Rational Inattention in Macroeconomics

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Introduction: Rational inattention in macroeconomics

- RI is the idea that people cannot process all available information and they allocate attention optimally.

- It is a simple, plausible assumption that may help us close the gap between benchmark macro models and the data.

- Solving RI problems in dynamic models and solving DSGE models with RI is challenging, but the literature has made significant progress.

Plan of this talk

- Attention problem in Sims (2003).
- Analytical results in Maćkowiak and Wiederholt (2009).
- Analytical results in Maćkowiak, Matějka, and Wiederholt (2018).
- RBC model with RI in Maćkowiak and Wiederholt (2020).
- Other recent advances in dynamic RI.

Attention problem in Sims (2003)

$$\min_{b,c} E[(x_t - x_t^*)^2]$$

subject to

$$x_t^* = \sum_{s=0}^{\infty} a_s \varepsilon_{t-s}$$

$$x_t = \sum_{s=0}^{\infty} b_s \varepsilon_{t-s} + \sum_{s=0}^{\infty} c_s \psi_{t-s}$$

$$\lim_{T\to\infty} \frac{1}{T}[H(x^{*T}) - H(x^{*T}|x^T)] \leq \kappa$$

ε_t, ψ_t indepedent Gaussian white noise

Analytical results in Maćkowiak and Wiederholt (2009)

Suppose x_t^* follows an AR(1) process, $a_s = \rho^s a_0$.

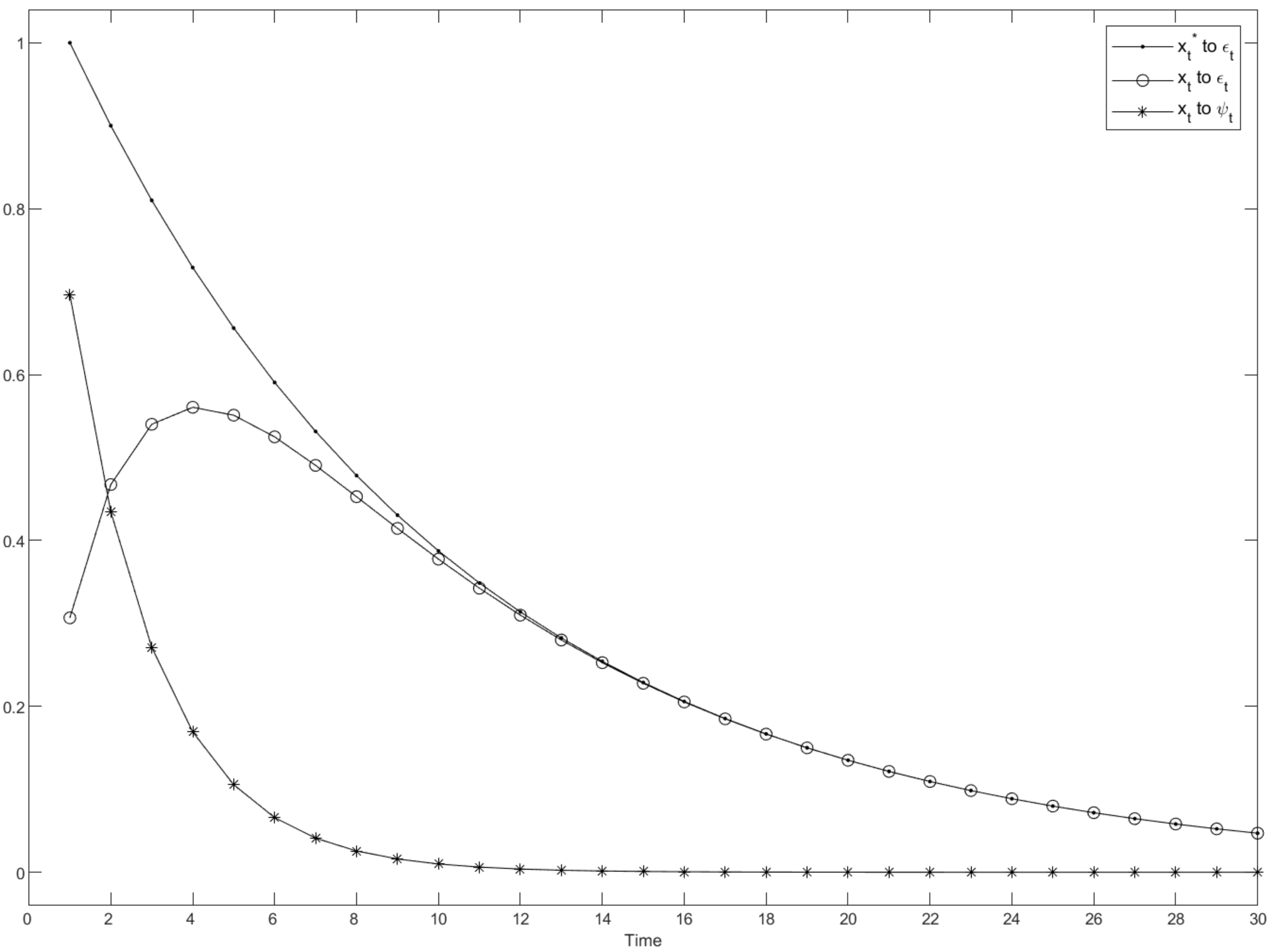
- Then

$$b_s = \left[\rho^s - \frac{1}{2^{2\kappa}}\left(\frac{\rho}{2^{2\kappa}}\right)^s\right] a_0, \qquad c_s = \sqrt{\frac{1}{2^{2\kappa}}\frac{2^{2\kappa}-1}{2^{2\kappa}-\rho^2}}\left(\frac{\rho}{2^{2\kappa}}\right)^s a_0$$

- Also,

$$x_t = E\left[x_t^* | \mathcal{I}_t\right]$$

$$\mathcal{I}_t = \mathcal{I}_{-1} \cup \left\{s_0, s_1, \ldots, s_t\right\} \quad \text{with} \quad s_t = x_t^* + \psi_t$$



x_t^* to ϵ_t
x_t to ϵ_t
x_t to ψ_t
0
0.2
0.4
0.6
0.8
1
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
Time

Attention problem in MMW (2018)

$$\min_{A,B,\Sigma_\psi} E[(x_t - x_t^*)^2]$$

subject to

$$x_t^* = \phi_1 x_{t-1}^* + \ldots + \phi_p x_{t-p}^* + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q}$$

$$x_t = E\left[x_t^* | \mathcal{I}_t\right]$$

$$\mathcal{I}_t = \mathcal{I}_{-1} \cup \left\{s_0, s_1, \ldots, s_t\right\}$$

$$s_t = A\bar{x}_t^* + B\bar{\varepsilon}_t + \psi_t$$

$$\lim_{T\to\infty} \frac{1}{T}[H(x^{*T}) - H(x^{*T}|s^T)] \leq \kappa$$

Analytical results

- Any optimal signal is a signal about the state vector

$$\xi_t = (x_t^*, \ldots, x_{t-(p-1)}^*, \varepsilon_t, \ldots, \varepsilon_{t-(q-1)})'$$

- The optimum can be attained with a one-dimensional signal.
- Special case: If $x_t^* = \phi_1 x_{t-1}^* + \phi_2 x_{t-2}^* + \varepsilon_t$, the optimal signal is

$$s_t = g_1 x_t^* + g_2 x_{t-1}^* + \psi_t \quad \text{with} \quad g_2 \neq 0$$

  - which can also be written

$$s_t' = \omega x_t^* + (1-\omega)(\phi_1 x_t^* + \phi_2 x_{t-1}^*) + \psi_t' \quad \text{with} \quad 1-\omega \neq 0$$

Solving for an optimal signal

1. **State-space representation**:

$$\xi_{t+1} = F\xi_t + v_{t+1}$$

$$s_t = g'\xi_t + \psi_t$$

2. **Attention constraint:**

$$H(\xi_t | s^{t-1}) - H(\xi_t | s^t) \leq \kappa$$

where the left-hand side equals

$$\frac{1}{2}\log_2\left(\frac{\det \Sigma_1}{\det \Sigma_0}\right) = \frac{1}{2}\log_2\left(\frac{g'\Sigma_1 g}{\sigma_\psi^2} + 1\right)$$

Σ_0 (Σ_1) is the steady-state conditional variance-covariance matrix of ξ_t given $\mathcal{I}_t$ ($\mathcal{I}_{t-1}$).

Solving for an optimal signal

3. **Optimization problem**:

$$\min_{g,\sigma_\psi^2} \Sigma_0^{(1,1)} \quad \text{subject to} \quad \frac{1}{2}\log_2\left(\frac{g'\Sigma_1 g}{\sigma_\psi^2}+1\right)=\kappa$$

   and the usual Kalman filter equations for Σ_0 and Σ_1.

Or let $\lambda > 0$ be the marginal cost of attention and solve

$$\min_{g,\sigma_\psi^2} \Sigma_0^{(1,1)} + \frac{\lambda}{2}\log_2\left(\frac{g'\Sigma_1 g}{\sigma_\psi^2}+1\right)$$

subject to the Kalman filter equations for Σ_0 and Σ_1.

Model in Maćkowiak and Wiederholt (2020) - technology

- A continuum of firms indexed by $i \in [0, 1]$.
- Production: $Y_{it} = e^{a_t} K_{it-1}^{\alpha} L_{it}^{\phi} N_i^{1-\alpha-\phi}$ $\qquad \alpha \geq 0,\ \phi \geq 0,\ \alpha + \phi < 1$
- Capital accumulation: $K_{it} - K_{it-1} = I_{it} - \delta K_{it-1}$ $\qquad \delta \in (0, 1]$
- Dividends: $D_{it} = Y_{it} - W_t L_{it} - I_{it}$
- TFP: $a_t = \rho a_{t-1} + \sigma \varepsilon_{t-h}$ $\qquad \varepsilon_t \sim N(0, 1),\ \sigma > 0,\ \rho \in (0, 1)$
    - $h = 0$ is a standard productivity shock
    - $h \geq 1$ is a news shock

Model - preferences, budget constraint, market clearing

- A continuum of households indexed by $j \in [0, 1]$.
- Preferences: $U(C_{jt}, L_{jt}) = \frac{C_{jt}^{1-\gamma}-1}{1-\gamma} - \frac{L_{jt}^{1+\eta}}{1+\eta} \qquad \gamma > 0,\ \eta \geq 0,\ \beta \in (0, 1)$
- Budget: $V_t Q_{jt} - V_t Q_{jt-1} = W_t L_{jt} + D_t Q_{jt-1} - C_{jt} \qquad D_t = \int_0^1 D_{it} di$
- Market clearing: The wage adjusts so that labor demand equals labor supply ($\int_0^1 L_{it} di = \int_0^1 L_{jt} dj$) and the price of a share adjusts so that demand for shares equals supply of shares ($\int_0^1 Q_{jt} dj = 1$).

Definition of equilibrium

- In periods $t = 0, 1, 2, \ldots$
  - Firms maximize given their information sets.
  - Households maximize given their information sets.
  - Markets clear.
  - Agents' perceived law of motion of the economy equals the actual law of motion of the economy (rational expectations).
- In period $t = -1$, each firm chooses an optimal signal process.
- In period $t = -1$, firms receive a long sequence of signals such that the prior variance-covariance matrix of the state vector in $t = 0$ equals the steady-state prior variance-covariance matrix of the state vector.

Attention problem of a firm (no capital)

$$\min_{g,\sigma_\psi^2} \sum_{t=0}^{\infty} \beta^t \left\{ E_{i,-1}\left[\frac{\phi(1-\phi)}{2}\left(l_{it}-l_{it}^*\right)^2\right] + \lambda I\left(\xi_t; s_{it}|\mathcal{I}_{it-1}\right)\right\}$$

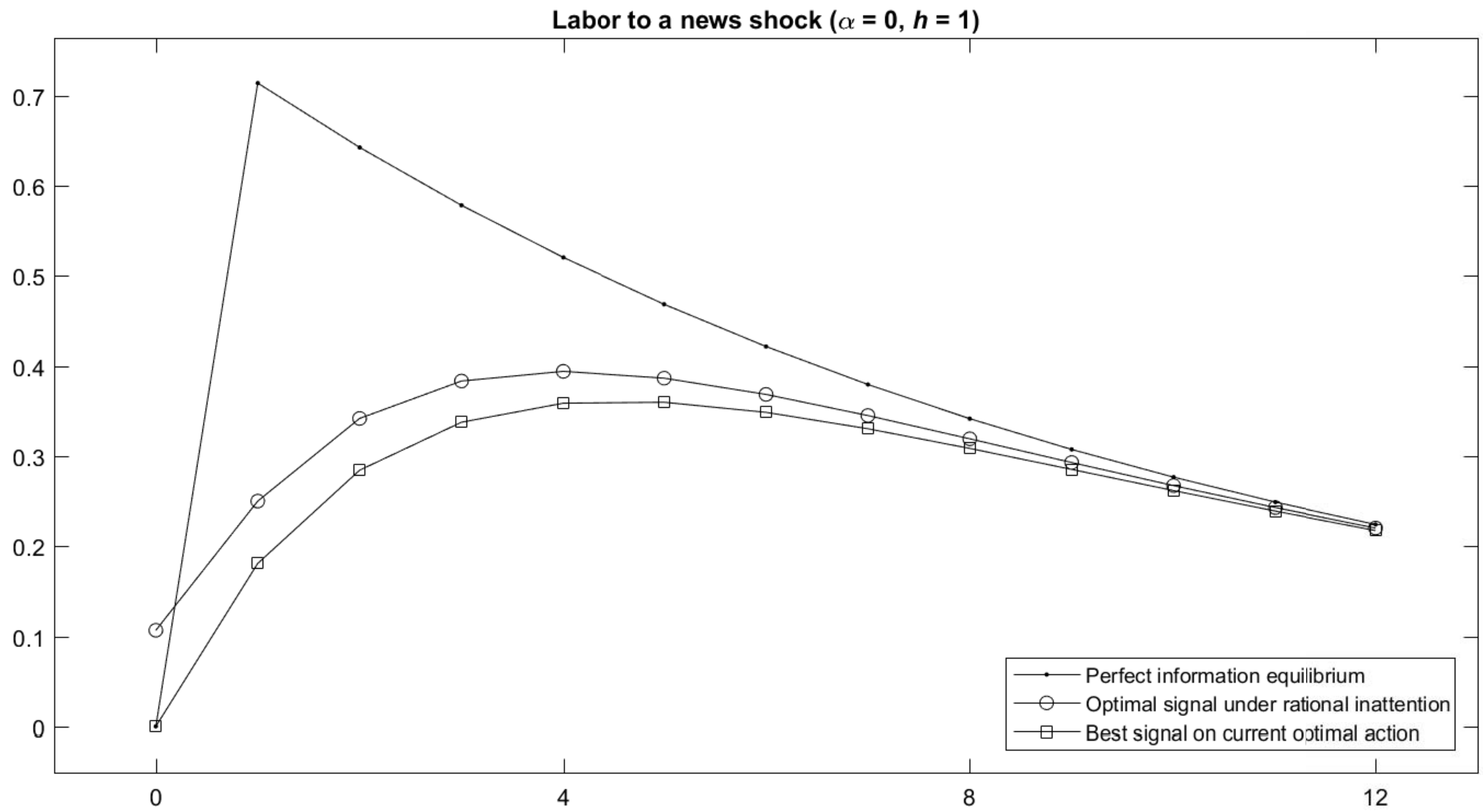
subject to

$$\xi_{t+1} = F\xi_t + v_{t+1}$$

$$l_{it} = E\left[l_{it}^*|\mathcal{I}_{it}\right]$$

$$\mathcal{I}_{it} = \mathcal{I}_{it-1} \cup \{s_{it}\}$$

$$s_{it} = g'\xi_t + \psi_{it}$$



Labor to a news shock (α = 0, h = 1)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
4
8
12
Perfect information equilibrium
Optimal signal under rational inattention
Best signal on current optimal action

Attention problem of a firm

$$x_t = \begin{pmatrix} k_{it} \\ l_{it} - \frac{\alpha}{1-\phi} k_{it-1} \end{pmatrix} \quad x_t^* = \begin{pmatrix} \frac{1}{1-\alpha-\phi} \begin{bmatrix} E_t a_{t+1} - \phi E_t w_{t+1} \\ -(1-\phi) \frac{\gamma E_t (c_{t+1} - c_t)}{1-\beta(1-\delta)} \end{bmatrix} \\ \frac{1}{1-\phi} (a_t - w_t) \end{pmatrix}$$

$$\min_{G, \Sigma_\psi} \sum_{t=0}^{\infty} \beta^t \left\{ E_{i,-1} \left[\frac{1}{2} (x_t - x_t^*)' \Theta (x_t - x_t^*) \right] + \lambda I (\xi_t; s_{it} | \mathcal{I}_{it-1}) \right\}$$

subject to

$$\xi_{t+1} = F \xi_t + v_{t+1}$$

$$x_{it} = E \left[x_{it}^* | \mathcal{I}_{it} \right]$$

$$\mathcal{I}_{it} = \mathcal{I}_{it-1} \cup \{ s_{it} \}$$

$$s_{it} = G' \xi_t + \psi_{it}$$

Parameter values

- Technology:
  - production function: $\alpha = 0.33$, $\phi = 0.65$
  - capital accumulation: $\delta = 0.025$
  - TFP: $\rho = 0.9$, $\sigma = 0.01$
- Preferences:
  - $\gamma = 1$, $\eta = 0$, $\beta = 0.99$
- Marginal cost of attention:
  - We set $\lambda = (1/10,000)$ times steady-state output.
  - For this value of λ, the equilibrium expected per period profit loss from suboptimal actions equals $(4/100,000)$ of steady-state output.
  - Following Coibion and Gorodnichenko (2015), we also regress the ex-post average forecast error on the ex-ante average forecast revision in SPF output forecast data and in simulated output forecast data. The regression coefficients are $0.76\,(0.30)$ and 1.07, respectively.

Figure 3: Impulse responses to a productivity shock

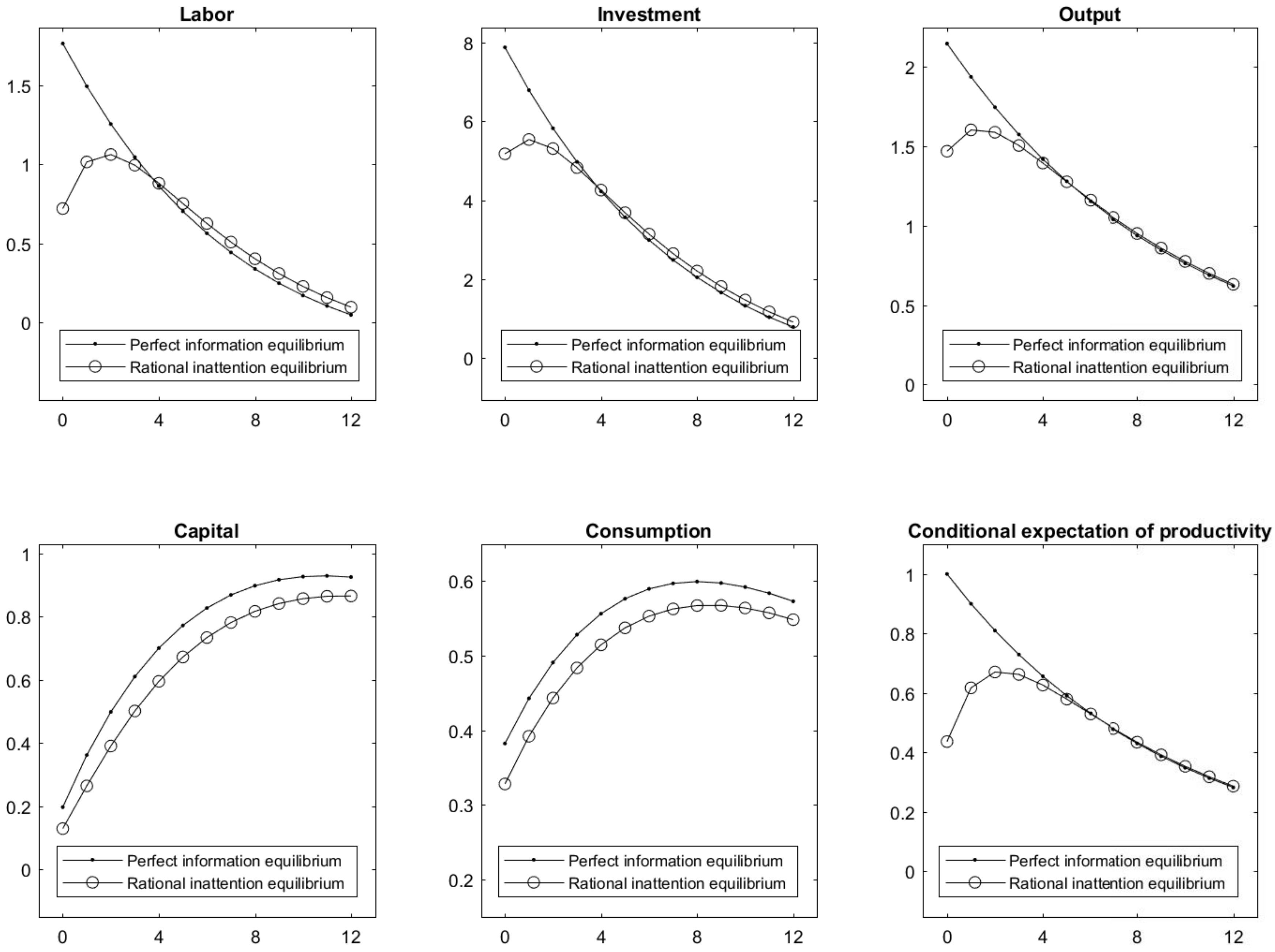
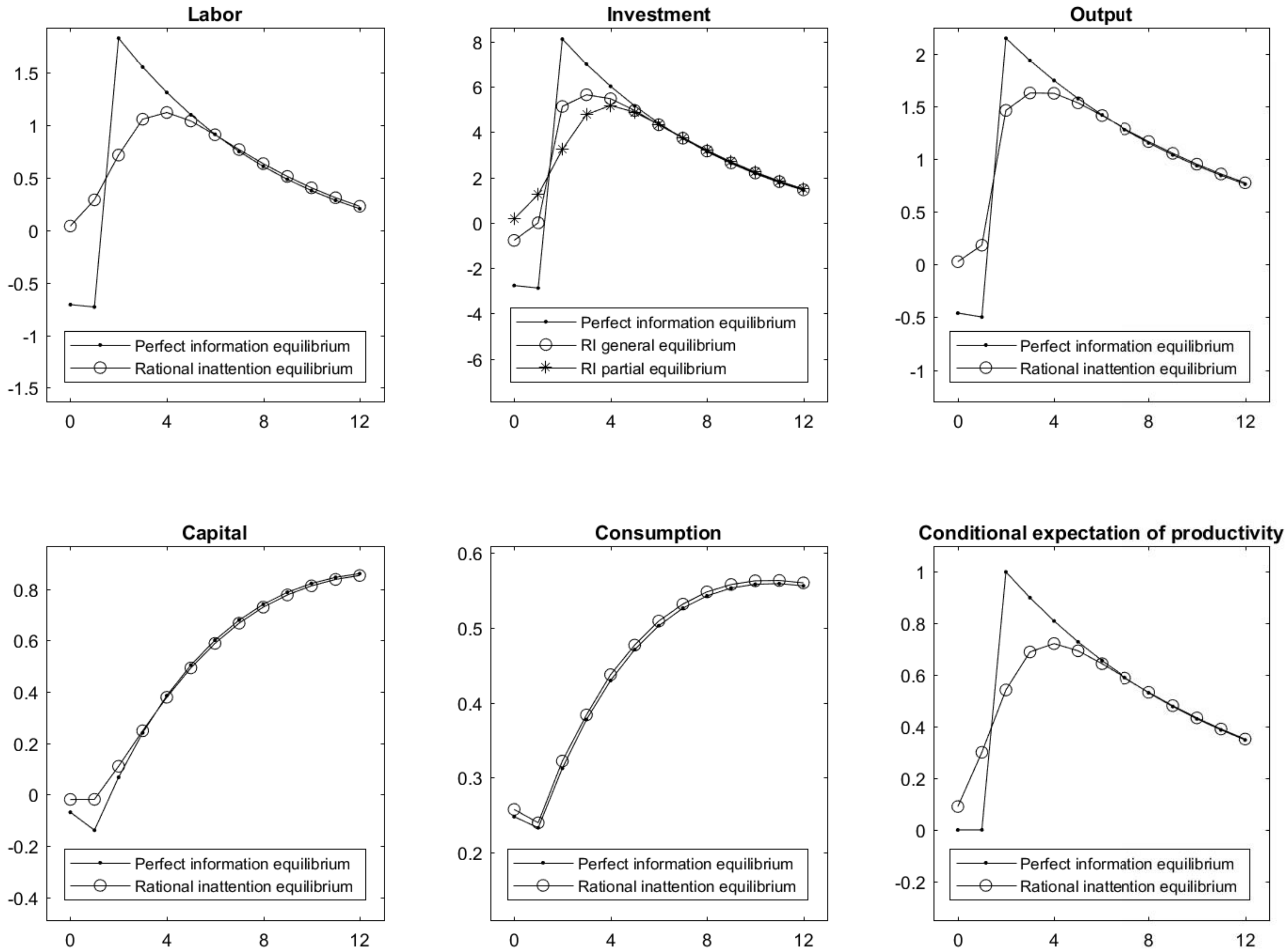


Table 1: Business cycle statistics

	Data	Model, $h = 0$	
		Perfect information	Rational inattention
Relative standard deviation			
σ_c/σ_y	0.55	0.56	0.59
σ_l/σ_y	0.92	0.66	0.57
σ_i/σ_y	2.88	3.05	2.93
σ_a/σ_y	0.52	0.46	0.51
Correlation			
$\rho_{c,y}$	0.78	0.78	0.81
$\rho_{l,y}$	0.85	0.85	0.83
$\rho_{i,y}$	0.90	0.93	0.92
$\rho_{a,y}$	0.40	1.00	0.99
First-order serial correlation			
Δc	0.27	0.23	0.28
Δl	0.41	-0.06	0.46
Δi	0.35	-0.06	0.14
Δy	0.30	-0.05	0.13
Δa	-0.06	-0.05	-0.05

Data: United States, 1955Q1-2007Q4, from Eusepi and Preston (2011).
Model: Unconditional moments computed from the equilibrium MA representation of each variable.

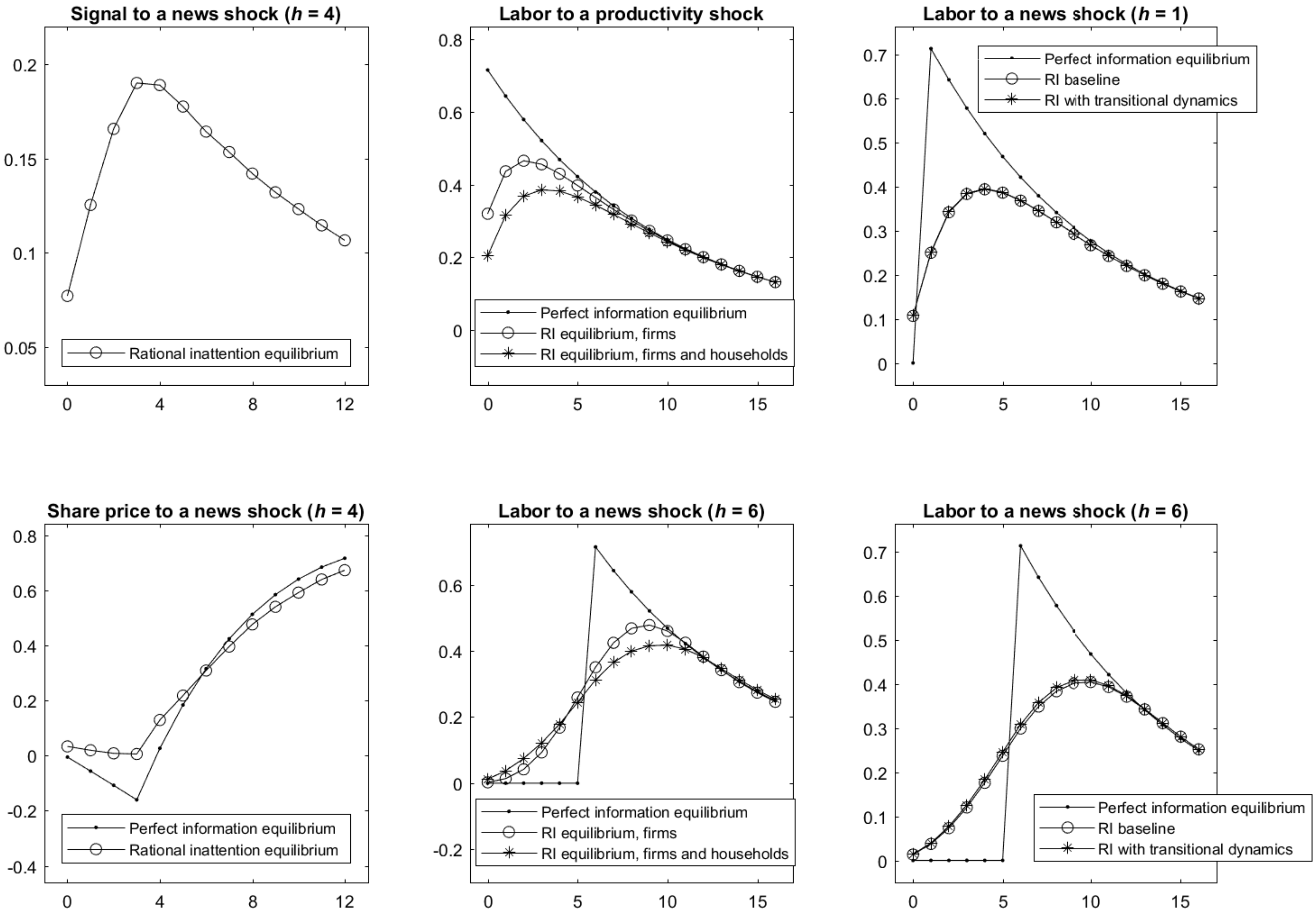
Figure 4: Impulse responses to a news shock (h = 2)



Adding RI by households

- To begin: no capital, no trade in shares. When households become subject to RI:
    - they supply less labor on impact of a positive productivity shock.
    - they supply *more* labor on impact of a positive news shock.
- Introduce capital. When households become subject to RI, they consume less on impact of a positive productivity (news) shock – they save more, and they supply more labor.

Figure 6: Additional impulse responses



Literature: Recent advances in dynamic RI

- Steiner, Stewart, and Matějka (2017).
- Jurado (2020).
- Afrouzi and Yang (2020).
- Miao, Wu, and Young (2020).

Conclusions

- In a dynamic model, RI causes a combination of delay in actions and forward-looking actions.
- In a neoclassical economy, RI induces persistence and helps produce comovement after news shocks.
- The recent advances suggest that RI in macroeconomics is an exciting research area.