Rational Inattention in Macroeconomics

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Introduction: Rational inattention in macroeconomics

- RI is the idea that people cannot process all available information and they allocate attention optimally.

- It is a simple, plausible assumption that may help us close the gap between benchmark macro models and the data.

- Solving RI problems in dynamic models and solving DSGE models with RI is challenging, but the literature has made significant progress.
Plan of this talk

- Analytical results in Maćkowiak and Wiederholt (2009).
- Analytical results in Maćkowiak, Matějka, and Wiederholt (2018).
- RBC model with RI in Maćkowiak and Wiederholt (2020).
- Other recent advances in dynamic RI.

\[
\min_{b,c} E[(x_t - x_t^*)^2]
\]

subject to

\[
x_t^* = \sum_{s=0}^{\infty} a_s \varepsilon_{t-s}
\]

\[
x_t = \sum_{s=0}^{\infty} b_s \varepsilon_{t-s} + \sum_{s=0}^{\infty} c_s \psi_{t-s}
\]

\[
\lim_{T \to \infty} \frac{1}{T} \left[ H(x_t^T) - H(x_t^T | x^T) \right] \leq \kappa
\]

\[\varepsilon_t, \psi_t \text{ independent Gaussian white noise}\]
Suppose $x^*_t$ follows an AR(1) process, $a_s = \rho^s a_0$.

- Then
  
  $$b_s = \left[ \rho^s - \frac{1}{2^2\kappa} \left( \frac{\rho}{2^2\kappa} \right)^s \right] a_0, \quad c_s = \sqrt{\frac{1}{2^2\kappa} \frac{2^{2\kappa} - 1}{2^{2\kappa} - \rho^2} \left( \frac{\rho}{2^2\kappa} \right)^s} a_0$$

- Also,
  
  $$x_t = E [x^*_t | \mathcal{I}_t]$$

  $$\mathcal{I}_t = \mathcal{I}_{t-1} \cup \{s_0, s_1, \ldots, s_t\} \quad \text{with} \quad s_t = x^*_t + \psi_t$$
Attention problem in MMW (2018)

\[
\min_{A, B, \Sigma, \psi} E[(x_t - x_t^*)^2]
\]

subject to

\[
x_t^* = \phi_1 x_{t-1}^* + \ldots + \phi_p x_{t-p}^* + \epsilon_t + \theta_1 \epsilon_{t-1} + \ldots + \theta_q \epsilon_{t-q}
\]

\[
x_t = E[x_t^* | I_t]
\]

\[
I_t = I_{t-1} \cup \{s_0, s_1, ..., s_t\}
\]

\[
s_t = A x_t^* + B \bar{\epsilon}_t + \psi_t
\]

\[
\lim_{T \to \infty} \frac{1}{T} [H(x_t^T) - H(x_t^T | s_t^T)] \leq \kappa
\]


Any optimal signal is a signal about the state vector

\[ \tilde{\zeta}_t = (x_t^*, \ldots, x_{t-(p-1)}^*, \varepsilon_t, \ldots, \varepsilon_{t-(q-1)})' \]

The optimum can be attained with a one-dimensional signal.

Special case: If \( x_t^* = \phi_1 x_{t-1}^* + \phi_2 x_{t-2}^* + \varepsilon_t \), the optimal signal is

\[ s_t = g_1 x_t^* + g_2 x_{t-1}^* + \psi_t \quad \text{with} \quad g_2 \neq 0 \]

which can also be written

\[ s'_t = \omega x_t^* + (1 - \omega)(\phi_1 x_t^* + \phi_2 x_{t-1}^*) + \psi'_t \quad \text{with} \quad 1 - \omega \neq 0 \]
Solving for an optimal signal

1. **State-space representation:**

\[
\xi_{t+1} = F\xi_t + \nu_{t+1}
\]

\[
s_t = g'\xi_t + \psi_t
\]

2. **Attention constraint:**

\[
H(\xi_t|s^{t-1}) - H(\xi_t|s^t) \leq \kappa
\]

where the left-hand side equals

\[
\frac{1}{2} \log_2 \left( \frac{\det \Sigma_1}{\det \Sigma_0} \right) = \frac{1}{2} \log_2 \left( \frac{g'\Sigma_1 g}{\sigma^2_\psi} + 1 \right)
\]

\[\Sigma_0 \ (\Sigma_1)\] is the steady-state conditional variance-covariance matrix of \(\xi_t\) given \(I_t \ (I_{t-1})\).
3. Optimization problem:

\[
\min_{g, \sigma^2_\psi} \Sigma_0^{(1,1)} \quad \text{subject to} \quad \frac{1}{2} \log_2 \left( \frac{g' \Sigma_1 g}{\sigma^2_\psi} + 1 \right) = \kappa
\]

and the usual Kalman filter equations for \( \Sigma_0 \) and \( \Sigma_1 \).

Or let \( \lambda > 0 \) be the marginal cost of attention and solve

\[
\min_{g, \sigma^2_\psi} \Sigma_0^{(1,1)} + \frac{\lambda}{2} \log_2 \left( \frac{g' \Sigma_1 g}{\sigma^2_\psi} + 1 \right)
\]

subject to the Kalman filter equations for \( \Sigma_0 \) and \( \Sigma_1 \).
A continuum of firms indexed by $i \in [0, 1]$.

Production: $Y_{it} = e^{a_t} K_{it-1}^\alpha L_{it}^\phi N_i^{1-\alpha-\phi}$ \quad $\alpha \geq 0$, $\phi \geq 0$, $\alpha + \phi < 1$

Capital accumulation: $K_{it} - K_{it-1} = I_{it} - \delta K_{it-1}$ \quad $\delta \in (0, 1]$

Dividends: $D_{it} = Y_{it} - W_t L_{it} - I_{it}$

TFP: $a_t = \rho a_{t-1} + \sigma \varepsilon_{t-h}$ \quad $\varepsilon_t \sim N(0, 1)$, $\sigma > 0$, $\rho \in (0, 1)$

$h = 0$ is a standard productivity shock

$h \geq 1$ is a news shock
A continuum of households indexed by \( j \in [0, 1] \).

Preferences: 
\[
U (C_{jt}, L_{jt}) = \frac{C_{jt}^{1-\gamma} - 1}{1-\gamma} - \frac{L_{jt}^{1+\eta} - 1}{1+\eta} \quad \gamma > 0, \ \eta \geq 0, \ \beta \in (0, 1)
\]

Budget: 
\[
V_t Q_{jt} - V_{t-1} Q_{jt} = W_t L_t + D_t Q_{jt-1} - C_{jt} \quad D_t = \int_0^1 D_{it} \, di
\]

Market clearing: The wage adjusts so that labor demand equals labor
supply \((\int_0^1 L_{it} \, di = \int_0^1 L_{jt} \, dj)\) and the price of a share adjusts so that
demand for shares equals supply of shares \((\int_0^1 Q_{jt} \, dj = 1)\).
Definition of equilibrium

- In periods $t = 0, 1, 2, \ldots$
  - Firms maximize given their information sets.
  - Households maximize given their information sets.
  - Markets clear.
  - Agents’ perceived law of motion of the economy equals the actual law of motion of the economy (rational expectations).

- In period $t = -1$, each firm chooses an optimal signal process.

- In period $t = -1$, firms receive a long sequence of signals such that the prior variance-covariance matrix of the state vector in $t = 0$ equals the steady-state prior variance-covariance matrix of the state vector.
Attention problem of a firm (no capital)

\[
\min_{g, \sigma^2_\psi} \sum_{t=0}^{\infty} \beta^t \left\{ E_{i,-1} \left[ \frac{\phi (1 - \phi)}{2} (l_{it} - l^*_{it})^2 \right] + \lambda I (\xi_t; s_{it} | \mathcal{I}_{it-1}) \right\}
\]

subject to

\[
\begin{align*}
\xi_{t+1} &= F \xi_t + v_{t+1} \\
l_{it} &= E [l^*_{it} | \mathcal{I}_{it}] \\
\mathcal{I}_{it} &= \mathcal{I}_{it-1} \cup \{ s_{it} \} \\
s_{it} &= g' \xi_t + \psi_{it}
\end{align*}
\]
Labor to a news shock ($\alpha = 0$, $h = 1$)
Attention problem of a firm

\[ x_t = \left( I_{it} - \frac{k_{it}}{1 - \phi} k_{it-1} \right) \quad x_t^* = \left( \frac{1}{1 - \alpha - \phi} \left[ \frac{E_t a_{t+1} - \phi E_t w_{t+1}}{1 - \beta (1 - \delta)} \right] - (1 - \phi) \frac{\gamma E_t (c_{t+1} - c_t)}{1 - \beta (1 - \delta)} \right) \]

\[
\min_{G, \Sigma \psi} \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2} (x_t - x_t^*)' \Theta (x_t - x_t^*) \right\} + \lambda I \left( \zeta_t; s_{it} | \mathcal{I}_{it-1} \right) \\
\text{subject to}
\]

\[ \zeta_{t+1} = F \zeta_t + \nu_{t+1} \]

\[ x_{it} = E \left[ x_{it}^* | \mathcal{I}_{it} \right] \]

\[ \mathcal{I}_{it} = \mathcal{I}_{it-1} \cup \{ s_{it} \} \]

\[ s_{it} = G' \zeta_t + \psi_{it} \]
Parameter values

- Technology:
  - production function: $\alpha = 0.33$, $\phi = 0.65$
  - capital accumulation: $\delta = 0.025$
  - TFP: $\rho = 0.9$, $\sigma = 0.01$

- Preferences:
  - $\gamma = 1$, $\eta = 0$, $\beta = 0.99$

- Marginal cost of attention:
  - We set $\lambda = (1/10,000)$ times steady-state output.
  - For this value of $\lambda$, the equilibrium expected per period profit loss from suboptimal actions equals $(4/100,000)$ of steady-state output.
  - Following Coibion and Gorodnichenko (2015), we also regress the ex-post average forecast error on the ex-ante average forecast revision in SPF output forecast data and in simulated output forecast data. The regression coefficients are 0.76 (0.30) and 1.07, respectively.
Figure 3: Impulse responses to a productivity shock

- **Labor**
  - Perfect information equilibrium
  - Rational inattention equilibrium

- **Investment**
  - Perfect information equilibrium
  - Rational inattention equilibrium

- **Output**
  - Perfect information equilibrium
  - Rational inattention equilibrium

- **Capital**
  - Perfect information equilibrium
  - Rational inattention equilibrium

- **Consumption**
  - Perfect information equilibrium
  - Rational inattention equilibrium

- **Conditional expectation of productivity**
  - Perfect information equilibrium
  - Rational inattention equilibrium
### Table 1: Business cycle statistics

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Perfect information</th>
<th>Rational inattention</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Relative standard deviation</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_c/\sigma_y$</td>
<td>0.55</td>
<td>0.56</td>
<td>0.59</td>
</tr>
<tr>
<td>$\sigma_l/\sigma_y$</td>
<td>0.92</td>
<td>0.66</td>
<td>0.57</td>
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<tr>
<td>$\sigma_i/\sigma_y$</td>
<td>2.88</td>
<td>3.05</td>
<td>2.93</td>
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<tr>
<td>$\sigma_a/\sigma_y$</td>
<td>0.52</td>
<td>0.46</td>
<td>0.51</td>
</tr>
<tr>
<td><strong>Correlation</strong></td>
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</tr>
<tr>
<td>$\rho_{c,y}$</td>
<td>0.78</td>
<td>0.78</td>
<td>0.81</td>
</tr>
<tr>
<td>$\rho_{l,y}$</td>
<td>0.85</td>
<td>0.85</td>
<td>0.83</td>
</tr>
<tr>
<td>$\rho_{i,y}$</td>
<td>0.90</td>
<td>0.93</td>
<td>0.92</td>
</tr>
<tr>
<td>$\rho_{a,y}$</td>
<td>0.40</td>
<td>1.00</td>
<td>0.99</td>
</tr>
<tr>
<td><strong>First-order serial correlation</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta c$</td>
<td>0.27</td>
<td>0.23</td>
<td>0.28</td>
</tr>
<tr>
<td>$\Delta l$</td>
<td>0.41</td>
<td>-0.06</td>
<td>0.46</td>
</tr>
<tr>
<td>$\Delta i$</td>
<td>0.35</td>
<td>-0.06</td>
<td>0.14</td>
</tr>
<tr>
<td>$\Delta y$</td>
<td>0.30</td>
<td>-0.05</td>
<td>0.13</td>
</tr>
<tr>
<td>$\Delta a$</td>
<td>-0.06</td>
<td>-0.05</td>
<td>-0.05</td>
</tr>
</tbody>
</table>

Model: Unconditional moments computed from the equilibrium MA representation of each variable.
Figure 4: Impulse responses to a news shock ($h = 2$)
Adding RI by households

To begin: no capital, no trade in shares. When households become subject to RI:
- they supply less labor on impact of a positive productivity shock.
- they supply more labor on impact of a positive news shock.

Introduce capital. When households become subject to RI, they consume less on impact of a positive productivity (news) shock – they save more, and they supply more labor.
Figure 6: Additional impulse responses

Signal to a news shock ($h = 4$)

Labor to a productivity shock

Labor to a news shock ($h = 1$)

Share price to a news shock ($h = 4$)

Labor to a news shock ($h = 6$)

Labor to a news shock ($h = 6$)
Literature: Recent advances in dynamic RI

- Steiner, Stewart, and Matějka (2017).
- Jurado (2020).
- Miao, Wu, and Young (2020).
Conclusions

- In a dynamic model, RI causes a combination of delay in actions and forward-looking actions.

- In a neoclassical economy, RI induces persistence and helps produce comovement after news shocks.

- The recent advances suggest that RI in macroeconomics is an exciting research area.