Banking Complexity in the Global Economy

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Abstract

International lending flows are often intermediated through banking hubs and complex multi-national routing. We develop a dynamic stochastic general equilibrium model where global banks choose the path of direct or indirect lending through partner institutions in multiple countries. We show how conflating locational loan flows with ultimate lending biases results both by attributing ultimate lending to banking hubs, and by missing ultimate lending that occurs indirectly via third countries. We next study the effects of banking complexity. Intense indirect lending allows countries to bypass shocked lending routes via alternative countries; however, it dilutes their ability to diversify sources of funds after shocks. The quantitative analysis reveals that banking complexity can exacerbate credit and output instability when countries feature heterogeneous banking efficiency.
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1 Introduction

Banks' global cross-border claims amount to more than 35 trillions of US dollars for the first time since 2009, recovering from the post-Global Financial Crisis retrenchment (Claessens, 2017). An unknown but potentially substantial share of lending flows are intermediated between source and destination countries, through the presence of a ramified network of bank affiliates (Allen, Gu, and Kowalewski, 2013) and complex financial routes through one or more third countries (Coppola, Maggiori, Neiman, and Schreger, 2021). The prevalence of these indirect flows obscures the ultimate source of lending supply and demand from official statistics on the bilateral flows of funds. Further, little is known about the role of these indirect banking linkages in the global propagation of shocks.

We develop an N-country dynamic stochastic general equilibrium model of lending where globally active banks choose the path of lending through an endogenously formed network of affiliates or partner institutions in multiple countries. Banking hubs arise endogenously as central nodes in the financial intermediation network. The model provides a framework to reconcile observable international statistics with theoretical models of banking gravity. It generates a set of bilateral *locational* flows of funds that conceptually matches aggregate (observable) Bank of International Settlement (BIS) locational banking statistics (LBS), as distinct from the ultimate demand and supply of lending, or *ultimate* lending. The model shows how conflating BIS LBS flows with ultimate demand and supply biases empirical results, and can even result in sign reversals. Moreover, the model simulations reveal that accounting for indirect lending flows is crucial for understanding the propagation of shocks and the impact of global banking on aggregate fluctuations.

In our economy, in each country, banks produce loans using deposits and loan officer labor (e.g. as in Goodfriend and McCallum (2007)), and offer loan contracts internationally to firms. Extending loans internationally is costly and banks use their heterogeneous global networks to minimize these costs. They can either lend funds directly to firms in destination countries, or they can choose to lend indirectly through one or more third countries. Indirect lending can lower costs when, for example, subsidiaries or partners in third countries can more cheaply acquire information on borrowers in destination countries. For instance, a US multinational bank may transfer funds to a subsidiary in the Netherlands whose loan officers better know (and hence more effectively lend to) borrowing firms in Belgium. Overall, banks choose the cheapest option for their liquidity to reach final loan demand.

Following Allen and Arkolakis (2022), we model the choice of lending paths through intermediate countries by assuming that path costs are a product of all bilateral intermediation frictions–*edge costs*—incurred along the path as well as path-idiosyncratic costs. This allows us to aggregate the path choices of individual banks and to derive analytic expressions for the share of all bank lending from each origin country i, to each destination country j, using each intermediate country k. The general equilibrium of the model generates a gravity equation where the bilateral gravity friction is a non-geographic, endogenous outcome that corresponds to bilateral network proximity. The model yields a closed-form expression relating the ultimate origin-destination (supply-demand) lending, the cost structure of the network, and the locational flows of funds, equivalent of the BIS LBS.

The model allows not only to simulate the response of the global economy to shocks at nodes, corresponding to TFP or banking shocks in one country, but also to edges connecting nodes, e.g. reflecting sanctions limiting a single cross-border loan flow. In both cases, we focus on how locational loan flows, ultimate origin-destination lending, as well as salient macroeconomic variables (investment and output) are affected, and contrast their responses to those obtained in a network-free model (i.e., where all loans flow directly to the destination country).

Using the model, we first show how empirical specifications conflating the BIS LBS data with ultimate origin-destination lending biases results in two directions: first, by missing indirect flows between the countries, and second by counting funds not originated or destined for the countries as lending. We show these biases in our first simulation exercise, examining the propagation of a TFP shock hitting one country. While unaffected countries substitute lending towards affected countries, as would be predicted in a model without indirect flows, locational flows tell a different story: bilateral flows between unaffected countries *decrease*, picking up the reduction in loans to and from the shocked country flowing (indirectly) through unaffected countries.

Next, we explore the global impact of a shock to a single connection (edge) in the network. Here, not only can the qualitative predictions of the effects on bilateral loan flows differ from a model without indirect lending, but, further, locational flows can move in opposition to the ultimate lending flows and incorrectly confirm the no-network predictions.[1] These theoretical and simulated results underscore the need to disentangle locational flows from ultimate lending and may help explain third-country effects as empirically documented in Hale et al. (2020) and Kalemli-Ozcan et al. (2013).

We then operationalize our edge shocks to examine the influence of banking complexity on the international propagation of shocks. We consider complexity in the form of policy or technology changes that foster denser international networks supporting more indirect international lending.[2] The effects of banking complexity

[1]More precisely, for any single *node* shock in our setting, an isomorphic single shock exists in a network model without indirect linkages. However, the same is not true for edge shocks.

[2]In our model, our notion of financial complexity corresponds to changes in the parameter governing the dispersion and relative importance of path productivity in intermediation. We also consider the effects of financial integration due to reductions in overall bilateral intermediation costs.

are multifaceted in our context: on the one hand, a complex banking network can make countries more resilient to shocks to lending costs by offering alternative paths through third countries. On the other hand, third countries' usage of indirect lending exposes their lending routes to shocks as well, inhibiting their ability to act as substitute sources of liquidity to shock-affected countries. Put differently, global financial complexity can increase the scope for diversification of lending *pathways*, allowing lending to evade increases in bilateral costs by substituting towards other indirect lending channels. However, it can reduce the scope for diversification of *sources*: as more banks rely on global partners in more countries for cross-border lending, an increase in cost in one location is more likely to impact the ability of third country banks to move liquidity internationally.

To quantitatively evaluate the relative strengths of the above mechanisms, we repeat the edge shock experiments in a series of increasingly complex environments. As the banking network becomes more complex (more indirect linkages), the benefit of improved path diversification dominates the reduced ability of the network to offer lending source diversification. On balance, in our baseline calibration, financial complexity moderates the drop in bilateral lending flows caused by an edge shock, thereby stabilizing investment and output in the shocked countries. We also obtain, however, that the relative strengths of the above mechanisms and, hence the overall effect of financial complexity, can differ when countries feature highly heterogeneous banking efficiency or when their banking productivity responds endogenously to network connectivity. Intuitively, when the banking sectors of third countries are highly efficient, the dilution of source diversification induced by a more complex banking network is particularly harmful. We reach similar conclusions when we experiment with edge shocks hitting multiple bilateral links. In such scenarios, deeper financial complexity, i.e. stronger indirect links, can have a destabilizing effect, amplifying rather than mitigating shocks.

Our first contribution is to the literature on the geography of banking and the determinants and aggregate implications of international lending flows. The BIS LBS is the most extensive source of international banking statistics. This has prompted many studies on the macroeconomics of banking to use locational lending data. However, the proliferation of indirect lending, due to the expansion of large multinational banking conglomerates and international syndicated lending markets as well as the growing exploitation of tax havens (Coppola et al., 2021), has resulted in an increasing misalignment between these statistics and ultimate bilateral lending. This may lead to biased conclusions about the effects of global banking on the international transmission of shocks (e.g. Kalemli-Ozcan et al. (2013)). Our DSGE model of indirect lending provides a conceptual framework for the relationship between ultimate and locational flows. We analytically describe the nature of these biases and study quantitatively their direction and magnitude following shocks.[3]

Our second contribution is to the literature on the macroeconomic effects of global banking and banking integration. Kalemli-Ozcan et al. (2013), Niepmann (2015) , Cao et al. (2021), and Morelli et al. (2022) stress that larger multinational banking groups can increase the exposure of countries to shocks hitting a common multinational lender. However, financial integration can also allow for better diversification of funding sources in the aftermath of shocks hitting individual countries. We highlight the consequences of financial complexity: the greater availability of alternative indirect lending paths and the possible dilution of the ability of countries to serve as alternative sources of funding. Our analysis is in spirit related to Fillat et al. (2018), who investigate how the branches and subsidiary composition of multinational banks affects the international transmission of shocks.[4] Our emphasis is not on the structure of branches and subsidiaries, but on the distinction between direct and indirect bank lending.

Finally, the paper broadly relates to the theoretical and empirical literature on financial and production networks. Studies explore the international transmission of the Lehman Brothers collapse via syndicated loans (De Haas and Van Horen, 2012), monetary policy transmission via production networks, business cycle synchronization via trade linkages (Juvenal and Monteiro, 2017), or the role of trade credit in amplifying financial shocks (Altinoglu, 2021). We embed an endogenously formed banking network in a DSGE multi-country model of banking and study the aggregate consequences of financial complexity through this framework. This enables us, for example, to study the effects of network edge shocks in a dynamic general equilibrium setting, which captures a large class of potential scenarios.[5] In this dimension, the analysis also speaks to a set of papers that explore financial contagion (Acemoglu, Ozdaglar, and Tahbaz-Salehi, 2015, Allen and Gale, 2000, Elliott, Golub, and Jackson, 2014). We define and focus on a different aspect of integration related to internal networks of global conglomerate banks exploiting indirect lending. We then study scenarios related to disruptions to international financial linkages, rather than country (node) shocks. This leads us to disentangle distinct mechanisms, path and source diversification, and the dispersion of indirect lending as driving forces of shock propagation.

[3]A related group of studies examine the existence of gravity in banking. Using BIS LBS data, Buch (2005) and Papaioannou (2009) find that distance is a relevant predictor of cross-border bank lending. See also Portes and Rey (2005), Aviat and Coeurdacier (2007), Buch, Neugebauer, and Schröder (2013). However, Delatte, Capelle-blancard, and Bouvatier (2017) show that an empirical gravity equation for banking does a poor job of rationalizing the data.

[4]For empirical studies, see, e.g., Cetorelli and Goldberg (2012) and De Haas and Van Horen (2012).

[5]Oberfield (2018) provides a partial equilibrium model where producers probabilistically match with upstream and downstream firms, giving rise to an endogenous input-output structure. Acemoglu and Azar (2020) allow for an endogenous choice of intermediate goods in an input-output framework; however they do not explicitly take into account network paths.

The remainder of the paper is organized as follows. In Section 2 we set up the model and solve for agents' decisions. Section 3 studies the equilibrium. In Section 4, we present the model calibration and assess the biases resulting from conflating locational and ultimate flows. Section 5 studies the effects of banking complexity. In Section 6 we consider extensions. Section 7 concludes. Technical derivations and proofs are relegated to the Online Appendix.

2 Model Setup

We present a discrete time dynamic general equilibrium model with endogenous international banking linkages. In this section, we specify agents' problems and study the funding decisions of the economy's heterogeneous banks. Banks' route choice problem and the network equilibrium are examined in subsequent sections.

In each country $i \in \mathcal{N}$ there are four sectors: the household sector, the firm (goods production) sector, the capital production sector, and the banking sector (comprising banking consulting firms and banks). All agents are owned by households, who supply labor to the goods production and banking sectors and consume a single non-tradable final good, produced by competitive domestic firms. Capital and labor are immobile across countries. Firms hire labor from households, purchase physical capital from capital producers, and demand a diversified aggregate loan to finance their capital investment. Banking consulting firms acquire the individual loans varieties $\omega \in \Omega$ through competitive international markets and produce the aggregate loan demanded by the firms via CES bundling.[6] Risk-neutral banks produce loans using households' deposits and labor (e.g., as loan officers) and offer debt contracts to consulting firms internationally through cost-minimizing lending paths.

2.1 Households

Households earn a wage rate w_t^H on labor supplied to the goods sector (H_t). They also earn a wage rate w_t^M on labor supplied to the banking sector as loan monitoring activity (M_t). Further, they earn a gross rate of return $(1+R_t^D)$ on deposit holdings D_t. They use their funds for consumption C_t and saving through deposit holdings, solving:

$$\max_{\{C_{i,t},H_{i,t},M_{i,t}\}_{t\geq 0}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_{i,t} - k_H \frac{H_{i,t}^{1+\epsilon}}{1+\epsilon} - k_M \frac{M_{i,t}^{1+\varphi}}{1+\varphi} \right)$$

$$s.t. \quad C_{i,t} + D_{i,t} = (1+R_{i,t-1}^D) D_{i,t-1} + w_{i,t}^H H_{i,t} + w_{i,t}^M M_{i,t} + \Pi_{i,t}, \qquad (1)$$

[6] The framework can capture the range of financial products that firms need from banks, e.g. credit lines and term financing, among others, or reflect sectoral specialization of financial institutions.

where ϵ is the inverse Frisch elasticity for labor supplied to the production of goods and φ is the inverse Frisch elasticity for labor supplied to banking activities. The parameters k_H and k_M govern the disutility from labor in the two sectors. The net transfers received by households comprise profits from owning firms ($\Pi^F_{i,t}$), banks ($\Pi^B_{i,t}$) and capital producers ($\Pi^K_{i,t}$).

Households maximize their lifetime utility by choosing consumption $C_{i,t}$, deposit holdings $D_{i,t}$, labor supply $H_{i,t}$ to firms, and labor supply $M_{i,t}$ to banks.[7] Henceforth, we denote by $\Lambda_{t,t+j} = \beta^t \frac{u'_{C_{t+j}}}{u'_{C_t}}$ the households' stochastic discount factor and, for notational simplicity, drop the subscript on the wage in the banking sector ($w^M = w$).

2.2 Capital Producers and Firms

Capital producers Capital producers invest in $I_{i,t}$ units of capital goods at the cost of $I_t\left[1 + f(I_{i,t}/I_{i,t-1})\right]$ units of consumption goods, where the continuous, convex function $f(\cdot)$, with $f(1) = 0$, $f'(1) = 0$, captures the adjustment cost in the capital-producing technology. They choose the amount of new capital $I_{i,t}$ to maximize the present discounted value of lifetime profits:

$$\max_{I_{i,t}} \qquad \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{i,0,t} \left\{ P^K_{i,t} I_{i,t} - \left[1 + f\left(\frac{I_{i,t}}{I_{i,t-1}}\right)\right] I_{i,t} \right\}. \tag{2}$$

$\Lambda_{t,t+j} = \beta^t \frac{u'_{C_{t+j}}}{u'_{C_t}}$ is the stochastic discount factor, since households own the capital producers and are the recipients of any profits $\Pi^K_{i,t}$. The first order condition

$$P^K_{i,t} = \left\{1 + f\left(\frac{I_{i,t}}{I_{i,t-1}}\right) + \frac{I_{i,t}}{I_{i,t-1}} f'\left(\frac{I_{i,t}}{I_{i,t-1}}\right)\right\} - \mathbb{E}_t \Lambda_{i,t,t+1} \left(\frac{I_{i,t+1}}{I_{i,t}}\right)^2 f'\left(\frac{I_{i,t+1}}{I_{i,t}}\right) \tag{3}$$

sets the price of capital goods $P^K_{i,t}$ equal to the marginal cost of capital production.

Firms The representative firm uses labor $H_{i,t}$ and capital $K_{i,t-1}$ to produce final good $Y_{i,t}$ via an increasing, concave, and constant returns to scale technology. Firms must obtain loans to finance purchases of capital. They do so by demanding bundled loans $X_{i,t}$ from banking consulting firms.[8]

Firms in country i maximize the discounted sum of dividends distributed to households, subject to the budget constraint (4), the technological constraint (5), the

[7] Households' and firms' optimizing conditions are reported in the Apppendix.

[8] This is similar to Craig and Ma (2020), where firms delegate their borrowing from lending banks to larger, diversified intermediary banks.

law of accumulation of the capital stock (6), and the financing constraint (7):

$$\max_{\{H_{i,t},K_{i,t},X_{i,t},I_{i,t}\}} \mathbb{E}_0 \sum_{j=0}^{\infty} \Lambda_{i,t,t+j+1} \Pi^F_{i,t}$$

$$s.t. \quad \Pi^F_{i,t} + P^K_{i,t} I_{i,t} + (1 + R^X_{i,t-1}) X_{i,t-1} = Y_{i,t} + X_{i,t} - w^H_{i,t} H_{i,t}, \tag{4}$$

$$Y_{i,t}(K_{i,t-1}, H_{i,t}) = A_{i,t} K^{\alpha}_{i,t-1} H^{1-\alpha}_{i,t}, \tag{5}$$

$$K_{i,t} = (1-\delta) K_{i,t-1} + I_{i,t}, \tag{6}$$

$$X_{i,t} = P^K_{i,t} I_{i,t}. \tag{7}$$

$R^X_{i,t}$ denotes the net interest rate on loan bundles, δ is the capital depreciation rate and $A_{i,t}$ captures the aggregate total factor productivity (TFP).

2.3 The Banking Sector

The banking sector comprises banking consulting firms and banks. Banks collect deposits from households and lend to domestic and foreign consulting banks, which aggregate loans and extend them to firms. There are two facets of the bank lending technology. First, banks produce loans using deposits and monitoring effort. Second, loans can reach ultimate demand in destination countries directly or indirectly through lending paths, p, involving third countries.

Consulting firms In each country, competitive consulting firms combine loans of type ω from the lowest-cost international suppliers to produce an aggregate non-traded loan $X_{i,t}$ using a Dixit-Stiglitz technology

$$X_{j,t} = \left(\sum_i \int_{\omega \in \Omega} x_{ij,t}(\omega)^{\frac{\sigma-1}{\sigma}} \, d\omega \right)^{\frac{\sigma}{\sigma-1}}, \tag{8}$$

where $\sigma > 1$ is the constant elasticity of substitution across loan varieties.

Banks For each loan variety ω, there is a set of risk-neutral, competitive banks in each country i.[9] We refer to banks by their loan variety index ω. To maintain tractability, we posit that banks return dividends to households at the end of each period. This framework (combined with the i.i.d. assumption introduced below) is effectively isomorphic to a setting where banks exit in every period, as in Boissay,

[9] ω banks could be differentiated in terms of maturity, industry, or other contract characteristics. As in Eaton and Kortum (2002), ω banks within countries are homogeneous, including their idiosyncratic path choices. This can be relaxed by considering a large-N number of banks of each ω type with heterogeneous networks available to them, i.e. heterogeneous path choices. The banking sector could be alternatively modeled via monopolistic competition, as in Gerali, Neri, Sessa, and Signoretti (2010). We provide a Melitz (2003) version of the banking partial equilibrium in the supplement.

Collard, and Smets (2016).[10] We accordingly drop the time subscript from the bank problem.

Loan production Building on Goodfriend and McCallum (2007), banks produce loans by combining monitoring hours $M_i(\omega)$ and deposits $D_i(\omega)$ via a Leontief production technology.[11] They maximize net cash flows taking as given the banking sector wage w and the net interest rate on deposits R^D. A loan $x_{ij}(\omega)$ offered to a firm in country j entails a route-dependent intermediation cost $\tau_{ij}(\omega, p) \geq 1$, modeled as an interest rate markup.[12]

Since the choice of the inputs does not affect the lending path optimality, and viceversa, we present the two subproblems (loan production and loan path selection) sequentially. Here, we describe banks' input choice problem in loan production. Substituting the usual asset and liabilities balance, a bank's problem reduces to maximizing net cash flows subject to the loan production function:

$$\max_{M_i(\omega), D_i(\omega)} \sum_j \frac{r_{ij}(\omega)}{\tau_{ij}(\omega, p)} y_{ij}^X(\omega) - w_i M_i(\omega) - R_i^D D_i(\omega) \tag{9}$$

$$s.t. \quad y_i^X(\omega) = \min\{z_i M_i(\omega), D_i(\omega)\} \tag{10}$$

where z_i is a country-specific monitoring productivity in the banking sector.[13] Competition implies that banks from i charge a net interest rate in j equal to their marginal cost:

$$r_{ij}(\omega, p) = c_i \tau_{ij}(\omega, p) \tag{11}$$

where the unit loan production cost, c_i, net of the bilateral intermediation friction, satisfies:[14]

$$c_i = \frac{w_i}{z_i} + R_i^D. \tag{12}$$

[10] In our model, their setting would imply that banks are born and collect deposits in $t-1$, hire workers in period t, and die at the end of period t, so that deposit rates, wages, and profits are payed to households in period t. Differently from Boissay, Collard, and Smets (2016) (BCS), the Eaton and Kortum (2002) structure implies that shocks are realized before banks choose inputs. This implies that individual banks do not face input mismatches, neither for liquidity (as in BCS) nor for labor. The model could also be augmented with an explicit interbank market, of which we provide an example in the supplement.

[11] Our model can be related to the two-tier Gerali, Neri, Sessa, and Signoretti (2010) banking structure, since the Leontief production function makes the two inputs (deposits and loan officers) complements and both necessary for the issuance of loans. A Cobb-Douglas loan production technology as in Goodfriend and McCallum (2007) would imply $c_i = a\,(w_i/z_i)^{\zeta}\left(R_i^D\right)^{1-\zeta}$ with a being the usual constant.

[12] This explicitly incorporates the cost into the banks' optimization problem, without resorting to the notion of iceberg losses as in standard gravity models.

[13] In our main exposition we treat z_i as exogenous. In Section 6 we relax the exogeneity assumption by making it network-dependent.

[14] All banks have the same technology, hence $c_i(\omega) = c_i$.

The ex-ante bilateral intermediation friction $\tau_{ij}(\omega, p)$ is both bank ω and path p specific. In the next section, we describe how we obtain observed interest rates $r_{ij}(\omega, p)$ and equilibrium quantities via banks' optimal routing decisions.

Loan path selection The decision facing banks of how to move funds internationally closely follows Allen and Arkolakis (2022). It is useful to introduce here some network terminology. A graph (network) G is a finite ordered pair of disjoint sets $(\mathcal{N}, E)$ such that E is a subset of the set $\mathcal{N}^{(2)}$ of unordered pairs of $\mathcal{N}$, the set of vertices (nodes, countries, or locations), and E is the set of edges, where an edge $\{k, l\}$ joins the nodes k and l in $\mathcal{N}$.

Moving liquidity across each edge, from any country (node) k to any country (node) l, is costly and incurs an intermediation friction $e_{kl} > 1$. These costs are primitive parameters of the model. They reflect country or country-pair characteristics such as distance, language, or other characteristics which affect monitoring or regulatory frictions, thus increasing the cost of lending. In Appendix A we microfound these "reduced form" markups as intermediation and monitoring costs of intermediary agents at each node.[15] It is important to note that because lending in equilibrium will often be indirect, these costs do not correspond to bilateral frictions in gravity models. Rather, the latter will arise endogenously through the banks' route choice problem as the average bilateral lending friction.

A path, or route, is a graph $p \in P$ which consists of an ordered sequence of nodes of length K_p and a set of edges that connects the nodes, following the sequence, i.e. $\mathcal{N}(P) = \{i, k_1, \ldots, j\}$, and $E(P) = \{\{i, k_1\}, \{k_1, k_2\}, \ldots, \{k_{N-1}, j\}\}$. To minimize a loan's intermediation cost $\tau_{ij}(\omega, p)$, a bank chooses the lowest-cost path through which to send the loan. Banks in i can extend a loan to j through the direct path $p_{direct} = \{i, j\}$ or indirectly through one or more intermediary locations k comprising a path of length greater than 1. Specifically, a bank ω's cost $\tau_{ij}(p, \omega)$ of sending funds from i to j through path p can be decomposed into the path-specific series of edge cost markups that are shared by all banks and a bank-specific cost:

$$\tau_{ij}(p, \omega) \equiv \frac{\tilde{\tau}_{ij}(p)}{\xi_{ij}(p, \omega)}. \tag{13}$$

$\tilde{\tau}_{ij}(p) \equiv \prod_{k=1}^{K_p} e_{k-1,k}$ is the (deterministic, shared) portion of costs related to bilateral liquidity transfers and $\xi_{ij}(p, \omega)$ is the intermediation cost of moving a loan of a given type ω from i to j that is specific to the organizational structure of affiliates associated with p. For example, these costs may capture the difficulty a given set of loan officers and report structures face in managing and monitoring loans of a specific type or

[15] In the main text, intermediation wages are paid in the originating country. In the Appendix, we explore extensions where intermediation requires that labor and wages are paid along the path, similar to Antràs and De Gortari (2020).

industry.[16] We further assume $\xi_{ij}(p,\omega)$ is an i.i.d. realization of draws from a Fréchet distribution with shape parameter $\theta > \max\{1, \sigma - 1\}$: $F(\xi_{ij}(p,\omega)) = \exp\left\{-\xi^{-\theta}\right\}$.[17] The shape parameter $\theta > \max\{1, \sigma - 1\}$ governs comparative advantage through the degree of heterogeneity across loans. When θ is large, banks have similar relative costs between paths. When it is small, the bank-specific portion of intermediation costs dominates.

ω-banks at i offering loans to j choose the least cost path to send their loan, such that:

$$\tau_{ij}(\omega) \equiv \min_{p \in P} \tau_{ij}(p,\omega). \tag{14}$$

3 Equilibrium

We first derive the partial equilibrium probabilities of loan and route choices, then distinct equilibrium gravity equations for locational and ultimate liquidity flows, and finally close the model.

3.1 Path Probabilities

After banks identify least-cost paths, consulting firms in j choose the lowest-cost supplier of loan ω from all countries $i \in I$. Using the standard Eaton and Kortum (2002) method, but summing across loan types for each path, we obtain the following:

Lemma 1 (Gravity probability)**.** *The probability that borrowers in country j choose to obtain a loan from country i through a path p is:*

$$\lambda_{ijp\omega} = \frac{\left[c_i \tilde{\tau}_{ij}(p)\right]^{-\theta}}{\sum_l c_l^{-\theta} \sum_{p \in G} \left[\tilde{\tau}_{lj}(p)\right]^{-\theta}}. \tag{15}$$

Proof. See appendix B.2.

Similar to Eaton and Kortum (2002), across all loan types, this is also the share of all loans extended in country j that come from i and take path p, λ_{ijp}. Overall, countries with lower loan production marginal costs and with paths incurring lower bilateral intermediation costs will account for a larger fraction of the loans extended to a country j.

[16] An alternative would be to model the edge frictions as additive. We prefer the multiplicative form which is consistent with intermediary interest markups.

[17] The i.i.d. assumption allows to maintain tractability and obtain analytical results.

3.2 Network Costs

We can define the term $\sum_{p\in G_{ij}} \tilde{\tau}_{ij}(p)^{-\theta}$ in the denominator of (15) as the *bilateral network cost*:

$$\tau_{ij} \equiv \sum_{p\in G_{ij}} \tilde{\tau}_{ij}(p)^{-\theta} \propto E(\tau_{ij}(p)) \tag{16}$$

τ_{ij} proportional to the average or expected bilateral cost, summing over all the possible paths' costs conditional on selection.It is a single measure that provides a summary of direct and indirect frictions between i and j. Reordering paths by length K, we can consider this sum as capturing K-th order connections between i and j, and as such, it is inversely related to the (measure of the) network proximity between i and j:

Lemma 2 (Network Costs)**.** *Let A be the inverse cost matrix where the ijth element of A is e_{ij}, and let b_{ij} denote an element of the matrix $B \equiv (\mathbf{I}-A)^{-1}$. For a constant γ*

$$\tau_{ij} = \gamma b_{ij}^{-1/\theta}. \tag{17}$$

Proof. See appendix B.3.

In the remaining of the paper, we sometimes prefer working directly with the matrices A and B rather than with the edge costs e or the network costs τ. With a slight abuse of interpretation, we call these edge and network probabilities, respectively.

3.3 Ultimate and Locational Gravity

We now describe how the above network probabilities (and hence, the network costs τ) give rise to aggregate variables: ultimate (X_{ij}) and locational (Ξ_{ij}) bilateral loan flows, and the aggregate interest rate that firms face in a given country, R_j^X.

We first derive a closed-form expression for the composite loan interest rate $R_{j,t}^X$:

$$R_j^X = \vartheta\left(\sum_i \left(c_i\tau_{ij}\right)^{-\theta}\right)^{-\frac{1}{\theta}} \tag{18}$$

where $\vartheta = \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)^{\frac{1}{1-\sigma}}$ is a constant.[18] The derivation is in appendix B.5.

Ultimate gravity Starting from lemma 1, summing across routes and using the network probabilities, we obtain the share of total loans demanded in country j obtained

[18]Note that in a one-country setup the aggregate interest would be equal - simplifying the weights - to $R_i^X = w_i + R_i^D$, and we would obtain a simple expression similar to the interest rate spreads in Goodfriend and McCallum (2007) or Gerali, Neri, Sessa, and Signoretti (2010).

from country i:

$$\lambda_{ij} = \frac{\left(c_i \tau_{ij}\right)^{-\theta}}{\sum_l \left(c_l \tau_{lj}\right)^{-\theta}} = \frac{\left(c_i \tau_{ij}\right)^{-\theta}}{\vartheta^{\theta} R_j^{-\theta}}. \tag{19}$$

Because our model conforms to the assumptions in Allen et al. (2020), we can obtain a relationship between these shares and aggregate loan demand: $X_{ij} = \lambda_{ij} X_j$. Jointly, this recovers a network gravity equation:

$$X_{ij} = \vartheta^{-\theta} R_j^{\theta} X_j c_i^{-\theta} \tau_{ij}^{-\theta}. \tag{20}$$

As usual, the gravity equation relates bilateral ultimate lending to loan demand in j, $R_j^{\theta} X_j$, loan production efficiency in i, $c_i^{-\theta}$, and a bilateral friction. However, recalling that $\tau_{ij}^{-\theta} = \left(\sum_{p \in G_{ij}} \prod_{k=1}^{K_p} e_{k-1,k}(p)^{-\theta}\right)$, rather than a primitive, this bilateral friction is the average cost of lending across paths of different length through the network. It accounts for the effect of the complexity of the entire banking network on ultimate bilateral lending, including the possibility of different path lengths and third country effects, using the network probability τ_{ij}.

Locational gravity Bank loans can take a direct path from country i to country j, as well as an indirect path, through a series of countries $k, l, \ldots, K$. In this section, we derive a *locational* gravity equation, i.e. the amount of loans that go through an edge e_{kl}, without necessarily originating at k nor stopping at l. Let $\psi_{kl|ij}$ denote the conditional probability of a bank ω serving country j from county i, and going through countries k and l. We can prove the following:

Lemma 3. *The probability of any loan product ω going through a kl edge, conditional on being originated in i and ultimately extended in j is*

$$\psi_{kl|ij} = \frac{b_{ik} a_{kl} b_{lj}}{b_{ij}} = \left(\gamma \frac{\tau_{ij}}{\tau_{ik} e_{kl} \tau_{lj}}\right)^{\theta}. \tag{21}$$

Proof. See appendix B.4.

The first expression for the conditional probability $\psi_{kl|ij}$ can be visualized in Figure 1. The link probability, a_{kl}, is the probability of going from k to l, conditional on the rest of the banking network. The network probabilities b, instead, capture all the possible paths from origin country i to the intermediate step k (upstream), and all the possible paths from the intermediate step l to the destination country j (downstream); where upstream and downstream are relative to the intermediate step kl. The second expression for $\psi_{kl|ij}$, written in terms of costs, relates the probability of using kl to the cost e_{kl} and the average network cost of moving loans from i to k and from l to j.

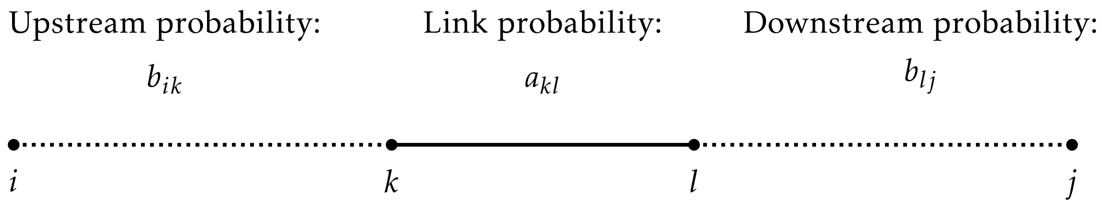


Figure 1: Locational gravity: probability interpretation.

$\psi_{kl|ij}$ is a microfounded, probabilistic version of the edge betweeness centrality formula, such that the importance of an edge depends on the amount of shortest lending paths that go through it.[19]

We can now obtain the total volume of locational loan flows between k and l, Ξ_{kl}, summing over all the origin i and destination j countries:

$$\Xi_{kl} = \sum_i \sum_j X_{ij} \psi_{kl|ij} = \sum_i \sum_j X_{ij} \frac{b_{ik} a_{kl} b_{lj}}{b_{ij}}. \tag{22}$$

This locational loan volume includes loan flows bound for l and those continuing onward to other destination countries. As in Ganapati, Wong, and Ziv (2020), locational loan flows Ξ are a function of loan intermediation costs and direct loan flows X only. The effects on loan flows of country-specific features such as loan production efficiency and equilibrium loan rates, as well as shocks to these, are subsumed by X, and affect locational flows proportionately on all routes according to $\psi_{kl|ij}$. Since, following Allen and Arkolakis (2022), there is a unique set of banking direct loan flows X consistent with observed country characteristics, market clearing conditions, and banks' optimization, there exists a unique locational matrix Ξ of banking flows.

Is the United Kingdom the same as the Caymen Islands? The equilibrium delivers a set of both ultimate (X_{ij}) and locational (Ξ_{ij}) bilateral loan flows. A country with

[19]To express node betweeness centrality, we can we present a triangular ikj version of the same, which is more intuitive: Ξ_{ikj} is the probability of a loan going from country origin i to destination country j through a third country k:

$$\Xi_{ikj} = \sum_l \psi_{kl|ij} X_{ij} = \sum_l \psi_{kl|ij} \lambda_{ij} X_j = \psi_{k|ij} \lambda_{ij} X_j$$

where the second line uses the definition of the bilateral loan flows, as the product of loan demand and the share λ. This is related to equation 7 in Arkolakis, Ramondo, Rodríguez-Clare, and Yeaple (2018) model of multinational production.

a solid presence of internationally active banks, e.g. Italy, would be characterized by higher direct loan flows $X_{\text{ITA},j}$. Countries with a developed banking sector that also act as banking hubs, e.g. Great Britain, would originate *and* intermediate loans, hence exhibiting higher ultimate flows $X_{\text{UK},j}$ *and* locational flows, both outward $\Xi_{\text{UK},l}$ and inward $\Xi_{k,\text{UK}}$. Tax-haven countries, instead, would be characterized by lower ultimate flows, both inward and outward, while exhibiting higher locational flows: they generate a small volume of loans, but they facilitate the intermediation of loans through their jurisdiction.

The comparison between locational loan flows and loans originated in a country allows to distinguish different scenarios. For example, one could consider the statistic $(\Xi_{ik} + \Xi_{kj})/Y_k^X$, which measures the total flow of locational loans in and out of country k relative to the total loans originated in country k. Two countries (nodes), such as the United Kingdom and the Caymens, could both have large locational loan flows going through them (i.e., a high value of $(\Xi_{ik} + \Xi_{kj})$) but exhibit a very different value of this statistic. The Caymens would have a high value of the above ratio, as it intermediates a large volume of international loans while producing very few; Great Britain would instead produce a substantial amount of loans, besides acting as an intermediary node in the global banking network. Hence, it would feature a lower value of the ratio.

3.4 Closing the Model

We define the aggregate loan supply and the aggregate demand of the banking sector for labor and deposits by aggregating across the individual loan varieties:

$$Y_{i,t}^X = \int_\Omega y_{i,t}^X(\omega)d\omega, \qquad M_{i,t} = \int_\Omega M_{i,t}(\omega)d\omega, \qquad D_{i,t} = \int_\Omega D_{i,t}(\omega)d\omega.$$

Aggregate demand for monitoring hours (loan officers) and deposits are, respectively:

$$M_{i,t} = Y_{i,t}^X/z_i. \qquad D_{i,t} = Y_{i,t}^X. \tag{23}$$

The cost of loan generation in country i is absorbed globally:

$$w_{i,t}M_{i,t} + R_{i,t}^D D_{i,t} = \sum_j \lambda_{ij,t} R_{j,t}^X X_{j,t}. \tag{24}$$

Aggregate transfers to households equal the sum of the profits of final good firms, capital producers and banks:

$$\Pi_{i,t} = Y_{i,t} - w_{i,t}^H H_{i,t} - (1 + R_{i,t-1}^X)X_{i,t-1} + P_{i,t}^K I_{i,t} - I_{i,t}\left[1 + f\left(\frac{I_{i,t}}{I_{i,t-1}}\right)\right]. \tag{25}$$

A competitive equilibrium is defined in the usual way: all agents optimize taking prices as given, and the markets for goods, capital, deposits, aggregate and individual loans, and both types of labor clear.[20]

4 Model Analysis

In this section, we study the mechanisms of propagation of shocks to nodes and edges through the banking network both through the model and simulations. We derive then illustrate the bias that arises from observing locational loan flows in place of ultimate lending. Locational loan flows can distort the measure of - or even reverse the sign of - a shock's impact on ultimate lending.

In what follows, when simulating the dynamic behavior of our model economy following shocks, we consider a framework with $\mathcal{N} = 3$ countries, and note whenever we expect implications to vary for a scenario with $\mathcal{N} > 3$. This allows us to disentangle the key mechanisms in a tractable and transparent way.

4.1 Baseline Calibration

Table 1 presents calibrated values for parameters common to all countries. Agnostically, we calibrate the parameters of the countries symmetrically. As a baseline, we also set intermediation frictions to be equal. The model is calibrated to quarterly frequency and solved numerically by locally approximating around the non-stochastic steady state.

We use fairly standard parameters for preferences and technologies. The Frisch elasticity of labor supply is set to 4 in both the final goods and the banking sectors, in line with the suggestion by Chetty, Guren, Manoli, and Weber (2011) for macro models. The discount factor is calibrated to 0.9975, implying a yearly steady state deposit rate (R^D) of around 1%, as in Goodfriend and McCallum (2007). In the final goods sector, the depreciation rate of capital is set to $\delta = 0.025$ implying an annual depreciation rate of 10%. In the capital producing sector, we specify the investment adjustment function as $f(I) = -\eta/2\,(I_t/I_{t-1} - 1)^2$, with $\eta = 1.728$, as in Gertler, Kiyotaki, and Queralto (2012). In the banking sector, we set the elasticity of substitution across loan product varieties at $\sigma = 1.471$ as in Gerali, Neri, Sessa, and Signoretti (2010). We initially set the Fréchet parameter to $\theta = 2$ so that, in conjunction with the chosen values for edge costs, we obtain sufficient short-run dispersion of lending routes as well as plausible values of cross-border relative to domestic lending (see below). In section 5 we investigate the role of θ and the sensitivity of the results to its value.

[20] The social resource constraint can be omitted by Walras law.

The remaining parameters are the labor disutility parameters (k_i^H and k_i^M) and the monitoring productivities z_i. We pick these parameters to jointly hit the following three targets: total hours worked ($M+H$) equal to 1/3 of GDP, a ratio of the wage in the banking sector relative to the wage in the goods producing sector of four thirds, and a steady-state spread between the loan and deposit rates of 100 basis points.[21]

Table 1: Calibration of selected common parameters

Description	Symbol	Value
Preferences		
Household discount factor	β	0.960
Inverse Frisch elasticity for H	ϵ	0.250
Inverse Frisch elasticity for M	φ	0.250
Firms		
Capital share of output	α	0.330
Capital depreciation	δ	0.025
Inverse elasticity of net investment to the price of capital	η	1.728
Banks		
Loans elasticity of substitution	σ	1.471
Fréchet shape parameter	θ	2.000

The model also requires calibration of the 3×3 matrix A of bilateral edge costs. As noted, we posit symmetry in lending intermediation frictions across the three countries. The calibrated values of the edge cost parameters $e_{ij}, i, j \in \{1,3\}$ must satisfy the condition that the spectral radius of the matrix A (defined in lemma 2) is less than 1.[22] We calibrate the common value of the edge cost parameters so that, together with θ, we match two ratios for lending flows: a ratio between cross border and domestic lending of 3 to 7; and a ratio for direct to indirect lending so that, on average, for every dollar of observed locational lending, slightly more than 50% are ultimate. We discuss the role of different network topologies in Section 5.3.

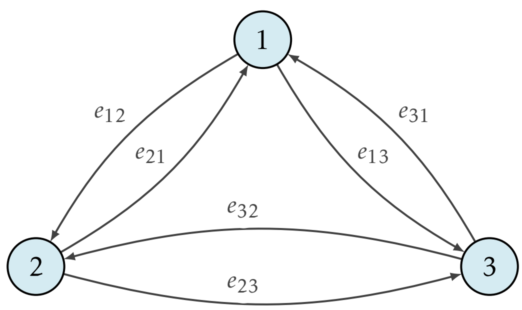


Figure 2: A 3-country set of edge costs

[21] In the US data the hourly wage is around \$40 in the financial sector, \$30 in manufactoring. Similar ratios apply to the Euro area.

[22] This is in order to apply the geometric sum that leads to matrix B. A sufficient condition for the spectral radius being less than 1 is $\sum_j e_{ij}^{-\theta} < 1$ for all i, which is satisfied in all calibrations.

4.2 Edge and Node Shocks

In what follows we will consider both shocks to nodes and edges. We will study the responses of loan flows as well as the responses of salient macroeconomic variables, including investment, labor, and output.

Node shocks can encompass, for example, disturbances to firm TFP or banking efficiency of a country. Edge shocks consist of changes to bilateral intermediation costs e_{ij} between country pairs. Allowing for edge shocks is crucial for studying changes in bilateral frictions such as the introduction of financial sanctions (e.g. limitations to Russia's access to SWIFT) or new bilateral financial regulations (e.g., Brexit). Node shocks do not affect the network structure. That means that their general equilibrium propagation through the network to other nodes functions isomorphically to how propagation would occur in a model without indirect lending, where each bilateral friction exactly equals the bilateral network cost in our model. By contrast, a change in a single edge cost e_{ij} determines a change in the edge probability $a_{ij} = e_{ij}^{-\theta}$. The presence of path and indirect loan flows in turn implies a change in the entire matrix of network probabilities, via $B = (\mathbb{I} - A)^{-1}$, and thus of all network costs τ_{ij}.

4.3 Confounding Ultimate and Locational Loan Flows

In what follows, we investigate the importance of distinguishing between ultimate and locational loan flows, first in the context of a node shock and then of an edge shock. The empirical literature has often preferred locational data for two reasons: cross-sectional and time-span availability, and the claim that locational flows better capture bilateral exposures, contagion, or synchronization. On the other hand, empirical studies (e.g. Hale et al., 2020) encounter major puzzles when using such locational loan flows, as discussed below.

Recalling equation (22), we can write the bias when using locational loan flows Ξ_{kl} in place of ultimate flows X_{kl}:

$$\Xi_{kl} - X_{kl} = \underbrace{\sum_{\{i,j\}\backslash\{k,l\}} X_{ij}\psi_{kl|ij}}_{\substack{\text{locational flows associated} \\ \text{with indirect lending}}} - \underbrace{X_{kl}(1-\psi_{kl|kl})}_{\substack{\text{ultimate flows not} \\ \text{transmitted through } kl}} . \tag{26}$$

The first term is the total amount of loans from all sources i to all destinations j, through kl. It represents a non-negative error that comes from attributing locational loan flows through kl to ultimate kl lending. Such error, for example, would be large in the case of tax haven countries with low loan origination. The second term represents flows that originate in k for destination l, hence ultimate loan flows, but that take longer paths. This is a negative error coming from undercounting kl lending that circumvents kl by using other network nodes. Overall, the more a node k is used as a hub for l, the more empirical work using Ξ_{kl} will overstate lending flows from k to l; the more lending from k to l flows through alternative hubs, the more locational flows will understate lending. The sign of the net error for each edge kl is ultimately a function of the network structure. The direction of the bias arising from these errors will be a function of the correlation between the net error and any variable of interest.

4.3.1 The case of a node shock (TFP)

We first explore the bias from equation (26) when a country is hit by a negative TFP shock, and the matrix of bilateral costs is unchanged. Changes to bilateral loan flows happen via general equilibrium effects which involve changes in domestic wages or interest rates which affect ultimate and *then* locational loan flows.

Proposition 1 (Node shock Impact on Ultimate and Locational Flows)**.** *A node shock (e.g., TFP) in country k results in the following change in ultimate and locational loan flows:*

$$\frac{\partial X_{ij}}{\partial TFP_k} = \frac{\partial X_j}{\partial TFP_k}\lambda_{ij} + X_j\frac{\partial \lambda_{ij}}{\partial TFP_k}, \qquad \frac{\partial \Xi_{k',l'}}{\partial TFP_k} = \sum_{i,j}\frac{\partial X_{ij}}{\partial TFP_k}\psi_{k'l'|ij}. \tag{27}$$

Hence, the bias from observing locational loan flows is:

$$\sum_{\{i,j\}\setminus\{k,l\}}\left(\underbrace{\frac{\partial X_j}{\partial TFP_{k'}}\lambda_{ij}}_{\substack{\text{GE effects on}\\ \text{indirect flows}}} + \underbrace{X_j\frac{\partial \lambda_{ij}}{\partial TFP_{k'}}}_{\substack{\text{Indirect}\\ \text{substitution lending}}}\right)\psi_{kl|ij} - \left(\underbrace{\frac{\partial X_l}{\partial TFP_{k'}}\lambda_{kl} + X_l\frac{\partial \lambda_{kl}}{\partial TFP_{k'}}}_{\substack{\text{Effect on flows not}\\ \text{transmitted through } kl}}\right)(1-\psi_{kl|kl}). \tag{28}$$

The impact of a node (TFP) shock on ultimate loan flows $\frac{\partial X_{ij}}{\partial TFP_k}$ would look the same in a standard gravity model. However, this is due to the fact that in our model the complexity of the network is captured via expected costs τ_{ij} which, in turn, affect the loan shares λ_{ij}. Equation (27), in turn, shows that the impact on the locational loan flows between k' and l' is the weighted average effect of the impact on the ultimate flows, with weights being the network relevance of the edge $k'l'$ in the paths starting in all i to all destinations j. Finally, equation (28) shows how observed locational loan flows belie the true bilateral effects of the shock. When the shock incentivizes to move loans through a link or substituted lending to move more funds indirectly through that link, the bias tends to be positive, while the true bilateral effects are understated to the extent that funding avoids flowing through that bilateral

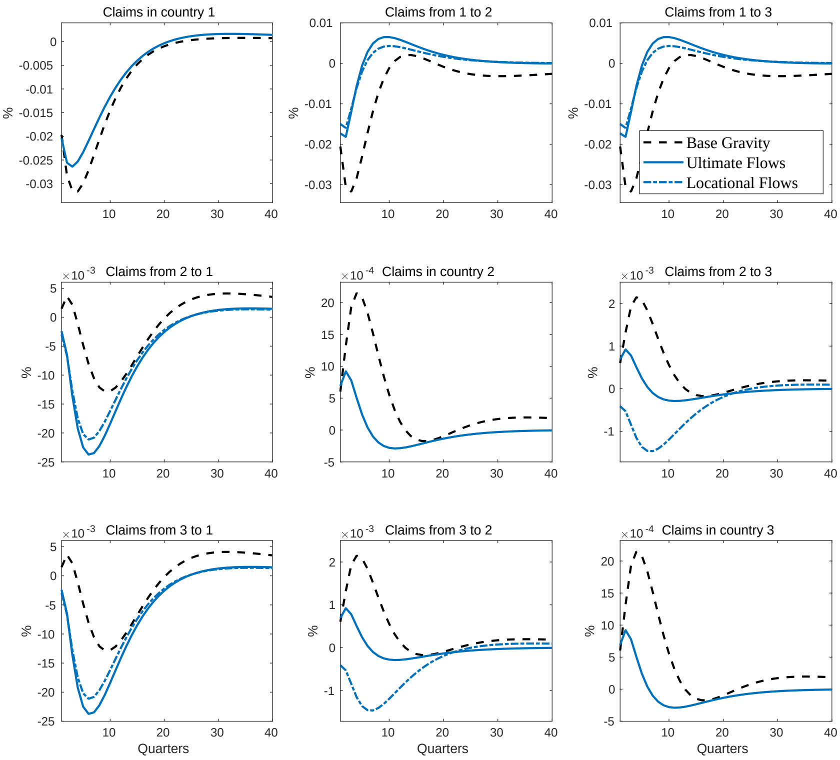


Figure 3: Bilateral Ultimate and Locational Lending, Node Shock

link.[23]

Illustrating the proposition, Figure 3 shows the impulse responses for all loan flows to a 1 percent negative TFP shock in country 1. The black, dashed lines correspond to loan flows in the base gravity case, i.e. in an economy with the same bilateral frictions but without network paths and locational flows.[24] The solid blue lines correspond to ultimate flows determined by equation (20), while the blue dotted lines correspond to locational flows determined by equation (22).

As expected, both our model and the comparison economy agree on the direction of the shock's effect on the ultimate loan flows. As standard in the IRBC literature, a negative TFP shock in country 1 reduces investment, and hence the amount of domestic credit, X_{11}. Absent other frictions[25] it also leads to a credit flight, that is, a decrease in X_{21} and X_{31}. Remarkably, the figure, particularly the response of the loan flows between non-shocked countries (2 and 3), reveals the bias resulting from using locational loan flow data to proxy for ultimate lending, as shown in equation (28). Observed locational data would suggest a decrease in lending. However, this decrease does not correspond to the ultimate loan flows: country 2 diverts lending away from now-less-productive firms in 1 towards firms in country 3. In contrast

[23] This interpretation ignores differential intensity of link usage through loops in paths.

[24] An alternative would be to calibrate the bilateral intermediation frictions in the comparison economy to exactly match the bilateral network costs.

[25] For example, different collateral liquidation technologies, as in Cao, Minetti, Olivero, and Romanini (2021).

with this increase in lending from 2 to 3 due to substitution, the net effect on the locational loan flows moves in the opposite direction. Because part of the ultimate loans to and from 1 flow indirectly through the 2 to 3 link, the decrease in lending to and from 1 results in observed decreases in locational loan flows between 2 and 3. This effect may help explain the negative third-country effects documented empirically by Hale et al. (2020).

4.3.2 The case of an edge shock (temporary sanctions)

We next consider the case of an edge shock, a counterfactual exercise where bilateral costs are increased (e.g., as a result of financial sanctions or decoupling).

Proposition 2 (Edge shock Impact on Flows). *An edge shock e_{kl} results in the following change in ultimate loan flows:*

$$\frac{\partial X_{ij}}{\partial e_{kl}} = \underbrace{\frac{\partial X_j}{\partial e_{kl}}\lambda_{ij} + X_j \frac{\partial c_i^{-\theta}}{\partial e_{kl}} \frac{\lambda_{ij}}{c_i^{-\theta}} + X_j \frac{\partial R_j^{-\theta}}{\partial e_{kl}} \frac{\lambda_{ij}}{R_j^{-\theta}}}_{\text{GE effects}} + \underbrace{X_j \frac{\partial \tau_{ij}^{-\theta}}{\partial e_{kl}} \frac{\lambda_{ij}}{\tau_{ij}^{-\theta}}}_{\text{network effects}} \tag{29}$$

and the following change in locational flows:

$$\frac{\partial \Xi_{k',l'}}{\partial e_{kl}} = \underbrace{\sum_{i,j} \frac{\partial X_{ij}}{\partial e_{kl}}}_{\text{average effect}} \psi_{k'l'|ij} + \underbrace{\sum_{i,j} X_{ij} \frac{\partial \psi_{k'l'|ij}}{\partial e_{kl}}}_{\text{diversion}}. \tag{30}$$

The first term in (29) is the effect of a change in intermediation frictions on total loan demand X_j. The second and third terms capture the effects through the marginal loan production cost and interest rates. The final term captures the changes through bilateral network costs. In off-the-shelf gravity models, the bilateral network cost in the last term would be replaced with (direct) bilateral frictions, and be zero unless $i = k$, $j = l$. This would bias the first three terms, with the direction of bias depending on the shape of the network.

Equation (30) separates the effect of an edge shock at kl on locational loan flows along edge $k'l'$ into two weighted averages: the average effect on ultimate loan flows for all bilateral pairs, weighted by the proportion of each flow using $k'l'$, and the effect on the proportion of ultimate loan flows using $k'l'$, weighted by the size of those ultimate flows. The first term, expanded above in equation (29), is the total effect of the edge shock on all lending. The second term represents the extent to which the edge shock diverts loan flows through the network towards or away from $k'l'$.

To illustrate the proposition, we experiment with an unexpected ten percent pos-

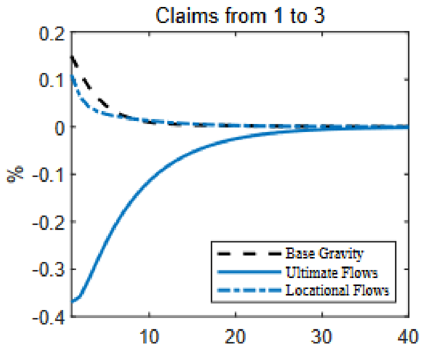


Figure 4: Ultimate and Locational Loans from 1 to 3, Edge Shock

itive shock to $e_{1,2}$. To fix ideas, we can think of country 1 as representing the EU bloc, country 2 Russia, and country 3 other BRICS countries. The shock could capture financial sanctions imposed on lending from the EU to Russia. As would be expected, the loan flows X_{12} decrease.[26] Figure 4 focuses on the effects between countries 1 (EU) and 3 (BRICS), between which there is no direct shock.[27] Loans from the EU bloc (country 1) to BRICS countries (country 3) also decrease, as the shock also impedes indirect lending between the two through Russia, raising the average cost of bilateral lending. The baseline gravity model, though, cannot account for indirect lending and predicts an increase in lending due to the diversion of loan flows from 2 (Russia) to 3 (BRICS) countries. In a twist, locational loan flows confirm the comparison model with no indirect lending for the wrong reasons. While naive observers would attribute these observed flows to ultimate loan flows, this increase is due to EU banks' funding reaching Russia through BRICS countries, and not to lending to BRICS countries as substitution.

The response of real variables In Figure 5 we display the impulse responses of salient real variables to an edge shock in our economy and in the comparison economy without network lending. The responses align with expectations: the negative edge shock depresses investment and output in all economies. On the other hand, the drop in investment and output in countries 2 and 3 is mitigated in our framework relative to the comparison, while the opposite occurs for the investment and output drop in country 1. In our framework country 3 appears to act less as a substitute source of liquidity for country 2, which insulates its firms from an outflow of loans toward country 2. Moreover, country 2 overall appears to benefit from the network structure. Country 1, on other other hand, experiences a bigger outflow of loans toward country 2.

[26] The full set of impulse responses can be found in the Appendix.

[27] Appendix Figure A1 plots all 9 bilateral flows. Appendix Figure A2 plots bilateral lending flows for permanent edge shocks.

In what follows, we rationalize these observations by examining how indirect lending affects the transmission of shocks through the banking network.

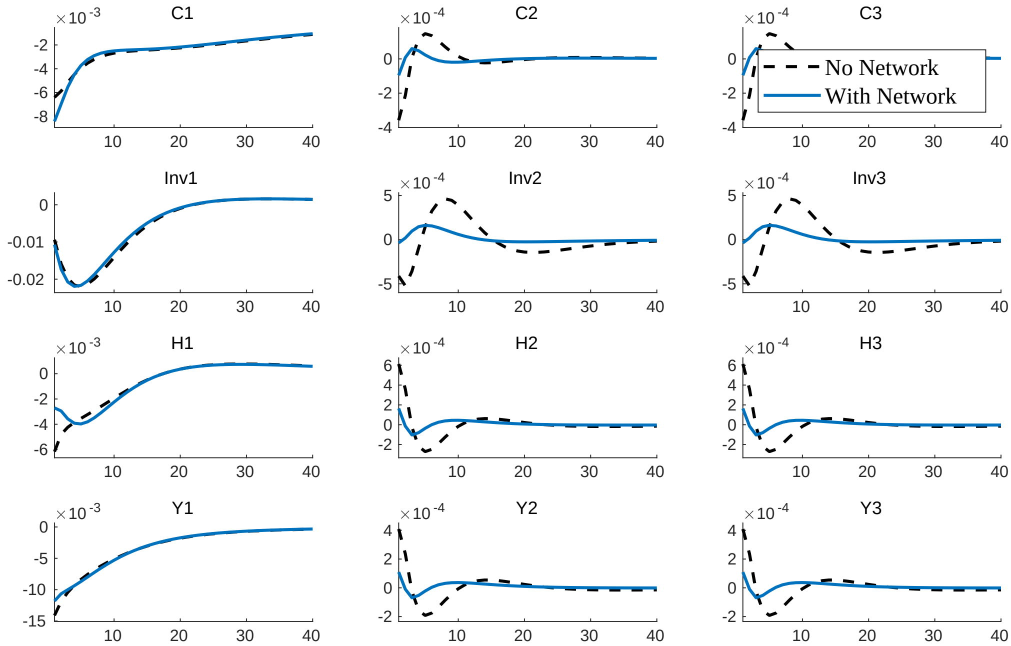


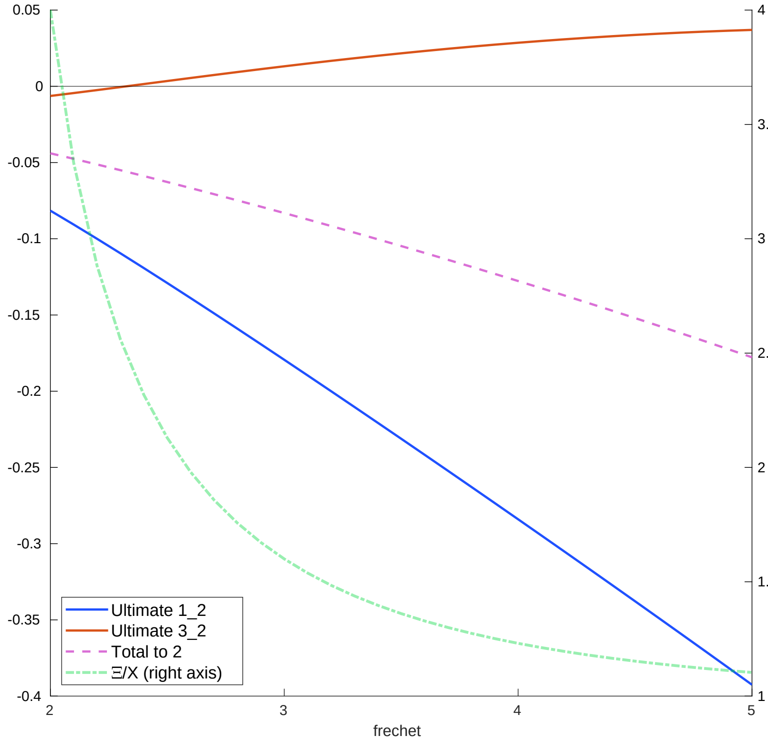
Figure 5: Responses of Real Variables to Edge Shock

5 Banking Complexity

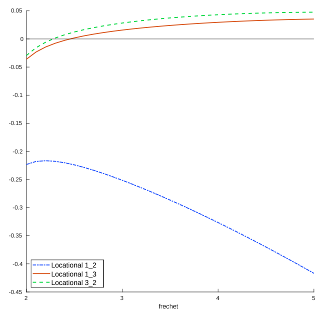
In this section, we explore how banking complexity (the prevalence of indirect lending) affects shock propagation. We imagine banking complexity as stemming from policy or technology changes affecting multinational and international banking activity such as facilitating local bank branching or reducing restrictions on cross-national bank mergers or large private syndicated loan networks.

Specifically, we repeat the edge shock exercise while gradually changing the value of θ, the parameter governing the relative importance of idiosyncratic draws and the prevalence of indirect lending. Lower θ values generate a thicker tail of idiosyncratic draws and corresponds to a world where such banking networks are more active. An alternative way to capture financial transformations would be adjusting the edge cost matrix, through general reductions in cross-border banking frictions (financial integration). We explore this in Section 5.3.

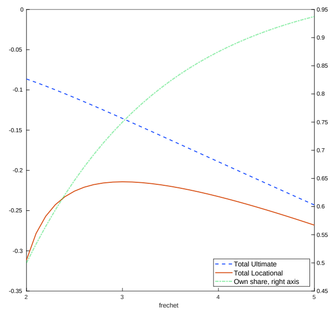
Greater financial complexity generates two competing forces. On the one hand, it allows for greater substitution of lending paths through the network: bank lending can circumvent a shocked link between two countries, which mitigates the impact and cross-border transmission of the shock. On the other hand, financial complexity leads to larger network spillovers, and a shock can be amplified because more lending



(a) Ultimate Flows



(b) Locational flows



(c) Total cross-border flows

Figure 6: Banking Complexity and the Response of Loan Flows to Edge Shocks

relationships across more countries are exposed to the shock through the network.

5.1 Source and Path Diversification

Lending to country 2 In Figure 6a, we graph the immediate impact of the shock to edge cost $e_{1,2}$ on ultimate lending flows to country 2 using values of θ ranging from 2 to 5, at which point nearly all locational flows are ultimate lending and the domestic lending share increases to above 90%. In Appendix Figure A3 we show robustness to considering the average response over the first four periods after the shock. The blue line in Figure 6a represents $X_{1,2}$, ultimate lending from 1 to 2, with 2 being more costly to reach following the edge shock. The orange line represents $X_{3,2}$, ultimate lending from 3 to 2, with country 3 representing country 2's alternative source of credit. The dashed red line represents total ultimate lending to country 2.

Consider first ultimate lending from country 1 to country 2 (blue line). Unsurprisingly, for any value of θ, such loan flow shrinks following the shock. Yet, this drop is less pronounced for lower values of the Frechét parameter, that is, greater financial complexity (i.e. when the global banking network features larger indirect loan flows). To gain insights into this effect, we open up the final term in equation

(29). Substituting for the effect on 1 to 2 lending, the network term becomes

$$\underbrace{X_2 \frac{\partial \tau_{12}^{-\theta}}{\partial e_{12}} \frac{\lambda_{12}}{\tau_{12}^{-\theta}}}_{\text{network effects}} = -X_{12} \left[\underbrace{\frac{\theta}{e_{12}} + \frac{\theta}{\tau_{11}} \frac{d\tau_{11}}{de_{12}} + \frac{\theta}{\tau_{22}} \frac{d\tau_{22}}{de_{12}}}_{\text{direct effect}} + \underbrace{\frac{1}{\psi_{12|12}} \frac{d\psi_{12|12}}{de_{12}}}_{\text{path diversification}} \right] \tag{31}$$

where the first three terms capture the direct effect of the shock on the network-average cost of lending from 1 to 2 through the direct link, and the fourth term captures the change in the probability that a given unit of loans uses a path with the direct link. When indirect liquidity paths are important (θ is relatively low), country 1 will be less exposed to the heightened cost of the direct link with country 2, that is, the proportion of paths using the direct link, and hence the first three terms, will be lower. Further, the banking network gives banks in country 1 the ability to shift loans to paths which avoid the direct link, as captured by the fourth term: *path diversification* achieved via indirect linkages of the banking network.

If an interconnected world with large indirect loan flows can benefit from a higher diversification of lending paths, it can, however, lose along an alternative dimension: *source diversification*. Considering ultimate lending from country 3 to country 2 (orange line in Figure 6a). At relatively high values of the Frechét parameter (i.e., in a poorly integrated world with little indirect lending), loan flows respond positively to the $1,2$ edge shock, reflecting a source-diversification mechanism: increased interest rates in country 2 induce country 3 to act as an alternative source of credit, supplanting the drain of liquidity from country 1. However, in a financially complex economy with more indirect lending (i.e., low θ), this substitution effect is weakened. The network effect term from Equation (29), which would not appear in a model without networks, shows the exposure of loans from 3 to 2 (those flowing through country 1) to the $1,2$ edge shock. As above, some portion of these newly exposed lending flows can divert back to a direct path. However, on net the network effect must be negative, undermining the ability of country 3 to act as an alternative source of funding. Indeed, as we move *left* from $\theta = 5$ to $\theta = 2$ along the graph, ultimate lending from 3 to 2 shrinks, rather than increases, following the edge shock.

The overall effect of the shock on the ultimate loan flows to country 2 (red line in Figure 6a) depends on how the impact of path diversification compares with that of source diversification. For lower values of the Frechét parameter, that is, when the global banking network features thicker indirect lending paths, the overall loan flow to country 2 drops less, suggesting that the gain in loan path diversification outweighs the loss in loan source diversification. Below, however, we will see that this conclusion depends both on the nature of the shock and on the relative efficiency of the banking sectors of the individual countries. In particular, the relative strengths of the above mechanisms can be reversed when considering shocks to multiple edges or scenarios where substitute lending sources are relatively more efficient in loan production.

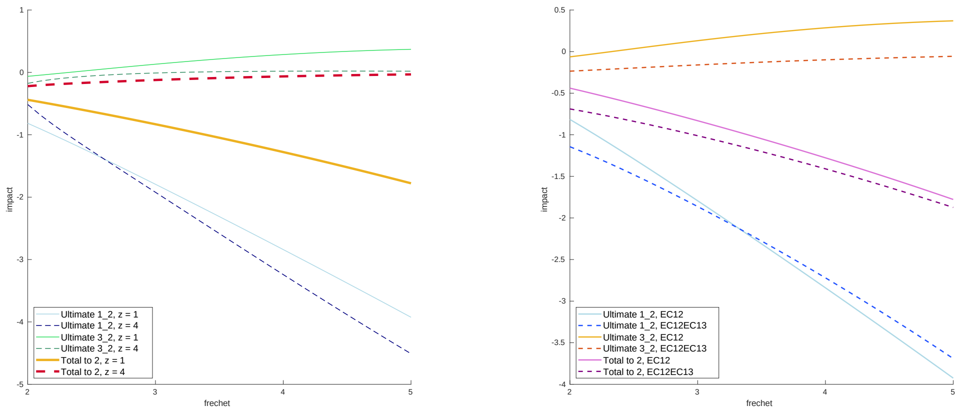
In section 4, we found that the conventional approach of looking at locational loan flows can lead to distorted conclusions about the mechanisms of transmission of shocks. Those same biases highlighted in section 4 can lead to distorted conclusions about the consequences of banking complexity by confounding the two forces, path and source diversification. Figure 6b shows that in this specification, while for the most part the direction of change in locational and ultimate bilateral loan flows are consistent, at high degrees of banking complexity, further complexity appears to exacerbate shocks by reducing locational lending from 1 to 2. This impression belies the continued stabilizing role of loan path diversification.

Global loan flows How does banking complexity affect the overall volume of global lending following an edge shock? As shown in Figure 6c, the effect of the $1,2$ edge shock on the global ultimate and locational loan flows is negative, and stronger for higher values of θ (lower banking complexity). This reflects the interaction of the aforementioned countervailing forces. High-θ, poorly integrated environments are less resilient; the low degree of dispersion in lending routes prevents countries from circumventing shocked links. Indeed, following the shock, country 3 fails to become a hub for 1 to 2 lending. Maximal source diversification, with loans from 1 to 3 replacing those extended by 2, and 3 replacing country 1 as a lending source for 2, moderates the impact of the shock but cannot offset the poor loan path diversification.

From a policy perspective, environments that foster larger, ramified global banking groups and syndicated lending consortia allow for a banking system which can more effectively circumvent disturbances to international lending by flexibly rerouting lending through alternative networks. However, precisely because such large networks are more common, a single disturbance also generates implications for many more countries which would otherwise would not be directly impacted. More integrated settings, with more intense indirect lending, mean that disturbances in any part of the banking network can have spillovers on lending efficiency between a broad range of countries. The net effect on global banking complexity depends on the relative magnitude of these forces. As we see below, that is ambiguous a priori and network-dependent.

5.2 When Can Banking Complexity be Destabilizing?

In this section, we explore the race between the path and source diversification forces. We first modify our calibration then the nature of the shocks in order to see if the



(a) Heterogeneous Banking Efficiencies **(b)** Multiple Edge Shocks

Figure 7: Banking Complexity and Loan Responses, Alternative Scenarios

conclusion in the previous section on the consequences of banking complexity can be reversed.

Heterogeneous banking efficiency Countries can differ significantly in the size and efficiency of their banking sectors. Here we show that such heterogeneity can shift the relative strengths of the two competing forces, loan path and loan source diversification, altering the impact of an edge shock in the global banking network. To this end, we allow for cross-country differences in the value of the productivity of loan officers in monitoring loans, z_i. If third countries unaffected by a shock feature a highly productive banking sector, their diminished ability to act as alternative sources of liquidity can be particularly harmful and even outweigh the benefits of loan path diversification.

In Figure 7a we re-consider the effects of a shock to the edge cost $e_{1,2}$ when country 3 features bank loan officers with twice the productivity (i.e., higher z_i than in countries 1 and 2). The effects of the shock on the loan flows from 1 to 2 and from 3 to 2 are qualitatively similar to the baseline scenario. Indeed, the parameter of the Fréchet distribution, and hence the degree of indirectness of the banking flows, exerts a similar influence on the change of the ultimate loan flows from 1 to 2 and from 3 to 2 induced by the shock. What is instead sharply different from the baseline is the way path diversification and source diversification are weighted against each other. When country 3 is highly efficient in producing loans, losing it as a source of credit exerts a large depressing impact on the total ultimate loans to country 2. While country 1 continues to be able to exploit country 3 as a tertiary node towards country 2, this path-diversification effect is now dominated by the reduced ability of country 3 to act as an alternative source of liquidity.

Overall, as illustrated by the figure, banking complexity leads here to greater shock amplification; a global banking network with more indirect linkages (lower θ)

now exhibits a larger drop in the ultimate loan flows to country 2, as shown by the upward-sloping red line. This suggests that banking complexity can have a stabilizing influence for shocks between countries with similar degree of development of the banking sector but can have a destabilizing effect for shocks where such countries are third countries with significantly different banking efficiency.

Multiple edge shocks In Figure 7b we consider a shock that raises the costs of lending from country 1 to both countries 2 and 3 (i.e. an increase in $e_{1,2}$ and $e_{1,3}$, respectively). This double-edge shock can be thought as a tightening of banking regulation in country 1 that increases the cost of foreign lending for banks in 1. While banking complexity weakens source diversification as before, the benefit of a more complex banking network in terms of path diversification is now reduced. Intuitively, due to the increased costs of lending along $e_{1,3}$, banks in country 1 find it more difficult to circumvent the higher costs of lending directly to 2 by lending indirectly through 3. In Figure 7b the negatively sloped solid black line is now flatter (recall that this line represents the change in the ultimate loans from 1 to 2, for different values of the Fréchet parameter). On the other hand, in a more complex banking network the economy will suffer from a loss of source diversification just like in the scenario of a one-edge shock (observe the positively sloped red line). The total effects of banking complexity remain similar to the baseline scenario. The loss of source diversification is not large enough to flip the slope of the solid red line in Figure 7b, and the qualitative implications remain.

5.3 An Alternative Characterization of Banking Integration

Above we conceptualize banking integration as a set of policies allowing for richer, more complex international banking networks. Banking integration may instead reflect policies which reduce international banking frictions more broadly, for any type of financial specialization (ω). In our framework, the latter can be captured through reduced edge costs $e_{i,j}$.

We revisit the $e_{1,2}$ edge shock exercise by allowing for different initial levels of integration (bilateral edge costs), then reduce edge costs to obtain a more integrated network (symmetric edge costs). We consider two scenarios. In a first scenario, we start with poor integration between countries 1 and 3. Integrating 1 and 3 (i.e. reducing the steady state value of $e_{1,3}$) achieves better path diversification following the $e_{1,2}$ shock, as 1 can now circumvent the higher edge cost $e_{1,2}$ through country 3. In Figure 8a, the dashed blue line (which represents 1 to 2 lending under various θ values before integration) jumps to the solid blue when 3 is integrated. However, the same integration depresses source diversification: the green dashed line drops to the green solid line. Intuitively, country 3 was not initially able to exploit indirect lending, and never exposed to the edge shock, but now integrated, its lending to 2 suffers from the $e_{1,2}$ shock. On net, the two effects offset each other, and banking integration has negligible influence on the response of total ultimate lending to country 2 following the shock.

In the second scenario, we start with poor integration between countries 3 and 2, and reduce the steady state value of $e_{3,2}$ until we achieve a symmetric network. In this case, both source and path diversification strengthen: in Figure 8b integrating 3 pushes all dashed lines up to the solid lines. Intuitively, the lack of integration between 3 and 2 inhibited both the use of 3 as an indirect link between 1 and 2 and the use of 3 as an alternative source of funding for 2. After integration, the obstacles to both path and source diversification get removed.

In general, integrating two spokes is unambiguously stabilizing for shocks originating from hubs, but ambiguous and potentially destabilizing for shocks originating at the spokes.

6 Extensions: Endogenous Monitoring

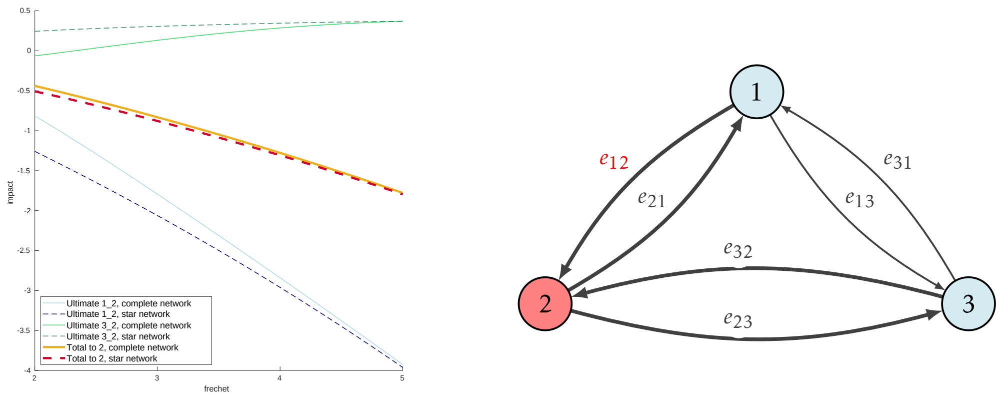
In the main version of the model we assume banks' monitoring productivities z_i to be exogenous and fixed. However, banks' efficiency in loan production and extension may be affected by experience accumulated through the activity of liquidity intermediation across countries.[28] In this section, we allow for endogenous loan monitoring productivity. Specifically, we let monitoring productivities be scaled by an exponential factor which depends on the geography of intermediation costs. The scale factor corresponds to the sum of inward and outward excess locational flows with respect to the country's steady state value. This reduced-form approach can be rationalized by a learning process such that monitoring loan officers acquire information and experienced based on the number of transaction they process. To capture the persistence of the centrality, we posit that the scale affects monitoring productivity with a lag, such that

$$z_{i,t} = \psi z_i * \exp\left(\tilde{\Xi}^{\text{IN}}_{i,t-1} + \tilde{\Xi}^{\text{OUT}}_{i,t-1}\right)$$

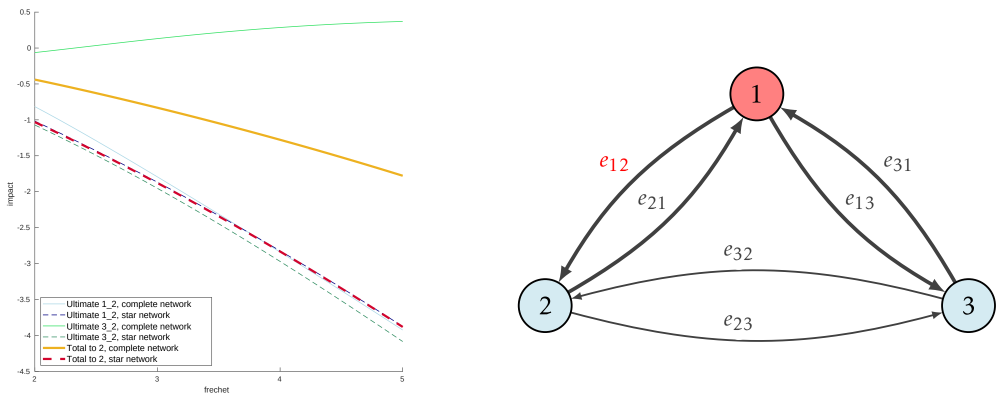
where z_i is the steady-state loan monitoring efficiency of the baseline economy, and ψ is a positive parameter. We calibrate the edge cost matrix as in the baseline analysis, setting $e_{i,j} = 2$.

Figure 9 plots the responses of the bilateral claims following a shock to the edge cost $e_{1,2}$. The response of the ultimate loan flows between countries 1 and 2 (solid

[28] In Appendix A we explicitly model intermediation as a loan officer's task. Alternatively, scale could affect global banking if loan officers' loan production and extension experience impacts their intermediation efficiency, which would endogenize the process of hub formation. This is explored in the Appendix's second part.



(a) Central node: 2



(b) Central node: 1

Figure 8: Financial Integration via Lower Bilateral Frictions

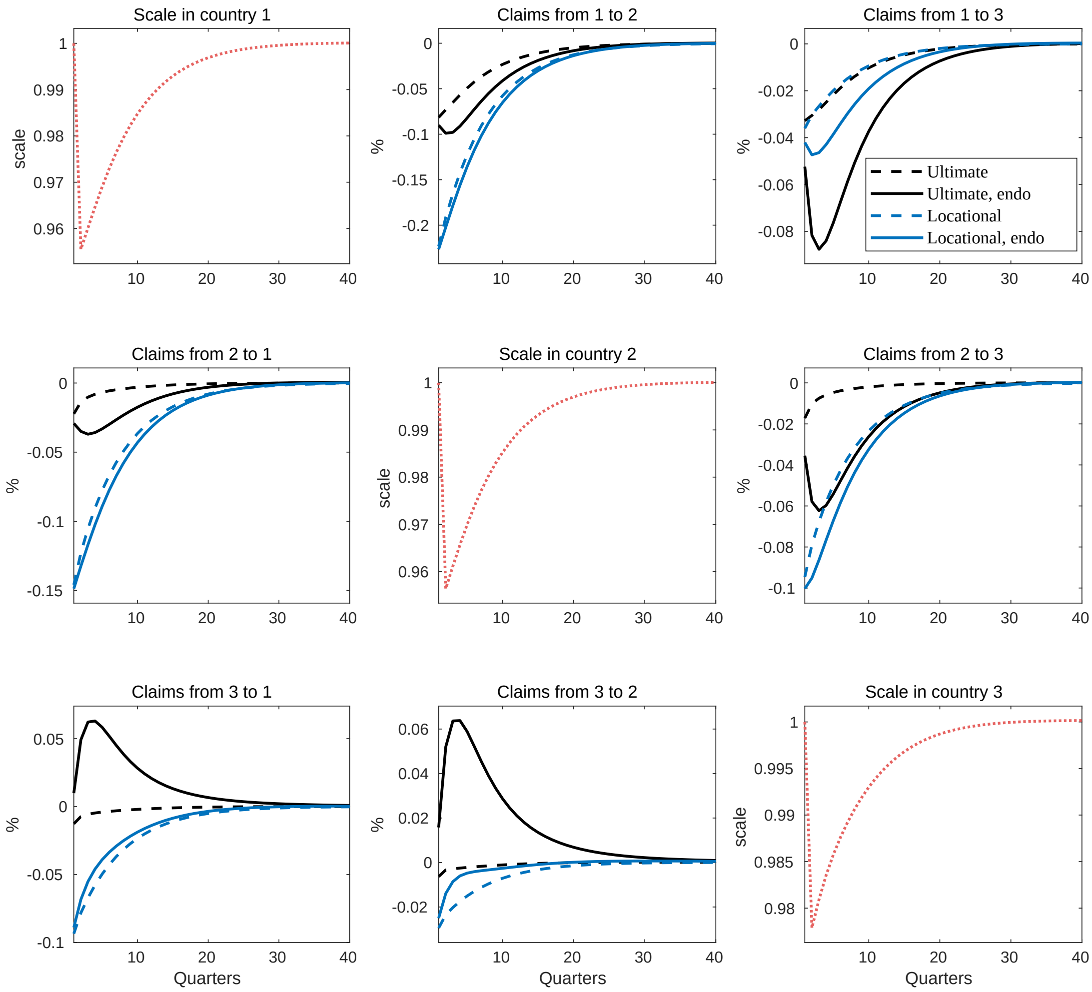


Figure 9: Complexity, Endogenous Banking Productivity

black line) qualitatively mirrors that obtained in the baseline scenario with exogenous bank productivity (dashed black line). However, the drop in the endogenous banks' productivity triggered by the reduction in loan intermediation shifts that response downward. Perhaps more surprisingly, the ultimate loan flows from country 3 to country 2 now display some increase following the shock, in contrast with what happens in the baseline scenario. Intuitively, country 3 becomes a stronger intermediation hub for loan flows from 1 to 2 (path diversification) and, because of this increased loan intermediation, tends to gain in monitoring efficiency and to produce more loans, benefiting also country 2. Interestingly, this suggests that in this extended setting path diversification could enhance third countries' ability to serve as alternative sources of funds, that is, to some extent path and source diversification could exhibit some degree of complementarity.

7 Conclusion

This paper presents a dynamic general equilibrium model with multi-country banking flows to account for the substantial fraction of international lending that is intermediated through banking hubs and complex multi-national routing. Our contributions are twofold. First, our model rationalizes observable international statistics. It generates a set of bilateral *locational* flows of funds that conceptually matches aggregate (BIS LBS) statistics, as distinct from the *ultimate* demand and supply of bank credit. We show how empirical specifications conflating the BIS LBS data with ultimate origin-destination lending biases results and can distort or even reverse empirical patterns.

Second, we show that indirect banking linkages are crucial to understanding how shocks to international frictions (e.g., financial regulation changes or the introduction of financial sanctions) propagate through the network. We find that accounting for indirect banking links unveils new tradeoffs when considering banking complexity. While a more interconnected banking network characterized by thicker indirect links eases the use of alternative paths for reaching destination countries (path diversification), it can undermine countries' ability to diversify the sources of international liquidity (source diversification). Overall, the model yields nuanced implications for macroprudential and regulatory policies: it reveals that the overall net effect of banking complexity on the propagation of shocks depends on the nature of the shocks and on the relative efficiency of the banking systems of the different countries.

The analysis leaves relevant questions open. While our framework captures a rich structure of network paths, it abstracts from complex general equilibrium interactions between banking intermediation costs and aggregate variables that may be relevant for international lending flows, such as the price of international collateral and financial assets. We leave this and other issues to future research.

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Online Appendices

These Online Appendices contain further microfoundations for intermediation costs (Appendix A), proofs of the model (Appendix B), and additional figures (Appendix C).

A Further Microfoundations

In Section 2.3, we modeled (deterministic) edge costs as e_{kl}. One interpretation of this cost is, as iceberg costs in the trade literature, an additional friction resulting in higher effective marginal cost of monitoring from origin i to destination j. In this section, we provide a microfoundation of this friction along these lines.

A.1 Bank Intermediation and Edge Costs along the Path

In this microfoundation, the banking sector of each country performs two functions: production of loans and intermediation of liquidity between countries. In line with the literature on multinational banks' internal capital markets, we specify liquidity intermediation as an inflow of transfers from loan-originating countries and an outflow of transfers towards ultimate destination countries. This mimics the case of the internal capital markets of multinational banking groups which use their own subsidiaries in third countries to reach final destination countries.

The problem of a representative bank is now augmented in two ways. On the constraints side, the bank faces N intermediation constraints (one for each destination country) which relate transfers inwards to transfers outwards, according to a 1:1 technology (i.e., each unit of transfers outwards must be matched by a unit of transfers inwards). This N-constraints specification embeds the idea that banks cannot divert funds committed for specific destinations to cheaper destinations (receiving higher edge cost payments and expending lower edge cost payments to such cheaper destinations). On the objective function side, the bank pays costs for operating the liquidity intermediation technology (e.g., for monitoring and managing transfers in and out and for matching them with each other). The bank also receives payments from loan-originating countries aimed at covering the expenses sustained in the intermediation process.

Formally the problem of a bank in node (country) country k reads:

$$\max_{M_k(\omega),D_k(\omega)} \sum_j \frac{r_{kj}(\omega)}{\tau_{kj}(\omega,p)} y^X_{kj}(\omega) - w_k M_k(\omega) - R^D_k D_k(\omega) - \sum_{j=1}^{N}\sum_{i=1}^{N} e_{ik}\tilde{\Xi}^j_{ik} r_{ij}(\omega) + \sum_{i=1}^{N} T_i$$

$$s.t. \quad y^X_k(\omega) = \min\{z_k M_k(\omega), D_k(\omega)\}$$

$$\sum_{i=1}^{N} \tilde{\Xi}^j_{ik} = \tilde{\Xi}_{kj} \qquad j = 1,...,N,$$

where $\tilde{\Xi}^j_{ik}$ denotes the transfers into country k from country i which are destined to country j; $\tilde{\Xi}_{kj}$ are the overall transfers from country k to country j; $e_{ik}\tilde{\Xi}^j_{ik}r_{ij}$ represents the edge costs sustained by the bank for intermediating the transfers $\tilde{\Xi}^j_{ik}$; and T_i is the total payment received from each origin country i for the purpose of covering the intermediation expenses sustained by banks in k. Two observations are in order about the intermediation costs. First, in this microfoundation the banks in k must sustain intermediation costs for managing transfers inwards (the analysis would be similar if costs had to be sustained for managing transfers outwards). Second, as implied by the the main text analysis, the payments T_i received from the loan-originating countries effectively cover all the expenses sustained for intermediation costs.

Finally, note that the problem above rules out that transfers are used in the loan production technology, as this would effectively imply that transfers are seized by the intermediating banking system of country k. In a similar vein, allowing for local deposits to be used in the intermediation activity would imply that epsilon draws are no longer forced to be associated with origin banks and their specific source of funds.

A.2 Bank Labor and Edge Costs

Below, we lay out a slightly richer microfoundation where edge costs are costs paid to labor for monitoring and managing intermediated liquidity at travelled nodes. For notational simplicity, in what follows we drop the time subscript t.

In addition to origination labor costs, banks choosing route p now pay monitoring costs for loan intermediation. As before, the deterministic elements of intermediation frictions are:

$$\tilde{\tau}_{ij}(p) = \prod_{k=1}^{K_p} e_{k-1,k}.$$

However, now $e_{k-1,k}$ reflects endogenous labor costs for loan intermediation:

$$e_{k-1,k} = \frac{w^I_k}{z^I_{k-1,k}}$$

where w_k^I is the wage paid in country k to workers engaged in (monitoring) loan intermediation activities, and z_k^I is the node's bilateral monitoring efficiency in intermediation activities involving transfers from country $k-1$.

The intermediation constraints faced by a bank at node k now become:

$$\sum_{i=1}^{N} \min\left\{z_{i,k}^I m_{ik}^I(\omega), \tilde{\Xi}_{ik}^j(\omega)\right\} = \tilde{\Xi}_{kj} \qquad j = 1, \ldots, N,$$

where m_{ik}^I is the amount of labor employed in monitoring transfers from i to k. Here, bilateral frictions are due to the need of paying for a loan's monitoring in its travel along p. Effectively, loan production is global, and the loan production function is only the domestic portion of loan production.

In a scenario in which labor for intermediation activities and for loan production activities are differentiated from each other, the total demand for labor in intermediation activities reads

$$M_k^I = \sum_i m_{ik}^I = \sum_i \sum_j \frac{\tilde{\Xi}_{ik}^j}{z_{i,k}^I}.$$

The supply of labor for intermediation activities comes from households, whose modified problem reads:

$$\begin{aligned}
\max_{\{C_{k,t}, H_{k,t}, M_{k,t}, M_{k,t}^I\}_{t\geq 0}} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_{k,t}^M - k_H \frac{H_{k,t}^{1+\epsilon}}{1+\epsilon} - k_M \frac{M_{k,t}^{1+\varphi}}{1+\varphi} - k_I \frac{M_{k,t}^{I\ 1+\eta}}{1+\eta} \right) \\
s.t. \quad & C_{k,t} + D_{k,t} = (1 + R_{k,t-1}^D) D_{k,t-1} + w_{k,t}^H H_{k,t} + w_{k,t}^M M_{k,t} + w_{k,t}^I M_{k,t}^I + \Pi_{k,t}.
\end{aligned}$$

Thus, households also derive disutility from a third type of labor (for bank intermediation tasks) and receive wages on such labor. In this setup, edge costs are endogenous to banking wages for intermediations tasks, $w_{k,t}^I$. Increased demand for using k as a node can raise banking wages for intermediation activities and affect edge costs, as long as $\eta > 0$. However, in a setting where $\eta = 0$ and households suffer from linear disutility in performing intermediation tasks, the wage rate $w_{k,t}^I$ for intermediation will be fixed (possibly normalized to a value consistent with the data), and the analysis would be exactly as in the main text.

B Model Derivations

In this section, with some definitional abuse, we use price and interest rate interchangeably.

B.1 Details on Households and Firms

Households' first order conditions read

$$[C_{i,t}]: \quad 1 = \mathbb{E}_t \Lambda_{i,t,t+1}(1 + R^D_{i,t}), \tag{B.1}$$

$$[H_{i,t}]: \quad k_H H^{\epsilon}_{i,t} = \frac{w^H_{i,t}}{C_{i,t}}, \tag{B.2}$$

$$[M_{i,t}]: \quad k_M M^{\varphi}_{i,t} = \frac{w^M_{i,t}}{C_{i,t}}, \tag{B.3}$$

Firms' first order conditions, in turn, read:

$$[H^D_{i,t}]: \quad \frac{(1-\alpha)Y_{i,t}}{H_{i,t}} = w^H_{i,t}, \tag{B.4}$$

$$[K_{i,t}]: \quad -P^K_{i,t}(1 + R^X_{i,t-1}) + \mathbb{E}_t\left[\Lambda_{i,t,t+1}\left((1-\delta)P^K_{i,t+1}(1 + R^X_{i,t}) + \frac{\alpha Y_{i,t+1}}{K_{i,t}}\right)\right] = 0. \tag{B.5}$$

B.2 Proof of Lemma 1 - Gravity Path Probability

Firms receive bids for financing their capital investments. Banks are competitive and each bank from country i and industry ω makes firms face the same interest rate P^X_i. Hence, price is given by:

$$p^X_{ij}(\omega) = c^X_i \ \tau_{ij}(\omega). \tag{B.6}$$

The goal is to derive the probability that a route is the lowest-cost route from i to j for loan product ω and country i is the lowest-cost supplier of loan product ω to j. We want to know the probability that any given loan ω is sent from i to j on a specific route p. Firms choose the lowest-cost route from i to j for ω from all routes $p \in G$ and firms in j choose the lowest-cost supplier of loan ω from all countries $i \in I$. We will observe ω being sent on a route from i to j if the final price of ω including both the marginal cost of loan production and the shipping cost on that route from i to j, $p_{ijnr}(\omega)$, is lower than all the other prices of loan ω from all the other country-route combinations.

Therefore, we will find i) the probability that a country i provides loans to country j at the lowest price; ii) the price of the loan that a country i actually pays to country j is independent of j's characteristics.

B.2.1 Lenders

The *unconditional* probability that taking a route p to lend from country i to j for a given loan product ω costs less than a constant τ is:

$$\begin{aligned} H_{ijp\omega}(\tau) \equiv & \Pr\Big(\tau_{ij}(p,\omega) \leq \tau\Big) \\ = & 1 - \exp\left\{-\left[\frac{\tilde{\tau}_{ij}(p)}{\tau}\right]^{-\theta}\right\}. \end{aligned} \tag{B.7}$$

Because the technology is i.i.d across types, this probability will be the same for all loan products $\omega \in \Omega$.

So far we have considered the potential intermediation cost. However, we do not observe bilateral ex-ante costs, but the cost that each country applies ex-post, after choosing the cheapest path. The probability that, *conditional* on banks choosing the least cost route, the cost in ω is less than some constant τ is given by:

$$H_{ij\omega}(\tau) \equiv \Pr\Big(\tau_{ij}(\omega) \leq \tau\Big),$$

which, after some algebra, yields

$$1 - \exp\left\{-\tau^{\theta}\sum_{p\in G}\left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right\}. \tag{B.8}$$

To summarize, this is the probability that, given that banks choose the lower cost route, the cost is below a certain value.

B.2.2 Borrowers

Similar to equation D.3, the probability that the price is below a certain constant is the following:

$$\begin{aligned} G_{ijp\omega}(r) \equiv & \Pr\Big(r_{ij}(p,\omega) \leq r\Big) \\ = & 1 - \exp\left\{-\left[c_i\frac{\tilde{\tau}_{ij}(p)}{r}\right]^{-\theta}\right\}. \end{aligned} \tag{B.9}$$

Firms minimize the price they pay across countries and routes:

$$
\begin{aligned}
G_{j\omega}(r) \equiv &\Pr\left(\min_{i\in I, p\in G} r_{ij}(p,\omega) \leq r\right) \\
= &1 - \exp\left\{-r^{\theta} \sum_{i\in I} c_i^{-\theta} \sum_{p\in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right\}.
\end{aligned} \tag{B.10}
$$

B.2.3 Market Making

Finally, we can combine the two sides of the market, i.e. the probability that a firm in country j chooses to borrow from a bank of country i, and that the route from country i to j is the minimal cost route. In other words, we compute the probability that, picking any other route-country pair, the price will be higher than the optimal one.

$$
\begin{aligned}
\pi_{ijp\omega} \equiv &\Pr\left(r_{ij}(p,\omega) \leq \min_{k\neq i,\, s\neq p} r_{kj}(s,\omega)\right) \\
= &\frac{\left[c_i \tilde{\tau}_{ij}(p)\right]^{-\theta}}{\sum_{i'\in I} c_i'^{-\theta} \sum_{p\in G} \left[\tilde{\tau}_{i'j}(p)\right]^{-\theta}}.
\end{aligned} \tag{B.11}
$$

By the law of large numbers, given the continuum of loan products, this is also the share of all loans extended from i to j in industry ω and that take route p, $\lambda_{ijp\omega}$.

B.3 Proof of Lemma 2 - Expected Cost

Assume banks choose the route that minimizes the cost of sending a loan from country i to country j. Let A be the inverse cost matrix defined above, and each element b_{ij} of the matrix $B \equiv (\mathbf{I} - A)^{-1}$. Then, the network cost τ_{ij} is:

$$\tau_{ij} = \gamma b_{ij}^{-1/\theta}$$

where

$$b_{ij} = \sum_{p\in G_{ij}} \tilde{\tau}_{ij}(p)^{-\theta} \equiv \sum_{p\in G_{ij}} \prod_{k=1}^{K_p} e_{k-1,k}(p)^{-\theta}.$$

B.3.1 Expected Cost

The cost between locations i and j is the expected trade cost τ_{ij} from i to j across all lenders:

$$
\begin{aligned}
\tau_{ij} \equiv \mathbb{E}_{\omega}\left[\tau_{ij}(p)\right] &= \int_{p\in G_{ij}} \tau_{ijp}(\omega)\, dp \\
&= \int_0^{\infty} \tau \, dH_{ij\omega}(\tau) \quad \text{by distribution B.8} \\
&= \Gamma\left(\frac{1+\theta}{\theta}\right)\left[\sum_{p\in G_{ij}} \tilde{\tau}_{ijp}^{-\theta}\right]^{-1/\theta}. \qquad \text{(B.12)}
\end{aligned}
$$

B.3.2 Expected Cost with Paths

Let $\gamma \equiv \Gamma\left(\frac{1+\theta}{\theta}\right)$. Following Allen and Arkolakis (2022) and taking into account the length of the path, and all possible lengths:

$$
\begin{aligned}
\tau_{ij}^{-\theta} &= \gamma^{-\theta} \sum_{p\in G_{ij}} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta} \\
&= \gamma^{-\theta} \sum_{K=0}^{\infty} \sum_{p\in G_{ij}(K)} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta} \\
&= \gamma^{-\theta} \sum_{K=0}^{\infty} \sum_{p\in G_{ij}(K)} \prod_{k=1}^{K} a_{ij} \qquad \text{defining } e_{k-1,k}^{-\theta} \equiv a_{ij} \\
&= \gamma^{-\theta} \sum_{K=0}^{\infty} A_{ij}^{K}.
\end{aligned}
$$

Assuming that the spectral radius of A is less than one, then:[29]

$$
\sum_{K=0}^{\infty} A^{K} = (\mathbf{I} - A)^{-1} \equiv B. \qquad \text{(B.13)}
$$

Hence:

$$
\tau_{ij} = \gamma b_{ij}^{-1/\theta} \Leftrightarrow b_{ij} = \sum_{p\in G_{ij}} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}. \qquad \text{(B.14)}
$$

[29] AA19: "a sufficient condition for the spectral radius being less than one is if $\sum_j e_{ij}^{-\theta} < 1$ for all i. This will necessarily be the case if either trade costs between connected locations are sufficiently large, the adjacency matrix is sufficiently sparse, or the heterogeneity across traders is sufficiently small (i.e. θ is sufficiently large."

B.4 Proof of Lemma 3 - Locational Gravity

The probability of going through an edge tk, conditional on origin i and destination j, is:

$$
\begin{aligned}
\psi_{kl|ij} =& \sum_{K=0}^{\infty} \sum_{p \in G_{ij}^{kl}(K)} \frac{\tilde{\tau}_{ij}(p)^{-\theta}}{\sum_{K=0}^{\infty} \sum_{p \in G_{ij}(K)} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}} \\
=& \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{p \in G_{ij}^{kl}(K)} \tilde{\tau}_{ij}(p)^{-\theta} \\
=& \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{p \in G_{ij}^{kl}(K)} \prod_{k=1}^{K} e_{k-1,k}(p)^{-\theta} \\
=& \frac{1}{b_{ij}} \left(b_{ik} a_{kl} b_{lj}\right)
\end{aligned}
\tag{B.15}
$$

where in the last step we isolate the kl step and follow the matrix algebra in Allen and Arkolakis (2022), such that $\sum\limits_{K=0} \sum\limits_{L=0}^{K-1} A^L A A^{K-L-1} = (I-A)^{-1} A (I-A)^{-1}$.

The conditional probability is:

$$
\psi_{kl|ij} = \frac{b_{ik} a_{kl} b_{lj}}{b_{ij}} = \left(\gamma \frac{\tau_{ik} e_{kl} \tau_{lj}}{\tau_{ij}}\right)^{-\theta} \tag{B.16}
$$

where the last step was obtained by plugging the expected cost definition in (B.14).

B.5 Aggregate Interest Rate

Let $G_{ij}(\phi)$ be the Pareto (equilibrium) probability density function of the productivities of banks from country i that lend to country j such that the measure of banks from country i with productivity ϕ is $N_i \, dG_i(\phi)$. Then we can write the aggregate interest rate in j as:

$$
R_j = \vartheta \left(\sum_i c_i^{-\theta} b_{ij} \right)^{-\frac{1}{\theta}}
$$

where $\vartheta = \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)^{\frac{1}{1-\sigma}}$. In fact,

$$
\begin{aligned}
R_j^{1-\sigma} &= \int_{\Omega} r_{ij}(\omega)^{1-\sigma}\, d\omega \\
&= \int_0^{\infty} p^{1-\sigma}\, dG_j(p) \\
&= \int_0^{\infty} p^{1-\sigma} \frac{d}{dp}\left(1-\exp\{-p\Phi\}\right) dp \\
&= \Phi^{-\frac{1-\sigma}{\theta}} \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right) \\
R_j &= \vartheta\left(\sum_i c_i^{-\theta} b_{ij}\right)^{-\frac{1}{\theta}}
\end{aligned}
$$

C Appendix Figures

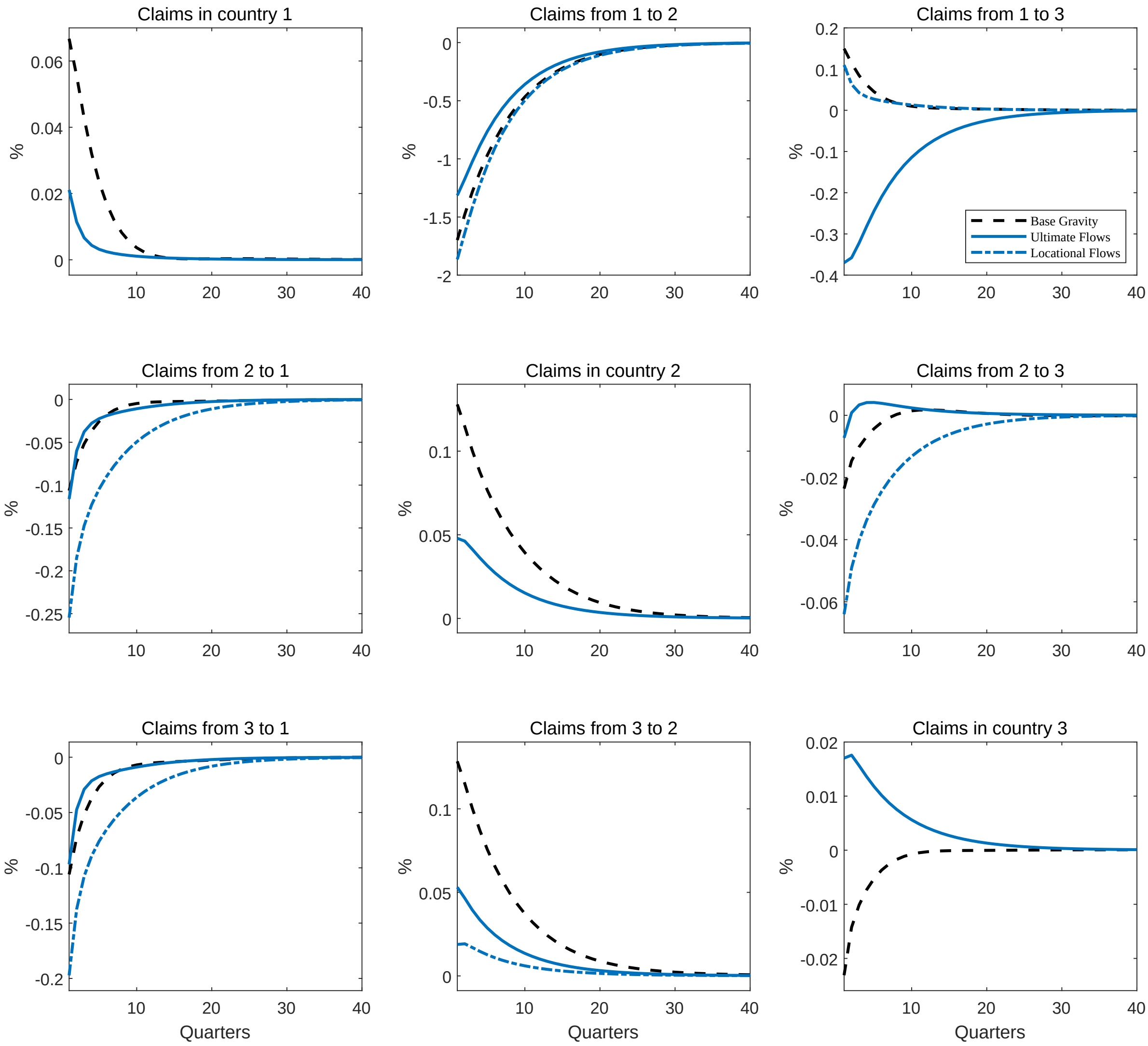

Figure A1: Edge Shock, Full Set of Loan Responses

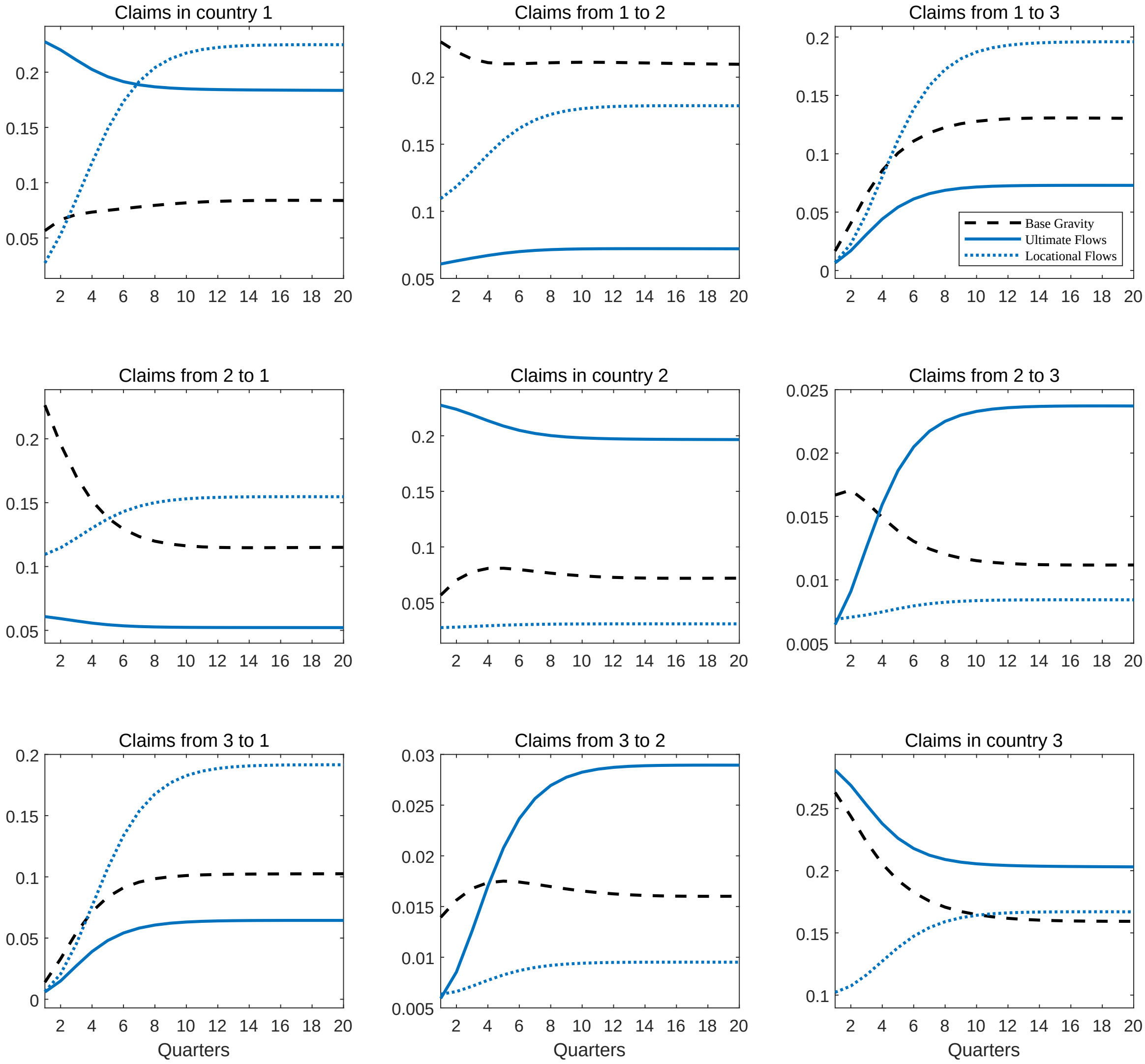

Figure A2: Loan Responses to a Permanent Edge Shock

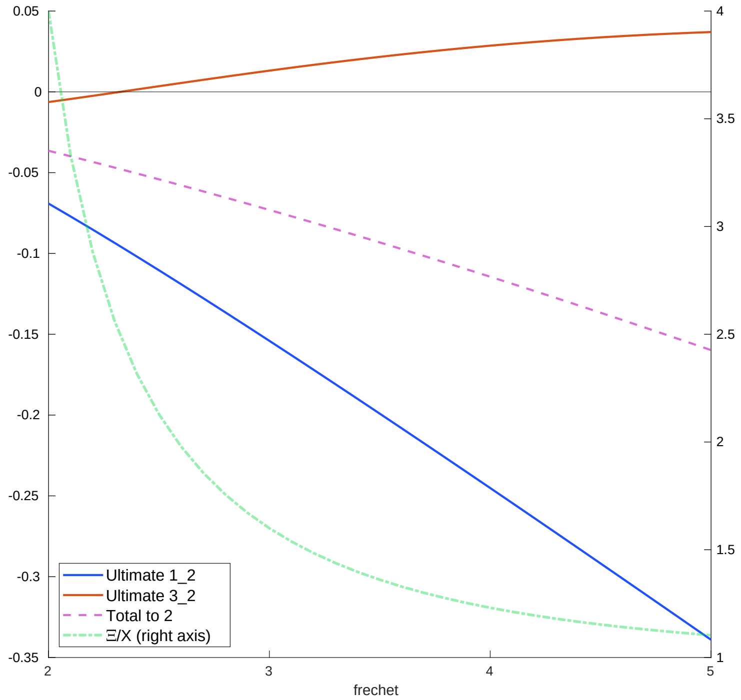

Figure A3: Complexity and Loan Responses to Edge Shocks, 4-Period Average

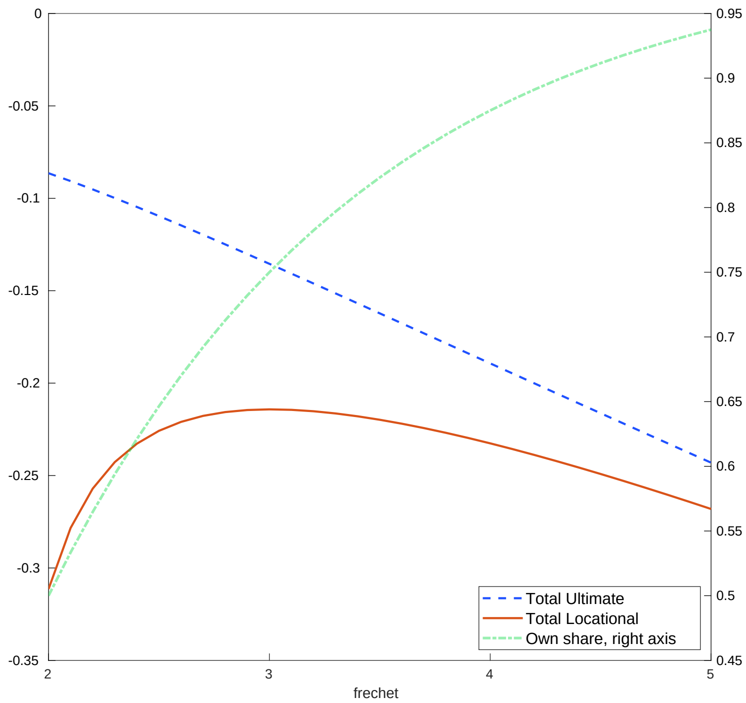

Figure A4: Banking Complexity, Totals and Own Share