A Model of the Asymmetric Transmission of Aggregate Shocks

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Abstract

I study the transmission of aggregate shocks in a New Keynesian model in which households' incomes are heterogeneously exposed to changes in aggregate income, and borrowing frictions limit opportunities for aggregate risk sharing. I analytically show that shock transmission is asymmetric: output responds more to contractionary shocks than to expansionary shocks of equal magnitude. Estimating key model parameters using the micro evidence on heterogeneous consumption exposures to changes in output generates asymmetric responses of output to monetary policy shocks that can explain at least 60% of the empirical asymmetry.

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1 Introduction

A growing body of papers studies how departing from the representative household paradigm affects the transmission of aggregate shocks in New Keynesian models. When households are no longer identical, some may be more exposed to the effects of aggregate shocks than others. For example, Guvenen and co-authors (2014, 2017) use high quality administrative income data for the US to document significant heterogeneity in how households' incomes co-move with business cycle movements in GDP.[1] However, this heterogeneity in the incidence of aggregate shocks is somewhat understudied by the existing literature, which focuses on the effects of idiosyncratic and uninsurable income risk on the transmission of aggregate shocks. In this paper, I contribute towards filling this gap.

I study the transmission of aggregate shocks when households' incomes are heterogeneously exposed to changes in aggregate income (output), and borrowing frictions limit the opportunities for risk sharing. I couple this model of the household sector with a standard New Keynesian supply-side: firms are monopolistically competitive and are subject to costly price adjustments, and nominal interest rates are set according to a Taylor rule. In this setting, I explore the transmission mechanism of aggregate shocks both theoretically and numerically.

Using a simple version of my model, I analytically establish my main novel result: output responds more to contractionary monetary policy shocks than to expansionary shocks of equal magnitude. This result follows from the fact that households' incomes are heterogeneously exposed to changes in aggregate income, and that borrowing frictions prevent households from fully sharing this aggregate income risk.

In the presence of binding borrowing constraints, only households with the strongest incentive to save respond to interest rate changes on the margin. I refer to these households as savers. In equilibrium, savers are unconstrained, and their consumption response is governed by a standard Euler equation.

As a simple example, suppose that the elasticity of intertemporal substitution (EIS) is one, and consider a transitory shock to the real interest rate of $+1\%$. The Euler equation states that, in response to this shock, saver households reduce their current consumption by 1%. Likewise, a shock to the real interest rate of -1% causes saver households to increase their current consumption by 1%.

In contrast, borrowing-constrained households necessarily respond asymmetrically to positive and negative interest rate changes. For a household to be borrowing-constrained in response to

[1] As an example of this heterogeneity, figure 4 in the appendix reproduces Figure 2 in Guvenen et al. (2017), which plots the β_g coefficients from the pooled OLS regression

$$\Delta \log y_{i,t} = \alpha_g + \beta_g \Delta \log Y_t + u_{i,t}$$

where $y_{i,t}$ is worker i's income in period t as reported on her W-2 form, Y_t is GDP, and the groups $\{g\}$ are the gender-specific (Panel A) or age-specific (Panel B) percentiles of the permanent income distribution computed using incomes in periods $t-6$ to $t-2$. In both cases, there is significant heterogeneity in the estimated β_g coefficients across groups.

a 1% interest rate hike, her equilibrium drop in current consumption must be greater than 1%. If this were not the case, she would use the asset to save in order to achieve the unconstrained response of a 1% drop in consumption.

Conversely, for a household to be borrowing-constrained in response to a 1% interest rate cut, her equilibrium increase in current consumption must be smaller than 1%. If not, she would use the asset to save in order to achieve the unconstrained response of a 1% increase in consumption.

Combining the consumption responses of savers and borrowing-constrained households implies that on average, households decrease their current consumption by more than 1% in response to a 1% interest rate hike, but increase it by less than 1% in response to a 1% interest rate cut. Therefore, in equilibrium, output (equal to aggregate consumption) must respond more to contractionary monetary policy shocks (interest rate hikes) than to expansionary monetary policy shocks of equal magnitude.

I establish analytically that this mechanism applies to the transmission of two other aggregate shocks commonly used in the New Keynesian literature: TFP shocks and cost-push shocks (direct shocks to inflation). In each case, the transmission of the shock that increases output in equilibrium is weaker than the transmission of an equal and opposite shock that decreases output. I also study the responses of inflation to these shocks and show that the direction of the asymmetry depends the type of shock hitting the economy: inflation inherits the output asymmetry in response to monetary policy shocks, but exhibits the opposite asymmetry pattern for TFP and cost-push shocks.

I confirm that output response asymmetry is robust to the introduction of idiosyncratic risk. I show that, in the empirically relevant case of very persistent idiosyncratic shocks, the asymmetry of the output responses to monetary policy shocks is unaffected by idiosyncratic risk. Intuitively, when idiosyncratic shocks are very persistent, a household does not expect her consumption to change for idiosyncratic reasons, so that her incentive to borrow or save is driven mainly by her income sensitivity to changes in output, as in the case without idiosyncratic risk.

Similarly, I show that output response asymmetry occurs when households have heterogeneous EISs. In this case, binding borrowing constraints restrict the increase in consumption of high EIS households in response to expansionary shocks, and also prevent low EIS households from achieving small consumption declines in response to contractionary shocks.

The size of the output response asymmetry is determined by the highest and lowest sensitivities of household consumption to equilibrium changes in output. Intuitively, a larger range of sensitivities implies less asset trading in equilibrium and hence tighter borrowing constraints, which causes a larger asymmetry. Using micro data on household consumption from the Consumer Expenditure Survey, I estimate a lower bound for the ratio of these sensitivities of 2.5. Inserting this ratio into the model then implies that the output response to a contractionary monetary policy shock is two and a half times larger than the response to an expansionary monetary policy shocks of equal size.

I finish by showing that an asymmetry of this magnitude is consistent with the macro-econometric evidence for asymmetric output responses to monetary policy shocks. Using local projection methods (Jorda, 2005), I estimate that the maximal response of output to a 1% contractionary monetary policy shock is approximately four times larger than the maximal response to a 1% expansionary shock. Therefore, the quantitative mechanism is capable of explaining at least 60% of the empirical asymmetry.

Related Literature I contribute to a growing literature that studies how departing from the representative household paradigm affects the transmission of monetary policy and other aggregate shocks in New Keynesian models. A large body of work has replaced the representative household with the assumption that households face idiosyncratic and uninsurable income risk that causes ex-ante identical households to experience ex-post heterogeneous time paths of income and consumption. This class of Heterogeneous Agent New Keynesian (HANK) models has been used to study the decomposition of monetary transmission (Auclert (2017) and Kaplan et al. (2018)), the power of forward guidance (McKay et al. (2016) and Werning (2015)), and the determinacy of interest rate rules (Acharya and Dogra, 2018), among other issues. Relative to these papers, I consider a novel dimension of household heterogeneity, and study its implications for the transmission of a variety of aggregate shocks, not limited to monetary policy.

An important exception in the extant literature, and the closest forebear to my paper, is Bilbiie (2018), who analyzes the transmission of monetary policy in a tractable class of Two Agent New Keynesian (TANK) models. In this class of models, the first set of households do not face any frictions in the asset market, while the second set face severe frictions that prevent them from both borrowing and saving. These households thus live “hand-to-mouth”, and consume their entire income in each period with a marginal propensity to consume of one. Using this set up, Bilbiie shows how the response of output to interest rate changes is amplified when changes in aggregate income fall mainly on the hand-to-mouth households, and dampened otherwise.

While Bilbiie’s model also features heterogeneous income exposures, the responses of output remain symmetric in his framework. The lack of asymmetry follows from the extreme way in which asset markets are modeled in TANK frameworks: the first group of households has complete access to asset markets, while the second is completely barred from borrowing or saving any amount. This assumption implies that only the first group of households adjust their consumption in response to interest rate changes. Hence, the equilibrium output response simply coincides with the consumption response of this group, and is therefore symmetric in the sign of the interest rate change. I show that relaxing this assumption so that all households only face constraints to borrowing generates asymmetric transmission of aggregate shocks that aligns well with the macro-econometric evidence for such asymmetry.

Finally, in contemporaneous work, Patterson (2018) argues that contractionary shocks are amplified when households who are highly exposed to the fall in aggregate income also have high marginal propensities to consume (MPC), a fact that she documents in the data. My results complement this empirical finding by providing a structural theory of the amplification of contractionary shocks, and by exploring its consequences for expansionary shocks, thus establishing my key result on asymmetric transmission.

The paper proceeds as follows: section 2 describes the economic environment. Section 3 establishes my main theoretical result on asymmetric output responses, and discusses extensions and robustness. I estimate key model parameters in section 4, and compare the implied output response asymmetry to the empirical evidence in section 5. Section 6 concludes.

2 Environment

The economy is populated by a unit mass of households indexed by $i \in [0,1]$. Each household has preferences over her infinite sequence of final good consumption $\{c_{i,t}\}$ and labor supply hours $\{n_{i,t}\}$ given by

$$\mathbb{E}_1\left[\sum_{t=1}^{\infty}\delta^{t-1}u\left(c_{i,t},n_{i,t}\right)\right]$$

where $\delta \in (0,1)$ is a time discount factor, $\mathbb{E}_t$ is an expectations operator conditioned on time t information, and u is strictly increasing and concave in c and strictly decreasing and concave in n.

If household i works for $n_{i,t}$ hours, she supplies $\theta_{i,t}n_{i,t}$ units of effective labor, where $\theta_{i,t}$ is her labor productivity and is subject to idiosyncratic shocks, as described below. Effective labor earns the nominal wage $P_t w_t$ where P_t is the nominal price of the final consumption good, and w_t is the real wage. In addition to wage income, household i receives a fixed share s_i of dividends from intermediate goods firms d_t measured in consumption units.[2]

When households have heterogeneous dividend shares and labor productivities, their incomes are heterogeneously sensitive to changes in aggregate income. For example, if total wage income increases more than total dividend income, households with higher labor productivities will benefit more than households with higher dividend shares. In order to smooth these heterogeneous exposures to aggregate income fluctuations and to insure against idiosyncratic income shocks, I assume that households can trade a nominal, risk-less one period bond $b_{i,t}$, that earns the real rate r_t given by

$$1+r_t=\frac{1+\iota_{t-1}}{1+\pi_t}$$

where ι_t is the nominal interest rate, and $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$ is inflation. All households begin with zero assets, $b_{i,0}=0$ for all i.

In this economy, households face income risk due to idiosyncratic and aggregate shocks. Without additional restrictions on the structure of financial markets, households could achieve large

[2]Interpreting s_i as a household's equity holdings, I assume that trading equity is sufficiently costly so that households do not trade at business cycle frequencies in response to aggregate shocks.

amounts of self-insurance against these shocks using only the 1 period bond (Krusell and Smith, 1998). Therefore, I follow the literature on incomplete markets and heterogeneous households and assume that households are subject to ad hoc borrowing constraints,

$$b_{i,t} \geq -\underline{b}_{i,t}$$

where $\underline{b}_{i,t} \geq 0$ for all i, t. The fact that the constraint may depend on time and the identity of each household captures, in reduced form, the fact that different households may face borrowing frictions of varying severity at different points in time. For example, higher income households are likely to face less stringent restrictions on their borrowing capacity than lower income households.

In sum, and taking all prices and dividends as given, household i solves

$$\max_{\{c_{i,t}, n_{i,t}, b_{i,t}\}_t} \mathbb{E}_1 \left[\sum_{t=1}^{\infty} \delta^{t-1} u\left(c_{i,t}, n_{i,t}\right) \right]$$

subject to

$$c_{i,t} + b_{i,t} = w_t \theta_{i,t} n_{i,t} + s_i d_t + (1 + r_t)\, b_{i,t-1}$$

$$b_{i,t} \geq -\underline{b}_{i,t}$$

$$b_{i,0} = 0$$

$$\theta_{i,0} = \theta_i$$

A representative competitive final good firm packages the unit mass of intermediate goods indexed by $j \in [0, 1]$, using the CES production function

$$Y_t = \left(\int_0^1 y_t\,(j)^{\frac{\Phi_t - 1}{\Phi_t}}\, dj \right)^{\frac{\Phi t}{\Phi t - 1}}$$

where $\Phi_t > 1$ is the elasticity of substitution across intermediate inputs, and is subject to aggregate shocks. Taking the price of each input and the price of the final good as given, the firm solves

$$\max_{\{y\}} P_t \left(\int_0^1 y_t\,(j)^{\frac{\Phi_t - 1}{\Phi_t}}\, dj \right)^{\frac{\Phi t}{\Phi_t - 1}} - \int_0^1 p_t\,(j)\, y_t\,(j)\, dj$$

Optimization yields a demand function for each intermediate good

$$y_t\,(j) = \left(\frac{p_t\,(j)}{P_t} \right)^{-\Phi_t} Y_t$$

and a nominal price index

$$P_t = \left(\int_0^1 p_t\,(j)^{1-\Phi_t}\, dj \right)^{\frac{1}{1-\Phi_t}}$$

Each intermediate good j is produced by a monopolistically competitive firm employing effective labor $E_t(j)$ in the production function

$$y_t(j) = A_t E_t(j)$$

where A_t is aggregate TFP and is also subject to aggregate shocks. Each firm faces its own demand curve, and chooses its path of prices to maximize profits subject to quadratic price adjustment costs (Rotemberg, 1982):

$$\max_{p(j)} \mathbb{E}_1 \left[\sum_{t=1}^{\infty} \delta^{t-1} \left(p_t(j) y_t(j) - P_t w_t E_t(j) - \frac{\xi^p}{2} \left(\frac{p_t(j)}{p_{t-1}(j)} - 1 \right)^2 P_t Y_t \right) \right]$$

subject to

$$y_t(j) = \left(\frac{p_t(j)}{P_t} \right)^{-\Phi_t} Y_t$$

$$y_t(j) = A_t E_t(j)$$

$$p_0(j) = P_0$$

where, for simplicity, I assume that firms discount future profits using the discount factor of households.[3] I focus on the symmetric equilibrium in which $p_t(j) = P_t$, $y_t(j) = Y_t$, and $E_t(j) = E_t$ for all $j \in [0,1]$. In this case, the aggregate dividend in period t is given by

$$d_t = Y_t \left(1 - \frac{w_t}{A_t} - \frac{\xi^p}{2} \pi_t^2 \right)$$

A monetary authority sets the nominal interest rate on the asset according to the Taylor rule

$$1 + \iota_t = \frac{1}{\delta} (1 + \pi_t)^{\phi_\pi} \left(\frac{Y_t}{Y^T} \right)^{\phi_y} e^{v_t}$$

where $\phi_\pi > 1$, $\phi_y \geq 0$, v_t is a monetary policy shock, and Y^T is some fixed target level of aggregate income (output).[4]

In each period, the labor, final good, and bond market must clear:

$$E_t = \int_0^1 \theta_{i,t} n_{i,t} di$$

$$\int_0^1 c_{i,t} di = Y_t \left(1 - \frac{\xi^p}{2} \pi_t^2 \right)$$

$$\int_0^1 b_{i,t} di = 0$$

Households are subject to idiosyncratic shocks to their labor productivity. Specifically, labor

[3] This assumption is innocuous for my theoretical results since I approximate around equilibria in which the true stochastic discount factor is constant.

[4] Throughout the paper, I refer to output and aggregate income interchangeably.

productivity for household i follows the AR(1) process

$$\log \theta_{i,t} = \rho_\theta \log \theta_{i,t-1} + (1-\rho_\theta) \log \theta_i + \epsilon_{i,t}$$

where $\rho_\theta \in (0,1)$ and $\epsilon_{i,t}$ is an i.i.d. random variable with mean zero and variance σ_ϵ^2.

Aggregate shocks affect aggregate TFP, the elasticity of substitution among intermediate inputs (the source of so called "cost-push" shocks), and the innovations to monetary policy, all of which evolve as AR(1) processes,

$$\log A_t = \rho_a \log A_{t-1} + (1-\rho_a) \log \bar{A} + \epsilon_t^a$$

$$\log \Phi_t = \rho_\Phi \log \Phi_{t-1} + (1-\rho_\Phi) \log \bar{\Phi} + \epsilon_t^\Phi$$

$$v_t = \rho_v v_{t-1} + \epsilon_t^v$$

where $\rho_a, \rho_\Phi, \rho_v \in (0,1)$, and $\left\{\epsilon_t^a, \epsilon_t^\Phi, \epsilon_t^v\right\}_t$ are i.i.d. random variables, each with mean zero and variance Σ^2. Due to their effects on inflation, and following the New Keynesian convention, I refer to $\epsilon_t^\Phi < 0$ as a positive cost-push shock and $\epsilon_t^\Phi > 0$ as a negative cost-push shock.[5]

Competitive Equilibrium

Definition 1. Given initial conditions $\left\{\{b_{i,0}, \theta_{i,0}\}_i, P_0\right\}$, a competitive equilibrium is a sequence $\left\{\{c_{i,t}, n_{i,t}, b_{i,t}\}_{i,t}, \{y_t(j)\}_{j,t}, P_t, d_t, w_t, \iota_t\right\}_t$ such that

1. $\{c_{i,t}, n_{i,t}, b_{i,t}\}_{i,t}$ solve the household problem for each i.
2. $\{y_t(j)\}_{j,t}$ solve the final good firms' problem.
3. $\{P_t\}_t$ solve the intermediate goods firms' problem.
4. $\{d_t\}_t$ satisfies the dividend equation.
5. $\{\iota_t\}_t$ satisfies the Taylor rule.
6. Markets clear at every time $t \geq 1$.

3 Asymmetric Transmission of Aggregate Shocks

In this section, I obtain analytical results regarding the responses of output to aggregate shocks. I begin with the case of monetary policy shocks, and then discuss the extension to cost-push and TFP shocks. I also discuss inflation responses, and generalizations of the result to the inclusion of idiosyncratic risk, and heterogeneous preferences.

[5] Intuitively, $\epsilon_t^\Phi > 0$ increases the elasticity of demand faced by each monopolist producer, and hence causes firms to lower their prices.

3.1 Asymmetric Output Responses to Monetary Policy Shocks

In order to achieve tractability, I simplify the model economy in various dimensions.

Assumption 1. *Let*

$$\rho_a, \rho_\Phi, \rho_v = 0$$

$$\Sigma \to 0$$

$$u(c, n) = \frac{\left(c - \frac{n^{1+\varphi}}{1+\varphi}\right)^{1-\sigma}}{1-\sigma}$$

$$\sigma_\epsilon = 0$$

$$\underline{b}_{i,t} \to 0 \ \forall i, t$$

The first two conditions restrict aggregate stochastic parameters to be i.i.d. over time, and to have "small" variances so that local approximation techniques are valid.

The next two conditions restrict aspects of the household problem. The Greenwood et al. (1988) (GHH) specification of utility is common in the business cycle literature, and is tractable since it sets income effects on labor supply to zero. The forth condition sets idiosyncratic risk to zero so that $\theta_{i,t} = \theta_i$ for all i and t. I relax this restriction in section 3.4.

The final condition restricts trading in the asset market, and should be interpreted as follows: as the $\{\underline{b}_{i,t}\}$ parameters approach zero, the sizes of the asset positions taken by households who would borrow in response to a shock get closer to zero due to the binding borrowing constraint. Since the asset is in zero net supply, in general equilibrium, the asset positions of saving households must also get closer to zero. Therefore, the household budget constraint implies that, as the $\{\underline{b}_{i,t}\}$ parameters approach zero, the equilibrium consumption choices of a household become well approximated by her income choices. I study the equilibrium under this approximation.

Importantly, this condition is not the same as imposing autarky. Instead, the limit condition implies that households can take arbitrarily small positions in the asset in equilibrium. This then requires that prices and quantities adjust in equilibrium so that the asset market clears. In particular, the adjustment must be such that households who save in equilibrium optimally choose a vanishingly small asset position that offsets the vanishingly small positions taken by borrowing-constrained households. Therefore, this assumption buys tractability without losing the key transmission mechanism from interest rates to savings choices.[6]

Deterministic Competitive Equilibrium Under assumption 1, I can define a deterministic competitive equilibrium as a competitive equilibrium when all aggregate shocks are set to

[6] Werning (2015) uses a similar assumption to analyze how the cyclicality of idiosyncratic risk affects the power of forward guidance.

zero in all periods. In this equilibrium, inflation is always zero, aggregate prices and quantities are constant, and all household choices of consumption, labor supply, and asset positions are fixed over time since they do not face any risk.

Definition 2. Suppose that assumption 1 holds, and assume that there are no aggregate shocks. Then, given initial conditions $\left\{\{b_{i,0}, \theta_{i,0}\}_i, P_0\right\}$, a deterministic competitive equilibrium is a sequence $\left\{\{c_i, n_i, b_i\}_i, \{y(j)\}_j, P, d, w, \iota\right\}$ such that

1. $\{c_i, n_i, b_i\}_i$ solve the household problem for each i.
2. $\{y(j)\}_j$ solve the final good firms' problem.
3. $P = P_0$ solves the intermediate goods firms' problem.
4. d satisfies the dividend equation.
5. Markets clear at every time $t \geq 1$.

I consider the dynamics of the economy in response to aggregate shocks around this deterministic competitive equilibrium.[7] Formally, the following lemma condenses the economy's equilibrium dynamics to a set of necessary and sufficient conditions expressed as log deviations around the deterministic competitive equilibrium, where I use $\hat{x}_t = \log x_t - \log x$ to denote such a deviation. The proofs of this and all other results are contained in the appendix.

Lemma 1. *Under assumption 1, the economy's first order equilibrium dynamics in response to monetary policy shocks satisfy the system*

$$\iota_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t + \epsilon_t^v$$

$$\pi_t = \frac{\bar{\Phi} - 1}{\xi^p} \varphi \hat{y}_t + \delta \mathbb{E}_t [\pi_{t+1}]$$

$$\min_i \left\{\mathbb{E}_t \left[\hat{\tilde{c}}_{i,t+1}\right] - \hat{\tilde{c}}_{i,t}\right\} = \frac{1}{\sigma} (\iota_t - \mathbb{E}_t [\pi_{t+1}] - \rho)$$

$$\hat{\tilde{c}}_{i,t} = \beta_i^y \hat{y}_t \ \forall i$$

where $\rho = -\log \delta$, $\{\beta_i^y\}_i$ depend only on model primitives, and $\tilde{c}$ is consumption net of the disutility of labor supply, $\tilde{c} = c - \frac{n^{1+\varphi}}{1+\varphi}$.

$\phi_\pi > 1$ and $\phi_y \geq 0$ are sufficient to ensure that the system has a unique steady state, $\hat{y}_t = 0$, $\pi_t = 0$, $\hat{\tilde{c}}_{i,t} = 0 \ \forall i$.

The first two equations of lemma 1 are standard features of New Keynesian models. The first equation is the Taylor rule for the nominal interest rate where the target level of output Y^T is set to the level in the deterministic competitive equilibrium. The second equation is the

[7] In deterministic economies with heterogeneous households, the wealth distribution may be indeterminate (Sorger, 2000). However, my restriction that $\underline{b}_{i,t} \to 0$ imposes that all households hold zero assets in all periods, thus breaking the indeterminacy, and ensuring uniqueness of the deterministic competitive equilibrium.

New Keynesian Phillips Curve (NKPC) linking inflation, and output. Intuitively, if output is higher today holding, firms face higher marginal costs of labor ceteris paribus, and so will optimally choose to raise their prices, leading to inflation.

The third equation captures the effect of binding borrowing constraints on the equilibrium. I emphasize that this equation does not require the no-borrowing limit restriction in assumption 1. It only requires that the borrowing constraints bind for some positive measure of households in each period.

In the presence of binding borrowing constraints, only households with the strongest incentive to save are unconstrained in equilibrium, and respond to interest rate changes on the margin. All other households are borrowing-constrained and are unresponsive to marginal interest rate changes. This logic is captured by the equation

$$\min_i \left\{ \mathbb{E}_t \left[\hat{\tilde{c}}_{i,t+1} \right] - \hat{\tilde{c}}_{i,t} \right\} = \frac{1}{\sigma} \left(\iota_t - \mathbb{E}_t \left[\pi_{t+1} \right] - \rho \right)$$

which states that only the Euler equations of households with the lowest consumption growth (i.e. households who save on the margin in equilibrium) hold with equality.[8] I refer to these households as savers.[9]

In order to simplify the exposition, I invoke the restriction of i.i.d. monetary policy shocks in assumption 1. This restriction implies that $\mathbb{E}_t \left[\hat{\tilde{c}}_{i,t+1} \right] = 0$ for all i, and that shocks only affect contemporaneous consumption. I extend the analysis to persistent shocks in section 4.

Given this, the Euler equation for unconstrained saver households has the usual interpretation: a 1% increase in the real interest rate translates into a $\frac{1}{\sigma}$% decrease in the current consumption of saver households, where $\frac{1}{\sigma}$ is the elasticity of intertemporal substitution (EIS). Note that the consumption response of savers is symmetric in the sign of the interest rate change.

Borrowing-constrained households, however, necessarily respond asymmetrically to positive and negative interest rate changes. In response to a 1% interest rate hike, borrowing-constrained households must experience a drop in current consumption of more than $\frac{1}{\sigma}$%. If not, they would not want to borrow on the margin to reduce their drop in consumption towards the unconstrained response of $\frac{1}{\sigma}$%.

Conversely, in response to an interest rate cut, borrowing-constrained households must experience an increase in current consumption of less than $\frac{1}{\sigma}$%. If not, they would not want to borrow on the margin to increase their equilibrium current consumption towards the unconstrained response of $\frac{1}{\sigma}$%.

Combining the consumption responses of savers and borrowing-constrained households implies that on average, households decrease their current consumption by more than $\frac{1}{\sigma}$% in response

[8] I refer to $\tilde{c}$ as "consumption" instead of "consumption net of the disutility of labor supply" for ease of exposition. This slight abuse of definitions does not affect the intuition for any of the results.

[9] Note that the solution to the min operator need not be unique. Multiple households $i \in [0,1]$ may be unconstrained in period t. Since all of these households will choose the same expected consumption growth in equilibrium, I only need to know the growth for a single $i^* \in \arg\min_i \left\{ \mathbb{E}_t \left[\hat{\tilde{c}}_{i,t+1} \right] - \hat{\tilde{c}}_{i,t} \right\}$.

to a 1% interest rate hike, but increase it by less than $\frac{1}{\sigma}$% in response to a 1% interest rate cut. This suggests that, in equilibrium, output must respond more to contractionary monetary policy shocks than to expansionary monetary policy shocks.

In order to pin down this output response asymmetry, the forth equation maps the heterogeneous sensitivities of household income to changes in output into heterogeneous sensitivities of household consumption to changes in output

$$\hat{\tilde{c}}_{i,t} = \beta_i^y \hat{y}_t \ \forall i$$

where the sensitivity coefficients $\{\beta_i^y\}$ measure the per cent change in household i's consumption for a 1% change in output in equilibrium. Intuitively, the pattern of $\{\beta_i^y\}$ depends on the underlying heterogeneity in income sensitivities, and the tightness of the borrowing constraints that restrict asset trading in equilibrium. For example, in the absence of any asset trading frictions, households would fully insulate their consumption from their heterogeneous income sensitivities, and β_i^y would be fixed across i.

Under assumption 1 however, the no-borrowing limit restriction implies the other extreme: a household's consumption inherits the sensitivity of her income to changes in output. Heterogeneity in consumption sensitivities therefore reflects heterogeneity in the underlying income sensitivities.

There are two sources of heterogeneity in the household income sensitivities, which are most clearly seen using the explicit expression for β_i^y given by

$$\beta_i^y = \frac{\varphi \frac{\bar{\Phi}-1}{\bar{\Phi}} \frac{\theta_i^{\frac{1+\varphi}{\varphi}}}{\Theta} + s_i \left(\frac{1}{\bar{\Phi}} - \varphi \frac{\bar{\Phi}-1}{\bar{\Phi}} \right)}{\frac{\varphi}{1+\varphi} \frac{\bar{\Phi}-1}{\bar{\Phi}} \frac{\theta_i^{\frac{1+\varphi}{\varphi}}}{\Theta} + s_i \frac{1}{\bar{\Phi}}}$$

where $\Theta = \int_0^1 \theta_i^{\frac{1+\varphi}{\varphi}} di$. First, when output increases, households with higher labor productivities receive more of the corresponding increase in wage income, so that β_i^y is increasing in θ_i. Second, since TFP is fixed in the case of monetary policy shocks, higher wages cause the dividend share of aggregate income to decline, so that β_i^y is decreasing in s_i.

It is simple to show that $\beta_i^y > 0$ when the labor productivity effect dominates the dividend effect on household income and hence consumption,

$$\beta_i^y > 0 \iff \frac{\theta_i^{\frac{1+\varphi}{\varphi}}}{\Theta} > s_i \left(1 - \frac{1}{\varphi \left(\bar{\Phi} - 1 \right)} \right)$$

Anticipating the empirical results, which find positive consumption sensitivities for all i, I assume that this condition holds for all i from now on.

Given the consumption response of saver households $i = S$, the equilibrium output response must mechanically satisfy

$$\hat{y}_t = \frac{\hat{\hat{c}}_{S,t}}{\beta_S^y}$$

Hence, if β_S^y is different for contractionary and expansionary shocks, then the equilibrium path of output will be shock-dependent, and asymmetric, as demonstrated by the following example.

A Numerical Example Let there be two household types, $i \in \{1, 2\}$, and assume that prices are fixed, $\xi^p \to +\infty$, so that the central bank directly controls the real interest rate. Suppose that $\phi_y = 0$, and $\frac{1}{\sigma} = 1$, and assume that group 1 households' consumption is more sensitive to changes in output than group 2 households' consumption, $\beta_1^y = 2$, and $\beta_2^y = 0.5$.[10]

Given real interest rate shocks of $\epsilon_t^v = \pm 1\%$, the responses of output are found by solving the system

$$\min_i \left\{-\hat{\hat{c}}_{i,t}\right\} = \frac{1}{\sigma}\epsilon_t^v$$

$$\hat{\hat{c}}_{i,t} = \beta_i^y \hat{y}_t \ \forall i$$

where I have used $\mathbb{E}_t\left[\hat{\hat{c}}_{i,t+1}\right] = 0$ because the interest rate shock is i.i.d. over time.

When the real interest rate increases by 1%, saver households reduce their consumption by 1% because $\frac{1}{\sigma} = 1$. Since all other households are borrowing-constrained, their equilibrium reductions in consumption are greater than 1%. Hence, the equilibrium reduction in output must be greater than 1%.

Specifically, given output falls in equilibrium, group 2 households must be the savers since they experience the smallest drop in current consumption and therefore have the lowest expected consumption growth. Hence, $\hat{c}_{2,t} = -1\%$. Inverting the sensitivity equation for group 2 households, $\hat{\hat{c}}_{2,t} = 0.5\hat{y}_t$, implies that $\hat{y}_t = \frac{-1\%}{0.5} = -2\%$, so that output falls by 2% in equilibrium.

When the real interest rate decreases by 1%, saver households increase their consumption by 1%. Since all other households are borrowing-constrained, their equilibrium increase in consumption is less than 1%. Hence, the equilibrium increase in output must be less than 1%.

Specifically, given output rises in equilibrium, group 1 households must be the savers since they experience the largest increase in current consumption and therefore have the lowest expected consumption growth. Hence, $\hat{c}_{1,t} = 1\%$. Inverting the sensitivity equation for group 1 households, $\hat{c}_{1,t} = 2\hat{y}_t$, implies that $\hat{y}_t = 0.5\%$ so that output increases by 0.5% in equilibrium.

Therefore, output responds more to contractionary monetary policy shocks than to expansionary monetary policy shocks. Furthermore, because the saver households always adjust their consumption growth by 1% in equilibrium, the ratio of the contractionary response to the

[10] Technically, the steady state of the system is no longer unique when $\xi^p \to +\infty$ and $\phi_y = 0$. However, I abstract from this complication for the purposes of this example.

expansionary response coincides with the ratio of consumption sensitivity coefficients. I come back to this connection in section 4.

In general, we have the following closed-form representation of the asymmetric output responses to monetary policy shocks.

Proposition 1. *Under assumption 1, the first order equilibrium dynamics of output in response to monetary policy shocks are given by*

$$\hat{y}_t = \begin{cases} -\frac{1}{\underline{\beta}+\frac{1}{\sigma}\phi_\pi\frac{\bar{\Phi}-1}{\xi^p}\varphi+\frac{1}{\sigma}\phi_y}\frac{1}{\sigma}\epsilon_t^v & \text{if} \quad \epsilon_t^v > 0 \\ \\ -\frac{1}{\bar{\beta}+\frac{1}{\sigma}\phi_\pi\frac{\bar{\Phi}-1}{\xi^p}\varphi+\frac{1}{\sigma}\phi_y}\frac{1}{\sigma}\epsilon_t^v & \text{if} \quad \epsilon_t^v < 0 \end{cases}$$

where

$$\bar{\beta} = \max_i \{\beta_i^y\}$$

$$\underline{\beta} = \min_i \{\beta_i^y\}$$

$\bar{\beta} > \underline{\beta}$ implies that output responds more to positive (contractionary) monetary policy shocks than to negative (expansionary) monetary policy shocks of equal magnitude. In other words, output responds asymmetrically to monetary policy shocks.[11]

In response to an expansionary monetary policy shock, interest rates fall, and saver households increase their current consumption. By virtue of being borrowing-constrained in equilibrium, all other households must increase their consumption by less than savers. Therefore, the equilibrium increase in output is smaller than the consumption response of savers alone. Equivalently, saver households' consumption increase is the most sensitive to the increase in output in equilibrium, as captured by the forth equation of the lemma, evaluated for the saver household $i = S$,

$$\hat{\hat{c}}_{S,t} = \bar{\beta}\hat{y}_t, \ \bar{\beta} = \max_i \{\beta_i^y\}$$

In response to a contractionary monetary policy shock, saver households decrease their current consumption. By virtue of being borrowing-constrained in equilibrium, all other households must experience a larger decrease in their consumption than savers do. Therefore, the equilibrium decrease in output is larger than the consumption response of savers alone. Equivalently, saver households' consumption is the least sensitive to changes in output in equilibrium,

$$\hat{\hat{c}}_{S,t} = \underline{\beta}\hat{y}_t, \ \underline{\beta} = \min_i \{\beta_i^y\}$$

[11] The equilibrium dynamics exist as long as $\underline{\beta} + \frac{1}{\sigma}\phi_\pi\frac{\bar{\Phi}-1}{\xi^p}\varphi + \frac{1}{\sigma}\phi_y > 0$, which is implied by $\beta_i^y > 0$ for all i. If this condition fails, then there does not exist an equilibrium output response to a contractionary monetary policy shock, $\epsilon_t^v > 0$, of the piece-wise linear form presented in proposition 1. However, given $\underline{\beta}$, parameters ϕ_π, ϕ_y, and ξ^p can always be chosen to ensure that the condition holds.

Since the size of the consumption response of savers is the same for both positive and negative equilibrium real interest rate changes, output must respond more to contractionary monetary policy shocks than to expansionary monetary policy shocks.

For completeness, I note that in the knife-edge case of $\bar{\beta} = \underline{\beta}$, there is no asymmetry. In this case, all households' incomes are equally sensitive to changes in aggregate income so that there is no incentive for households to trade the asset in response to an aggregate shock. Therefore, borrowing constraints do not play a role in determining the equilibrium response of output to monetary policy shocks.

3.2 Other Aggregate Shocks

It is simple to derive the equivalent of proposition 1 for the cases of cost-push shocks and TFP shocks. In both cases, the same asymmetry emerges: output responds more to shocks that increase interest rates and lower output, and less to shocks that decrease interest rates and increase output. I briefly sketch the intuition below. Full details of the analysis are in the appendix.

The asymmetry of cost-push shocks follows from the fact that a positive cost-push shock creates inflation which causes interest rates to rise via the Taylor rule, while a negative cost-push shock causes interest rates to fall. These interest rate movements then initiate the same mechanism as above, causing output to respond more to the contractionary movement than to the expansionary movement.

Similarly, a positive TFP shock causes deflation and hence lower interest rates, while a negative TFP shock creates inflation and higher interest rates. Therefore, the same mechanism implies that output will respond more to negative (contractionary) TFP shocks than to positive (expansionary) TFP shocks.

3.3 Inflation Responses

Solving the system of equations in lemma 1 yields equilibrium responses of both output and inflation. In contrast to output, the direction of the asymmetry of inflation is shock-dependent. I outline the key economic mechanisms below, and relegate the derivations to the appendix.

In the case of monetary policy shocks, inflation inherits the asymmetry of output. This occurs because the response of inflation is entirely determined by the response of output via the logic of the NKPC: higher output implies higher marginal costs which causes firms to increase their prices, thus raising inflation. Therefore, inflation moves in the same direction as output in response to monetary shocks. Since output responds more to contractionary monetary shocks than to expansionary shocks, inflation inherits this asymmetry.

In the cases of cost-push and TFP shocks, the asymmetry of the inflation response is the opposite to that of output: inflation responds more to shocks that increase output. This occurs because the responses of inflation are determined by two forces. First, the movement of output affects inflation via the NKPC as in the monetary shock case. Second, both cost-push and TFP shocks directly affect firms' marginal costs and so directly affect inflation (positive cost-push shocks increase marginal costs and inflation, while positive TFP shocks lower marginal costs and inflation). Crucially, this effect pushes inflation in the opposite direction to the movement of output such that the first effect offsets the second. The strength of this offsetting force inherits the asymmetry of output's responses to both types of shock so that inflation responds more overall when output responds less and the offsetting force is weaker. Therefore inflation responds with the opposite asymmetry to output.

3.4 Idiosyncratic Shocks

It is straightforward to extend the analysis of this section to the case in which idiosyncratic risk is "small", so that $\sigma_\epsilon \to 0$. In this case, the asymmetry of output responses to monetary policy shocks depends on the persistence of idiosyncratic shocks. In particular, when uninsurable idiosyncratic shocks are very persistence ($\rho_\theta \to 1$), the output response asymmetry is the same as in the case without idiosyncratic risk, so that proposition 1 still applies. I outline the key economic mechanisms below, and relegate the derivations to the appendix.

In the presence of idiosyncratic risk, there is an additional channel through which households can have low consumption growth, and hence be savers in equilibrium. When a household expects to experience a drop in her idiosyncratic labor productivity, her consumption growth will be low to the extent that the drop in labor productivity is uninsurable and hence transmits to her consumption. Importantly, this novel channel is independent of aggregate shocks hitting the economy, and so cannot be a source of asymmetry.

However, when the process for uninsurable labor productivity shocks is very persistent ($\rho_\theta \to 1$), households expect their labor productivity to remain approximately constant across consecutive periods. This renders the novel savings channel inactive. As a result, savings choices are entirely determined by household income sensitivities to changes in output. Therefore, the responses of output to monetary policy shocks are identical to the economy without idiosyncratic risk.

The empirical evidence suggests that the process for idiosyncratic shocks to labor income has a very persistent component, so that $\rho_\theta \to 1$ is a good approximation to reality (see, for example, Storesletten et al. (2004) and Guvenen et al. (2016)). Furthermore, related evidence suggests that households are very well insured against shocks to the transitory component (Blundell et al. (2008) and Heathcote et al. (2014)), so that these shocks do not affect consumption growth computations.

In order to obtain analytical insights, I assume that the idiosyncratic risk is "small" so that first order approximations are valid. This approach rules out second order phenomena, such as the cyclicality of idiosyncratic income risk, that may also effect the responses of output to monetary policy shocks (and other aggregate shocks). These effects are the focus of Werning

(2015) and Acharya and Dogra (2018), who show how the cyclicality of idiosyncratic income risk affects the size of the output responses to both positive and negative monetary policy shocks. Importantly, this effect is symmetric in the sign of the shock, and so does not affect the asymmetry that I am interested in.

3.5 Heterogeneous Preferences

In my benchmark model, I follow the New Keynesian literature and assume that households have homogeneous EISs given by $\frac{1}{\sigma}$.[12] However, the asset pricing literature has suggested that heterogeneous EISs may help to reconcile macroeconomic models and asset pricing facts (Guvenen, 2009), and have empirical grounding (Vissing-Jorgensen (2002), Guvenen (2006)). In the case of heterogeneous EISs, the asymmetry still emerges, but for different reasons. I outline the key mechanism below, and relegate the derivations to the appendix.

When households have heterogeneous EISs, their consumption responses to changes in the real interest rate are heterogeneous. In equilibrium, these different consumption responses occur via asset trading. For example, in response to a contractionary monetary policy shock, households with larger EISs will save, lending to households with smaller EISs in equilibrium.

However, binding borrowing constraints limit the asset trading that occurs in equilibrium and causes output to respond asymmetrically. Consider a contractionary shock. When the real interest rate falls, households try to decrease their consumption by an amount dictated by their EIS. However, households with small EISs become borrowing-constrained in equilibrium and so experience larger consumption drops than they would in the absence of the constraint. Therefore, the overall equilibrium decrease in output is amplified.

When interest rates rise after an expansionary shock, households increase their consumption. In this case, households with large EISs become borrowing-constrained in equilibrium, and so experience smaller consumption increases then they would were they not constrained. Therefore, the overall equilibrium increase in output is dampened, thus creating asymmetric responses of output to monetary policy shocks.

4 Numerical Exercise

In this section, I quantitatively assess the asymmetry of output responses to monetary policy shocks. I first use my theoretical insights to highlight the key parameters that I need to measure to quantify the asymmetry, and then turn to parameter estimation.

My estimates of the highest and lowest sensitivities of household consumption to changes in output imply that the output response to a contractionary monetary policy shock is two and a half times the size of the response to an expansionary shock of the same magnitude. This result compares favorably to the macro-econometric evidence for asymmetry in section 5.

[12] Technically, $\frac{1}{\sigma}$ is the EIS for "net consumption" when preferences are of the GHH form. However, abstracting from this complication does not affect the intuition.

4.1 Sufficient Statistics for Output Response Asymmetry

The theoretical analysis has shown that output response asymmetry arises in equilibrium when two conditions hold: some households are borrowing-constrained, and households have heterogeneous consumption sensitivities to changes in output in equilibrium. While proposition 1 was derived under assumption 1, the intuition suggests that such stringent restrictions are not necessary to generate output response asymmetry more generally.

In this section, I consider a system of equations that is motivated by the system in lemma 1, but does not have such a restrictive structural interpretation attached to it. In particular, rather than deriving the sensitivities of household consumption to changes in output from primitives using strong structural assumptions, I specify the sensitivities as reduced-form parameters that have a similar interpretation to their structural counterparts, but are estimable using micro data on household consumption.

Let $\mathcal{S}$ be the system

$$\iota_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t + v_t$$

$$\pi_t = \kappa_y \hat{y}_t + \delta \mathbb{E}_t \left[\pi_{t+1}\right]$$

$$\min_i \left\{\mathbb{E}_t \left[\hat{c}_{i,t+1}\right] - \hat{c}_{i,t}\right\} = \frac{1}{\sigma} \left(\iota_t - \mathbb{E}_t \left[\pi_{t+1}\right] - \rho\right)$$

$$\hat{c}_{i,t} = \beta_i^{c,v} \hat{y}_t \ \forall i$$

that describes the equilibrium dynamics of output $\hat{y}_t$, inflation π_t, nominal interest rates ι_t, and household consumption $\hat{c}_{i,t}$, in response to monetary policy shocks v_t.

Even in the absence of an underlying structural model, the first three equations in system $\mathcal{S}$ have clear interpretations as a Taylor rule, NKPC, and Euler equation in the presence of binding borrowing constraints.

Furthermore, it is straightforward to interpret the $\beta_i^{c,v}$ parameters as mapping heterogeneous sensitivities of household income to changes in output into heterogeneous sensitivities of household consumption to changes in output via asset market trading. For example, if $\beta_i^{c,v}$ is fixed for all i, then households would be fully sharing the incidence of aggregate shocks, which would indicate that income sensitivities are homogeneous or that borrowing constraints are slack. As the $\beta_i^{c,v}$ parameters become more heterogeneous, it is reasonable to interpret this as evidence for a combination of heterogeneous income sensitivities and restricted asset market trading caused by binding borrowing constraints. In this sense, the $\{\beta_i^{c,v}\}$ coefficients are the theoretical analogs of the $\{\beta_i^y\}$ coefficients when the no-borrowing restriction is relaxed.[13]

In the empirically relevant case of persistent monetary policy shocks (Gertler and Karadi (2015) and Christiano et al. (2005)), the inherent non-linearity of system $\mathcal{S}$ prevents me from studying the full equilibrium dynamics. Therefore, I instead derive the responses of output to

[13] I assume the consumption sensitivities are fixed over time for each household. While this is a reduced-form assumption, it is consistent with the subsequent empirical strategy since the micro data on consumption are not rich enough to permit credible estimation of time-varying sensitivities.

one-time, zero probability ("MIT") monetary policy shocks of the form $v_t = \rho_v^{t-1} v_1$, where v_1 is the zero-probability monetary policy shock, and $\rho_v \in (0,1)$ is a persistence parameter. This approach is tractable since once the shock hits, the economy transitions deterministically back to the steady state of the system.

Proposition 2. *In system $\mathcal{S}$, the response of output to a one time, zero probability monetary policy shock v_1 with persistence ρ_v, is given by*

$$\hat{y}_t = \begin{cases} -\frac{1}{(1-\rho_v)\underline{\beta}^{c,v} + \frac{1}{\sigma}(\phi_\pi - \rho_v)\frac{\kappa_y}{1-\delta\rho_v} + \frac{1}{\sigma}\phi_y}\frac{1}{\sigma}\rho_v^{t-1} v_1 & \text{if} \quad v_1 > 0 \\ \\ -\frac{1}{(1-\rho_v)\bar{\beta}^{c,v} + \frac{1}{\sigma}(\phi_\pi - \rho_v)\frac{\kappa_y}{1-\delta\rho_v} + \frac{1}{\sigma}\phi_y}\frac{1}{\sigma}\rho_v^{t-1} v_1 & \text{if} \quad v_1 < 0 \end{cases}$$

where

$$\underline{\beta}^{c,v} = \min_i \{\beta_i^{c,v}\}$$

$$\bar{\beta}^{c,v} = \max_i \{\beta_i^{c,v}\}$$

As in the case of i.i.d shocks, output responds more to the contractionary monetary policy shock than to the expansionary shock. Appealing to the structural interpretation of the system yields the same intuition as before: when $v_1 > 0$, the decrease in savers' consumption is the smallest among all households, so that output falls a lot. When $v_1 < 0$, the increase in savers' consumption is the largest among all households so that output increases a little.

In order to assess the quantitative magnitude of the output response asymmetry, I define the ratio of the contractionary response to the expansionary response,

$$\mathcal{R} = \frac{(1-\rho_v)\bar{\beta}^{c,v} + \frac{1}{\sigma}(\phi_\pi - \rho_v)\frac{\kappa_y}{1-\delta\rho_v} + \frac{1}{\sigma}\phi_y}{(1-\rho_v)\underline{\beta}^{c,v} + \frac{1}{\sigma}(\phi_\pi - \rho_v)\frac{\kappa_y}{1-\delta\rho_v} + \frac{1}{\sigma}\phi_y}$$

The key parameters for quantifying the asymmetry are $\bar{\beta}^{c,v}$ and $\underline{\beta}^{c,v}$, which measure the highest and lowest equilibrium sensitivities of household consumption to changes in output. In particular, when $\frac{1}{\sigma}(\phi_\pi - \rho_v)\frac{\kappa_y}{1-\delta\rho_v} + \frac{1}{\sigma}\phi_y = 0$, $\mathcal{R} = \frac{\bar{\beta}^{c,v}}{\underline{\beta}^{c,v}}$.

The parameters $\bar{\beta}^{c,v}$ and $\underline{\beta}^{c,v}$ are "sufficient statistics" for computing the output response asymmetry (Chetty, 2009).[14] In other words, to compute $\mathcal{R}$, I only need to know the values of $\bar{\beta}^{c,v}$ and $\underline{\beta}^{c,v}$, and do not need quantitative information on the underlying structural mechanism that generates them. In my setting, this means that I do not need to know quantitative details concerning borrowing constraints or the heterogeneity of income sensitivities. This is convenient because I can estimate $\bar{\beta}^{c,v}$ and $\underline{\beta}^{c,v}$ directly using micro data on household consumption, and then plug these estimates into $\mathcal{R}$ to immediately quantify the asymmetry.

[14]Sufficient statistics approaches have recently become popular in macroeconomics. See, for example, Auclert and Rognlie (2017).

4.2 Parameter Estimation

In principle, estimates for $\bar{\beta}^{c,v}$ and $\underline{\beta}^{c,v}$ can be recovered using data on household consumption and output. Formally, let $\{c_{i,t}\}_{i,t}$ and $\{Y_t\}_t$ be data on household consumption and output respectively, and consider the linear regression

$$\Delta \log c_{i,t} = \alpha_i + \beta_i \Delta \log Y_t + u_{i,t}$$

where $\{u_{i,t}\}$ are idiosyncratic shocks uncorrelated with changes in output, and running the regression in growth rates rather than in levels controls for long run growth trends in household consumption and output.

As it stands, this specification has two issues: shock conditioning, and data feasibility.

Shock Conditioning In order to measure the sensitivities of household consumption to changes in output driven by monetary policy shocks, the variation in $\Delta \log Y_t$ must be due to monetary policy shocks only. However, the variation in raw output data is driven by multiple aggregate shocks hitting the economy simultaneously in each period. Running the above regression would therefore result in estimates of $\{\beta_i\}$ that measure the sensitivity of household consumption to changes in output driven by multiple shocks, and would not correspond to the theoretical parameters $\{\beta_i^{c,v}\}$.

In order to alleviate this issue, I first project the output data onto a set of identified, lagged monetary policy shocks (described in more detail below), $Z_t = \left(\epsilon_{t-1}^v, ..., \epsilon_{t-L}^v\right)$,

$$\Delta \log Y_t = \alpha_y + Z_t' \gamma + e_t$$

so that the fitted values $\left\{\Delta \hat{\log} Y_t\right\}$ capture the variation in $\Delta \log Y_t$ driven by monetary policy shocks only.[15]

Then, using these fitted values, I estimate the regression

$$\Delta \log c_{i,t} = \alpha_i + \beta_i \Delta \hat{\log} Y_t + u_{i,t}$$

which correctly identifies β_i as the sensitivity of household consumption to changes in output driven by monetary policy shocks only. Given these estimates $\left\{\hat{\beta}_i\right\}$, the key asymmetry parameters are estimated as $\bar{\beta}^{c,v} = \max_i \left\{\hat{\beta}_i\right\}$ and $\underline{\beta}^{c,v} = \min_i \left\{\hat{\beta}_i\right\}$.

Intuitively, this process amounts to Two-Stage-Least-Squares (2SLS) estimation, where the first stage extracts the variation in $\Delta \log Y_t$ due to monetary policy shocks only, and the second stage estimates the household sensitivity parameters using this variation alone.

[15]My theoretical results suggest that $\Delta \log Y_t$ should depend non-linearly on the history of monetary policy shocks. However, for the purposes of extracting the variation in $\Delta \log Y_t$ driven by monetary policy shocks, I abstract from this complication. I investigate non-linear responses in section 5.

Household Consumption Data There do not exist data that contain precise measures of consumption at the household level and business cycle frequency. The Consumer Expenditure Survey (CEX) is the closest substitute, but is known to have measurement error problems (Aguiar and Bils, 2015), and only features the same household for four consecutive quarters.

In order to alleviate these issues, I group households together within the CEX data and estimate pooled OLS regressions instead. This approach helps to mitigate the effects of measurement error in the cross-section, and creates longer synthetic panels in the time series dimension. Similar methods are common among analyses that use CEX data to analyze trends and fluctuations in household consumption (see, for example, Parker and Vissing-Jorgensen (2009), Primiceri and van Rens (2009), and De Giorgi and Gambetti (2017)).

The choice of grouping naturally affects the estimates obtained from running regressions at the group level. Formally, let $\mathcal{G}$ be a surjective function that maps household i in period t, i.e. the household-period tuple (i,t), into a finite set of groups $\{1,2,...,G\}$. $\mathcal{G}$ represents an arbitrary group formation process, and nests fixed group assignment as a special case, $\mathcal{G}(i,t)$ fixed for all t.

Given a choice of $\mathcal{G}$ function, consider the pooled OLS regression for a group $g \in \{1,2,...,G\}$,

$$\Delta \log c_{i,t} = \alpha_g + \beta_g \Delta \hat{\log} Y_t + e_{i,t}$$

where the pooling occurs over the set $\{(i,t) : \mathcal{G}(i,t) = g\}$ of household-periods assigned to group g. Estimating this regression for each group implies that the key parameters for quantifying the asymmetry can be estimated as $\bar{\beta}^{c,v} = \max_g \left\{\hat{\beta}_g\right\}$ and $\underline{\beta}^{c,v} = \min_g \left\{\hat{\beta}_g\right\}$.

When the $\mathcal{G}$ function assigns each household i to a fixed group over time, the implied asymmetry parameters will always be weakly bounded by the true asymmetry parameters $\max_i \{\beta_i\}$ and $\min_i \{\beta_i\}$. Therefore, pooled OLS using fixed group assignments will always weakly underestimate the true asymmetry.

Proposition 3. *Suppose the model for household consumption growth is given by*

$$\Delta \log c_{i,t} = \alpha_i + \beta_i \Delta \hat{\log} Y_t + u_{i,t}$$

If $\mathcal{G}$ does not depend on t for all i, then the asymmetry parameters implied by the pooled OLS regressions

$$\Delta \log c_{i,t} = \alpha_g + \beta_g \Delta \hat{\log} Y_t + e_{i,t}$$

are weakly bounded by $\max_i \{\beta_i\}$ *and* $\min \{\beta_i\}$*, i.e.*

$$\max_g \left\{plim_{T\to\infty} \hat{\beta}_g\right\} \leq \max_i \{\beta_i\}$$

$$\min_g \left\{plim_{T\to\infty} \hat{\beta}_g\right\} \geq \min_i \{\beta_i\}$$

Intuitively, when group assignments are fixed over time, the estimated consumption exposure of a group g is a convex combination of the consumption exposures of each household in that group. Therefore, each group's consumption exposure is weakly smaller than the largest household exposure, and weakly larger than the smallest household exposure. This immediately says that the asymmetry implied by the estimates must be bounded by the true asymmetry at the household level.

When $\mathcal{G}$ assigns households to different groups over time, it is difficult to say whether the implied asymmetry from pooled OLS over- or underestimates the true asymmetry. As an extreme example, suppose that $\beta_i = 1$ for all i (so that the true asymmetry is nil) and consider the following assignment process for a fixed group g. When $\Delta \hat{\log} Y_t > 0$, assign households with the highest consumption growths to group g. When $\Delta \hat{\log} Y_t < 0$, assign households with the lowest consumption growths to group g. Such a process will result in an estimate of $\hat{\beta}_g$ much larger than 1, due to the selection bias created by the assignment mechanism's dependence on idiosyncratic shocks, and will therefore overestimate the true asymmetry. Furthermore, the opposite assignment process will clearly result in an underestimate of the true asymmetry.

In light of this discussion, I choose as a benchmark, an assignment mechanism that is fixed over time, so that the estimated asymmetry is known to be a lower bound on the true asymmetry (in the limit $T \to \infty$). In practice, this amount to defining groups based on household characteristics that are fixed in the sample of households that I observe.

4.3 Data

Monetary Policy Shocks In order to extract the variation in $\Delta \log Y_t$ driven by monetary policy shocks, I follow Coibon et al. (2017), who use the methods introduced by Romer and Romer (2004) to identify innovations to monetary policy that are orthogonal to economic conditions. Formally, the authors run the regression

$$\Delta FFR_t = x_t' \Gamma + \epsilon_t^v$$

where ΔFFR_t is the change in the federal funds rate from period $t-1$ to t, and x_t is a vector of controls that contains forecasts of GDP growth, inflation, and the unemployment rate taken from the Greenbooks at each Federal Open Market Committee meeting. The residuals from this regression, $\{\hat{\epsilon}_t^v\}$, are then taken as the series of monetary policy shocks, with the interpretation that $\hat{\epsilon}_t^v > 0$ is a contractionary shock, and $\hat{\epsilon}_t^v < 0$ is an expansionary shock.

Using this method, Coibon et al. (2017) generate a series of monetary policy shocks at monthly and quarterly frequencies from 1969 to 2008, which I plot in figure 1. The shocks are evenly spread over positive and negative values, and are very volatile during the Volcker disinflation period in the early 1980s.

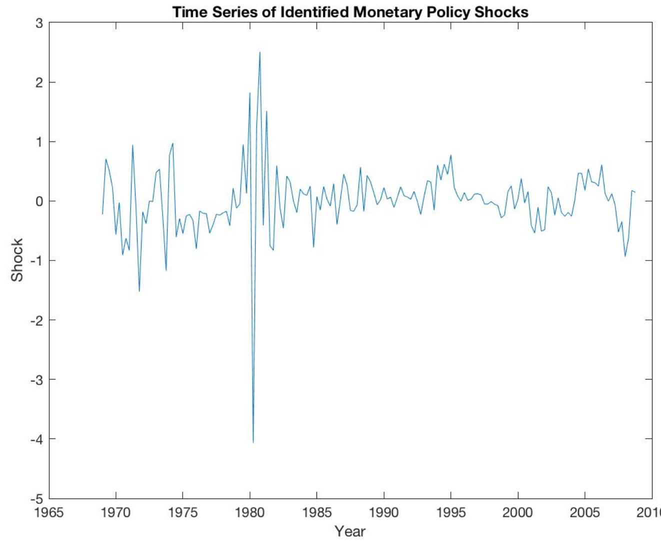

Figure 1: Identified Monetary Policy Shocks from Coibon et al. (2017). The authors run the regression $\Delta FFR = x_t'\gamma + \epsilon_t$ where ΔFFR_t is the change in the federal funds rate from period $t-1$ to t, and x_t is a vector of controls that contains forecasts of GDP growth, inflation, and the unemployment rate taken from the Greenbooks at each Federal Open Market Committee meeting. The residuals from this regression, $\{\hat{\epsilon}_t\}$, are then taken as the series of monetary policy shocks, with the t interpretation that $\hat{\epsilon}_t > 0$ is a contractionary shock, and $\hat{\epsilon}_t < 0$ is an expansionary shock.

Consumption and Output Data I use the CEX surveys from 1996 to 2009 to measure consumption of non-durables and services at the household level. I follow the literature (for example, Attanasio et al., 2009) and define non-durable and services consumption as total expenditures on food, services, heating fuel, public and private transport, personal care, and clothing and footwear.[16] I deflate nominal expenditures using the personal consumption expenditure price deflator. I also restrict the sample to urban households, not in student status, where the household head is of working age (25-64), and only consider households who respond to all four interview waves.[17]

Each household reports their consumption four times at three month intervals. From these reports, I compute three quarterly growth rates of log consumption for each household. Since different households are interviewed each month, I have quarterly growth rates of household consumption, available at a monthly frequency.

Proposition 3 suggests that grouping households together based on a fixed attribute is a useful benchmark to estimate a lower bound on the asymmetry coefficients. In the CEX data, the best candidate for this is the level of education of the household head.[18] Over the year long

[16]My results are robust to variations in this definition.

[17]In order to remove consumption variation caused by factors outside of my model, I first regress log real consumption on a polynomial in age of the household head, family size, and number of children under the age of eighteen, and use the residuals from this regression as my measures of household consumption.

[18]The very short panel nature of the CEX data implies that other potential fixed attributes such as permanent

cycle during which the household reports consumption, the education level of the household head is fixed and is certainly exogenous to changes in output over the same period. Therefore, I sort households into five groups based on the education level of the household head: less than high-school, high-school, some college, full college, and beyond college (advanced degree).

As my measure of output, I use quarterly growth rates of per-capita personal consumption expenditures of non-durable goods and services (at a monthly frequency), taken from the NIPA, deflated using the personal consumption expenditure price deflator.

My choice to use growth in per-capita personal consumption expenditures as the right-hand side variable reflects two considerations. First, the theoretical models I have studied in this paper have all abstracted from capital investment and government spending, so that aggregate consumption is the theoretically consistent measure of total output. Second, unlike measures of GDP, personal consumption expenditures are available at a monthly frequency, which enables me to exploit all of the variation in the micro-data and to maintain a reasonable sample size.

Empirical Specification For the first stage regression

$$\Delta \log Y_t = Z_t' \gamma + e_t$$

I project $\Delta \log Y_t$ onto a vector of the ninety six most recent identified monetary policy shocks $Z_t = \left(\hat{\epsilon}^v_{t-1}, ..., \hat{\epsilon}^v_{t-96}\right)$. This allows the effects of monetary policy shocks to persist for up to eight years, which more than captures the empirically relevant range of persistence of up to two to three years (Gertler and Karadi (2015) and Christiano et al. (2005)). My results are robust to variations in the lag length.

All regressions are weighted using the CEX survey weights provided in the data sets.

4.4 Results

Table 1 shows the estimated coefficient $\hat{\beta}_g$ for each education group, together with its standard error, which I cluster at the household level, and total sample size. The estimated coefficients display a pronounced "U-shape" with respect to education. A 1% increase in the growth of aggregate consumption caused by monetary policy shocks is associated with a 3.34% increase in the consumption growth of households with an advanced degree, but a 1.36% increase in the consumption growth of households with only a high-school diploma.

income are difficult to plausibly compute. Education is of course likely to be correlated with this and other fixed attributes.

	Less than High-School	High-School	Some College	College	Advanced Degree
$\hat{\beta}_g$	1.55	1.36	1.57	2.99	3.34
s.e.	1.22	0.78	0.71	0.77	1.13
n	9,621	21,396	27,025	19,206	11,022

Table 1: Estimated $\left\{\hat{\beta}_g\right\}$ exposure coefficients across household groups with different education levels using monthly data over the period 1996-2008. Standard errors are clustered at the household level.

These results imply an estimate for the sensitivity ratio of $\frac{\bar{\beta}^{c,v}}{\underline{\beta}^{c,v}} \approx 2.5$. Therefore, the most sensitive households are approximately two and a half times as sensitive to changes in aggregate consumption than the least sensitive households.

This finding is in line with previous studies of heterogeneous consumption sensitivities. For example, Parker and Vissing-Jorgensen (2009) group households in period t by their consumption level in period $t-1$, and find a sensitivity ratio of 5. While this estimate is larger than the lower bound of 2.5, the grouping strategy fails the conditions in proposition 3 so that it likely yields a biased estimate the true sensitivity ratio.

The "U-shaped" pattern of sensitivities is also consistent with the evidence on heterogeneous income sensitivities. For example, Guvenen et al. (2017) run a similar regression using worker level income data and unconditional variation in GDP growth across percentiles of the permanent income distribution, and find a "U-shaped" pattern of sensitivities such that the highest and lowest permanent income workers are the most sensitive to unconditional changes in GDP growth (see figure 4). This finding supports the theory that borrowing constraints cause household consumption to inherit the sensitivity of household income to changes in output.

4.5 Quantitative Assessment

Given estimates for $\bar{\beta}^{c,v}$ and $\underline{\beta}^{c,v}$, the other key parameter in $\mathcal{R}$ is the slope of the NKPC, κ_y. In order to set κ_y, I appeal to the empirical evidence from the literatures on inflation forecasting and estimation of the NKPC.

Both of these literatures suggest that κ_y is very small. The forecasting literature suggests that $\kappa_y = 0$ is very plausible (Atkeson and Ohanian, 2001), while the estimation literature tends to find κ_y around 0.05, but with a decent dose of uncertainty (Schorfheide, 2008). Therefore, as a convenient benchmark, I set $\kappa_y = 0$.

When $\kappa_y = 0$, the asymmetry ratio is $\mathcal{R} = 2.5$. Therefore, the output response to a contractionary monetary policy shock is two and a half times as large as the output response to an expansionary monetary policy shock of equal magnitude. I compare this asymmetry to the macro evidence for asymmetry in the next section.

For completeness, figure 2 plots $\mathcal{R}$ as a function of κ_y using a standard calibration of the other parameters.[19] The ratio declines as κ_y increases, but remains above two throughout the range, which covers the most plausible values of κ_y away from zero.

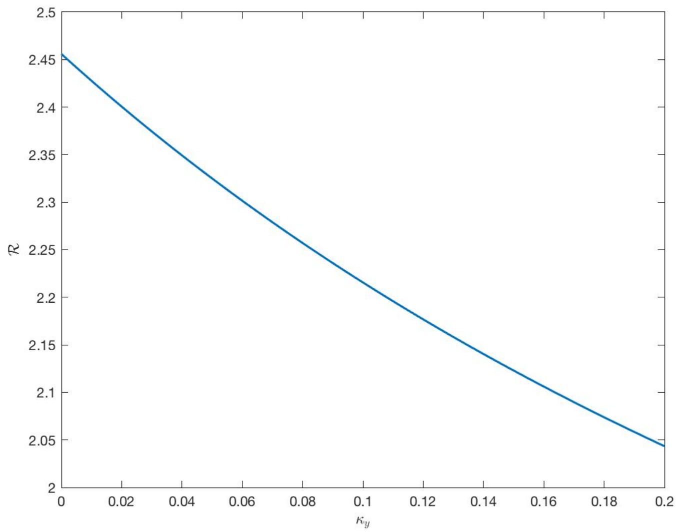

Figure 2: Asymmetry ratio $\mathcal{R}$ as a function of κ_y when $\rho_v = 0.6$, $\phi_\pi = 1.25$, $\phi_y = 0$, $\sigma = 1.5$, and $\delta = 0.995$.

Intuitively, when output increases after an expansionary shock, $\kappa_y > 0$ implies that inflation also increases. Higher inflation causes high nominal rates via the Taylor rule, which offsets some of the initial expansionary shock. The same logic implies that $\kappa_y > 0$ causes deflation to offset the contractionary shock. Since the initial output response is larger for a contractionary shock, the offsetting force is larger too, which shrinks the overall asymmetry.

5 Empirical Evidence of Monetary Policy Asymmetry

The micro evidence on heterogeneous consumption sensitivities implies that contractionary monetary policy shocks are two and a half times more powerful than expansionary monetary policy shocks. In this section, I show that this result is in line with the macro-econometric evidence for asymmetric monetary policy transmission. Specifically, I use local projection methods (Jorda, 2005) to demonstrate that contractionary monetary policy shocks are approximately four times more powerful than expansionary shocks.[20]

[19] I set $\rho_v = 0.6$ to reflect the quarterly persistence of monetary policy shocks estimated in the data (Christiano et al. (2005), Gertler and Karadi (2015)). I set $\phi_\pi = 1.25$ and $\phi_y = 0$, which is a commonly used specification for the Taylor rule, and set $\frac{1}{\sigma} = 0.67$ in line with estimates for the EIS (Vissing-Jorgensen, 2002). Finally, I set $\delta = 0.995$, which is consistent with an annual real interest rate of 2%.

[20] The literature on asymmetric monetary policy goes back to at least Cover (1992) and DeLong and Summers (1988), who both find contractionary shocks are more powerful than expansionary shocks. More recently, Angrist et al. (2013), and Barnichon and Matthes (2016), introduce novel methodologies to measure asymmetric effects, and also find that contractionary monetary policy shocks are more powerful than expansionary shocks.

5.1 Empirical Specification

I follow Jorda (2005), and estimate the impulse response of output to monetary policy shocks using local projection methods. Formally, I estimate the specification

$$y_{t+h} = \alpha^h + \beta^{h,+} \max\{\hat{\epsilon}_t^v, 0\} + \beta^{h,-} \min\{\hat{\epsilon}_t^v, 0\} + \sum_{l=0}^{L} \gamma_{y,l}^h y_{t-l} + \sum_{l=1}^{L} \gamma_{FFR,l}^h FFR_{t-l} + u_{t+h}^h$$

for horizons $h = 1, ..., H$. Here, $\{y_t\}$ is linearly de-trended output (in logs), $\{\hat{\epsilon}_t^v\}$ is the series of identified monetary policy shocks, and $\{FFR_t\}$ is the federal funds rate. The estimated coefficients $\left\{\hat{\beta}^{h,+}\right\}_1^H$ and $\left\{\hat{\beta}^{h,-}\right\}_1^H$ are the impulse responses of y to positive and negative shocks of unit size respectively.

I use quarterly frequency data over the period 1969 - 2008. In order to be consistent with the micro-data evidence, I use per-capita aggregate consumption of non-durables and services as my measure of output. I set $L = 1$, and note that the inclusion of contemporaneous aggregate consumption as a regressor is consistent with the convention that monetary policy shocks only affect measures of aggregate demand with a 1 period delay (Christiano et al., 1999). Finally, I estimate the system of equations over $h = 1, ..., H$ jointly, and compute Driscoll-Kraay (1998) standard errors that are robust to arbitrary serial and cross-sectional correlation across time and horizons.

5.2 Results

Figure 3 plots the estimated impulse responses of output to contractionary (positive) and expansionary (negative) monetary policy shocks of 1% size over fifteen quarters. The dashed lines are 90% confidence intervals. For ease of comparison, I have multiplied the expansionary response by -1. Both impulse responses exhibit the “U-shape” that is a common feature of output responses to monetary policy shocks (Christiano et al., 1999).[21]

[21] Since my simple model does not contain ingredients such as consumption habits or investment frictions that are typically found in medium-scale DSGE models, it cannot generate the “hump-shaped” impulse responses found in the data.

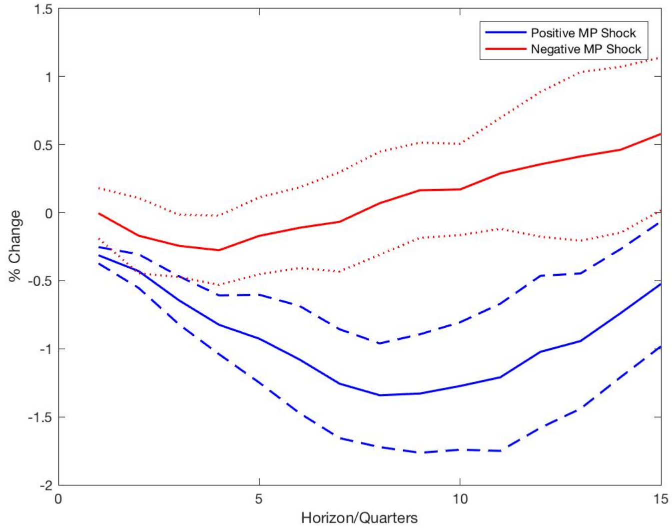

Figure 3: Impulse responses of aggregate consumption (from NIPA) estimated using local projection methods. The dashed lines are 90% confidence intervals computed using Driscoll-Kraay standard errors.

The contractionary shock generates a maximum response that is approximately four times as large as the maximum response to an expansionary shock. The asymmetry is statistically significant after about one year, by which time the effect of the expansionary shock has started to died out, but the contractionary shock is still causing further declines in output.

As a simple metric of comparison, I compare the ratio of the maximum responses in the data to the ratio of responses in the model, $\mathcal{R}$. According to this metric, the asymmetry estimated in the macro data is reasonably consistent with the asymmetry implied by the micro-data. The fact that the sensitivity ratio implied by the micro data is a lower bound implies that a quantitative version of model can explain at least 60% of the asymmetry found in the macro data, and could plausibly explain much more if we can estimate the true exposure ratio at a more granular level of household heterogeneity than education.

5.3 Robustness Checks

Here, I show that the asymmetric responses of output to monetary policy shocks are robust to regression specifications with different lag and control variable structures, sample restrictions that exclude the Volcker disinflation period, and when I change the dependent variable to GDP. All figures are in the appendix.

My baseline choice of $L = 1$ is optimal according to the Bayesian Information Criterion (BIC) given by

$$T \log\left(RSS/T\right) + k \log T$$

where RSS is the residual sum of squares from the regressions and T is the sample length. I also consider the Akaike Information Criterion (AIC), which is given by

$$T \log \left(RSS/T \right) + 2k$$

and also suggests an optimal choice of $L = 1$. Furthermore, figure 5 plots the impulse responses for $L \in \{2, 3, 4, 5\}$, and shows that the asymmetry is similar to the baseline specification in all cases.

The baseline regression includes aggregate demand and the federal funds rate as control variables. However, most New Keynesian models imply that inflation is also determined as part of the equilibrium system, and so affects the path of aggregate demand. To this end, figure 6 plots the impulse responses with inflation (measured by the Personal Consumption Expenditure deflator) as an additional control variable that follows the same lag structure as aggregate demand. The asymmetry is essentially unchanged.

It is well known that the Volcker disinflation period in the early 1980s resulted in volatile monetary policy, as exhibited by the large shocks in figure 1. While these shocks provide useful variation in the explanatory variable, it is useful to check that they are not the driving force behind the result. Therefore, in figure 7 I plot the impulse responses from the baseline regression having restricted the sample to 1985Q1 onwards, thus dropping the entire Volcker episode. While the smaller sample results in much wider confidence intervals, the asymmetry is still clear to see, with contractionary shocks having twice the effect of expansionary shocks. Note that in this case, the micro evidence can explain all of the asymmetry.

Finally, I run the baseline regression with real GDP as the dependent variable instead of aggregate consumption. Figure 8 plots the impulse responses, which exhibit similar levels of asymmetry, although they are slightly more noisily estimated.

6 Conclusion

When output falls in response to a contractionary monetary policy shock, the decrease in consumption of saver households is necessarily the smallest among all households. Therefore, the fall in output is greater than the response of saver households alone. In contrast, when output increases in response to an expansionary monetary policy shock, the increase in consumption of saver households is necessarily the largest among all households. Therefore, the increase in output is smaller than the response of saver households alone. Hence, output responds more to contractionary monetary shocks than to expansionary shocks of equal magnitude.

The micro-data suggests that the largest sensitivity of household consumption to changes in output is at least two and half times the size of the smallest sensitivity. When inflation is unresponsive to changes in output, output should respond two and a half times as much to contractionary shocks than to expansionary shocks. This quantitative result can therefore explain at least 60% of the asymmetry found in the macro data.

The mechanism in this paper applies to any aggregate shock. It would therefore be interesting to investigate the asymmetric transmission of other aggregate shocks, and to see how well the model does at explaining the asymmetry.

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A Additional Figures

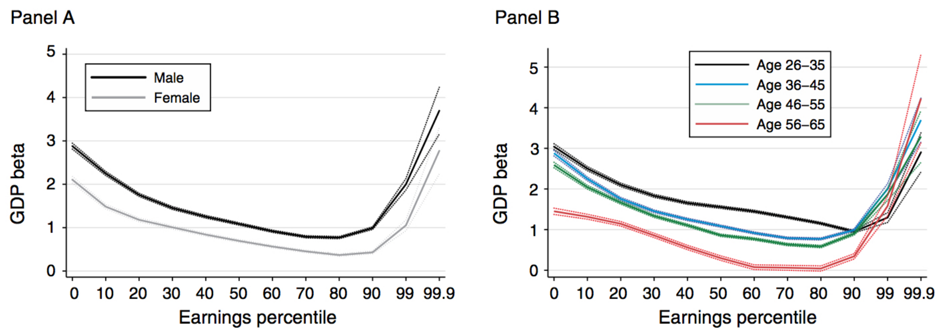

FIGURE 1. GDP BETA AT AGE 36–45 BY GENDER AND FOR MALES BY AGE GROUP

Notes: Earnings percentiles (conditional on gender and age group) are based on average real earnings from year $t-6$ to $t-2$, while real earnings growth used to estimate GDP beta is from year $t-1$ to t. The dotted lines represent the 95 percent confidence interval.

Figure 4: Reproduction of Figure 1 from Guvenen et al. (2017), which plots the β_g coefficients from the pooled OLS regression $\Delta \log y_{i,t} = \alpha_g + \beta_g \Delta \log Y_t + u_{i,t}$, where $y_{i,t}$ is worker i's income in period t as reported on her W-2 form, Y_t is GDP, and the groups $\{g\}$ are the gender-specific (Panel A) or age-specific (Panel B) percentiles of the permanent income distribution computed using incomes in periods $t-6$ to $t-2$.

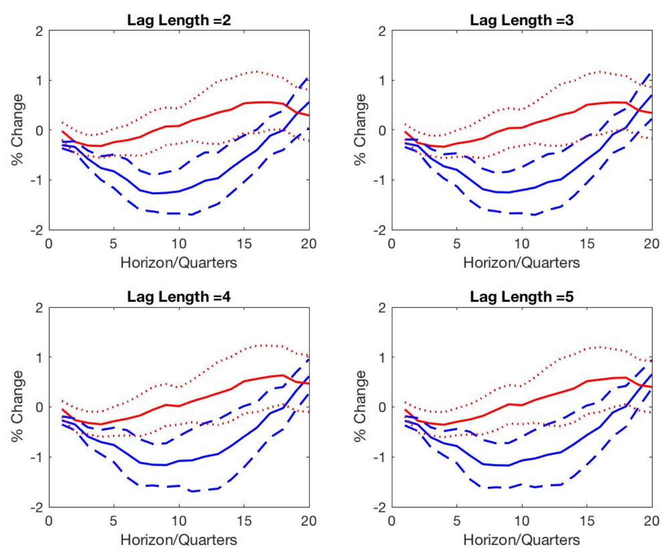

Figure 5: Impulse responses of real GDP (from NIPA) estimated using local projection methods and different lag structures.

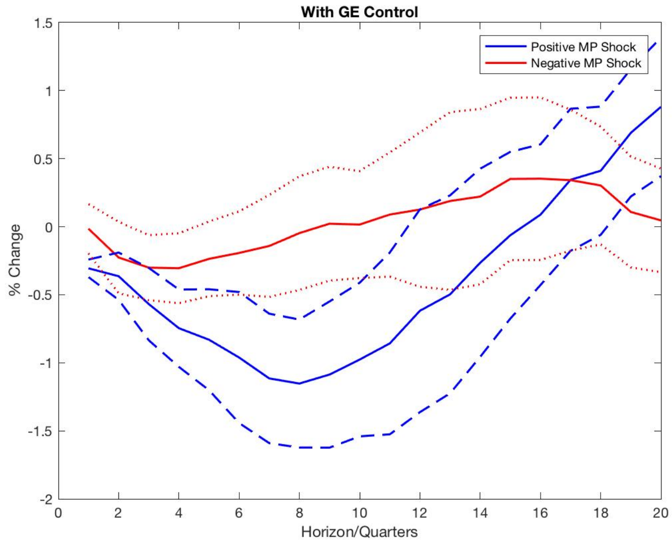

Figure 6: Impulse responses of real GDP (from NIPA) estimated using local projection methods with inflation as an additional control variable.

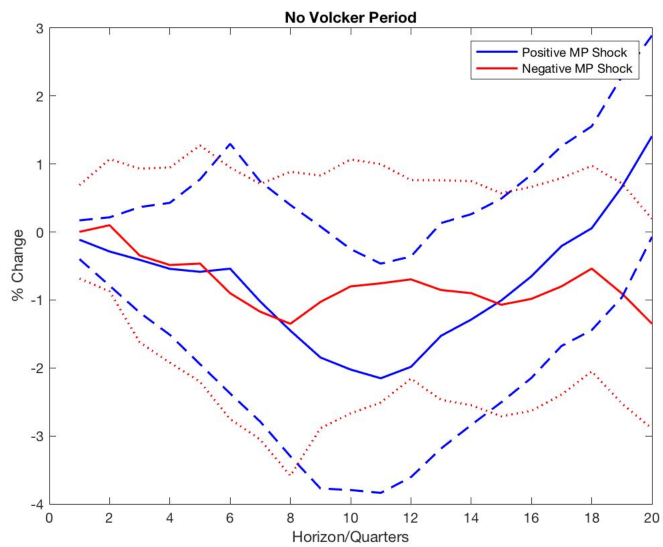

Figure 7: Impulse responses of real GDP (from NIPA) estimated using local projection methods, using only the post-Volcker sample.

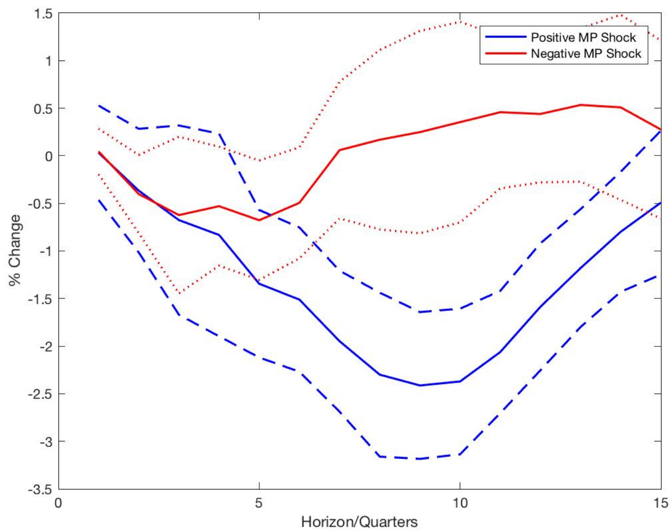

Figure 8: Impulse responses of real aggregate consumption of non-durables and services (from NIPA) estimated using local projection methods.

B Additional Results for Section 3

Proposition 4. *Under assumption 1, the first order equilibrium dynamics of output in response to cost-push shocks are given by*

$$\hat{y}_t = \begin{cases} \dfrac{\phi_\pi \frac{1}{\xi^p}}{\bar{\beta} + \frac{1}{\sigma}\phi_\pi \frac{\Phi - 1}{\xi^p}\varphi + \frac{1}{\sigma}\phi_y} \frac{1}{\sigma}\epsilon_t^\Phi & \text{if} \quad \epsilon_t^\Phi > 0 \\ \\ \dfrac{\phi_\pi \frac{1}{\xi^p}}{\underline{\beta} + \frac{1}{\sigma}\phi_\pi \frac{\Phi - 1}{\xi^p}\varphi + \frac{1}{\sigma}\phi_y} \frac{1}{\sigma}\epsilon_t^\Phi & \text{if} \quad \epsilon_t^\Phi < 0 \end{cases}$$

where

$$\bar{\beta} = \max_i \{\beta_i^y\}$$

$$\underline{\beta} = \min_i \{\beta_i^y\}$$

Proposition 4 shows the equilibrium dynamics of output as a function of the contemporaneous cost-push shock, and the heterogeneity in exposures of household income to aggregate income captured by the set of coefficients $\{\beta_i^y\}$. Similar to the monetary shock case, the dynamics of output depend on the sign of the cost-push shock, and the fact that $\bar{\beta} > \underline{\beta}$ implies that output responds more to positive cost-push shocks ($\epsilon_t^\Phi < 0$) than to negative cost-push shocks ($\epsilon_t^\Phi > 0$).

Proposition 5. *Under assumption 1, the first order equilibrium dynamics of output in re-*

sponse to TFP shocks are given by

$$\hat{y}_t = \begin{cases} c_y^+ \epsilon_t^a & \text{if} \quad \epsilon_t^a > 0 \\ \\ c_y^- \epsilon_t^a & \text{if} \quad \epsilon_t^a < 0 \end{cases}$$

where

$$c_y^+ = \frac{\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}(1+\varphi)}{\bar{\beta}^{TFP,+} + \frac{1}{\sigma}\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \frac{1}{\sigma}\phi_y} \frac{1}{\sigma} > 0$$

$$c_y^- = \frac{\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}(1+\varphi)}{\underline{\beta}^{TFP,-} + \frac{1}{\sigma}\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \frac{1}{\sigma}\phi_y} \frac{1}{\sigma} > c_y^+$$

$$\bar{\beta}^{TFP,+} = \max_i \left\{ \beta_i^y + \frac{\beta_i^a}{c_y^+} \right\}$$

$$\underline{\beta}^{TFP,-} = \min_i \left\{ \beta_i^y + \frac{\beta_i^a}{c_y^-} \right\}$$

Proposition 5 shows the equilibrium dynamics of aggregate output as a function of the contemporaneous TFP shock, and the heterogeneity in exposures of household consumption (and hence income) to aggregate income captured by the set of coefficients $\left\{\beta_i^y + \frac{\beta_i^a}{c_y^{a,+}}\right\}$ and $\left\{\beta_i^y + \frac{\beta_i^a}{c_y^{a,-}}\right\}$. Relative to the previous analyses, an additional complication stems from the fact that TFP shocks directly affect households' incomes since an increase in TFP increases the non-labor share of income and hence raises the exposure of households with large dividend shares. Therefore, the coefficients $c_y^{a,-}$, $c_y^{a,+}$, $\bar{\beta}^{TFP}$, and $\underline{\beta}^{TFP}$ must be determined jointly unlike in the previous analyses where the $\{\beta_i^y\}$ coefficients were direct functions of primitives. Similar to the other shocks, the dynamics of output depend on the sign of the TFP shock, and the fact that $c_y^- > c_y^+$ implies that output responds more to negative TFP shocks than to positive TFP shocks.[22]

Proposition 6. *Under assumption 1, the first order equilibrium dynamics of inflation in*

[22]Technically, it is possible that $c_y^+, c_y^- < 0$ due to the complementarity between consumption and labor supply created by my assumption of GHH preferences. Specifically, when TFP increases, there is a direct effect on labor supply: holding aggregate demand fixed, labor supply falls since TFP is higher. Given GHH preferences, this fall in labor supply causes net consumption to increase. Holding interest rates fixed, this causes households to lower their demand for consumption in order to maintain a smooth time path of marginal utility. Therefore, a positive TFP shock has a direct contractionary effect on output when households have GHH preferences. Given this effect is purely a result of my special assumption on preferences, I assume that parameters are such that this channel does not dominate the the usual effects that are the focus of my analysis, so that $c_y^+, c_y^- > 0$.

response to monetary policy shocks are given by

$$\pi_t = \begin{cases} -\frac{\frac{\bar{\Phi}-1}{\xi^p}\varphi}{\underline{\beta}+\frac{1}{\sigma}\phi_\pi\frac{\bar{\Phi}-1}{\xi^p}\varphi+\frac{1}{\sigma}\phi_y}\frac{1}{\sigma}\epsilon_t^v & if \quad \epsilon_t^v > 0 \\ \\ -\frac{\frac{\bar{\Phi}-1}{\xi^p}\varphi}{\bar{\beta}+\frac{1}{\sigma}\phi_\pi\frac{\bar{\Phi}-1}{\xi^p}\varphi+\frac{1}{\sigma}\phi_y}\frac{1}{\sigma}\epsilon_t^v & if \quad \epsilon_t^v < 0 \end{cases}$$

where

$$\bar{\beta} = \max_i \{\beta_i^y\}$$

$$\underline{\beta} = \min_i \{\beta_i^y\}$$

Proposition 6 shows the equilibrium dynamics of inflation as a function of the contemporaneous monetary shock, and the heterogeneity in exposures of household income to aggregate income captured by the set of coefficients $\{\beta_i^y\}$. Similar to output, the dynamics of inflation depend on the sign of the monetary shock, and the fact that $\bar{\beta}^{MP} > \underline{\beta}^{MP}$ implies that inflation responds more to positive (contractionary) monetary shocks than to negative (expansionary) shocks, thus mimicking the asymmetry of output.

Proposition 7. *Under assumption 1, the first order equilibrium dynamics of inflation in response to cost-push shocks are given by*

$$\pi_t = \begin{cases} -\frac{1}{\xi^p}\left(\frac{\bar{\beta}^{CP}+\frac{1}{\sigma}\phi_y}{\bar{\beta}^{CP}+\frac{1}{\sigma}\phi_\pi\frac{\bar{\Phi}-1}{\xi^p}\varphi+\frac{1}{\sigma}\phi_y}\right)\epsilon_t^\Phi & if \quad \epsilon_t^\Phi > 0 \\ \\ -\frac{1}{\xi^p}\left(\frac{\underline{\beta}^{CP}+\frac{1}{\sigma}\phi_y}{\underline{\beta}^{CP}+\frac{1}{\sigma}\phi_\pi\frac{\bar{\Phi}-1}{\xi^p}\varphi+\frac{1}{\sigma}\phi_y}\right)\epsilon_t^\Phi & if \quad \epsilon_t^\Phi < 0 \end{cases}$$

where

$$\bar{\beta}^{CP} = \max_i \{\beta_i^y\}$$

$$\underline{\beta}^{CP} = \min_i \{\beta_i^y\}$$

Proposition 7 shows the equilibrium dynamics of inflation as a function of the contemporaneous cost-push shock, and the heterogeneity in exposures of household income to aggregate income captured by the set of coefficients $\{\beta_i^y\}$. Similar to output, the dynamics of inflation depend on the sign of the shock, and the fact that $\bar{\beta}^{CP} > \underline{\beta}^{CP}$ implies that inflation responds more to negative cost-push shocks ($\epsilon_t^\Phi > 0$) than to positive cost-push shocks. Hence inflation exhibits the opposite asymmetry to output.

Proposition 8. *Under assumption 1, the first order equilibrium dynamics of inflation in* *response to TFP shocks are given by*

$$\pi_t = \begin{cases} c_\pi^+ \epsilon_t^a & \text{if} \quad \epsilon_t^a > 0 \\ \\ c_\pi^- \epsilon_t^a & \text{if} \quad \epsilon_t^a < 0 \end{cases}$$

where

$$c_\pi^+ = -\frac{\bar{\Phi}-1}{\xi^p}(1+\varphi)\frac{\bar{\beta}^{TFP,+} + \frac{1}{\sigma}\phi_y}{\bar{\beta}^{TFP,+} + \frac{1}{\sigma}\left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \phi_y\right)} < 0$$

$$c_\pi^- = -\frac{\bar{\Phi}-1}{\xi^p}(1+\varphi)\frac{\underline{\beta}^{TFP,-} + \frac{1}{\sigma}\phi_y}{\underline{\beta}^{TFP,-} + \frac{1}{\sigma}\left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \phi_y\right)} > c_y^+$$

$$\bar{\beta}^{TFP,+} = \max_i \left\{\beta_i^y + \frac{\beta_i^a}{c_y^+}\right\}$$

$$\underline{\beta}^{TFP,-} = \min_i \left\{\beta_i^y + \frac{\beta_i^a}{c_y^-}\right\}$$

Proposition 8 shows the equilibrium dynamics of inflation as a function of the contemporaneous TFP shock, and the heterogeneity in exposures of household income to aggregate income captured by the set of coefficients $\left\{\beta_i^y + \frac{\beta_i^a}{c_y^+}\right\}$ and $\left\{\beta_i^y + \frac{\beta_i^a}{c_y^-}\right\}$. Similar to output, the dynamics of inflation depend on the sign of the shock, and the fact that $c_\pi^+ < c_\pi^-$ implies that inflation responds more to positive TFP shocks than to negative TFP shocks. Hence inflation exhibits the opposite asymmetry to output.

B.1 Idiosyncratic Risk

In this section, I relax the restriction that $\sigma_\epsilon = 0$, so that households experience idiosyncratic shocks to their labor productivity $\theta_{i,t}$. In order to maintain tractability, I assume instead that the idiosyncratic risk is "small", and study the economy in the limit $\sigma_\epsilon \to 0$. In addition, I assume that all households have the same average productivity level, $\theta_i = \bar{\theta}$ for all i. Since the response of output to interest rate changes lies at the heart of all of the asymmetry results, I focus on the case of monetary policy shocks.

For clarity, I state the assumption I require for tractability, which replaces assumption 1. All common restrictions have the same interpretation as before.

Assumption 2. *Let*

$$\rho_a, \rho_\Phi, \rho_v = 0$$

$$\Sigma \to 0$$

$$u(c,n) = \frac{\left(c - \frac{n^{1+\varphi}}{1+\varphi}\right)^{1-\sigma}}{1-\sigma}$$

$$\sigma_\epsilon \to 0 \ \forall i$$

$$\theta_i = \bar{\theta} \ \forall i$$

$$\underline{b}_{i,t} \to 0 \ \forall i, t$$

Stationary Competitive Equilibrium When all aggregate shocks are set to zero in all periods, I can define a stationary competitive equilibrium. In this equilibrium, aggregate quantities and prices are constant over time, while households' choices of consumption and labor supply change over time as a function of their labor productivity, which is subject to idiosyncratic shocks.

Under assumption 2, there is a unique ergodic distribution of labor productivities in the economy, Λ_θ. Since the no-borrowing restriction implies that the wealth distribution is degenerate in equilibrium, Λ_θ is sufficient to compute cross-sectional averages of household consumption, labor supply, and income variables. Hence, the stationary competitive equilibrium is unique.

Definition 3. Under assumption 2, and given initial conditions $\left\{\{b_{i,0}, \theta_{i,0}\}_i, P_0\right\}$ where $\theta_{i,0} \sim \Lambda_\theta$ and $b_{i,0} = 0$ for all i, the unique stationary competitive equilibrium is a sequence $\left\{\{c_{i,t}, n_{i,t}, b_{i,t}\}_{i,t}, \{y(j)\}_j, P, d, w, \iota\right\}$ such that

1. $\{c_{i,t}, n_{i,t}, b_{i,t}\}_{i,t}$ solve the household problem for each i.
2. $\{y(j)\}_j$ solve the final good firms' problem.
3. $P = P_0$ solves the intermediate goods firms' problem.
4. d satisfies the dividend equation.
5. Markets clear at every time $t \geq 1$.

In order to derive analytical results, I consider the dynamics of the economy in response to aggregate shocks around this unique stationary equilibrium. Formally, the following lemma condenses the economy to a set of four equations, analogous to lemma 1 for the case without idiosyncratic risk. Similar to that case, aggregate variables are expressed in log deviations around their values in the stationary equilibrium. Variables for household i are expressed as linear functions of these log deviations and also of log deviations of her labor productivity $\theta_{i,t}$

from its mean value θ_i. The approximation in the labor productivity dimension is valid since I work in the neighborhood of zero idiosyncratic risk, $\sigma_\epsilon \to 0$.

Lemma 2. *Under assumption 2, and to first order, the economy admits the following representation:*

$$\iota_t = r + \phi_\pi \pi_t + \phi_y \hat{y}_t + \epsilon_t^v$$

$$\pi_t = \frac{\bar{\Phi} - 1}{\xi^p} \varphi \hat{y}_t - \frac{\bar{\Phi} - 1}{\xi^p} (1 + \varphi) \epsilon_t^a - \frac{1}{\xi^p} \epsilon_t^\Phi + \delta \mathbb{E}_t [\pi_{t+1}]$$

$$\min_i \left\{ \mathbb{E}_t \left[\hat{\tilde{c}}_{i,t+1} \right] - \hat{\tilde{c}}_{i,t} \right\} = \frac{1}{\sigma} (\iota_t - \mathbb{E}_t [\pi_{t+1}] - \rho)$$

$$\hat{\tilde{c}}_{i,t} = \beta_i^y \hat{y}_t + \beta_i^a \epsilon_t^a + \beta_i^\theta \hat{\theta}_{i,t} \ \forall i$$

where r is the real interest rate in the stationary competitive equilibrium, $\left\{ \beta_i^y, \beta_i^a, \beta_i^\theta \right\}_i$ depend only on model primitives, and where $\tilde{c}$ is consumption net of the disutility of labor supply, $\tilde{c} = c - \frac{n^{1+\varphi}}{1+\varphi}$.

$\phi_\pi > 1$ and $\phi_y \geq 0$ are sufficient to ensure that the system has a unique steady state, $\hat{y}_t = 0$, $\pi_t = 0$, $\hat{\tilde{c}}_{i,t} = \beta_i^\theta \hat{\theta}_{i,t}$ $\forall i$.

Remark 1. The stationary real interest rate is given by

$$r = \rho - \sigma (1 - \rho_\theta) \max_i \left\{ \beta_i^\theta \hat{\theta}_{i,t} \right\} < \rho$$

In the presence of idiosyncratic labor productivity risk and binding borrowing constraints, households use the asset to build a buffer stock of savings as a self insurance mechanism. This increases the demand for the asset, and drives down the equilibrium real interest rate below the discount rate of households, $r < \rho$.

The empirical evidence suggests that the process for idiosyncratic shocks to labor income has a very persistent component (see, for example, Storesletten et al. (2004) and Guvenen et al. (2016)). In my setting, this is the case in which $\rho_\theta \to 1$. Conveniently, this limit case also allows me to solve the system explicitly for the responses of output to monetary policy shocks.

Proposition 9. *Under assumption 2, and in the limit as $\rho_\theta \to 1$, the first order equilibrium dynamics of output in response to monetary policy shocks are given by*

$$\hat{y}_t = \begin{cases} -\frac{1}{\underline{\beta}+\frac{1}{\sigma}\phi_\pi\frac{\bar{\Phi}-1}{\xi^p}\varphi+\frac{1}{\sigma}\phi_y}\frac{1}{\sigma}\epsilon_t^v & \text{if} \quad \epsilon_t^v > 0 \\ \\ -\frac{1}{\bar{\beta}+\frac{1}{\sigma}\phi_\pi\frac{\bar{\Phi}-1}{\xi^p}\varphi+\frac{1}{\sigma}\phi_y}\frac{1}{\sigma}\epsilon_t^v & \text{if} \quad \epsilon_t^v < 0 \end{cases}$$

where

$$\bar{\beta} = \max_i \{\beta_i^y\}$$
$$\underline{\beta} = \min_i \{\beta_i^y\}$$

Proposition 9 shows that the responses of output are identical to the case without idiosyncratic risk. Therefore, the asymmetry is robust to the inclusion of idiosyncratic shocks that are very persistent.

In the presence of idiosyncratic risk, there are two distinct channels through which households can have low consumption growth, and hence be savers in equilibrium. The first is the source of asymmetry that I have analyzed in earlier sections. When households' incomes are heterogeneously exposed to changes in aggregate income, binding borrowing constraints prevent the equalization of consumption sensitivities of output changes, and cause contractionary shocks to have larger effects than expansionary shocks.

The novel channel is due to idiosyncratic risk and is independent of heterogeneous income exposures, and so cannot be a source of asymmetry. When a household expects to experience a drop in her labor productivity, her consumption growth will be low to the extent that the drop in labor productivity is uninsurable and hence transmits to her consumption.

When the process for labor productivity is very persistent ($\rho_\theta \to 1$), households expect their labor productivity to remain approximately constant across consecutive periods. This renders the novel savings channel inactive. As a result, savings choices are entirely determined by exposures to changes in output. In this case, the responses of output to monetary policy shocks are identical to the economy without idiosyncratic risk, as shown by the proposition.

Idiosyncratic shocks are often modeled as the sum of a persistent and transitory component. However, the empirical evidence suggests that households are very well insured against transitory shocks (Blundell et al. (2008) and Heathcote et al. (2014)), so that they do not affect consumption growth computations. Therefore, adding transitory shocks to labor productivity with a sufficiently rich set of contracts to provide insurance against them would complicate the model greatly without providing additional insights.

B.2 Heterogeneous Preferences

Let household i have preferences given by

$$u_i(c,n) = \frac{\left(c - \frac{n^{1+\varphi}}{1+\varphi}\right)^{1-\sigma_i}}{1-\sigma_i}$$

where $\sigma_i > 0$ for all i. The follow lemma summarizes the dynamics of the economy, and is the natural extension of lemma 1.

Lemma 3. *Under assumption 1, the economy's first order equilibrium dynamics in response to monetary policy shocks satisfy the system*

$$\iota_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t + \epsilon_t^v$$

$$\pi_t = \frac{\bar{\Phi} - 1}{\xi^p} \varphi \hat{y}_t + \delta \mathbb{E}_t [\pi_{t+1}]$$

$$\min_i \left\{ \sigma_i \left(\mathbb{E}_t \left[\hat{\tilde{c}}_{i,t+1} \right] - \hat{\tilde{c}}_{i,t} \right) \right\} = \iota_t - \mathbb{E}_t [\pi_{t+1}] - \rho$$

$$\hat{\tilde{c}}_{i,t} = \beta_i^y \hat{y}_t \ \forall i$$

where $\rho = -\log\delta$, $\{\beta_i^y\}_i$ *depend only on model primitives, and* $\tilde{c}$ *is consumption net of the disutility of labor supply,* $\tilde{c} = c - \frac{n^{1+\varphi}}{1+\varphi}$.

$\phi_\pi > 1$ *and* $\phi_y \geq 0$ *are sufficient to ensure that the system has a unique steady state,* $\hat{y}_t = 0$, $\pi_t = 0$, $\hat{\tilde{c}}_{i,t} = 0 \ \forall i$.

Given this lemma, we have the following closed-form representation of the asymmetric output responses to monetary policy shocks.

Proposition 10. *Under assumption 1, and when households have heterogeneous* σ_i *parameters, the first order equilibrium dynamics of output in response to monetary policy shocks are given by*

$$\hat{y}_t = \begin{cases} -\frac{1}{\underline{\beta\sigma} + \phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \phi_y} \epsilon_t^v & \text{if} \quad \epsilon_t^v > 0 \\ \\ -\frac{1}{\bar{\beta\sigma} + \phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \phi_y} \epsilon_t^v & \text{if} \quad \epsilon_t^v < 0 \end{cases}$$

where

$$\bar{\beta\sigma} = \max_i \{\sigma_i \beta_i^y\}$$

$$\underline{\beta\sigma} = \min_i \{\sigma_i \beta_i^y\}$$

Hence output responds more to contractionary monetary policy shocks than to expansionary shocks of equal magnitude. Clearly in the case $\sigma_i = \sigma$ for all i, the result simplifies to proposition 1.

C Proofs of Analytical Results

Proof of Lemma 1 I prove the lemma for the general case of all shocks.

The first equation is simply the Taylor rule for nominal interest rates, and is derived by taking natural logs of

$$1+\iota_t = \frac{1}{\delta}\left(1+\pi_t\right)^{\phi_\pi}\left(\frac{Y_t}{Y^T}\right)^{\phi_y} e^{v_t}$$

and using $\log\left(1+x\right) \approx x$ for small x.

In order to derive the remaining equations, I first derive an expression for household income, $y_{i,t}$. Under GHH preferences, labor supply of household i is given by

$$n_{i,t} = \theta_{i,t}^{\frac{1}{\varphi}} w_t^{\frac{1}{\varphi}}$$

so that total household income is given by

$$y_{i,t} = w_t^{\frac{1+\varphi}{\varphi}} \theta_{i,t}^{\frac{1+\varphi}{\varphi}} + s_i d_t$$

Aggregating the labor supply condition over all households, and using the production function yields an expression for aggregate output,

$$Y_t = A_t w_t^{\frac{1}{\varphi}} \int_0^1 \theta_{i,t}^{\frac{1+\varphi}{\varphi}} di$$

so that the real wage is given by

$$w_t = \left(\frac{Y_t}{A_t \int_0^1 \theta_{i,t}^{\frac{1+\varphi}{\varphi}} di}\right)^{\varphi}$$

To first order, resource costs of inflation are zero. Hence dividends are given by

$$d_t = Y_t\left(1 - \frac{w_t}{A_t}\right)$$

Defining

$$\Theta_t = \int_0^1 \theta_{k,t}^{\frac{1+\varphi}{\varphi}} dk$$

and substituting these expressions into the equation for household income yields

$$y_{i,t} = \left(\frac{Y_t}{A_t\Theta_t}\right)^{1+\varphi}\Theta_t\left(\frac{\theta_{i,t}^{\frac{1+\varphi}{\varphi}}}{\Theta_t} - s_i\right) + s_i Y_t$$

Using $\theta_{i,t} = \theta_i$ for all t implies that

$$y_{i,t} = \left(\frac{Y_t}{A_t\Theta}\right)^{1+\varphi}\Theta\left(\frac{\theta_i^{\frac{1+\varphi}{\varphi}}}{\Theta} - s_i\right) + s_i Y_t$$

can be expressed as a function only of Y_t and A_t,

$$y_{i,t} = f_i\left(Y_t, A_t\right)$$

where the function index i stems from cross-sectional heterogeneity in θ_i and s_i.

The second equation is the New Keynesian Phillips Curve (NKPC), and follows from two steps. First, log linearizing the FOC of the intermediate goods firms' problem around the zero inflation deterministic competitive equilibrium yields

$$\pi_t = \frac{\Phi_t}{\xi^p}\frac{w_t}{A_t} - \frac{\Phi_t - 1}{\xi^p} + \delta\mathbb{E}_t\left[\pi_{t+1}\right]$$

where the product terms are approximated by

$$\frac{\Phi_t}{\xi^p}\frac{w_t}{A_t} = \frac{\bar{\Phi}}{\xi^p}\frac{\bar{\Phi}-1}{\bar{\Phi}} + \frac{\bar{\Phi}-1}{\xi^p}\left(\log\Phi_t - \log\bar{\Phi}\right) + \frac{\bar{\Phi}-1}{\xi^p}\left(\log\frac{w_t}{A_t} - \log\frac{\bar{\Phi}-1}{\bar{\Phi}}\right)$$

and

$$\frac{\Phi_t - 1}{\xi^p} = \frac{\bar{\Phi}-1}{\xi^p} + \frac{\bar{\Phi}}{\xi^p}\left(\log\Phi_t - \log\bar{\Phi}\right)$$

so that

$$\pi_t = \frac{\bar{\Phi}-1}{\xi^p}\left(\log\frac{w_t}{A_t} - \log\frac{\bar{\Phi}-1}{\bar{\Phi}}\right) - \frac{1}{\xi^p}\left(\log\Phi_t - \log\bar{\Phi}\right) + \delta\mathbb{E}_t\left[\pi_{t+1}\right]$$

Second, aggregation of the household labor supply condition stemming from GHH preferences yields

$$w_t = \left(\frac{Y_t}{A_t\Theta_t}\right)^{\varphi}$$

Taking logs yields

$$\log w_t - \log A_t = \varphi\log Y_t - (1+\varphi)\log A_t - \varphi\log\Theta_t$$

Using $\Theta_t = \Theta$, a first-order Taylor expansion of this equation around the deterministic competitive equilibrium then yields

$$\log\frac{w_t}{A_t} - \log\frac{\bar{\Phi}-1}{\bar{\Phi}} = \varphi\hat{y}_t - (1+\varphi)\,\hat{a}_t$$

Substituting this into the NKPC yields

$$\pi_t = \frac{\bar{\Phi}-1}{\xi^p}\varphi\hat{y}_t - \frac{\bar{\Phi}-1}{\xi^p}(1+\varphi)\,\hat{a}_t - \frac{1}{\xi^p}\left(\log\Phi_t - \log\bar{\Phi}\right) + \delta\mathbb{E}_t\left[\pi_{t+1}\right]$$

Next, consider the third equation, which is the Euler equation for the economy. To derive this equation, note that the Euler equation for household i is given by

$$\tilde{c}_{i,t}^{-\sigma} \geq \delta\mathbb{E}_t\left[\tilde{c}_{i,t+1}^{-\sigma}\left(1+r_{t+1}\right)\right]$$

where the inequality is strict if the borrowing constraint binds, and $\tilde{c} = c - \frac{n^{1+\varphi}}{1+\varphi}$ is consumption net of the disutility of labor supply (this occurs due to the GHH preference specification).

Therefore, the Euler equation features a "distortion" only if household i would like to borrow in equilibrium,

$$\tilde{c}_{i,t}^{-\sigma} > \delta\mathbb{E}_t\left[\tilde{c}_{i,t+1}^{-\sigma}\left(1+r_{t+1}\right)\right] \iff b_{i,t} < 0$$

Hence, in equilibrium, there exists a household $i^*(t)$ such that

$$1 = \delta\mathbb{E}_t\left[\left(1+r_{t+1}\right)\left(\frac{\tilde{c}_{i^*(t),t+1}}{\tilde{c}_{i^*(t),t}}\right)^{-\sigma}\right] \geq \delta\mathbb{E}_t\left[\left(1+r_{t+1}\right)\left(\frac{\tilde{c}_{i,t+1}}{\tilde{c}_{i,t}}\right)^{-\sigma}\right]$$

for all $i \neq i^*(t)$, where $i^*(t)$ satisfies

$$i^*(t) \in \arg\max_i \mathbb{E}_t\left[\left(1+r_{t+1}\right)\left(\frac{\tilde{c}_{i^*(t),t+1}}{\tilde{c}_{i^*(t),t}}\right)^{-\sigma}\right]$$

The aggregate Euler equation is therefore given by

$$1 = \delta\max_i \mathbb{E}_t\left[\left(1+r_{t+1}\right)\left(\frac{\tilde{c}_{i,t+1}}{\tilde{c}_{i,t}}\right)^{-\sigma}\right]$$

Taking logs and using the first-order approximation $\log \mathbb{E}_t [x_{t+1}] \approx \mathbb{E}_t [\log x_{t+1}]$ yields

$$0 = \log \delta + \max_i \left\{ \mathbb{E}_t [\log (1 + r_{t+1})] - \sigma \mathbb{E}_t \left[\log \left(\frac{\tilde{c}_{i,t+1}}{\tilde{c}_{i,t}} \right) \right] \right\}$$

Writing $\rho = -\log \delta$ and using the approximation $\log(1+r) \approx r$ together with the definition of the real interest rate simplifies the equation to

$$0 = i_t - \mathbb{E}_t [\pi_{t+1}] - \rho - \sigma \min_i \{ \mathbb{E}_t [\log \tilde{c}_{i,t+1}] - \log \tilde{c}_{i,t} \}$$

where I have also used the fact that

$$\max_i \{-X_i\} = -\min_i \{X_i\}$$

Using the log deviation around the deterministic competitive equilibrium

$$\hat{\tilde{c}}_{i,t} = \log \tilde{c}_{i,t} - \log \tilde{c}_i$$

yields the third equation

$$\min_i \left\{ \mathbb{E}_t \left[\hat{\tilde{c}}_{i,t+1} \right] - \hat{\tilde{c}}_{i,t} \right\} = \frac{1}{\sigma} (i_t - \mathbb{E}_t [\pi_{t+1}] - \rho)$$

For the final equation, use the fact that in equilibrium

$$\tilde{c}_{i,t} = y_{i,t} - \frac{\theta_{i,t}^{\frac{1+\varphi}{\varphi}} w_t^{\frac{1+\varphi}{\varphi}}}{1+\varphi}$$

so that

$$\tilde{c}_{i,t} = y_{i,t} - \frac{\theta_{i,t}^{\frac{1+\varphi}{\varphi}} \left(\frac{Y_t}{A_t \Theta_t} \right)^{1+\varphi}}{1+\varphi}$$

Therefore, when $\theta_{i,t} = \theta_i$,

$$\log \tilde{c}_{i,t} = \log \left(y_{i,t} - \frac{\theta_i^{\frac{1+\varphi}{\varphi}} \left(\frac{Y_t}{A_t \Theta} \right)^{1+\varphi}}{1+\varphi} \right)$$

which can be linearly approximated around the deterministic competitive equilibrium as

$$\hat{\tilde{c}}_{i,t} = \beta_i^y \hat{y}_t + \beta_i^a \epsilon_t^a$$

for some coefficients β_i^y and β_i^a that depend only model primitives, as required to complete the representation.

Note that

$$\log \tilde{c}_{i,t} = \log \left(y_{i,t} - \frac{\theta_i^{\frac{1+\varphi}{\varphi}} \left(\frac{Y_t}{A_t \Theta} \right)^{1+\varphi}}{1+\varphi} \right)$$

implies

$$\log \tilde{c}_{i,t} = \log \left(\left(\frac{Y_t}{A_t \Theta} \right)^{1+\varphi} \Theta \left(\frac{\theta_i^{\frac{1+\varphi}{\varphi}}}{\Theta} \frac{\varphi}{1+\varphi} - s_i \right) + s_i Y_t \right)$$

which has a first order expansion

$$\hat{\tilde{c}}_{i,t} \approx \frac{(1+\varphi) \left(\frac{Y}{A\Theta} \right)^{1+\varphi} \Theta \left(\frac{\theta_i^{\frac{1+\varphi}{\varphi}}}{\Theta} \frac{\varphi}{1+\varphi} - s_i \right) + s_i Y}{\left(\frac{Y}{A\Theta} \right)^{1+\varphi} \Theta \left(\frac{\theta_i^{\frac{1+\varphi}{\varphi}}}{\Theta} \frac{\varphi}{1+\varphi} - s_i \right) + s_i Y} \hat{y}_t - \frac{(1+\varphi) \left(\frac{Y}{A\Theta} \right)^{1+\varphi} \Theta \left(\frac{\theta_i^{\frac{1+\varphi}{\varphi}}}{\Theta} \frac{\varphi}{1+\varphi} - s_i \right)}{\left(\frac{Y}{A\Theta} \right)^{1+\varphi} \Theta \left(\frac{\theta_i^{\frac{1+\varphi}{\varphi}}}{\Theta} \frac{\varphi}{1+\varphi} - s_i \right) + s_i Y} \epsilon_t^a$$

In the deterministic competitive equilibrium,

$$w = \frac{\bar{\Phi} - 1}{\bar{\Phi}} \bar{A} = \left(\frac{Y}{A\Theta} \right)^{\varphi}$$

$$\left(\frac{\bar{\Phi}-1}{\bar{\Phi}}\bar{A}\right)^{\frac{1+\varphi}{\varphi}} = \left(\frac{Y}{A\Theta}\right)^{1+\varphi}$$

$$Y = \left(\frac{\bar{\Phi}-1}{\bar{\Phi}}\bar{A}\right)^{\frac{1}{\varphi}} \bar{A}\Theta$$

so that

$$\hat{\tilde{c}}_{i,t} \approx \frac{(1+\varphi)\left(\frac{\bar{\Phi}-1}{\bar{\Phi}}\bar{A}\right)^{\frac{1+\varphi}{\varphi}} \Theta\left(\frac{\theta_i^{\frac{1+\varphi}{\varphi}}}{\Theta}\frac{\varphi}{1+\varphi} - s_i\right) + s_i\left(\frac{\bar{\Phi}-1}{\bar{\Phi}}\bar{A}\right)^{\frac{1}{\varphi}}\bar{A}\Theta}{\left(\frac{\bar{\Phi}-1}{\bar{\Phi}}\bar{A}\right)^{\frac{1+\varphi}{\varphi}} \Theta\left(\frac{\theta_i^{\frac{1+\varphi}{\varphi}}}{\Theta}\frac{\varphi}{1+\varphi} - s_i\right) + s_i\left(\frac{\bar{\Phi}-1}{\bar{\Phi}}\bar{A}\right)^{\frac{1}{\varphi}}\bar{A}\Theta}\hat{y}_t - \frac{(1+\varphi)\left(\frac{\bar{\Phi}-1}{\bar{\Phi}}\bar{A}\right)^{\frac{1+\varphi}{\varphi}} \Theta\left(\frac{\theta_i^{\frac{1+\varphi}{\varphi}}}{\Theta}\frac{\varphi}{1+\varphi} - s_i\right)}{\left(\frac{\bar{\Phi}-1}{\bar{\Phi}}\bar{A}\right)^{\frac{1+\varphi}{\varphi}} \Theta\left(\frac{\theta_i^{\frac{1+\varphi}{\varphi}}}{\Theta}\frac{\varphi}{1+\varphi} - s_i\right) + s_i\left(\frac{\bar{\Phi}-1}{\bar{\Phi}}\bar{A}\right)^{\frac{1}{\varphi}}\bar{A}\Theta}\epsilon_t^a$$

Hence

$$\beta_i^y = \frac{\varphi\frac{\bar{\Phi}-1}{\bar{\Phi}}\frac{\theta_i^{\frac{1+\varphi}{\varphi}}}{\Theta} + s_i\left(1-(1+\varphi)\frac{\bar{\Phi}-1}{\bar{\Phi}}\right)}{\frac{\bar{\Phi}-1}{\bar{\Phi}}\frac{\theta_i^{\frac{1+\varphi}{\varphi}}}{\Theta}\frac{\varphi}{1+\varphi} + s_i\frac{1}{\bar{\Phi}}}$$

$$\beta_i^a = -\frac{\varphi\frac{\bar{\Phi}-1}{\bar{\Phi}}\frac{\theta_i^{\frac{1+\varphi}{\varphi}}}{\Theta} - (1+\varphi)\frac{\bar{\Phi}-1}{\bar{\Phi}}s_i}{\frac{\bar{\Phi}-1}{\bar{\Phi}}\frac{\theta_i^{\frac{1+\varphi}{\varphi}}}{\Theta}\frac{\varphi}{1+\varphi} + s_i\frac{1}{\bar{\Phi}}}$$

so that

$$\beta_i^y > 0 \iff \frac{\theta_i^{\frac{1+\varphi}{\varphi}}}{\Theta} > s_i\left(1 - \frac{1}{\varphi\left(\bar{\Phi}-1\right)}\right)$$

$$\beta_i^a > 0 \iff \frac{\theta_i^{\frac{1+\varphi}{\varphi}}}{\Theta} < s_i\frac{1+\varphi}{\varphi}$$

I assume that both conditions are satisfied. □

Proof of Propositions 1 and 6 Recall the representation

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t + \epsilon_t^v$$

$$\pi_t = \frac{\bar{\Phi}-1}{\xi^p}\varphi\hat{y}_t + \delta\mathbb{E}_t\left[\pi_{t+1}\right]$$

$$\min_i\left\{\mathbb{E}_t\left[\hat{\tilde{c}}_{i,t+1}\right] - \hat{\tilde{c}}_{i,t}\right\} = \frac{1}{\sigma}\left(i_t - \mathbb{E}_t\left[\pi_{t+1}\right] - \rho\right)$$

$$\hat{\tilde{c}}_{i,t} = \beta_i^y \hat{y}_t$$

where I have set $\epsilon_t^a = \epsilon_t^\Phi = 0$ by assumption.

Consider $\epsilon_t^v > 0$, and suppose that the solution takes the form

$$\hat{y}_t = c_y^+ \epsilon_t^v$$

$$\pi_t = c_\pi^+ \epsilon_t^v$$

where $c_y^+ < 0$. Substituting these guesses into the system and simplifying yields

$$c_\pi^+ = \frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^+$$

$$\min_i\left\{-\beta_i^y c_y^+ \epsilon_t^v\right\} = \frac{1}{\sigma}\left(\phi_\pi c_\pi^+ \epsilon_t^v + \phi_y c_y^+ \epsilon_t^v + \epsilon_t^v\right)$$

so that

$$\min_i\left\{-\beta_i^y c_y^+ \epsilon_t^v\right\} = \frac{1}{\sigma}\left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^+ + \phi_y c_y^+ + 1\right)\epsilon_t^v$$

By the supposition, $-c_y^+ \epsilon_t^v > 0$ so that

$$-c_y^+ \epsilon_t^v \underline{\beta} = \frac{1}{\sigma}\left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^+ + \phi_y c_y^+ + 1\right)\epsilon_t^v$$

where

$$\underline{\beta} = \min_i \{\beta_i^y\}$$

Hence

$$c_y^+ = -\frac{1}{\underline{\beta} + \frac{1}{\sigma}\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \frac{1}{\sigma}\phi_y}\frac{1}{\sigma}$$

$$c_\pi^+ = -\frac{1}{\underline{\beta} + \frac{1}{\sigma}\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \frac{1}{\sigma}\phi_y}\frac{1}{\sigma}\frac{\bar{\Phi}-1}{\xi^p}\varphi$$

where $c_y^+ < 0$ since

$$\underline{\beta} + \frac{1}{\sigma}\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \frac{1}{\sigma}\phi_y > 0$$

For completeness, suppose $c_y^+ > 0$. The same steps yield

$$\min_i \left\{-\beta_i^y c_y^+ \epsilon_t^v\right\} = \frac{1}{\sigma}\left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^+ + \phi_y c_y^+ + 1\right)\epsilon_t^v$$

By supposition, $c_y^+ \epsilon_t^v > 0$ so that

$$-c_y^+ \epsilon_t^v \bar{\beta} = \frac{1}{\sigma}\left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^+ + \phi_y c_y^+ + 1\right)\epsilon_t^v$$

where

$$\bar{\beta} = \max_i \{\beta_i^y\} > 0$$

Hence

$$c_y^+ = -\frac{1}{\bar{\beta} + \frac{1}{\sigma}\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \frac{1}{\sigma}\phi_y}\frac{1}{\sigma} < 0$$

where the inequality follows from

$$\bar{\beta} + \frac{1}{\sigma}\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \frac{1}{\sigma}\phi_y > 0$$

Therefore, we have a contradiction.

Now consider $\epsilon_t^v < 0$, and suppose that the solution takes the form

$$\hat{y}_t = c_y^- \epsilon_t^v$$

$$\pi_t = c_\pi^- \epsilon_t^v$$

where $c_y^- < 0$. Substituting these guesses into the system and simplifying yields

$$c_\pi^- = \frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^- > 0$$

$$\min_i \left\{-\beta_i^y c_y^- \epsilon_t^v\right\} = \frac{1}{\sigma}\left(\phi_\pi c_\pi^- \epsilon_t^v + \phi_y c_y^- \epsilon_t^v + \epsilon_t^v\right)$$

so that

$$\min_i \left\{-\beta_i^y c_y^- \epsilon_t^v\right\} = \frac{1}{\sigma}\left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^- + \phi_y c_y^- + 1\right)\epsilon_t^v$$

By the supposition, $c_y^- \epsilon_t^v > 0$ so that

$$-c_y^- \epsilon_t^v \bar{\beta} = \frac{1}{\sigma}\left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^- + \phi_y c_y^- + 1\right)\epsilon_t^v$$

where

$$\bar{\beta} = \max_i \{\beta_i^y\}$$

Hence

$$c_y^- = -\frac{1}{\bar{\beta} + \frac{1}{\sigma}\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \frac{1}{\sigma}\phi_y}\frac{1}{\sigma} < 0$$

$$c_\pi^+ = -\frac{1}{\bar{\beta} + \frac{1}{\sigma}\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \frac{1}{\sigma}\phi_y}\frac{1}{\sigma}\frac{\bar{\Phi}-1}{\xi^p}\varphi_\Theta < 0$$

as required.

For completeness, suppose that $c_y^- > 0$ so that $-c_y^- \epsilon_t^v > 0$ and the same steps as above lead to

$$-c_y^- \epsilon_t^v \underline{\beta} = \frac{1}{\sigma}\left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^- + \phi_y c_y^- + 1\right)\epsilon_t^v$$

so that

$$c_y^- = -\frac{1}{\underline{\beta} + \frac{1}{\sigma}\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \frac{1}{\sigma}\phi_y}\frac{1}{\sigma} < 0$$

which is a contradiction. □

Proof of Propositions 4 and 7 Recall the representation

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t$$

$$\pi_t = \frac{\bar{\Phi}-1}{\xi^p}\varphi\hat{y}_t - \frac{1}{\xi^p}\epsilon_t^\Phi + \delta\mathbb{E}_t\left[\pi_{t+1}\right]$$

$$\min_i\left\{\mathbb{E}_t\left[\hat{\hat{c}}_{i,t+1}\right] - \hat{\hat{c}}_{i,t}\right\} = \frac{1}{\sigma}\left(i_t - \mathbb{E}_t\left[\pi_{t+1}\right] - \rho\right)$$

$$\hat{\hat{c}}_{i,t} = \beta_i^y \hat{y}_t$$

where I have set $\epsilon_t^a = \epsilon_t^v = 0$ by assumption.

Consider $\epsilon_t^\Phi > 0$, and suppose that the solution takes the form

$$\hat{y}_t = c_y^+ \epsilon_t^\Phi$$

$$\pi_t = c_\pi^+ \epsilon_t^\Phi$$

where $c_y^+ > 0$. Substituting these guesses into the system and simplifying yields

$$c_\pi^+ = \frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^+ - \frac{1}{\xi^p}$$

$$\min_i\left\{-\beta_i^y c_y^+ \epsilon_t^\Phi\right\} = \frac{1}{\sigma}\left(\phi_\pi c_\pi^+ \epsilon_t^\Phi + \phi_y c_y^+ \epsilon_t^\Phi\right)$$

so that

$$\min_i\left\{-\beta_i^y c_y^+ \epsilon_t^\Phi\right\} = \frac{1}{\sigma}\left(\phi_\pi\left(\frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^+ - \frac{1}{\xi^p}\right) + \phi_y c_y^+\right)\epsilon_t^\Phi$$

By supposition, $c_y^+ \epsilon_t^\Phi > 0$ so that

$$-c_y^+ \epsilon_t^\Phi \bar{\beta} = \frac{1}{\sigma}\left(\phi_\pi\left(\frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^+ - \frac{1}{\xi^p}\right) + \phi_y c_y^+\right)\epsilon_t^\Phi$$

where

$$\bar{\beta} = \max_i\left\{\beta_i^y\right\} > 0$$

Hence

$$c_y^+ = \frac{\phi_\pi \frac{1}{\xi^p}}{\bar{\beta} + \frac{1}{\sigma}\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \frac{1}{\sigma}\phi_y}\frac{1}{\sigma} > 0$$

$$c_\pi^+ = -\frac{1}{\xi^p}\left(\frac{\bar{\beta} + \frac{1}{\sigma}\phi_y}{\bar{\beta} + \frac{1}{\sigma}\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \frac{1}{\sigma}\phi_y}\right)$$

as required.

For completeness, suppose that $c_y^+ < 0$ so that $-c_y^+ \epsilon_t^\Phi > 0$ and

$$c_y^+ = \frac{\phi_\pi \frac{1}{\xi^p}}{\underline{\beta} + \frac{1}{\sigma}\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \frac{1}{\sigma}\phi_y} \frac{1}{\sigma} > 0$$

where

$$\underline{\beta} = \min_i \{\beta_i^y\}$$

which is a contradiction.

Now consider $\epsilon_t^\Phi < 0$, and suppose that the solution takes the form

$$\hat{y}_t = c_y^- \epsilon_t^\Phi$$

$$\pi_t = c_\pi^- \epsilon_t^\Phi$$

where $c_y^- > 0$. Substituting these guesses into the system and simplifying yields

$$c_\pi^- = \frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^- - \frac{1}{\xi^p}$$

$$\min_i \left\{-\beta_i^y c_y^- \epsilon_t^\Phi\right\} = \frac{1}{\sigma}\left(\phi_\pi c_\pi^- \epsilon_t^\Phi + \phi_y c_y^- \epsilon_t^\Phi\right)$$

so that

$$\min_i \left\{-\beta_i^y c_y^- \epsilon_t^\Phi\right\} = \frac{1}{\sigma}\left(\phi_\pi \left(\frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^- - \frac{1}{\xi^p}\right) + \phi_y c_y^-\right)\epsilon_t^\Phi$$

By supposition, $-c_y^- \epsilon_t^\Phi > 0$ so that

$$-c_y^- \epsilon_t^\Phi \underline{\beta}^{CP} = \frac{1}{\sigma}\left(\phi_\pi \left(\frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^- - \frac{1}{\xi^p}\right) + \phi_y c_y^-\right)\epsilon_t^\Phi$$

where

$$\underline{\beta} = \min_i \{\beta_i^y\}$$

Hence

$$c_y^- = \frac{\phi_\pi \frac{1}{\xi^p}}{\underline{\beta}^{CP} + \frac{1}{\sigma}\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \frac{1}{\sigma}\phi_y} \frac{1}{\sigma} > 0$$

$$c_\pi^+ = -\frac{1}{\xi^p}\left(\frac{\underline{\beta}^{CP} + \frac{1}{\sigma}\phi_y}{\underline{\beta}^{CP} + \frac{1}{\sigma}\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \frac{1}{\sigma}\phi_y}\right)$$

as required.

For completeness, suppose that $c_y^- < 0$ so that $c_y^- \epsilon_t^\Phi > 0$ and

$$c_y^- = \frac{\phi_\pi \frac{1}{\xi^p}}{\bar{\beta} + \frac{1}{\sigma}\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \frac{1}{\sigma}\phi_y} \frac{1}{\sigma} > 0$$

where

$$\bar{\beta} = \max_i \{\beta_i^y\}$$

which is a contradiction. □

Proof of Propositions 5 and 8 Recall the representation

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t$$

$$\pi_t = \frac{\bar{\Phi}-1}{\xi^p}\varphi \hat{y}_t - \frac{\bar{\Phi}-1}{\xi^p}(1+\varphi)\epsilon_t^a + \delta \mathbb{E}_t[\pi_{t+1}]$$

$$\min_i \left\{\mathbb{E}_t\left[\hat{\tilde{c}}_{i,t+1}\right] - \hat{\tilde{c}}_{i,t}\right\} = \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}] - \rho)$$

$$\hat{\tilde{c}}_{i,t} = \beta_i^y \hat{y}_t + \beta_i^a \epsilon_t^a$$

where I have set $\epsilon_t^{\Phi} = \epsilon_t^{v} = 0$ by assumption.

Consider $\epsilon_t^a > 0$, and suppose that the solution is given by

$$\hat{y}_t = c_y^+ \epsilon_t^a$$

$$\pi_t = c_\pi^+ \epsilon_t^a$$

where $c_y^+ > 0$. Substituting these guesses into the the system and simplifying yields

$$c_\pi^+ = \frac{\bar{\Phi} - 1}{\xi^p} \varphi c_y^+ - \frac{\bar{\Phi} - 1}{\xi^p} (1 + \varphi)$$

$$\min_i \left\{ - \left(\beta_i^y c_y^+ + \beta_i^a \right) \epsilon_t^a \right\} = \frac{1}{\sigma} \left(\phi_\pi c_\pi^+ + \phi_y c_y^+ \right) \epsilon_t^a$$

so that

$$\min_i \left\{ - \left(\beta_i^y c_y^+ + \beta_i^a \right) \epsilon_t^a \right\} = \frac{1}{\sigma} \left(\phi_\pi \frac{\bar{\Phi} - 1}{\xi^p} \varphi c_y^+ - \phi_\pi \frac{\bar{\Phi} - 1}{\xi^p} (1 + \varphi) + \phi_y c_y^+ \right) \epsilon_t^a$$

Now define $\beta_i^{TFP,+} = \beta_i^y + \frac{\beta_i^a}{c_y^+}$ so that

$$\min_i \left\{ -\beta_i^{TFP} c_y^+ \epsilon_t^a \right\} = \frac{1}{\sigma} \left(\phi_\pi \frac{\bar{\Phi} - 1}{\xi^p} \varphi c_y^+ - \phi_\pi \frac{\bar{\Phi} - 1}{\xi^p} (1 + \varphi) + \phi_y c_y^+ \right) \epsilon_t^a$$

where $c_y^+ \epsilon_t^a > 0$ by the supposition, so that

$$c_y^+ = \frac{\phi_\pi \frac{\bar{\Phi}-1}{\xi^p} (1 + \varphi)}{\bar{\beta}^{TFP,+} + \frac{1}{\sigma} \left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p} \varphi + \phi_y \right)} \frac{1}{\sigma}$$

$$c_\pi^+ = -\frac{\bar{\Phi} - 1}{\xi^p} (1 + \varphi) \frac{\bar{\beta}^{TFP,+} + \frac{1}{\sigma} \phi_y}{\bar{\beta}^{TFP,+} + \frac{1}{\sigma} \left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p} \varphi \Theta + \phi_y \right)}$$

where

$$\bar{\beta}^{TFP,+} = \max_i \left\{ \beta_i^y + \frac{\beta_i^a}{c_y^+} \right\}$$

To ensure that $c_y^+ > 0$, first rewrite

$$c_y^+ = \frac{\phi_\pi \frac{\bar{\Phi}-1}{\xi^p} (1 + \varphi) \frac{1}{\sigma} - \beta_{i^*}^a}{\beta_{i^*}^y + \frac{1}{\sigma} \left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p} \varphi + \phi_y \right)}$$

where

$$i^* = \arg\max_i \left\{ \beta_i^y + \frac{\beta_i^a}{c_y^+} \right\}$$

Then, I impose the restriction

$$\beta_i^a < \phi_\pi \frac{\bar{\Phi} - 1}{\xi^p} (1 + \varphi) \frac{1}{\sigma}$$

for all i, which guarantees $c_y^+ > 0$ as required. This restriction requirement is purely a result of using GHH preferences: a positive TFP shock lowers labor supply ceteris paribus, which boosts a household's net consumption due to the strong complementarity between labor supply and consumption embodied by GHH preferences. If this complementarity is strong enough, output can fall in response to a positive TFP shock in equilibrium since households are happy to consume less but also work less. Since this feature of GHH preferences is not the focus of my analysis, I rule it out by assumption.

Now consider $\epsilon_t^a < 0$, and suppose that the solution is given by

$$\hat{y}_t = c_y^- \epsilon_t^a$$

$$\pi_t = c_\pi^- \epsilon_t^a$$

where $c_y^- > 0$. Substituting these guesses into the the system and simplifying yields

$$c_\pi^- = \frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^- - \frac{\bar{\Phi}-1}{\xi^p}(1+\varphi)$$

$$\min_i \left\{-\left(\beta_i^y c_y^- + \beta_i^a\right)\epsilon_t^a\right\} = \frac{1}{\sigma}\left(\phi_\pi c_\pi^- + \phi_y c_y^-\right)\epsilon_t^a$$

Now define $\beta_i^{TFP,-} = \beta_i^y + \frac{\beta_i^a}{c_y^-}$ so that

$$\min_i \left\{-\beta_i^{TFP,-} c_y^- \epsilon_t^a\right\} = \frac{1}{\sigma}\left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^- - \phi_\pi \frac{\bar{\Phi}-1}{\xi^p}(1+\varphi) + \phi_y c_y^-\right)\epsilon_t^a$$

where $-c_y^- \epsilon_t^a > 0$ by the supposition, so that

$$c_y^- = \frac{\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}(1+\varphi)}{\underline{\beta}^{TFP,-} + \frac{1}{\sigma}\left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \phi_y\right)}\frac{1}{\sigma} > 0$$

$$c_\pi^- = -\frac{\bar{\Phi}-1}{\xi^p}(1+\varphi)\frac{\underline{\beta}^{TFP,-} + \frac{1}{\sigma}\phi_y}{\underline{\beta}^{TFP,-} + \frac{1}{\sigma}\left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \phi_y\right)}$$

where

$$\underline{\beta}^{TFP,-} = \min_i \left\{\beta_i^y + \frac{\beta_i^a}{c_y^-}\right\}$$

To ensure that $c_y^- > 0$, rewrite

$$c_y^- = \frac{\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}(1+\varphi)\frac{1}{\sigma} - \beta_{i^{**}}^a}{\beta_{i^{**}}^y + \frac{1}{\sigma}\left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \phi_y\right)}$$

where

$$i^{**} = \arg\min_i \left\{\beta_i^y + \frac{\beta_i^a}{c_y^-}\right\}$$

The restriction

$$\beta_i^a < \phi_\pi \frac{\bar{\Phi}-1}{\xi^p}(1+\varphi)\frac{1}{\sigma}$$

for all i then ensures that $c_y^- > 0$.

Finally, in order to prove that $c_y^- > c_y^+$, define the functions

$$g^+(c) = \frac{\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}(1+\varphi)}{\sigma \max_i \left\{\beta_i^y + \frac{\beta_i^a}{c}\right\} + \phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \phi_y} - c$$

$$g^-(c) = \frac{\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}(1+\varphi)}{\sigma \min_i \left\{\beta_i^y + \frac{\beta_i^a}{c}\right\} + \phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \phi_y} - c$$

where $g^-(c) > g^+(c)$ for all $c > 0$, and the coefficients c_y^- and c_y^+ satisfy

$$g^-\left(c_y^-\right) = 0$$

$$g^+\left(c_y^+\right) = 0$$

Then, the fact that $g^-(c) > g^+(c)$ implies that $c_y^+ < c_y^-$ as required. □

Proof of Lemma 2 The first equation is simply the Taylor rule for nominal interest rates. In order to derive the remaining equations, I first derive an expression for household income, $y_{i,t}$. Under GHH preferences, labor supply of household i is given by

$$n_{i,t} = \theta_{i,t}^{\frac{1}{\varphi}} w_t^{\frac{1}{\varphi}}$$

so that total household income is given by

$$y_{i,t} = w_t^{\frac{1+\varphi}{\varphi}} \theta_{i,t}^{\frac{1+\varphi}{\varphi}} + s_i d_t$$

Aggregating the labor supply condition over all households, and using the production function yields an expression for aggregate output,

$$Y_t = A_t w_t^{\frac{1}{\varphi}} \int_0^1 \theta_{i,t}^{\frac{1+\varphi}{\varphi}} di$$

so that the real wage is given by

$$w_t = \left(\frac{Y_t}{A_t \int_0^1 \theta_{i,t}^{\frac{1+\varphi}{\varphi}} di} \right)^{\varphi}$$

To first order, resource costs of inflation are zero. Hence dividends are given by

$$d_t = Y_t \left(1 - \frac{w_t}{A_t} \right)$$

Defining

$$\Theta_t = \int_0^1 \theta_{k,t}^{\frac{1+\varphi}{\varphi}} dk$$

and substituting these expressions into the equation for household income yields

$$y_{i,t} = \left(\frac{Y_t}{A_t \Theta_t} \right)^{1+\varphi} \Theta_t \left(\frac{\theta_{i,t}^{\frac{1+\varphi}{\varphi}}}{\Theta_t} - s_i \right) + s_i Y_t$$

In the stationary competitive equilibrium, $\Theta_t = \Theta$ by construction, so that

$$y_{i,t} = \left(\frac{Y_t}{A_t \Theta} \right)^{1+\varphi} \Theta \left(\frac{\theta_{i,t}^{\frac{1+\varphi}{\varphi}}}{\Theta} - s_i \right) + s_i Y_t$$

The second equation is the New Keynesian Phillips Curve (NKPC), and follows from two steps. First, log linearizing the FOC of the intermediate goods firms' problem around the zero inflation stationary competitive equilibrium yields

$$\pi_t = \frac{\Phi_t}{\xi^p} \frac{w_t}{A_t} - \frac{\Phi_t - 1}{\xi^p} + \delta \mathbb{E}_t \left[\pi_{t+1} \right]$$

where the product terms are approximated by

$$\frac{\Phi_t}{\xi^p} \frac{w_t}{A_t} = \frac{\bar{\Phi}}{\xi^p} \frac{\bar{\Phi} - 1}{\bar{\Phi}} + \frac{\bar{\Phi} - 1}{\xi^p} \left(\log \Phi_t - \log \bar{\Phi} \right) + \frac{\bar{\Phi} - 1}{\xi^p} \left(\log \frac{w_t}{A_t} - \log \frac{\bar{\Phi} - 1}{\bar{\Phi}} \right)$$

and

$$\frac{\Phi_t - 1}{\xi^p} = \frac{\bar{\Phi} - 1}{\xi^p} + \frac{\bar{\Phi}}{\xi^p} \left(\log \Phi_t - \log \bar{\Phi} \right)$$

so that

$$\pi_t = \frac{\bar{\Phi} - 1}{\xi^p} \left(\log \frac{w_t}{A_t} - \log \frac{\bar{\Phi} - 1}{\bar{\Phi}} \right) - \frac{1}{\xi^p} \left(\log \Phi_t - \log \bar{\Phi} \right) + \delta \mathbb{E}_t \left[\pi_{t+1} \right]$$

Second, aggregation of the household labor supply condition stemming from GHH preferences yields

$$w_t = \left(\frac{Y_t}{A_t \Theta_t} \right)^{\varphi}$$

Taking logs yields

$$\log w_t - \log A_t = \varphi \log Y_t - (1 + \varphi) \log A_t - \varphi \log \Theta_t$$

Around the stationary competitive equilibrium, $\Theta_t = \Theta$, and a first-order Taylor expansion of this equation yields

$$\log \frac{w_t}{A_t} - \log \frac{\bar{\Phi} - 1}{\bar{\Phi}} = \varphi \hat{y}_t - (1 + \varphi) \hat{a}_t$$

Substituting this into the NKPC yields

$$\pi_t = \frac{\bar{\Phi}-1}{\xi^p}\varphi \hat{y}_t - \frac{\bar{\Phi}-1}{\xi^p}(1+\varphi)\hat{a}_t - \frac{1}{\xi^p}\left(\log \Phi_t - \log \bar{\Phi}\right) + \delta \mathbb{E}_t\left[\pi_{t+1}\right]$$

Next, consider the third equation, which is the Euler equation for the economy. To derive this equation, note that the Euler equation for household i is given by

$$\tilde{c}_{i,t}^{-\sigma} \geq \delta \mathbb{E}_t\left[\tilde{c}_{i,t+1}^{-\sigma}(1+r_{t+1})\right]$$

where the inequality is strict if the borrowing constraint binds, and $\tilde{c} = c - \frac{n^{1+\varphi}}{1+\varphi}$ is consumption net of the disutility of labor supply (this occurs due to the GHH preference specification).

Therefore, the Euler equation features a "distortion" only if household i would like to borrow in equilibrium,

$$\tilde{c}_{i,t}^{-\sigma} > \delta \mathbb{E}_t\left[\tilde{c}_{i,t+1}^{-\sigma}(1+r_{t+1})\right] \iff b_{i,t} < 0$$

Hence, in equilibrium, there exists a household $i^*(t)$ such that

$$1 = \delta \mathbb{E}_t\left[(1+r_{t+1})\left(\frac{\tilde{c}_{i^*(t),t+1}}{\tilde{c}_{i^*(t),t}}\right)^{-\sigma}\right] \geq \delta \mathbb{E}_t\left[(1+r_{t+1})\left(\frac{\tilde{c}_{i,t+1}}{\tilde{c}_{i,t}}\right)^{-\sigma}\right]$$

for all $i \neq i^*(t)$, where $i^*(t)$ satisfies

$$i^*(t) \in \arg\max_i \mathbb{E}_t\left[(1+r_{t+1})\left(\frac{\tilde{c}_{i^*(t),t+1}}{\tilde{c}_{i^*(t),t}}\right)^{-\sigma}\right]$$

The aggregate Euler equation is therefore given by

$$1 = \delta \max_i \mathbb{E}_t\left[(1+r_{t+1})\left(\frac{\tilde{c}_{i,t+1}}{\tilde{c}_{i,t}}\right)^{-\sigma}\right]$$

Taking logs and using the first-order approximation $\log \mathbb{E}_t[x_{t+1}] \approx \mathbb{E}_t[\log x_{t+1}]$ yields

$$0 = \log \delta + \max_i \left\{\mathbb{E}_t\left[\log(1+r_{t+1})\right] - \sigma \mathbb{E}_t\left[\log\left(\frac{\tilde{c}_{i,t+1}}{\tilde{c}_{i,t}}\right)\right]\right\}$$

Writing $\rho = -\log\delta$ and using the approximation $\log(1+r) \approx r$ together with the definition of the real interest rate simplifies the equation to

$$0 = i_t - \mathbb{E}_t[\pi_{t+1}] - \rho - \sigma \min_i \left\{\mathbb{E}_t\left[\log \tilde{c}_{i,t+1}\right] - \log \tilde{c}_{i,t}\right\}$$

where I have also used the fact that

$$\max_i \{-X_i\} = -\min_i \{X_i\}$$

Using the log deviation around the stationary competitive equilibrium and mean labor productivity level

$$\hat{\tilde{c}}_{i,t} = \log \tilde{c}_{i,t} - \log \tilde{c}_i$$

yields the third equation

$$\min_i \left\{\mathbb{E}_t\left[\hat{\tilde{c}}_{i,t+1}\right] - \hat{\tilde{c}}_{i,t}\right\} = \frac{1}{\sigma}\left(i_t - \mathbb{E}_t[\pi_{t+1}] - \rho\right)$$

For the final equation, use the fact that in equilibrium

$$\tilde{c}_{i,t} = y_{i,t} - \frac{\theta_{i,t}^{\frac{1+\varphi}{\varphi}} w_t^{\frac{1+\varphi}{\varphi}}}{1+\varphi}$$

so that

$$\tilde{c}_{i,t} = y_{i,t} - \frac{\theta_{i,t}^{\frac{1+\varphi}{\varphi}} \left(\frac{Y_t}{A_t \Theta}\right)^{1+\varphi}}{1+\varphi}$$

Therefore,

$$\log \tilde{c}_{i,t} = \log \left(y_{i,t} - \frac{\theta_{i}^{\frac{1+\varphi}{\varphi}} \left(\frac{Y_t}{A_t \Theta}\right)^{1+\varphi}}{1+\varphi} \right)$$

which can be linearly approximated around the stationary competitive equilibrium and mean labor productivity level as

$$\hat{\tilde{c}}_{i,t} = \beta_i^y \hat{y}_t + \beta_i^a \epsilon_t^a + \beta_i^\theta \hat{\theta}_{i,t}$$

for some coefficients β_i^y, β_i^a, and β_i^θ that depend only model primitives, as required to complete the representation. $\square$

Proof of Remark 1 In the stationary competitive equilibrium, the economy is summarized by the two equations

$$\min_i \left\{ \mathbb{E}_t \left[\hat{\tilde{c}}_{i,t+1} \right] - \hat{\tilde{c}}_{i,t} \right\} = \frac{1}{\sigma} (r - \rho)$$

$$\hat{\tilde{c}}_{i,t} = \beta_i^\theta \hat{\theta}_{i,t} \; \forall i$$

so that

$$\min_i \left\{ \beta_i^\theta \left(\mathbb{E}_t \left[\hat{\theta}_{i,t+1} \right] - \hat{\theta}_{i,t} \right) \right\} = \frac{1}{\sigma} (r - \rho)$$

Since

$$\hat{\theta}_{i,t} = \rho_\theta \hat{\theta}_{i,t-1} + \epsilon_{i,t}$$

we have

$$-(1-\rho_\theta) \max_i \left\{ \beta_i^\theta \hat{\theta}_{i,t} \right\} = \frac{1}{\sigma} (r - \rho)$$

$$r = \rho - \sigma (1-\rho_\theta) \max_i \left\{ \beta_i^\theta \hat{\theta}_{i,t} \right\} < \rho$$

as required. $\square$

Proof of Proposition 9 Recall the system

$$\iota_t = r + \phi_\pi \pi_t + \phi_y \hat{y}_t + \epsilon_t^v$$

$$\pi_t = \frac{\bar{\Phi} - 1}{\xi^p} \varphi \hat{y}_t + \delta \mathbb{E}_t [\pi_{t+1}]$$

$$\min_i \left\{ \mathbb{E}_t \left[\hat{\tilde{c}}_{i,t+1} \right] - \hat{\tilde{c}}_{i,t} \right\} = \frac{1}{\sigma} (\iota_t - \mathbb{E}_t [\pi_{t+1}] - \rho)$$

$$\hat{\tilde{c}}_{i,t} = \beta_i^y \hat{y}_t + \beta_i^\theta \hat{\theta}_{i,t} \; \forall i$$

where I have set $\epsilon_t^a = \epsilon_t^\Phi = 0$ by assumption. Substitution yields

$$\min_i \left\{ \beta_i^y (\mathbb{E}_t [\hat{y}_{t+1}] - \hat{y}_t) + \beta_i^\theta \left(\mathbb{E}_t \left[\hat{\theta}_{i,t+1} \right] - \hat{\theta}_{i,t} \right) \right\} = \frac{1}{\sigma} \left(\phi_\pi \frac{\bar{\Phi} - 1}{\xi^p} \varphi \hat{y}_t + \phi_y \hat{y}_t + \epsilon_t^v + r - \rho \right)$$

Suppose $\epsilon_t^v > 0$. Guess a solution of the form $\hat{y}_t = c_y^+ \epsilon_t^v$ with $c_y^+ < 0$. Using this and the fact that

$$\hat{\theta}_{i,t+1} = \rho_\theta \hat{\theta}_{i,t} + \epsilon_{i,t+1}$$

$$-\frac{1}{\sigma} (r - \rho) = (1-\rho_\theta) \max_j \left\{ \beta_j^\theta \hat{\theta}_{j,t} \right\}$$

yields

$$\min_i \left\{ -\beta_i^y c_y^+ \epsilon_t^v + (1-\rho_\theta) \left(\max_j \left\{ \beta_j^\theta \hat{\theta}_{j,t} \right\} - \beta_i^\theta \hat{\theta}_{i,t} \right) \right\} = \frac{1}{\sigma} \left(\phi_\pi \frac{\bar{\Phi} - 1}{\xi^p} \varphi + \phi_y \right) c_y^+ \epsilon_t^v + \frac{1}{\sigma} \epsilon_t^v$$

Taking the limit $\rho_\theta \to 1$ then implies that $(1-\rho_\theta)\left(\max_j\left\{\beta_j^\theta \hat{\theta}_{j,t}\right\} - \beta_i^\theta \hat{\theta}_{i,t}\right)$ is of second order so that the equation becomes

$$\min_i \left\{-\beta_i^y c_y^+ \epsilon_t^v\right\} = \frac{1}{\sigma}\left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \phi_y\right) c_y^+ \epsilon_t^v + \frac{1}{\sigma}\epsilon_t^v$$

which implies

$$c_y^+ = -\frac{1}{\underline{\beta} + \frac{1}{\sigma}\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \frac{1}{\sigma}\phi_y}\frac{1}{\sigma}$$

where

$$\underline{\beta} = \min_i \left\{\beta_i^y\right\}$$

Now suppose $\epsilon_t^v < 0$. Guess a solution of the form $\hat{y}_t = c_y^- \epsilon_t^v$ with $c_y^- < 0$. Using this and the fact that

$$\hat{\theta}_{i,t+1} = \rho_\theta \hat{\theta}_{i,t} + \epsilon_{i,t+1}$$

$$-\frac{1}{\sigma}(r-\rho) = (1-\rho_\theta)\max_j\left\{\beta_j^\theta \hat{\theta}_{j,t}\right\}$$

yields

$$\min_i\left\{-\beta_i^y c_y^- \epsilon_t^v + (1-\rho_\theta)\left(\max_j\left\{\beta_j^\theta \hat{\theta}_{j,t}\right\} - \beta_i^\theta \hat{\theta}_{i,t}\right)\right\} = \frac{1}{\sigma}\left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \phi_y\right) c_y^- \epsilon_t^v + \frac{1}{\sigma}\epsilon_t^v$$

Taking the limit $\rho_\theta \to 1$ then implies that $(1-\rho_\theta)\left(\max_j\left\{\beta_j^\theta \hat{\theta}_{j,t}\right\} - \beta_i^\theta \hat{\theta}_{i,t}\right)$ is of second order so that the equation becomes

$$\min_i \left\{-\beta_i^y c_y^- \epsilon_t^v\right\} = \frac{1}{\sigma}\left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \phi_y\right) c_y^- \epsilon_t^v + \frac{1}{\sigma}\epsilon_t^v$$

which implies

$$c_y^- = -\frac{1}{\bar{\beta} + \frac{1}{\sigma}\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \frac{1}{\sigma}\phi_y}\frac{1}{\sigma}$$

where

$$\bar{\beta} = \max_i \left\{\beta_i^y\right\}$$

as required. □

Proof of Proposition 10 Recall the system

$$\iota_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t + \epsilon_t^v$$

$$\pi_t = \frac{\bar{\Phi}-1}{\xi^p}\varphi \hat{y}_t + \delta \mathbb{E}_t\left[\pi_{t+1}\right]$$

$$\min_i\left\{\sigma_i\left(\mathbb{E}_t\left[\hat{\tilde{c}}_{i,t+1}\right] - \hat{\tilde{c}}_{i,t}\right)\right\} = \left(\iota_t - \mathbb{E}_t\left[\pi_{t+1}\right] - \rho\right)$$

$$\hat{\tilde{c}}_{i,t} = \beta_i^y \hat{y}_t \;\; \forall i$$

Consider $\epsilon_t^v > 0$, and suppose that the solution takes the form

$$\hat{y}_t = c_y^+ \epsilon_t^v$$

$$\pi_t = c_\pi^+ \epsilon_t^v$$

where $c_y^+ < 0$. Substituting these guesses into the system and simplifying yields

$$c_\pi^+ = \frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^+$$

$$\min_i\left\{-\sigma_i \beta_i^y c_y^+ \epsilon_t^v\right\} = \left(\phi_\pi c_\pi^+ \epsilon_t^v + \phi_y c_y^+ \epsilon_t^v + \epsilon_t^v\right)$$

so that

$$\min_i\left\{-\sigma_i \beta_i^y c_y^+ \epsilon_t^v\right\} = \left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^+ + \phi_y c_y^+ + 1\right)\epsilon_t^v$$

By the supposition, $-c_y^+\epsilon_t^v > 0$ so that

$$-c_y^+\epsilon_t^v \underline{\beta\sigma} = \left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^+ + \phi_y c_y^+ + 1\right)\epsilon_t^v$$

where

$$\underline{\beta\sigma} = \min_i \{\sigma_i \beta_i^y\}$$

Hence

$$c_y^+ = -\frac{1}{\underline{\beta\sigma} + \phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \phi_y}$$

$$c_\pi^+ = -\frac{1}{\underline{\beta\sigma} + \phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \phi_y}\frac{\bar{\Phi}-1}{\xi^p}\varphi$$

where $c_y^+ < 0$ since

$$\underline{\beta\sigma} + \phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \phi_y > 0$$

For completeness, suppose $c_y^+ > 0$. The same steps yield

$$\min_i \left\{-\sigma_i \beta_i^y c_y^+ \epsilon_t^v\right\} = \left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^+ + \phi_y c_y^+ + 1\right)\epsilon_t^v$$

By supposition, $c_y^+\epsilon_t^v > 0$ so that

$$-c_y^+\epsilon_t^v \bar{\beta\sigma} = \left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^+ + \phi_y c_y^+ + 1\right)\epsilon_t^v$$

where

$$\bar{\beta\sigma} = \max_i \{\sigma_i \beta_i^y\} > 0$$

Hence

$$c_y^+ = -\frac{1}{\bar{\beta\sigma} + \phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \phi_y} < 0$$

where the inequality follows from

$$\bar{\beta\sigma} + \phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \phi_y > 0$$

Therefore, we have a contradiction.

Now consider $\epsilon_t^v < 0$, and suppose that the solution takes the form

$$\hat{y}_t = c_y^- \epsilon_t^v$$

$$\pi_t = c_\pi^- \epsilon_t^v$$

where $c_y^- < 0$. Substituting these guesses into the system and simplifying yields

$$c_\pi^- = \frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^- > 0$$

$$\min_i \left\{-\sigma_i \beta_i^y c_y^- \epsilon_t^v\right\} = \left(\phi_\pi c_\pi^- \epsilon_t^v + \phi_y c_y^- \epsilon_t^v + \epsilon_t^v\right)$$

so that

$$\min_i \left\{-\sigma_i \beta_i^y c_y^- \epsilon_t^v\right\} = \left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^- + \phi_y c_y^- + 1\right)\epsilon_t^v$$

By the supposition, $c_y^-\epsilon_t^v > 0$ so that

$$-c_y^-\epsilon_t^v \bar{\beta\sigma} = \left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^- + \phi_y c_y^- + 1\right)\epsilon_t^v$$

where

$$\bar{\beta\sigma} = \max_i \{\sigma_i \beta_i^y\}$$

Hence

$$c_y^- = -\frac{1}{\bar{\beta}\sigma + \phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \phi_y} < 0$$

$$c_\pi^+ = -\frac{1}{\bar{\beta}\sigma + \phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \phi_y} \frac{\bar{\Phi}-1}{\xi^p}\varphi_\Theta < 0$$

as required.

For completeness, suppose that $c_y^- > 0$ so that $-c_y^- \epsilon_t^v > 0$ and the same steps as above lead to

$$-c_y^- \epsilon_t^v \underline{\beta}\sigma = \left(\phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi c_y^- + \phi_y c_y^- + 1\right)\epsilon_t^v$$

so that

$$c_y^- = -\frac{1}{\underline{\beta}\sigma + \phi_\pi \frac{\bar{\Phi}-1}{\xi^p}\varphi + \phi_y} < 0$$

which is a contradiction. □

Proof of Proposition 2 Recall the system

$$\iota_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t + v_t$$

$$\pi_t = \kappa_y \hat{y}_t + \delta \mathbb{E}_t\left[\pi_{t+1}\right]$$

$$\min_i \left\{\mathbb{E}_t\left[\hat{c}_{i,t+1}\right] - \hat{c}_{i,t}\right\} = \frac{1}{\sigma}\left(\iota_t - \mathbb{E}_t\left[\pi_{t+1}\right] - \rho\right)$$

$$\hat{c}_{i,t} = \beta_i^{c,v}\hat{y}_t \ \forall i$$

Consider $v_1 > 0$, and guess linear solution $\hat{y}_t = c_y^+ v_t$, $\pi_t = c_\pi^+ v_t$ with $c_y^+ < 0$. Substitution yields

$$c_\pi^+ = \frac{\kappa_y}{1-\delta\rho_v} c_y^+$$

$$\min_i \left\{\left(\beta_i^{c,v}\rho_v - \beta_{i,t}^{c,v}\right) c_y^+ v_t\right\} = \frac{1}{\sigma}\left(\left(\phi_\pi - \rho_v\right)\frac{\kappa_y}{1-\delta\rho_v}c_y^+ + \phi_y c_y^+ + 1\right) v_t$$

$-c_y^+ v_t > 0$ implies

$$c_y^+ = -\frac{1}{\left(1-\rho_v\right)\underline{\beta}^{c,v} + \frac{1}{\sigma}\left(\phi_\pi - \rho_v\right)\frac{\kappa_y}{1-\delta\rho_v} + \frac{1}{\sigma}\phi_y}\frac{1}{\sigma}$$

where

$$\underline{\beta}^{c,v} = \min_i \left\{\beta_i^{c,v}\right\}$$

Suppose $v_1 < 0$, and guess linear solution $\hat{y}_t = c_y^- v_t$, $\pi_t = c_\pi^- v_t$ with $c_y^- < 0$. Substitution yields

$$\min_i \left\{\left(\rho_v - 1\right)\beta_i^{c,v} c_y^- v_t\right\} = \frac{1}{\sigma}\left(\left(\phi_\pi - \rho_v\right)\frac{\kappa_y}{1-\delta\rho_v}c_y^- + \phi_y c_y^- + 1\right) v_t$$

where $\left(1-\rho_v\right) c_y^- v_t > 0$ implies

$$c_y^- = -\frac{1}{\left(1-\rho_v\right)\bar{\beta}^{c,v} + \frac{1}{\sigma}\left(\phi_\pi - \rho_v\right)\frac{\kappa_y}{1-\delta\rho_v} + \frac{1}{\sigma}\phi_y}\frac{1}{\sigma}$$

where

$$\bar{\beta}^{c,v} = \max_i \left\{\beta_i^{c,v}\right\}$$

as required. □

Proof of Proposition 3 Pooled OLS estimation for group g yields

$$\hat{\beta}_g = \frac{\sum_t \sum_i \left(\Delta \hat{\log} Y_t - \bar{y}\right) \Delta \log c_{i,t}}{\sum_t \sum_i \left(\Delta \hat{\log} Y_t - \bar{y}\right)^2}$$

where $\bar{y} = \frac{1}{T} \sum \Delta \hat{\log} Y_t$, and the summation over i is read as "sum over all households i such that $\mathcal{G}(i,t) = g$". Substituting in the true model for household consumption yields

$$\hat{\beta}_g = \frac{\sum_t \sum_i \left(\Delta \hat{\log} Y_t - \bar{y}\right)\left(\alpha_i + \beta_i \Delta \hat{\log} Y_t + u_{i,t}\right)}{\sum_t \sum_i \left(\Delta \hat{\log} Y_t - \bar{y}\right)^2}$$

$$\hat{\beta}_g = \frac{\sum_t \sum_i \left(\alpha_i \Delta \hat{\log} Y_t + \beta_i \left(\Delta \hat{\log} Y_t\right)^2 + u_{i,t} \Delta \hat{\log} Y_t - \bar{y}\alpha_i - \beta_i \bar{y} \Delta \hat{\log} Y_t - \bar{y} u_{i,t}\right)}{\sum_t \sum_i \left(\Delta \hat{\log} Y_t - \bar{y}\right)^2}$$

$$\hat{\beta}_g = \frac{\sum_t \Delta \hat{\log} Y_t \sum_i \alpha_i + \sum_t \left(\Delta \hat{\log} Y_t\right)^2 \sum_i \beta_i + \sum_t \Delta \hat{\log} Y_t \sum_i u_{i,t} - \bar{y} \sum_t \sum_i \alpha_i - \bar{y} \sum_t \Delta \hat{\log} Y_t \sum_i \beta_i - \bar{y} \sum_t \sum_i u_{i,t}}{\sum_t \sum_i \left(\Delta \hat{\log} Y_t - \bar{y}\right)^2}$$

$$\hat{\beta}_g = \frac{\sum_t \Delta \hat{\log} Y_t \alpha_{g,t} + \sum_t \left(\Delta \hat{\log} Y_t\right)^2 \beta_{g,t} + \sum_t \Delta \hat{\log} Y_t u_{g,t} - \bar{y} \sum_t \alpha_{g,t} - \bar{y} \sum_t \Delta \hat{\log} Y_t \beta_{g,t} - \bar{y} \sum_t u_{g,t}}{\frac{1}{n} \sum_t \sum_i \left(\Delta \hat{\log} Y_t - \bar{y}\right)^2}$$

where

$$\alpha_{g,t} = \frac{1}{n} \sum_{i \in g,t} \alpha_i$$

$$\beta_{g,t} = \frac{1}{n} \sum_{i \in g,t} \beta_i$$

$$u_{g,t} = \frac{1}{n} \sum_{i \in g,t} u_{i,t}$$

are parameter means over households in group g in period t. Note that $\beta_{g,t} \in (\min_i \beta_i, \max_i \beta_i)$ by definition.

Continuing,

$$\hat{\beta}_g = \frac{\frac{1}{T} \sum_t \left(\Delta \hat{\log} Y_t\right)\left(\Delta \hat{\log} Y_t\right) \beta_{g,t} - \bar{y} \frac{1}{T} \sum_t \Delta \hat{\log} Y_t \beta_{g,t} + \frac{1}{T} \sum_t \Delta \hat{\log} Y_t \alpha_{g,t} - \bar{y} \frac{1}{T} \sum_t \alpha_{g,t} + \frac{1}{T} \sum_t \Delta \hat{\log} Y_t u_{g,t} - \bar{y} \frac{1}{T} \sum_t u_{g,t}}{\frac{1}{n} \sum_t \sum_i \left(\Delta \hat{\log} Y_t - \bar{y}\right)^2}$$

Hence as $T \to \infty$, we can apply a suitable Law of Large Numbers to obtain

$$\hat{\beta}_g \to^p \frac{Cov\left(\Delta \hat{\log} Y_t, \Delta \hat{\log} Y_t \beta_{g,t}\right) + Cov\left(\Delta \hat{\log} Y_t, \alpha_{g,t} + u_{g,t}\right)}{V\left[\Delta \hat{\log} Y_t\right]}$$

where, if $\mathcal{G}$ does not alter group assignments over time (i.e. is exogenous to changes in $\Delta \hat{\log} Y_t$), then

$$Cov\left(\Delta \hat{\log} Y_t, \alpha_{g,t} + u_{g,t}\right) = 0$$

$$Cov\left(\Delta \hat{\log} Y_t, \Delta \hat{\log} Y_t \beta_{g,t}\right) = V\left[\Delta \hat{\log} Y_t\right] \frac{1}{n} \sum_{i \in g} \beta_i$$

so that

$$\hat{\beta}_g \to^p \frac{1}{n} \sum_{i \in g} \beta_i \in \left(\min_i \{\beta_i\}, \min_i \{\beta_i\}\right)$$

as required. □