Capturing Macroeconomic Tail Risks with Bayesian Vector Autoregressions

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The results presented here do not necessarily represent the views of the Federal Reserve Bank of Cleveland or the Federal Reserve System.

February 2020
How do we measure (negative) tail risks to macroeconomic outcomes?

- Rapidly growing lit focused on GDP growth and quantile regression: Adrian, Boyarchenko, and Giannone (2019, ABG), Adrian, et al. (2018), Giglio, Kelly, and Pruitt (2016)
  - Focus on risks associated with poor financial conditions
Introduction

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- Rapidly growing lit focused on GDP growth and quantile regression: Adrian, Boyarchenko, and Giannone (2019, ABG), Adrian, et al. (2018), Giglio, Kelly, and Pruitt (2016)
  - Focus on risks associated with poor financial conditions
- Some work considers risks to unemployment (e.g., Galbraith and van Norden 2019, Kiley 2018) or inflation
- Some work considers other methods, such as copula
- Other work drills deeper to better understand tail risks: Loria, Matthes, and Zhang (2019) examine drivers of left tail
Interest in tail risks reflects a perception of asymmetries in distributions of outcomes

- Precedents: Markov switching and threshold models, plus literature on asymmetries in unemployment
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- Some recent research has focused on evidence of skewness in GDP growth: Kozeniauskas, Orlik, and Veldkamp (2018), Orlik and Veldkamp (2015), Jensen, et al. (2020)
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- Monetary policymakers have commonly treated forecast distributions as being potentially asymmetric, at some points in time
Important distinction: asymmetries in conditional vs. unconditional distributions

- ABG and others sometime refer explicitly to conditional: recessions associated with left-skewed distributions
- But one pattern that has been emphasized — downside risk varying more than upside — could occur with predictive distributions that are symmetric
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- But one pattern that has been emphasized — downside risk varying more than upside — could occur with predictive distributions that are symmetric
- Need simultaneous mean and variance shifts
- Simple example: shift from $N(0,1)$ to $N(-2,4) \Rightarrow$ 95% quantile edges down from 1.65 to 1.29, whereas 5% quantile drops from -1.65 to -5.29.
We examine the ability of BVARs with stochastic volatility (SV) to capture tail risks in macroeconomic forecast distributions and outcomes.

- BVARs have a known track record and can be used for a range of forecasting purposes.
- BVAR-SV models commonly improve on the point and density forecast accuracy of their homoskedastic counterparts.
We examine the ability of BVARs with stochastic volatility (SV) to capture tail risks in macroeconomic forecast distributions and outcomes.

- BVARs have a known track record and can be used for a range of forecasting purposes.
- BVAR-SV models commonly improve on the point and density forecast accuracy of their homoskedastic counterparts.

BVAR-SV models have the potential to capture time-varying tail risks:

- Due to simultaneous shifts in conditional means and variances.
- Even though conditional distributions are symmetric (1-step, not necessarily multi-step).
Innovation volatility estimate: BVAR-SV model, N=5
(standard deviation)

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Our models:

- Include 4 primary macroeconomic indicators and the NFCI indicator of financial conditions
- Focus on risks to GDP growth
- Include conventional BVAR-SV and a BVAR with a generalized factor structure to volatility in which the common factor is a function of past financial conditions
- Robustness checks:
  - Models with 2 and 15 variables
  - Models replacing NFCI with an indicator of financial volatility
  - Forecasting unemployment instead of GDP growth
We provide more formal evaluations of asymmetries and risk forecasts than has much of the recent literature.

- Formal tests of skewness in the data, BVAR residuals, and forecast errors.
- Formal scoring of quantile and expected shortfall forecasts:
  - Conventional quantile scoring function.
  - More recent joint scoring function for the quantile and its associated expected shortfall.
- BVAR-SV vs. ABG-style quantile regression.
Related contemporaneous paper: Caldara, Scotti, and Zhong (2019)

- Our focus is more on forecasting with VARs like those common in the literature, including formal scoring comparisons.
- They focus more on using a bivariate VAR with an explicit correlation in shocks to levels and volatilities to produce time-varying asymmetries in conditional predictive distributions.
Introduction

Main findings

- Formal statistical evidence of skewness in output growth is generally weak.
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- QR-based approaches can come with some challenges in macro data samples: quantile crossing and coefficient variability
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- Formal statistical evidence of skewness in output growth is generally weak
- QR-based approaches can come with some challenges in macro data samples: quantile crossing and coefficient variability
- BVAR-SV models are able to capture time variation in output tail risks — with downside risks more variable than upside risks — like that emphasized in ABG
  - SV crucial
- BVAR-SV and BVAR-GFSV models score about as well as QR for downside tail risks
Outline

1. Models
1 Models
2 Data
3 Forecast Metrics
Outline

1. Models
2. Data
3. Forecast Metrics
4. Empirical Results
1. Models
2. Data
3. Forecast Metrics
4. Empirical Results
5. Conclusions
$y_t = \sum_{i=1}^{p} \Pi_i y_{t-i} + \nu_t$

$\nu_t = A^{-1}\Lambda_t^{0.5}\epsilon_t, \quad \epsilon_t \sim N(0, I_n), \quad \Lambda_t \equiv \text{diag}(\lambda_{1,t}, \ldots, \lambda_{n,t})$

$\ln(\lambda_{i,t}) = \gamma_{0,i} + \gamma_{1,i} \ln(\lambda_{i,t-1}) + \nu_{i,t}, \quad i = 1, \ldots, n$

$\nu_t \equiv (\nu_{1,t}, \nu_{2,t}, \ldots, \nu_{n,t})' \sim N(0, \Phi)$

- $\Lambda_t$ contains the time-varying variances of conditionally Gaussian shocks
- $A$ is uni-triangular
- Reduced-form VCV is $\text{var}(\nu_t) \equiv \Sigma_t = A^{-1}\Lambda_tA^{-1'}$
Models: BVAR-GFSV model

Incorporates a factor structure of volatility in a VAR-SV (CCM 2016, 2017) and links the volatility factor to the lagged NFCI

- Allows a link of poor financial conditions to elevated macroeconomic uncertainty and volatility, capturing the basic idea of ABG
- Each variable’s log vol. follows a linear factor model with a common uncertainty factor $\ln m_t$ and an idiosyncratic component $\ln h_{i,t}$
- Generalizes a much simpler model of a robustness check in ABG

\[
y_t = \sum_{i=1}^{p} \Pi_i y_{t-i} + A^{-1} \Lambda_t^{0.5} \epsilon_t
\]

\[
\ln \lambda_{i,t} = \beta_{m,i} \ln m_t + \ln h_{i,t}, \; i = 1, \ldots, n
\]

\[
\ln m_t = \sum_{i=1}^{p_m} \delta_{m,i} \ln m_{t-i} + \delta_f \text{NFCI}_{t-1} + u_{m,t}, \; u_{m,t} \sim iid \; N(0, \phi_m)
\]

\[
\ln h_{i,t} = \gamma_{i,0} + \gamma_{i,1} \ln h_{i,t-1} + e_{i,t}, \; i = 1, \ldots, n
\]
Models: Priors and Estimation

SV Priors:
- VAR coefficients $\Pi$: Normal, with Minnesota-style prior, means of 0
- $A$, row by row: Normal, prior mean of 0 and variance $10 \cdot I$
- SV process: Normal, with $\gamma_{i,1}$ having mean of 0.9 and st. dev. of 0.2
- $\Phi$: IW with mean of $0.03 \cdot I$ and 10 df

Estimation:
- BVAR-SV estimated with a Gibbs sampler
- GFSV’s volatility factor is sampled with a particle Gibbs step
- We use 5000 retained draws from the posterior predictive distribution
- Symmetry? 1-step ahead predictive distributions are symmetric
- Multi-step predictive distributions don’t have to be symmetric but are, empirically speaking
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Step 1 of 2: Conventional QR estimation

\[ y_{t+h}^{(h)} = x_t' \beta + \epsilon_{t+h} \]

\[ \hat{\beta} = \arg\min_{\beta} \sum_{t=1}^{T-h} \left( \tau \cdot 1(y_{t+h}^{(h)} \geq x_t' \beta) |y_{t+h}^{(h)} - x_t' \beta| + (1 - \tau) \cdot 1(y_{t+h}^{(h)} < x_t' \beta) |y_{t+h}^{(h)} - x_t' \beta| \right) \]

- \( h = \) forecast horizon of either 1 or 4 quarters
- \( y_t = 400 \Delta \ln \text{GDP}_t, \quad y_{t+h}^{(h)} \equiv h^{-1} \sum_{i=1}^{h} y_{t+i} \)
- \( x_t \) includes a constant, \( y_t \), and \( \text{NFCI}_t \)
- Model estimated for quantiles of \( \tau = 0.05, 0.25, 0.75, 0.95, \) and 0.5.
Models: Quantile Regression

Step 1 of 2: Conventional QR estimation

\[ y_{t+h}^{(h)} = x_t' \beta + \epsilon_{t+h} \]

\[ \hat{\beta} = \arg\min_{\beta} \sum_{t=1}^{T-h} \left( \tau \cdot 1_{(y_{t+h}^{(h)} \geq x_t' \beta)} |y_{t+h}^{(h)} - x_t' \beta| + (1 - \tau) \cdot 1_{(y_{t+h}^{(h)} < x_t' \beta)} |y_{t+h}^{(h)} - x_t' \beta| \right) \]

- \( h = \) forecast horizon of either 1 or 4 quarters
- \( y_t = 400 \Delta \ln GDP_t, \quad y_{t+h}^{(h)} \equiv h^{-1} \sum_{i=1}^{h} y_{t+i} \)
- \( x_t \) includes a constant, \( y_t \), and NFCI\(_t\)
- Model estimated for quantiles of \( \tau = 0.05, 0.25, 0.75, 0.95, \) and 0.5.

Step 2: Smooth the estimated quantile function by fitting a skewed-\( t \) distribution

- Fit to quantiles of \( \tau = 0.05, 0.25, 0.75, 0.9 \)
- Use fitted distribution for computing expected shortfall, etc.
VARs include 5 variables: GDP growth, unemployment rate, inflation (GDP deflator), federal funds rate, NFCI

- Results similar with 2 and 15 variables
- Alternative financial measure and longer sample: turbulence measure of volatility from Giglio, Kelly, and Pruitt (2016)
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We consider both in-sample and real-time, out-of-sample forecasts of GDP growth (and unemployment in a robustness check)

- We abstract from the real-time aspect of the NFCI
- In real-time evaluation, actuals = 1st release available in RTDSM
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Samples:

- Estimation with NFCI uses data starting in 1971, ending in 2018
- Estimation with turbulence uses data starting in 1959, ending in 2011
- Real time forecasts start in 1985 with NFCI and 1972 with turbulence
We consider both in-sample and OOS forecasts for small samples of bad outcomes and comparability to ABG
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Basic checks: RMSEs of point forecasts and log scores of forecasts
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Basic checks: RMSEs of point forecasts and log scores of forecasts

Tail risks to GDP growth evaluated at $\tau = 5\%$
- Results similar at $\tau = 10\%$
5% quantile score

\[
QS_{t+h} = (y_{t+h} - Q_{\tau,t+h})(\tau - 1_{(y_{t+h} \leq Q_{\tau,t+h})}),
\]

where \( Q_{\tau,t+h} = \) forecast quantile at quantile \( \tau = 0.05 \)

- 5% quantile \( \equiv \) value at risk (VaR)
Forecast Metrics

5% quantile score

\[ QS_{t+h} = (y_{t+h} - Q_{\tau,t+h})(\tau - 1_{(y_{t+h} \leq Q_{\tau,t+h})}) , \]

where \( Q_{\tau,t+h} = \) forecast quantile at quantile \( \tau = 0.05 \)

- 5% quantile \( \equiv \) value at risk (VaR)

Expected shortfall (ES) and long-rise (LR)

- Shortfall = \( E(\)GDP growth in 5% tail\()\)
- Long-rise = \( E(\)GDP growth in 95% tail\()\)
- BVAR-SV: easily computed with draws from predictive distribution
- QR: Computed with complete skew-\( t \) density functions from smoothing step
Forecast Metrics

Joint VaR-ES score:

- VaR-ES can be jointly elicited, but ES by itself cannot (Fissler and Ziegel 2016)
- We use the joint score of Fissler, Ziegel, and Gneiting (2015)

\[
S_{t+h} = Q_{\tau, t+h} \cdot (1_{(y_{t+h} \leq Q_{\tau, t+h})} - \tau) - y_{t+h} \cdot 1_{(y_{t+h} \leq Q_{\tau, t+h})} \\
+ \frac{e^{\text{ES}_{\tau, t+h}}}{1 + e^{\text{ES}_{\tau, t+h}}} \left(\text{ES}_{\tau, t+h} - Q_{\tau, t+h} + \tau^{-1} (Q_{\tau, t+h} - y_{t+h}) 1_{(y_{t+h} \leq Q_{\tau, t+h})}\right) \\
+ \ln \frac{2}{1 + e^{\text{ES}_{\tau, t+h}}} 
\]
Results: Roadmap

Note: BVAR-GFSV estimates generally very similar to BVAR-SV

Tests of skewness
Results: Roadmap

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Tests of skewness

Practical challenges with QR in small samples
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Estimate of ES and LR: In-sample and out-of-sample forecasts
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Formal assessment of forecast accuracy
Results: Roadmap

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Tests of skewness

Practical challenges with QR in small samples

Estimate of ES and LR: In-sample and out-of-sample forecasts

Formal assessment of forecast accuracy

Robustness checks

- Alternative measure of financial conditions: turbulence
- Alternative measure of economic activity: $\Delta$ unemployment rate
### Results: Skewness

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<th>Data, 1972-2018</th>
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<th>Bai-Ng</th>
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<tr>
<td>GDP growth</td>
<td>−0.364</td>
<td>−0.865</td>
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<tr>
<td>Unemployment</td>
<td>0.683</td>
<td>0.644</td>
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<td>GDP inflation</td>
<td>1.402</td>
<td>1.995**</td>
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<td>Fed funds rate</td>
<td>0.709</td>
<td>0.787</td>
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<tr>
<td>NFCI</td>
<td>1.979</td>
<td>2.016**</td>
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- Right column provides Bai and Ng (2005) time-series robust test statistic for skewness
- Raw data: skewness statistics are often large, but not necessarily statistically significant
Results: Skewness

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<td><strong>BVAR-SV resid., 1972-2018</strong></td>
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<td>GDP growth</td>
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<td>Unemployment</td>
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<td>Fed funds rate</td>
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<td>0.644</td>
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<td>−0.258</td>
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<td><strong>BVAR-SV resid./SV, 1972-2018</strong></td>
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<td>GDP growth</td>
<td>0.129</td>
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<td>Unemployment</td>
<td>0.326</td>
<td>2.783***</td>
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<td>Fed funds rate</td>
<td>−0.119</td>
<td>−0.754</td>
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<tr>
<td>NFCI</td>
<td>0.341</td>
<td>2.557**</td>
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- BVAR-SV residuals: No evidence of skewness
- Normalized BVAR-SV residuals: Evidence of skewness increases some
## Results: Skewness

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<tr>
<td>Unemployment</td>
<td>0.850</td>
<td>1.560</td>
</tr>
<tr>
<td>GDP inflation</td>
<td>−0.367</td>
<td>−1.839*</td>
</tr>
<tr>
<td>Fed funds rate</td>
<td>−0.115</td>
<td>−0.260</td>
</tr>
<tr>
<td>NFCI</td>
<td>1.632</td>
<td>0.826</td>
</tr>
</tbody>
</table>

- OOS forecast errors: A little evidence of skewness
Results: Skewness

- BVAR-SV residuals: Notable departures from normality, esp. for NFCI and FFR
Normalized BVAR-SV residuals: With normalization, departures from normality not so obvious
Results: Skewness

OOS BVAR-SV forecast errors, 1-step: As in not-normalized residuals, there are notable departures from normality
Results: Skewness

QQ plots of OOS forecast errors from BVAR-SV model, N=5
forecast horizon = 4

- GDP
- P-GDP
- NFCI
- UR
- FFR

OOS BVAR-SV forecast errors, 4-step: Again, there are notable departures from normality
In OOS estimates for GDP growth using turbulence (1-step), the 75th and 95th quantiles cross in two periods.
In OOS estimates for GDP growth (1-step), the coefficient on \( NFCI_{t-1} \) can change sharply with the sample, esp. for the tail quantiles.
Results: Empirical challenges with QR

Small samples: Common to use extremal quantile methods for bias correction and inference

- Rule of thumb from Chernozhukov, Fernandez-Val, and Kaji (2017): extremal quantile methods should be used when $\tau T/k \leq 15$ to 20, where $\tau =$ quantile, $T =$ sample size, and $k =$ # regressors.

- GDP-NFCI application: with $T = 160$, $\tau = 0.05$, and $k = 3$, $\tau T/k \approx 2.7$
In-sample forecast estimates display asymmetries highlighted by ABG:
ES variability > LR variability
Pattern stronger at $h = 1$ than $h = 4$
BVAR-SV comparable to QR, esp. at $h=1$
Asymmetries still present but a little weaker OOS than in-sample

QR-based estimates have some **upside** asymmetries in the 1990s

OOS compared to IS: BVAR-SV estimates noisier; QR estimates less variable
Results: Predictive distributions

What drives the pattern in the BVAR estimates?

- Need SV to get conditional variance to move over time
- Need financial conditions
What drives the pattern in the BVAR estimates?

- Need SV to get conditional variance to move over time
- Need financial conditions

Monte Carlo experiments replicate variability of ES compared to LR

- Bivariate BVAR-GFSV based on GDP growth and NFCI
- BVAR-SV performance comparable to BVAR-GFSV
Results: Accuracy of in-sample forecasts of GDP growth

<table>
<thead>
<tr>
<th></th>
<th>BVAR-SV/QR</th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$h = 1Q$</td>
<td>$h = 4Q$</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.943</td>
<td>0.962</td>
</tr>
<tr>
<td>Log score</td>
<td>0.044*</td>
<td>0.031</td>
</tr>
<tr>
<td>5% QS</td>
<td>0.982</td>
<td>1.081</td>
</tr>
<tr>
<td>5% VaR-ES</td>
<td>−0.073</td>
<td>−0.143</td>
</tr>
</tbody>
</table>

|                     |           |              |
| RMSE                | 0.942     | 0.958        | 0.988      | 1.013        |
| Log score           | 0.051**   | 0.053        | 0.019      | −0.010       |
| 5% QS               | 0.965     | 1.081        | 0.980      | 1.067        |
| 5% VaR-ES           | −0.016    | −0.073       | −0.032     | −0.112       |

- Conventional point and density forecasts: BVARs and QR broadly similar in accuracy
- Quantile score: Same
- VaR-ES score: Same, although GFSV sometimes better than SV and closer to QR performance
Results: Accuracy of out-of-sample forecasts of GDP growth

<table>
<thead>
<tr>
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<tbody>
<tr>
<td></td>
<td>$h = 1Q$</td>
<td>$h = 4Q$</td>
<td>$h = 1Q$</td>
<td>$h = 4Q$</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.875*</td>
<td>0.873</td>
<td>0.867</td>
<td>0.819</td>
</tr>
<tr>
<td>Log score</td>
<td>0.062*</td>
<td>0.146</td>
<td>0.065</td>
<td>0.162</td>
</tr>
<tr>
<td>5% QS</td>
<td>1.099</td>
<td>0.877</td>
<td>1.158</td>
<td>0.767</td>
</tr>
<tr>
<td>5% VaR-ES</td>
<td>−0.251*</td>
<td>0.420</td>
<td>−0.355*</td>
<td>0.591</td>
</tr>
</tbody>
</table>

|                  |               |                  |                  |                  |
| RMSE             | 0.874**        | 0.872            | 0.860*           | 0.821            |
| Log score        | 0.111***       | 0.176            | 0.122***         | 0.186            |
| 5% QS            | 0.982          | 0.801            | 0.989            | 0.689            |
| 5% VaR-ES        | −0.104         | 0.651            | −0.135           | 0.782            |

- OOS: In conventional point and density forecasts, BVARs beat QR
- Quantile score: BVAR-GFSV as good as or better than QR, BVAR-SV a little more mixed
- VaR-ES score: BVAR-GFSV comparable to QR, BVAR-SV not quite as good as GFSV
QR doesn’t seem to offer any advantages over BVAR-SV or BVAR-GFSV specifications
Results: GDP growth forecast takeaways

QR doesn’t seem to offer any advantages over BVAR-SV or BVAR-GFSV specifications

QR itself simple, but:

- 2nd step for smoothing adds complexity
- Need to estimate separate models for each quantile-horizon combination
QR doesn’t seem to offer any advantages over BVAR-SV or BVAR-GFSV specifications

QR itself simple, but:
- 2nd step for smoothing adds complexity
- Need to estimate separate models for each quantile-horizon combination

BVARs with time-varying volatility offer a viable alternative
- BVARs broadly useful for forecasting (point forecasts, scenario analysis, etc.)
- Single model can cover all variables, horizons, and quantiles of interest
- Need time-varying volatility to get shifts in conditional variances
Robustness results using turbulence: Predictive distributions

Expected longrise and shortfall: GDP growth
baseline QR vs. turbu. QR

1-step ahead forecasts
baseline QR longrise  turbu. QR longrise
baseline QR shortfall  turbu. QR shortfall

4-step ahead forecasts
baseline QR longrise  turbu. QR longrise
baseline QR shortfall  turbu. QR shortfall

- QR: Replacing NFCI with turbulence greatly increases ES variability (even more so OOS)
Robustness results using turbulence: Predictive distributions

Expected longrise and shortfall: GDP growth
turbu. BVAR-SV vs. baseline BVAR-SV

1-step ahead forecasts
turbu. SV longrise
turbu. SV shortfall
baseline SV longrise
baseline SV shortfall

4-step ahead forecasts
turbu. SV longrise
turbu. SV shortfall
baseline SV longrise
baseline SV shortfall

- BVAR-SV: Estimates similar with turbulence as compared to NFCI
Robustness results using unemployment: Predictive distributions

In-sample estimates show considerable time variation in upside risk
LR more variable than ES
Contours of QR and BVAR-SV estimates are similar
Evidence of asymmetries in LR vs. ES is somewhat weaker out-of-sample

Considerable choppiness of QR estimate in 1990s
Robustness results using unemployment: Accuracy of in-sample forecasts

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<tbody>
<tr>
<td>5% QS</td>
<td>1.010</td>
<td>0.825</td>
</tr>
<tr>
<td>5% VaR-ES</td>
<td>0.060</td>
<td>0.092</td>
</tr>
</tbody>
</table>

- In-sample tail risk forecasts: SV and GFSV estimates about as accurate as QR
Robustness results using unemployment: Accuracy of out-of-sample forecasts

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</thead>
<tbody>
<tr>
<td>5% QS</td>
<td>1.092</td>
<td>1.125</td>
</tr>
<tr>
<td>5% VaR-ES</td>
<td>−0.014</td>
<td>−0.069</td>
</tr>
</tbody>
</table>

- Out-of-sample tail risk forecasts: SV and GFSV estimates about as accurate as QR
In general, a BVAR-SV model performs as well as quantile regression in measuring and forecasting tail risks to economic activity.

- Key features for BVAR: time-varying volatility and inclusion of financial conditions.
- Captures simultaneous shifts in conditional means and variances.
In general, a BVAR-SV model performs as well as quantile regression in measuring and forecasting tail risks to economic activity.

Key features for BVAR: time-varying volatility and inclusion of financial conditions.
Captures simultaneous shifts in conditional means and variances.

Key findings:
- Statistical evidence of skewness in output growth is generally weak.
- QR-based approaches can come with some challenges in macro data samples: quantile crossing and coefficient variability.
- BVAR-SV models are able to capture time variation in output tail risks — with downside risks more variable than upside risks — like that emphasized in ABG.
- BVAR-SV scores as well as QR for downside tail risks.