
| Mean $(ATT)$ | 25.83 | 0.12 | 0.56 | 0.53 | -0.10 | 0.56 |
| Gini SWF | 16.89 | 0.06 | 0.36 | – | 0.25 | 0.36 |
| $\Delta_M$ | – | 0.08 | 0.37 | – | -0.07 | 0.37 |
| $\Delta_I^\dagger$ | – | -0.28 | 0.30 | – | -0.32 | 0.00 |

† The bounds on $\Delta_I$ outerset bounds

Remarks:

- CS bounds are proportionately small, but positive for ATT and Gini SWF
- DiD and DistDiD provide opposite signs for ATT

# Empirical Illustration: Bounds on Lower-Tail Gini SWTT

| | $\hat{F}_{Y_1\|D=1}$ | SWTT | | | |
|---|---|---|---|---|---|
| | | CS-LB | CS-UB | DistDiD | CiC-PE |
| Lower-tail Mean $(ATT(u))$ | | | | | |
| $u = 0.01$ | 3.63 | 1.59 | 1.68 | 1.90 | 1.68 |
| $u = 0.025$ | 6.15 | 0.95 | 1.06 | 1.06 | 1.06 |
| $u = 0.05$ | 7.57 | 0.60 | 0.91 | 1.06 | 0.91 |
| Lower-tail Gini SWF $(SWTT_\omega(u) = \Delta_M(u) - \Delta_I(u))$ | | | | | |
| $u = 0.01$ | 2.10 | 1.39 | 1.44 | 1.53 | 1.44 |
| $\Delta_M(u)$ | – | 0.92 | 0.97 | 1.10 | 0.97 |
| $\Delta_I(u)^\dagger$ | – | -0.53 | -0.42 | -0.43 | -0.47 |
| $u = 0.025$ | 4.58 | 1.26 | 1.34 | 1.44 | 1.34 |
| $\Delta_M(u)$ | – | 0.71 | 0.79 | 0.79 | 0.79 |
| $\Delta_I(u)^\dagger$ | – | -0.63 | -0.47 | -0.65 | -0.55 |
| $u = 0.05$ | 6.38 | 0.85 | 1.04 | 1.01 | 1.04 |
| $\Delta_M(u)$ | – | 0.51 | 0.76 | 0.63 | 0.76 |
| $\Delta_I(u)^\dagger$ | – | -0.54 | -0.09 | -0.38 | -0.28 |

Remarks:

- CS bounds suggest substantive positive increases in lower-tail means and Gini SWF
- Decomposition of lower-tail Gini SWF suggests positive bounds on $\Delta_M$ and negative (outerset) bounds) $\Delta_I$ for lowest values of $u$

# Empirical Illustration: Bounds on Parameters from Cengiz et al (2019)

| | $\hat{F}_{Y_1\mid D=1}$ | CS-LB | CS-UB | DistDiD | CiC-PE |
|---|---|---|---|---|---|
| $\Delta b$ | 2.27% | -2.89% | 0.31% | -1.18% | -2.89% |
| $\Delta a$ ($\overline{W}=11$) | 16.61% | -0.93% | 2.31% | 1.71% | 2.27% |
| $\Delta e$ ($\overline{W}=11$) | 18.88% | -0.62% | -0.59% | 0.53% | -0.62% |

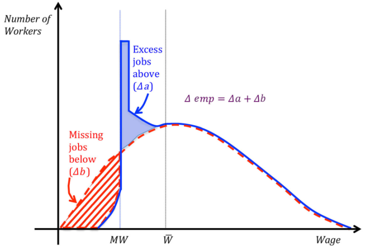


FIGURE I

**Remark**: Our bounds are consistent with the conceptual framework in Cengiz et al (2019)

# Conclusion

- Regulatory policies induce behavioral responses that lead to mass points in outcome distributions

- We propose a unifying partial identification result for the counterfactual distribution in DiD designs:
  - our method is invariant to monotone transformations of the outcome
  - applies to any type of outcome distribution, whether continuous, discrete, or mixed
  - valid under a Copula Stability (CS) assumption

- Our bounds on the counterfactual distribution can be used to bound a broad class of SWTTs

- We illustrate the empirical relevance of our approach and the SWTT parameters in the context of a recent minimum-wage study (Cengiz et al 2019)

# Conclusion

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THANK YOU!

# Appendix

# (Sub)copula definition and Sklar's theorem

The horizontal copula at $q$ is $C_{Y_{t0},D}(F_{Y_{t0}}(y), q) = F_{Y_{t0},D}(y, 0)$.

**Sklar's Theorem**: There exists a unique subcopula $C : \overline{Ran} F_{Y_{t0}} \times \{0, q, 1\} \to [0, 1]$:

$$F_{Y_{t0},D}(y, 0) \quad = \quad C_{Y_{t0},D}(F_{Y_{t0}}(y), q), y \in [-\infty, \infty].$$

- The subcopula $C_{Y_{t0},D}(u, q)$ is uniquely identified from $F_{Y_{t0},D}$ for $u \in \overline{Ran} F_{Y_{t0}}$
- If $\overline{Ran} F_{Y_{t0}} = [0, 1]$, the subcopula $C_{Y_{t0},D}(u, q)$ is a copula and unique for $u \in [0, 1]$

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# (Sub)copula definition and Sklar's theorem

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**Definition of a Subcopula** (Nelson 2006)
A two-dimensional subcopula is a function $C$ with the following properties:

1. $DomC = S_1 \times S_2$, where $S_1$ and $S_2$ are subsets of $[0, 1]$ containing $0$ and $1$;
2. For all $u, u' \in S_1$, and $v, v' \in S_2$ such that $u \leq u'$, and $v \leq v'$, we have:
$$C(u', v') + C(u, v) \geq C(u', v) + C(u, v');$$
3. $C(0, v) = C(u, 0) = 0$ for all $(u, v) \in S_1 \times S_2$, and $C(1, v) = v$, $C(u, 1) = u$ for all $(u, v) \in S_1 \times S_2$.

A copula is a subcopula with $S_1 = S_2 = [0, 1]$

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# Callaway and Li (2019)

Main Identification Assumptions

- Distributional DiD Assumption:

$$\Delta Y_{0t} \perp D$$

- Copula stability between changes and levels:

$$C_{\Delta Y_{0t}, Y_{0(t-1)}|D=1}(\cdot, \cdot) = C_{\Delta Y_{0(t-1)}, Y_{0(t-2)}|D=1}(\cdot, \cdot)$$

Note: Identification result requires *3 time periods of panel data* as well as additional regularity conditions (continuity of random variables)

Back to Contributions

Back to Assumptions

# Main Identifying Assumption: Copula Stability

**Comparison to PT in Gaussian Example**

$$\begin{cases} Y_0 &= Y_{00} \\ Y_1 &= \eta D + Y_{10} \\ D &= \mathbb{1}\{\eta \geq 0\} \end{cases}$$

where

$$\begin{pmatrix} Y_{00} \\ Y_{10} \\ \eta \end{pmatrix} \sim N(0, \Sigma) \quad \text{with } \Sigma = \begin{pmatrix} \sigma_0^2 & \delta\sigma_0\sigma_1 & \rho_0\sigma_0 \\ \delta\sigma_0\sigma_1 & \sigma_1^2 & \rho_1\sigma_1 \\ \rho_0\sigma_0 & \rho_1\sigma_1 & 1 \end{pmatrix}.$$

- Copula Stability: $\rho_0 = \rho_1$
- 1-Parallel Trends: $\rho_0\sigma_0 = \rho_1\sigma_1$

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## Binary Example with Multidimensional Heterogeneity

Suppose that

$$
\begin{aligned}
Y_{t0} &= 1 - \mathbb{1}\{U_t \leq c_t, \tilde{U}_t \leq \tilde{c}_t\}, \quad t = 0, 1, \\
D &= \mathbb{1}\{V > q\},
\end{aligned}
$$

where $(V, U_0, U_1, \tilde{U}_0, \tilde{U}_1)$ is a latent random vector, and $(c_t, \tilde{c}_t)$ is a constant vector. For simplicity, we normalize $U_t$, $\tilde{U}_t$ and $V$ to be uniformly distributed on $[0, 1]$.

Suppose $C_{U_t,\tilde{U}_t,V}(u, \tilde{u}, v) = C_t\left(C_{U_t,\tilde{U}_t}(u, \tilde{u}), v\right)$ where $C_t$ and $C_{U_t,\tilde{U}_t}$ are two-dimensional Archimedean copulas. Define $C_{Y_{t0},D}(u, q) \equiv C_t(u, q)$. Then, the stability of the copula of $(U_t, \tilde{U}_t, V)$ implies the stability of the copula of $(Y_{t0}, D)$.

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# Selection on Lagged Outcomes

Consider the following model

$$\begin{cases} Y_0 & = & Y_{00}, \\ Y_1 & = & Y_{11} D + Y_{10}(1-D), \\ D & = & \mathbb{1}\{Y_{00} > c\}. \end{cases}$$

Assume that $Y_{00}$ is continuous and has strictly increasing cdf. Then, from the Sklar theorem, we have for any $u \in [0,1]$ and $q \equiv \mathbb{P}(D=0) = F_{Y_{00}}(c)$

$$\begin{aligned} C_{Y_{00},D}(u,q) &= \mathbb{P}\left(Y_{00} \leq Q_{Y_{00}}^{\mathbb{R},-}(u), D \leq Q_D^{\mathbb{R},-}(q)\right) = \mathbb{P}\left(F_{Y_{00}}(Y_{00}) \leq F_{Y_{00}}(Q_{Y_{00}}^{\mathbb{R},-}(u)), D = 0\right), \\ &= \mathbb{P}\left(F_{Y_{00}}(Y_{00}) \leq u, Y_{00} \leq c\right) \\ &= \mathbb{P}\left(F_{Y_{00}}(Y_{00}) \leq u, F_{Y_{00}}(Y_{00}) \leq q\right) \\ &= \mathbb{P}\left(F_{Y_{00}}(Y_{00}) \leq \min(u,q)\right) = \min(u,q) \ \text{ since } F_{Y_{00}}(Y_{00}) \sim \mathcal{U}_{[0,1]}. \end{aligned}$$

**Remarks**:

- Selection on lagged outcomes requires a specific dependence structure that rules out our strict monotonicity assumption
- Note however that selection on outcomes implies unconfoundedness which should be used to identify the counterfactual distribution

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# Parallel to PT

**Parallel Trends**: stable difference in expected outcome between control group and marginal distribution

$$\mathbb{E}[Y_{t0}|D=1] \quad - \quad \mathbb{E}[Y_{t0}|D=0] = \Delta$$

$$\Updownarrow$$

$$\mathbb{E}[Y_{t0}|D=0] \quad = \quad \mathbb{E}[Y_{t0}] - \widetilde{\Delta}, \quad \text{for } t = 0, 1$$

where $\widetilde{\Delta} = -\Delta(1-q)$

Object of Interest: $\mathbb{E}[Y_{10}|D=1] = \frac{1}{1-q}\left(\mathbb{E}[Y_{10}] - q\mathbb{E}[Y_{10}|D=0]\right)$

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# Proof Sketch

- For any RV $X$, we have the following sharp bounds:

$$F_X\left(Q_X^{\mathbb{R},+}(u)-\right) \quad \leq \quad u \leq F_X\left(Q_X^{\mathbb{R},-}(u)\right) \text{ for all } u \in [0,1]$$

- Apply inequality with $X = Y_0|D=0$ and $u = F_{Y_1|D=0}(y)$ for $y \in \mathbb{Y}_{10|0}$, which yields the following

$$F_{Y_0|D=0}\left(\underline{y}\right) \quad \leq \quad F_{Y_1|D=0}(y) \leq F_{Y_0|D=0}\left(\overline{y}\right),$$

where $\underline{y} < Q^{\mathbb{R},+}_{Y_0|D=0}(F_{Y_1|D=0}(y))$ and $\overline{y} = Q^{\mathbb{R},-}_{Y_0|D=0}(F_{Y_1|D=0}(y))$.

$$\begin{array}{lrcl}
\text{(Sklar)} & C_{Y_0,D}\left(F_{Y_0}\left(\underline{y}\right),q\right) & \leq & C_{Y_{10},D}\left(F_{Y_{10}}(y),q\right) \leq C_{Y_0,D}\left(F_{Y_0}\left(\overline{y}\right),q\right), \\
\text{(CS)} & C_{Y_0,D}\left(F_{Y_0}\left(\underline{y}\right),q\right) & \leq & C_{Y_0,D}\left(F_{Y_{10}}(y),q\right) \leq C_{Y_0,D}\left(F_{Y_0}\left(\overline{y}\right),q\right), \\
\text{(Strictly} \uparrow \text{C)} & F_{Y_0}\left(\underline{y}\right) & \leq & F_{Y_{10}}(y) \leq F_{Y_0}\left(\overline{y}\right).
\end{array}$$

Back

# Proof Sketch

- Apply monotonic transformation $v - C_{Y_0,D}(v,q)$ on the last inequality and divide by $p$:

$$F_{Y_0|D=1}\left(\underline{y}\right) \quad \leq \quad F_{Y_{10}|D=1}(y) \leq F_{Y_0|D=1}\left(\overline{y}\right).$$

- Take supremum over $\underline{y} < Q^{\mathbb{R},+}_{Y_0|D=0}(F_{Y_1|D=0}(y))$ for $y \in \mathbb{Y}_{10|0}$:

$$F_{Y_0|D=1}\left(Q^{\mathbb{R},+}_{Y_0|D=0}\left(F_{Y_1|D=0}(y)\right)-\right) \leq F_{Y_{10}|D=1}(y) \leq F_{Y_0|D=1}\left(Q^{\mathbb{R},-}_{Y_0|D=0}\left(F_{Y_1|D=0}(y)\right)\right).$$

- Then we apply a transformation to ensure the bounds are right-continuous:

$$\begin{aligned}
&\limsup_{\tilde{y}\downarrow y}\left\{F_{Y_0|D=1}\left(Q^{\mathbb{R},+}_{Y_0|D=0}\left(F_{Y_1|D=0}(t)\right)-\right) : t \leq \tilde{y} \;\&\; t \in \mathbb{Y}_{10|0} \cup \{-\infty\}\right\} \\
\leq \quad & F_{Y_{10}|D=1}(y) \leq \limsup_{\tilde{y}\downarrow y}\left\{F_{Y_0|D=1}\left(Q^{\mathbb{R},-}_{Y_0|D=0}\left(F_{Y_1|D=0}(t)\right)\right) : t \leq \tilde{y} \;\&\; t \in \mathbb{Y}_{10|0} \cup \{-\infty\}\right\}, \quad y \in \mathbb{R}.
\end{aligned}$$

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# DGPs

We generate the conditional potential outcome distirbution by the following

$$F_{Y_{t0}|D=0}(y) = \frac{1}{q} C_{Y_0,D}(F_{Y_{t0}}(y), q) \quad (1)$$

$$F_{Y_{t0}|D=1}(y) = \frac{1}{p} \left(F_{Y_{t0}}(y) - C_{Y_0,D}(F_{Y_{t0}}(y), q)\right) \quad (2)$$

where $F_{Y_{t0}}$ is the marginal distribution and $C_{Y_0,D}(u,q) = (max(u^{-\theta} + q^{-\theta} - 1, 0))^{-1/\theta}$.
Our baseline results rely on $\theta = 1$, which fulfils our strict monotonicity condition on the horizontal copula.

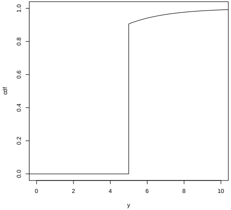
Table: Numerical Examples: Outcome Distributions

| | |
|---|---|
| I. Left-censoring | $\begin{cases} F_{Y_{t0}}(y) = \begin{cases} 0 & \text{if } y < c_t \\ \Lambda_t(y) & \text{if } y \geq c_t \end{cases}, \\ \text{where } \Lambda_t(\cdot) \text{ is the } \chi^2 \text{ cdf with } k_t \text{ degrees of freedom.} \end{cases}$ |
| II. Poisson | $F_{Y_{t0}}(y) = \Pi_t(y)$, where $\Pi_t(\cdot)$ is the Poisson cdf with mean $\lambda_t$. |
| III. Right-censoring | $\begin{cases} F_{Y_{t0}}(y) = \begin{cases} \Lambda_t(y) & \text{if } y < c_t \\ 1 & \text{if } y \geq c_t \end{cases}, \\ \text{where } \Lambda_t(\cdot) \text{ is the } \chi^2 \text{ cdf with } k_t \text{ degrees of freedom.} \end{cases}$ |
| IV. Bunching | $\begin{cases} F_{Y_{t0}}(y) = \begin{cases} \Phi_t(y) & \text{if } y \notin [c_t, w_t) \\ \Phi_t(c_t) + b_t(\Phi_t(w_t) - \Phi_t(c_t)) & \text{if } y = c_t \\ \Phi_t(c_t) + b_t(\Phi_t(w_t) - \Phi_t(c_t)) + (1 - b_t)(\Phi_t(y) - \Phi_t(c_t) & \text{if } y \in (c_t, w_t) \end{cases} \\ \text{where } \Phi_t(.) \text{ is the standard normal cdf with mean } \mu_t \text{ and standard deviation } \sigma_t. \end{cases}$ |
| V. Minimum Wage | $F_{Y_{t0}}(y) = F_{Y^*_{t0}}(y) + (1 - b_t)(F_{Y^*_{t0}}(y) - F_{Y^*_{t0}}(\underline{w}))\mathbb{1}\{y \in (\underline{w}, c]\}$, where $Y^*_{td} \sim \chi^2(k_{td})$ for $(t,d) \in \{(0,0),(1,0)\}$. |

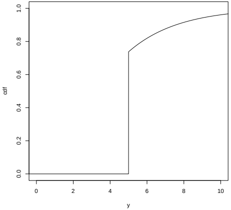
Back to MW Ex | Back to Num Examples | Right-censoring ($c_0 = c_1$) | Left-censoring ($c_0 \neq c_1$) | Poisson | Right-censoring | Bunching

# Bottom-coding: $c_0 = c_1 = 5$, $k_0 = 3$, $k_1 = 5$

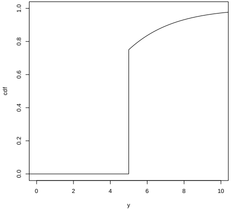
$F_{Y_{00}|D=0}$



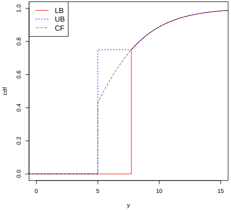

$F_{Y_{10}|D=0}$




$F_{Y_{00}|D=1}$




CS Bounds on $F_{Y_{10}|D=1}$

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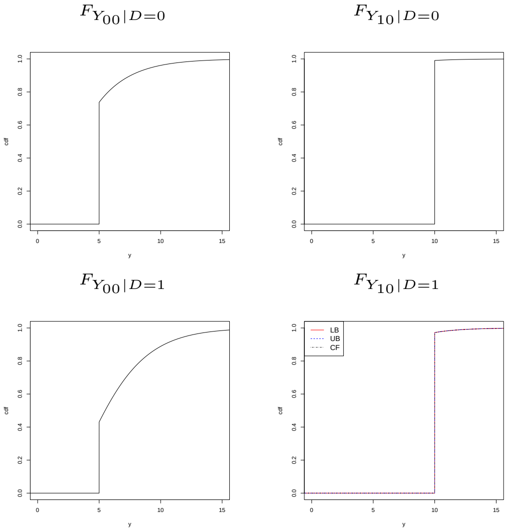
# Bottom-coding: $c_0 = 5$, $c_1 = 10$, $k_0 = 5$, $k_1 = 3$

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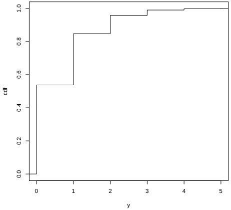
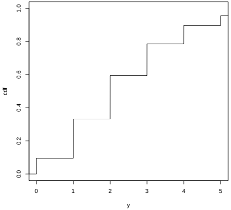
# Poisson with time-varying parameter

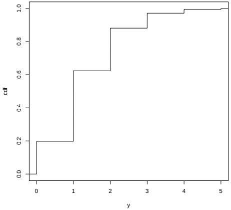
$F_{Y_{00}|D=0}$



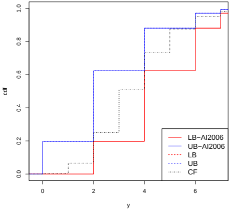

$F_{Y_{10}|D=0}$




$F_{Y_{00}|D=1}$




CS Bounds on $F_{Y_{10}|D=1}$

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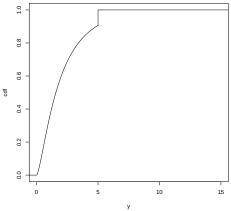
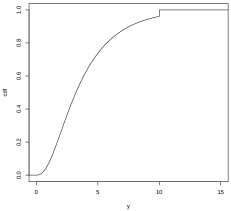
# Top-coding Example with Time-varying Cutoff

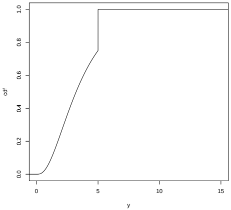
$F_{Y_{00}|D=0}$



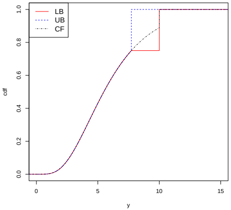

$F_{Y_{10}|D=0}$




$F_{Y_{00}|D=1}$




CS Bounds on $F_{Y_{10}|D=1}$

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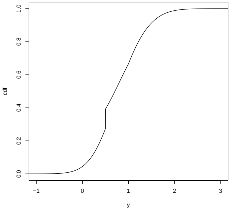
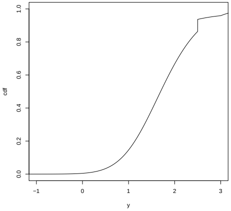
# Bunching Example with Time-varying Cutoff and Proportion

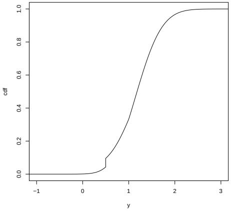
$F_{Y_{00}|D=0}$



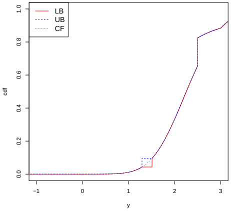

$F_{Y_{10}|D=0}$




$F_{Y_{00}|D=1}$




CS Bounds on $F_{Y_{10}|D=1}$

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# CS and CiC Assumptions

## Claim (Continuous, Strictly Increasing CDF)

*Assume the cdfs* $F_{Y_{t0}}(.), t \in \{0,1\}$ *are continuous and strictly increasing, then the two following statements are equivalent:*

(i) $C_{Y_{00},D}(u,q) = C_{Y_{10},D}(u,q)$ *for all* $u \in [0,1]$.

(ii) *There exist two strictly increasing functions* $h_t(.) = Q^{\mathbb{R},-}_{Y_{t0}}(\cdot)$ *and* $U_{t0} = F_{Y_{t0}}(Y_{t0}) \sim \mathcal{U}[0,1]$ *for* $t \in \{0,1\}$, *such that* $Y_{t0} = h_t(U_{t0})$ *and* $U_{00}|D = d \sim U_{10}|D = d$ *for* $d \in \{0,1\}$.

*Intuition*: If $F_{Y_{t0}}(\cdot)$ is continuous and strictly increasing, then

$$Y_{t0} = Q^{\mathbb{R},-}_{Y_{t0}}\left(F_{Y_{t0}}(Y_{t0})\right)$$

# CS and CiC Assumptions

## Claim (Continuous, Strictly Increasing CDF)

*Assume the cdfs* $F_{Y_{t0}}(.), t \in \{0,1\}$ *are continuous and strictly increasing, then the two following statements are equivalent:*

(i) $C_{Y_{00},D}(u,q) = C_{Y_{10},D}(u,q)$ *for all* $u \in [0,1]$.

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*Intuition*: If $F_{Y_{t0}}(\cdot)$ is continuous and strictly increasing, then

$$Y_{t0} = Q^{\mathbb{R},-}_{Y_{t0}}\left(F_{Y_{t0}}(Y_{t0})\right) = Q^{\mathbb{R},-}_{Y_{t0}}\left(U_{t0}\right)$$

where $U_{t0} = F_{Y_{t0}}(Y_{t0}) \sim \mathcal{U}[0,1]$ has a time-invariant distribution by definition

$$\text{Stability of } C_{Y_{t0},D}(\cdot,q) \Leftrightarrow \text{Stability of } U_t|D = d$$

Note: Result extends to continuous outcomes where the above representation of $Y_{t0}$ holds

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# Binary Example with Multidimensional Heterogeneity

Suppose that

$$\begin{aligned} Y_{t0} &= 1 - \mathbb{1}\{U_t \leq c_t, \tilde{U}_t \leq \tilde{c}_t\}, \quad t = 0, 1, \\ D &= \mathbb{1}\{V > q\}, \end{aligned}$$

where $(V, U_0, U_1, \tilde{U}_0, \tilde{U}_1)$ is a latent random vector, and $(c_t, \tilde{c}_t)$ is a constant vector. For simplicity, we normalize $U_t$, $\tilde{U}_t$ and $V$ to be uniformly distributed on $[0, 1]$.

Suppose $C_{U_t, \tilde{U}_t, V}(u, \tilde{u}, v) = C_t\left(C_{U_t, \tilde{U}_t}(u, \tilde{u}), v\right)$ where $C_t$ and $C_{U_t, \tilde{U}_t}$ are two-dimensional Archimedean copulas. Define $C_{Y_{t0}, D}(u, q) \equiv C_t(u, q)$. Then, the stability of the copula of $(U_t, \tilde{U}_t, V)$ implies the stability of the copula of $(Y_{t0}, D)$.

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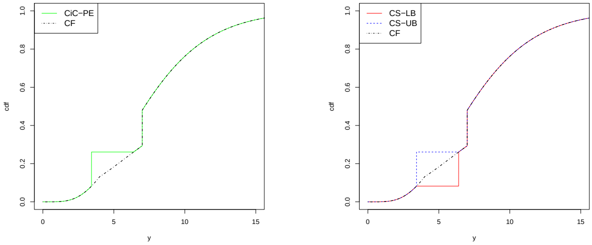
# CiC Point-Estimand in the Minimum-Wage Numerical Example

CiC Point-Estimand using $t \in \{0, 1\}$ CS Bounds using $t \in \{0, 1\}$




**Remarks**

- CiC point-estimand equals CS upper bound
- CiC upper and lower bounds for discrete outcomes are equal to each other

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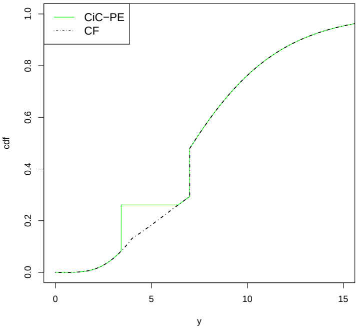
# CiC Bounds for Discrete Outcomes in the Minimum Wage Numerical Example

- CiC upper and lower bounds will be equal and not equal to the counterfactual outcome distribution




$$F^{UB}_{CiC}(s) = \underbrace{F_{Y_0|D=1}}\big(\underbrace{Q^{\mathbb{Y}_{0|0},-}_{Y_0|D=0}(F_{Y_1|D=0}(y))}\big)$$

$$||\qquad\qquad ||$$

$$||\qquad\qquad ||$$

$$F^{LB}_{CiC}(s) = \overbrace{F_{Y_0|D=1}}\big(\overbrace{Q^{\mathbb{Y}_{0|0},+}_{Y_0|D=0}(F_{Y_1|D=0}(s))}\big)$$

for $s \in \mathbb{Y}_{0|1}$

- CS lower bound differs from CiC lower bound

$$F^{LB}(s) = F_{Y_0|D=1}\left(Q^{\mathbb{R},+}_{Y_0|D=0}\left(F_{Y_{1|D=0}}(s)\right)-\right) = \mathbb{P}\left(Y_0 < Q^{\mathbb{R},+}_{Y_0|D=0}(F_{Y_{1|D=0}}(s))|D=1\right)$$ for $s \in \mathbb{Y}_{0|1}$

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# Construction of Outcome Variable in Cengiz et al (2019)

Our outcome variable is $Y_{ist}$ denote the wage reported by survey respondent $i$ in state $s$ in quarter $t$

The outcome variable in Cengiz et al (2019) is the employment-to-population ratio in \$ 0.25 wage bins $\{Bin_j\}_{j=1}^{J}$

$$empbin_{st}^{j} = \frac{\sum_{i=1}^{n_{st}} 1\{Y_{ist} \in Bin_j\} earnwt_{ist}}{\sum_{i=1}^{n_{st}} earnwt_{ist}}$$

where $earnwt_{ist}$ is the earnings weight for each survey respondent and $n_{st}$ is the number of survey respondents in state $s$ in quarter $t$, which equals the total population above 16 in the state that year

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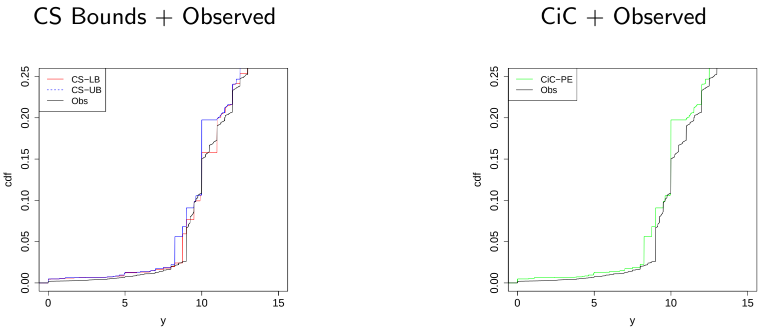
# CiC Point Estimator in Application




*Notes*: *Obs* denotes $F_{Y_{11}|D=1}$. *CF-LB*/*CF-UB* denote the CS LB/UB on $F_{Y_{10}|D=1}$. *CiC-PE* denotes the CiC point estimand that assumes continuity of the observed distribution.

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