Convenience Yield, Inflation Expectations, and Public Debt Growth*

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Abstract

We present new facts on how convenience yields fluctuate with macroeconomic variables and fiscal policy. We find the convenience yield of long-term Treasuries is negatively correlated with inflation expectations, and inflation expectations predict future debt-to-GDP growth at different horizons. To rationalize these findings, we incorporate the convenience yield into a staggered-price model with a non-Ricardian fiscal policy. The government finances deficit shocks partially through higher inflation and partially through more borrowing in the future, which reduces the convenience yield today. The feedback loop between the convenience yield and future debt supply amplifies the effect of fiscal shocks on the cost of government borrowing and its debt balances. We further verify this channel in the data using empirically constructed exogenous deficit shocks.

Keywords: Convenience yield; inflation expectation; debt-to-GDP ratio; fiscal policy
JEL codes: E31; G12; E62; E63

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1 Introduction

As of 2023, the U.S. government debt stands at 33 trillion, nearly 120 percent of GDP. The cost of funding U.S. debt has historically been low because Treasuries benefit from a convenience yield. That is, investors are willing to accept Treasuries for lower yields compared to other assets because of their safety and liquidity. The convenience yield averages around 0.88 percentage points, saving the U.S. government billions of dollars in interest rate expenses.[1] As the U.S. debt continues expanding, it is important to understand how convenience yields fluctuate with macroeconomic conditions.

In addition to the factors accounted for in the literature (e.g., Krishnamurthy and Vissing-Jorgensen, 2012; Nagel, 2016), we find that the convenience yield of long-term Treasuries is negatively correlated with long-run inflation expectations in cyclical variations, as shown in Figure 1.[2] This correlation is significant even after controlling for economic fundamentals, short-term interest rates, and the current debt-to-GDP ratio. This fact is surprising because the convenience yield is, by construction, in *real* terms: it is constructed as the spread between the yields of Treasuries and matched securities.

What could be the driver behind the correlation between the convenience yield of long-term Treasuries and inflation expectations? Do inflation expectations reflect some deeper determinant of the convenience yield? We look at the role of fiscal policy and U.S. public debt growth. We find that higher inflation expectations forecast higher public debt growth rates, suggesting that inflation expectations contain information about future government fiscal policy. The convenience yield is not only a function of current government debt supply, but it also reflects expected *future* debt supply. The effect of future debt supply on current convenience yields is particularly strong when the asset providing convenience service has a longer maturity. Thus, fiscal policy could be the nexus between the convenience yield and inflation expectations.

To shed light on the role of fiscal policy, we further show that given an exogenous government deficit shock, long-run inflation expectations increase, the convenience yield drops, and future debt growth increases. These findings suggest beliefs about future fiscal policy are indeed one of the drivers of the correlation between the convenience yield and inflation expectations.

We rationalize the empirical findings using a staggered-price model in which long-term government debt provides convenience benefits. Given a government deficit shock, the government finances the deficit by both increasing market-borrowing and partially inflating away the debt. As a result, expected inflation and expected future debt both rise, leading to lower

[1] This is based on the spread between the AAA bonds from Moody's and 20yr Tbond; see Table 1.

[2] Interestingly, we do not find any correlation between the convenience yield of short-term Treasuries and inflation expectations.

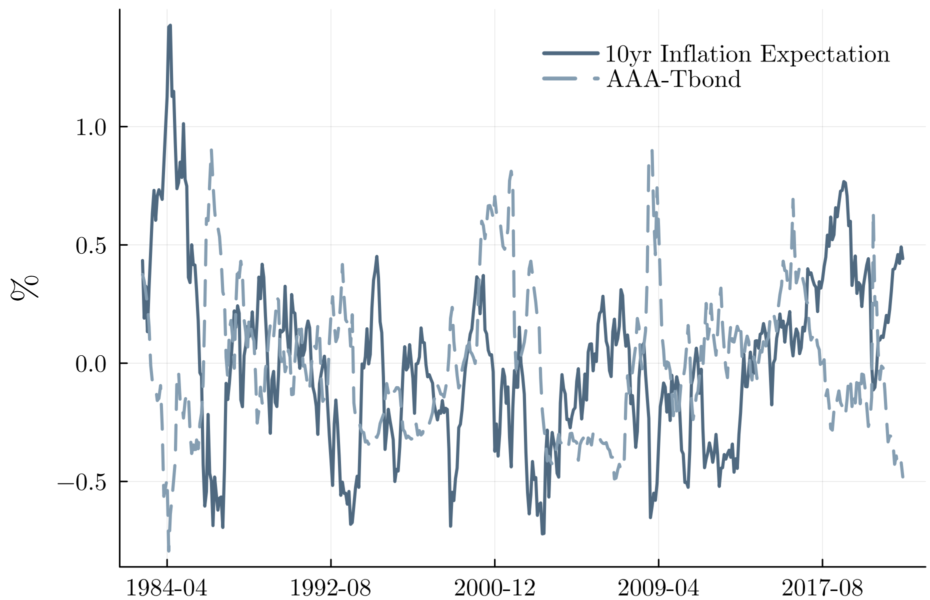

Figure 1: Negative Correlation between Inflation Expectations and AAA Spread

Notes. AAA-Tbond is the difference between Moody's Seasoned Aaa corporate bond yield and 20-year T-bond yield. Moody's Seasoned Aaa corporate bond yield is based on bonds with maturities of 20 years and above. The 10-year inflation expectation series is from the Federal Reserve Bank of Cleveland. Both series are linearly detrended and at a monthly frequency.

convenience yields today. Furthermore, a lower convenience yield implies higher borrowing costs for the government, creating a feedback loop that amplifies the effect of deficit shocks on the cost of funding and debt accumulation.

To the best of our knowledge, our paper is the first to connect the dynamics among the convenience yield, inflation expectations, and government debt. In particular, we highlight how the convenience yield today is affected by investors' belief about *future* safe asset supply. We provide a unified explanation based on standard New Keynesian assumptions and non-Ricardian fiscal policy.[3] During the COVID-19 pandemic, governments worldwide spent trillions of dollars to support their economies. A large amount of government spending is expected to raise future debt. At the same time, inflation expectations are steadily rising globally. Given the current situation, it is particularly important to understand how government actions affect borrowing costs going forward. Our results shed light on this issue through the impact on the convenience yield.

We now describe our approach in detail. Following Krishnamurthy and Vissing-Jorgensen (2012), we measure the convenience yield as the difference between the AAA corporate bond

[3]The "non-Ricardian" fiscal policy refers to the language in Woodford (2003), in which fiscal policy is not neutral in affecting inflation and output.

rate and the T-bond rate. We also construct a maturity-matched AAA-Treasury spread from ICE bond indices to rule out confounding factors related to term structure movements.[4] Our baseline measure for inflation expectation is the 10-year inflation swap rates, available since July 2004. To obtain longer time-series data, we also use the inflation expectations published by the Federal Reserve Bank of Cleveland, available since 1984. All data series are at a monthly frequency.

We regress the change in the convenience yield on the change in inflation expectations, controlling for variables approximating the fundamental states of the economy and monetary policy, such as the CBOE volatility index (VIX), the current growth rate of public debt, the growth rate of industrial production, inflation risk premium, and the effective federal funds rate. We find that a one percentage point increase in long-run inflation expectations is associated with a 20 basis points reduction in the convenience yield for long-term assets. Our estimate suggests a conditional relationship between convenience yield and inflation expectations, controlling for economic fundamentals and monetary policy.

Next, we show that inflation expectations strongly predict future public debt growth. We regress the privately held debt-to-GDP growth rate on inflation expectations, again controlling for contemporaneous output growth, inflation risk, current debt-to-GDP ratio, and the effective federal funds rate. We find that a one percentage point increase in 10-year inflation expectations predicts 14.6% higher growth in the debt-to-GDP ratio over the next year, 49.4% higher growth over the next 5 years, and 51.7% higher growth over the next decade.

To study the role of fiscal policy, we further examine how these variables respond to government deficit shocks. We construct exogenous government deficit shocks following Ramey (2011): a deficit shock is measured by the difference between realized federal spending and expected spending from the Survey of Professional Forecasters (provided by the Federal Reserve Bank of Philadelphia). Consistent with the correlation patterns, when government spending is higher than expected, inflation expectations increase. Meanwhile, the convenience yield drops, and future (ex-post) debt increases.

In the last section of the paper, we develop a model to rationalize our findings by introducing convenience yields into an otherwise standard New Keynesian model with non-Ricardian fiscal policy. Our calibrated model can match the empirical correlations among the convenience yield, inflation expectations, and debt growth reasonably well. Following the literature, households directly derive utility from holding safe and liquid assets such as long-term government debt.[5] This value comes either from liquidity, used as collateral, or

[4]We also use the Refcorp-Treasury spread, defined in Section 2, as a credit-risk-free measure of the convenience yield and show that our results are robust.

[5]See, e.g., Krishnamurthy and Vissing-Jorgensen (2012), Sunderam (2014), Nagel (2016), and He and Song (2021).

from special preferences for safe storage of value. In a special case of the model, we derive a closed-form result in which the convenience yield today decreases with the current real debt-to-GDP ratio and increases with the future path of expected government surpluses.

In our model, given a deficit shock, the government's debt balance increases in the medium to long run, leading to a lower convenience yield today. We assume a fiscal rule such that the proportional income tax rate positively responds to the overall debt balance. So when debt increases, the tax rate and the surplus automatically increase to pay down the debt. As the tax rate increases, the marginal cost of labor increases, which pushes up inflation and inflation expectations.[6] Hence, the deficit shock is financed partially by future surpluses and partially by higher inflation and inflation expectations.

Such policy pattern is consistent with the empirical findings in Berndt, Lustig and Yeltekin (2012). We generate a series of impulse responses to verify that our model can jointly explain the negative correlation between inflation expectations and the convenience yield, as well as the predictability of inflation expectations to the future debt-to-GDP growth rate. Quantitatively, the model-produced correlations between inflation expectations, convenience yield, and future debt growth are also close to the empirical estimates from the data. We focus on deficit shocks in the model to match the reduced-form coefficients, because in the reduced-form exercise, we have already conditioned on a wide range of other variables representing shocks to economic fundamentals and monetary policy.

We highlight the feedback loop between the convenience yield and the government debt supply generated by our mechanism. Given a deficit shock, a lower convenience yield implies higher borrowing costs for the government, and thus the government has to issue more debt in the future to finance its deficit. More future debt supply in turn results in an even lower convenience yield today. The response from the convenience yield implies that the government faces a less elastic long-run demand curve than otherwise when issuing debt. As a result, it amplifies the effect of deficit shocks on the costs of funding and debt balances.

Our explanation is not limited by the specific assumption of the fiscal policy. In many New Keynesian models, as long as Ricardian equivalence breaks down, it is also possible to generate such correlation patterns.[7] Alternatively, we also consider a formulation of policy rules in the spirit of the fiscal theory of price level (FTPL) by assuming active fiscal policy and lump-sum taxes in Appendix F. In this alternative setup, we show the key channels and results still hold.

[6] We assume the monetary policy follows a standard Taylor rule, which does not fully offset the fiscal effect on inflation and inflation expectations.

[7] Fiscal policies are non-Ricardian in a large set of New Keynesian models with a standard active monetary policy, such as overlapping-generation models (e.g., Ascari and Rankin, 2013), models with bounded rationality (e.g., Gabaix, 2020; Xie, 2020; Woodford and Xie, 2022) or with imperfect knowledge and learning (e.g., Eusepi and Preston, 2018), and the class of heterogeneous agent New Keynesian (HANK) models (e.g., Kaplan, Moll and Violante, 2018).

Different from the existing literature that focuses on the long-run trend of convenience yield, we study its cyclical pattern and how it correlates with macroeconomic variables. In particular, we take a dynamic perspective in which the convenience yield depends on the expected *future* path of debt balances. The rest of the paper is organized as follows. The rest of this section reviews the literature. Section 2 explains the data we use in detail and Section 3 shows the reduced-form evidence. Section 4 presents the theoretical model and quantitative analyses. Section 5 concludes.

1.1 Literature Review

Our paper contributes to the literature on safe assets and convenience yields (Longstaff, 2004; Bartolini et al., 2010; Coleman, Lundblad and Bansal, 2011; Gorton, Lewellen and Metrick, 2012; Gorton, 2017; Carlson and Wheelock, 2018). Krishnamurthy and Vissing-Jorgensen (2012) establish that the size of the convenience yield is decreasing in the supply of Treasuries relative to the size of the economy, and Nagel (2016) looks at how the opportunity cost of holding money affects the convenience yield on all short-term near-money assets. Du, Im and Schreger (2018) find that U.S. Treasury bonds hold a premium internationally and Valchev (2020) employs this premium to explain the failure of uncovered interest parity (UIP). Yet, a recent study He, Nagel and Song (2021) provides evidence that such premium may be declining during the COVID-19 crisis. Recent literature also finds that assets other than government debt may provide convenience services as well (Mota, 2021; He and Song, 2021). Li and Krishnamurthy (2022) measure different forms of safe assets and estimate the substitutability among different assets that provide convenience services. Cieslak, Li and Pflueger (2023) focus on the changing relationship between the convenience yield and realized inflation over the last century and attribute the changes to different drivers of inflation. Most papers in this literature focus on those contemporaneous factors that contribute to the convenience yield. Our paper instead takes a dynamic perspective: we focus on the convenience yield of long-term assets and study its forward-looking feature.

Our work is also closely related to the literature on government fiscal conditions and borrowing costs. Jiang et al. (2021) study bond convenience yields in a currency union when countries have different fiscal conditions. Interestingly, on eurozone bond yields, they find that a larger fraction of variation is explained by convenience yields rather than default spreads. This is direct evidence that the convenience yield matters for the government's borrowing cost. Berndt, Lustig and Yeltekin (2012) argue that 9% of unexpected spending shocks are absorbed by debt revaluation, and 70% is absorbed by the primary surplus. Nguyen (2021) finds that the debt-to-GDP ratio strongly correlates with the slope of the Treasury yield curve, and he rationalizes this finding through a model with endogenous growth and inflation. His work focuses on the yield premium of Treasury bonds, whereas we study the convenience yield component of Treasuries. Lastly, Elenev et al. (2021) explores how different monetary policies can reduce fiscal risks. Although we do not analyze the interactions between monetary and fiscal policies in this paper, the framework we adopt can be extended to study the interaction effects.

The model generates a convenience yield for certain assets by adopting the common assumption in the literature that households derive utility directly from holding them. Feenstra (1986) shows that it is equivalent to modeling a transaction cost in the budget constraint. The literature has also modeled safe assets as a result of coordination (He, Krishnamurthy and Milbradt, 2019) or as information insensitive assets (Gorton and Pennacchi, 1990). We focus on the interaction between fiscal policy and the price of a general group of safe assets instead of focusing on why some assets are viewed as safe whereas others are not.

From the macro perspective, Caballero (2006) and Caballero and Farhi (2017), among others, have studied the macroeconomic impact of safe asset shortages. Del Negro et al. (2017) argue that the low interest rates in the U.S. are primarily a result of the safety and liquidity premium. Christensen (2021) studies how unconventional monetary policy affects the convenience yield. But none have focused on the safe assets' connection with inflation and fiscal policy.

Our paper also builds on the macro-finance literature that studies the connection between bond returns and inflation. This literature focuses on bond-stock return comovements that help to identify when an inflation shock is associated with good fundamental news and when the opposite is true (e.g., Baele, Bekaert and Inghelbrecht, 2010; David and Veronesi, 2013; Kang and Pflueger, 2015; Campbell, Sunderam and Viceira, 2017; Campbell, Pflueger and Viceira, 2019). Our result holds before and after the bond-stock covariance sign switched, suggesting that the correlation between inflation and the convenience yield is not driven by economic fundamental movements.

Our paper is also closely connected with the literature on fiscal theories, particularly recent developments in the fiscal theory of the price level (FTPL). Jiang et al. (2019) propose valuation puzzles for US government debt and document a bubble component, and Brunnermeier, Merkel and Sannikov (2020) further develop a version of the FTPL with a bubble term. In contrast, Cochrane (2022*a*) claims that the puzzle disappears by imposing an S-shaped response in primary surpluses to policy shocks. By considering non-Ricaridan fiscal policy and introducing convenience benefits, our paper rationalizes the empirically observed relationship between the convenience yield and inflation expectations.

Lastly, the logic in our paper resembles the convenience yield and price backwardation in commodity pricing. Price backwardation refers to the fact that the spot price of a storable commodity is higher than its near-future price. Working (1948), Brennan (1958), and Telser (1958) argue that producers and firms hold inventories because they derive convenience services from holding physical stocks that at least partially offset the inventory costs. Considine and Larson (2001) and Carter and Giha (2007) also show empirically that such convenience yield constitutes a significant part of price backwardation. The convenience yield is inversely related to the supply of inventories, similar to our case, where the convenience yield of Treasuries is also inversely related to the expected future debt balance. However, there is a major difference regarding the time horizon. Since the commodity literature has typically looked at the difference between the spot price and the near-future price, what matters is the contemporaneous supply. However, in the case of long-term Treasury bonds, we argue that the future expected debt supply also affects current convenience yields, quantitatively even more important.

2 Data

We first introduce and discuss the data used in our empirical analysis. Details of the data construction as well as the sources for all variables can be found in Appendix A.

Measures of inflation expectations. We use the 10-year inflation swap rate as our baseline measure of inflation expectations for Fact 1: inflation expectations are negatively correlated with the convenience yield. An inflation swap is a financial derivative in which one party pays a predetermined fixed rate cash flow while the other party pays a floating rate linked to the consumer price index (CPI). It provides a market-based estimation for the average inflation rate over the contract period. We choose 10-year inflation expectations as our baseline because it is the most commonly used measure of long-term inflation expectations, and 10-year inflation swaps are also one of the most liquid contracts in the inflation swap market. Our results are also robust at other horizons.

Inflation swap rates have many advantages over alternative measures of inflation expectations, such as surveys or Treasury inflation-protected securities (TIPS). First, inflation swaps are directly traded on financial markets, reflecting the real-time model-free inflation expectations by the market participants. Second, they are also highly liquid and thus less prone to liquidity shocks relative to TIPS (Haubrich, Pennacchi and Ritchken, 2012; Fleming and Sporn, 2013).

Inflation swap rates also have two limitations, however. First, the data series is relatively short. Inflation swap rates are only available since July 2004. This drawback is especially restrictive for our second analysis—predicting future debt growth at the 10-year horizon—leaving few observations in the sample. Second, when investors are risk-averse and inflation is stochastic, investors demand compensation for inflation risks, so the swap rate reflects not only average inflation expectations but also the premium for the inflation risks. To overcome these limitations, we also use the inflation expectations estimated by the Federal Reserve Bank of Cleveland as an alternative measure. The Cleveland Fed utilizes the information in survey forecasts by Blue Chip and the Survey of Professional Forecasters, nominal yields, and inflation swaps to estimate an affine term structure model. With a structure model, they are able to identify both the average inflation expectations and the inflation risk premium. Their inflation expectations series goes back to 1982 at a monthly frequency. Details of their model and estimation approach can be found in Haubrich, Pennacchi and Ritchken (2012).

Figure 2 plots the two measures of 10-year inflation expectations. These two series are consistent in both the long-run trends and the direction of short-run fluctuations, although they often differ in levels. Both the inflation swap rate and the Cleveland Fed estimates show a secular decline in long-run inflation expectations within the sample period until very recently, when inflation expectations picked up after the COVID-19 recession.

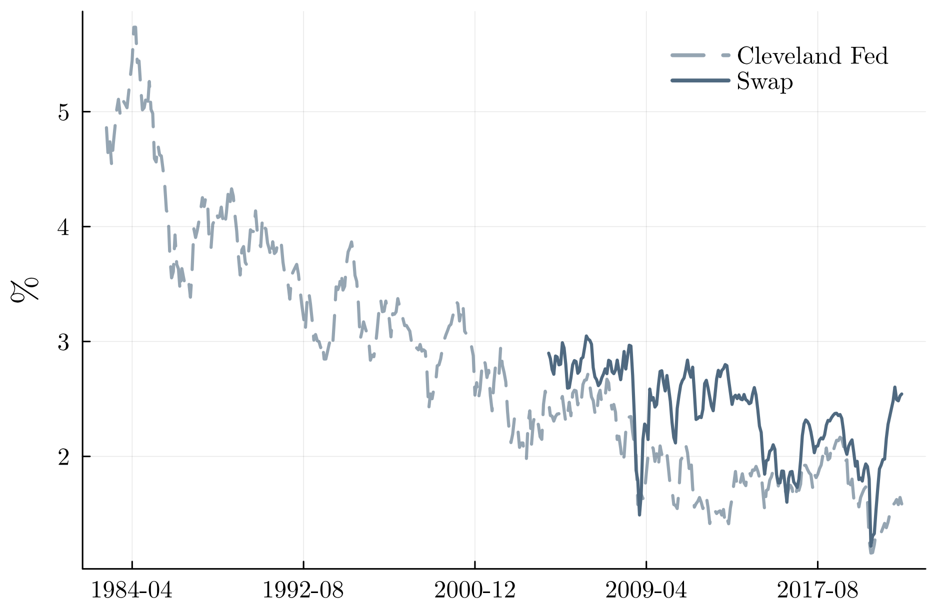

Figure 2: Two Measures of Inflation Expectations

Notes. This figure plots the two measures of 10-year inflation expectations at a monthly frequency. The solid black line indicates the data series constructed from inflation swaps, available since July 2004; the dashed grey line indicates the data series constructed by the Cleveland Fed, available since January 1982.

Measures of the convenience yield. We use the AAA-Treasury spread as our baseline measurement of the convenience yield. We obtain the AAA yield from Moody's daily yield averages for Aaa bonds,[8] and the Treasury yield is the market yield on U.S. Treasuries at 20-year constant maturity. Both sequences are retrieved via the FRED database at the St. Louis Fed.[9] As the default risk for AAA bonds is very small, the literature has attributed the difference in the spreads to the convenience yield (Krishnamurthy and Vissing-Jorgensen, 2012). Indeed, as shown in Table 1, the AAA-Treasury spread is, on average, about 88 basis points over our sample period. Because of the low historical default probability of AAA bonds, structural models often find it hard to explain this spread of nearly one percentage point with only credit risks (e.g., Huang and Huang, 2012; Feldhütter and Schaefer, 2018).

To address the concern about default risk in AAA bonds, we use an alternative measure of the convenience yield: the Refcorp-Treasury spread. Refcorp stands for Resolution Funding Corporation, a government agency created by the Financial Institutions Reform, Recovery, and Enforcement Act of 1989 (FIRREA). Different from other agency securities, Refcorps are backed by the Treasury and subject to the same taxation as Treasury bonds, so the Refcorp-Treasury spread has no default concern and is only driven by convenience services carried by Treasury bonds.

Public debt. Our measures of public debt are obtained from the Federal Reserve Bank of Dallas, which provides the market value and the par value of privately held government debt at the monthly frequency since 1942. To be consistent with our model, we use the market value of debt normalized by (nominal) GDP as the baseline measure for the real debt burden. More discussions on these measures can be found in Section 3.1.2.

Other economic variables. The following variables are controlled in regressions when applicable: industrial production, CBOE volatility index (VIX), public debt (measured in par value), the effective federal funds rate, and the inflation risk premium. All the control variables are from the FRED database except for the inflation risk premium, which is estimated by Haubrich, Pennacchi and Ritchken (2012). To construct government fiscal shocks, we use predicted government spending data in the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia. Other economic series such as GDP and CPI are from the Bureau of Economic Analysis (BEA).

Summary statistics. Table 1 presents the summary statistics of the key variables, in levels and in first differences, all measured in annualized percentage points. The variable

[8]The formal release name is "Moody's Daily Corporate Bond Yield Averages." It is based on bonds with maturities of 20 years and above.

[9]One may be concerned that the Moody's AAA yield index is constructed from bonds with maturities greater than 20 years so it has a longer duration than the 20-year Treasury. In Appendix B, we show that our results are robust to using the 30-year Treasury as a benchmark.

$E\pi$ is our baseline measure of the 10-year inflation expectations swap rate, from July 2004 onward. The 10-year inflation expectations averaged 2.39 percentage points throughout this sample period. Although the U.S. has not seen major inflation or deflation episodes in this century, there are still meaningful variations in the 10-year inflation expectations: the standard deviation is 0.39 percentage points. The Cleveland Fed's 10-year inflation expectations series dates back to as early as the 1980s, when the U.S. was just out of the Great Inflation period and inflation expectations were as high as 5.74 percentage points.

The average AAA-Tbond spread is 0.88 percentage points in the sample, and that for the Refcorp spread is smaller but still meaningful (0.48 percentage points). Both convenience yield measures peaked during the Great Recession, with 1.87 percentage points for the AAA spread and 1.82 percentage points for the Refcorp spread.

Table 1: Summary Statistics

		Level				**First Diff.**			
Variable	**Time Period**	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**	**Min**	**Max**
$E\pi$	04/07-21/09	2.39	0.39	1.22	3.05	-0.0	0.15	-0.64	0.52
$E\pi$ (Fed)	83/01-21/09	2.79	1.02	1.16	5.74	-0.01	0.12	-0.43	0.36
10yr Tbond	83/01-21/09	5.29	2.85	0.62	13.56	-0.02	0.29	-1.13	1.09
20yr Tbond	83/01-21/09	5.70	2.71	1.06	13.54	-0.02	0.28	-1.14	0.95
AAA	83/01-21/09	6.59	2.54	2.14	13.55	-0.02	0.23	-1.01	0.79
Refcorp	91/04-21/09	4.64	1.77	1.40	8.83	-0.02	0.26	-1.23	0.90
AAA-Tbond	83/01-21/09	0.88	0.33	-0.14	1.87	-0.0	0.13	-0.61	0.65
Refcorp-Tbond	91/04-21/09	0.48	0.24	0.15	1.82	-0.0	0.09	-0.31	0.51

Notes. This table reports summary statistics for key variables at the monthly frequency. Variables are measured in percentage points. Statistics in both levels and first differences are reported separately. For each variable, we report its available time period in our sample. $E\pi$ is our baseline measure of 10-year inflation expectations from inflation swaps. $E\pi$ (Fed) is the estimated 10-year expected inflation by the Cleveland Fed. 10yr (20yr) Tbond is the market yield on U.S. Treasuries at 10-year (20-year) constant maturity (DGS10/20 on FRED), and AAA is the Moody's seasoned Aaa corporate bond yield (DAAA on FRED). Refcorp refers to the yield on RefCorp bonds with 10-year maturity. AAA-Tbond is the spread between the AAA and 20yr Tbond, and Refcorp-Tbond refers to the spread between the Refcorp and 10yr Tbond, to match maturities.

3 Reduced-Form Evidence

We present our empirical results in this section. We show that the convenience yield on long-term Treasury bonds is negatively correlated with inflation expectations (Fact 1), despite being measured in real terms. Furthermore, we find that inflation expectations predict future debt growth (Fact 2), suggesting that inflation expectations reflect the market's belief about the government's fiscal policy going forward. To shed light on the role of fiscal policy, we construct government deficit shocks following the public finance literature and show that government deficit shocks indeed lead to higher inflation expectations, higher debt growth, and a lower convenience yield today.

3.1 New Stylized Facts

3.1.1 Inflation Expectations Negatively Comove with the Convenience Yield

Baseline results. We investigate the comovement of inflation expectations and the convenience yield by running the following time-series regression at the monthly frequency:

$$\Delta cy_t = \beta_0 + \beta \Delta E_t \pi_t^{t+10} + \beta_X \Delta X_t + \epsilon_t. \quad (3.1)$$

In our baseline design, we take first differences for each variable (denoted by Δ) to remove any potential trends.[10] Variable cy_t denotes the convenience yield in month t, and $E_t\pi_t^{t+10}$ is 10-year average inflation expectations at month t. We use inflation swaps for $E_t\pi_t^{t+10}$ in the baseline and the Cleveland Fed estimates when a longer sample is needed. Due to data availability reasons, our analysis focuses on the period after 1990s.[11]

The term X_t is a vector of control variables that generally fall into three categories. First, we control for cyclicality using the VIX, industrial production, and an indicator for recessions. The VIX is the implied volatility of S&P 500 index options on CBOE, which is a widely used indicator of financial stress. Periods of financial market turmoil tend to coincide with high levels of the VIX. Industrial production controls for the fundamental cyclical drivers of both inflation expectations and the convenience yield. The recession indicator equals one during recessions dated by the NBER Business Cycle Dating Committee and zero otherwise.

Second, we control for the inflation risk premium using estimates produced by the Cleveland Fed (Haubrich, Pennacchi and Ritchken, 2012). Periods with high inflation expectations often coincide with high inflation risks. Moreover, the fear of inflation risks is also directly priced in our baseline measure, the inflation swap rate. As inflation risks may affect corporate bond spreads via other channels, such as the default probability (Kang and Pflueger, 2015), we use the inflation risk premium as a proxy for the perceived inflation risks by investors.

Finally, we also control for other determinants of the convenience yield (i.e., the effective federal funds rate and the *contemporaneous* public debt supply). Krishnamurthy and Vissing-Jorgensen (2012) show that the convenience yield is decreasing in the supply of public debt.[12]

[10] In Appendix B, we also use the Hodrick-Prescott filter to remove the low-frequency movement and perform the regressions using the residuals. The results are robust. Our baseline results are also robust to regressions in levels.

[11] Moreover, the behavior of long-run inflation expectations changed during the 1980s, in which it has become more "anchored" in the post-1990 period.

[12] When as a control, we use the par value of public debt to avoid diluting the variation in inflation expectations.

Nagel (2016) further shows that at least in the short term, the elasticity of substitution between convenience services from T-bills and money is very high, so the effective federal funds rate, as the opportunity cost of money (or the "convenience yield" of money), is highly correlated with the convenience yield of 3-month T-bills. By controlling for both the effective federal funds rate and the contemporaneous debt supply in our regressions, we show that inflation expectations capture a new determinant of the convenience yield.[13] With these controls, any correlation we find between the convenience yield and inflation expectation is a conditional relationship after controlling for the macroeconomic state, monetary policy, and contemporaneous debt supply.

Table 2 presents our baseline regression results. All columns are in first differences, as noted by Δ. Columns (1) and (2) report results from regressing the 20-year Treasury yield and the AAA yield on 10-year inflation expectations, respectively. On average, a one percentage point increase in 10-year inflation expectations is related to an 82 basis points (bps) increase in the 20-year Treasury yield and a 47 bps increase in AAA. These two columns rule out the potential concern of extreme market segmentation: only Treasuries get repriced when inflation expectations rise, but AAA yields do not, so the AAA-Treasury spread falls mechanically. Columns (1) and (2) show that this is not the case. Both AAA and Treasuries respond to inflation expectations, but AAA yields are less sensitive.

Table 2: The AAA-Treasury Spread and Inflation Expectations

	Δ 20yr Tbond	Δ AAA	Δ AAA-Tbond			
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta E\pi$	0.82	0.47	-0.35	-0.23	-0.40	-0.27
	(0.13)	(0.13)	(0.08)	(0.08)	(0.06)	(0.07)
Measure	Swap	Swap	Swap	Swap	Fed	Fed
Controls	False	False	False	True	True	True
Sample	04/08-21/09	04/08-21/09	04/08-21/09	04/08-21/09	90/01-21/09	90/01-01/06
N	206	206	206	206	380	137
R^2	0.29	0.12	0.20	0.34	0.30	0.34

Notes. Newey-West standard errors (bandwidths optimally chosen following Newey and West (1994)) are shown in parentheses. Δ indicates first differences at the monthly level. In Columns (1)-(4), inflation expectations are proxied by the 10-year inflation swap rate for the sample period from July 2004 to September 2021, and in Columns (5)-(6), by the Cleveland Fed inflation expectations (Haubrich, Pennacchi and Ritchken, 2012) for a longer sample period. Control variables in Columns (4)-(6) include industrial production, the VIX, the effective federal funds rate, contemporaneous public debt supply, the inflation risk premium, and the recession indicator. Controls are also first differenced (or in growth rates) whenever applicable.

Column (3) shows the effect on the convenience yield by regressing the AAA-Treasury spread on inflation expectations. Consistent with the pattern in Figure 1, unconditionally, a one percentage point increase in 10-year inflation expectations is related to a reduction

[13] Our results are also robust to further controls, such as the slope in the yield curve.

of 35 bps in the AAA-Treasury spread. The effect is significant both economically and statistically. Column (4) controls for the relevant covariates. The coefficient in front of inflation expectations is stable and still significant at the 1% level.

In Appendix B.4, we use the short-term liquidity premium as a placebo test. As we explain in Section 4, our mechanism works through debt growth over relatively long-run horizons, and short-term Treasuries mature before any meaningful change in the public debt level. Consistent with this prediction, we do not detect significant comovements between the short-term convenience yield and inflation expectations. This result also suggests that our finding cannot be fully explained by the fluctuation in liquidity demand, because the liquidity demand channel should affect the short-term convenience yield more and one should observe a stronger correlation between short-term convenience yield and inflation expectation than the long-term convenience yield.

A natural explanation for the negative correlation between the convenience yield and inflation expectations is the "flight-to-safety" hypothesis. This argument suggests that during economic downturns, inflation expectations drop, and demand for Treasuries as safe assets increases, which implies a higher convenience yield. Several pieces of evidence suggest that the flight-to-safety hypothesis is not the main driver of our results.

First, whenever applicable, we control for cyclicality and financial market sentiments using industrial production growth and the VIX. Second, our results are robust when we measure the convenience yield as differences in yields between two equally safe assets (that is, Refcorp and Treasury bonds). It implies flight to safety cannot be the sole factor. Finally, the negative correlation exists even in periods when long-term Treasuries *suffer* from flight to safety. Campbell, Pflueger and Viceira (2019) find that before 2001Q3, the bond-stock correlation was actually positive, so long-term Treasuries dropped in value during downturns. Column (5) in Table 2 shows that inflation expectations are negatively correlated with the convenience yield by including this sample period in which Treasuries comove positively with the stocks.[14] Here we have switched to the Cleveland Fed estimates of inflation expectations to extend the sample back to the last century. Column (6) only looks at the sample period before June 2001. The point estimate is comparable to that in the whole sample and is also statistically significant despite the smaller sample size.

Dynamic effects. To study the dynamic effect of the correlation, we employ a local projection design following Jordà (2005):

$$cy_{t+h} = \sum_{i=0}^{p} \beta_{i,h}(E\pi)_{t-i} + \sum_{j=0}^{p} \alpha_{j,h} cy_{t-j} + \beta_{X,h} X_t + \beta_0 + \beta_1 t + \epsilon_{t+h}, \tag{3.2}$$

[14] In Columns (5)-(6) of Table 2, the sample period starts from 1990 due to the availability of the VIX in control variables. The results are robust without controls in the longer sample back to 1982.

where $E\pi_t$ is the 10-year inflation swap rate in period t and cy_{t+h} is the AAA-Treasury spread h-periods ahead. The coefficient $\beta_{0,h}$ gives the dynamic responses in the convenience yield h-periods ahead when there is a one percentage point increase in inflation expectations today. We include a linear trend t in the regression to avoid a spurious correlation caused by common trends. The maximum lag p is chosen at two months according to the Bayesian information criterion (BIC).

Figure 3 plots the estimated $\beta_{0,h}$ for horizons from 0 to 6 months together with the 95% confidence interval. The negative response in the convenience yield persists at least for two quarters: the estimated contemporaneous response $\beta_{0,0}$ is -0.53, and after 6 months the coefficient is still $\beta_{0,6} = -0.33$, larger than the estimate from the first-difference regression in Column (4) of Table 2. The long-lasting response in the convenience yield suggests that the disconnection between yields of AAA and Treasuries is unlikely a result of market segmentation in the short run.

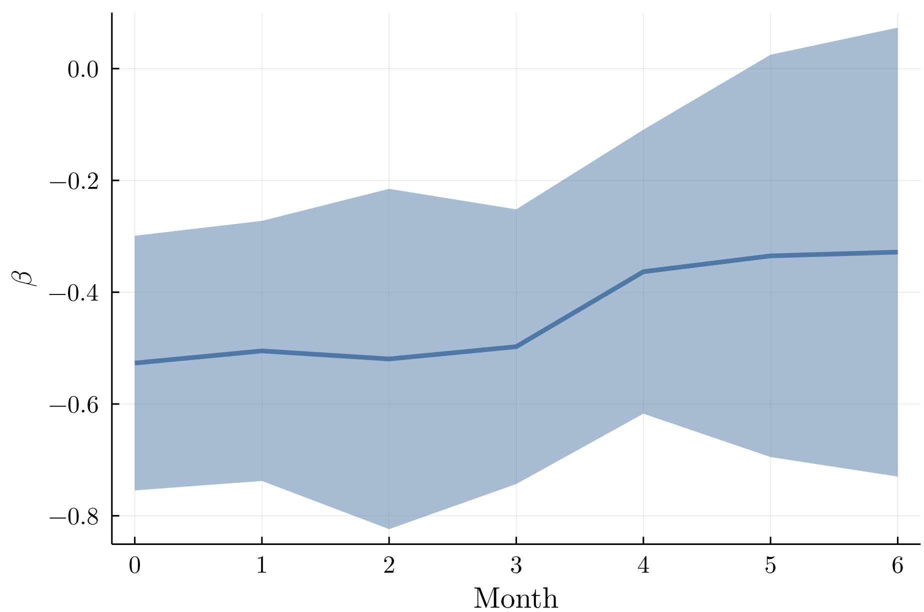

Figure 3: Local Projection of AAA-Tbond on Inflation Expectations

Notes. This figure plots the estimated $\beta_{0,h}$ from equation (3.2), that is, the impulse response of cy_t, proxied by the AAA-Tbond spread, to a one-unit innovation to $E\pi$, proxied by the 10-year inflation swap rate. The shaded area represents the 95% confidence interval with Newey-West standard errors (bandwidths optimally chosen following Newey and West (1994)).

Alternative measures of the convenience yield. Though very small, the default risk of AAA corporate bonds is still a component of the AAA-Treasury spread. To show that our results are not only driven by time-varying default risks, we turn to other default-risk-free measures of the convenience yield. We discuss the Refcorp-Treasury spread in detail here and leave results using other measures to Appendix B.

Unlike other agency bonds that are often still subject to small credit risks, Refcorp is fully guaranteed by the Treasury under FIRREA, so they have exactly the same credit risk as Treasury bonds. Refcorp bonds are also subject to the same taxation as Treasuries.[15] Therefore, the spread between Refcorp bonds and Treasuries should be free from default concerns and can be solely attributed to the convenience premium on Treasuries.

Table 3 presents regressions of the Refcorp-Treasury spread on inflation expectations, following the same design as in equation (3.1). To match the horizon of inflation expectations, both the Refcorp and the Treasury have a maturity of 10 years. The estimated coefficients are significant across different specifications. In Column (1), we regress the Refcorp-Treasury spread univariately on inflation expectations measured by the inflation swap rate. The point estimate implies that a one percentage point change in inflation expectations translates into a reduction of 13 bps in the Refcorp-Treasury spread, or 54% of its long-run standard deviation. Column (2) adds controls to the same regression. After controlling for covariates, the point estimate suggests a reduction of 10 bps (or 42% of its long-run standard deviation) in the Refcorp-Treasury spread for a one percentage point increase in inflation expectations. Column (3) extends the sample back to April 1993 using the Cleveland Fed inflation expectations. The results are similar to those in Column (2).

During recessions, especially during severe financial turmoil, investors may prefer to hold highly liquid securities such as Treasuries rather than less-liquid but equally safe Refcorp bonds, increasing the convenience yield on Treasuries in a flight to liquidity. In Column (4), we run the same regression using only the non-recession sample. The coefficient is stable and, if anything, becomes stronger.

Additional robust checks. In Appendix B, we perform further robustness checks and rule out several other potential theories. In Appendix B.1, we use Treasury bonds with different maturities as the benchmark yield to make sure that the variation is not driven by differences in time to maturity. We also use the option-adjusted spread to account for the callable options on corporate bonds. In Appendix B.2, we use bond-CDS spreads as an alternative default-risk-free measure of the convenience yield, and it is also negatively correlated with inflation expectations.[16] Finally, our results are stable when we filter out the low-frequency movement using the Hodrick-Prescott filter in Appendix B.3.

[15] See Longstaff (2004) for a more thorough discussion on the institutional details.

[16] Here we match the maturities of the corporate bond, its CDS, and the Treasury bond. Thus, our result is not driven by maturity differences between corporate bonds and Treasury bonds.

Table 3: The Refcorp-Treasury Spread and Inflation Expectations

	Δ Refcorp - TBond			
	(1)	(2)	(3)	(4)
$\Delta E\pi$	-0.13	-0.09	-0.11	-0.20
	(0.04)	(0.02)	(0.06)	(0.06)
Measure	Swap	Swap	Fed	Fed
Controls	False	True	True	True
Sample	04/08-21/09	04/08-21/09	91/04-21/09	No Recession
N	206	206	365	336
R^2	0.04	0.14	0.11	0.06

Notes. Newey-West standard errors (bandwidths optimally chosen following Newey and West (1994)) are shown in parentheses. Δ indicates first differences at the monthly level. In Columns (1)-(2), inflation expectations are proxied by the 10-year inflation swap rate for the sample of 2004-2021, and in Columns (3)-(4), by inflation expectations estimated by the Cleveland Fed (Haubrich, Pennacchi and Ritchken, 2012). Control variables for Columns (2)-(4) include industrial production, the VIX, the effective federal funds rate, contemporaneous public debt supply, the inflation risk premium, and the recession indicator. Controls are also first differenced (or in growth rates) whenever applicable.

3.1.2 Inflation Expectations Positively Predict Future Public Debt Growth

We next provide evidence that inflation expectations predict future debt growth. To include a longer sample size with sufficient variations in the debt-to-GDP ratio, we use inflation expectations from the Cleveland Fed, which dates back to 1982. Since GDP is reported quarterly, we base our analysis on the quarterly frequency in this section.[17]

Figure 4 plots the level of debt-to-GDP ratio since 1982 and the 10-year-forward cumulative growth rates, measured by the market value and the par value, respectively. We only include privately held debt in both measures. During this period, the debt burden of the U.S. government has been steadily increasing, from only 20% of GDP in 1982 to more than 80% after the forceful fiscal response to the COVID-19 shock in 2020. The market value measure shows slightly larger cyclical patterns than the par value measure, reflecting time-varying discount rates over business cycles. Nevertheless, panel (b) in Figure 4 shows the differences in 10-year growth rates between the two measures are small. Therefore, over a long horizon such as 10 years, most of the variations in the market value are from the variations in the par value (that is, from the active management of the fiscal authority).

In the remaining part of this section, we use the market value of privately held debt divided by GDP as our baseline measure for the debt-to-GDP ratio. This choice is motivated by the

[17] Our results are also robust at the monthly level where GDP is linearly interpolated in-between quarter ends.

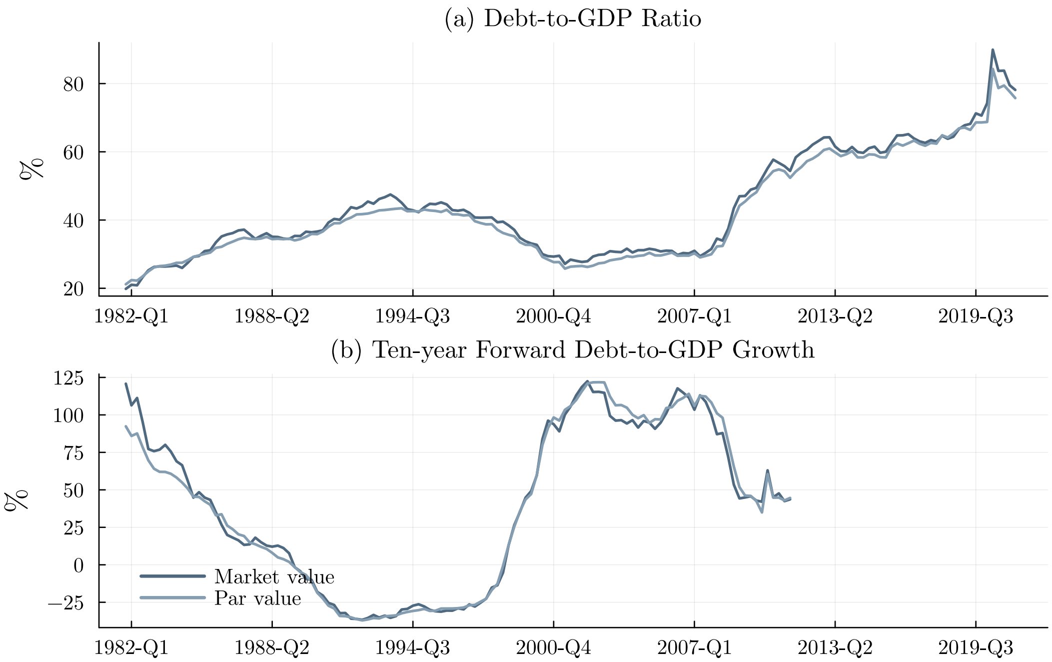

Figure 4: Debt-to-GDP Ratio and Forward Growth Rates

Notes. Panel (a) presents the raw series of the debt-to-GDP ratio, where debt is measured by privately held public debt, both in market value and in par value. Panel (b) presents the 10-year-forward growth rates of the two measures.

economics of the convenience yield: households gain convenience services such as collateral values or regulatory relief from the market value of the public debt rather than the par value. The demand for such services scales with the level of economic activities, so we scale all the measures by GDP. To address the concern that market value is endogenously determined by discount rates, or the concern that variations of the debt-to-GDP ratio come from GDP instead of the debt growth, we also measure the debt growth by the growth rate of par values, normalized by CPI (but not scaled by GDP). The results are presented in Appendix C.1 and are similar to the baseline.

Figure 5 plots the inflation expectations (left y-axis) and debt-to-GDP growth rates (right y-axis) for one-year-ahead (panel a) and 10-year-ahead (panel b), respectively. All series are linearly detrended to remove correlations caused by common trends. As shown in panel (a), one-year inflation expectations predict the one-year debt-to-GDP growth at a high frequency well. An increase in one-year inflation expectations is often accompanied by higher debt-to-GDP growth one-year forward. Predicting the 10-year debt-to-GDP growth rate is much more demanding. Nevertheless, in the long run, we find that periods when inflation expectations are high are often associated with higher debt-to-GDP growth in the next 10 years.

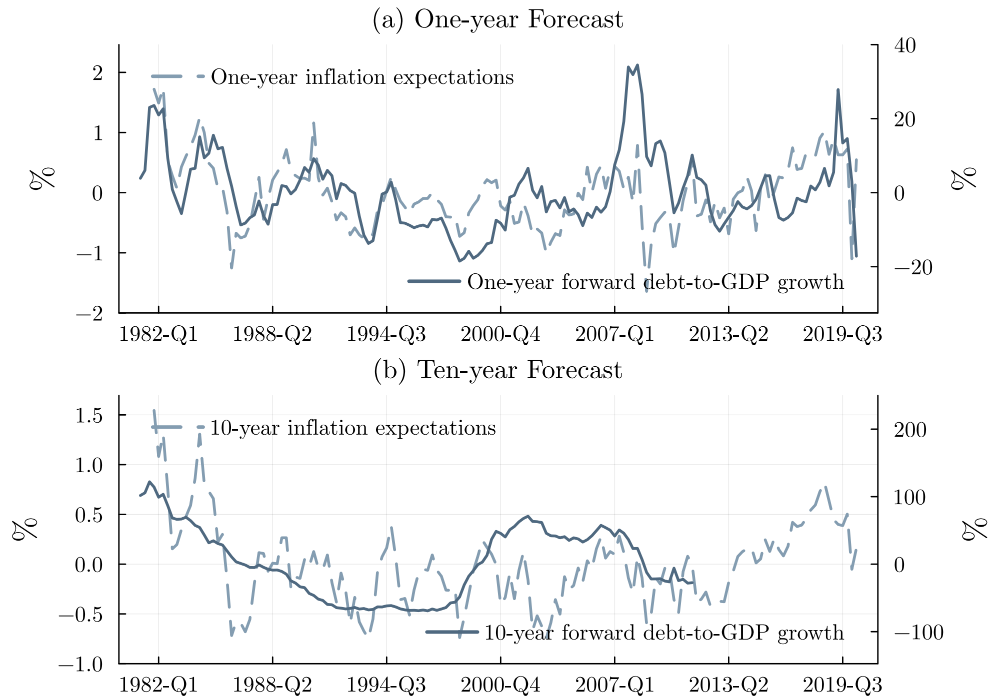

Figure 5: Inflation Expectations and Future Debt Growth

Notes. The light dotted lines plot the one-year (10-year) inflation expectations at the end of each quarter, and the dark solid lines plot the one-year (10-year) forward growth rate of the debt-to-GDP ratio. Debt is measured by the market value of privately held public debt. All sequences are linearly detrended to avoid common trends.

We now formally test the predictability using regressions. The specification is as follows:

$$g_t^{t+h}(V/Y) = \beta_h E_t \pi_t^{t+10} + \beta_X X_t + \epsilon_t, \tag{3.3}$$

where $g_t^{t+h}(V/Y)$ denotes the growth rate of debt-to-GDP from t to $t+h$, $E_t \pi_t^{t+10}$ denotes 10-year inflation expectations,[18] and X_t denotes control variables, including contemporaneous output growth, the inflation risk premium, the debt-to-GDP ratio, and the effective federal funds rate. As before, the correlation we find from the regression between inflation expectation and debt growth is a conditional relationship. To avoid comovements from common trends, we employ two approaches: linear detrended variables, or taking first differences as the specifications in Section 3.1.1. Both methods produce significant results, as shown in Table 4.

Table 4 presents the coefficient β_h for horizons equal to 1 year, 5 years, and 10 years. Columns (1)-(3) present the results with linear detrending. A one percentage point increase

[18] We also match the horizons of inflation expectations to the debt-to-GDP ratio and the results are similar. We use 10-year inflation expectations to be consistent with previous sections.

in 10-year inflation expectations predicts 14.6% higher growth in debt-to-GDP over the next year, 49.4% higher growth over the next 5 years, and 51.7% higher growth over the next decade, conditional on output, the current level of public debt, and monetary policy rate. All results are statistically significant. Columns (4)-(6) present the results in the first differences, i.e., we regress $\Delta g_t^{t+h}(V/Y)$ on $\Delta E_t \pi_t^{t+10}$ with the same controls as before.

We further conduct a panel of robustness checks. We leave the full reports to Appendix C and briefly summarize the main results here. As pointed out by Stambaugh (1999) and Boudoukh, Israel and Richardson (2021), predictive regressions in a finite sample are subject to small sample bias. We correct for this bias using the formula by Boudoukh, Israel and Richardson (2021) and present the unbiased estimates in Appendix C.2. The differences are small. In addition, Appendix C.1 presents the results when debt growth is measured by the real growth rate of the par value of privately held debt. The results remain unchanged. Appendix C.3 uses the realized inflation as a proxy for inflation expectations to extend the sample back to 1947. Consistent with Cochrane (2022*b*), we find that realized inflation also predicts future debt growth. However, when put in regressions together with inflation expectations, realized inflation loses its predictive power, whereas the coefficients on inflation expectations are not affected.

Table 4: Inflation Expectations Predict Future Debt/GDP Growth

	Linearly Detrended			First Differences		
	1yr	5yr	10yr	1yr	5yr	10yr
	(1)	(2)	(3)	(4)	(5)	(6)
$E\pi$	14.59	49.38	51.69			
	(2.62)	(12.54)	(14.86)			
$\Delta E\pi$				10.18	9.58	6.87
				(1.35)	(2.59)	(3.19)
Sample	81Q4-20Q2	81Q4-16Q2	81Q4-11Q2	81Q4-20Q2	81Q4-16Q2	81Q4-11Q2
Control	True	True	True	True	True	True
N	155	139	119	154	138	118
R^2	0.30	0.48	0.82	0.35	0.32	0.25

Notes. Newey-West standard errors (bandwidths optimally chosen following Newey and West (1994)) are shown in parentheses. The predicted variables are future debt/GDP cumulative growth rates, where debt is measured by the market value of privately held public debt. Columns (1)-(3) report the regression results with linearly detrended variables, and Columns (4)-(6) report regression results under first differences. Horizons are indicated in each column. The predictor variable is 10-year inflation expectations estimated by the Cleveland Fed (Haubrich, Pennacchi and Ritchken, 2012). In all columns, we control for output growth, the ratio of par value of debt to GDP, the inflation risk premium, and the effective federal funds rate, all at time t.

3.2 The Impact of Fiscal Shocks

Considering the link between inflation expectations and future debt growth, one may naturally conjecture that fiscal policy plays some role in driving our empirical findings. To shed light on the potential mechanism and the role of fiscal policy, we construct fiscal deficit shocks and study the joint response of inflation expectations, the convenience yield, and future debt issuance.

The main difficulty in identifying fiscal shocks is to separate out the exogenous unexpected component of government spending. Following Ramey (2011), we use the Survey of Professional Forecasters to separate the "predicted" versus "unexpected" component of government spending. In particular, a shock is defined as the difference between forecasted real federal spending growth and realized federal spending growth.

We obtain predicted government spending using the Survey of Professional Forecasters from the Philadelphia Fed, while other economic series such as GDP and CPI are retrieved from the National Income and Product Accounts (NIPA) tables provided by the BEA. From 1990Q1 to 2021Q2, the mean and median of such government spending shocks are both -0.02% with a standard deviation of 0.014. See Figure 6 for the series of shocks over time.

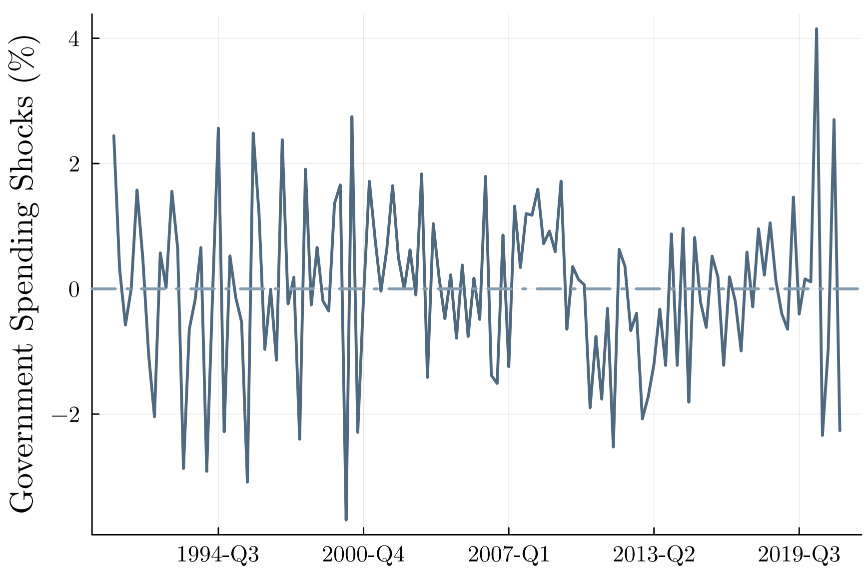

Figure 6: Government Spending Shocks

Notes. We extend government spending shocks using the Survey of Professional Forecasters, following Ramey (2011).

To examine the impact of fiscal stimulus, we regress the convenience yield, inflation expectations, and future debt growth on exogenous government spending shocks. Table 5 reports the results after controlling for output growth, the VIX, the effective federal funds rate, and a recession indicator. We find that an increase in government spending leads to significantly higher expected inflation, higher growth of future debt at both 5-year and 10-year horizons, as well as a lower convenience yield measured by the AAA-Treasury spread. The results suggest that an unexpected government spending shock is financed partially by inflating away some current debt and issuing more debt in the future. Motivated by such evidence, we next provide a model to quantify the role of fiscal shocks in generating the empirically observed correlation among inflation expectations, the convenience yield, and debt growth.

Table 5: The Effects of Government Spending Shocks

	Δ AAA-Tbond	$\Delta E\pi$	$\Delta_t^{t+5yr}\left(\frac{V}{Y}\right)$	$\Delta_t^{t+10yr}\left(\frac{V}{Y}\right)$
	(1)	(2)	(3)	(4)
Fiscal shock	-0.05	0.03	5.29	4.74
	(0.01)	(0.01)	(2.53)	(2.76)
Sample	90Q1-21Q2	90Q1-21Q2	90Q1-16Q2	90Q1-11Q2
Controls	Yes	Yes	Yes	Yes
N	126	126	106	86
R^2	0.10	0.09	0.33	0.59

Notes. In all columns, we control for output growth, the VIX, the effective federal funds rate, and a recession indicator. The fiscal shocks are government spending shocks constructed by the Survey of Professional Forecasters, following the approach in Ramey (2011). They are defined as the differences between the forecasted real federal spending growth and realized federal spending growth in percentage terms. $g_t^{t+h}(V/Y)$ denotes debt-to-GDP growth rate between t and $t+h$ years. Newey-West standard errors (bandwidths optimally chosen following Newey and West (1994)) are shown in parentheses.

4 A Model of the Convenience Yield and Fiscal Policy

To illustrate the role of fiscal policy, particularly the future supply of government bonds, in affecting the convenience yield and inflation expectations, we build a New Keynesian model in which government bonds such as Treasury securities also provide convenience benefits. We regard assets with such a feature as convenience assets. The wedge between the returns on convenience and non-convenience assets is the convenience yield. Our model rationalizes the two empirical facts documented in Section 3.1. Furthermore, whereas the model is calibrated to match the aggregate dynamics, the model simulations also match the empirical findings well by doing the same regressions, serving as an external validity check.

We assume an active monetary policy following the Taylor rule as in a standard New Keynesian model, while assuming the fiscal authority finances its deficits by collecting proportional income taxes. The proportional tax breaks Ricardian equivalence, thus fiscal and monetary policy jointly determine the convenience yield, inflation, and inflation expectations.[19] We also consider an alternative formulation of policy rules in the spirit of the fiscal theory of price level (FTPL) by assuming active fiscal policy and lump-sum taxes. In this alternative setup, we show the key channels/results still hold. Details can be found in Appendix F.

The economy consists of four types of agents: representative households; firms (owned by the households) that produce output; a fiscal authority (the government) that specifies fiscal policy, a rule of collecting primary surpluses through proportional income taxes from households, which implicitly controls the supply of government bonds; and a central bank that sets the short-term risk-free nominal interest rate. The only asset that provides convenience benefits is long-term government-issued bonds.[20]

4.1 Long-Term Government Bonds

To model the long-term government bonds, we follow the approach developed in Woodford (2001) by assuming only one type of bond issued by the government: perpetuities with coupon payments of 1, k, k^2, and so on. Let Q_t^B denote the time-t price of a new issue. Given the time pattern of the perpetuity payment, the new issue price Q_t^B summarizes the prices at all maturities (e.g., kQ_t^B is the time-t price of the perpetuity issued in period $t-1$). The duration on these bonds is given by $h = (1-k)^{-1}$, and the (gross) yield to maturity is given by $R_t^B = (Q_t^B)^{-1} + k$. The advantage of modeling such decaying coupon bonds is that it abstracts from the heterogeneous maturities of outstanding government bonds and one only needs to track the total outstanding bonds rather than individual issues.

Let N_t denote the number of new perpetuities issued at time t. At time t, the government's nominal liability on past issues is

$$B_{t-1} = \sum_{j=1}^{\infty} k^{j-1} N_{t-j},$$

[19] The assumption of a proportional income tax is common in the literature, e.g., Erceg and Lindé (2014). Alternatively, one can assume lump-sum taxation while keep breaking Ricardian equivalence by assuming either overlapping generation (OLG), bounded rationality, imperfect knowledge and learning, or the existence of hand-to-mouth agents. Nevertheless, the key channels in our baseline model still go through under these alternative setups.

[20] Short-term near-money assets could also provide convenience benefits (e.g., Greenwood, Hanson and Stein, 2015; Nagel, 2016), but they are not the focus of this paper. Nevertheless, the way we model long-term convenience assets can also nest the analyses of short-term convenience assets, as discussed in Section 4.3.

which implies the newly issued debt at time t is

$$N_t = B_t - kB_{t-1}.$$

Thus, the government budget constraint is given by

$$\frac{B_{t-1}}{P_t} = S_t + \frac{Q_t^B(B_t - kB_{t-1})}{P_t}, \tag{4.1}$$

where S_t is the real value of surplus and P_t is the aggregate price level. The left-hand side of equation (4.1) is the real coupon liability from all past issuances due in period t, which needs to be financed through either raising surplus or issuing new liabilities.[21] The government collects surplus S_t by levying a proportional income tax on households.

The government budget constraint (4.1) can be rewritten into the real value of outstanding debt:

$$\frac{V_{t-1}}{\Pi_t}\frac{1 + kQ_t^B}{Q_{t-1}^B} = S_t + V_t, \tag{4.2}$$

where $V_t \equiv Q_t^B B_t/P_t$ is the real (market) value of all outstanding debt and $\Pi_t = P_t/P_{t-1}$ is inflation at time t.

4.2 Households

An infinitely lived representative household maximizes the objective

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(C_t) - \Psi(H_t) + \mu(V_t)],$$

where C_t is consumption, H_t is labor supply, and V_t is the real value of long-term government bonds, which carry a convenience premium.

The household budget constraint expressed in real terms is

$$\begin{aligned}\frac{Q_t^B(B_t - kB_{t-1})}{P_t} &+ \frac{Q_t^D(D_t - kD_{t-1})}{P_t} + \frac{L_t}{P_t} + C_t \\ &= (1 - \mathcal{T}_t)\frac{W_t H_t}{P_t} + \frac{B_{t-1}}{P_t} + \frac{D_{t-1}}{P_t} + \frac{L_{t-1}R_{t-1}}{P_t} + (1 - \mathcal{T}_t)\Phi_t,\end{aligned}$$

where D_t is the holdings of perpetuities that have the same payment structure with government bonds but provide no convenience benefits, Q_t^D is the new issue price of this asset, L_t is the holdings of short-term risk-free nominal bonds with an (nominal) interest rate R_t. $\mathcal{T}_t$

[21] We assume that the government is always capable of paying its liabilities and thus abstract from sovereign default.

is the income tax levied by the government. W_t is the household's nominal wage and Φ_t is the (real) lump-sum dividends transferred from firms to households.[22]

The only purpose of introducing D_t is to calculate the convenience yield between the two long-term assets that share the same payment structure but differ in convenience benefits. We assume assets D_t and L_t are traded only among the households with zero net supply.

The household's first-order conditions with respect to consumption and labor yield

$$u_c(C_t) = E_t[\beta R_t \frac{u_c(C_{t+1})}{\Pi_{t+1}}], \tag{4.3}$$

$$\frac{\Psi_h(H_t)}{u_c(C_t)} = (1 - \mathcal{T}_t)\frac{W_t}{P_t}. \tag{4.4}$$

By applying a first-order approximation, the household's first-order conditions with respect to asset balances B_t and D_t yield

$$\log R_t = \frac{\mu_v(V_t)}{u_c(C_t)} + E_t \log(\frac{kQ^B_{t+1} + 1}{Q^B_t}), \tag{4.5}$$

and

$$\log R_t = E_t \log(\frac{kQ^D_{t+1} + 1}{Q^D_t}). \tag{4.6}$$

Derivation of conditions (4.5)-(4.6) can be found in Appendix D.1. The left-hand side of these two equations represents the marginal cost of holding one-period risk-free nominal bonds (which is determined by the central bank), whereas the right-hand side represents the marginal benefit, in terms of utility, of holding long-term assets with and without convenience benefits.

4.3 The Convenience Yield

Before closing the model, we first show the expression of the convenience yield on long-term government-issued assets, which only originates from the optimality conditions of households and the government's budget constraint. Although we have not yet introduced the supply side of the economy, we will utilize the market clearing condition that the total output Y_t of the economy equals households' total consumption C_t to simplify expressions. This must hold as a market clearing condition in this closed economy, once we further introduce firm production in Section 4.4.

We now derive a closed-form expression of the convenience yield by log-linearizing the equilibrium conditions around the steady state to a first-order approximation. We denote

[22] The capital income tax rate on Φ_t can be different from the one of wage income. For simplicity, we assume they are the same.

log-deviations from the steady state by lowercase letters unless otherwise stated, using the following notations:

$$i_t \equiv \log(R_t/\bar{R}), \qquad v_t \equiv \log(\frac{V_t}{Y_t}/\frac{\bar{V}}{\bar{Y}}), \qquad y_t \equiv \log(Y_t/\bar{Y}),$$

$$q_t^B \equiv \log(Q_t^B/\bar{Q}^B), \qquad q_t^D \equiv \log(Q_t^D/\bar{Q}^D), \qquad i_{t+1}^B \equiv \log(\frac{kQ_{t+1}^B+1}{Q_t^B}/\frac{k\bar{Q}^B+1}{\bar{Q}^B}).$$

Given the market clearing condition $Y_t = C_t$, log-linearizing equations (4.5) and (4.6) yields

$$i_t = -\gamma\sigma_v^{-1}[v_t + y_t] + \gamma\sigma_c^{-1}y_t + kE_t q_{t+1}^B - q_t^B \tag{4.7}$$

and

$$i_t = kE_t q_{t+1}^D - q_t^D, \tag{4.8}$$

where $\sigma_v \equiv -\mu_v(\bar{V})/(\mu_{vv}(\bar{V})\bar{V}) > 0$ is the intertemporal elasticity of substitution (IES) of household convenience benefits and $\sigma_c \equiv -u_c(\bar{C})/(u_{cc}(\bar{C})\bar{C}) > 0$ is the IES of household expenditure. $\gamma \equiv \mu_v(\bar{V})/u_c(\bar{C})$ measures the marginal benefits of holding one more unit of convenience assets relative to the marginal benefits of consumption at the steady state. Details of the derivations can be found in Appendix D.2.

Given the nominal yields of long-term bonds with and without convenience benefits are

$$R_t^B = (Q_t^B)^{-1} + k, \qquad R_t^D = (Q_t^D)^{-1} + k,$$

respectively, after log-linearization, we have

$$r_t^B = -(\bar{Q}^B)^{-1}q_t^B, \qquad r_t^D = -(\bar{Q}^D)^{-1}q_t^D, \tag{4.9}$$

where $r_t^B \equiv \log(R_t^B/\bar{R}^B)$ and $r_t^D \equiv \log(R_t^D/\bar{R}^D)$.[23]

Thus by combining (4.7)-(4.9), the convenience yield $cy_t \equiv r_t^D - r_t^B$ is given by

$$cy_t = kE_t(cy_{t+1}) + (1-k)[-\gamma\sigma_v^{-1}v_t + \gamma(\sigma_c^{-1} - \sigma_v^{-1})y_t]. \tag{4.10}$$

Details of deriving (4.10) can be found in Appendix D.2. Given the perpetuity coupon payment satisfies $0 < k < 1$, equation (4.10) then implies Proposition 1.

Proposition 1 (Impact of government bond supply on convenience yield) *The convenience yield on government-issued perpetuities is equal to the discounted sum of all current*

[23]Here we have used the steady-state relationship $\bar{R}^B = (k\bar{Q}^B + 1)/\bar{Q}^B$ and $\bar{R}^D = (k\bar{Q}^D + 1)/\bar{Q}^D$. We also assume $\bar{R}^B - 1$ and $\bar{R}^D - 1$ are at the scale of first order.

and expected future government real debt balances (scaled by aggregate output) plus the discounted sum of all current and expected future output gaps; that is,

$$cy_t = (1-k)\sum_{j=0}^{\infty} k^j \gamma E_t[-\sigma_v^{-1} v_{t+j} + (\sigma_c^{-1} - \sigma_v^{-1}) y_{t+j}]. \tag{4.11}$$

Proposition 1 nests the analyses for both the long-term and short-term convenience yields simultaneously by simply varying the value of k. When k is close to one, expression (4.11) is equivalent to the long-term convenience yield modeled in Krishnamurthy and Vissing-Jorgensen (2012) but under the assumption of separable utilities for the convenience benefits and consumption. When $k = 0$, the perpetuities collapse to one-period riskless nominal bonds, and thus it corresponds to the analyses of the short-term convenience yield in models such as Greenwood, Hanson and Stein (2015) and Nagel (2016).[24]

Expression (4.11) indicates that higher *future* government real debt balances (that is, debt-to-GDP ratios) lead to a lower convenience yield on long-term government bonds. If long-run inflation expectations partially reflect the *future* debt-to-GDP ratio, which is a stylized fact documented in Section 3.1.2, long-run inflation expectations will then be negatively correlated with the long-term convenience yield, as found in Section 3.1.1.

Next, to better understand the right-hand side of expression (4.11), we further solve for the determination of future government real debt balances. For simplicity, assume that the IES of the convenience benefits $\mu(\cdot)$ and consumption utility $u(\cdot)$ are both equal to one; that is, $\sigma_c = \sigma_v = 1$. Proposition 1 then implies that the long-term convenience yield is a function of only current and expected future government real debt balances.

The Euler equation (4.3) yields

$$y_t = E_t y_{t+1} - \sigma_c(i_t - E_t \pi_{t+1}), \tag{4.12}$$

and the government budget constraint (4.2) yields

$$\rho v_{t+1} = v_t + i_{t+1}^B - \pi_{t+1} - s_{t+1} - (y_{t+1} - y_t), \tag{4.13}$$

where i_{t+1}^B is the log-deviation of the one-period nominal return on government-issued bonds and s_{t+1} is the scaled real primary surplus-to-output ratio. $\rho \equiv e^{-\bar{i}^B} < 1$ is a constant.[25] Then by plugging equations (4.7) and (4.12) into (4.13), and also noting that $\sigma_c = \sigma_v = 1$, we get the expected government real debt balance at any time $t + j + 1$ satisfying the following

[24]The convenience yield expression in (4.11) with the assumption of $k = 0$ corresponds to a special case in Nagel (2016) in which money and short-term Treasury securities are not substitutes.

[25]Details of deriving (4.13) follow the appendix in Cochrane (2022*b*).

evolution process:

$$E_t v_{t+j+1} = \frac{1}{\rho} E_t[(1+\gamma)v_{t+j} - s_{t+j+1}] \tag{4.14}$$

for any $j \geq 0$. For derivation, see Appendix D.3.

Finally, by iterating (4.14) from time $t+j+1$ to time t, we can obtain the expected future real debt balance of the government $E_t v_{t+j+1}$. It equals the summation of two terms: the current debt-to-GDP ratio v_t and the expected future flows of government surpluses $\{E_t s_{t+n}\}_{n=1}^{j+1}$.[26] Then, by substituting v_{t+j+1} in the expression of the convenience yield (4.11), we turn Proposition 1 into Proposition 2.

Proposition 2 (Impact of government surpluses on convenience yield) *Given the intertemporal elasticities of substitution of the convenience benefits and consumption utility are both equal to one (i.e., $\sigma_c = \sigma_v = 1$), the convenience yield on government-issued perpetuities is equal to the summation of the current real debt balance and the discounted sum of all expected future government surpluses; that is,*

$$cy_t = \eta_v v_t + \sum_{j=1}^{\infty} \eta_{s,j} E_t s_{t+j}, \tag{4.15}$$

where the constant coefficients are given by

$$\eta_v \equiv -\gamma(1-k)\sum_{j=0}^{\infty} k^j (\frac{1+\gamma}{\rho})^j < 0,$$
$$\eta_{s,j} \equiv \gamma \frac{1-k}{\rho} \sum_{n=j}^{\infty} k^n (\frac{1+\gamma}{\rho})^{n-1} > 0.$$

Proposition 2 indicates that the convenience yield on long-term government bonds is affected by two parts: the current real value of the debt-to-GDP ratio and future government surpluses. Conditional on the current real debt-to-GDP ratio, a rise in the expected future government surplus increases the convenience yield. How does it link to the empirical exercises in Section 3.1.1? It implies that, by controlling for the current debt-to-GDP ratio and other control variables such as proxies for economic fundamentals and monetary policy, long-run inflation expectations must be correlated with the long-term convenience yield through changes in expected future government surpluses. Indeed, in the full-fledged model, inflation expectations are negatively correlated with the expected future flow of government surpluses upon a deficit shock, driven by the fiscal policy rule.

[26] The detailed expression of $E_t v_{t+j+1}$ can be found in Appendix D.3.

Lastly, one may be concerned that the assumption of unit intertemporal elasticity in Proposition 2 is too restrictive. First, it is common to set the values of the IES at one for closed-form results (e.g., Nagel, 2016). A large literature has estimated the IES of consumption with an estimate not far from one, ranging from values smaller than one to larger than one.[27] Nevertheless, to address this concern, we relax this assumption after laying out the full-fledged model, and the numerical exercise shows that a negative shock to government surpluses (a so-called deficit shock) results in a positive response in long-run inflation and inflation expectations, as well as a negative response in the long-term convenience yield. Thus, the association between long-run inflation expectations and the long-term convenience yield via the channel of government surpluses generally holds.

4.4 Firm Production

We model firm production facing monopolistic competition and being subject to sticky prices, as in a standard New Keynesian model. The nominal rigidity induces sluggish adjustments in prices rather than abrupt jumps and its associated Phillips curve links inflation expectations to current inflation.

Consider a continuum of firms indexed by $j \in [0, 1]$. Each firm produces a differentiated good but follows the same production technology:

$$Y_t(j) = A_t H_t(j)^{1-\phi} \equiv f(H_t(j)),$$

where A_t represents productivity and $H_t(j)$ is the labor input. ϕ is a constant, measuring the degree of returns to scale. For simplicity, we abstract from productivity shock and assume A_t is fixed.

The representative household purchases differentiated outputs from firms for final consumption, following a CES aggregator

$$C_t \equiv (\int_0^1 C_t(j)^{1-\frac{1}{\epsilon}} dj)^{\frac{\epsilon}{\epsilon-1}},$$

where ϵ is the elasticity of substitution across differentiated goods and $C_t(j)$ represents the quantity of good j consumed by the household in period t. The optimality problem of the household to maximize final consumption C_t for any given level of expenditures yields a demand function of C_j as

$$C_t(j) = (\frac{P_t(j)}{P_t})^{-\epsilon} C_t \tag{4.16}$$

for all $j \in [0, 1]$, where $P_t(j)$ is the price of good j.

[27] See, among others, Bansal and Yaron (2004), Gruber (2013), and Best et al. (2020).

Firms face the same demand schedule (4.16) and take the aggregate price level P_t and consumption C_t as given. They are owned by households and thus transfer their profits, if any, to households via dividends.

Following Calvo (1983), we assume that each firm could reset its prices only with probability $1-\lambda$ in any given period, independently from the time since its last adjustment. Then in aggregation, a $1-\lambda$ fraction of firms reset their prices in each period, while the rest have to keep their prices unchanged.

Together with the market clearing condition $C_t(j) = Y_t(j)$, the optimal pricing problem of firms yields the New Keynesian Phillips curve[28]

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \psi \widehat{mc}_t, \tag{4.17}$$

where $\widehat{mc}_t$ denotes the log-deviation of the economy's average real marginal cost of production from the steady state. $\psi \equiv \frac{(1-\lambda)(1-\beta\lambda)}{\lambda} \frac{1-\phi}{1-\phi+\phi\epsilon}$ is a constant measuring the degree to which inflation responds to the changes in the real marginal cost.

The average real marginal cost is

$$MC_t = \frac{W_t}{P_t f'(H_t)}. \tag{4.18}$$

By substituting the labor supply condition (4.4) into (4.18) and taking log-linearization, we have

$$\widehat{mc}_t = \Lambda y_t + \tau_t, \tag{4.19}$$

where $\tau_t \equiv \log(1 + \mathcal{T}_t / 1 + \bar{\mathcal{T}})$ denotes the log-deviation of income tax from the steady state. Variable $\Lambda \equiv \frac{1}{\sigma_c} + \frac{1+\phi\sigma_h}{\sigma_h(1-\phi)}$ is a constant, where $\sigma_h \equiv \Psi_h(\bar{H})/(\Psi_{hh}(\bar{H})\bar{H}) > 0$ is the IES of household labor supply.

Thus, by plugging expression (4.19) into (4.17), the Phillips curve can be rewritten as

$$\pi_t = \kappa y_t + \beta E_t \pi_{t+1} + \psi \tau_t, \tag{4.20}$$

where parameter $\kappa \equiv \psi \Lambda$ represents the slope of the Phillips curve. Expression (4.20) shows that the income tax behaves like a cost-push shock, that is, an increase in the tax rate leads to rising inflation and inflation expectations.

4.5 Fiscal and Monetary Policies

The government sets the fiscal policy via a rule of collecting income taxes, while the central bank sets the rule of short-term nominal interest rates. We assume the fiscal and monetary

[28] For derivations, see Galí (2015, chap. 3).

policies follow a simple linear form. For the monetary policy, that is

$$i_t = \theta_{i\pi}\pi_t + \theta_{iy}y_t + u_t^i, \tag{4.21}$$

where u_t^i represents an exogenous monetary policy shock.

For the fiscal policy, the income tax follows

$$\tau_t = \theta_{s\pi}\pi_t + \theta_{sy}y_t + \alpha v_{t-1}, \tag{4.22}$$

where $\theta_{s\pi}$ and θ_{sy} capture the policy response to inflation and output, respectively, and α measures the degree to which the government respects its debt responsibility. The government surplus satisfies[29]

$$s_t = \tau_t + u_t^s, \tag{4.23}$$

where u_t^s represents an exogenous fiscal shock.

Both the monetary policy shock u_t^i and fiscal shock u_t^s follow AR(1) processes:

$$u_{t+1}^i = \rho_i u_t^i + \epsilon_{t+1}^i, \qquad u_{t+1}^s = \rho_s u_t^s + \epsilon_{t+1}^s,$$

where ϵ_{t+1}^i and ϵ_{t+1}^s are i.i.d. innovations with mean zero.

By utilizing the fiscal rule (4.22), together with (4.23), and assuming $\theta_{s\pi} = \theta_{sy} = 0$, we can further rewrite the expression of the convenience yield in Proposition 2 as the summation of two terms: the current real value of the debt-to-GDP ratio and the exogenous surplus shock. That is,

$$cy_t = \phi_v v_t + \sum_{j=1}^{\infty} \phi_{s,j} E_t u_{t+j}^s, \tag{4.24}$$

where parameters $\phi_v < 0$ and $\phi_{s,j} > 0$ for any $j \geq 1$. Detailed proof can be found in Appendix D.3.

Equation (4.24) shows that the coefficients before the surplus shocks are positive, hence a deficit shock (negative u_t^s) reduces the convenience yield. Meanwhile, since the government has to increase future taxes to finance today's deficit, the increased tax rate leads to higher inflation and inflation expectations through the Phillips curve. Thus, by controlling for the current debt-to-GDP ratio as in the empirical exercises in Section 3, long-run inflation expectations are negatively correlated with the long-term convenience yield via the surplus channel.

Given the policy rules, we can characterize the full equilibrium. Details of the system of

[29] Abstract from fiscal shock u_t^s, the real government surplus S_t satisfies $S_t = \mathcal{T}_t(\frac{W_t H_t}{P_t} + \Phi_t) = \mathcal{T}_t Y_t$, where the second equality comes from the firm's balance sheet condition. By log-linearization, we have $s_t = \tau_t$, that is, an increase in income tax is equivalent to increasing government surpluses.

equilibrium conditions can be found in Appendix D.4.

4.6 Quantitative Analyses

In this section, we calibrate the model and demonstrate the effect of fiscal shocks on convenience yield. The quantitative results confirm the negative association between inflation expectations and the convenience yield via the channel of expected future government surpluses and debt supply. In Appendix E, we also consider the effects of monetary policy shocks and cost-push shocks, and the negative association between inflation expectations and the convenience yield still holds.

4.6.1 Numerical Calibration

We calibrate the model at the quarterly frequency. Table 6 reports the calibrated parameter values in the baseline model. The household subjective discount factor β is set to 0.995, implying an annual risk-free real interest rate of 1.92%, as estimated in Jordà et al. (2019) for the US economy since 1870 (excluding world war periods). The IES of household consumption σ_c is set to the standard value of 0.5 in the literature (e.g., Arellano, 2008; Cochrane, 2022*a*).[30] We further set the IES of convenience benefits equal to that of household consumption ($\sigma_v = \sigma_c$), following Nagel (2016).

The discount factor ρ of long-term government bonds is set to 0.987, matching the 10-year Tbond return of 5.29% in the empirical sample (see Table 1). Similarly, the coupon payment decay rate k is 0.975, so that the duration of long-term government bonds is 10 years. The steady-state ratio of marginal benefits of convenience to consumption γ satisfies $\gamma = (\bar{R}^D - \bar{R}^B)/\bar{R}^B = \bar{cy}/\bar{R}^B$, and thus it is set to 0.0045 to match the steady-state convenience yield of 0.48%, measured by the Refcorp-Tbond spread in Table 1. We target the Refcorp-Tbond spread because it is free from default risk. The slope of the Phillips curve is set to $\kappa = 0.31$, following the estimation in Barnichon and Mesters (2020). As a common practice in the literature, we also set $\lambda = 0.75$, implying an average duration of four quarters for a subsequent price adjustment, with a constant-returns-to-scale technology of production $\phi = 0$.

For the parameters characterizing fiscal and monetary policies, as well as the process of fiscal shock, we follow the estimates of a monetary-led policy regime for the U.S. economy estimated in Bianchi and Melosi (2017). The tax response to real public debt α is set to be 0.07. The parameters in the monetary policy rule (4.21) are set to $\theta_{i\pi} = 1.6$ and $\theta_{iy} = 0.5$

[30] The long-run risk literature estimates a larger value for the IES by separating the IES with relative risk aversion (e.g., Schorfheide, Song and Yaron, 2017). Nevertheless, our results regarding the convenience yield are robust to different values of the IES.

and the parameters in the fiscal policy rule (4.22) are set to $\theta_{s\pi} = 0$ and $\theta_{sy} = 0.28$. The shock process on surplus is set to $\rho_s = 0.96$ and $\sigma_s = 0.005$.

Table 6: Calibrated Parameters

Variable	Description	Value
β	Subjective discount factor	0.995
σ	IES of household consumption	0.5
ψ	IES of convenience benefits	0.5
ρ	One minus steady-state long-term government bond return	0.987
k	Coupon payment decay rate	0.975
γ	Steady-state ratio of marginal benefits of convenience to consumption	0.0045
κ	Slope of the Phillips curve	0.31
λ	An average duration of 4 quarters in adjacent price adjustments	0.75
Fiscal Policy Parameters:		
α	Surplus response to real government debt	0.07
$\theta_{s\pi}$	Surplus response to inflation	0
θ_{sy}	Surplus response to output	0.28
ρ_s	Persistence of surplus shock	0.96
σ_s	Std of surplus shock	0.005
Monetary Policy Parameters:		
$\theta_{i\pi}$	Policy (interest) rate response to inflation	1.6
θ_{iy}	Policy (interest) rate response to output	0.5

Notes. IES denotes the intertemporal elasticity of substitution.

4.6.2 Impulse Responses to a Fiscal Deficit Shock

Figure 7 shows the impulse response of various endogenous variables to a one percentage point deficit shock, $\epsilon_1^s = -1\%$, occurring in period $t = 1$. The left-top panel plots the exogenous path of surplus shock u_t^s, which is highly persistent since ρ_s is close to one.

Following a deficit shock, the real public debt balance v_t initially drops and then soon climbs to a positive level because of the persistent negative surplus shock. The initial drop in debt balance is caused by the temporary decline in the nominal return on government bonds i_t^B.[31] This drop in turn results in a lower tax rate as required by the fiscal rule, leading to a temporary mild deflation and output improvement.[32] After the initial periods, the real

[31] The initial decline of i_t^B comes from a lower market price of all outstanding debt on the impact of the shock. Intuitively, after the unexpected deficit shock occurs, bond buyers demand higher returns on those long-term government bonds going forward, which implies a downward jump in market price when the deficit shock is realized. Cochrane (2022*b*) provides a detailed explanation for this feature of pricing long-term bonds.

[32] This temporary effect relies on the calibration of bond duration. When the bond duration is relatively

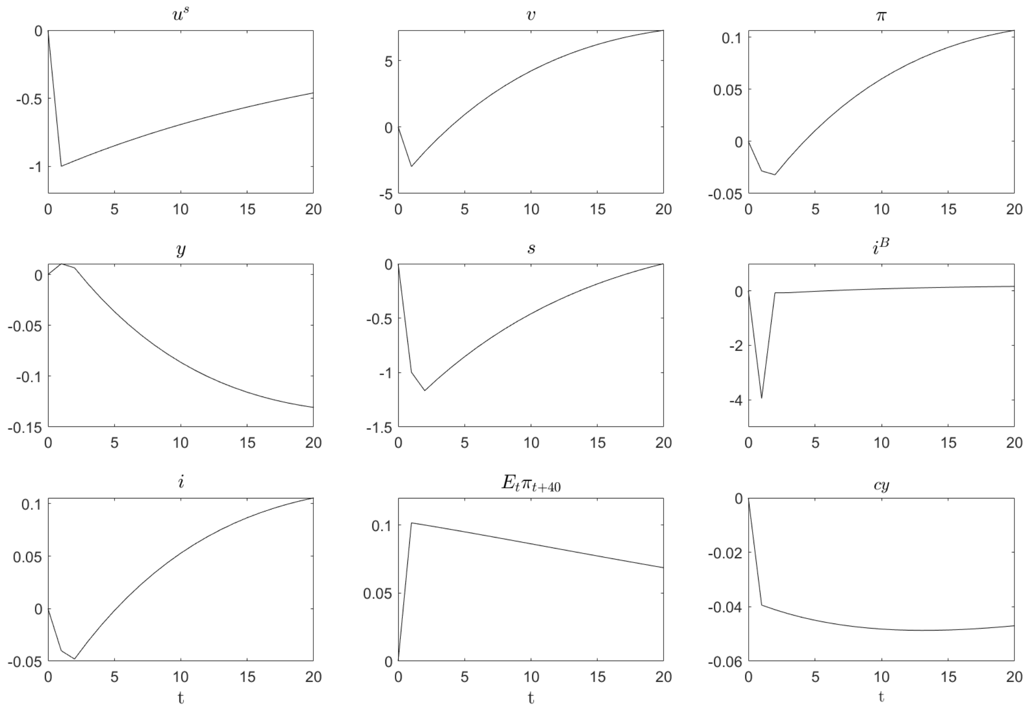

Figure 7: Impulse Responses to a Fiscal Deficit Shock

Notes. The figure plots the impulse response of various endogenous variables to a deficit shock, $\epsilon_1^s = -1\%$, occurring in period $t = 1$. The top row shows the exogenous shock process u_t^s, and the responses of the real value of government debt v_t and inflation π_t. The middle row shows the responses of output y_t, government surplus s_t, and short-term nominal return on government-issued bond i_t^B. The bottom row shows the responses of nominal interest rate i_t, long-run inflation expectations $E_t\pi_{t+40}$, and the long-term convenience yield cy_t. The x-axis is in quarters. All numbers are reported in percentages.

debt reverts to positive values because the direct effect from deficit shock dominates in the medium to long run.

As the real debt balance increases, the government has to increase the income tax to finance its accumulated deficit, following the fiscal policy rule. This pushes up inflation as implied by the Phillips curve. The central bank reacts to the increased inflation by raising the nominal interest rate, leading to output contraction.

The bottom panels in Figure 7 further show the impulse responses of long-run inflation expectation $E_t\pi_{t+40}$ (which corresponds to the 10-year inflation expectation in the empirical analysis as our calibration is done at the quarterly frequency) and the convenience yield on long-term government bonds cy_t. The higher future real debt results in higher future income tax rates and lower convenience benefits of government bonds. The former implies higher

smaller, e.g., one or two quarters, inflation and real debt balance immediately increase and output decreases when the shock occurs. The impulse response for such two cases can be found in Appendix E.

inflation expectations, while the latter yields a lower convenience yield.

Thus, a higher inflation expectation reflects more future government debt, which is associated with a lower convenience yield for long-term assets. Conditional on shocks on fundamentals and the federal funds rate being controlled for, as in the empirical exercises, the theoretical model confirms the negative correlation between long-run inflation expectations and the long-term convenience yield in facing fiscal shocks through the channel of future debt issuance.

Finally, when the increase in income tax dominates the effect from the initial negative surplus shock, the surplus will pick up and become positive, which in turn starts to reduce the debt balance. Eventually, in the long run, all the endogenous variables converge back to the steady state. Thus, the responses of both government surplus and debt balance feature an S-shape (Figure 7 only plots a few beginning periods for illustration).

4.6.3 Model Prediction on the Empirical Findings

We next examine the model prediction by conducting the same empirical regressions in Section 3.1 using model-simulated data. The model-simulated correlations between the convenience yield, inflation expectations, and debt growth are quantitatively similar to the empirical counterparts, even though we are not targeting those moments explicitly.

To investigate the negative correlation between the convenience yield and inflation expectations, we run the following regression using model-simulated data and get

$$\Delta cy_t = \underset{(0.001)}{-0.465} \Delta E_t \pi_t^{t+10} + \epsilon_t, \tag{4.25}$$

where cy_t represents the convenience yield calibrated to the Refcorp-Tbond spread with an average 10-year duration and $E_t \pi_t^{t+10}$ represents the average inflation expectations over the 10 years following period t.[33] Δ indicates taking first differences. Newey and West (1994) standard errors are reported in parentheses.

The estimated coefficient -0.465 from model simulations falls quantitatively within the range of its empirically estimated counterparts, which varies from -0.09 to -0.47. This range comes from different methods of decomposing the data from its underlying trend in the empirical specification; see Table 3 and Table B.5 (in Appendix B.3) for more details. Although there are other shocks in the data, the correlation between the convenience yield and inflation expectations that we are matching is a conditional relationship, after controlling for other shocks (proxied by VIX, output growth, etc.). Hence, even though we only consider

[33] In the quarterly model, $E_t \pi_t^{t+10}$ is calculated by the average of inflation expectations from period $t+1$ to $t+40$. In the regression exercise, we simulate the model for $10,000$ periods and burn the first half of the sample.

fiscal shocks in the model, the model simulated correlation is comparable to the reduced-form estimates in Section 3.

We also run a similar regression for the relationship between the ex-post growth rate of the debt-to-GDP ratio and long-run inflation expectations, which yields

$$\Delta g_t^{t+10}(v_t) = \underset{(0.26)}{7.88} \Delta E_t \pi_t^{t+10} + \epsilon_t, \tag{4.26}$$

where $g_t^{t+10}(v_t)$ denotes the ex post growth rate of debt-to-GDP ratio v_t over the following 10 years since period t. In a quarterly model, $g_t^{t+10}(v_t) \equiv v_{t+40} - v_t$. The estimated coefficient 7.88 also has a quantitatively similar size to our empirical estimate, which is about 6.87, as documented in Table 4.

To sum up, though not perfectly, the model generates similar responses to the estimates of interest to us in the empirical analyses of Section 3.1.

4.6.4 A Feedback Loop between the Convenience Yield and Public Debt Supply

Thus far, our model has laid out the main mechanism for the two empirical observations. First, upon a government deficit shock, the government both increases its future borrowings and partially inflates away its deficit, and in consequence, an increase in inflation expectation $E\pi$ is a proxy for more future debt supply v. Second, a higher future debt supply v reduces the convenience yield cy, thus implying a negative association between inflation expectations and the convenience yield.

The feedback loop between the convenience yield and government debt supply amplifies the effect of deficit shocks on the government's funding cost and debt dynamics. Specifically, following the aforementioned mechanism, a lower convenience yield implies higher borrowing costs for the government, and thus the government has to issue more debt in the future to finance its deficit. More future debt supply in turn results in an even lower convenience yield today. This feedback loop implies that the government faces a less elastic demand curve of public debt than otherwise, hence any effects on funding costs and debt accumulation are amplified in all horizons.

To illustrate this feedback channel, we consider a counterfactual scenario in which government bonds provide no convenience benefits, that is, $\gamma = 0$. Figure 8 shows the impulse response to a one percentage point deficit shock, $\epsilon_1^s = -1\%$, in cases with and without convenience benefits (dashed and solid lines), respectively. Panel (a) plots the interest rate on long-term government bond i_t^B, panel (b) plots the real public debt v_t, and panel (c) plots the convenience yield.

Figure 8 shows that convenience benefits lead to lower borrowing costs in government bonds and less debt accumulation in the short term compared with the other case. However,

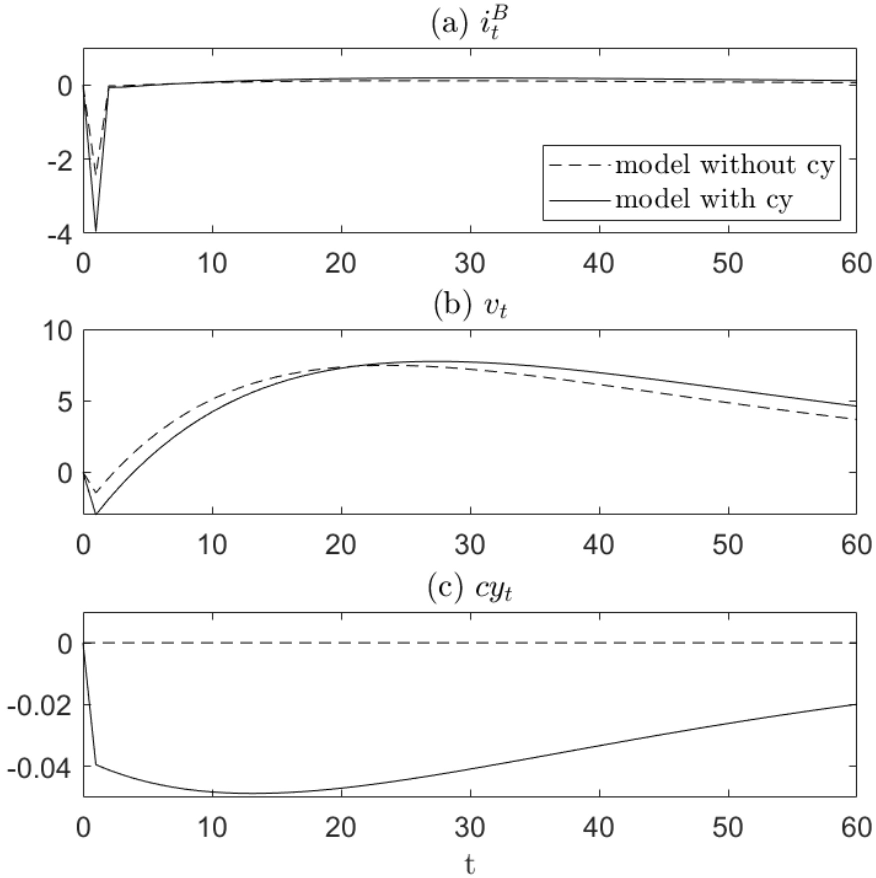

Figure 8: Impulse Responses to a Fiscal Deficit Shock

Notes. This figure plots the impulse response to a one percentage point deficit shock, $\epsilon_1^s = -1\%$. The dashed line represents the case without convenience benefits of government bonds, whereas the solid line represents the case with convenience benefits. Panel (a) plots the interest rate on long-term government bond i_t^B, panel (b) plots the real public debt v_t, and panel (c) plots the convenience yield cy_t. In the case of no convenience benefits, all model parameters remain the same except for $\gamma = 0$. The x-axis is in quarters. All numbers are reported in percentages.

in the long run, the feedback loop between convenience yield and debt supply leads to higher borrowing costs and more debt supply. In other words, the effects of the shock are amplified in all horizons. Moreover, the convenience yield reduces the short-term debt burden of the government at the cost of increasing the long-term debt burden.

As a numerical example, we compare the change in real value of coupon payment B_{t-1}/P_t between the case with convenience yield and the case without. In response to a one percentage point deficit shock, the increase in coupon payment at the end of 10 years is 10.51% in the case with convenience yield, whereas the increase would only be 7.96% in the case without convenience yield. Even though the convenience yield reduces the cost of borrowing on average, it increases the sensitivity of debt balance to deficit shocks.

5 Conclusion

In this paper, we present two new empirical findings and provide a New Keynesian model to rationalize both findings quantitatively. We first show that the convenience yield on long-term Treasuries is negatively correlated with long-run inflation expectations, controlling for fundamentals, the current debt-to-GDP ratio, and short-term interest rates. Second, we demonstrate that higher inflation expectations today are associated with higher government debt-to-GDP growth going forward.

To explain these findings, we introduce convenience yields into an otherwise standard general equilibrium model with sticky prices and income taxes. The model yields three insights. First, the convenience yield on long-term government debt is the discounted sum of all future convenience services provided. Hence, the convenience yield is negatively correlated with the future debt supply. Second, a fiscal deficit shock is partially funded by higher future debt issuance and partially by inflating away the debt burden. In consequence, inflation is positively correlated with the future debt supply, which in turn is negatively associated with the convenience yield. Third, the negative association between the convenience yield and the future debt supply features a feedback loop. The calibrated model can match the empirical correlations reasonably well among variables of interest.

Different from existing research, we highlight the dynamic feature embedded in the convenience yield relating to inflation expectations and government fiscal policies. In particular, we show that fiscal policy could also affect government borrowing costs through the channel of convenience yields. It would also be interesting to formally quantify the size of our channel, but doing so will require seriously estimating the policy rules, which would be subject to the issue of observational equivalence, as argued in Cochrane (2022*a*). We thus leave this topic for future study when evaluating different policies. Our findings also have important implications for the interaction between monetary policy and the government's fiscal space. When the monetary authority manages inflation expectations, the action will have non-trivial consequences on the government's cost of borrowing. One natural question is to think about optimal policy designs to minimize government borrowing costs in this environment. We leave these important questions for future study.

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Online Appendix for "Convenience Yield, Inflation Expectations, and Public Debt Growth"

A Data Sources and Variable Construction

A.1 Daily Data

Below we list the sources and construction of the major variables used in our empirical investigation. Most of the variables are from daily market prices or indices. Unless otherwise noted, data represent the close value of each day. We aggregate them to the monthly level as follows. For variables in levels, we take the average of daily observations over each month; for first differences, we use the differences in the last trading days in each month.

- Inflation swap rates: We obtain inflation swap rates from Bloomberg under the mnemonics "USSWIT[Y] Curncy," where [Y] is the maturity for inflation swap rates. We use the 10-year inflation swap rate in most of the empirical investigation unless otherwise specified.
- Treasury yields: We use market yields on U.S. Treasury securities at [X]-year constant maturity from FRED with tickers DGS[X], where [X] represents maturity.
- AAA yield: We use Moody's Seasoned Aaa Corporate Bond Yield, retrieved from FRED under ticker DAAA. This index is constructed based on bonds with the AAA rating and maturities of 20 years and above. In the baseline, we use the 20-year Treasury yield as the benchmark yield to compute the spread, and in Appendix B.1 we also use 10-year and 30-year Treasuries as robustness checks.
- Refcorp bond yield: To match the horizon of inflation expectations, we use the yield of the 10-year Refcorp bond, retrieved from Bloomberg fair value curves under the mnemonics "C09110Y Index."
- Bond spread net of CDS: we get the transaction-level bond spread from TRACE Enhanced historical data and bond maturity and rating information from WRDS. Daily credit default swap rates are obtained from Markit.
- Cleveland Fed inflation expectations: In several specifications, we use the inflation expectations estimated by the Cleveland Fed to have longer sequences as early as 1983. Estimates are retrieved from `https://www.clevelandfed.org/our-research/indicators-and-data/inflation-expectations.aspx`. See Haubrich, Pennacchi and Ritchken (2012) for details of the underlying model and estimation. In their dataset,

variables are estimated at the beginning of each month, whereas our other variables are all at the end of each month. Therefore, we shift one month backward for all variables from the Cleveland Fed to be consistent with other variables.

- Public debt supply: We measure the public debt supply using the historical data from the website of the Federal Reserve Bank of Dallas, available at https://www.dallasfed.org/research/econdata/govdebt#tab3. We use the sequences for privately held debt all the time. We use the market value as the benchmark for forecasting regressions and use the par value measure for controls in Section 3.1.1 and for robustness checks in Section 3.1.2. The public debt supply is normalized by either nominal GDP or CPI to reflect the real debt burden.

Figure A.1 plots the raw data of the convenience yield, measured by AAA-Tbond and the 10-year inflation expectations from the Fed.

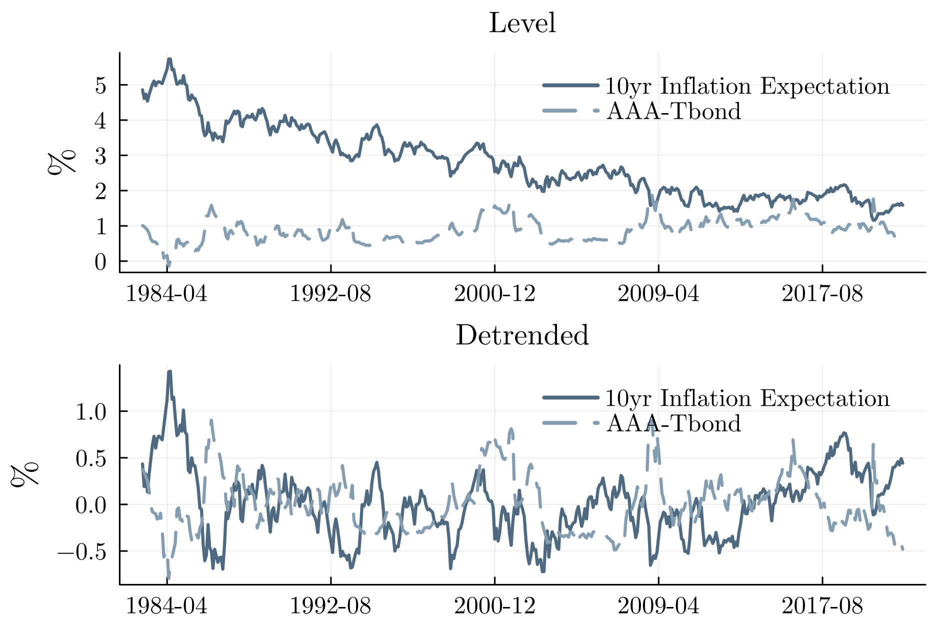

Figure A.1: Negative Correlation between Inflation Expectations and AAA Spread

Notes. AAA-Tbond is the difference between Moody's Seasoned Aaa corporate bond yield and 20-year T-bond yield. Moody's Seasoned Aaa corporate bond yield is based on bonds with maturities of 20 years and above. The 10-year inflation expectation series is from the Federal Reserve Bank of Cleveland.

Below we list the sources and construction of control variables used across specifications:

- Inflation risk premium: When inflation is stochastic, the inflation swap rate also contains the inflation risk premium. Therefore, we also control for those risk premia estimated by the Cleveland Fed. The data sequences are obtained from https://www.clevelandfed.org/our-research/indicators-and-data/inflation-expectations.aspx.

- Industrial production (or GDP at the quarterly level): We use the Industrial Production Index (INDPRO from FRED) as a proxy for real outputs at the monthly level. The industrial production index is an economic indicator that measures real output for all facilities located in the U.S. manufacturing and mining sector and electric and gas utilities (excluding those in U.S. territories). At the quarterly level, we use the real GDP from FRED (GDPC1).

- VIX: VIX is the CBOE Volatility Index, which measures market expectations of near-term volatility conveyed by stock index option prices. It is a common measure in the literature for the financial market sentiment. We retrieve the VIX at the daily level from FRED under ticker VIXCLS.

- Effective federal funds rate: Following Nagel (2016), we use the effective federal funds rate to control for the opportunity cost of money. Data are from FRED under ticker DFF.

- Recession indicator: We construct a recession indicator that equals one during recessions, dated by the NBER Business Cycle Dating Committee, and zero otherwise.

Table A.1: Summary Statistics for Control Variables

		Level				First Diff.			
Variable	**Time Period**	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**	**Min**	**Max**
Inflation risk premium	83/01-21/09	0.41	0.07	-0.12	0.59	0.0	0.05	-0.55	0.18
log(Ind. Prod.)	83/01-21/09	4.4	0.23	3.86	4.65	0.0	0.01	-0.15	0.06
VIX	90/01-21/09	19.49	7.69	10.13	62.67	-0.01	4.59	-19.39	21.27

Notes. This table reports summary statistics of control variables at the monthly frequency. Variables are measured in percentage points. Statistics in both levels and first differences are reported separately. For each variable, we report its available time period in our sample.

In Section B, we check the robustness of our results using other measures of convenience yields. We list the data sources here and defer the discussion to later sections.

- AAA Option-Adjusted Spread (OAS): We use the AAA-OAS as an alternative measure of the convenience yield to address the embedded callable option in corporate bonds. The sequence is constructed by ICE BofA and retrieved via FRED (ticker BAMLC0A1CAAA). From the description on FRED: "This subset includes all securities with a given investment grade rating AAA. The ICE BofA OASs are the calculated spreads between a computed OAS index of all bonds in a given rating category and a spot Treasury curve. An OAS index is constructed using each constituent bond's OAS, weighted by market capitalization."

- Near-10-year AAA yield: As a measure of the 10-year convenience yield, we also construct a spread between a near-10-year AAA yield index and 10-year Treasury yield. The near-10-year AAA yield index is constructed as the average of the 7- to 10-year AAA average yield and the 10- to 15-year AAA average yield. Both indices are from ICE indices and retrieved via Bloomberg under the mnemonics "C4A1 index" and "C8A1 index."

B Additional Tests for the Correlation between Inflation Expectations and the Convenience Yield

B.1 Term Structure and Embedded Callable Options

A plausible but uninteresting explanation for the negative comovement between the AAA-Tbond spread and inflation expectations is that it is a result of the different payoff structures of AAA corporate bonds and Treasuries. They indeed have several subtle but important differences. First, the AAA yield index from Moody's is based on bonds with maturities of 20 years and above, whereas the benchmark Treasury is chosen at a 20-year constant maturity to match the horizon for inflation expectations. One may be concerned that different term structures are driving our results. Second, corporate bonds may have an embedded callable option that gives the issuer the right to "call" or buy back its existing bonds prior to maturity when interest rates decline, whereas most of the U.S. Treasuries in our sample are non-callable.[34] The callable option may also explain the differential levels of exposures to inflation expectations of AAA bonds and Treasuries.

To address these concerns, we use the AAA-Treasury option-adjusted spread (OAS) as the dependent variable. The AAA-OASs adjust for the value of the embedded callable option in AAA corporate bonds. In computing the OAS, the cash flow of bonds is discounted using the whole Treasury yield curve plus a constant spread, so the concern about term structure is also addressed. The data source and description can be found in Appendix A.1. We regress AAA-OAS on inflation expectations, and the results are presented in Column (1) of Table B.2. If anything, the coefficient for option-adjusted spread is stronger than our baseline estimate (reproduced in Column (3)).

[34] Certain bonds issued before 1985 were also embedded with a callable option. See https://www.treasurydirect.gov/indiv/research/indepth/tbonds/res_tbond_call.htm

Column (2) and Column (4) of Table B.2 use 10-year and 30-year Treasuries as the benchmark to compute the AAA-Treasury spread. Compared to the OAS, they serve as model-free robustness checks to address the concern about term structure. The coefficients are comparable to the baseline results in Column (3).

Table B.2: Option-Adjusted Spread and Different Maturities

	Δ AAA-OAS	Δ AAA-10yTbond	Δ AAA-Tbond	Δ AAA-30yTbond	Δ 10y AAA-Tbond
	(1)	(2)	(3)	(4)	(5)
$\Delta E\pi$	-0.34	-0.20	-0.23	-0.21	-0.22
	(0.09)	(0.07)	(0.08)	(0.05)	(0.08)
Measure	Swap	Swap	Swap	Swap	Swap
Control	true	true	true	true	true
N	206	206	206	206	205
R^2	0.37	0.29	0.34	0.42	0.24

Notes. Newey-West standard errors (bandwidths optimally chosen following Newey and West (1994)) are shown in parentheses. Δ indicates first differences at the monthly level. In all columns, we use the inflation swap rate as the measure for inflation expectations. In Column (1) the dependent variable is the AAA option-adjusted spread from ICE BofA. Column (4) in Table 2 uses the 10-year Treasury as the benchmark to compute the AAA-Treasury spread. Columns (3) and (4) use 20-year and 30-year Treasuries as the benchmark, respectively. Across all columns, the control variables include industrial production, the VIX, the effective federal funds rate, contemporaneous public debt supply, the inflation risk premium, and the recession indicator. Controls are also first differenced (or in growth rates) whenever applicable.

B.2 The Bond-CDS Spread and Inflation Expectations

Table B.3: Summary Statistics for Bond-CDS spreads

		Level				**First Diff.**			
Variable	**Time Period**	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**	**Min**	**Max**
AA bond-CDS spread	05Q2-19Q2	0.27	0.34	-0.09	1.54	-0.0	0.16	-0.57	0.61
BBB-AA basis-spread	05Q2-19Q2	0.35	0.42	0.04	2.28	0.0	0.26	-0.98	1.39

Notes. This table reports summary statistics for bond-CDS spreads at the quarterly frequency. Variables are measured in percentage points. Statistics in both levels and first differences are reported separately. For each variable, we report its available time period in our sample. See equations (B.1) and (B.2) for the definitions.

A credit default swap, or CDS, is a swap contract that allows an investor to "swap" the default risk of some reference assets, such as corporate bonds, with another investor. The buyer of a CDS pays an ongoing premium to the seller until the maturity date of the contract in exchange for the insurance against credit defaulting. By buying a corporate bond and a maturity-matched CDS, an investor can almost replicate the cash flow from a risk-free bond, such as the U.S. Treasury with the same maturity.[35]

[35] In implementation, the synthetic bond and a Treasury security still have subtle differences. For example, the CDS may be subject to counterparty risks.

Using CDSs, we are able to take the default component out of corporate bond spreads. For each bond i, we construct its *bond-CDS spread*, defined as the bond-Treasury spread net of the CDS spread; that is,

$$\text{bond-CDS spread}_{i,t} = (y_{i,t} - y_{Treasury,t}) - CDS_{i,t}, \tag{B.1}$$

where the maturities of both the CDS and the Treasury are matched to bond i. Notice that our definition of the bond-CDS spread is the opposite of the conventional *CDS-bond basis* commonly used in the literature. By our definition, the sign of the bond-CDS spread is consistent with our other measures of the convenience yield: a positive number indicates that Treasuries have a higher price (a lower spread) relative to the other assets.

In a frictionless world without special demand for convenience services and liquidity concerns, the bond-Treasury spread should be zero by the no-arbitrage condition. Nevertheless, positive bond-CDS spreads (or a negative CDS basis) are a well-documented empirical fact in the literature, suggesting that Treasuries carry a premium relative to the synthetic risk-free bond.

The source of the premium may be multifaceted. It can be from the illiquidity of corporate bonds (Longstaff, Mithal and Neis, 2005; Li and Yu, 2021) or the relative "safety premium" of Treasuries above that of corporate bonds. For now, we bundle different sources of the premium together as the convenience yield of Treasuries relative to corporate bonds and refer readers to the discussion of the exact feature of safe assets to Section 4.[36]

High-quality corporate bonds can also carry a safety premium relative to risky bonds. This observation motivates Krishnamurthy and Vissing-Jorgensen (2012) to use the BBB-AAA spread as an alternative measure of the safety premium. Mota (2021) further refines this measure by isolating the non-credit-risk component using CDSs. Following this literature, we construct the BBB-AA basis spread as the difference between the BBB bond-CDS spread and the AA bond-CDS spread. Notice that the Treasury yield is canceled out in the BBB-AA basis-spread (see equation (B.2)). Therefore, the BBB-AA basis spread is a measure of the premium on AA bonds relative to BBB bonds, independent of Treasury yields:

$$\begin{aligned} \text{BBB-AA basis-spread}_t &= \text{bond-CDS spread}_{BBB,t} - \text{bond-CDS spread}_{AA,t} \\ &= (y_{BBB,t} - CDS_{BBB,t}) - (y_{AA,t} - CDS_{AA,t}) \end{aligned} \tag{B.2}$$

If AA bond yields indeed contain a premium relative to BBB bond yields, the BBB-AA basis spread would be positive. Moreover, if the premia share a similar micro-foundation

[36]Technically, the positive spreads can also originate from the imperfect replication of risk-free bonds due to the frictions on the CDS market, such as counterparty risks, the imperfect matching of the payoff structure, and so on. However, these differences are in general small (e.g., Arora, Gandhi and Longstaff (2012) show the counterparty risk is "vanishingly small"), so we do not take them into account in our analysis.

with the U.S. Treasury premium, then the BBB-AA basis spread should also comove with inflation expectations in the same way as the convenience yield on Treasuries does. This is exactly what we find.

Table B.3 reports summary statistics for bond-CDS spreads. Table B.4 presents regressions of bond-CDS spreads on inflation expectations at the quarterly level. We switch to a quarterly frequency to include more matched bond-CDS pairs. The sample is from 2005Q2 to 2019Q2. Inflation expectations are measured by the 10-year inflation swap rates. In Column (1), we regress the AA bond-CDS spread on inflation expectations without covariates. The point estimate is negative, consistent with our baseline results. It shows that a one percentage point increase in inflation expectations is associated with a 24 bps decrease in the AA bond-CDS spread, or 72% of its standard deviation. The effect is significant both economically and statistically. In Column (2), we add controls to the regression. The point estimate is slightly smaller, reduced to 17 bps, but still highly significant ($p < 0.05$).

Columns (3) and (4) investigate the relationship between the BBB-AA basis spread and inflation expectations. As discussed above, it captures the relative premium of AA to BBB. Column (3) shows the result of the univariate regression. The coefficient shows that a one percentage point increase in inflation expectations translates into a 33 bps (or 0.79 s.d.) decrease in the BBB-AA basis spread. In other words, with a one percentage point increase in inflation expectations, AA bonds appreciate relative to BBB bonds by 33 bps. The coefficient is statistically significant ($p < 0.01$). In Column (4), we add usual controls to the regression, which reduces the point estimate to -8 bps. The point estimate still points to the correct direction, though it is no longer statistically significant.

B.3 Hodrick-Prescott Filter

Throughout the main text, we use time-differencing to remove potential spuriously correlated stochastic trends. We show that our results are robust to an alternative approach also commonly used in the literature: the Hodrick-Prescott (HP) filter.

For each variable, we run the HP filter with the suggested monthly smoothing parameter 129,600. We then take the deviation of each variable from its low-frequency trends as the regressands and regressors. Table B.5 reproduces our baseline results using HP-filtered sequences. For notation convenience, we continue to use Δ (only in this table) to denote the deviation from filtered trends. Across different specifications, the point estimates are negative and comparable to the baseline results in Table 2.

Table B.4: Bond-CDS Spreads and Inflation Expectations

	Δ AA-Tbond		Δ BBB-AA	
	(1)	(2)	(3)	(4)
$\Delta E\pi$	-0.24	-0.17	-0.33	-0.08
	(0.05)	(0.05)	(0.12)	(0.05)
Measure	Swap	Swap	Swap	Swap
Controls	False	True	False	True
Sample	05Q2-19Q2	05Q2-19Q2	05Q2-19Q2	05Q2-19Q2
N	56	56	56	56
R^2	0.20	0.53	0.15	0.48

Notes. Newey-West standard errors (bandwidths optimally chosen following Newey and West (1994)) are shown in parentheses. Δ indicates first differences at the quarterly level. The sample is from 2005Q1 to 2019Q2. In Columns (1)-(2), the dependent variable is the average bond-CDS spread for AA bonds. In Columns (3)-(4), the dependent variable is the average BBB-AA basis-spread, defined in Eq. (B.2). Control variables for Columns (2) and (4) include quarterly fixed effects, a recession indicator, and first differences in the (log of) real GDP, the VIX, debt-to-GDP ratio, the effective federal funds rate, and the inflation risk premium.

Table B.5: The Convenience Yield and Inflation Expectations (HP Filtered)

	Δ AAA-Tbond		Δ Refcorp - TBond	
	(1)	(2)	(3)	(4)
$\Delta E\pi$	-0.60	-0.26	-0.47	-0.21
	(0.10)	(0.06)	(0.17)	(0.08)
Measure	Swap	Swap	Swap	Swap
Controls	False	True	False	True
Sample	04/07-21/09	04/07-21/09	04/07-21/09	04/07-21/09
N	207	207	207	207
R^2	0.51	0.79	0.35	0.65

Notes. Newey-West standard errors (bandwidths optimally chosen following Newey and West (1994)) are shown in parentheses. Δ indicates the deviation from the trends filtered by the HP filter. The smoothing parameter is set at the suggested monthly level of 129,600. In Columns (1)-(2), the dependent variables are changes in the AAA-Tbond spread. In Columns (3)-(4), the dependent variables are the Refcorp-Tbond spread. Control variables in Columns (2) and (4) include industrial production, the VIX, the effective federal funds rate, contemporaneous public debt supply, the inflation risk premium, and the recession indicator. Variables are filtered using the HP filter when applicable.

B.4 Short-Term Liquidity Premia

As a placebo test, the short-term liquidity premium on Treasuries is not sensitive to inflation expectations. This is consistent with our mechanism discussed in Section 4, which hinges on the supply of Treasuries over the maturity of long-term Treasuries. Short-term Treasuries mature before any meaningful changes to the public debt level and therefore are not exposed to this channel.

We measure the short-term liquidity premium by the 3-month Repo-Tbill spread, introduced and motivated by Nagel (2016). The repo rate is the interest rate for a three-month term interbank loan that is collateralized with a portfolio of U.S. Treasury securities, so the repo rate is virtually free of any credit risk component. It is illiquid because the investment is locked during the three-month term. On the other hand, three-month T-bills are among the most liquid Treasury securities. The spreads between the illiquid rate and the T-bill yield capture the convenience service provided by the T-bills.[37]

Table B.6 presents the regression results of short-term liquidity premia on inflation expectations in the same specification as in our baseline. To match the horizon of the short-term liquidity premium, we use one-year inflation expectations as the predictor. In Columns (1)-(2), inflation expectations are measured with one-year inflation swap rates, and in Columns (3)-(4), they are measured by the Cleveland Fed estimates. Columns (1) and (3) present the results from univariate regressions, and in Columns (2) and (4), we add in the usual controls. Notably, first differences in the federal funds rate, the VIX, and debt supply are controlled, as emphasized in Krishnamurthy and Vissing-Jorgensen (2012) and Nagel (2016). Across all specifications, the coefficients are small and insignificant.

C Additional Tests for the Predictability of Future Debt Growth

C.1 Alternative Measures for Debt Growth

Because the market value of public debt contains information on discount rates, the growth rate of the market value may reflect the change in discount rates, including changes in inflation expectations and the convenience yield. Therefore, regressing the growth of the market value on inflation expectations may capture the autocorrelation of inflation expectations. Moreover, as GDP is the denominator in our baseline measure, the forecastability can also capture the cyclicality of the GDP. Both concerns are quantitatively insignificant over the

[37] Sources and the construction of dependent variables are listed in Appendix A. Interested readers are referred to Nagel (2016) for a detailed discussion.

Table B.6: Short-term Liquidity Premia and Inflation Expectations

	Δ (Repo - Tbill)			
	(1)	(2)	(3)	(4)
$\Delta E\pi$ (1yr)	0.02	0.01	0.03	0.04
	(0.03)	(0.02)	(0.02)	(0.02)
Measure	Swap	Swap	Fed	Fed
Controls	False	True	False	True
Sample	04/08-21/09	04/08-21/09	91/06-21/09	91/06-21/09
N	206	206	364	364
R^2	0.01	0.13	0.01	0.04

Notes. Newey-West standard errors (bandwidths optimally chosen following Newey and West (1994)) are shown in parentheses. Δ indicates first differences at the monthly level. In Columns (1)-(2), we use the one-year inflation swap rates, and in Columns (3)-(4), we use the Cleveland Fed inflation expectations. In all columns, control variables for Columns (2) and (4) include industrial production, the VIX, the effective federal funds rate, contemporaneous public debt supply, the inflation risk premium, and the recession indicator. Controls are also first differenced (or in growth rates) whenever applicable.

long horizon, as we have shown in Section 3.1.2. Here we formally address this concern by using an alternative measure: the real growth rate of the par value of privately held debt, constructed as the growth rate of the nominal par value minus the realized inflation over the same period.

Table C.7 presents the same regression as in Section 3.1.2 but with the real par value growth as the predicted variable. The coefficients are significant at 1% level and are comparable with those in Table 4.

C.2 Bias-Adjusted Estimates

The literature has found that predictive regressions may have small sample biases when innovations to the predictor and the predicted variables are correlated (Stambaugh, 1999; Boudoukh, Israel and Richardson, 2021). Using the formula provided by Boudoukh, Israel and Richardson (2021), we compute the biases in the small sample under the null hypothesis that $\beta_h = 0$ and plot the bias-adjusted coefficients in Figure C.2. The biases are relatively small relative to the point estimates. Our results are still robust after adjusting for biases.

C.3 The Predictive Power of the Realized Inflation

Since inflation expectations could be highly correlated with realized inflation (the correlation in our sample is 0.33), we can use the realized inflation as a proxy for inflation expectations to

Table C.7: Inflation Expectations Predict Future Real Debt Growth

	1yr	5yr	10yr	1yr	5yr	10yr
	(1)	(2)	(3)	(4)	(5)	(6)
$E\pi$	4.81	31.32	51.34	8.77	48.69	55.10
	(2.08)	(8.22)	(20.83)	(2.31)	(10.85)	(17.20)
Sample	81Q4-20Q3	81Q4-16Q3	81Q4-11Q3	81Q4-20Q3	81Q4-16Q3	81Q4-11Q3
Control	False	False	False	True	True	True
N	156	140	120	156	140	120
R^2	0.06	0.16	0.15	0.22	0.52	0.81

Notes. Newey-West standard errors (bandwidths optimally chosen following Newey and West (1994)) are shown in parentheses. The predicted variables are future real debt cumulative growth rates, where debt is measured by the *par value* of privately held public debt, normalized by CPI. Both variables are linearly detrended. Horizons are indicated in each column. The predictor variable is 10-year inflation expectations estimated by the Cleveland Fed (Haubrich, Pennacchi and Ritchken, 2012). Columns (1)-(3) perform the univariate prediction, and Columns (4)-(6) control for output growth, the par value of debt to GDP, the inflation risk premium, and the effective federal funds rate, all at time t.

extend our sample further back to 1947. The realized inflation is calculated as the annualized quarterly growth rate of CPI for urban consumers (ticker CPIUCSL in the FRED database). Columns (1)-(3) in Table C.8 use the realized inflation to predict future debt growth since 1947, controlling for contemporaneous debt-to-GDP ratio, GDP growth, and a linear trend. The realized inflation also significantly predicts future debt growth. The coefficients are smaller than inflation expectations in the baseline regression, since the realized inflation is based on each quarter whereas inflation expectations are over 10 years in the future. These results mirror the findings of Cochrane (2022*b*), that is, an innovation to inflation coincides with a rise in the value of public debt in the future.

Columns (4)-(6) perform a horse race between the realized inflation and inflation expectations. After controlling for inflation expectations, the realized inflation loses its predictive power for the 10-year debt-to-GDP growth. For shorter horizons, the point estimates of the realized inflation are still positive but no longer statistically significant. These results suggest that inflation expectations have better predictive powers than realized inflation.

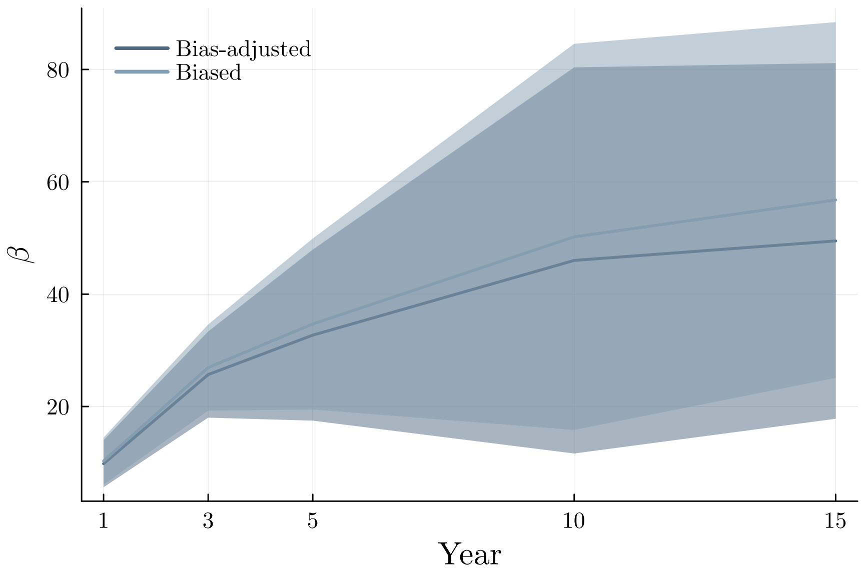

Figure C.2: Bias-adjusted Estimates

Notes. This figure plots the estimates of β_h in Eq. 3.3 and the bias-adjusted estimates under the null hypothesis, using the formula from Boudoukh, Israel and Richardson (2021). The shaded area represents the 95% confidence interval with Newey-West standard errors (bandwidths optimally chosen following Newey and West (1994)).

Table C.8: Predictive Regressions with the Realized Inflation

	1yr	5yr	10yr	1yr	5yr	10yr
	(1)	(2)	(3)	(4)	(5)	(6)
π	0.32	2.21	3.31	0.48	1.58	-0.84
	(0.19)	(0.78)	(1.05)	(0.38)	(0.74)	(0.98)
$E\pi$				10.35	33.08	30.99
				(1.65)	(7.53)	(11.61)
Sample	47Q2-20Q2	47Q2-16Q2	47Q2-11Q2	81Q4-20Q2	81Q4-16Q2	81Q4-11Q2
Control	True	True	True	True	True	True
N	293	277	257	155	139	119
R^2	0.22	0.46	0.60	0.25	0.51	0.82

Notes. Newey-West standard errors (bandwidths optimally chosen following Newey and West (1994)) are shown in parentheses. The predicted variables are future debt-to-GDP cumulative growth rates, where debt is measured by the market value of privately held public debt. Both variables are linearly detrended. Horizons are indicated in each column. The predictor variable is the realized inflation, calculated as the annualized quarterly growth rate of CPI for urban consumers (CPIUCSL) and the 10-year inflation expectations estimated by the Cleveland Fed (Haubrich, Pennacchi and Ritchken, 2012). All columns control for output growth and the par value of debt to GDP at time t.

D Proofs for the Model

D.1 The First-Order Conditions of the Household's Problem

The first-order conditions of the household's problem with respect to L_t, B_t, and D_t are

$$1 = E_t[\beta \frac{u_c(C_{t+1})}{u_c(C_t)} \frac{R_t}{\Pi_{t+1}}], \tag{D.1}$$

$$1 - \frac{\mu_v(V_t)}{u_c(C_t)} = E_t[\beta \frac{u_c(C_{t+1})}{u_c(C_t)} \frac{kQ^B_{t+1} + 1}{Q^B_t} \frac{1}{\Pi_{t+1}}], \tag{D.2}$$

$$1 = E_t[\beta \frac{u_c(C_{t+1})}{u_c(C_t)} \frac{kD^D_{t+1} + 1}{Q^D_t} \frac{1}{\Pi_{t+1}}]. \tag{D.3}$$

To a first-order approximation, the IS curve (D.1) and the demand function of convenience assets (D.2) yield

$$\log R_t = \frac{\mu_v(V_t)}{u_c(C_t)} + E_t \log(\frac{kQ^B_{t+1} + 1}{Q^B_t}). \tag{D.4}$$

Similarly, the demand function of non-convenience assets (D.3) with (D.1) yields

$$\log R_t = E_t \log(\frac{kQ^D_{t+1} + 1}{Q^D_t}). \tag{D.5}$$

D.2 Log-Linearized F.O.C.s of Households

Here we show the steps of deriving equations (4.7), (4.8), and (4.10) in the household's first-order conditions. First, denote i^B_{t+1} as

$$i^B_{t+1} \equiv \log(\frac{kQ^B_{t+1} + 1}{Q^B_t} / \frac{k\bar{Q}^B + 1}{\bar{Q}^B}),$$

and then by log-linearization, we have

$$i^B_{t+1} = \frac{k\bar{Q}^B}{k\bar{Q}^B + 1} q^B_{t+1} - q^B_t = kq^B_{t+1} - q^B_t, \tag{D.6}$$

where we have used the steady-state relationship $1 + \bar{i}^B = (k\bar{Q}^B + 1)/\bar{Q}^B$ in the second equality (note that $\bar{i}^B$ is at the scale of first order).

To get equation (4.7), we log-linearize equation (4.5) and utilize the market clearing condition $Y_t = C_t$; that is,

$$\begin{aligned}
\log R_t - log\bar{R} &= \frac{\mu_v(V_t)}{u_c(C_t)} - \frac{\mu_v(\bar{V})}{u_c(\bar{C})} + E_t \log(\frac{kQ^B_{t+1}+1}{Q^B_t}) - \log(\frac{k\bar{Q}^B+1}{\bar{Q}^B}) \\
\Rightarrow i_t &= \frac{\mu_v(\bar{V})[1+\frac{\mu_{vv}(\bar{V})\bar{V}}{\mu_v(\bar{V})}\frac{V_t-\bar{V}}{\bar{V}}]}{u_c(\bar{C})[1+\frac{u_{cc}(\bar{C})\bar{C}}{u_c(\bar{C})}\frac{C_t-\bar{C}}{\bar{C}}]} - \frac{\mu_v(\bar{V})}{u_c(\bar{C})} + E_t i^B_{t+1} \\
&= \frac{\mu_v(\bar{V})[1-\sigma_v^{-1}(v_t+y_t)]}{u_c(\bar{C})[1-\sigma_c^{-1}y_t]} - \frac{\mu_v(\bar{V})}{u_c(\bar{C})} + kE_t q^B_{t+1} - q^B_t \\
&= \frac{\mu_v(\bar{V})}{u_c(\bar{C})}[1-\sigma_v^{-1}(v_t+y_t)+\sigma_c^{-1}y_t] - \frac{\mu_v(\bar{V})}{u_c(\bar{C})} + kE_t q^B_{t+1} - q^B_t \\
&= -\gamma\sigma_v^{-1}[v_t+y_t] + \gamma\sigma_c^{-1}y_t + kE_t q^B_{t+1} - q^B_t,
\end{aligned}$$

where

$$\sigma_v \equiv -\frac{\mu_v(\bar{V})}{\mu_{vv}(\bar{V})\bar{V}} > 0, \qquad \sigma_c \equiv -\frac{u_c(\bar{C})}{u_{cc}(\bar{C})\bar{C}} > 0, \qquad \gamma \equiv \frac{\mu_v(\bar{V})}{u_c(\bar{C})}.$$

Similarly, we log-linearize (4.6) to derive equation (4.8); that is,

$$\begin{aligned}
i_t &= \frac{k\bar{Q}^D}{k\bar{Q}^D+1} E_t q^D_{t+1} - q^D_t \\
&= kE_t q^D_{t+1} - q^D_t.
\end{aligned}$$

Finally, for equation (4.10), we have the convenience yield $cy_t \equiv r^D_t - r^B_t$, given by

$$\begin{aligned}
cy_t &= kE_t(cy_{t+1}) + (\bar{Q}^D)^{-1}[-\gamma\sigma_v^{-1}v_t + \gamma(\sigma_c^{-1}-\sigma_v^{-1})y_t] \\
&= kE_t(cy_{t+1}) + (1+\bar{i}-k)[-\gamma\sigma_v^{-1}v_t + \gamma(\sigma_c^{-1}-\sigma_v^{-1})y_t] \\
&= kE_t(cy_{t+1}) + (1-k)[-\gamma\sigma_v^{-1}v_t + \gamma(\sigma_c^{-1}-\sigma_v^{-1})y_t].
\end{aligned}$$

Note that in the first equality, we have assumed $1/\bar{Q}^B \approx 1/\bar{Q}^D$ (to the first-order approximation) in the steady state; in the second equality, we have used the steady-state relationship $1+\bar{i} = (k\bar{Q}^D+1)/\bar{Q}^D$.

D.3 Expression of Expected Future Government Debt Balances

To nail down the expression of future government real debt balances, we need to log-linearize the government budget constraint (4.2), the Euler equation (4.3), and the trade-off condition between holding convenience assets and short-term riskless assets (D.2). In the derivation here, we also include aggregate demand shock for generality.

The government budget constraint (4.2) yields

$$\rho v_{t+1} = v_t + i^B_{t+1} - \pi_{t+1} - s_{t+1} - (y_{t+1} - y_t), \tag{D.7}$$

where s_{t+1} is the scaled real primary surplus-to-output ratio and $\rho \equiv e^{-\bar{i}^B} < 1$ is a constant.

Similarly, the IS curve (4.3) yields

$$y_t - u^d_t = E_t(y_{t+1} - u^d_{t+1}) - \sigma_c(i_t - E_t\pi_{t+1}), \tag{D.8}$$

where $u^d_{t+1} = \rho_d u^d_t + \epsilon^d_{t+1}$ denotes an aggregate demand shock (such as a preference shock) and ϵ^d_{t+1} is an i.i.d. innovation with mean zero; the trade-off condition of holding convenience assets (D.2) yields

$$i_t = -\gamma\sigma_v^{-1}[v_t + y_t] + \gamma\sigma_c^{-1}y_t + E_t i^B_{t+1}. \tag{D.9}$$

Combining (D.7)-(D.9) and also noting that $\sigma_c = \sigma_v = 1$, we have the expected government real debt balance in any time $t + j + 1$ satisfying

$$E_t v_{t+j+1} = \frac{1}{\rho}E_t[(1+\gamma)v_{t+j} - s_{t+j+1} - (u^d_{t+j+1} - u^d_{t+j})] \tag{D.10}$$

for any $j \geq 0$, and then by iterating this equation to time t, it yields

$$E_t v_{t+j+1} = (\frac{1+\gamma}{\rho})^{j+1} v_t - \left[\sum_{n=1}^{j+1} \frac{1}{\rho}(\frac{1+\gamma}{\rho})^{j+1-n} E_t s_{t+n}\right] + \frac{1-\rho_d}{\rho\rho_d - (1+\gamma)}\left[\rho_d^{j+1} - (\frac{1+\gamma}{\rho})^{j+1}\right] u^d_t. \tag{D.11}$$

Equation (D.11) shows that the expected future real debt balance of the government equals the summation of three terms: the current debt-to-GDP ratio, the future flows of government surpluses, and the contemporaneous demand shock.

Plugging expression (D.11) into equation (4.11) and noting $\sigma_c = \sigma_v = 1$, we have

$$cy_t = \eta_v v_t + \eta_d u^d_t + \sum_{j=1}^{\infty} \eta_{s,j} E_t s_{t+j},$$

where the constant coefficients are given by

$$\eta_v \equiv -\gamma(1-k)\sum_{j=0}^{\infty} k^j (\frac{1+\gamma}{\rho})^j < 0,$$

$$\eta_d \equiv -\gamma(1-k)\frac{1-\rho_d}{\rho\rho_d-(1+\gamma)}\sum_{j=1}^{\infty} k^j [\rho_d^j - (\frac{1+\gamma}{\rho})^j] < 0,$$

$$\eta_{s,j} \equiv \gamma\frac{1-k}{\rho}\sum_{n=j}^{\infty} k^n (\frac{1+\gamma}{\rho})^{n-1} > 0.$$

Now further consider a fiscal policy rule in which $s_{t+1} = \alpha v_t + u^s_{t+1}$. By substituting s_{t+j+1} into (D.10), we have

$$E_t v_{t+j+1} = \frac{1}{\rho}E_t[(1+\gamma-\alpha)v_{t+j} - u^s_{t+j+1} - (u^d_{t+j+1} - u^d_{t+j})]$$

for any $j \geq 0$, and similarly, by iterating this equation to time t, it yields

$$E_t v_{t+j+1} = (\frac{1+\gamma-\alpha}{\rho})^{j+1} v_t - \left[\sum_{n=1}^{j+1}\frac{1}{\rho}(\frac{1+\gamma-\alpha}{\rho})^{j+1-n} E_t u^s_{t+n}\right]$$
$$+\frac{1-\rho_d}{\rho\rho_d-(1+\gamma-\alpha)}\left[\rho_d^{j+1} - (\frac{1+\gamma-\alpha}{\rho})^{j+1}\right] u^d_t.$$

Thus, in such a case, the convenience yield is given by

$$cy_t = \phi_v v_t + \phi_d u^d_t + \sum_{j=1}^{\infty}\phi_{s,j} E_t u^s_{t+j},$$

where the constant coefficients are given by

$$\phi_v \equiv -\gamma(1-k)\sum_{j=0}^{\infty} k^j (\frac{1+\gamma-\alpha}{\rho})^j < 0,$$

$$\phi_d \equiv -\gamma(1-k)\frac{1-\rho_d}{\rho\rho_d-(1+\gamma-\alpha)}\sum_{j=1}^{\infty} k^j [\rho_d^j - (\frac{1+\gamma-\alpha}{\rho})^j] < 0,$$

$$\phi_{s,j} \equiv \gamma\frac{1-k}{\rho}\sum_{n=j}^{\infty} k^n (\frac{1+\gamma-\alpha}{\rho})^{n-1} > 0.$$

D.4 Equilibrium Characterization

Denote the log-deviation from steady-state values by lowercase letters. Market clearing conditions require $y_t = c_t$. Given those first-order conditions from the households optimality problem (D.6)-(D.9), the New Keynesian Phillips curve (4.20), the policy rules (4.21)-(4.22), the government budget constraint (4.13), and the expression for the convenience yield (4.10), the equilibrium is fully characterized by the following system of equations:

$$y_t = E_t y_{t+1} - \sigma_c(i_t - E_t \pi_{t+1}) \tag{D.12}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + \psi \tau_t \tag{D.13}$$

$$E_t i^B_{t+1} = i_t - \gamma[-\sigma_v^{-1} v_t + (\sigma_c^{-1} - \sigma_v^{-1}) y_t] \tag{D.14}$$

$$i^B_{t+1} = k q^B_{t+1} - q^B_t \tag{D.15}$$

$$i_t = \theta_{i\pi}\pi_t + \theta_{iy} y_t + u^i_t \tag{D.16}$$

$$\tau_{t+1} = \theta_{s\pi}\pi_{t+1} + \theta_{sy} y_{t+1} + \alpha v_t \tag{D.17}$$

$$s_{t+1} = \tau_{t+1} + u^s_{t+1} \tag{D.18}$$

$$\rho v_{t+1} = v_t + i^B_{t+1} - \pi_{t+1} - s_{t+1} - (y_{t+1} - y_t) \tag{D.19}$$

$$0 = \lim_{T\to\infty} \rho^T E_t v_{t+T} \tag{D.20}$$

$$c y_t = k E_t(c y_{t+1}) + (1-k)\gamma[-\sigma_v^{-1} v_t + (\sigma_c^{-1} - \sigma_v^{-1}) y_t] \tag{D.21}$$

$$u^s_{t+1} = \rho_s u^s_t + \epsilon^s_{t+1}. \tag{D.22}$$

Here we only include the fiscal shock and monetary policy shock, but one can easily extend the set of shocks to incorporate aggregate demand or supply shocks.

E Robustness of Model Prediction

E.1 Impulse Responses to Monetary Policy and Cost-Push Shocks

Figure E.3 shows the impulse response of various endogenous variables to a one percentage point expansionary monetary policy shock, $\epsilon^i_1 = -1\%$, occurring in period $t = 1$. Following Bianchi and Melosi (2017), the persistence of monetary policy shock is set to $\rho_i = 0.87$. All the variables are shown similarly as in Figure 7. Given the expansionary monetary policy shock, both inflation and output rise. The government's real debt balance declines as it is being inflated away. As a result, convenience yield increases due to the lower supply of future government bonds. Long-run inflation expectation slightly drops due to the overshooting of inflation response in the long run. Figure E.3 illustrates the negative association between long-run inflation expectation $E_t\pi_{t+40}$ and the convenience yield on long-term government

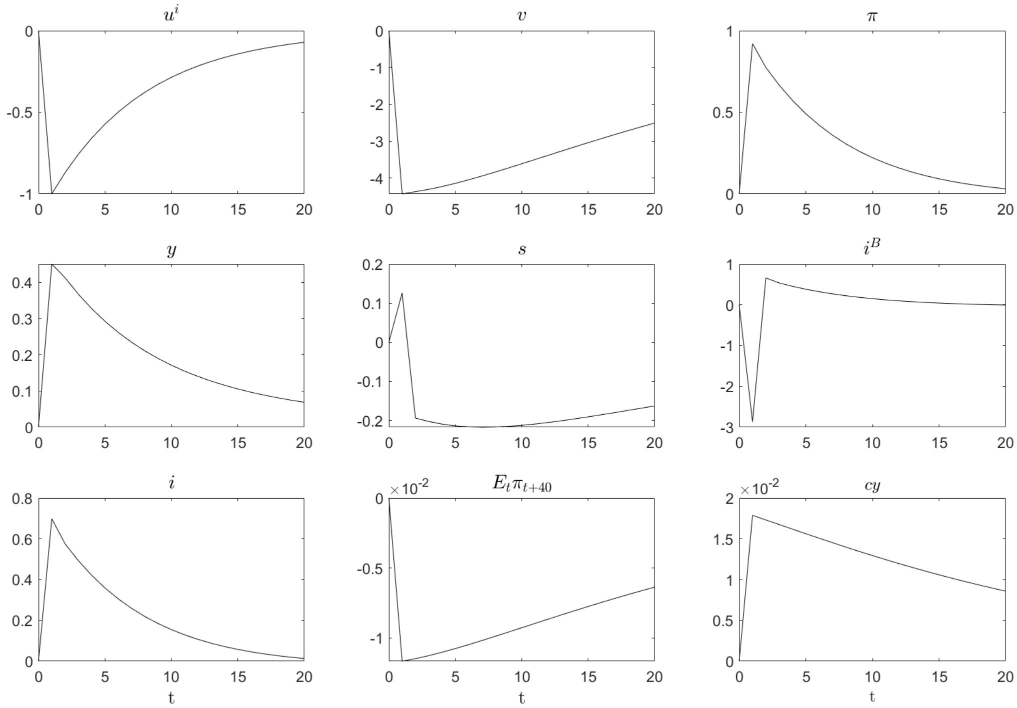

Figure E.3: Impulse Responses to a Monetary Policy Shock

Notes. The figure plots the impulse response of various endogenous variables to an expansionary monetary policy shock, $\epsilon_1^i = -1\%$, occurring in period $t = 1$. The top row shows the exogenous shock process u_t^i, and the responses of the real value of government debt v_t and inflation π_t. The middle row shows the responses of output gap y_t, government surplus s_t, and short-term nominal return on government-issued bond i_t^B. The bottom row shows the responses of nominal interest rate i_t, long-run inflation expectations $E_t\pi_{t+40}$, and the long-term convenience yield cy_t. The x-axis is in quarters. All numbers are reported in percentages.

bonds cy_t.

Consider a cost-shock in the Phillips curve:

$$\pi_t = \kappa y_t + \beta E_t \pi_{t+1} + \psi \tau_t + u_t^p,$$

where u_t^p is an exogenous cost-push shock following AR(1) process, that is, $u_t^p = \rho_p u_{t-1}^p + \epsilon_t^p$ with ϵ_t^p being i.i.d. innovations. Following Bianchi and Melosi (2017), the persistence of cost-push shock is set to $\rho_p = 0.48$.

Figure E.4 shows the impulse response of various endogenous variables to a one percentage point increase in the cost-push shock, $\epsilon_1^p = 1\%$, occurring in period $t = 1$. Inflation rises whereas output drops. Similar to the case of monetary policy shock, the government's real debt balance declines as it is being inflated away. Convenience yield increases due to the lower supply of future government bonds and long-run inflation expectation slightly drops

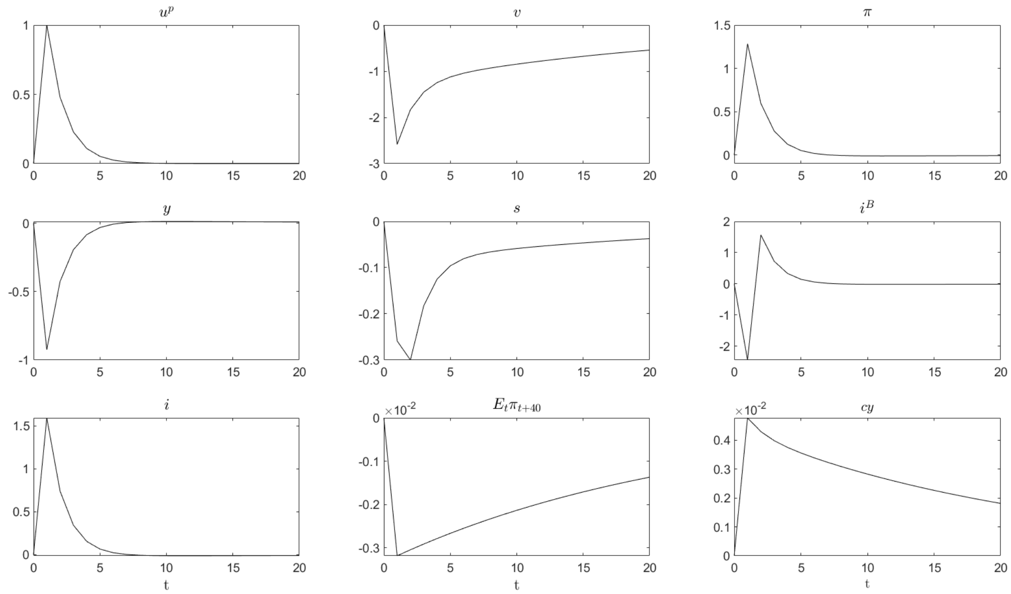

Figure E.4: Impulse Responses to a Cost-Push Shock

Notes. The figure plots the impulse response of various endogenous variables to a positive cost-push shock, $\epsilon_1^p = 1\%$, occurring in period $t = 1$. The top row shows the exogenous shock process u_t^p, and the responses of the real value of government debt v_t and inflation π_t. The middle row shows the responses of output gap y_t, government surplus s_t, and short-term nominal return on government-issued bond i_t^B. The bottom row shows the responses of nominal interest rate i_t, long-run inflation expectations $E_t\pi_{t+40}$, and the long-term convenience yield cy_t. The x-axis is in quarters. All numbers are reported in percentages.

due to the overshooting of inflation. Figure E.4 illustrates the negative association between long-run inflation expectation $E_t\pi_{t+40}$ and the convenience yield on long-term government bonds cy_t.

E.2 Impulse Response to a Fiscal Deficit Shock with Different Bond Durations

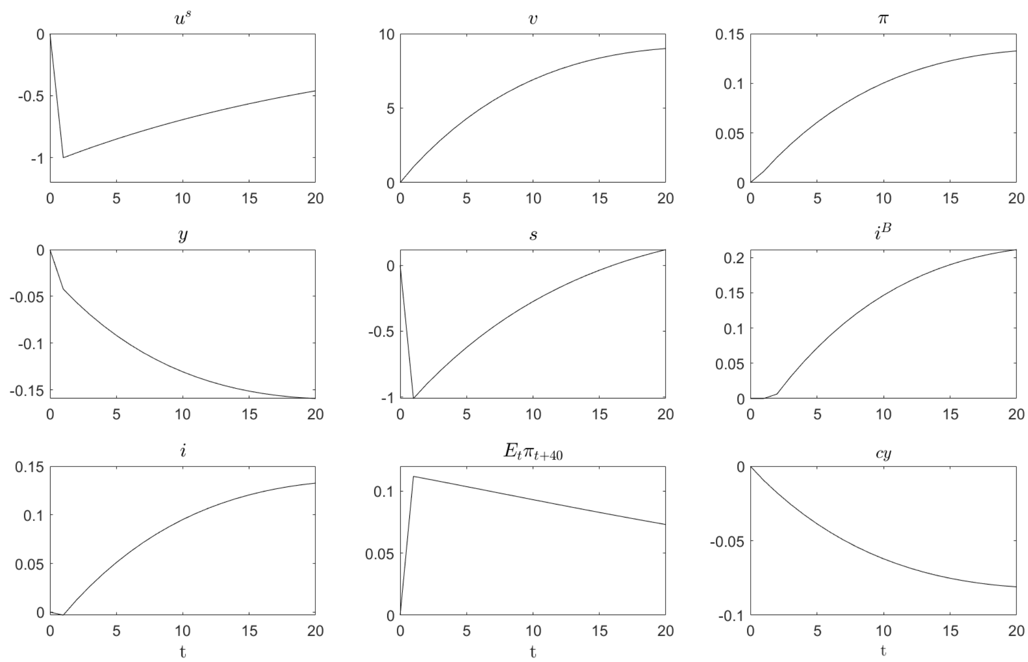

Figure E.5: Impulse Responses to a Fiscal Deficit Shock (One-Quarter Duration)

Notes. The figure plots the impulse response of various endogenous variables to a deficit shock, $\epsilon_1^s = -1\%$, occurring in period $t = 1$. All variables are shown similarly as in Figure 7 except that the government bond duration is set to be one quarter (that is, $k = 0$). The x-axis is in quarters. All numbers are reported in percentages.

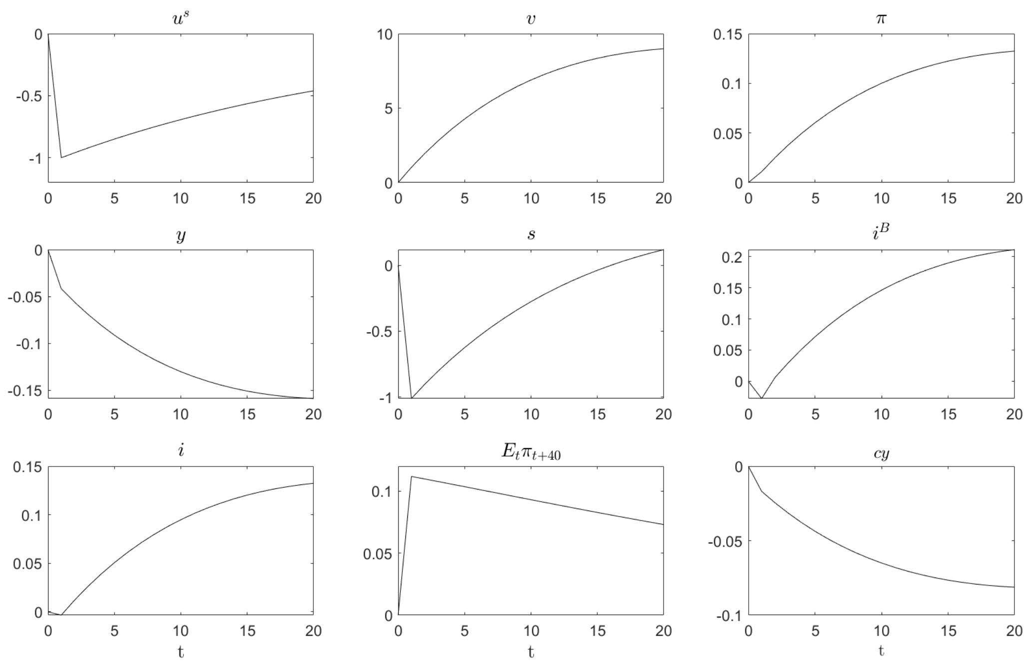

Figure E.6: Impulse Responses to a Fiscal Deficit Shock (Two-Quarters Duration)

Notes. The figure plots the impulse response of various endogenous variables to a deficit shock, $\epsilon_1^s = -1\%$, occurring in period $t = 1$. All variables are shown similarly as in Figure 7 except that the government bond duration is set to be two quarters (that is, $k = 0.5$). The x-axis is in quarters. All numbers are reported in percentages.

F Alternative Formulation with Active Fiscal Policy

In this section, we consider an alternative formulation of the model with policy rules following the fiscal theory of price level (FTPL) by assuming active fiscal policy and lump-sum taxes. We show that the main results in Section 4 still hold.

We follow the approach of modeling an active fiscal policy developed in Cochrane (2022*a*). Cochrane (2022*a*,*b*) argue that this framework compared with other alternative setups of active fiscal policy has the advantage of allowing for a more realistic government debt and surplus process and thus is able to produce more reasonable responses to policy shocks.

F.1 Fiscal and Monetary Policies

The government sets the fiscal policy through a rule of collecting primary surpluses, where the surplus is imposed as a lump-sum tax on households. That being said, the proportional income tax $\mathcal{T}_t = 0$ in Section 4 is zero. Despite this difference, the equilibrium conditions of households and firms remain the same.

The central bank sets the rule of the nominal interest rate on short-term riskless bonds. As commonly adopted in the literature, we assume that fiscal and monetary policies follow a simple linear form. For the monetary policy, that is

$$i_t = \theta_{i\pi}\pi_t + \theta_{iy}y_t + u_t^i, \tag{F.1}$$

and for the fiscal policy, that is

$$s_{t+1} = \theta_{s\pi}\pi_{t+1} + \theta_{sy}y_{t+1} + \alpha v_t^* + u_{t+1}^s, \tag{F.2}$$

where the group of parameters θ captures the policy responsiveness to various endogenous variables. Both the exogenous monetary shock u_t^i and fiscal shock u_t^s follow AR(1) processes:

$$u_{t+1}^i = \rho_i u_t^i + \epsilon_{t+1}^i, \qquad u_{t+1}^s = \rho_s u_t^s + \epsilon_{t+1}^s,$$

where ϵ_{t+1}^i and ϵ_{t+1}^s are i.i.d. innovations with mean zero.

The setup of fiscal rule (F.2) follows Cochrane (2022*a*). In particular, v_t^* in (F.2) represents the *target level* of the real balance of government debt, which in equilibrium equals the market value of government debt v_t.[38] Here $\alpha > 0$ captures to what extent the government respects its liability by collecting surpluses to pay back its debt. Together with the shock processes, this policy rule yields an S-shape response in surpluses to fiscal shocks: upon a deficit shock u_t^s, the surplus initially drops, and the value of public debt increases; the increased debt, in

[38] The proof of $v_t^* = v_t$ in equilibrium can be found in Appendix F.2.

turn, drives up the surplus as a result of the fiscal policy rule, and as the original shock u_t^s dies out, surpluses will rise above the steady-state level to pay down the debt.

One may need to be careful about the concept of v_t^*. Although the target variable v_t^* equals v_t in equilibrium, v_t^* differs from v_t in the sense that v_t^* increases with the inflation targeted by the fiscal authority, whereas v_t may not. This makes the fiscal policy active in determining the price level. We briefly discuss the intuition below. The law of motion for v_t^* is set to be the same as that for v_t in (4.13) except for the realized inflation π_t being replaced by the inflation target π_t^*; that is,

$$\rho v_{t+1}^* = v_t^* + i_{t+1}^B - \pi_{t+1}^* - s_{t+1} - (y_{t+1} - y_t). \tag{F.3}$$

Moreover, the inflation target π_t^* directly responds to the innovation ϵ_{t+1}^s in the surplus shock such that

$$\Delta E_{t+1}\pi_{t+1}^* = -\beta_s \epsilon_{t+1}^s, \tag{F.4}$$

where the operator $\Delta E_{t+1} \equiv E_{t+1} - E_t$ denotes the unexpected change at time $t+1$. Here the exogenous parameter $\beta_s > 0$ measures the unexpected change in the inflation target in response to fiscal shocks. In equilibrium, the realized inflation is consistent with the inflation target.[39] Thus, with a positive value of β_s, a deficit shock leads to an increase in inflation and inflation expectations; furthermore, a small value of $\beta_s > 0$ indicates the situation in which the deficit is partially financed by inflating debt away and partially by future borrowing, consistent with the empirical findings in Berndt, Lustig and Yeltekin (2012).[40]

Cochrane (2022*a*) argues that this setup of fiscal policy (F.2)-(F.4) allows for a more (empirically) reasonable response of government debt, which is a variable of keen interest to us, in facing deficit shocks: a deficit *raises* instead of *lowers* the value of debt. Furthermore, as we illustrate in the numerical exercise in Appendix F.3, a deficit shock also yields a positive response in inflation and inflation expectations, and a negative response in the convenience yield as a result of the increase in the path of future public debt.

F.2 Equilibrium Characterization

Denote the log-deviation from steady-state values by lowercase letters. Market clearing conditions require $y_t = c_t$. The equilibrium is fully characterized by the following system of

[39] To see this, notice that if realized inflation differs from the inflation target, v_t will also deviate from v_t^*. However, since the fiscal rule does not respond to v_t, this deviation will accumulate and eventually lead to an explosive path of public debt, violating the transversality condition.

[40] The parameter β_s in equilibrium gauges the extent to which a deficit shock is financed by the effect of inflating away. When $\beta_s = 0$, any deficit is fully repaid by the following surpluses, and thus there is no change in inflation at all. A higher value of β_s indicates that the government is able to inflate away more debt in facing a deficit shock.

equations:

$$y_t = E_t y_{t+1} - \sigma_c(i_t - E_t\pi_{t+1}) \tag{F.5}$$

$$\pi_t = \beta E_t\pi_{t+1} + \kappa y_t \tag{F.6}$$

$$E_t i^B_{t+1} = i_t - \gamma[-\sigma_v^{-1} v_t + (\sigma_c^{-1} - \sigma_v^{-1}) y_t] \tag{F.7}$$

$$i^B_{t+1} = k q^B_{t+1} - q^B_t \tag{F.8}$$

$$i_t = \theta_{i\pi}\pi_t + \theta_{iy} y_t + u^i_t \tag{F.9}$$

$$s_{t+1} = \theta_{s\pi}\pi_{t+1} + \theta_{sy} y_{t+1} + \alpha v^*_t + u^s_{t+1} \tag{F.10}$$

$$\rho v^*_{t+1} = v^*_t + i^B_{t+1} - \pi^*_{t+1} - s_{t+1} - (y_{t+1} - y_t) \tag{F.11}$$

$$E_t\pi^*_{t+1} = E_t\pi_{t+1} \tag{F.12}$$

$$\Delta E_{t+1}\pi^*_{t+1} = -\beta_s\epsilon^s_{t+1} \tag{F.13}$$

$$\rho v_{t+1} = v_t + i^B_{t+1} - \pi_{t+1} - s_{t+1} - (y_{t+1} - y_t) \tag{F.14}$$

$$0 = \lim_{T\to\infty} \rho^T E_t v_{t+T} \tag{F.15}$$

$$c y_t = k E_t(c y_{t+1}) + (1-k)\gamma[-\sigma_v^{-1} v_t + (\sigma_c^{-1} - \sigma_v^{-1}) y_t] \tag{F.16}$$

$$u^s_{t+1} = \rho_s u^s_t + \epsilon^s_{t+1}, \tag{F.17}$$

where v^*_t represents the target level of the government real debt balance in period t and π^*_t represents the target level of inflation. The operator $\Delta E_{t+1} \equiv E_{t+1} - E_t$ denotes the unexpected change at time $t+1$.

Equation (F.10) generates an S-shaped response of government surpluses to fiscal shocks, which is a more realistic description of the real data. The purpose of introducing those target (latent) variables and equations (F.11)-(F.13) is to allow the economy to finance a deficit shock partially through inflating the debt away and partially through future borrowing (which is captured by the value of β_s). More specifically, equation (F.11) captures the evolution of the target level of the government real debt balance v^*_t. Together with the fiscal rule of collecting surpluses (F.10), it generates the desired S-shaped surplus response to a deficit shock. Equations (F.12) and (F.13) describe the latent variable of an inflation target, meaning that the expected inflation has to be consistent with the expected inflation target.

Note that equations (F.11) and (F.14) imply

$$\rho(v_{t+1} - v^*_{t+1}) = (v_t - v^*_t) - (\Delta E_t\pi_{t+1} - \Delta E_{t+1}\pi^*_t).$$

Together with the transversality condition (D.20), we have in equilibrium

$$v_t = v^*_t, \qquad \pi_{t+1} = \pi^*_{t+1}.$$

Thus, the equilibrium conditions (F.5)-(F.17) reduce to the following system of equations:[41]

$$
\begin{aligned}
y_t &= E_t y_{t+1} - \sigma_c(i_t - E_t\pi_{t+1}) \\
\pi_t &= \beta E_t\pi_{t+1} + \kappa y_t \\
E_t i^B_{t+1} &= i_t - \gamma[-\sigma_v^{-1}v_t + (\sigma_c^{-1} - \sigma_v^{-1})y_t] \\
i^B_{t+1} &= kq^B_{t+1} - q^B_t \\
i_t &= \theta_{i\pi}\pi_t + \theta_{iy}y_t + u^i_t \\
\Delta E_{t+1}\pi_{t+1} &= -\beta_s\epsilon^s_{t+1} - \beta_i\epsilon^i_{t+1} \\
s_{t+1} &= \theta_{s\pi}\pi_{t+1} + \theta_{sy}y_{t+1} + \alpha v_t + u^s_{t+1} \\
\rho v_{t+1} &= v_t + i^B_{t+1} - \pi_{t+1} - s_{t+1} - (y_{t+1} - y_t) \\
cy_t &= kE_t(cy_{t+1}) + (1-k)\gamma[-\sigma_v^{-1}v_t + (\sigma_c^{-1} - \sigma_v^{-1})y_t] \\
u^i_{t+1} &= \rho_i u^i_t + \epsilon^i_{t+1} \\
u^s_{t+1} &= \rho_s u^s_t + \epsilon^s_{t+1}.
\end{aligned}
$$

F.3 Quantitative Analyses

Same as in Section 4.6, we calibrate the model at the quarterly frequency. All the parameters are the same as in Table 6 except for the parameters in the policy rules.

For the parameters characterizing fiscal and monetary policies, we set the surplus response to real public debt α to be 0.04, as suggested by the regression coefficient of surplus on the debt-to-GDP ratio in Cochrane (2022*b*). The response of unexpected inflation to deficit shock β_s is set to 2.1, so that the initial response of annualized 10-year inflation expectation to a deficit shock in the model is 0.03, which matches the estimate of the response of long-run inflation expectations in Column (2) of Table 5. Following the estimates of a fiscal-led policy regime for the U.S. economy in Bianchi and Melosi (2017), we set the parameters in the fiscal policy rule (4.22) satisfying $\theta_{s\pi} = 0$ and $\theta_{sy} = 0.28$ and the parameters in the monetary policy rule (4.21) satisfying $\theta_{i\pi} = 0.63$ and $\theta_{iy} = 0.27$.

Figure F.7 shows the impulse response of various endogenous variables to a one percentage point deficit shock, $\epsilon^s_1 = -1\%$, occurring in period $t = 1$ under active fiscal policy. The variables are shown in a similar manner as in Figure 7. It shows that, following a deficit shock, inflation and output have an immediate positive response, indicating a stimulative effect of the deficit shock because of provoked inflation.

Real public debt initially drops and then climbs to a positive level because of the persistent negative surplus shock. Meanwhile, the accumulated debt balance slowly raises the surplus following the fiscal policy rule. When the debt level is high enough to dominate the negative

[41] The determinacy conditions of the equilibrium and the relevant proofs can be found in Appendix F.4.

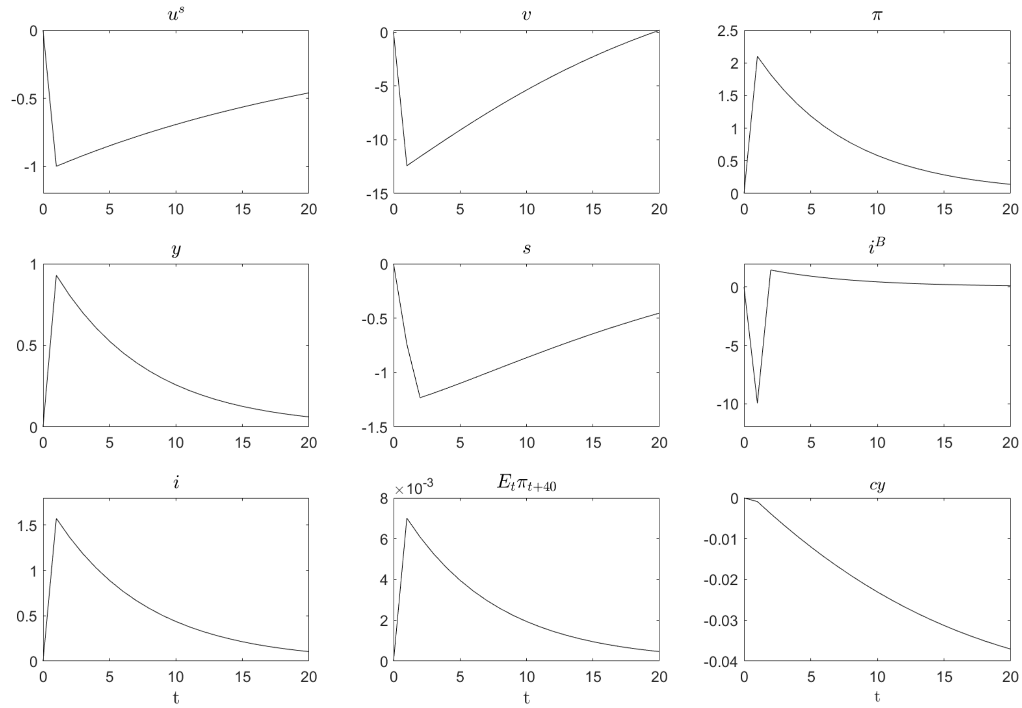

Figure F.7: Impulse Responses to a Fiscal Deficit Shock

Notes. The figure plots the impulse response of various endogenous variables to a deficit shock, $\epsilon_1^s = -1\%$, occurring in period $t = 1$. The top row shows the exogenous shock process u_t^s, and the responses of the real value of government debt v_t and inflation π_t. The middle row shows the responses of output gap y_t, government surplus s_t, and short-term nominal return on government-issued bond i_t^B. The bottom row shows the responses of nominal interest rate i_t, long-run inflation expectations $E_t\pi_{t+40}$, and the long-term convenience yield cy_t. The x-axis is in quarters. All numbers are reported in percentages.

surplus shock, the surplus will pick up and become positive, which in turn starts to reduce the debt balance. Eventually, in the long run, all the endogenous variables converge back to the steady state. In consequence, the IRFs of both government surplus and debt feature an S-shape (Figure F.7 only plots a few beginning periods for illustration).

Figure F.7 also indicates higher inflation expectations associated with increased future real value of debt balances. The higher future real value of debt balances in turn implies a lower return on the convenience benefits of government bonds and thus a lower convenience yield. The bottom row confirms this intuition by comparing the responses of inflation expectations and the convenience yield on long-term government bonds. A positive response in inflation expectation is associated with a negative movement in the convenience yield.

F.4 Determinacy Condition of the Full Equilibrium

Here we characterize the determinacy condition of the full equilibrium under active fiscal policy. Denote δ as expectational errors in the equations that only tie down expectations. For example, $\delta_{\pi,t+1} = \pi_{t+1} - E_t\pi_{t+1}$.

We can rewrite the system of equations of the full equilibrium conditions in a recursive form of matrices as

$$\begin{pmatrix} 1 & \sigma & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -k & 0 & 0 & 0 & 0 \\ -\theta_{sy} & -\theta_{s\pi} & 0 & 0 & 1 & 0 & 0 & -1 \\ 1 & 1 & -1 & 0 & 1 & \rho & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_{t+1} \\ \pi_{t+1} \\ i^B_{t+1} \\ q_{t+1} \\ s_{t+1} \\ v_{t+1} \\ u^i_{t+1} \\ u^s_{t+1} \end{pmatrix}$$

$$= \begin{pmatrix} 1+\sigma\theta_{iy} & \sigma\theta_{i\pi} & 0 & 0 & 0 & 0 & \sigma & 0 \\ -\kappa & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \theta_{iy}+\gamma(\psi^{-1}-\sigma^{-1}) & \theta_{i\pi} & 0 & 0 & 0 & \gamma\psi^{-1} & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_s \end{pmatrix} \begin{pmatrix} y_t \\ \pi_t \\ i^B_t \\ q_t \\ s_t \\ v_t \\ u^i_t \\ u^s_t \end{pmatrix} + \begin{pmatrix} -\sigma\beta_i & -\sigma\beta_s \\ -\beta\beta_i & -\beta\beta_s \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \epsilon^i_{t+1} \\ \epsilon^s_{t+1} \end{pmatrix} + \begin{pmatrix} \delta_{y,t+1} \\ 0 \\ \delta_{i^B,t+1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \tag{F.18}$$

where we do not include cy_t in the matrix since it is uniquely pinned down as long as the equilibrium is stationary.

Denote

$$A \equiv \begin{pmatrix} 1 & \sigma & 0 & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -k & 0 & 0 \\ -\theta_{sy} & -\theta_{s\pi} & 0 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 1 & \rho \end{pmatrix}. \tag{F.19}$$

Then the inverse of the matrix on the left-hand side of (F.18) is

$$\begin{pmatrix} A^{-1} & -A^{-1}\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ 0 & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{pmatrix}. \tag{F.20}$$

Further denote

$$\tilde{A} \equiv \begin{pmatrix} 1+\sigma\theta_{iy} & \sigma\theta_{i\pi} & 0 & 0 & 0 & 0 \\ -\kappa & 1 & 0 & 0 & 0 & 0 \\ \theta_{iy}+\gamma(\psi^{-1}-\sigma^{-1}) & \theta_{i\pi} & 0 & 0 & 0 & \gamma\psi^{-1} \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} \begin{pmatrix} 1+\sigma\theta_{iy} & \sigma\theta_{i\pi} \\ -\kappa & 1 \end{pmatrix} & 0 \\ C & \begin{pmatrix} 0 & 0 & 0 & \gamma\psi^{-1} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & \alpha \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{pmatrix}. \tag{F.21}$$

Then the eigenvalues of Ω in $X_{t+1} = \Omega X_t$ are ρ_i, ρ_s and the eigenvalues of $A^{-1}\tilde{A}$, where

$$X_t \equiv \left(y_t, \pi_t, i_t^B, q_t, s_t, v_t, u_t^i, u_t^s\right)'.$$

We can further rewrite A^{-1} as

$$A^{-1} = \begin{pmatrix} \begin{pmatrix} 1 & -\frac{\sigma}{\beta} \\ 0 & \frac{1}{\beta} \end{pmatrix} & 0 \\ B & \begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{k} & -\frac{1}{k} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{\rho} & 0 & -\frac{1}{\rho} & \frac{1}{\rho} \end{pmatrix} \end{pmatrix}. \tag{F.22}$$

The eigenvalues of $A^{-1}\tilde{A}$ are then the eigenvalues $\{\lambda_1, \lambda_2\}$ of the upper block

$$\begin{pmatrix} 1 & -\frac{\sigma}{\beta} \\ 0 & \frac{1}{\beta} \end{pmatrix}\begin{pmatrix} 1+\sigma\theta_{iy} & \sigma\theta_{i\pi} \\ -\kappa & 1 \end{pmatrix} = \begin{pmatrix} 1+\sigma\theta_{iy}+\frac{\sigma}{\beta}\kappa & \sigma\theta_{i\pi}-\frac{\sigma}{\beta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{pmatrix} \tag{F.23}$$

and the eigenvalues of the lower block

$$\begin{pmatrix}1 & 0 & 0 & 0\\ \frac{1}{k} & -\frac{1}{k} & 0 & 0\\ 0 & 0 & 1 & 0\\ \frac{1}{\rho} & 0 & -\frac{1}{\rho} & \frac{1}{\rho}\end{pmatrix}\begin{pmatrix}0 & 0 & 0 & \gamma\psi^{-1}\\ 0 & -1 & 0 & 0\\ 0 & 0 & 0 & \alpha\\ 0 & 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}0 & 0 & 0 & \gamma\psi^{-1}\\ 0 & \frac{1}{k} & 0 & \frac{\gamma\psi^{-1}}{k}\\ 0 & 0 & 0 & \alpha\\ 0 & 0 & 0 & \frac{\gamma\psi^{-1}}{\rho}-\frac{\alpha}{\rho}+\frac{1}{\rho}\end{pmatrix}, \tag{F.24}$$

where the eigenvalues of the lower block are $\{0, 0, \frac{1}{k}, \frac{\gamma\psi^{-1}}{\rho} - \frac{\alpha}{\rho} + \frac{1}{\rho}\}$.

To sum up, the eigenvalues of the system are: $\{\rho_i, \rho_s, 0, 0, \frac{1}{k}, \frac{\gamma\psi^{-1}}{\rho} - \frac{\alpha}{\rho} + \frac{1}{\rho}, \lambda_1, \lambda_2\}$, where λ_1 and λ_2 (with $\lambda_1 < \lambda_2$) are solutions to

$$\mathcal{P}(\lambda) \equiv \lambda^2 - (\frac{1}{\beta} + 1 + \sigma\theta_{iy} + \frac{\sigma}{\beta}\kappa)\lambda + \frac{1 + \sigma\theta_{iy} + \sigma\kappa\theta_{i\pi}}{\beta} = 0. \tag{F.25}$$

With two linearly independent expectational errors, the equilibrium is determinant if and only if there are two eigenvalues outside the unit circle. Given $1/k > 1$, the equilibrium is determined if and only if $\gamma\psi^{-1} - \alpha + 1 < \rho$ and $|\lambda_1| < 1 < |\lambda_2|$.[42]

Now we focus on the equivalent condition of $|\lambda_1| < 1 < |\lambda_2|$ and show that its necessary and sufficient condition is $\mathcal{P}(1) < 0$.

Note that $\theta_{iy}, \theta_{i\pi} > 0$ and $\sigma, \kappa > 0$ and $0 < \beta < 1$. It is obvious that $\mathcal{P}(1) < 0$ is a sufficient condition in which the two roots satisfy $0 < \lambda_1 < 1 < \lambda_2$. To show that it is also a necessary condition: conditional on that the two roots satisfy $|\lambda_1| < 1 < |\lambda_2|$, if the two roots form a complex pair, they must have a common modulus, and thus they must be either both outside the unit circle or both within the unit circle, which contradicts with $|\lambda_1| < 1 < |\lambda_2|$. So the two roots have to be both real roots, which must be the case $0 < \lambda_1 < 1 < \lambda_2$, requiring $\mathcal{P}(1) < 0$.

Therefore, the equilibrium is determined if and only if $\gamma\psi^{-1} - \alpha + 1 < \rho$ and

$$\mathcal{P}(1) = \theta_{iy}(\frac{1}{\beta} - 1) + \frac{\kappa}{\beta}(\theta_{ix} - 1) < 0.$$

[42] Here we ignore the boundary analysis as in Woodford (2003, appx. C).