Employer Credit Checks: Poverty Traps versus Matching Efficiency

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Abstract

We develop a framework to understand pre-employment credit screening through adverse selection in labor and credit markets. Workers differ in an unobservable characteristic that induces a positive correlation between labor productivity and repayment rates in credit markets. Firms therefore prefer to hire workers with good credit because it correlates with high productivity. A poverty trap may arise, in which an unemployed worker with poor credit has a low job finding rate, but cannot improve her credit without a job. In our calibrated economy, this manifests as a large and persistent wage loss from default, equivalent to 2.3% per month over ten years. Banning employer credit checks eliminates the poverty trap, but pools job seekers and reduces matching efficiency: average unemployment duration rises by 13% for the most productive workers after employers are banned from using credit histories to screen potential hires.

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"We want people who have bad credit to get good jobs. Then they are able to pay their bills, and get the bad credit report removed from their records. Unfortunately, the overuse of credit reports takes you down when you are down." Michael Barrett (State Senator, D-Lexington, MA).

1 Introduction

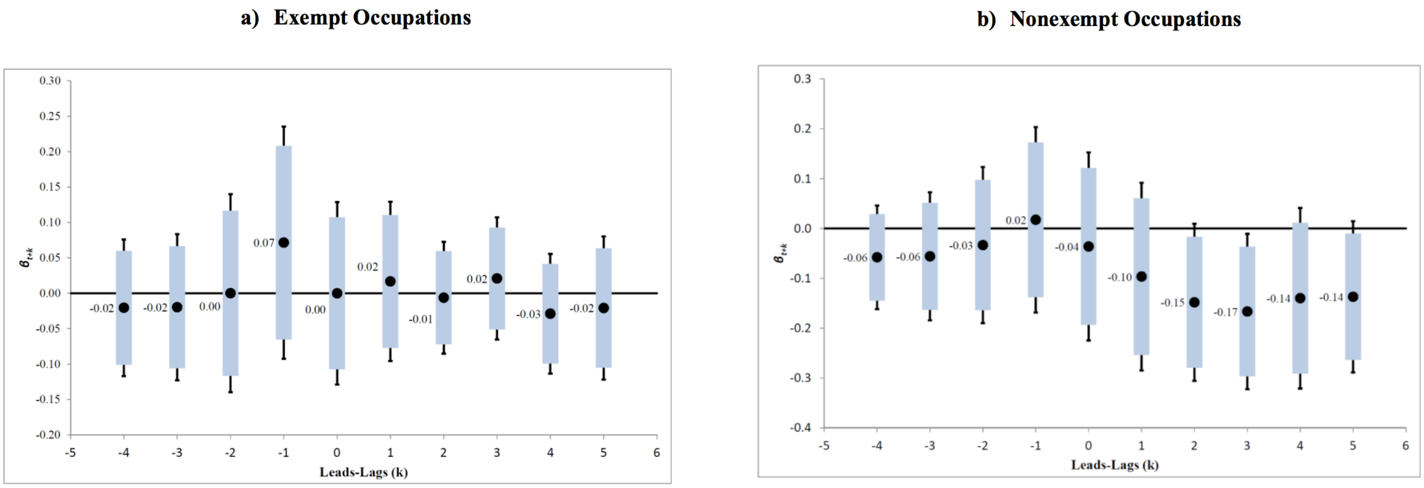
The three largest consumer credit agencies (Equifax Persona, Experian Employment Insight, and TransUnion PEER) market credit reports to employers, which include credit histories and public records (such as bankruptcy, liens and judgments). According to a Survey by the Society for Human Resource Management (2010), 60% of human resource representatives who were interviewed in 2009 indicated that their companies checked the credit of potential employees. Furthermore, a report by the policy think tank DEMOS found that 1 in 7 job applicants with bad credit had been denied employment because of their credit history (Traub [36]).

Until recently, pre-employment credit screening (PECS) was largely unregulated and remains so at the federal level – the FTC writes "As an employer, you may use consumer reports when you hire new employees and when you evaluate employees for promotion, reassignment, and retention as long as you comply with the Fair Credit Reporting Act (FCRA)."[1] However, since 2005, numerous state and federal laws have been introduced with the goal of limiting or banning employer credit checks and, as of 2018, eleven states have enacted such laws.[2] Legislators often express concern of a "poverty trap" arising due to employer credit checks: a worker loses her job and cannot pay her debts, which negatively impacts her credit report and thereby makes her unable to find a job. We build a model of unsecured credit and labor market search with adverse selection in which such poverty traps arise endogenously, which we use to assess the welfare consequences of policies to ban PECS.

A growing empirical literature seeks to estimate the effects of PECS on labor market outcomes. Examples include Ballance, Clifford and Shoag [2], Bartik and Nelson [3], and Cortes, Glover and Tasci [12], which is most directly related to this paper. Cortes, Glover and Tasci estimate a fall in posted vacancies following the implementation of employer credit check bans, but not in occupations that are exempted (jobs with access to financial or personal information). We reproduce their plots in Figure 1, showing

[1] http://www.ftc.gov/bcp/edu/pubs/business/credit/bus08.shtm

[2] The states with bans are CA, CO, CT, DE, HI, IL, MD, NV, OR, VT, WA.



(a) (b)

Notes: Regressions estimated for occupations $o \in \{exempt, affected\}$. Estimated equation is

$$\text{log vacancies}_{c,o,t} = \sum_{k=-4}^{5} \beta_k^o \text{BAN}_{c,o,t-k} + \text{FE}_t + \text{FE}_{o,c} + \varepsilon_{c,o,t},$$

where $BAN_{c,o,t} = 1$ if county c has a PECS ban in quarter t and occupation o is affected. Lead-lags are in quarters, with 5 representing more than one year post ban. Blue boxes are 90% confidence intervals. Exempt occupations are two-digit SOC codes representing Business and Financial (SOC-13), Legal (SOC-23), and Protective Services (SOC-33).

Figure 1: Effect of PECS Ban on Log-Vacancies

that affected occupations experience a significant decline in posted vacancies following the ban, which persists even after a year, whereas exempt occupations are unaffected. They also estimate an increase in delinquencies by subprime borrowers living in counties affected by employer credit check bans, which occurs in our model due to weakened repayment incentives. Their labor market estimates are directly related to the demand effect of our theory and their delinquency estimates confirm the possibility of a feedback from labor to credit markets.

Given the above empirical work on PECS, we develop a dynamic equilibrium model in order to understand the positive and normative implications of PECS. Our model features four key components: an unobservable characteristic that we model through hetregeneous time preferences (which creates adverse selection), an initial human capital investment (which is subject to moral hazard), labor search frictions, and unsecured credit with endogenous default. Employers value the PECS process because credit records are an externally verifiable and inexpensive signal about a *residual* component of labor productivity that is not observable *before* the worker is hired.[3] This component of productivity is the outcome of an *unobserved* investment in human capital, the cost of which is borne in disutility when young (i.e. studying at the library rather than going out with friends), but provides benefits later in life through higher wages. Patient workers therefore invest more in this human capital than do impatient. In equilibrium, patient (high productivity) workers are also less likely to default in response to unexpected expenditure shocks (health care bills, for example) since they value favorable future credit terms more than impatient types. This means that workers with higher credit scores have a higher expected match surplus and therefore generate higher profits post match, which in turn generates a tighter labor market for workers with good credit.

We make two assumptions to keep the labor market model tractable. First, we assume that all matches have positive surplus, so low-score matches generate low, but still positive, expected profits. Since our results depend on the job finding rate's sensitivity to the score rather than the exact point in the matching process at which the job finding rate is determined, we find this assumption innocuous.[4] Second, we assume that productivity is immediately learned by employers once a match is made. We view this as a technical assumption to retain tractability by avoiding asymmetric information during wage bargaining.[5] If learning was more gradual post-match, then we expect the job finding rate for job seekers with bad credit would be even lower than in our model, since their wages would begin higher; by symmetric reasoning, we expect job seekers with good credit would have higher job finding rates than in our model. This would only magnify the results in the paper.

Given that there appear to be interactions between labor and credit markets, we develop a rich model of credit markets with adverse selection. We model the credit market as a sequence of short-term loans, linked by the worker's score, which enters as a state variable representing the market belief that a worker is patient (and therefore low risk) given her history of repayments. Our short-term credit market equilibrium concept is from Netzer and Scheuer [31], which determines both interest rates and credit supply as the unique equilibrium of an extended form game played between lenders competing

[3]In our model, a credit record contains the borrower's history of debt repayment. This will map into a worker's ex-ante probability of being a patient type, which coincides with a higher ex-ante probability of repaying debt. We will therefore refer to the worker's "score" rather than report since it is this probability of being patient that is relevant for employers and lenders.

[4]If the surplus from an impatient worker was negative, then they would not be hired at all. With a positive surplus, they simply face a longer expected duration of unemployment.

[5]Jarosch and Pilossoph [22] make the same assumption in a model of bargaining with pre-match asymmetric information.

to make loans to borrowers with private information about their default rates.[6] This framework allows us to rationalize the credit market effects of credit scores and to study how the credit market responds to a PECS ban. First, the equilibrium contracts posted by lenders depend on the borrower's score because high-risk borrowers may be cross subsidized through lower interest rates, while higher scores relax credit constraints for low-risk borrowers. Second, the PECS ban affects individual repayment incentives and therefore the equilibrium credit market contracts (both interest rates and credit supply).

We then use this model as a laboratory to assess the effect of a policy that bans PECS (i.e. forces employers to ignore applicants' credit histories in the hiring decision). A PECS ban has both direct and indirect effects on the equilibrium. First, as expected by policy makers, there is a redistribution of labor market opportunity (and therefore welfare) from high to low credit score workers, which in equilibrium also translates into a redistribution from high to low productivity workers. However, there is also an indirect effect on repayment that lowers welfare for everyone. When credit scores are not used in the labor market, workers lose some of their incentives to repay debts. This leads to higher interest rates and less borrowing. This general equilibrium cost of PECS bans has not been considered by policy makers, even by those who advocate on behalf of lower income households with bad credit.

We proceed as follows. In Section 2, we place our paper in the context of the literatures on private information in both credit and labor markets. In Section 3 we describe the economic environment and in Section 4 we define, prove existence, and characterize equilibrium for our adverse selection environment as well as compare it to a full information version. In Section 5 we calibrate the economy and describe properties of the adverse selection equilibrium such as a poverty trap and quantify labor market inefficiencies. In Section 6 we study the welfare consequences of a ban on using credit checks in the labor market.

2 Related Literature

As discussed above, there is a growing empirical literature studying the effect of PECS bans on labor market outcomes in the U.S. Bartik and Nelson [3] use a statistical discrimination model to study the impact of PECS bans on different racial groups. They

[6]The first key feature of this game is that an equilibrium always exists. This would not be the case for low scores (i.e. when there are few high risk borrowers) in the competitive framework of Rothschild and Stiglitz [32].

find that the bans significantly reduce job-finding rates for blacks but that the results for Hispanics and whites are less conclusive. Their findings are consistent with PECS bans reducing the match quality of newly hired black job applicants (more high match-quality applicants are rejected and more low match-quality applicants are hired after the ban). Bos, Breza and Liberman [4] provide empirical evidence on the effect of credit history on labor market outcomes. They estimate a large negative effect of delinquencies on employment and earnings in Sweden and argue that their results are driven by pre-employment screening on the part of employers. Similarly, Friedberg, Hynes and Pattison [15] estimate an increase in job-finding rates for financially distressed households following PECS bans, which highlights the distributional effect of these laws and provides a key elasticity that our quantitative model matches.

Our paper contributes to the literature on asymmetric information in unsecured consumer credit markets with default. Some closely related papers include Athreya, Tam and Young [1], Chatterjee, et. al. [7], Chatterjee, et. al. [8], Livshits, MacGee and Tertilt [27], and Narajabad [30] so we briefly describe how our approach differs from theirs.[7] First, we include labor market search frictions as in Mortensen and Pissarides [29]. Second, we employ a different equilibrium concept in the credit market. This equilibrium, studied by Netzer and Scheuer [31], is the robust sub-game perfect equilibrium of a sequential game between firms competing to make short term loans to borrowers with private information about their default propensities. The salient assumption is that competitive lenders endogenously choose *both* the level of debt *and* the price at which it is offered as opposed to offering a risk adjusted competitive (break even) price for each *given* level of debt as in, for instance, Chatterjee, et. al. [8]. The equilibrium allocation of this game solves a constrained optimization problem with incentive compatibility constraints and the equilibria may feature cross-subsidization or even pooling.[8] We make a methodological contribution to the static model of Netzer and Scheuer by introducing a dynamic Bayesian type score upon which contracts are conditioned every period so that an individual's credit access varies over time in response to past behavior.

Our paper is also related to the literature on the effect of asset markets on labor markets. These papers focus on how financial status (i.e. ability to borrow or dis-save to fund current consumption) affect job-finding rates. Lentz and Tranaes [25] study

[7]The paper is also related to the reputation based model of Cole and Kehoe [9], who demonstrate how an exogenous utility loss in the labor market can incentivize sovereigns not to default in the credit market.

[8]We discuss the relationship between our allocations and the fully separating equilibria in Guerrieri, Shimer and Wright [17] in Section 4.3 where we present the programming problem.

the effect of precautionary savings on workers' search intensity and job-finding rates in partial equilibrium. Krusell, Mukoyama and Sahin [23] extend the Diamond-Mortensen-Pissarides general equilibrium model with random search and ex-post bargaining to include risk aversion and precautionary savings. While workers do not accumulate wealth in our model, credit access has a similar effect because it controls the worker's ability to smooth consumption and therefore their valuation of a job, which in turn affects finding rates and wages.

While we model the effect of credit histories on labor demand, a related literature uses changes in an individual's credit score to instrument for credit access in order to estimate labor supply response to credit. In a series of papers, Herkenhoff, Phillips and Cohen-Cole ([20], [21]) show that increased credit access leads workers to become more selective in their job search (accept longer unemployment durations in order to obtain higher post-employment wages) and more likely to start their own business. We do not model the search decision of unemployed workers, but note that in our model an unemployed worker with bad credit would have a strong incentive to find a job in order to begin rebuilding her credit history. Furthermore, a worker with bad credit has a weaker bargaining position, which is reflected in lower equilibrium wages (although quantitatively this effect is small).

Finally, we contribute to the literature on labor market discrimination and screening based on observable characteristics of workers. There are far too many papers to discuss fully so we simply relate our paper to one of the most relevant. Jarosch and Pilossoph [22] build a labor search model with ex-ante private information about worker productivity that is correlated with unemployment duration and therefore used to screen job seekers ex-ante. As in our paper, they also assume that the worker's type is revealed after matching, so that bargaining is under full information. We abstract from unemployment duration as a signal (since all matches have positive surplus in our model, duration provides no additional information about type beyond the credit report), but our model is also more general in some dimensions. Most importantly, our signal is directly affected by a worker's credit market decisions and the information context of the signal endogenously responds to labor market policies.

3 Environment

Time is discrete and infinite. Each period is split into two subperiods (i.e. a beginning and end of the month). The economy is composed of a large number of workers, firms, lenders, and the credit reporting agency.

A newborn starts life unemployed and draws a discount factor β_i, which determines her type $i \in \{H, L\}$. The probability the agent draws $\beta_H > \beta_L$ is given by π_H. We call a worker "patient" if her discount factor is β_H. A worker keeps her discount factor throughout her life and dies with probability δ. A newborn worker of type i makes a one-time choice of her human capital $h_i \in \{\underline{h}, \overline{h}\}$ at cost $\phi \times h_i$ where $\underline{h} < \overline{h}$.[9] The human capital choice is observed only by the agent and her eventual employer, but not by the eventual employer during the PECS hiring decision nor by lenders or the credit reporting agency. Since the cost of the human capital choice is born today and payoffs come in the future, patient workers will tend to accumulate more human capital in the equilibrium we consider.

In any period t, workers have one unit of time in the first subperiod and zero in the second subperiod. They can either be unemployed ($n_t = 0$) or employed ($n_t = 1$), which means they work for a firm. Worker preferences are represented by the function $\mathcal{U}(c_{1,t}, c_{2,t}, n_t) = c_{1,t} + z(1 - n_t) + \psi c_{2,t}$ with the unemployed getting $\mathcal{U}(0, 0, 1)$ and the employed getting $\mathcal{U}(c_{1,t}, c_{2,t}, 0)$ (i.e. the employed derive disutility from work). We assume that $\psi < 1$ so that workers prefer consumption in the first subperiod to the second. Since an unemployed worker does not receive income with which to repay debt, she cannot borrow, and hence her flow utility is simply z.

Once employed, a worker's human capital is observable to the firm. Production takes place in two stages: the worker puts in effort ($n_t = 1$) in the first subperiod which generates output $y_t = h_i n_t$ in the second subperiod. The worker and firm Nash bargain over her wage w_t in the first subperiod to be paid when her effort yields output in the second subperiod. The worker's bargaining weight is λ and her outside option is to walk away, receive z utility from leisure in this period, and to search for another match tomorrow. The outside option for the firm is to produce nothing this period and post another vacancy at cost κ (in equilibrium the firm's outside option will be zero due to free entry). The firm sells its second subperiod output, yielding period t profits of the firm given by $h_i - w_t$, which are valued as $\psi(h_i - w_t)$ in the first subperiod of t. After production, the worker and firm may exogenously separate with probability σ.

[9]Under our parametric assumptions, a patient household will choose $\overline{h}$ and an impatient will choose $\underline{h}$ in equilibrium, which generates a lower match surplus (and therefore lower job finding rate) for low-score workers. Other mechanisms could generate such a difference in match surpluses, such as impatient workers providing less effort or having higher separation rates. Direct moral hazard in the form of theft is also a possible reason for employers to check credit reports, but laws restricting PECS explicitly exempt jobs for which embezzlement is a concern, so this mechanism is less relevant for our policy experiment.

Since an employed worker is paid at the end of the period, if she wants to consume at the beginning of the period and has no savings, she can borrow Q_t from a lender.[10] When an employed worker borrows in the first subperiod, she is expected to repay the unsecured debt b_t once she is paid in the second subperiod, provided she does not default. In the second subperiod, however, an employed worker receives an expenditure shock, τ, drawn from a distribution with CDF $F(\tau)$. The expenditure shock is unobservable to anyone but the worker. Her choice of whether to repay in the second subperiod $d_t \in \{0, 1\}$ is recorded by a credit reporting agency. If the worker does not repay (i.e. $d_t = 1$) we say she is delinquent at time t and defaults at $t + 1$. Default bears a bankruptcy cost ϵ in the second subperiod at $t + 1$, which corresponds to both direct costs (legal fees), but is also a reduced form for higher costs borne in other markets due to bad credit (for example, higher insurance premiums, as explored in Chatterjee, Corbae and Rios-Rull [7]).

A credit reporting agency records the history of repayments by a worker, which is summarized by a score s_t. This score is the probability that a given worker is type H with discount factor β_H at the beginning of any period t. Given the prior s_t and the repayment decision d_t, the credit reporting agency updates the assessment of a worker's type s_{t+1} via Bayes Rule.[11] Since a patient worker cares about their future ability to borrow more than an impatient worker, repayment is a signal to a scorer that the worker is more likely to be a high type. Our type score s_t is therefore not directly comparable to empirical credit scores such as FICO, which orders repayment likelihood on an index from 300 to 850. However, we can rank people by their expected repayment rate within the model, which allows us to group them into credit ratings (subprime, prime, and super prime) based on their ordering in the population, as in the data.

Since a worker's type influences her human capital and default decisions, a worker's score may be used in hiring and lending decisions. We assume that matches between job seekers with score s_t, denoted $u(s_t)$, and firms posting vacancies for such workers, denoted $v(s_t)$, are governed by a constant returns to scale matching function, $M\big(u(s_t), v(s_t)\big)$. Therefore, an unemployed worker with score s_t matches with a firm with probability $f\big(\theta_t(s_t)\big) = \frac{M\big(u(s_t), v(s_t)\big)}{u(s_t)} = M\big(1, \frac{v(s_t)}{u(s_t)}\big)$. We will assume that a tighter

[10] We will develop the model without intertemporal savings, but will assume that $\beta_H \leq R^{-1}$ which, along with the linearity of preferences, ensures that households do not want to save.

[11] We assume that unemployed workers do not receive the expenditure shock since they have no income with which to pay it. If an unemployed worker received an i.i.d. expenditure shock, she would default with probability one, which would not provide any new information and their score would remain the same.

labor market (higher $\theta(s_t)$) increases the job finding rate for workers (i.e. $f'\big(\theta_t(s_t)\big) > 0$). The cost to a firm of posting a vacancy for workers with score s_t is denoted κ and the job filling rate is denoted $q(\theta_t(s_t))$, which is decreasing in tightness (i.e. $q'\big(\theta_t(s_t)\big) < 0$). Future profits of the firm are discounted at rate R^{-1}.

There are a large number of competitive lenders who have access to consumption goods in the first subperiod, for which they must pay an exogenously given worldwide interest rate of R in the second subperiod. Lenders observe each potential borrower's type score s_t and post a menu of contracts $\mathcal{C}_t(s_t) = \{(Q_{jt}(s_t), b_{jt}(s_t))\}_{j=1}^{J}$ which specifies an amount to be lent in the first subperiod (i.e. at the beginning of the month), Q_{jt}, and a promised repayment in the second subperiod (i.e. at the end of the month), b_{jt}. Lenders realize that households may default on their debt and the probability may differ by worker type, which affects their expected profits for a given contract. As in Netzer and Scheuer [31], after posting these menus the lenders observe all other menus posted and then may withdraw from the market at a cost k.[12]

Specifically, a large number of lenders play a game against one another by posting menus of contracts (including $(0, 0)$ so that a worker need not borrow) for each observable credit score $\mathcal{C}_t(s_t)$. The game has three stages, all of which occur in the beginning of the first subperiod of t:

Stage 1: Lenders simultaneously post menus of contracts.

Stage 2: Each lender observes all other menus from stage 1. Lenders simultaneously decide whether to withdraw from the market or remain. Withdrawal entails removing the lender's entire menu of contracts with a payoff of $-k$ (i.e. it is costly to withdraw).

Stage 3: Workers simultaneously choose the contract they most prefer.

To summarize the information structure, workers observe everything (i, h, s_t, τ_t). Before hiring a worker, a firm only observes the worker's score s_t, which we refer to as pre-employment credit screening. After hiring a worker, a firm observes her type i and human capital h. Lenders only observe the worker's score s_t. The credit reporting agency observes a worker's current score s_t and default decision d_t. Credit and labor markets are segmented in the sense that lenders and scorers cannot communicate with firms who know the worker's type after the hiring decision.

[12]The ability to withdraw contracts after observing all others posted is key to ensuring that an equilibrium exists, counter to purely competitive models with adverse selection. That the withdrawal of contracts is costly ensures that the equilibrium is unique.

Having described the environment for workers, firms, lenders, and credit reporting agencies, we now describe the timing of actions. Under the assumption that workers do not start the period with assets (which we will show is optimal by setting $\beta_L \leq \beta_H < R^{-1}$), a worker of type i with credit score s_t and human capital h begins the period either unemployed or employed.

For an unemployed worker:

1. Enjoy utility z_t from leisure $n_t = 0$.
2. Die with probability δ.
3. Surviving workers with score s_t are matched with a firm in labor sub-market s_t with probability $f\big(\theta_t(s_t)\big)$

For an employed worker:

1. First Subperiod:

1.1 Determine earnings w_t via Nash Bargaining and work $n_t = 1$.

1.2 Choose debt contract $\big(Q_{jt}(s_t), b_{jt}(s_t)\big)$ and consume Q_{jt}.

2. Second Subperiod:

2.1 Output $y_t = h_i \cdot n_t$ is created, from which earnings w_t are paid.

2.2 Draw expenditure shock τ_t from CDF $F(\tau_t)$

2.3 Choose whether to default $d_t \in \{0, 1\}$ and pay $(1 - d_t)(b_{jt} + \tau_t)$.

2.4 Type score updated $s_{t+1}(s_t, d_t)$.

2.5 Separate from employer exogenously with probability σ and die with probability δ.

4 Equilibrium

We now provide the decision problems for all agents in recursive form. To that end, we let variable x_t be denoted x and x_{t+1} be denoted x'. Further, to save on notation we denote $s_{t+1}(s_t, d_t)$ as s'_d and will use x_{i^*} in place of x_{i,h_i^*} whenever we are evaluating an equilibrium variable at the optimal human capital choice of an i type worker.

4.1 Worker Decisions

The value function for an unemployed worker of type i with human capital h and score s is given by

$$U_{i,h}(s) = z + (1-\delta)\beta_i \left[f\big(\theta(s)\big) W^*_{i,h}(s) + \Big(1 - f\big(\theta(s)\big)\Big) U^*_{i,h}(s) \right] \tag{1}$$

where $W^*_{i,h}(s)$ and $U^*_{i,h}(s)$ are the value functions evaluated at equilibrium credit contracts and wages, as described below. The unemployed worker receives current flow utility z and survives until the next period with probability $1-\delta$. She then transits to employment next period with probability $f(\theta(s))$ and remains unemployed with probability $1 - f(\theta(s))$. Note that, with no credit market activity, the unemployed worker's score remains constant. Furthermore, since job-finding rates are identical for both worker types conditional on score and all matches have positive surplus, scores are independent of the length of an unemployment spell or total number of spells.

The value function for an employed worker of type i with human capital h and score s who has chosen contract (Q, b) and wage w is given by

$$\begin{aligned} W_{i,h}(Q,b,w,s) &= Q + \psi w \\ &+ \psi \int_0^\infty \max_d \left[\beta_i (1-\delta) \Big(V_{i,h}\big(s'_d\big) - d\psi\epsilon \Big) - (1-d)(b+\tau) \right] dF(\tau), \end{aligned} \tag{2}$$

where we have introduced the intermediate value function:

$$V_{i,h}\big(s'_d\big) = \left[(1-\sigma) W^*_{i,h}\big(s'_d\big) + \sigma U^*_{i,h}\big(s'_d\big) \right]. \tag{3}$$

The first line in (2) reflects borrowing $Q(s)$ to pay for first subperiod consumption and the second subperiod wage w payment. The second line in (2) reflects the strategic decision of whether to go delinquent to avoid paying off $b + \tau$ in the second subperiod followed by default which bears bankruptcy cost ϵ the following period. Note that the scorer updates s'_d his assessment of the agent's type given the worker's default decision d.

Working backwards, we start by characterizing the worker's default choice, taking all other objects (in particular their contract choice) as given. The worker defaults if and only if:

$$\tau > \tau^*_{i,h}(s,b) \equiv \beta_i(1-\delta) \left[\psi\epsilon + V_{i,h}\big(s'_0\big) - V_{i,h}\big(s'_1\big) \right] - b \tag{4}$$

Thus, higher debt and higher expenditure shocks make default more likely. Furthermore, a lower discount factor and value from a good reputation (i.e. $V_{i,h}\big(s_0'\big) - V_{i,h}\big(s_1'\big)$) make default more likely. Using τ^*, after integrating by parts and some cancelation, this allows us to evaluate the integral in $W_{i,h}$ for given values of (Q, b, w):

$$W_{i,h}(Q,b,w,s) = Q + \psi w + \psi \int_0^{\tau^*_{i,h}(s,b)} F(\tau)d\tau + \psi\beta_i(1-\delta)\Big[V_{i,h}\big(s_1'\big) - \psi\epsilon\Big] \tag{5}$$

We can then write the worker's surplus (i.e. utility when employed versus unemployed) evaluated at the equilibrium contracts $(Q^*_{i,h}(s), b^*_{i,h}(s))$ as the difference:

$$W_{i,h}\big(Q^*_{i,h}(s), b^*_{i,h}(s), w, s\big) - U_{i,h}(s). \tag{6}$$

Finally, since a newborn begins life unemployed and there are only two values for human capital, her human capital choice must satisfy:

$$h_i^* = argmax_{h\in\{\underline{h},\overline{h}\}}\Big[\beta_i U_{i,h}(\pi_H) - \phi h\Big]. \tag{7}$$

In Theorems 1 and 2 below, we will assume that $\beta_L, \beta_H, \phi, \underline{h}$, and $\overline{h}$ are such that patient workers $(i = H)$ choose a high level of human capital $\overline{h}$ while impatient workers choose the low level of human capital $\underline{h}$.

4.2 Firm's Problem and Wage Determination

Recall that after a firm and worker are matched, the worker's type and human capital choice is observed by the firm. The value function for a firm matched with a worker of type i with human capital h and current type score s for a given wage w is:

$$J_{i,h}(w,s) = \psi\int_0^\infty \Big[h - w + R^{-1}(1-\sigma)(1-\delta)J_{i,h}\big(w^*_{i,h}(s_d'), s_d'\big)\Big]dF(\tau). \tag{8}$$

While s does not add information for the firm's inference about worker type, it influences the worker's bargaining position since it determines their credit contract and hence the worker's flow surplus from being employed. Since Nash Bargaining ensures that the firm receives a constant fraction of the match surplus as in (10) below, the firm's surplus will also depend on s *even though the firm knows i during bargaining.* Since free entry ensures that the firm's value of posting a vacancy is zero, the firm's surplus from a match is simply $J_{i,h}(w,s)$.

The wage is then determined by generalized Nash Bargaining in which the worker's bargaining weight is λ. The wage solves:

$$w^*_{i,h}(s) = \text{argmax}_w \left[W_{i,h}\big(Q^*_{i,h}(s), b^*_{i,h}(s), w, s\big) - U^*_{i,h}(s)\right]^\lambda J_{i,h}(w,s)^{1-\lambda} \tag{9}$$

Given that worker utility and firm profits are linear in earnings, (9) amounts to a simple splitting rule for the total surplus so that firms receive fraction $1-\lambda$, i.e.

$$J_{i,h}(w,s) = (1-\lambda)\Bigg(W_{i,h}\big(Q^*_{i,h}(s), b^*_{i,h}(s), w, s\big) + J_{i,h}(w,s) - U^*_{i,h}(s)\Bigg), \tag{10}$$

and the worker's surplus is fraction λ of the total. Note that the current wage does not directly affect the repayment decision or optimal debt choice of a household due to the linearity of preferences. If these choices were to depend on the wage, then the wage would affect both the size of the worker's surplus and the split of the total surplus, creating a nonconvexity that would complicate the analysis.

Firms post vacancies in labor "sub-markets" indexed by an unemployed worker's score s so that labor "sub-market" tightness is given by $\theta(s)$.[13] The expected profits from posting a vacancy must be equal to the cost of the vacancy in equilibrium:

$$\kappa = R^{-1} q\big(\theta(s)\big)\left[sJ^*_{H^*}\big(w^*_{H^*}(s), s\big) + (1-s)J^*_{L^*}\big(w^*_{L^*}(s), s\big)\right] \tag{11}$$

where, remember, $x_{i^*} = x_{i,h^*_i}$ and h^*_i is chosen in (7).[14]

4.3 Lender's Problem and Credit Contract Determination

Invoking Proposition 2 from Netzer and Scheuer [31], for sufficiently small $k > 0$ (i.e. $k \to 0$), the unique equilibrium to the lending game for credit sub-markets with score s is the two-contract menu $\{(Q_H(s), b_H(s)), (Q_L(s), b_L(s))\}$ that solves the following

[13] Our sub-markets are indexed by score rather than contract terms as in the models of directed search. A form of block recursivity, as in Menzio and Shi [28], exists when firms can screen using scores because the score corresponds to the fraction of good types with that score and hence firms do not need to know the entire distribution of workers over scores to evaluate the expected value of posting a vacancy in that sub-market.

[14] We write the free entry condition for a symmetric equilibrium in which individuals of each type choose the same h at birth. This simplifies exposition considerably and will be true in equilibrium.

constrained optimization problem:

$$\max_{\{Q_H,b_H,Q_L,b_L\}} Q_H \quad + \quad \psi \int_0^{\tau^*_{H^*}(s,b_H)} F(\tau)d\tau \tag{12}$$

s.t.

$$s\Big[-Q_H + R^{-1}F(\tau^*_{H^*}(s,b_H))b_H\Big] \quad + \tag{13}$$

$$(1-s)\Big[-Q_L + R^{-1}F\big(\tau^*_{L^*}(s,b_L)\big)b_L\Big] \geq 0$$

$$Q_L + \psi \int_0^{\tau^*_{L^*}(s,b_L)} F(\tau)d\tau \quad \geq \quad Q_H + \psi \int_0^{\tau^*_{L^*}(s,b_H)} F(\tau)d\tau \tag{14}$$

$$Q_H + \psi \int_0^{\tau^*_{H^*}(s,b_H)} F(\tau)d\tau \quad \geq \quad Q_L + \psi \int_0^{\tau^*_{H^*}(s,b_L)} F(\tau)d\tau \tag{15}$$

$$Q_L + \psi \int_0^{\tau^*_{L^*}(s,b_L)} F(\tau)d\tau \quad \geq \tag{16}$$

$$\max_b R^{-1}F\big(\tau^*_{L^*}(s,b)\big)b \quad + \quad \psi \int_0^{\tau^*_{L^*}(s,b)} F(\tau)d\tau.$$

This problem says that the credit contract for a worker whose score is s is designed to maximize the utility of the type H (low-risk) borrower subject to profitability, incentive compatibility, and participation constraints. The first constraint (13) says that the lender must make non-negative profits on the contract for each score. The first term is the profit (or loss) per type H borrowers' contract times the number of patient borrowers with score s. The second term is profit (or loss) for type L borrowers' contract times the number of impatient borrowers with score s. Note that (13) does not rule out cross-subsidization. The second and third inequalities ((14) and (15)) are incentive compatibility constraints. For instance, (14) says that impatient borrowers must choose the contract designed for them rather than the one designed for patient borrowers. The final constraint (16) says that an impatient borrower must get at least the utility from a credit contract that breaks even and maximizes her utility. That is, the equilibrium contract must give the impatient borrower at least her utility from her least cost separating contract, and will deliver strictly more utility if the contract cross subsidizes impatient borrowers.

We note some special properties of this game and its solution. First, we need a well defined solution for all credit scores, which would not be the case in the competitive model of Rothschild and Stiglitz [32]. In that model there would be no equilibrium for a score close enough to one, whereas in this model an equilibrium always exists.[15] The Netzer and Scheur equilibrium contract can be one of three types: least cost separating (denoted LCS), cross-subsidized separating (denoted CSS), or pooling (denoted PC). Unlike Rothschild and Stiglitz, cross-subsidization can occur in a Netzer and Scheuer equilibrium because lenders can withdraw their contracts. If another lender posted a contract that cream-skimmed (ie, attracted only patient borrowers) then the lender posting the cross-subsidizing contract would make losses and withdraw for sufficiently low k. Impatient households would then choose the cream-skimming contract, which would then cease to make profits. Second, we want a model in which workers care about their future scores *because their score improves credit contract terms (lower rates or looser constraints)* and the fact that credit contracts are cross-subsidizing or pooling for high scores ensures this. This would not be the case in a model in which the credit contracts were always least-cost separating, such as the competitive search model of Guerrieri, Shimer and Wright [17].[16] In that case, absent PECS, an individual's credit score would have no affect on their credit contract in equilibrium, which is inconsistent with data.[17] Finally, the Netzer and Scheuer equilibrium concept ensures that credit market allocations are always statically constrained efficient. In our calibration, most workers are patient and have scores in the region where the LCS contract is dominated by either the CSS or PC contracts, so the welfare gains from using the Netzer and Scheuer equilibrium can be substantial.

In order to understand how type score s affects the credit contract, we first consider the full-information allocation and then demonstrate the general form of optimal constrained allocations that arise for different scores. The full-information allocation is shown in Figure 2.[18] The patient worker chooses more debt and receives a lower inter-

[15]Non-existence follows from the standard argument of Rothschild and Stiglitz: the competitive equilibrium cannot include a pooling contract, since lenders could "cream skim" the patient borrowers by posting a contract with a slightly tighter borrowing constraint but lower interest rate. On the other hand, if there were very few impatient borrowers and all other lenders were offering separating contracts with borrowing limits then a lender could post a pooling contract and attract the entire market at a profit. Hence, there would be no competitive equilibrium.

[16]Their equilibrium concept also has search frictions and contract posting in the credit market and hence an extra endogenous variable. Their framework is directly comparable with the least-cost separating contracts in our work if the cost of posting credit contracts was taken to zero.

[17]For instance, in states with PECS bans, interest rates on debt would be independent of credit scores.

[18]The full information contract maximizes an employed borrower type i's utility subject to zero expected profits on the type i contract. This corresponds to maximizing $Q_i + \psi \int_0^{\tau^*_{i^*}(s,b_i)} F(\tau)d\tau$ (as in (12)) for each type i, subject to $Q_i \leq R^{-1}F(\tau^*_{i^*}(s,b_i))b_i$ (as in (13)). Graphically, this

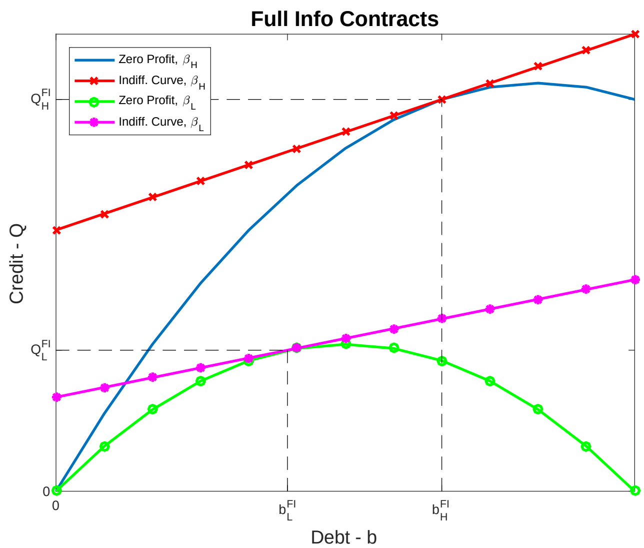


Figure 2: Full Information Example

est rate on this debt since she is less likely to default. But then, if type was private information, an impatient worker would choose the patient worker's contract, violating incentive compatibility in (14).

Figure 3 compares two different types of allocations under private information. In this case the impatient worker's incentive compatibility constraint (14) is binding (as well as their participation constraint (16)). The least cost separating (LCS) contracts are shown in the left box Figure 3a. These types of contracts arise for low scores (in our calibrated model, they arise for $s < 0.28$, whereas the median score is 0.69). The impatient borrower receives the same amount of debt as under full information and pays the risk-adjusted break-even interest rate. On the other hand, the patient borrower's contract is distorted because of the binding incentive compatibility constraint of the impatient worker. In particular, the patient borrower receives less debt than the impatient borrower, although her interest rate is still equal to the risk-adjusted break even rate on her loan. This puts the patient worker on a lower indifference curve than in Figure 2.

gives us indifference curves with slopes $\frac{dQ_i}{db_i} = \psi F\big(\tau^*_{i^*}(s,b_i)\big) \geq 0$ and isoprofit curves with slopes $\frac{dQ_i}{db_i} = R^{-1}\big[F\big(\tau^*_{i^*}(s,b_i)\big) - F'\big(\tau^*_{i^*}(s,b_i)\big)b_i\big]$. Since for a given (s,b), $\tau^*_{L^*}(s,b) < \tau^*_{H^*}(s,b)$, the slope of the type H indifference curve is greater than the slope of the type L. Furthermore, since the interest rate on these contracts is given by $\frac{b_i}{Q_i}$, the interest rate can be seen as the inverse of the slope of a ray from the origin to the contract point. This is analogous to the continuous asset version of Chatterjee, et. al. [6].

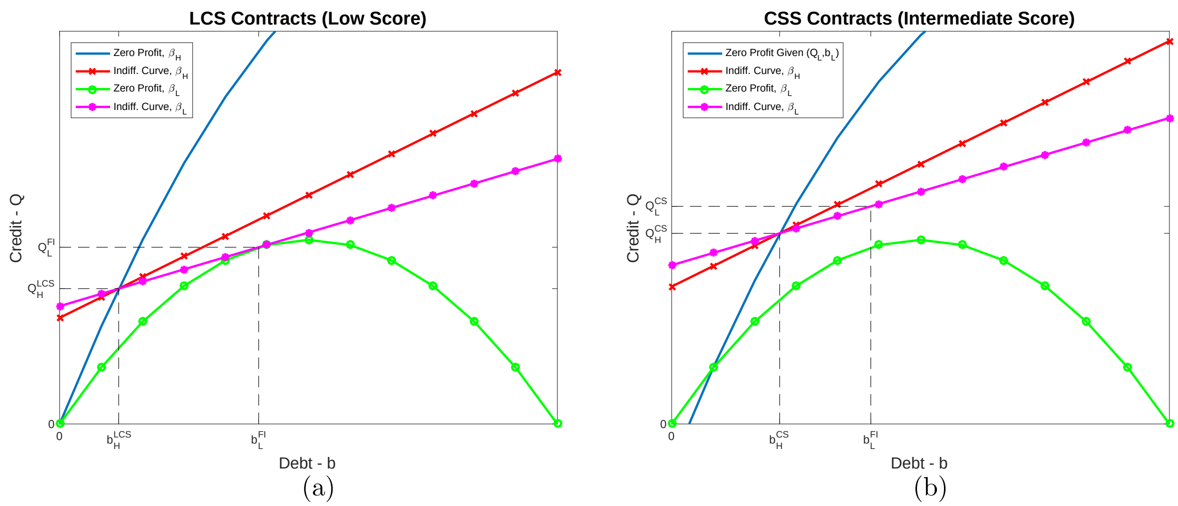
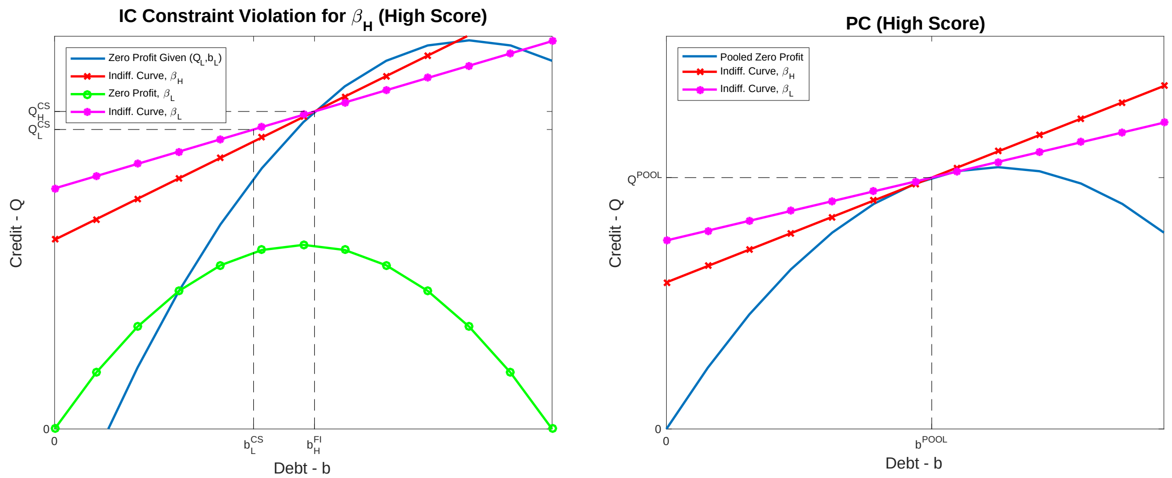


Figure 3: Least Cost (LCS) vs. Cross-Subsidized Separating (CSS) Contracts

As a worker's score rises the optimal contract switches from LCS to CSS. For CSS contracts, the impatient worker's participation constraint (16) is slack, because she still receives the full-information level of debt but pays a lower interest rate (illustrated by Q_L being above the impatient zero profit curve in Figure 3b). This moves the impatient borrower to a higher indifference curve, while shifting the effective zero-profit curve for patient borrowers downward by the total subsidy to impatient borrowers. The patient borrower's contract is given by the intersection of the impatient borrower's new indifference curve and the patient borrower's effective zero-profit curve. The CSS contract delivers more debt to the patient borrower than the LCS contract for the same score, but carries a higher interest rate than the LCS contract. The CSS contract dominates the LCS for intermediate scores ($0.28 \leq s < 0.42$ in our calibration) because the extra interest paid *per patient worker* to subsidize impatient workers is more than offset by the patient worker's utility gains from receiving more debt (e.g. loosening her credit limit).

The third contract type is pooling (PC), which can arise as s increases further (above 0.42 in our calibrated model) as the interest rate cross-subsidy to impatient workers becomes extremely generous. In this case, unlike the previous two, the patient household's incentive constraint (15) binds.[19] That this constraint binds can be seen in Figure 4a,

[19] In some settings, such as the constant risk model in Netzer and Scheuer, the high-type incentive



(a) (b)

Figure 4: Violation of Incentive Compatibility in CSS Vs. Pooling Contracts

where the interest rate paid by an impatient borrower in the CSS is so low that a patient borrower would prefer the impatient contract to the one prescribed to her. With so few impatient borrowers with a high score, the subsidy *per* impatient contract is too generous and the patient borrower would rather have the impatient borrower's subsidized rate, even though this gives her less credit. Therefore both incentive compatibility constraints bind, which means that the contract must be pooling (i.e. each type receives the same debt and interest rate). We find this contract by maximizing the utility of the patient borrower subject to the pooled zero-profit condition. Graphically, this is given by the tangency between the patient worker's indifference curve and the pooled zero-profit curve, as in Figure 4b.[20]

compatibility constraint never binds. This is not the case in our model because of our interaction of adverse selection and moral hazard, which means that default rates (and therefore the indifference curves and zero-profit curves) depend on debt for each borrower.

[20]The formula for the patient borrower's indifference curve is the same as before. The slope of the pooled zero-profit curve is given by $\frac{dQ}{db} = \frac{d}{db}\left\{R^{-1}\left[sF\big(\tau^*_{H^*}(s,b)\big) + (1-s)F\big(\tau^*_{L^*}(s,b)\big)\right]b\right\}$.

4.4 Type Scoring

Given the prior probability s that a worker is type H, the credit reporting agency forms a Bayesian posterior s' the worker is type H conditional on seeing whether she repays d:

$$s'_d(s) = \frac{F_d\Big(\tau^*_{H^*}\big(s, b^*_{H^*}(s)\big)\Big)s}{F_d\Big(\tau^*_{H^*}\big(s, b^*_{H^*}(s)\big)\Big)s + F_d\Big(\tau^*_{L^*}\big(s, b^*_{L^*}(s)\big)\Big)(1-s)}, \tag{17}$$

where the probability of receiving a shock lower than τ is given by $F_0(\tau) \equiv F(\tau)$ and the probability of receiving a shock larger than τ is given by $F_1(\tau) \equiv 1 - F(\tau)$.

Typically a credit score is a measure of how likely the borrower is to repay. In the context of our model, s is a "type" score. In equilibrium we can map s to a credit score (i.e. the probability of repayment given s) as follows:[21]

$$\Pr\big(d = 0|s\big) = F_0\Big(\tau^*_{H^*}\big(s, b^*_{H^*}(s)\big)\Big)s + F_0\Big(\tau^*_{L^*}\big(s, b^*_{L^*}(s)\big)\Big)(1-s). \tag{18}$$

4.5 Distributions

We denote the measure of workers of type i over employment status $n \in \{0, 1\}$ (where 1 denotes employed and 0 denotes unemployed) and score s in period t as $\mu_{i,n}(s)$. Given $\mu_{i,n}(s)$, we can compute $t+1$ measures (denoted $\mu'_{i,n}(S)$ for some set of scores S) using decision rules and the updating function (recalling that h^*_i is constant over time). For the employed we have:

$$\begin{aligned} \mu'_{i,1}(s') &= (1-\delta)\int_0^{s'} f\big(\theta(s)\big)d\mu_{i,0}(s) \qquad (19) \\ + \quad & (1-\delta)(1-\sigma)\int_0^1 \Big\{\mathbb{I}_{\{s'_0(s)\le s'\}}F_0\big(\tau^*_{i^*}(s, b^*_{i^*}(s))\big) + \mathbb{I}_{\{s'_1(s)\le s'\}}F_1\big(\tau^*_{i^*}(s, b^*_{i^*,}(s))\big)\Big\}d\mu_{i,1}(s). \end{aligned}$$

where $\mathbb{I}_{\{s'_d(s)\le s'\}}$ is an indicator function which takes the value one if $s'_d(s) \le s'$ and zero otherwise.

[21] Our score is consistent with credit scoring in reality, in that past actions in the credit market are used to forecast the likelihood of an individual defaulting on her debt (though her type). In our model, this is reflected by interest rates falling with credit rating, which we calibrate to be consistent with the data, as seen in Figure 9a. This is true even if the score is not highly predictive of a borrower's future likelihood of default after conditioning on other variables observed by an econometrician; since unobservable type does not change across an agent's lifetime, all that matters is that the score encapsulates something about the workers's type revealed by his history.

For the unemployed we have two regions. For scores lower than the population share of patient workers (i.e., for $s < \pi_H$):

$$\mu'_{i,0}(s') = (1-\delta)\int_0^{s'} \left[1 - f\big(\theta(s)\big)\right] d\mu_{i,0}(s) \tag{20}$$

$$+ \quad (1-\delta)\sigma \int_0^1 \Big\{ \mathbb{I}_{\{s'_0(s)\le s'\}} F_0\big(\tau^*_{i^*}(s, b^*_{i^*}(s))\big) + \mathbb{I}_{\{s'_1(s)\le s'\}} F_1\big(\tau^*_{i^*}(s, b^*_{i^*}(s))\big) \Big\} d\mu_{i,1}(s).$$

For scores above π_H we must add the newborns who start unemployed with $s = \pi_H$. That is, for $s \ge \pi_H$:

$$\mu'_{i,0}(s') = \delta + (1-\delta)\int_0^{s'} \left[1 - f\big(\theta(s)\big)\right] d\mu_{i,0}(s) \tag{21}$$

$$+ \quad (1-\delta)\sigma \int_0^1 \Big\{ \mathbb{I}_{\{s'_0(s)\le s'\}} F_0\big(\tau^*_{i^*}(s, b^*_{i^*}(s))\big) + \mathbb{I}_{\{s'_1(s)\le s'\}} F_1\big(\tau^*_{i^*}(s, b^*_{i^*}(s))\big) \Big\} d\mu_{i,1}(s).$$

4.6 Definition of Equilibrium

A steady-state Markov equilibrium consists of the following functions:

1. Worker value functions, $U^*_{i,h}(s), W^*_{i,h}(s)$, satisfy (1) and (2).
2. Default threshold functions, $\tau^*_{i,h}(s,b)$, satisfies (4).
3. Human capital investment, h^*_i, satisfies (7).
4. Firm value functions, $J_{i,h}(s)$, satisfies (8).
5. Wage functions, $w^*_{i,h}(s)$, satisfies (9).
6. Market tightness functions, $\theta^*(s)$, satisfies the free entry condition (7).
7. Credit market contracts, $\{(Q^*_{i,h}(s), b^*_{i,h}(s))\}_{i\in\{H,L\}}$, satisfy (12)-(16).
8. The updating function, s'_d, satisfies (17).
9. Stationary measures of each worker type over human capital levels and scores, $\mu^*_{i,1}(s), \mu^*_{i,0}(s)$ that satisfy (19) through (21) with $\mu'_{i,n}(s) = \mu_{i,n}(s) = \mu^*_{i,n}(s)$ for $n \in \{0,1\}$ and $i \in \{L, H\}$.

4.7 Full Information Equilibrium Characterization

We will define a poverty trap relative to the equilibrium outcomes of a full information model, so we provide a characterization. We first make parametric assumptions to guarantee that workers borrow within a period and do not save across periods (A.1), that the match surplus of both workers is positive (A.2), that credit contracts are unique (A.3), and that patient workers choose a high level of human capital while impatient workers choose a low level (A.4). We also ensure that all workers would repay some positive level of debt (A.5) and that all workers default with positive probability (A.6).

Assumption 1 .

A.1 $\psi < (\omega R)^{-1}$, $\beta_L < \beta_H \leq R^{-1}$

A.2 $z < \underline{h}$

A.3 $F''(\tau) \leq 0$

A.4 ϕ *and* β_L *are sufficiently small.*

A.5 $F\big(\beta_L(1-\delta)\psi\epsilon\big) > 0$

A.6 *The support of* τ *is unbounded above.*

In Appendix A we define a full-information equilibrium and prove the following:

Theorem 1 *Under Assumption 1, there exists a full information steady-state Markov equilibrium where* i *and* h *are publicly observable that is characterized by the following equations:*

$$h_H^* = \overline{h}, h_L^* \quad = \quad \underline{h} \tag{22}$$

$$\theta_H^* > \theta_L^* \rightarrow f(\theta_H^*) > f(\theta_L^*) \tag{23}$$

$$w_H > w_L \tag{24}$$

$$F_0\big(\tau_H^*(b_H^*)\big) > F_0\big(\tau_L^*(b_L^*)\big) \tag{25}$$

Importantly, with full information under the parametric restrictions in Assumption 1, patient workers choose higher human capital than impatient workers, have higher job finding rates (in (23)), have higher wages (in (24)), and have lower default rates ((25)) implies higher repayment rates for patient workers).

4.8 Existence of Private Information Equilibrium

We build an equilibrium in which patient households choose high human capital (i.e. $\overline{h}$), impatient households choose low human capital (i.e. $\underline{h}$), and repayment leads to a higher future score than does default due to Bayesian updating (i.e. updating function $s_0'(s) \geq s_1'(s)$ (with equality only when $s = 0$ or $s = 1$). Existence is complicated by the scoring functions, which are not contractions, and the programming problem generating credit contracts. We must therefore make additional technical assumptions to guarantee existence.

Theorem 2 *Under the restrictions in Assumption 1 as well as additional conditions on $F(\tau)$, ψ, ω, $R, \beta_L, \beta_H, f(\theta), q(\theta)$, and the programming problem in (12) through (16), there exists an equilibrium as defined in Section 4.6 with $h_L^* = \underline{h}$ and $h_H^* = \overline{h}$.*

The proof and additional conditions are in the appendix. The idea is to define a continuous operator mapping Lipschitz functions into themselves using the equilibrium conditions defined in Section 4.6. In the appendix in Section B, we define this operator, show how to find a Lipschitz space of functions for which the operator is a continuous self mapping, and then apply Schauder's fixed point theorem.

Economically speaking, existence requires that the marginal effect of default or repayment is sufficiently small so that the updating functions do not change rapidly across scores. This in turn requires that the odds-ratios for default and repayment are sufficiently independent of changes in score and continuation utilities, which in turn requires the same for the optimal contracts of each household type. We accomplish this by assuming that expenditure shocks are sufficiently volatile (i.e. $\sup_{\tau \geq 0} F'(\tau)$ is small) and that the slope of each Q_i and b_i with respect to s and $V_{i^*}(s_0'(s)) - V_{i^*}(s_1'(s))$ is sufficiently small.

5 Quantitative Exercise

To demonstrate how a poverty trap may arise and how markets respond to a policy banning PECS, we compute an equilibrium of the economy and then change the determination of market tightness so that it is independent of type score (consistent with a ban).[22]

[22]The algorithm for computing an equilibrium is available upon request.

5.1 Calibration

A model period is taken to be a month. We use a Cobb-Douglas matching technology so that the job-finding and filling rates are given by $f(\theta) = \theta^{\alpha}$ and $q(\theta) = \theta^{\alpha-1}$. We assume that expenditure shocks have an exponential CDF: $F(\tau) = 1 - e^{-\gamma\tau}$.[23] Once these functional forms are set, we must choose parameter values. Some values we set externally, while the remainder we choose to match data and model moments. The parameter values are listed in Table 1.

Table 1: Parameter Values

Externally Calibrated Parameters		
Parameter	Value	Source or Informative Moment
β_H	0.997	No inter-temporal savings condition
$R-1$	0.33%	Risk free rate 4%
δ	0.21%	45 Years in Market
α	0.50	Matching Elasticity[24]
λ	0.50	Hosios Condition
σ	2.6%	Separation Rate, Shimer (2005)
h_H	1	Normalization
z	0.4	Shimer (2005)
Internally Calibrated Parameters		
π_H	55.0%	Sub-prime through super prime rates, CFPB (2015)
ϵ	0.670	Sub-prime through super prime rates, CFPB (2015)
β_L	0.672	Sub-prime through super prime rates, CFPB (2015)
ψ	0.982	Debt to Labor Income, CFPB (2015)
h_L	0.572	Residual Earnings $50-10$, Lemieux (2006)
κ	1.45	Job-finding rate, Shimer (2005)
γ	13	Delinq. debt share, CFPB (2015)

Many of our parameters are taken from previous papers or otherwise calibrated externally. We choose the bargaining weight for workers so that the Hosios condition ($\lambda = \alpha$) holds. In a full information environment, the Hosios condition implies that total vacancies created is efficient. We will use that fact when comparing our results to a full information model of the labor market. While we cannot guarantee that the data represents a constrained efficient allocation, this ensures that our welfare results are not

[23]In order to guarantee model convergence, we include a small fixed probability of a shock that is too large to pay for any borrower. See the computational appendix for details.

[24]Hall [18] uses a value of $\alpha = 0.24$. Shimer [33] uses $\alpha = 0.72$. Other authors have used values in between, with many settling on 0.5. See Gertler and Trigari [16].

Table 2: Model Fit

Moment	Data Value	Model Value
Super Prime CC Rate, top 49%	0.87%	0.88%
Prime CC Rate, $34-50\%$	1.17%	1.20%
Sub-Prime CC Rate, $0-33\%$	1.60%	1.61%
Debt to Labor Income	21.24%	21.34%
Delinq. Rate	0.95%	0.92%
Residual Earnings $50-10$	0.57	0.57
Monthly Job Finding Rate	45.0%	45.1%

Note: Appendix 2 has definitions of model moments.

amplified due to beginning with an inefficient labor market equilibrium.

We have chosen moments on credit card debt from various sources, some of which are new to the quantitative household credit literature (to our knowledge). The average credit card rate and share of borrowers in each credit bracket are from the Consumer Financial Protection Bureau's "Consumer Credit Card Market" report [10]. The interest rates are "total costs of credit" for each credit bracket in 2015, less 2% for inflation, and reported as monthly rates. These are the most comparable numbers to the model interest rates, since some people pay all balances monthly in the data (and therefore do not pay interest) whereas everyone pays interest in the model.

We also use the CFPB's data to compute credit card debt to income and the share of debt that is defaulted upon. Total credit card debt was $779 Billion in 2015, which we divide by labor's share of average monthly GDP, which was $0.60 \times \$6.108$ Trillion. Finally, we use the CFPB's reported share of accounts that are more than three months past due as our measure of the delinquency rate.

Our moments on labor market outcomes are taken from economy wide reports since we do not have merged data with credit scores and earnings or job-finding rates. For the residual earnings $50-10$ ratio, we use the log of median earnings minus the log of the earnings of the tenth percentile, which is reported by Lemieux [24]. For the job finding rate we use the monthly rate implied by Shimer [33].

5.2 Properties of Stationary Equilibrium

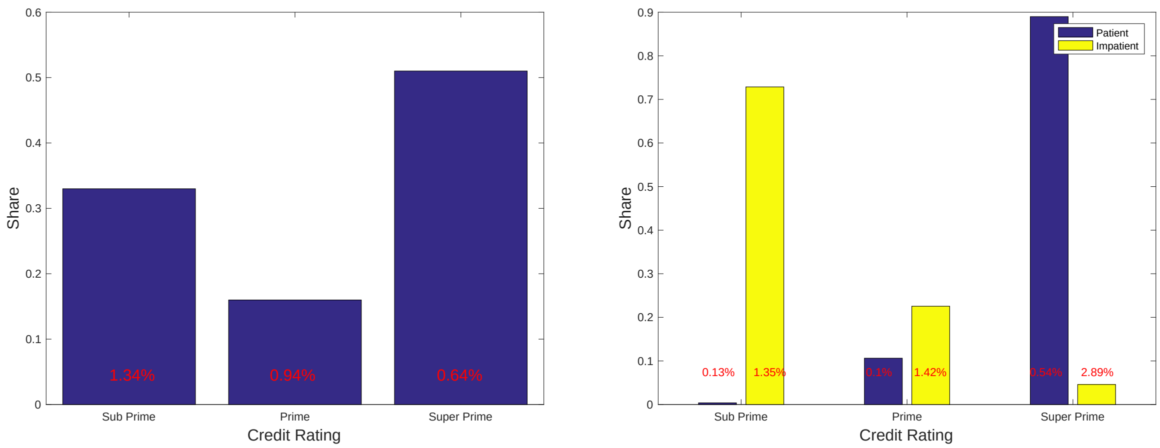
The equilibrium stationary distribution of workers over "type" scores and employment status is determined by the relative solvency and default rates of patient versus impatient workers, as well as job-finding rates. Since type scores are not directly observable, we construct a data comparable distribution by sorting borrowers by their default probability and then assigning credit ratings consistent with the empirical shares of households within each rating. This means that as in the data, the bottom third are labeled "sub prime", the next 15% are "prime" and the top 50% are "super prime". Figure 5a plots the histogram of workers over credit ratings constructed in this way.

While the population shares over credit ratings are defined to match the data, the share of workers of each type within each credit rating is endogenous – it depends on the relative default rates of each worker type in equilibrium. We plot these distributions in Figure 5b, where it is clear that the most impatient workers have sub prime credit, while less than 1% of patient workers have such poor credit since they only default due to extremely large expenditure shocks. Likewise, nearly 90% of patient workers have scores in the super prime range.

The composition of types over ratings determines the gradient of interest rates, default rates, and debt-to-income ratios with respect to credit rating. This can be understood by considering the average and type-specific default rates by credit rating, which we report in red text in Figures 5a and 5b. The average default rate is falling with credit rating, from 1.34% to 0.64%, but this is because the composition of borrowers in each group is changing, not because an individual always defaults less when her score is higher. For example, the average super-prime patient borrower actually defaults four times more than the average subprime patient borrower. This is because she receives much less credit when subprime and because she has a strong incentive to repay. In fact, a patient borrower in the prime category has the strongest incentive to repay and therefore the lowest average default rate because default generates the largest drop in score in the updating function in Figure 6b.[25]

The stationary distribution is derived from the law of motion for a worker's employment status and score, which depends on the job-finding rate for unemployed and the average change in score for employed workers. Figure 6a plots the job-finding rate $f(\theta(s))$, which is bounded below by the impatient worker's full information rate and above by the patient worker's (both of which are efficient under the Hosios condition). The finding rate rises monotonically for scores between zero and one, reflecting the rising surplus associated with patient (and more productive) workers. Since most unemployed patient workers have scores above 0.80 while most impatient are below 0.014, patient workers find jobs at a substantially higher rate than impatient on average. Of course,

[25]We plot all theoretical functions over the score range $0.01 - 0.99$ because these scores are never reached in theory.



(a) (b)

Notes: Unconditional shares are constructed to match the data, type-conditional are inferred from model. Red numbers are average default rates for workers in each rating, unconditional on type in Figure 5a and conditional on type in Figure 5a.

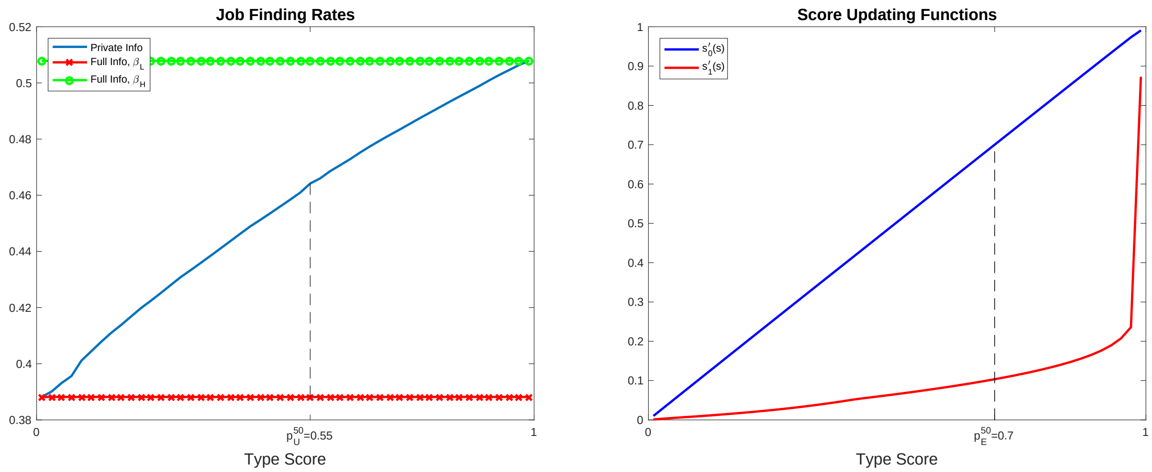
Figure 5: Histograms over Credit Ratings

some unlucky patient workers have substantially lower scores than average and therefore experience lower job-finding rates due to being pooled with the impatient. The median unemployed worker, marked by p_U^{50} on the graph, has a score of 0.55 and therefore a job finding rate of nearly 47%.[26]

The score updating functions are plotted in Figure 6b, the shape of which can be understood by the relative solvency and default rates of the two worker types. Because both worker types repay with a high probability at all scores, there is very little information revealed by repayment.[27] The score therefore updates very slowly in the positive direction, with $s_0'(s)$ just slightly above the forty-five degree line. However, the default rate for impatient workers is up to ten times times that of the patient. Therefore, observing a borrower default leads to a dramatic downward update of her score, thus $s_1'(s)$ is much lower than s for most scores. The median employed borrower has a score of

[26] Throughout, we use p^x to denote the x^{th} percentile of scores. If we condition on type or status then we use a subscript, so that the notation p_U^x is the score held by x^{th} percentile of the unemployed and p_H^x is the score held by the x^{th} percentile of high (patient) types. Likewise, p_{HU}^x is the score held by the x^{th} percentile of the patient unemployed.

[27] These rates are implied by the interest rate targets, which are relatively low relative to the risk-free rate.



(a) (b)

Notes: Vertical hashed lines mark median scores for unemployed workers in Figure 6a and for employed in Figure 6b. Functions are plotted on score range $0.01 - 0.99$.

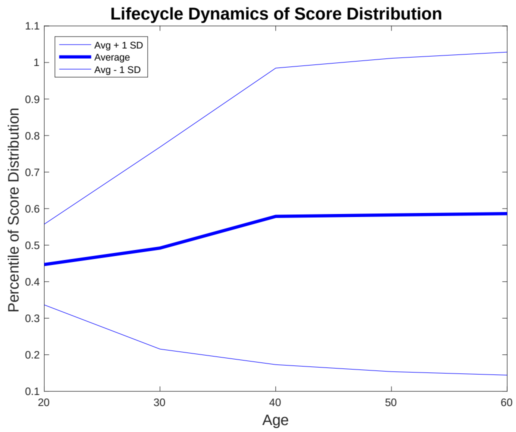
Figure 6: Job Finding Rates and Score Updates

0.69, implying that a default would reduce her score to 0.11 (the bottom third of scores in the stationary distribution).

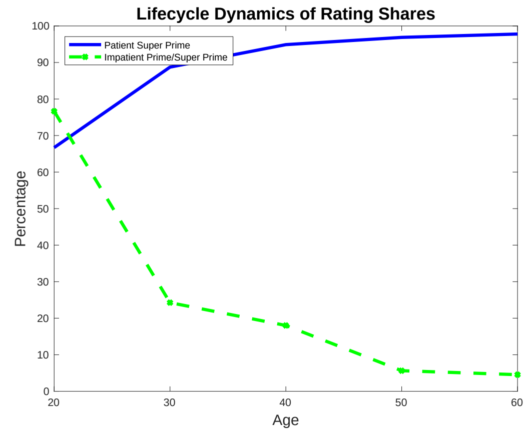
Our model also generates life-cycle profiles of credit ratings, which determines a worker's lifecycle of labor and credit market outcomes. Figure 7a plots the unconditional average credit score percentile by age, starting from $s = \pi_H$ for households entering the labor market at age 20, as well as the one standard-deviation spread around this average.[28] On average, older workers find themselves higher in the credit rating distribution than do younger workers. This occurs because workers separate by type the longer they survive, with patient workers' scores converging towards one and impatient towards zero. This separation is clear in Figure 7b, which shows that the share of patient workers who are super prime is rising with age while the share of impatient workers who are at least prime is falling. This tendency generates the rising spread in Figure 7a which implies a rising average of the rating in Figure 7a, since the cumulative distribution function is convex and our calibration has more patient types than impatient ($\pi_H = 0.55$). This

[28]Credit percentiles are averaged over ten year intervals. While newborns enter with $s = \pi_H = 0.55$ in our calibration, since the stationary distribution of scores is more heavily weighted to high scores, newborns only enter above 45% of the population.

results in a declining average finding rate, as shown in Figure 7c, since the job-finding curve is concave and the pool of unemployed becomes disproportionately populated by the low-productivity impatient over time.



(a)



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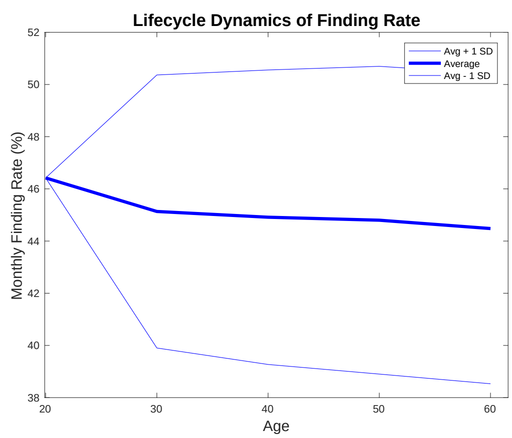


(c)

Notes: Figures generated by simulating $10,000$ individuals from birth. At each date there is a distribution of each endogenous variable, which we average over ten year intervals for averages in Figures 7a and 7c (i.e. mark at 20 represents average over $20-29$). Percentages are computed at the specific age in Figure 7b.

Figure 7: Lifecycle Dynamics In Baseline Economy

5.3 Covariance Between Earnings and Credit History

Our model generates a positive covariance between earnings and credit histories through two channels. First, unobservable heterogeneity in discount factors across types causes differences in both average credit rating and earnings. Patient workers have higher earnings than impatient workers for a given credit history and have better credit histories on average, which creates a positive correlation between credit score and earnings “across” types. Second, a worker of a given type with better credit has a larger threat point, since she knows that she can walk away from a match and find another with a high probability. This means that a better credit score causes higher wages “within” each worker type.

Figure 8 demonstrates these two covariances for our model calibration. On average, prime borrowers earn 20.4% more than sub prime and super prime earn an additional 34.4% than prime. Over 98% of this total covariance is driven by the “across” component, since patient workers earn roughly 76% more than impatient workers and represent a larger share of workers with good credit ratings. The remainder is determined by the “within” component, since moving from subprime to super prime increases earnings by 0.9% on average.

While there is no direct empirical counterpart to these numbers, there is a strong negative association between adverse credit events and residual earnings. We demonstrate this by estimating an earnings regression from the 2016 Survey of Consumer Finance, in which respondents answered three questions: Q1) whether they were ever delinquent on debt in 2015, Q2) whether they were ever delinquent on debt by more than two months, and Q3) whether they were ever turned down for a loan. We use the answers to these questions (1 = “yes”) to estimate the cross-sectional regression

$$\log \text{earnings}_i = \beta_1 Q1_i + \beta_2 Q2_i + \beta_3 Q3_i + \text{controls}_i + \varepsilon_i, \tag{26}$$

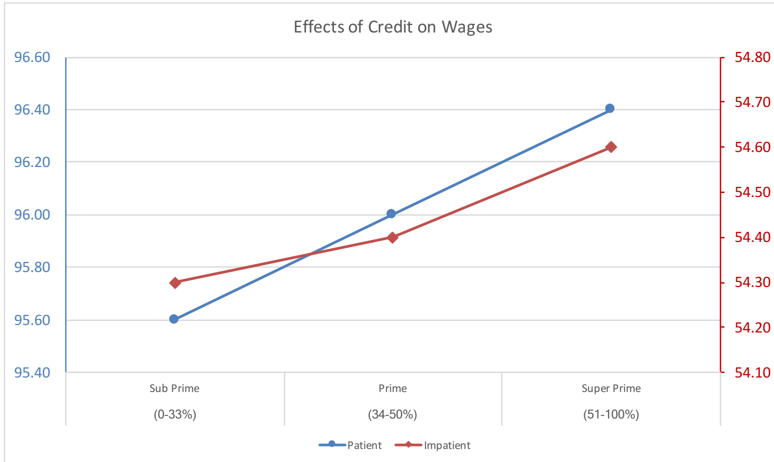
where controls include a quadratic function of age as well as dummies for years of education, gender, race, industry, and occupation. Table 3 reports our estimated β coefficients across various specifications. We consistently find a significantly large negative coefficient on adverse credit terms, with a magnitude ranging from 20.3% lower earnings for delinquency alone to 36.7% lower earnings for all three adverse events. These numbers are of similar magnitudes as our model's overall covariance between credit rating and earnings, although we do not know exactly how much these events would move someone's credit rating.

	Specification		
	(1)	(2)	(3)
Q1	-20.3^{***}	-14.7^{***}	-13.6^{***}
	(4.9)	(2.8)	(2.6)
Q2		-13.9^{*}	-12.7^{*}
		(1.9)	(1.7)
Q3			-10.4^{**}
			(2.2)
R^2	0.332	0.333	0.333
Obs	4451	4451	4451

Notes: Estimates from equation $\log earnings_i = \beta_1 Q1_i + \beta_2 Q2_i + \beta_3 Q3_i + CONTROLS_i$, where column (1) restricts $\beta_2 = \beta_3 = 0$ and column (2) restricts $\beta_3 = 0$. Questions are 1) were you ever delinquent on debt payments, Q2) were you ever delinquent by more than two months, and Q3) were you ever turned down for a loan. Parenthesis report absolute values of t-statistics. Significance levels represented as $*** = 1\%, ** = 5\%, * = 10\%$.

Table 3: Cross-Sectional Regression of Earnings on Credit Events

Finally, the fact that our "within" covariance is small is supported by estimates in Herkenhoff, Phillips, and Cohen-Cole [21], who report the average change in annual earnings for an individual one year before and after the removal of a bankruptcy flag from their credit report. This effectively isolates the effect of credit above and beyond any permanent worker type and turns out to be roughly 1% in their panel data (similar to our model finding that moving from subprime to super prime increases earnings by 0.9% on average).



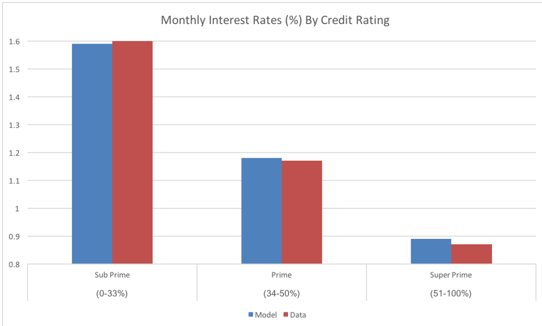
Notes: Average earnings by credit rating and worker type. Left vertical axis corresponds to patient workers and right vertical axis to impatient workers.

Figure 8: Credit and Wages

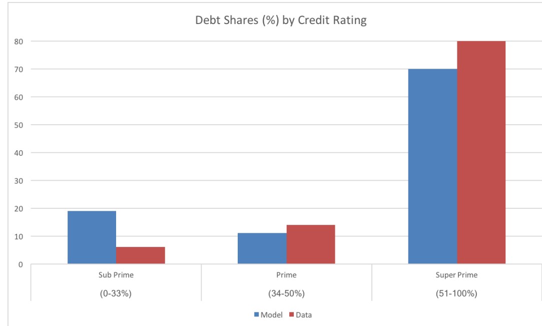
5.4 Fit in Other Dimensions

Our calibration is consistent with additional dimensions of the data not used to fit the model. Figure 9a reproduces the fit of the model's interest rates with data, while Figure 9b shows the shares of debt held by borrowers with each credit rating, both in the data and our model.[29] The fact that credit shares are increasing with rating is a success of the Netzer and Scheuer equilibrium concept and would not be generated by models in which credit contracts were least cost separating for all scores (since patient households would always have less debt than impatient households in such a model) to maintain incentive compatibility as is clear in Figure 3a.

[29]The data is from the Consumer Financial Protection Bureau's 2017 credit card report [11].



(a)



(b)

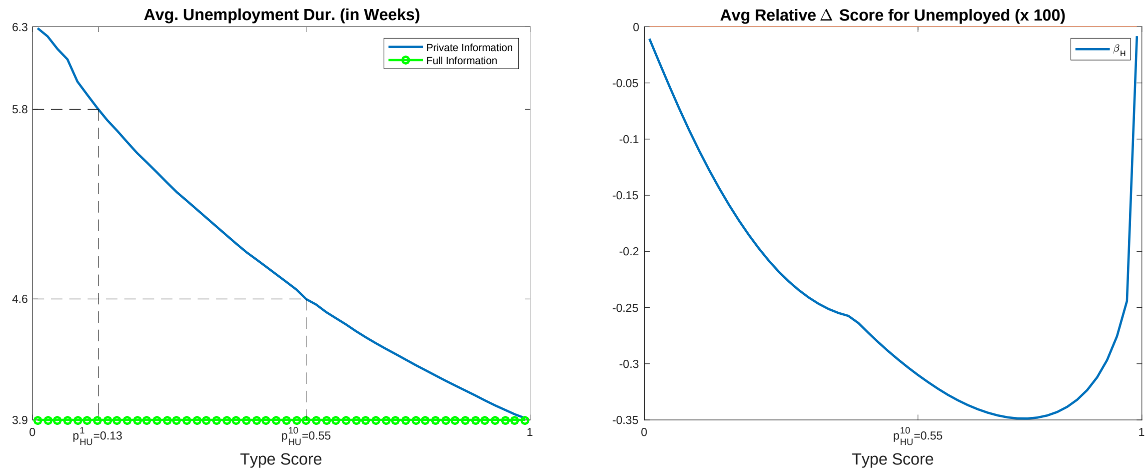
Notes: Model generated interest rates and debt shares relative to data. Figure 9a shows fit of model to moments chosen in calibration. Figure 9b compares model to empirical moments *not used* to fit model in calibration.

Figure 9: Average Interest Rates and Credit Usage by Rating

Furthermore, the policy experiment in Section 6 shows that our model closely matches the effect of PECS bans on the job finding rate of subprime workers. Friedberg, Hynes, and Pattison [15] estimate that workers in the bottom quintile of financial health enjoy a 25% decline in expected unemployment duration when PECS bans are enacted at the state level, while our calibrated model predicts that the bottom quintile of borrowers would enjoy a 27% reduction in unemployment duration. While the bottom quintile in our model is not precisely the same as the bottom quintile of Friedberg, Hynes, and Pattison [15], we are encouraged that our calibration predicts similar labor market effects of the PECS ban for financially distressed workers.

5.5 Poverty Traps

The definition of a poverty trap is not universally agreed upon, so we discuss two possible definitions. The first is a situation in which a worker's experience is made worse due to her credit score relative to an otherwise identical worker. In our case, this happens for the patient households. A patient worker who becomes unemployed with a bad score has a harder time finding a job than one who becomes unemployed with a good score. This leads to further divergence between the two, since the worker with good credit will find a job sooner *and therefore* have an even better credit score in the future. This is because employed patient workers experience an increase in their credit score on average while the unemployed do not. We say that the patient household is subject to a poverty trap because, on average, she experiences a decrease in her score (relative to being employed) and the decrease in score makes it harder to find a job in the next period.



(a) (b)

Notes: Figure 10a shows average unemployment duration as function of worker's score, relative to the efficient full information duration for a patient worker of 3.9 weeks. Hashed lines highlight duration for bottom 1% of the patient unemployed (5.8 weeks) and bottom 10% (4.6 weeks). Figure 10b plots change in score for patient individuals while unemployed minus average change in score when employed. Functions are plotted on score range $0.01 - 0.99$.

Figure 10: Poverty Trap for Patient Workers

We use two figures to understand how such a poverty trap may arise. Figure 10a uses the job-finding rates (as in Figure 6a) to compute the expected unemployment duration of an unemployed patient household as a function of her score s. It is falling with score, reflecting the fact that patient workers are more productive in equilibrium and tend to have higher scores. Note that there are some patient workers who end up with low scores, illustrated by the vertical bar at the tenth percentile. This is the first part of the poverty trap: an unlucky patient worker with a bad credit history has a hard time finding a job and therefore expects longer unemployment spells than if her score was higher.

We next look at the average change in a worker's score when unemployed relative to when she is employed.[30] Figure 10b plots this function for patient workers. On average,

[30] The average relative change in score is defined as:

$$\Delta(s) = s - F_0\left(\tau^*_{H^*}\left(s, b^*_{H^*}(s)\right)\right)s'_0(s) - F_1\left(\left(\tau^*_{H^*}\left(s, b^*_{H^*}(s)\right)\right)s'_1(s)\right.$$

The change while unemployed is 0 while the average change while employed is the negative of the above expression. Thus, the relative average change is $\Delta(s)$.

an employed patient worker experiences a rising score, while her score remains constant during an unemployment spell. It is evident from the figure that an unlucky patient worker with a low score therefore experiences a deterioration in her score relative to if she was employed, which reinforces the longer unemployment duration.

Another way of defining the poverty trap is relative to the full information equilibrium. The idea is that the job-finding rate for a worker with a low score may be strictly lower than if her human capital was observable. Again, consider Figure 6a and compare the finding rates between the private and full information economies. The patient worker experiences a lower job-finding rate for all $s < 1$ while the opposite is true for the impatient worker. For example, the bottom quintile of unemployed patient workers have scores below 0.68 and a job-finding rate below 47.9%, which is 3% below the full information rate of patient workers. Private information has the opposite effect for the impatient workers, 10% of whom have scores above 0.55 and therefore finding rates above 46.4%, which is 7.6% above their full information rate.

The extent of the poverty trap relative to full information depends on the patient worker's score. Using the score percentiles in Figure 10a we can say that the poverty trap adds just over two days to the median patient worker's unemployment duration, five days for the 25^{th} percentile, and just under a week for the lowest decile of patient job seekers.

A useful summary of the labor market impact of default can be computed as the present value of wages conditional on repayment minus the same value conditional on default. We compute these measures for each worker type and employment status, as well as the unconditional average, amortize them over 10 years, and report this measure relative to the average wage in Table 4. Our model predicts substantial expected wage losses from default through two mechanisms. First, the job-finding rate falls due to a lower score. Second, the worker's bargaining position becomes weaker and therefore their wages fall even conditional on being employed. The average across all worker types, scores, and employment statuses amounts to 2.34% of earnings in each month for ten years.

Table 4: Wage Losses From Default

	Employed	Unemployed	Overall
Patient (β_H)	3.19%	3.07%	3.18%
Impatient (β_L)	1.36%	0.66%	1.32%
Overall	1.93%	1.52%	2.34%

5.6 Labor Market Efficiency

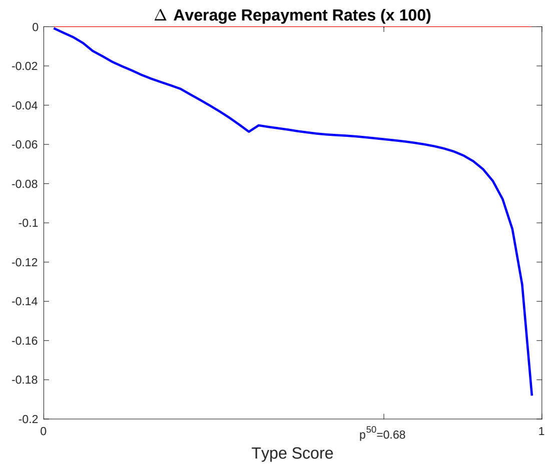
Since we have assumed that the Hosios condition holds, we know that the full-information finding rates are efficient. We can therefore define a measure of labor market efficiency by considering the average difference between each worker type's average finding rate in the economy with private information relative to the full information economy. For the patient households in the calibrated economy, the monthly job-finding rate averages 49.7%, which is 1.3 percentage points lower than the efficient 50.9%. On the other hand, impatient households have an inefficiently high job-finding rate. In the calibrated economy their monthly job-finding rate is 40.5%, which is 1.7 percentage points higher than the efficient rate.

6 Policy Experiment: Banning Credit Checks

We now solve the economy with the same parameters, except that vacancies cannot be conditioned on a worker's score which implies market tightness θ is independent of s. That is, we substitute $q(\theta)$ for $q(\theta(s))$ in the free entry condition in (11). While market tightness and the job-finding rate are therefore independent of s (and independent of β_i as before), match surplus and therefore bargained wages still depend on s since the worker's score affects her bargaining position post match. Credit markets operate as before the ban, except that the workers' incentives to repay endogenously fall: since default (which lowers a worker's credit score) does not affect the worker's job finding rate, there is less punishment associated with default.

The ban affects workers by changing equilibrium labor and credit market functions, which in turn affect lifecycle dynamics of credit ratings. This can be seen in Figure 12a, which shows that the score updating function for a defaulting worker is substantially less severe after the ban goes into effect. The ban reduces the *dynamic* incentive for

(a)

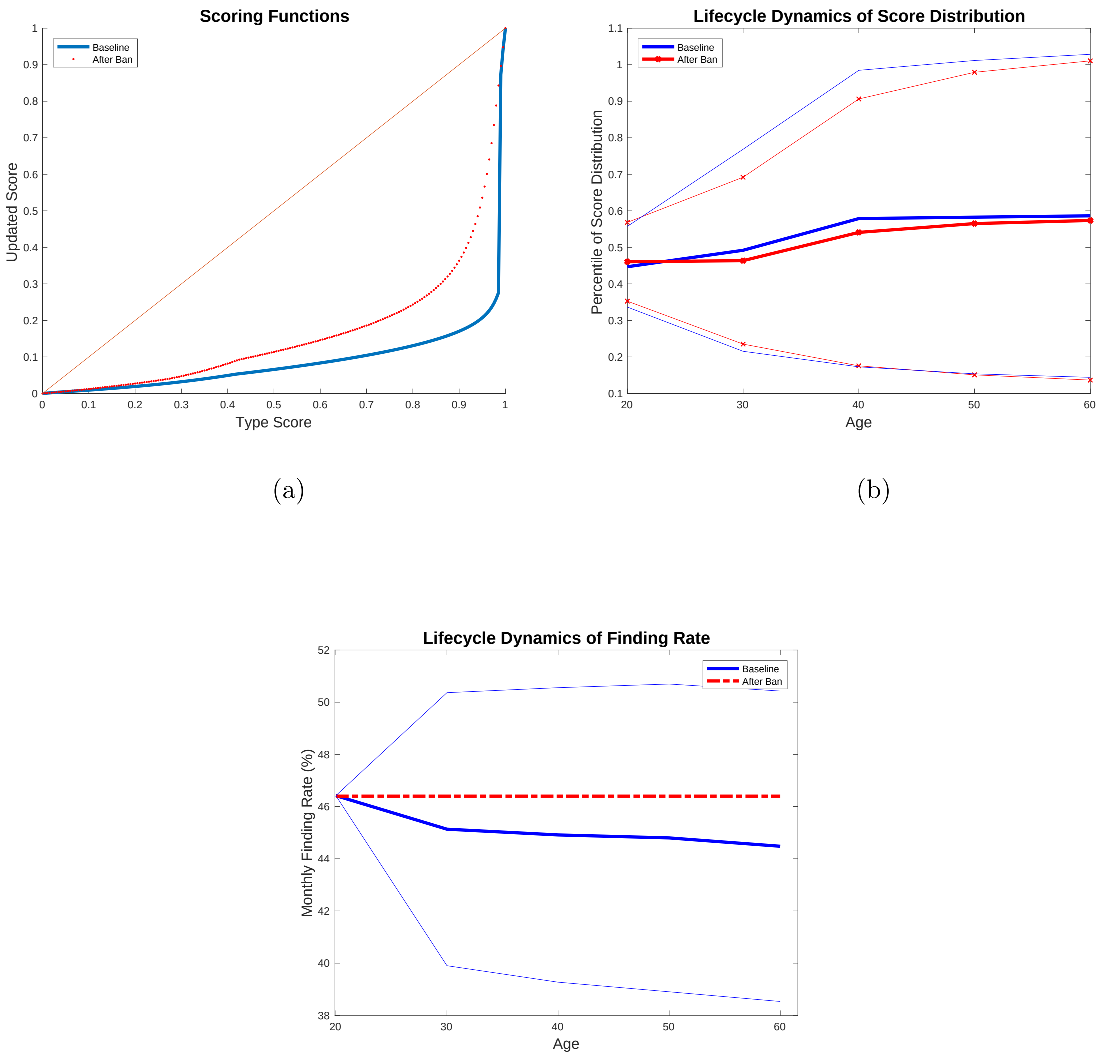


(b)

Notes: Model's predictions for unemployment duration and repayment rates in response to PECS ban. Functions are plotted on score range $0.01 - 0.99$.

Figure 11: Equilibrium Effects of Ban

workers to repay debts, which is much more important for the patient borrowers since the impatient discount the future heavily. This in turn affects default behavior for patient borrowers more than for impatient, which makes the default rates more similar for the two types. A flatter score updating function manifests as a flatter lifecycle profile of credit ratings, as seen in Figure 12b, where the post-ban average credit percentile grows less than in the baseline economy since separation occurs less rapidly. Finally, the job-finding rate is constant following the ban, which eliminates the effect of age on finding rates, as seen in Figure 12c.



(a) (b)

(c)

Notes: Lifecycle figures generated by simulating 10,000 individuals from birth. At each date there is a distribution of each endogenous variable, which we average over ten year intervals for averages in Figures 12b and 12c.

Figure 12: Lifecycle Dynamics Before & After Ban

The ban's effect on aggregate variables can be seen in Table 5. The average job-finding rate actually rises from 45.11% to 46.4%, which occurs for three reasons. First, the finding function is concave in scores, which means that the finding rate rises mechanically from pooling, keeping all other equilibrium variables constant. Second, the equilibrium unemployment pool's composition shifts towards higher productivity workers following the ban. This shift occurs because high-score workers find jobs at a higher rate in the baseline economy and the patient are disproportionately represented in the upper credit ratings. Therefore, the patient have shorter unemployment durations and make up a smaller fraction of the unemployed pool than they do in the population as a whole. Once the ban goes into effect, they have the same job-finding rate as everyone else, and therefore their share of the unemployed is the same as their share of the population.

Finally, post-match expected discounted profits rise on average after the ban because workers' threat points change. As shown in Figures 15c and 15d, the post-match profitability of employing a worker of either type with super prime credit rises, since these workers experience a deterioration in their threat points. On the other hand, the post-match profitability of employing a worker with prime or sub prime credit falls since these workers experience an increase in their threat points (i.e. they no longer suffer from low job finding rates due to their bad credit). On net, however, post-match expected discounted profits rise after the ban, since almost all patient households have excellent credit (post-match profits rise in 56.8% of matches overall, which is driven by an increase in 95% of matches with patient workers).[31]

The effects on job-finding rates differ substantially across the score distribution, as seen in Figure 11a.[32] We find that the job finding rate for workers with very low scores rises substantially, which causes the average duration of unemployment for the bottom quintile of workers to decline by 27%.

[31] The above increase in average post-match profits occurs after the ban goes into effect through changes in the equilibrium threat point of workers, which are taken as given during bargaining. This means that a given employer in the baseline economy (with employer credit checks) cannot increase profits by unilaterally ignoring the worker's credit report. Furthermore, there is no change in the cost of posting a vacancy, so the ex-ante expected profit from posting a vacancy is zero in both environments.

[32] We plot changes in the expected unemployment duration in Figure 11a since it is in more easily interpreted units (weeks). The relationship with the job finding rate is monotone - a higher finding rate implies a lower duration.

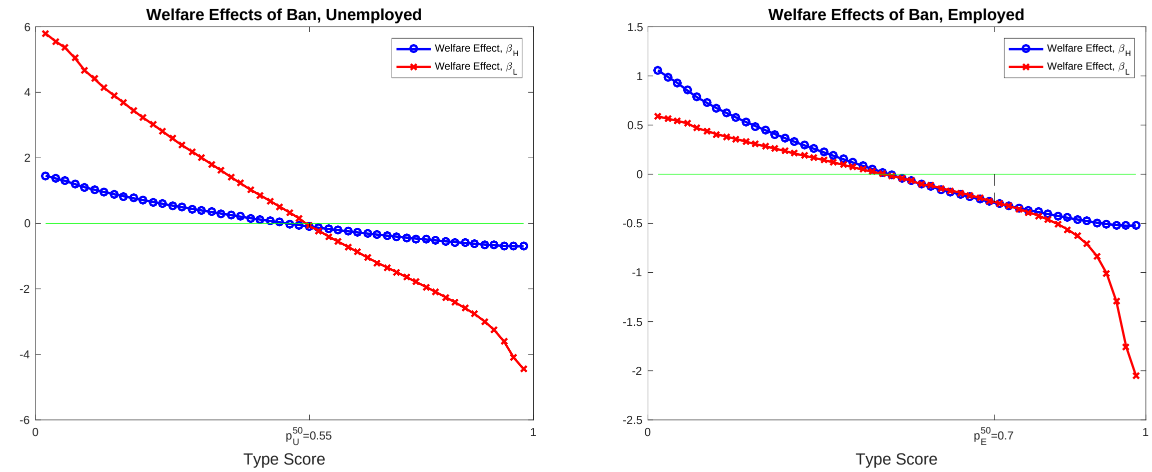
Table 5: Effect of Employer Credit Ban

Moment	Baseline	After Ban
Avg. Job Finding Rate	45.11%	46.4%
Median Job Finding Rate	46.42%	46.40%
Average Interest Rate	1.17%	1.18%
Average Debt to Income	21.34%	20.50%
Delinq. Rate	0.92%	0.93%

This exercise shows that banning PECS may actually increase the average job-finding rate, but still does so at the cost of labor market efficiency. This can be seen by the small fall in the median job-finding rate, which is due to a decline in the job-finding rate for almost all of the patient workers (who are 55% of the population in our baseline economy and tend to have high scores). In fact, the unemployment rate for patient workers *rises* from 5.4% to 5.7% following the ban. Relative to the efficient job-finding rate, the impatient worker's finding rate is 7.6% higher after the ban (in levels, it rises from 42.4% to 46.4%). On the other hand, patient workers are now pooled with more low-patience workers and therefore experience a more inefficiently low finding rate than in the economy with pre-employment credit screening. Their finding rates falls from 49.7% to 46.4%, which is 3.25% lower than the efficient level. If we average over these absolute changes, then the ban moves job-finding rates away from efficiency by 5.42%.

The ban also affects the credit market through the repayment decisions of borrowers, again seen in Table 5. The average interest rate rises from 1.17% to 1.18% as the average default rate rises from 0.92% to 0.93%. However, these incentive effects differ across worker types and states. Specifically, the patient worker's repayment rate falls more than the impatient, since they respond to dynamic incentives more in the first place. The new stationary equilibrium therefore features less separation of worker types by credit score (i.e. more workers of each type in the prime rating rather than impatient in subprime and patient in super prime). This causes a small decline in the average interest rate of prime borrowers, since a larger equilibrium share are patient. The overall rise in default causes a reduction in credit access, as the the average debt-to-income ratio falls from 21.34% to 20.50%. Again, the small aggregate changes mask larger changes at the micro level, as seen in Figure 11b.[33]

[33]Note that the change in default rate is zero at both $s = 0$ and $s = 1$ since these are absorbing scores and therefore the dynamic incentives to repay are zero for both types in both the baseline economy and



(a) (b)

Notes: Consumption equivalent welfare effects of being in an economy with PECS ban relative to baseline economy, by worker type, score, and employment status. Positive numbers represent a gain from the ban, negative numbers represent a loss.

Figure 13: Welfare Effects of Ban

It is important to note that, while banning PECS eliminates the poverty trap, most of the people with inefficiently low finding rates (i.e. unlucky, patient workers) experience lower job-finding rates. The pre-PECS ban equilibrium is nearly separating, with only 10% of impatient workers carrying scores below $s = 0.55$, which is the threshold for which durations rise post-ban (as seen in Figure 11a). On average, patient workers experience 3.8 days more unemployment following the ban, which is large when compared to the effects of much larger labor market policies. For example, Card and Levine [5] estimate that a thirteen week extension of unemployment benefits increases average unemployment duration by roughly one week.[34]

Furthermore, the net effect is a decrease in labor market efficiency in spite of the increase in average job-finding rates. This can be seen by comparing the finding rate after the ban to the full-information finding rates in Figure 6a. The full-information rates are efficient since our bargaining weight satisfies the Hosios condition. The economy with PECS experience partial separation through type scores, so on average each type has a

the one with PECS bans.

[34]We make this comparison to put the magnitude into context, not because they are directly comparable policies. Specifically, unemployment benefits likely work through labor supply rather than demand, as in our model.

job finding rate closer to their full information value than under the pooled finding rate that arises after the ban goes into effect.

Banning pre-employment credit screening also affects the size and split of rents after a match has occurred by affecting a worker's bargaining position. We demonstrate this in Figures 15a-15d. Prior to the ban, there is a clear positive effect of credit rating on wages for both worker types and, likewise, a downward effect on profits. Wages depend on the score because it affects the job-finding rate of unemployed workers. A higher credit score means that the worker would find a job faster if she was to walk away from her current match. One one hand, this means that the match surplus is smaller overall. However, it also means that the worker has a better bargaining position and therefore captures a larger share of the surplus as her credit score rises. The net effect causes wages to rise with credit score for a given worker type. Of course, the unconditional wage rises even faster with credit rating since patient workers have higher wages at all scores. The opposite profile appears in profits - conditional on worker type, profits are highest for workers with bad credit ratings. On the other hand, the level of profits is strictly higher for patient workers than for impatient, due to their higher labor productivities, which generates the positive profile of vacancies with respect to score.

Once the ban goes into place, job finding rates are no longer score specific, which means that a worker's outside option is less affected by her score. This leads to a near complete flattening of the wage profiles in Figures 15a and 15b and profit profiles in Figures 15a and 15b.[35] Relative to the baseline, this causes a decline in wages for workers with high scores but a rise in wages for sub prime and prime, while profits move in the opposite direction.

We next plot the net effect of the ban on welfare[36] for the unemployed in Figure 13a, since the direct change on market tightness and finding rates affects these workers. Workers with low type scores experience a gain in welfare, since they experience a higher job finding rate than when firms can discriminate based on score.[37] Furthermore, the patient workers gain more since they put a higher weight on finding a job due to their higher β. The welfare gains are falling for both worker types as scores rise, eventually

[35] Quantitatively, our wage profiles are flat to three decimal places and therefore appear as such in the plots, but do still vary in theory. Likewise, the discounted profit lines are quite flat, though less so than wages.

[36] See the appendix for the definition of these welfare measures.

[37] We can evaluate the welfare effects for workers at each score, even if the theoretical measure of them is zero. For example, we calculate the value function of patient workers at $s = 0$ and impatient at $s = 1$ when we solve the model. However, we omit these points from our plots because there are no workers who actually experience them in equilibrium.

	Patient	Impatient	Ex Ante
Employed	-0.40%	0.46%	
Unemployed	-0.54%	4.78%	
Average	-0.43%	0.72%	-0.10%

Table 6: Avg. Welfare Effects

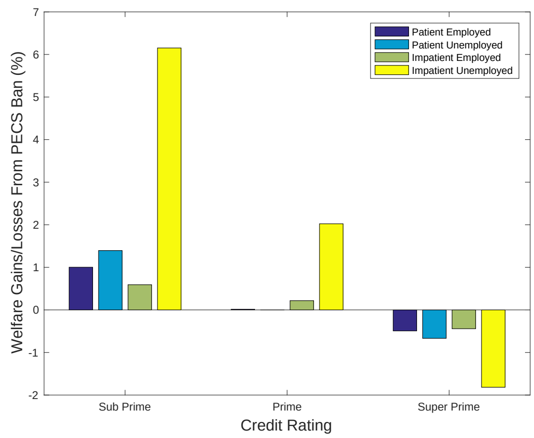


Figure 14: Welfare Effects by Credit Rating

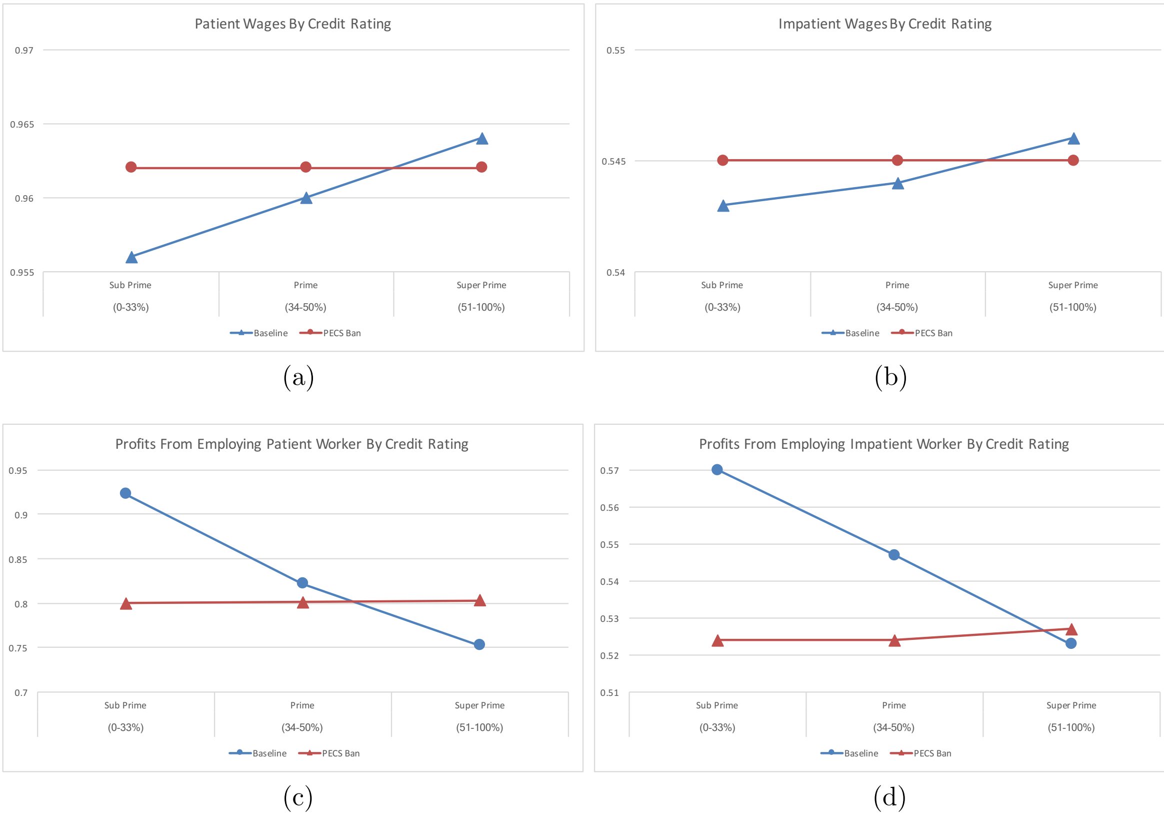
becoming negative for those with high scores. Likewise the welfare effect is positive but falling for employed workers, as seen in Figure 13b. On average, impatient workers gain from the ban and patient workers lose. The effects (both positive and negative) are magnified for unemployed workers since any change in job finding rates affects them immediately.

We summarize these conditional averages in Table 6.[38] In aggregate, only 43% of the population have a positive gain from banning PECS.[39] However, the distributional effects are substantial, with patient workers losing slightly on average (equivalent to $0.4-0.54\%$ of consumption each month), but impatient workers gaining a lot, especially the unemployed. If we consider the ex-ante lifetime utility of a worker before her type is realized (i.e. who has a π_H probability of being patient and will enter the economy as unemployed), then there is a welfare loss of 0.10% of monthly consumption for a worker born into the economy without PECS, relative to being born into an economy that allows them.

Even within a worker type and employment status, there is substantial heterogeneity in the welfare effect of banning PECS. We illustrate this in Figure 14, which shows that subprime workers gain from banning PECS no matter the worker's type or employment

[38]If private information persisted after hiring, then we would expect reduced expected profits due to overpaying the low-productivity type. This would make scores more valuable than in our baseline model. So, getting rid of PECS would have bigger negative effects on matching and welfare losses would be larger than what we are estimating.

[39]So the ban would be voted down.



(a) (b) (c) (d)

Notes: Wages and post-match expected discounted profits by worker type and credit rating before PECS ban and after.

Figure 15: Effect of PECS Ban on Bargaining

status, while the opposite is true for super prime workers, who lose from the ban regardless of type or status. In each case, the unemployed gains/losses are larger than the employed because they are immediately affected by changes in the job-finding rate. Furthermore, the patient employed have muted welfare changes since they greatly discount the effect of the ban on their future job finding rates when they become unemployed.

7 Conclusion

We have presented a theoretical foundation for why employers may use credit histories in the hiring process and how this practice can create a poverty trap. Our theory extends the workhorse Diamond-Mortensen-Pissarides model to include ex-ante private information about worker productivity, while also building a novel framework for including credit scores when borrowers have private information about their repayment rates.

Combining these two microeconomic models highlights the connection across markets in the presence of private information. Our complete theory of labor and credit contracts under adverse selection overcomes the Lucas Critique in our policy analysis, which we find to be important since the PECS ban's direct effects in the labor market spill over to the credit market. Our model also demonstrates that even when the aggregate effects of a policy may appear small (see Table 5), there can be large effects in the cross section (see Figure 11a) and on life cycle dynamics (see Figure 12b).

We have used our model to complement the empirical literature on the effect of banning PECS. Specifically, we address the effect on unmeasurable outcomes – labor market efficiency and welfare. Banning PECS increase the job finding rate of low-score workers, but these workers are predominantly low productivity. The opposite is true for high-score workers, who mostly have high productivity: they experience an increase in average unemployment duration of 3.8 days following the ban. While efficiency is unequivocally reduced, the welfare effects are more nuanced. Impatient workers, who tend to have relatively bad credit, gain substantially, the equivalent of 0.72% of monthly consumption. Patient workers, who are the majority, lose 0.42% of monthly consumption, so that only 43% of the population gains from the ban. We conclude that policy makers should consider the trade-off between equity and efficiency when considering PECS bans.

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A Full Info Equilibrium

The full information equilibrium consists of $(w_i^{fi}, \theta_i^{fi}, b_i^{fi}, Q_i^{fi}, h_i^{fi})_{i\in\{L,H\}}$ and associated values $W_i^{fi}, U_i^{fi}, J_i^{fi}$. This gives the following system of equations, assuming that $h_H^{fi} = \overline{h}$

and $h_L^{fi} = \underline{h}$:

$$\begin{aligned}
b_i^{fi} &= \text{argmax}_b \Bigg[R^{-1}F\Bigg(\beta_i(1-\delta)\psi\epsilon - b\Bigg)b + \psi\int_0^{\beta_i(1-\delta)\psi\epsilon - b} F(\tau)d\tau\Bigg] && (27)\\
Q_i^{fi} &= R^{-1}F\Bigg(\beta_i(1-\delta)\psi\epsilon - b_i^{fi}\Bigg)b_i^{fi} && (28)\\
J_i^{fi} &= \psi\Bigg[h - w_i^{fi} + R^{-1}(1-\delta)(1-\sigma)J_i^{fi}\Bigg] && (29)\\
\kappa &= R^{-1}q\Bigg(\theta_i^{fi}\Bigg)J_i^{fi} && (30)\\
W_i^{fi} &= && (31)\\
& Q_i^{fi} + \psi\Bigg[w_i^{fi} + \int_0^{\beta_i(1-\delta)\psi\epsilon - b_i^{fi}} F(\tau)d\tau + \beta_i(1-\delta)(V_i^{fi} - \psi\epsilon)\Bigg]\\
U_i^{fi} &= z + \beta_i(1-\delta)\psi\Bigg[f\big(\theta_i^{fi}\big)W_i^{fi} + \big[1 - f\big(\theta_i^{fi}\big)\big]U_i^{fi}\Bigg] && (32)\\
V_i^{fi} &= \sigma U_i^{fi} + (1-\sigma)W_i^{fi} && (33)\\
W_i^{fi} - U_i^{fi} &= \lambda\big[W_i^{fi} + J_i^{fi} - U_i^{fi}\big]. && (34)
\end{aligned}$$

A first useful result is that b_i^{fi} and Q_i^{fi} are independent of h and that the problem is concave whenever $F''(\tau) \leq 0$. This can be seen from the first order condition on b:

$$R\psi F(\beta_i(1-\delta)\psi - b_i^{fi}) = F(\beta_i(1-\delta)\psi - b_i^{fi}) - F'(\beta_i(1-\delta)\psi - b_i^{fi})b_i^{fi}. \tag{35}$$

The left hand side has intercept $\psi R < 1$ when $b = 0$ while the right hand side has an intercept of one. The slope of the left-hand side is $-R\psi F'(\beta_i(1-\delta)\psi - b_i^{fi})$ while the slope of the right-hand side is $-2F'(\beta_i(1-\delta)\psi - b_i^{fi}) + F''(\beta_i(1-\delta)\psi - b_i^{fi})$. We can guarantee that the right-hand side is steeper than the left-hand side by assuming $F''(\tau) \leq 0$, so the intersection will have $b_i^{fi} > 0$. Furthermore, this condition shows that b is increasing in $\beta_i(1-\delta)\psi$ (and therefore in β_i) since implicit differentiation gives:

$$\frac{\partial b}{\partial(\beta_i(1-\delta)\psi)} = \frac{(1-R\psi)F'(\beta_i(1-\delta)\psi - b) - F''(\beta_i(1-\delta)\psi - b)b}{(2-R\psi)F'(\beta_i(1-\delta)\psi - b) - F''(\beta_i(1-\delta)\psi - b)b} > 0 \tag{36}$$

and from this we can conclude that $F(\beta_H(1-\delta)\psi - b_H^{fi}) \geq F(\beta_L(1-\delta)\psi - b_L^{fi})$ by considering $\frac{\partial x}{\partial b}$ in

$$\frac{\partial x}{\partial b}\Bigg[(1-R\psi)F'(x) - F'(x)b\Bigg] = F'(x), \tag{37}$$

which means that the optimal b from this first order condition can only increase if the term inside of $F(\beta_i(1-\delta)\psi - b)$ increases.

We now consider a single worker of type i who chooses h freely, given the equilibrium $h_H^{fi} = \overline{h}$ and $h_L^{fi} = \underline{h}$. That is, the worker's type is public, but the deviation to h rather than h_i^{fi} is not, so that θ_i^{fi} is fixed. Using that Q and b are independent of h, we can write the resulting match surplus

$$S_i^{fi}(h) = \frac{\psi h - z + Q_i^{fi} + \psi\left[\int_0^{\beta_i(1-\delta)\psi\epsilon - b_i^{fi}} F(\tau)d\tau - \beta_i(1-\delta)\psi\epsilon\right]}{1-\beta_i(1-\delta)\psi(1-f\left(\theta_i^{fi}\right)-\sigma)\lambda - R^{-1}(1-\delta)\psi(1-\lambda)}, \tag{38}$$

and the value of being unemployed for a given h is

$$\tilde{U}_i^{fi}(h) = \frac{z+\beta_i(1-\delta)\psi f\left(\theta_i^{fi}\right)\lambda S_i^{fi}(h)}{1-\beta_i(1-\delta)\psi}. \tag{39}$$

We then define the function $\Delta_i = \beta_i \frac{\tilde{U}_i^{fi}(\overline{h}) - \tilde{U}_i^{fi}(\underline{h})}{\overline{h}-\underline{h}}$, which takes the form:

$$\Delta_i = \frac{\beta_i\psi}{1-\beta_i(1-\delta)\psi}\frac{\beta_i(1-\delta)\psi f(\theta_i^{fi})\lambda}{1-\beta_i(1-\delta)\psi\lambda(1-f(\theta_i^{fi})-\sigma)-R^{-1}(1-\delta)(1-\lambda)}. \tag{40}$$

We want to have $\Delta_H \geq \phi > \Delta_L$. Clearly $\Delta_i > 0$ whenever $\beta_i > 0$, so it is possible to set $\phi = \Delta_H$. We can then set β_L sufficiently close to zero to guarantee that $\phi > \Delta_L$. The only thing that keeps us from fully describing the condition in terms of parameters is the presence of $f(\theta_i^{fi})$, but these are just two numbers, so we can guarantee the correct choices of h if $\widetilde{\Delta}_H \geq \phi > \widetilde{\Delta}_L$, where

$$\widetilde{\Delta}_H = \min_{j\in\{H,L\},\ell\in\{H,L\}} \frac{\beta_i\psi}{1-\beta_i(1-\delta)\psi}\frac{\beta_i(1-\delta)\psi f(\theta_j^{fi})\lambda}{1-\beta_i(1-\delta)\psi\lambda(1-f(\theta_\ell^{fi})-\sigma)-R^{-1}(1-\delta)(1-\lambda)},$$

$$\widetilde{\Delta}_L = \max_{j\in\{H,L\},\ell\in\{H,L\}} \frac{\beta_i\psi}{1-\beta_i(1-\delta)\psi}\frac{\beta_i(1-\delta)\psi f(\theta_j^{fi})\lambda}{1-\beta_i(1-\delta)\psi\lambda(1-f(\theta_\ell^{fi})-\sigma)-R^{-1}(1-\delta)(1-\lambda)}.$$

We can therefore guarantee that $h_H^{fi} = \overline{h}$ and $h_L^{fi} = \underline{h}$. This ensures that the match surplus is higher for patient workers, since they have larger h and lower default rates (which allows them to have more consumption in the first sub-period via higher Q). The higher match surplus guarantees that $\theta_H^{fi} > \theta_L^{fi}$ from the zero-profit condition and the fact that $q(\theta)$ is decreasing in θ.

B Existence of Private Information Equilibrium

We first fix notation. The domain of all functions is $[0,1]$ throughout, so will be omitted. The family of bounded Lipschitz continuous functions on this domain with slopes bounded by M is given by

$$\mathcal{L}_M = \{h : [0,1] \to \mathbb{R} : \|h\| \text{ is bounded and } \forall (s_x, s_y), |h(s_x) - h(s_y)| \leq M|s_x - s_y|\},$$

where the norm of a function $f \in \mathcal{L}_M$ is given by $\|h\| = \max_{0\leq s\leq 1} |h(s)|$. We will work with a vector operator $T : \mathcal{L} \rightrightarrows \mathbb{C}$, where:

$$\mathcal{L} \equiv \Pi_{\ell=1}^{17} \mathcal{L}_{M_\ell}$$

and $\mathbb{C} \subset \mathcal{L}$ is the space of continuously bounded functions. The vector of norms for $v \in \mathbb{L}$ will be denoted by $\|v\|$ and the norm on this space is the Euclidian norm on this vector, denoted $\|v\|_2 = \sqrt{\|v\| \cdot \|v\|}$. We will verify that T is continuous and that $T : \mathcal{L} \rightrightarrows \mathcal{L}$. Denoting a vector of functions in $\mathcal{L}$ by

$$v = \left\langle W_H, W_L, U_H, U_L, \psi w_H, \psi w_L, J_H, J_L, \theta, s_0', s_1', \Gamma_0, \Gamma_1, b_H, b_L, Q_H, Q_L \right\rangle,$$

we define T element-by-element. We fix $h_H = \overline{h}$ and $h_L = \underline{h}$ throughout, so suppress these arguments, then verify that these choices are consistent under Assumption 1. Note that, for our proofs, we assume that there is a probability $p > 0$ that borrowers receive an expenditure shock $\tau = 0$ and a probability $(1-p)q > 0$ that the shock is sufficiently large that the worker cannot pay. First, the operators for the value functions are given by:

$$T_H^W(s) = T_1[v](s) \equiv \psi w_H(s) + Q_H(s) + p\Big[\beta_H(1-\delta)V_H\big(s_0'(s)\big) - \psi b_H(s)\Big] + \tag{41}$$
$$(1-p)\Big[\beta_H(1-\delta)V_H\big(s_1'(s)\big) - \psi\beta_H(1-\delta)\epsilon\Big] + (1-p)(1-q)\psi \int_0^{\lambda_H(s;v)-b_H(s)} F(\tau)d\tau,$$

$$T_L^W(s) = T_2[v](s) \equiv \psi w_L(s) + Q_L(s) + p\Big[\beta_L(1-\delta)V_L\big(s_0'(s)\big) - \psi b_L(s)\Big] + \tag{42}$$
$$(1-p)\Big[\beta_L(1-\delta)V_L\big(s_1'(s)\big) - \psi\beta_L(1-\delta)\epsilon\Big] + (1-p)(1-q)\psi \int_0^{\lambda_L(s;v)-b_L(s)} F(\tau)d\tau,$$

where the functions V_i and λ_i are defined by:

$$V_i(s) \equiv \sigma U_i(s) + (1-\sigma) W_i(s), \tag{43}$$

$$\lambda_i(s; v) \equiv \beta_i (1-\delta) \Big[\psi \epsilon + V_i\big(s_0'(s)\big) - V_i\big(s_1'(s)\big) \Big]. \tag{44}$$

The next two operator elements map into unemployment values and are then given by:

$$T_H^U(s) = T_3[v](s) \equiv z + \beta_H (1-\delta) \Big[f\big(\theta(s)\big) W_H(s) + \big(1 - f\big(\theta(s)\big)\big) U_H(s) \Big], \tag{45}$$

$$T_L^U(s) = T_4[v](s) \equiv z + \beta_L (1-\delta) \Big[f\big(\theta(s)\big) W_L(s) + \big(1 - f\big(\theta(s)\big)\big) U_L(s) \Big]. \tag{46}$$

The next two operator elements update wages from the definition of firm values:

$$T_H^w(s) = T_5[v](s) \equiv \psi \overline{h} - J_H(s) + R^{-1}(1-\sigma)(1-\delta) \sum_{d \in \{0,1\}} G_d\big(\lambda_H(s; v) - b_H(s)\big) J_H\big(s_d'(s)\big), \tag{47}$$

$$T_L^w(s) = T_6[v](s) \equiv \psi \underline{h} - J_L(s) + R^{-1}(1-\sigma)(1-\delta) \sum_{d \in \{0,1\}} G_d\big(\lambda_L(s; v) - b_L(s)\big) J_L\big(s_d'(s)\big). \tag{48}$$

The next two operator elements map into firm values using Nash Bargaining:

$$T_H^J(s) = T_7[v](s) \equiv (1-\lambda) \Big[W_H(s) - U_H(s) + J_H(s) \Big], \tag{49}$$

$$T_L^J(s) = T_8[v](s) \equiv (1-\lambda) \Big[W_L(s) - U_L(s) + J_L(s) \Big]. \tag{50}$$

The next operator element maps to market tightness:

$$T^\theta(s) = T_9[v](s) \equiv q^{-1}\left(\kappa R \Big[s J_H(s) + (1-s) J_L(s) \Big]^{-1} \right) \tag{51}$$

The next two operator maps to the scoring functions:

$$T_0^s(s) = T_{10}[v](s) \equiv \frac{s}{s + (1-s)\Gamma_0(s)}, \tag{52}$$

$$T_1^s(s) = T_{11}[v](s) \equiv \frac{s}{s + (1-s)\Gamma_1(s)}. \tag{53}$$

The next two operator elements are the definitions of each odds ratio used to update the scoring functions:

$$T_0^{\Gamma}(s) = T_{12}[v](s) \equiv \frac{p + (1-p)(1-q)F\big(\lambda_L(s;v) - b_L(s)\big)}{p + (1-p)(1-q)F\big(\lambda_H(s;v) - b_H(s)\big)},$$

$$T_1^{\Gamma}(s) = T_{13}[v](s) \equiv \frac{(1-p)\Bigg(q + (1-q)\Bigg(1 - F\big(\lambda_L(s;v) - b_L(s)\big)\Bigg)\Bigg)}{(1-p)\Bigg(q + (1-q)\Bigg(1 - F\big(\lambda_H(s;v) - b_H(s)\big)\Bigg)\Bigg)}.$$

The final four operator elements are the credit contracts that solve the optimization problem below, taking v as given.

$$\Big\langle T_{14}[v](s), T_{15}[v](s), T_{16}[v](s), T_{17}[v](s)\Big\rangle =$$

$$\text{argmax}_{\{b_H, b_L, Q_H, Q_L\}} Q_H \quad + \quad \psi\Bigg[(1-p)(1-q)\int_0^{\lambda_H(s;v)-b_H} F(\tau)d\tau - pb_H\Bigg]$$

s.t.

$$s\Bigg[-Q_H + (\omega R)^{-1} G_0\big(\lambda_H(s;v) - b_H\big)b_H\Bigg] \quad +$$

$$(1-s)\Bigg[-Q_L \quad + \quad (\omega R)^{-1} G_0\big(\lambda_L(s;v) - b_L\big)b_L\Bigg] \geq 0$$

$$Q_L + \psi\Bigg[(1-p)(1-q)\int_0^{\lambda_L(s;v)-b_L} F(\tau)d\tau - pb_L\Bigg] \quad \geq$$

$$Q_H \quad + \quad \psi\Bigg[(1-p)(1-q)\int_0^{\lambda_L(s;v)-b_H} F(\tau)d\tau - pb_H\Bigg]$$

$$Q_H + \psi\Bigg[(1-p)(1-q)\int_0^{\lambda_H(s;v)-b_H} F(\tau)d\tau - pb_H\Bigg] \quad \geq$$

$$Q_L \quad + \quad \psi\Bigg[(1-p)(1-q)\int_0^{\lambda_H(s;v)-b_L} F(\tau)d\tau - pb_L\Bigg]$$

$$Q_L + \psi\Bigg[(1-p)(1-q)\int_0^{\lambda_L(s;v)-b_L} F(\tau)d\tau - pb_L\Bigg] \quad \geq$$

$$\max_b \Bigg\{(\omega R)^{-1} G_0\big(\lambda_L(s;v) - b\big)b \quad + \quad \psi\Bigg[(1-p)(1-q)\int_0^{\lambda_L(s;v)-b} F(\tau)d\tau - pb\Bigg]\Bigg\}$$

We now study the slopes and bounds of each element of the operator and seek conditions to ensure that $T : \mathcal{L} \rightrightarrows \mathcal{L}$.

Lemma 1 *If the finding rate and its derivative have bounds, so that* $\sup_{\theta \geq 0} f(\theta) \leq B_f$ *and* $\sup_{\theta \geq 0} f'(\theta) \leq B_{f'}$,[40] *then for* $i = 1, 2, 3, 4$ *the functions* $T_i[v](s)$ *are Lipschitz continuous with bounding constant* $\tilde{M}_i$*:*

$$\begin{aligned}
\tilde{M}_1 &= M_H^w + M_H^Q + p\Big[\psi M_H^b + \beta_H(1-\delta)\big(\sigma M_H^U + (1-\sigma)M_H^W\big)M_0^s\Big] \\
&+ (1-p)\beta_H(1-\delta)\Big(\sigma M_H^U + (1-\sigma)M_H^W\Big)M_1^s \\
&+ (1-p)(1-q)\psi\Big[\beta_H(1-\delta)\big(\sigma M_H^U + (1-\sigma)M_H^W\big)(M_0^s + M_1^s) + M_H^b\Big], \\
\tilde{M}_2 &= M_L^w + M_L^Q + p\Big[\psi M_L^b + \beta_L(1-\delta)\big(\sigma M_L^U + (1-\sigma)M_L^W\big)M_0^s\Big] \\
&+ (1-p)\beta_L(1-\delta)\Big(\sigma M_L^U + (1-\sigma)M_L^W\Big)M_1^s \\
&+ (1-p)(1-q)\psi\Big[\beta_L(1-\delta)\big(\sigma M_L^U + (1-\sigma)M_L^W\big)(M_0^s + M_1^s) + M_L^b\Big], \\
\tilde{M}_3 &= \beta_H(1-\delta)\Big[B_f(M_H^W + M_H^U) + M^\theta \frac{B_{f'}\overline{h}}{1-\beta_H} + M_H^U\Big], \\
\tilde{M}_4 &= \beta_L(1-\delta)\Big[B_f(M_L^W + M_L^U) + M^\theta \frac{B_{f'}\underline{h}}{1-\beta_L} + M_L^U\Big].
\end{aligned}$$

If there exists some m_f *such that* $\sup_{\tau \geq 0} |F'(\tau)| \leq m_f$ *then the functions* $T_5[v]$ *and* $T_6[v]$

[40] Conceptually, we have $B_f = 1$, although not all matching functions used in practice satisfy this for $\theta \geq 0$. Furthermore, in practice it is enough that $f'(\theta)$ be bounded on a compact interval of θ which excludes zero.

are Lipschitz with constants:

$$\begin{aligned}
\tilde{M}_5 &= M_H^J + R^{-1}(1-\delta)(1-\sigma)\bigg\{\big(p+(1-p)(1-q)\big)M_H^J M_0^s \\
&+ (1-p)(1-q)m_f\frac{\overline{h}}{1-R^{-1}}\bigg(M_H^b + \beta_H(1-\delta)\big(\sigma M_H^U + (1-\sigma)M_H^W\big)(M_0^s + M_1^s)\bigg) \\
&+ (1-p)M_H^J M_1^s \\
&+ +(1-p)(1-q)m_f\frac{\overline{h}}{1-R^{-1}}\bigg(M_H^b + \beta_H(1-\delta)\big(\sigma M_H^U + (1-\sigma)M_H^W\big)(M_0^s + M_1^s)\bigg)\bigg\}, \\
\tilde{M}_6 &= M_L^J + R^{-1}(1-\delta)(1-\sigma)\bigg\{\big(p+(1-p)(1-q)\big)M_L^J M_0^s \\
&+ (1-p)(1-q)m_f\frac{\underline{h}}{1-R^{-1}}\bigg(M_L^b + \beta_L(1-\delta)\big(\sigma M_L^U + (1-\sigma)M_L^W\big)(M_0^s + M_1^s)\bigg) \\
&+ (1-p)M_L^J M_1^s \\
&+ +(1-p)(1-q)m_f\frac{\underline{h}}{1-R^{-1}}\bigg(M_L^b + \beta_L(1-\delta)\big(\sigma M_L^U + (1-\sigma)M_L^W\big)(M_0^s + M_1^s)\bigg)\bigg\}.
\end{aligned}$$

The operators T_7 and T_8 generate functions with Lipschitz constants:

$$\begin{aligned}
\tilde{M}_7 &= (1-\lambda)\big(M_H^W + M_H^U + M_H^J\big), \\
\tilde{M}_8 &= (1-\lambda)\big(M_L^W + M_L^U + M_L^J\big).
\end{aligned}$$

If there exists some m_q such that $\sup_{\theta\geq 0}|\frac{dq^{-1}(\theta)}{d\theta}| \leq m_q$,[41] *then the function $T_9[v]$ is Lipschitz with constant:*

$$\tilde{M}_9 = m_q\kappa R\max_{0\leq s\leq 1}\left|\left(\frac{1}{sJ_H(s)+(1-s)J_L(s)}\right)^2\right|\left[2M_L^J + M_H^J + \max_{0\leq s\leq 1}|J_H(s)-J_L(s)|\right].$$

The functions implied by the operators T_{10} and T_{11} are Lipschitz with constants:

$$\begin{aligned}
\tilde{M}_{10} &= \frac{M_0^\Gamma + \max_{0\leq s\leq 1}\Gamma_0(s)}{\min_{0\leq s\leq 1}|\big(s+(1-s)\Gamma_0(s)\big)^2|}, \\
\tilde{M}_{11} &= \frac{M_1^\Gamma + \max_{0\leq s\leq 1}\Gamma_1(s)}{\min_{0\leq s\leq 1}|\big(s+(1-s)\Gamma_1(s)\big)^2|}.
\end{aligned}$$

[41] As with $f'(\theta)$, we only require that $\frac{dq^{-1}(\theta)}{d\theta}$ be bounded on a compact interval of θ which excludes zero in practice.

If $\sup_\tau F'(\tau) \leq m_f$*, then the functions* $T_{16}[v]$ *and* $T_{17}[v]$ *are Lipschitz with constants:*

$$\begin{aligned}
\tilde{M}_{12} &= \frac{1}{p}(1-p)(1-q)m\Big[M_L^b + \beta_L(1-\delta)\big(\sigma M_L^U + (1-\sigma)M_L^W\big)\big(M_0^s + M_1^s\big)\Big] \\
&+ \left(\frac{p+(1-p)(1-q)}{p^2}\right)(1-p)(1-q)m\Big[M_H^b + \beta_H(1-\delta)\big(\sigma M_H^U + (1-\sigma)M_H^W\big)\big(M_0^s + M_1^s\big)\Big], \\
\tilde{M}_{13} &= \frac{1}{(1-p)q}(1-p)(1-q)m\Big[M_L^b + \beta_L(1-\delta)\big(\sigma M_L^U + (1-\sigma)M_L^W\big)\big(M_0^s + M_1^s\big)\Big] \\
&+ \left(\frac{1-p}{((1-p)q)^2}\right)(1-p)(1-q)m\Big[M_H^b + \beta_H(1-\delta)\big(\sigma M_H^U + (1-\sigma)M_H^W\big)\big(M_0^s + M_1^s\big)\Big].
\end{aligned}$$

Finally, for the general problem, the distances for $T_{14}, T_{15}, T_{16},$ *and* T_{17} *depend on the form and parameters of* F *and the parameters* $p, q, \kappa, R, \beta_L,$ *and* β_H*. We will therefore assume that there exist some values of* $M_H^b, M_L^b, M_H^Q, M_L^Q$ *such that* $\tilde{M}_{14} \leq M_H^b, \tilde{M}_{15} \leq M_L^b, \tilde{M}_{16} \leq M_H^Q,$ *and* $\tilde{M}_{17} \leq M_L^Q$*.*

Each of these is derived from the definition of a Lipschitz constant, the triangle inequality, and the mean value theorem. We can use these expressions to prove the following lemma:

Lemma 2 *Assume that there exists* m_f *such that* $\sup_{\tau\geq 0}|F'(\tau)| \leq m_f$*,* m_q *such that* $\sup_{\theta\geq 0}|\frac{dq^{-1}(\theta)}{d\theta}| \leq m_q$*,* B_f *and* $B_{f'}$ *such that* $\sup_{\theta\geq 0} f(\theta) \leq B_f$ *and* $\sup_{\theta\geq 0} f'(\theta) \leq B_{f'}$*. Further, assume that operators* $14-17$ *have* $\tilde{M}_i$ *as described above. If* $m_f, \beta_H, \beta_L, \psi, 1-\lambda,$ *and* κ *are sufficiently small, then there exists some vector of bounds,* M*, such that* $T : \mathcal{L} \rightrightarrows \mathcal{L}$*.*

Note that we must only find a vector M for which the operator is self mapping. We do this by construction for each operator element. For the first, we set M_i^w, M_i^W, and M_i^Q such that $M_i^w + M_i^Q < M_i^W$. Then we can set ψ, β_L, and β_H sufficiently low to ensure that $\tilde{M}_1 \leq M_H^W$ and $\tilde{M}_2 \leq M_L^W$. Ensuring that $\tilde{M}_3 \leq M_H^U$ and $\tilde{M}_4 \leq M_L^U$ requires again setting β_L and β_H sufficiently low. If we set $M_H^J < M_H^w$ and $M_L^J < M_L^w$ then $\tilde{M}_5 \leq M_H^w$ and $\tilde{M}_6 \leq M_L^w$ for R^{-1} sufficiently small. If we set $M_i^J \geq \frac{1-\lambda}{\lambda}\big(M_i^W + M_i^U\big)$, which can be guaranteed for sufficiently small λ, then $\tilde{M}_7 \leq M_H^U$ and $\tilde{M}_8 \leq M_L^U$. We can then set M^θ to the expression defining $\tilde{M}_9$, M_0^s to $\tilde{M}_{10}$, and M_1^s to $\tilde{M}_{11}$. Finally, for sufficiently small m we can guarantee that $\tilde{M}_{12} \leq M_0^\Gamma$ and $\tilde{M}_{13} \leq M_1^\Gamma$. By assumption we can find values of $M_H^b, M_L^b, M_H^Q, M_L^Q$ such that $\tilde{M}_{14} \leq M_H^b, \tilde{M}_{15} \leq M_L^b, \tilde{M}_{16} \leq M_H^Q$, and $\tilde{M}_{17} \leq M_L^Q$.

QED

In order to apply Schauder's fixed point theorem, we must also ensure that T is a continuous mapping. It is sufficient to check that, for each $s \in [0,1]$, and vector v_1, v_2 with each $v_i \in \mathcal{L}$, there exists some matrix Z such that $|T[v_1](s) - T[v_2](s)| \leq Z \cdot |v_1(s) - v_2(s)|$. For the first 13 elements of T this follows from the definition and applications of the triangle inequality and mean value theorem. For a general contracting problem we again assume that this is true. We now write the expressions for $\|T_i[v_x] - T_i[v_y]\|$ for $i = 1, 2, ...13$.

Lemma 3 *Take v_x and v_y in $\mathcal{L}_M$. If $\sup_{\theta \geq 0} f(\theta) \leq B_f$ and $\sup_{\theta \geq 0} f'(\theta) \leq B_{f'}$ for some $B_f, B_{f'}$, then for $i = 1, 2, 3, 4$, the distances $\|T_i[v_x] - T_i[v_y]\|$ are bounded by*

$$
\begin{aligned}
\|T_1[v_x] - T_1[v_y]\| \;\leq\;& \|\psi w_{H,x} - \psi w_{H,y}\| + \|Q_{H,x} - Q_{H,y}\| \\
&+ p\beta_H(1-\delta)(\sigma M_H^U + (1-\sigma)M_H^W)\Big(\|s'_{0,x} - s'_{0,y}\| + \psi\|b_{H,x} - b_{H,y}\|\Big) \\
&+ (1-p)\beta_H(1-\delta)(\sigma M_H^U + (1-\sigma)M_H^W)\|s'_{1,x} - s'_{1,y}\| \\
&+ \psi(1-p)(1-q)\Big(\|b_{H,x} - b_{H,y}\| + \sigma\|U_{H,x} - U_{H,y}\| + (1-\sigma)\|W_{H,x} - W_{H,y}\|\Big), \qquad (54)
\end{aligned}
$$

$$
\begin{aligned}
\|T_2[v_x] - T_2[v_y]\| \;\leq\;& \|\psi w_{L,x} - \psi w_{L,y}\| + \|Q_{L,x} - Q_{L,y}\| \\
&+ p\beta_L(1-\delta)(\sigma M_L^U + (1-\sigma)M_L^W)\Big(\|s'_{0,x} - s'_{0,y}\| + \psi\|b_{L,x} - b_{L,y}\|\Big) \\
&+ (1-p)\beta_L(1-\delta)(\sigma M_L^U + (1-\sigma)M_L^W)\|s'_{1,x} - s'_{1,y}\| \\
&+ \psi(1-p)(1-q)\Big(\|b_{L,x} - b_{L,y}\| + \sigma\|U_{L,x} - U_{L,y}\| + (1-\sigma)\|W_{L,x} - W_{L,y}\|\Big), \qquad (55)
\end{aligned}
$$

$$
\begin{aligned}
\|T_3[v_x] - T_3[v_y]\| \;\leq\;& \beta_H(1-\delta)\Big[(1+B_f)\|U_{H,x} - U_{H,y}\| \qquad (56) \\
&+ B_f\|W_{H,x} - W_{H,y}\| + \frac{1}{1-\beta_H}B_{f'}\|\theta_x - \theta_y\|\Big],
\end{aligned}
$$

$$
\begin{aligned}
\|T_4[v_x] - T_4[v_y]\| \;\leq\;& \beta_L(1-\delta)\Big[(1+B_f)\|U_{L,x} - U_{L,y}\| \qquad (57) \\
&+ B_f\|W_{L,x} - W_{L,y}\| + \frac{1}{1-\beta_L}B_{f'}\|\theta_x - \theta_y\|\Big].
\end{aligned}
$$

If there is some $m_f > 0$ such that $\sup_{\tau \geq 0} F'(\tau) \leq m_f$ then, for $i = 5, 6$, the distances

$\|T_i[v_x] - T_i[v_y]\|$ are bounded by

$$
\begin{aligned}
\|T_5[v_x] - T_5[v_y]\| \;\leq\;& \|J_{H,x} - J_{H,y}\| \qquad (58)\\
&+\; R^{-1}(1-\delta)(1-\sigma)\Bigg[M_H^J\Big(\|s'_{0,x} - s'_{0,y}\| + \|s'_{1,x} - s'_{1,y}\|\Big)\\
&+\; \frac{\overline{h}}{1-R^{-1}} m_f\Big((\sigma M_H^U + (1-\sigma)M_H^W)\big(\|s'_{0,x} - s'_{0,y}\| + \|s'_{1,x} - s'_{1,y}\|\big)\\
&+\; 2\|b_{H,x} - b_{H,y}\|\Big)\Bigg],\\
\|T_6[v_x] - T_6[v_y]\| \;\leq\;& \|J_{L,x} - J_{L,y}\| \qquad (59)\\
&+\; R^{-1}(1-\delta)(1-\sigma)\Bigg[M_L^J\Big(\|s'_{0,x} - s'_{0,y}\| + \|s'_{1,x} - s'_{1,y}\|\Big)\\
&+\; \frac{\underline{h}}{1-R^{-1}} m_f\Big((\sigma M_L^U + (1-\sigma)M_L^W)\big(\|s'_{0,x} - s'_{0,y}\| + \|s'_{1,x} - s'_{1,y}\|\big)\\
&+\; 2\|b_{L,x} - b_{L,y}\|\Big)\Bigg].
\end{aligned}
$$

For $i = 7, 8$, the distances $\|T_i[v_x] - T_i[v_y]\|$ are given by

$$\|T_7[v_x] - T_7[v_y]\| = (1-\lambda)\Bigg[\|W_{H,x} - W_{H,y}\| + \|U_{H,x} - U_{H,y}\| + \|J_{H,x} - J_{H,y}\|\Bigg], \quad (60)$$

$$\|T_8[v_x] - T_8[v_y]\| = (1-\lambda)\Bigg[\|W_{L,x} - W_{L,y}\| + \|U_{L,x} - U_{L,y}\| + \|J_{L,x} - J_{L,y}\|\Bigg]. \quad (61)$$

If there exists some m_q such that $\sup_{\theta\geq 0}|\frac{dq^{-1}(\theta)}{d\theta}| \leq m_q$ and $\underline{J} > 0$ such that $J_i(s) \geq \underline{J}$,[42] *then the distance $\|T_9[v_x] - T_9[v_y]\|$ is bounded by*

$$\|T_9[v_x] - T_9[v_y]\| \leq \kappa R m_q \underline{J}^{-2}\Bigg[\|J_{H,x} - J_{H,y}\| + \|J_{L,x} - J_{L,y}\|\Bigg]. \quad (62)$$

For $i = 10, 11$, the distances $\|T_i[v_x] - T_i[v_y]\|$ are bounded by

$$\|T_{10}[v_x] - T_{10}[v_y]\| \leq 0.25\left(\frac{p + (1-p)(1-q)}{p}\right)^2 \|\Gamma_{0,x} - \Gamma_{0,y}\|, \quad (63)$$

$$\|T_{11}[v_x] - T_{11}[v_y]\| \leq 0.25 q^{-2}\|\Gamma_{1,x} - \Gamma_{1,y}\|. \quad (64)$$

If there is some $m_f > 0$ such that $\sup_{\tau\geq 0} F'(\tau) \leq m_f$ then, for $i = 12, 13$, the distances

[42] Our assumption that $\underline{h} > z$ and $\lambda < 1$ ensures that such a lower bound on $J_i(s)$ can be imposed.

$\|T_i[v_x] - T_i[v_y]\|$ *are bounded by*

$$
\begin{aligned}
\|T_{12}[v_x] - T_{12}[v_y]\| \;\; \leq \;\; & \frac{p+(1-p)(1-q)}{p^2}(1-p)(1-q)m_f\Bigg[\|b_{H,x} - b_{H,y}\| \qquad (65)\\
+ \;\; & \beta_H(1-\delta)(\sigma M_H^U + (1-\sigma)M_H^W)\Big(\|s'_{0,x} - s'_{0,y}\| + \|s'_{1,x} - s'_{1,y}\|\Big)\Bigg]\\
+ \;\; & p^{-1}(1-p)(1-q)m_f\Bigg[\|b_{L,x} - b_{L,y}\|\\
+ \;\; & \beta_L(1-\delta)(\sigma M_L^U + (1-\sigma)M_H^U)\Big(\|s'_{0,x} - s'_{0,y}\| + \|s'_{1,x} - s'_{1,y}\|\Big)\Bigg],\\
\|T_{13}[v_x] - T_{13}[v_y]\| \;\; \leq \;\; & \frac{1-p}{((1-p)q)^2}(1-p)(1-q)m_f\Bigg[\|b_{H,x} - b_{H,y}\| \qquad (66)\\
+ \;\; & \beta_H(1-\delta)(\sigma M_H^U + (1-\sigma)M_H^W)\Big(\|s'_{0,x} - s'_{0,y}\| + \|s'_{1,x} - s'_{1,y}\|\Big)\Bigg]\\
+ \;\; & \frac{1}{(1-p)q}(1-p)(1-q)m_f\Bigg[\|b_{L,x} - b_{L,y}\|\\
+ \;\; & \beta_L(1-\delta)(\sigma M_L^U + (1-\sigma)M_H^U)\Big(\|s'_{0,x} - s'_{0,y}\| + \|s'_{1,x} - s'_{1,y}\|\Big)\Bigg].
\end{aligned}
$$

These are again derived algebraically by evaluating the difference $\|T_i[v_x]-T_i[v_y]\|$. These bounds immediately imply the following lemma:

Lemma 4 *Suppose that there exists some* 4×17 *matrix* z *such that, for any* v_x *and* v_y *in* $\mathcal{L}_M$*, for* $i = 14, 15, 16,$ *and* 17*,*

$$\|T_i[v_x] - T_i[v_y]\| \leq z\|v_x - v_y\|. \qquad (67)$$

Then, if the conditions stated for each bound to hold in Lemma 3 are satisfied, the operator $T : \mathcal{L}_M \rightrightarrows \mathbb{C}$ *is continuous.*

Let the assumptions of Lemma 4 hold and denote the matrix implied by the inequalities in Lemma 1 (after appending z for rows $14-17$) by Z. This means that for any v_x, v_y in $\mathcal{L}$ we have $\|T[v_x] - T[v_y]\| \leq Z\|v_x - v_y\|$. Denote z^* as the element of Z with the largest value. For any $\epsilon > 0$, the distance $\|T[v_x] - T[v_y]\|_2$ is smaller than ϵ for any pair v_x, v_y for which $\|v_x - v_y\|_2 < z^*\epsilon \equiv \delta$.

QED

We can now invoke Schauder's Fixed Point Theorem to ensure existence of an equilibrium under the conditions from these lemmas.

Theorem 3 *Under the conditions in Lemmas 1 and 3, there exists some* $v^* \in \mathcal{L}$ *such that* $\forall s \in [0,1] : T[v^*](s) = v^*(s)$.

We apply the statement of Schauder's Fixed Point Theorem in Stokey, et al [35]. The space $\mathcal{L}$ is closed, bounded, and convex. $T : \mathcal{L} \rightrightarrows \mathcal{L}$ is continuous by Lemma 3 and the image is an equicontinuous family by virtue of $\mathcal{L}$ being Lipschitz and T being self-mapping on $\mathcal{L}$.

QED

The final step is to verify that $h_L^* = \underline{h}$ and $h_H^* = \overline{h}$. We again consider a single agent of type i deviating to h instead of h_i^* and check that doing so is suboptimal for each type. This requires setting β_L, β_H, and ϕ so that:

$$U_{H,\overline{h}}(\pi_H) - U_{H,\underline{h}}(\pi_H) \geq \phi(\overline{h} - \underline{h}), \tag{68}$$

$$U_{L,\overline{h}}(\pi_H) - U_{L,\underline{h}}(\pi_H) < \phi(\overline{h} - \underline{h}). \tag{69}$$

For ϕ sufficiently low and $\beta_H > 0$, the top inequality holds. Driving $\beta_L \to 0$ then ensures the bottom inequality is satisfied.

C Definitions of Moments

For model moments we use the stationary distribution. For the average value of an endogenous variable $x_{i\ell}(s)$ where i is worker type, ℓ is worker employment status, and s is score we compute:

$$\bar{x} = \int_0^1 \sum_{i \in \{L,H\}} \sum_{\ell \in \{U,E\}} x_{i\ell}(s) d\mu_{i\ell}^*(s)$$

$$\bar{x}_i = \frac{\int_0^1 \sum_{\ell \in \{U,E\}} x_{i\ell}(s) d\mu_{i\ell}^*(s)}{\sum_\ell \mu_{i\ell}^*(1)}$$

$$\bar{x}_\ell = \frac{\int_0^1 \sum_{i \in \{U,E\}} x_{i\ell}(s) d\mu_{i\ell}^*(s)}{\sum_i \mu_{i\ell}^*(1)}$$

So, for example, the quarterly repayment rate is conditional on employment and is therefore defined as:

$$\frac{\int_0^1 \sum_{i \in \{L,H\}} G_0(\tau_i^*\big(s, b_i^*(s)\big) d\mu_{iE}^*(s)}{\sum_i \mu_{iE}^*(1)}$$

In order to compute percentiles of the score distribution, we first define the cumulative distribution for the level of aggregation of interest. For unconditional percentiles, we use CDF:

$$\mu^*(s) \equiv \sum_{i\in\{L,H\}} \sum_{\ell\in\{U,E\}} \mu^*_{i\ell}(s) \tag{70}$$

Unconditional percentiles are then found by first solving for the type score of that percentile. For percentile $x \in [0,1]$ we solve:

$$x = \mu^*(p^x) \tag{71}$$

Likewise we define conditional percentiles using the conditional cumulative distributions. So, for example, the x^{th} percentile of unemployed uses the CDF of unemployed households defined by:

$$\mu^*_U(s) \equiv \frac{\sum_{i\in\{L,H\}} \mu^*_{iU}(s)}{\sum_{i\in\{L,H\}} \mu^*_{iU}(1)} \tag{72}$$

which is then used to solve for p^x_U:

$$x = \mu^*_U(p^x_U) \tag{73}$$

These percentiles are used to report conditional means. We also use the stationary distribution to create distributions over other endogenous variables. For example, to compute a percentile of earnings we create a grid $\mathcal{W} \equiv \{w^*_i(s)|s \in \{s_0, s_1, ...s_N\}, i \in \{L,H\}\}$ and create the approximate probability distribution:

$$PDF^w\big(w^*_i(s_j)\big) \equiv \frac{\mu^*_{iE}(s_{j+1}) - \mu^*_{iE}(s_j)}{\sum_{i\in\{L,H\}} \mu^*_{iE}(1)} \tag{74}$$

We then arrange $\mathcal{W}$ in ascending order and for any $w \in \mathcal{W}$ create:

$$CDF^w(w) = \sum_{m\in\mathcal{W}, m\leq w} PDF^w(m) \tag{75}$$

And finally we use these approximate cumulative densities to compute percentiles of the earnings distribution.

For our welfare measures, we use the consumption equivalent concept. Since our preferences are linear, this corresponds to the percentage change in welfare. We ask "what fraction of total consumption in each period of the economy with employer credit checks would the worker exchange in order to switch to the economy without employer credit checks?" When this number is negative the household gains from the ban and when it is positive the household loses. We scale consumption in each sub-period in each date by a number $1 + \gamma_{ij}(s)$, where i is worker type and j is employment status. Denoting W^{nc} and U^{nc} as the value functions without employer credit checks, we define $\gamma_{ij}(s)$ by:

$$W_{ih_i^*}(s)[1 + \gamma_{iE}(s)] = W_{ih_i^*}^{nc}(s) \tag{76}$$

$$U_{ih_i^*}(s)[1 + \gamma_{iU}(s)] = U_{ih_i^*}^{nc}(s) \tag{77}$$